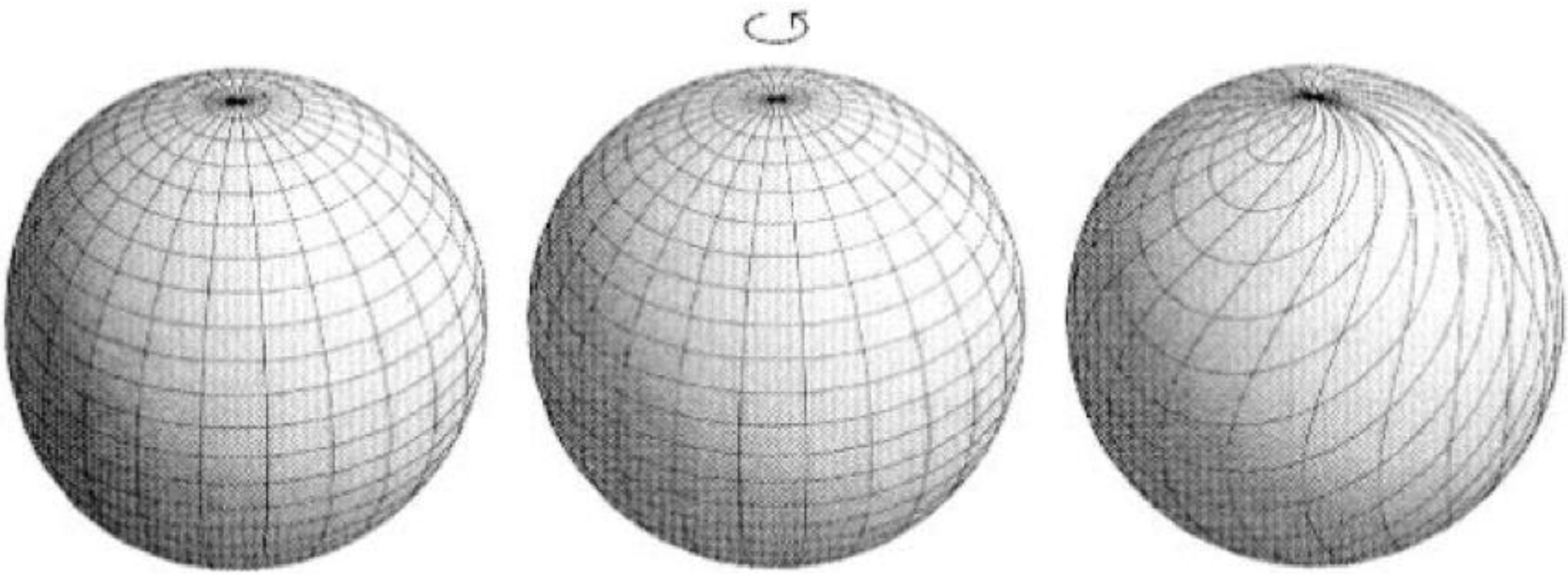
Symmetries in Physics



Andrea Banfi

University of Sussex

The Abdus Salam International Centre for Theoretical Physics
Physics Without Frontiers

IAEA International Atomic Energy Agency

unesco United Nations Educational, Scientific and Cultural Organization

Lecture 2: continuous symmetries

Part I: continuous symmetries

- Symmetries and conservation laws
- Conservation of energy, momentum and angular momentum
- Conservation of electric charge

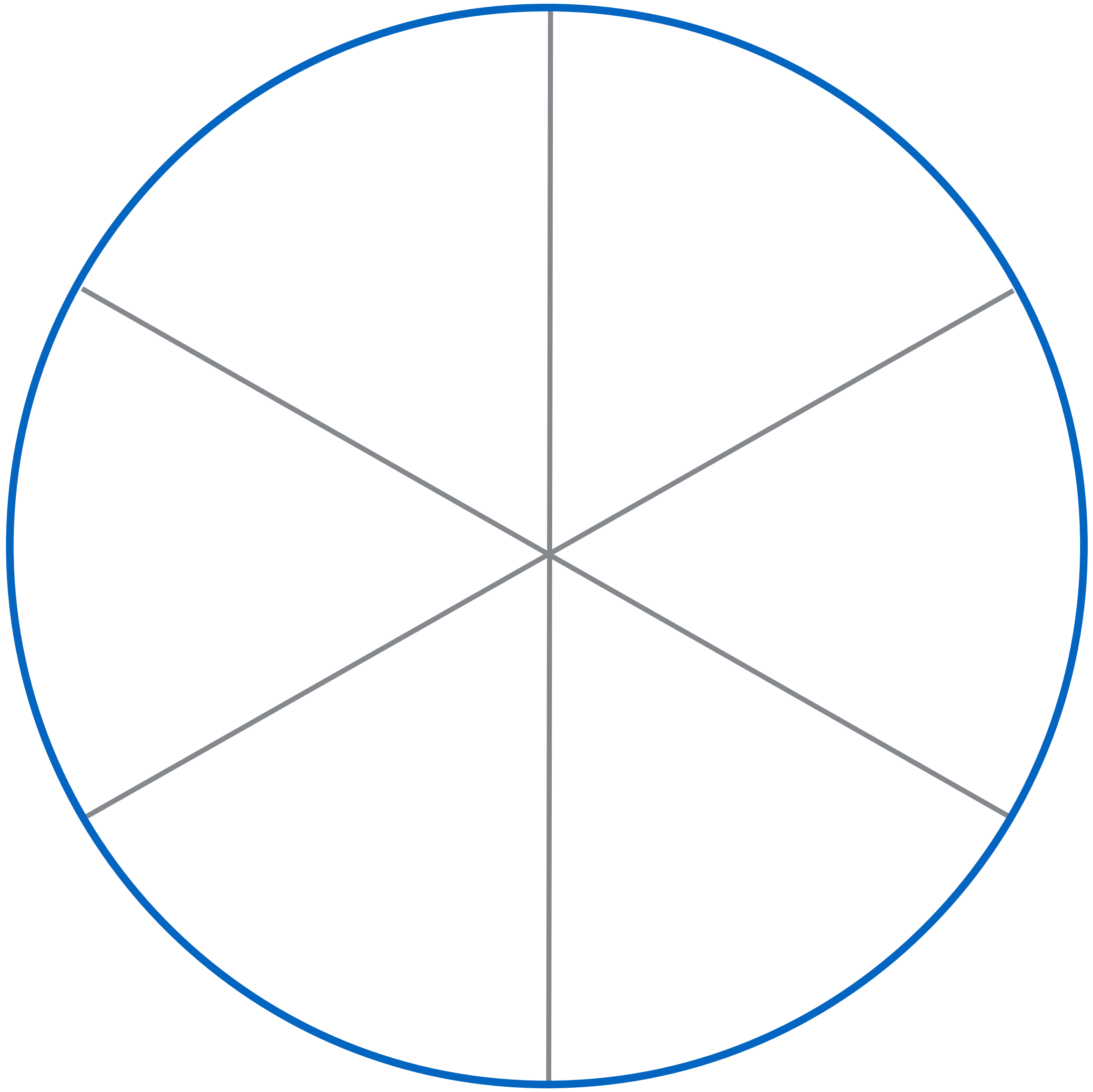
Part II: gauge symmetries

- Introduction to the gauge principle
- Electrodynamics as a gauge theory

Continuous symmetries and conservation laws

Continuous symmetries

There exist infinitely many transformations that leave an object invariant



Example: the circle is invariant under rotations by any angle

Noether's theorem

For any continuous symmetry of the Action there exists a corresponding conservation law

– Emmy Noether

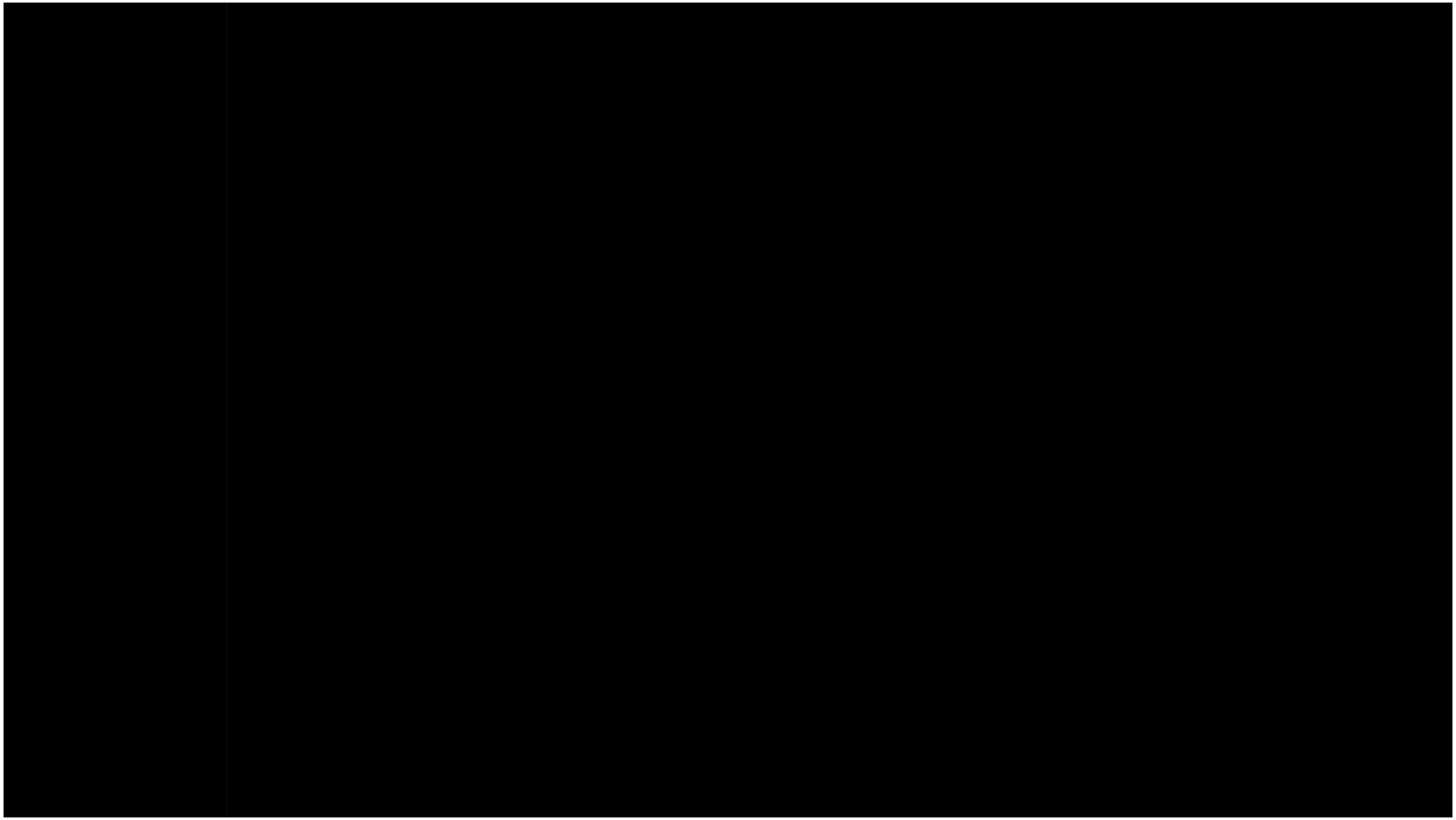
Action: the mathematical object encoding the physics of a given system

Time translations

- Invariance under time translations $\Rightarrow$ conservation of …….

Conservation of energy

- Invariance under time translations ⇒ conservation of energy



http://youtu.be/d4K6ATZSJwk?t=51s

Space translations

- Invariance under space translations $\Rightarrow$ conservation of ……..

Conservation of impulse

- Invariance under space translations ⇒ conservation of impulse (momentum)

http://youtu.be/MdwVrrnRaCE?t=1m23s

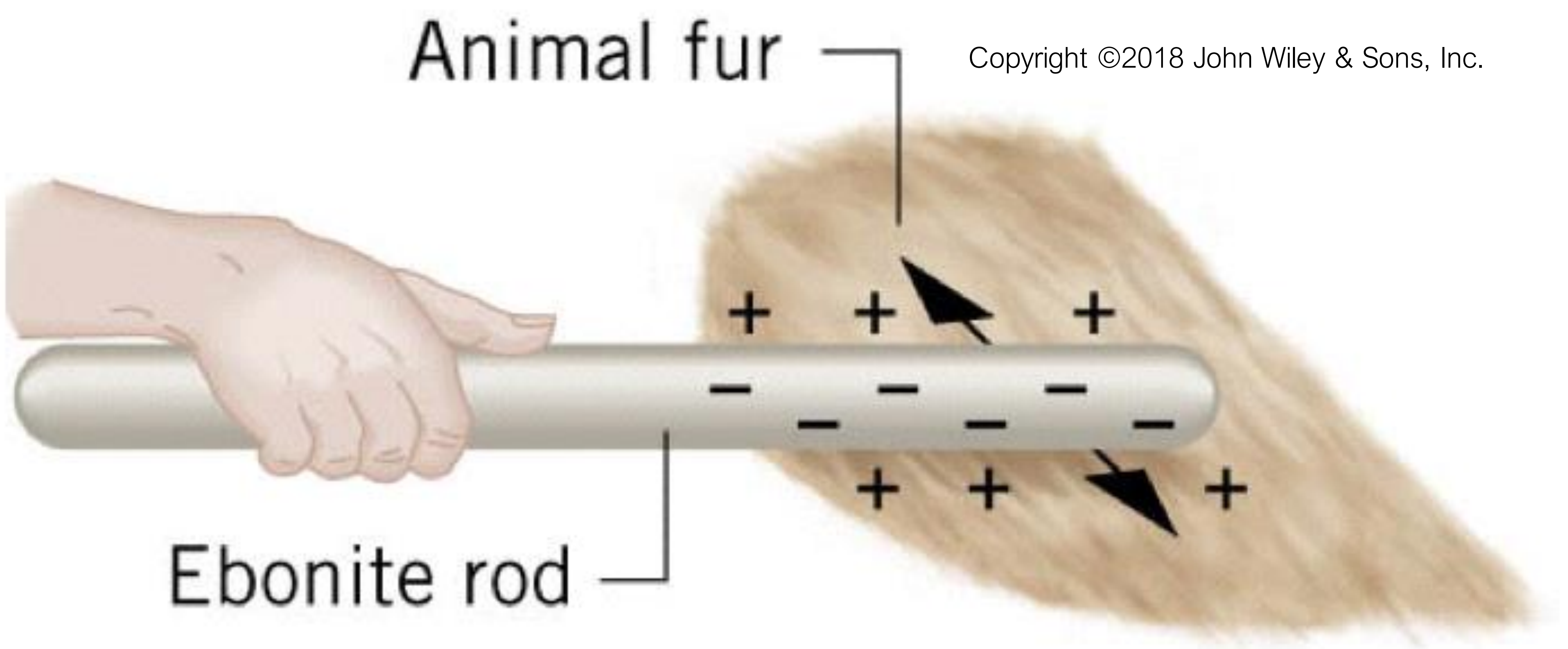
Invariance under rotations

- Invariance under rotations $\Rightarrow$ conservation of ………

Conservation of angular momentum

- Invariance under rotations $\Rightarrow$ conservation of angular momentum

Conservation of electric charge

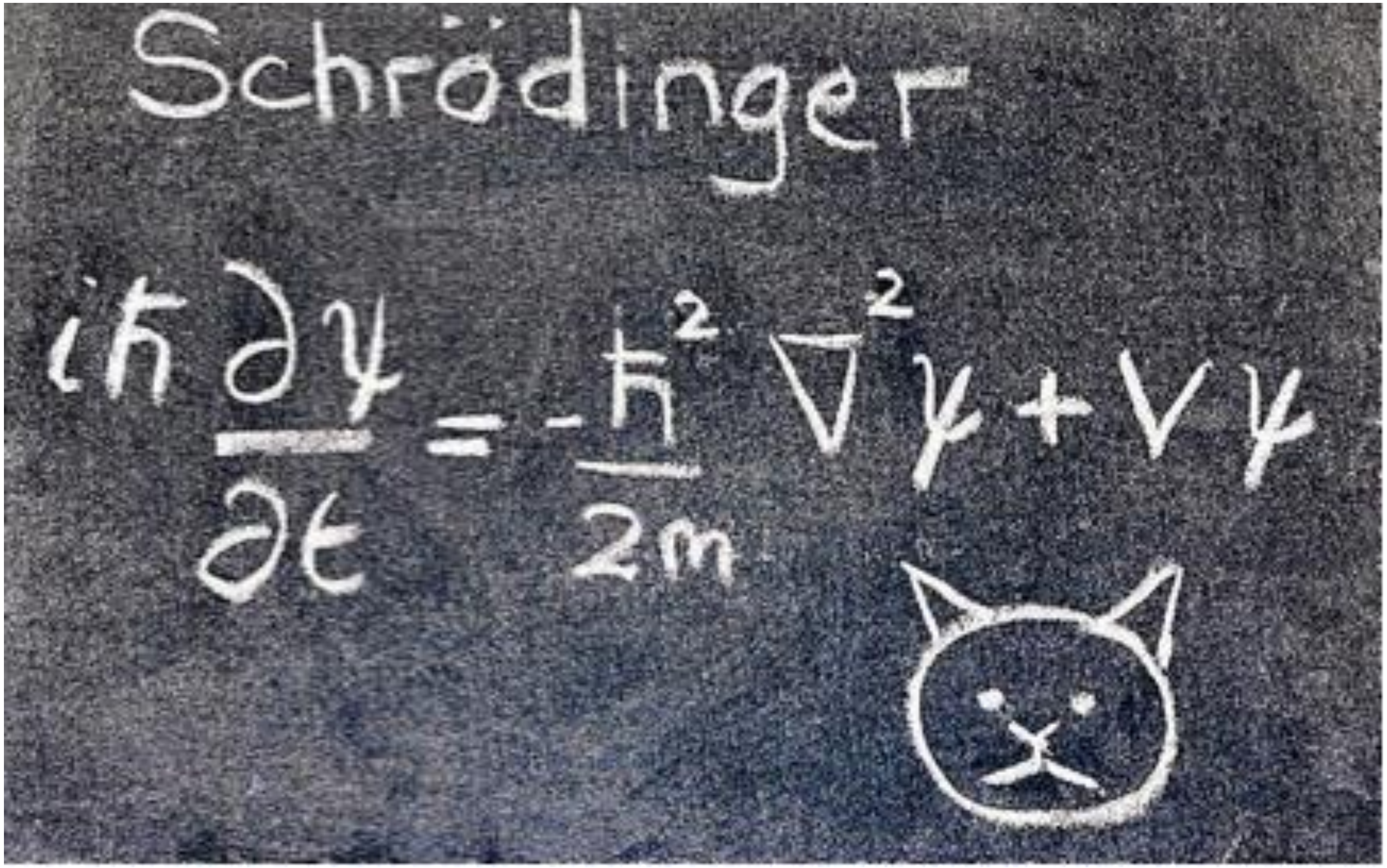


- During any process, the net electric charge of an isolated system remains constant (is conserved)

Is there a symmetry corresponding to conservation of electric charge?

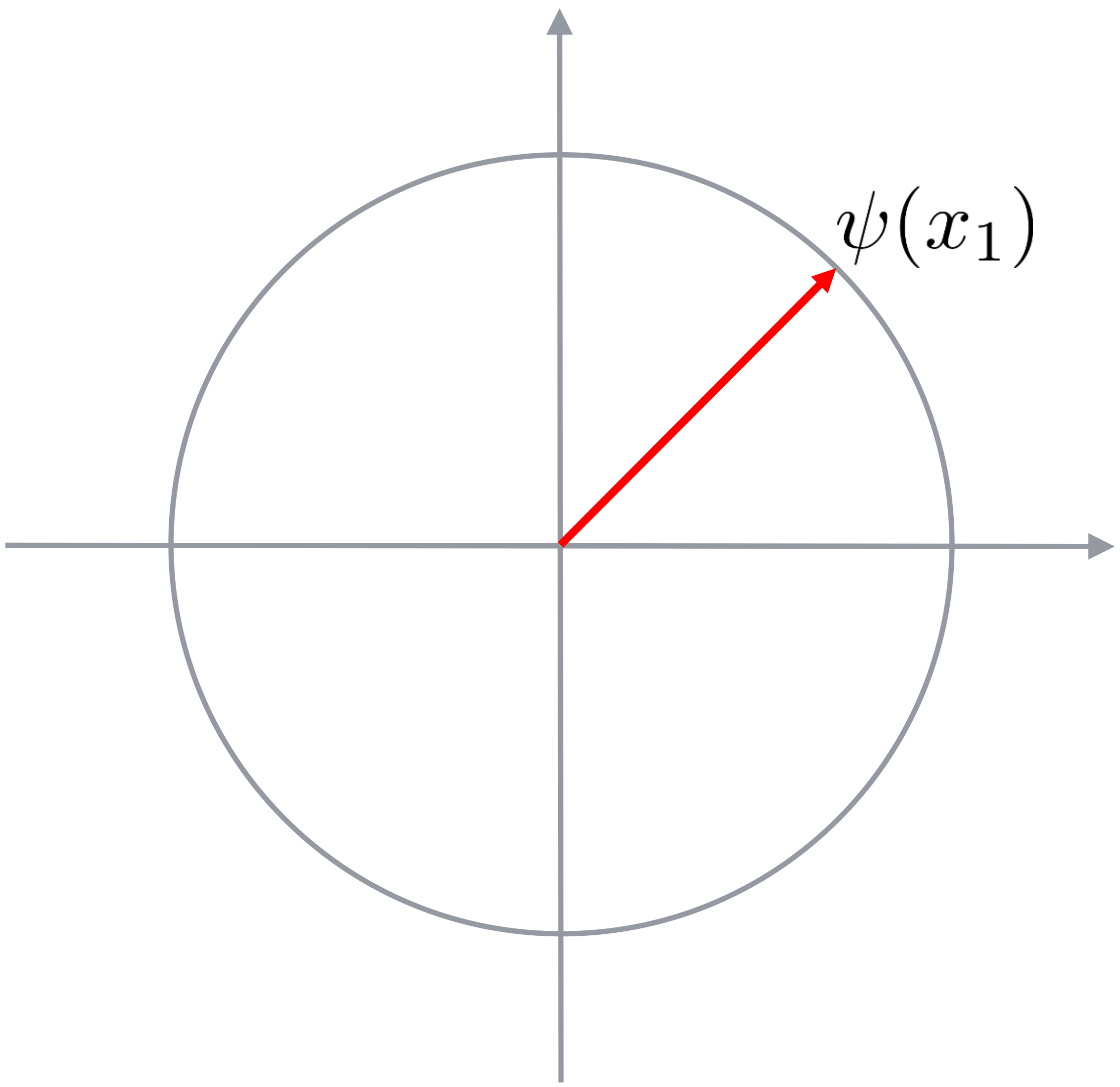
Wave functions

- In quantum mechanics, every physical system is described by a wave function $\psi(x)$, a solution of Schrödinger's equation

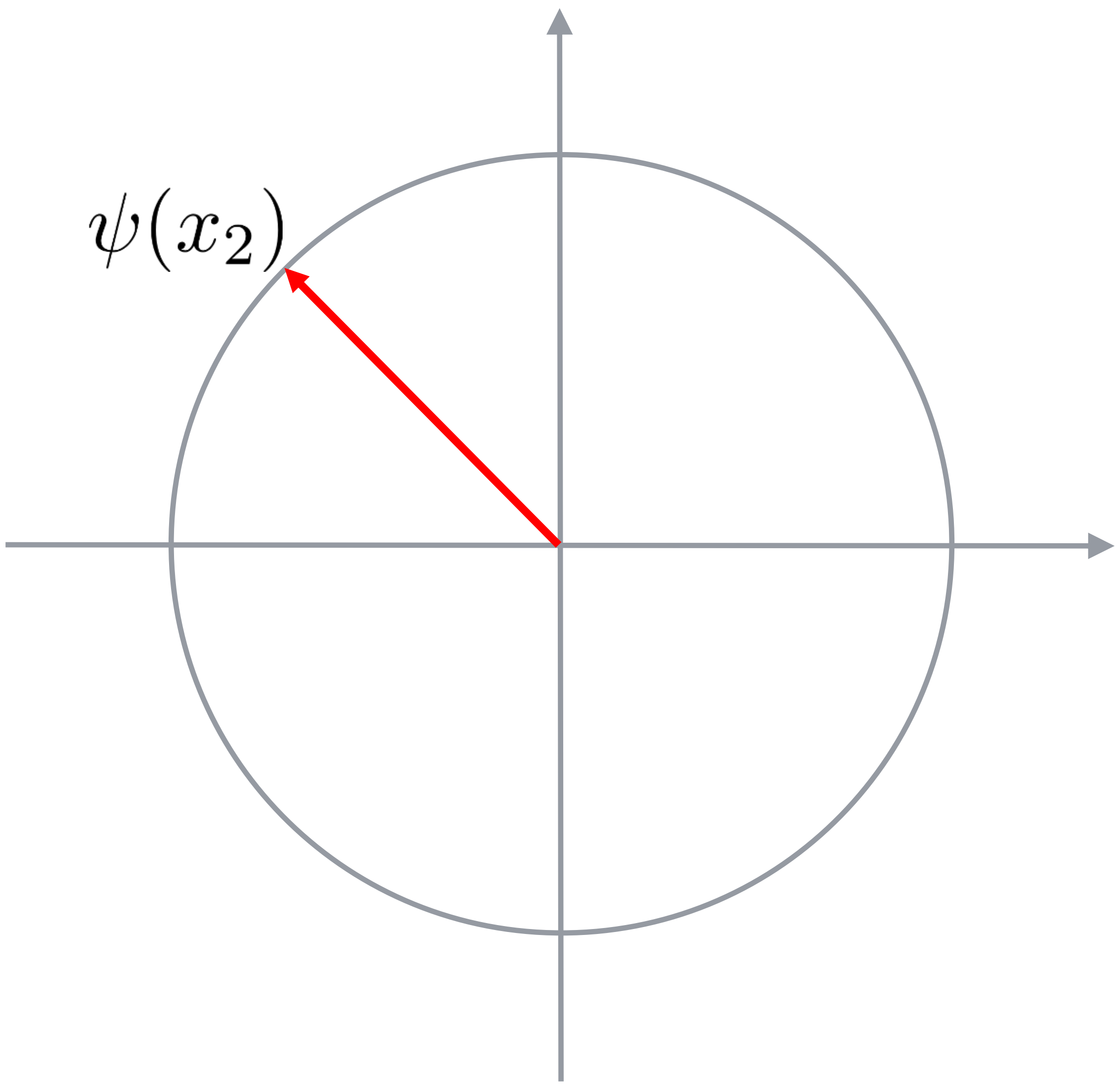


Wave functions

- Any wave function in quantum mechanics is a complex number, i.e. it can be represented as a point on a circle



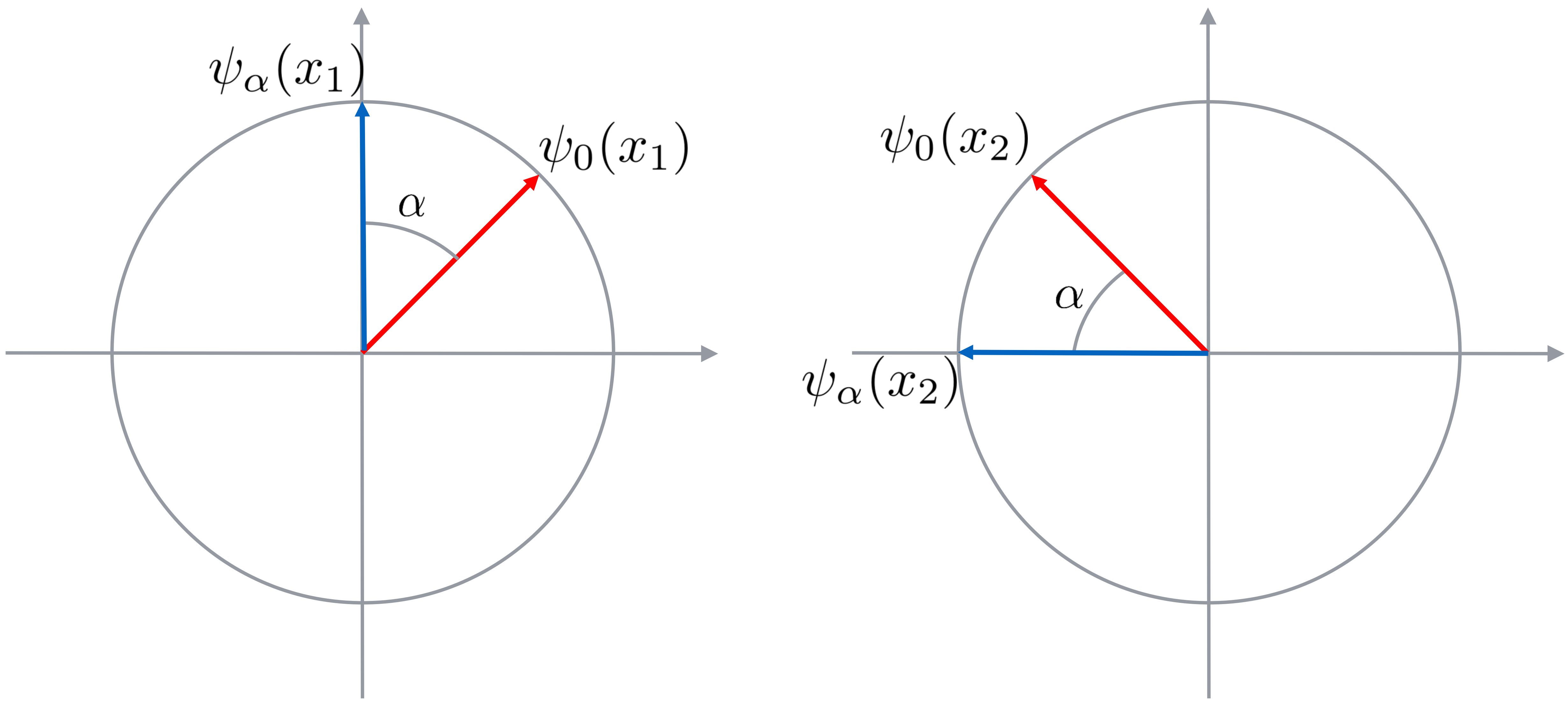
$x_1 = \text{Brighton}$



$x_2 = \text{Kanglung}$

Phase symmetry

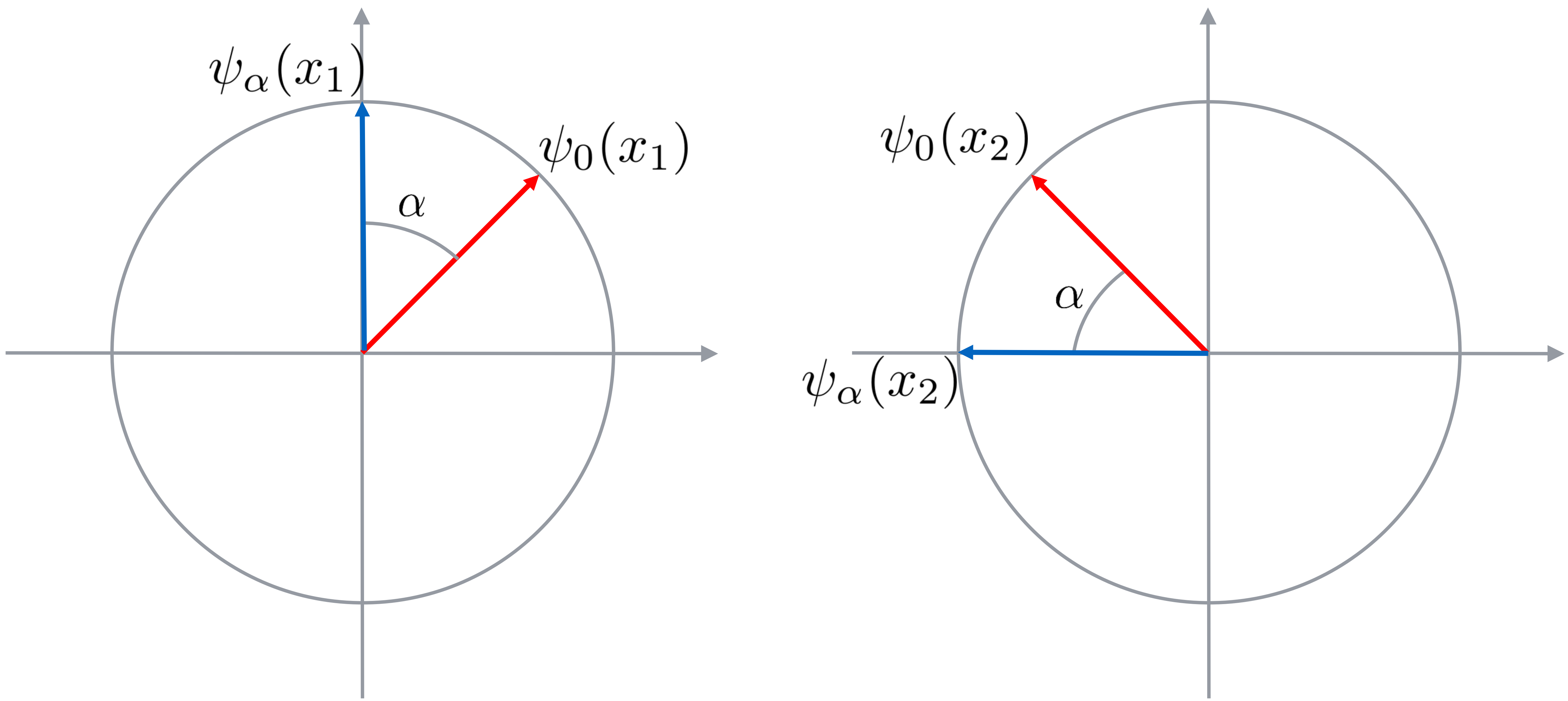
- Any wave function can be “rotated” at any point by the same angle α



$x_1 = \text{Brighton}$ $\qquad$ $x_2 = \text{Kanglung}$

Phase symmetry

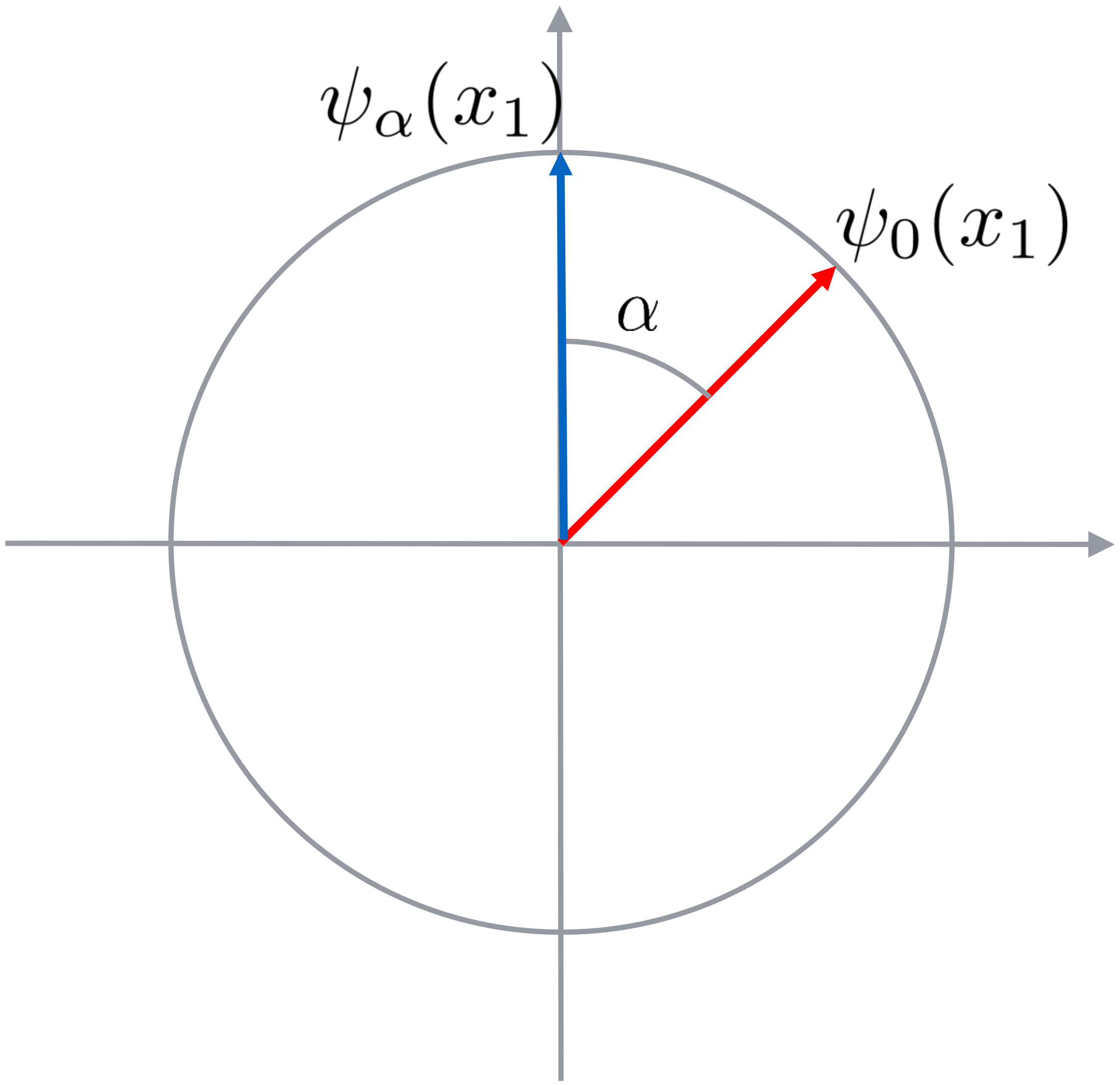
- The physics does not depend on the choice of the angle $\alpha \Rightarrow$ “phase” symmetry

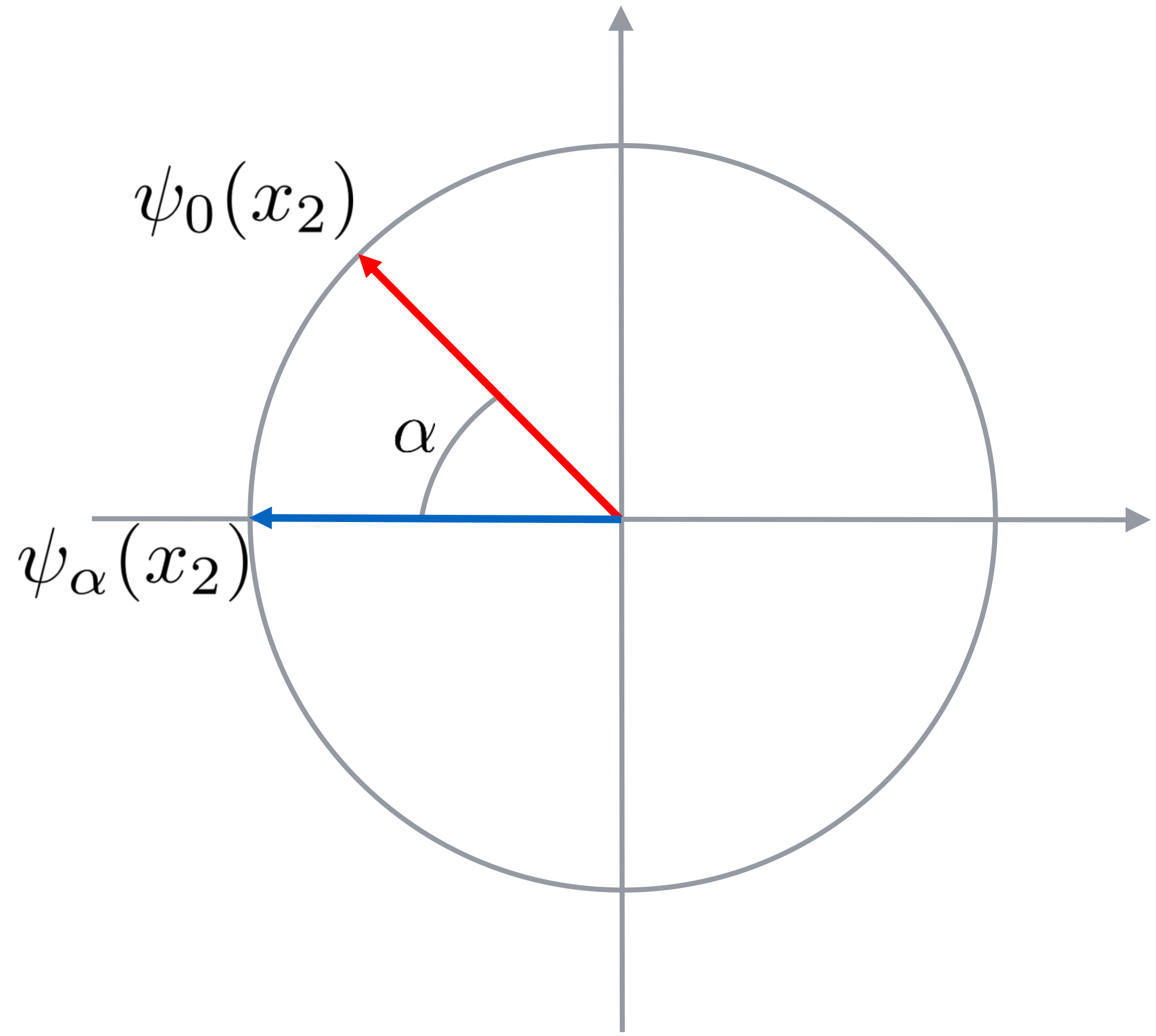


$x_1 = \text{Brighton}$ $x_2 = \text{Kanglung}$

Phase symmetry

- Phase symmetry is an internal symmetry, i.e. it is neither a symmetry of space nor of time





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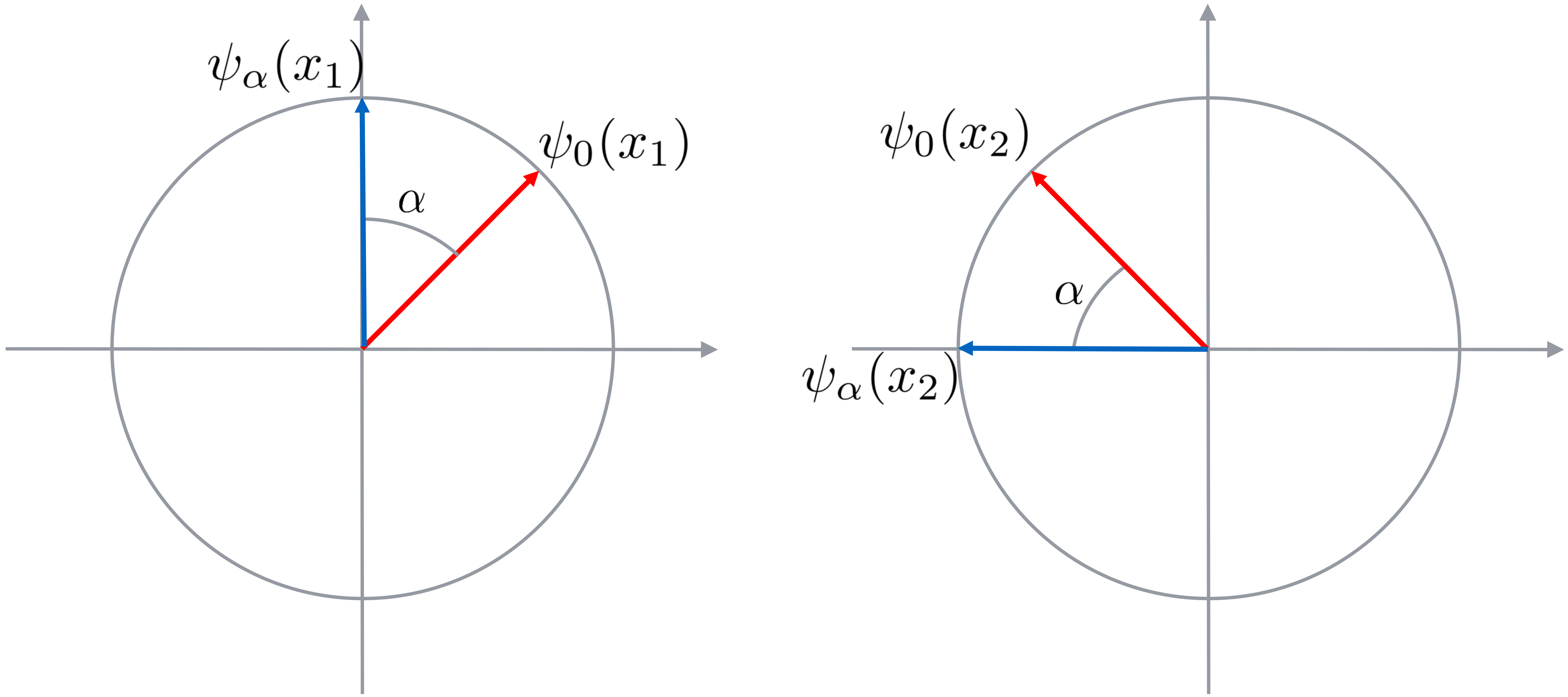

- The conserved quantity associated by Noether's theorem to phase symmetry is the electric charge

Phase symmetry

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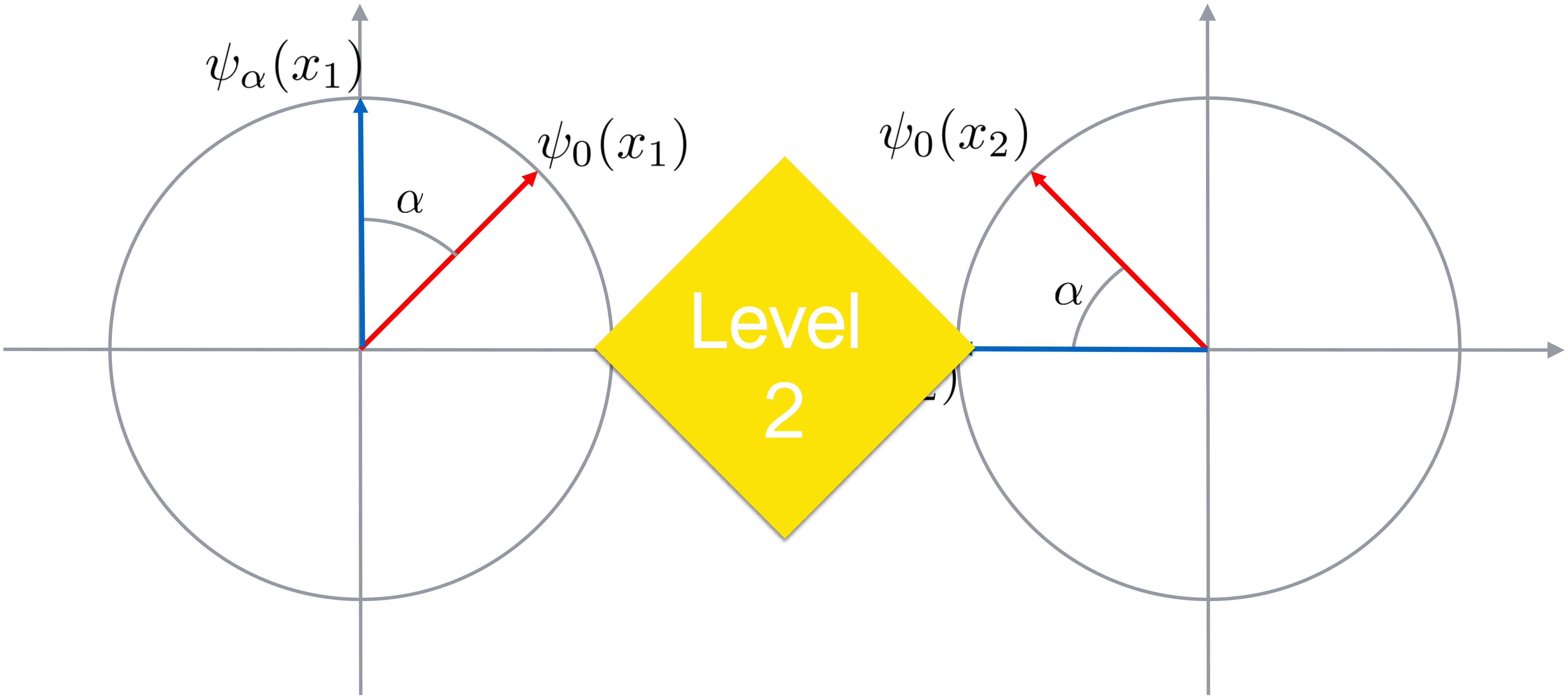
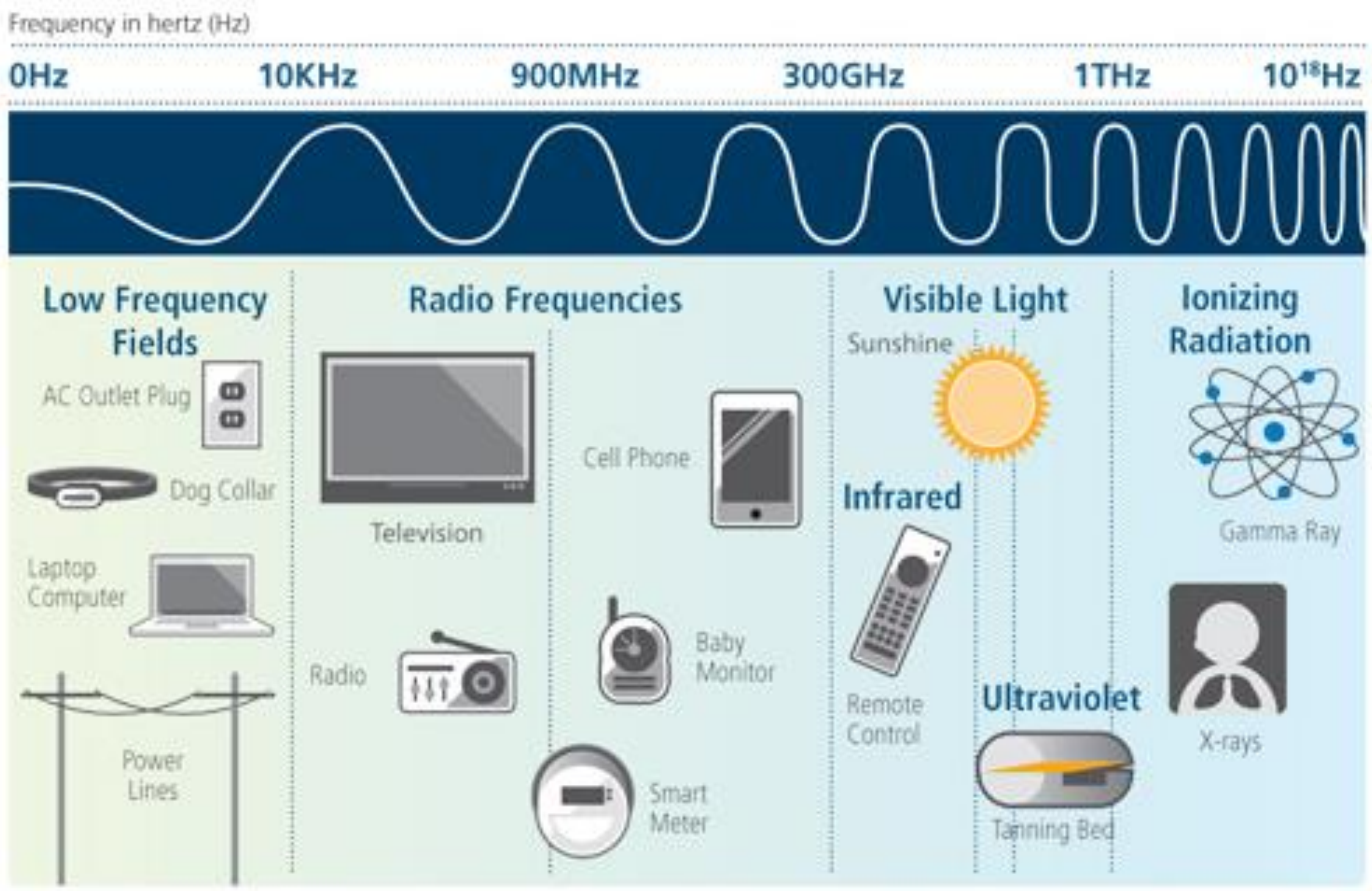


- The conserved quantity associated by Noether’s theorem to phase symmetry is the electric charge

Why do electrically charged particles exert forces between each other?

The electromagnetic field

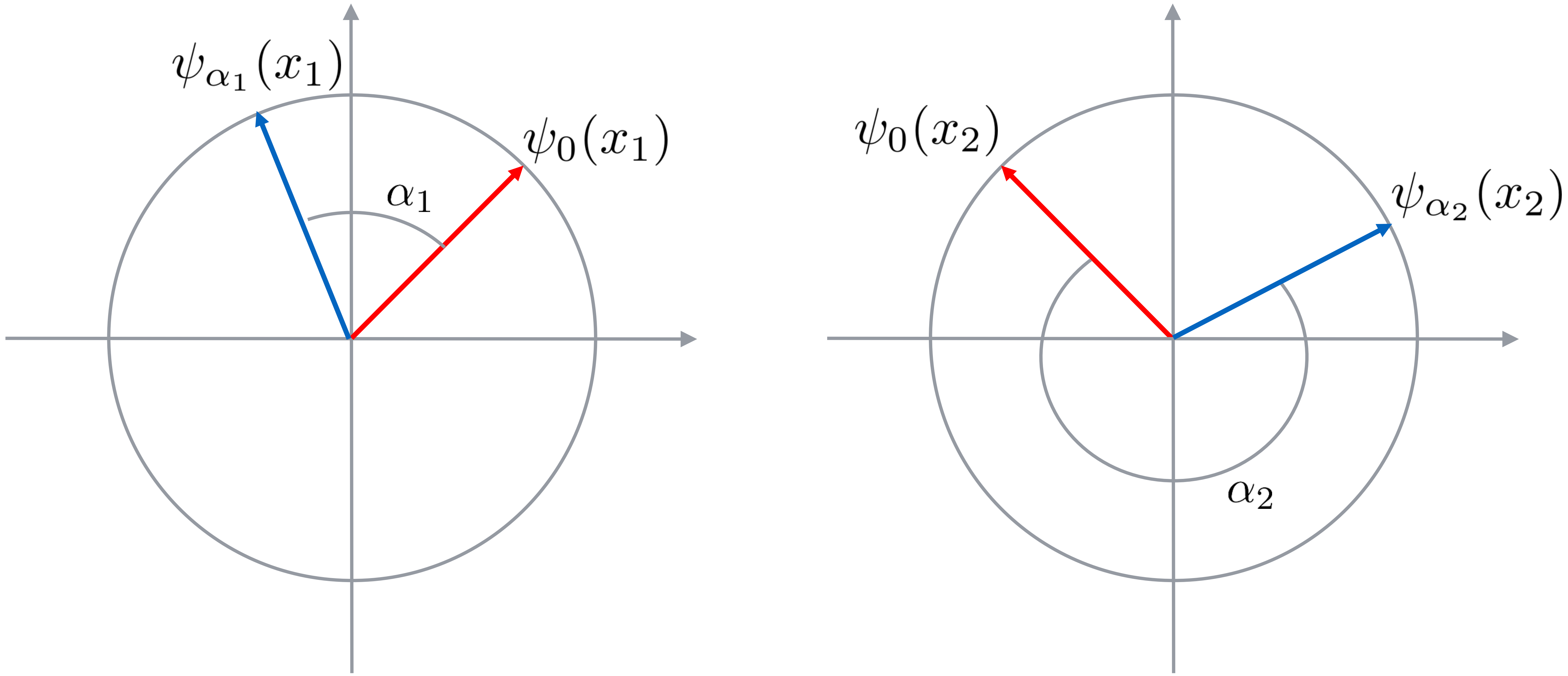
- Electric charges affect the properties of space and time, producing everywhere electromagnetic fields



- Electric charges experience forces in the presence of electromagnetic fields

Gauge symmetry

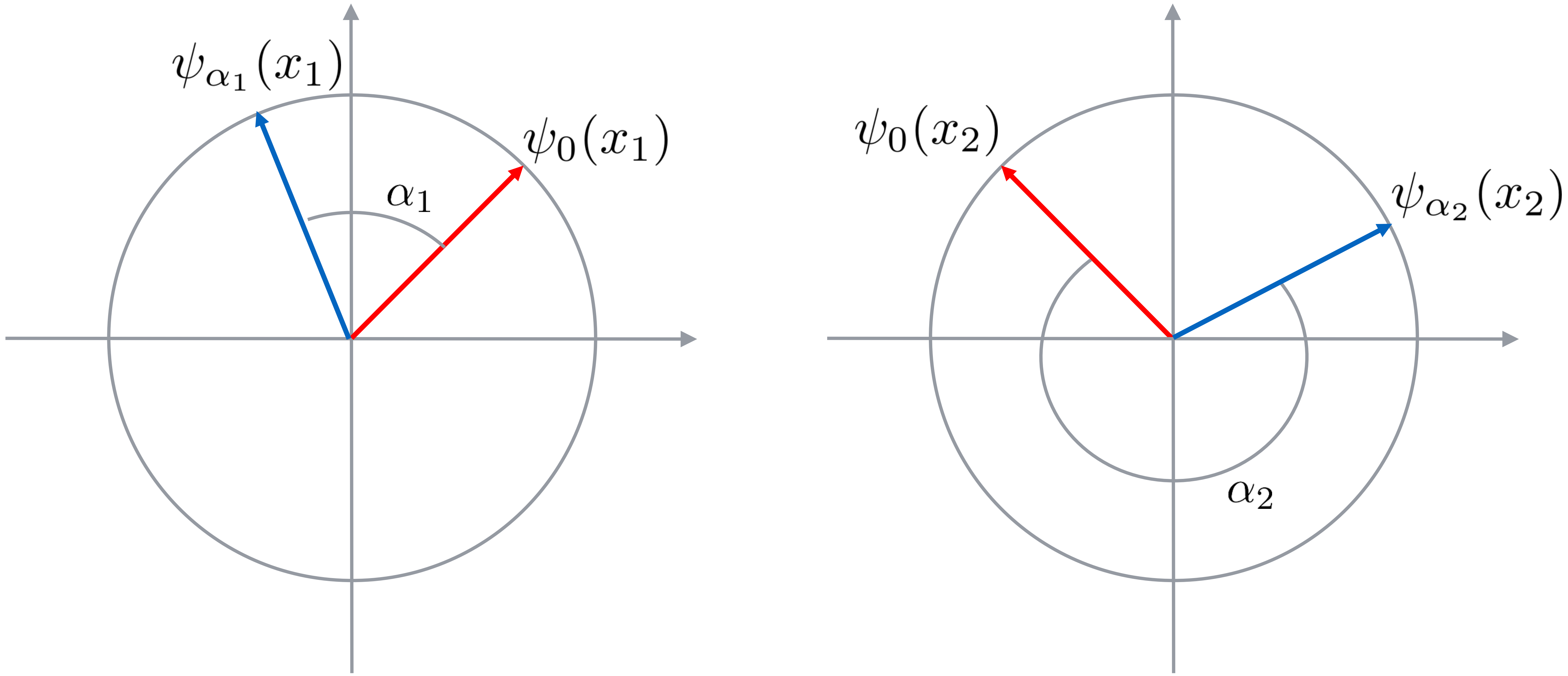
- Suppose that the wave function of an electrically charged particle "rotates" with a different angle for each point of space-time



- This transformation is "local", i.e. it is different in every point. Such transformations are commonly known as "gauge" transformations

Gauge symmetry

- Imposing that that physics is invariant under such "gauge" transformations requires the introduction of a new field, called the "gauge" field

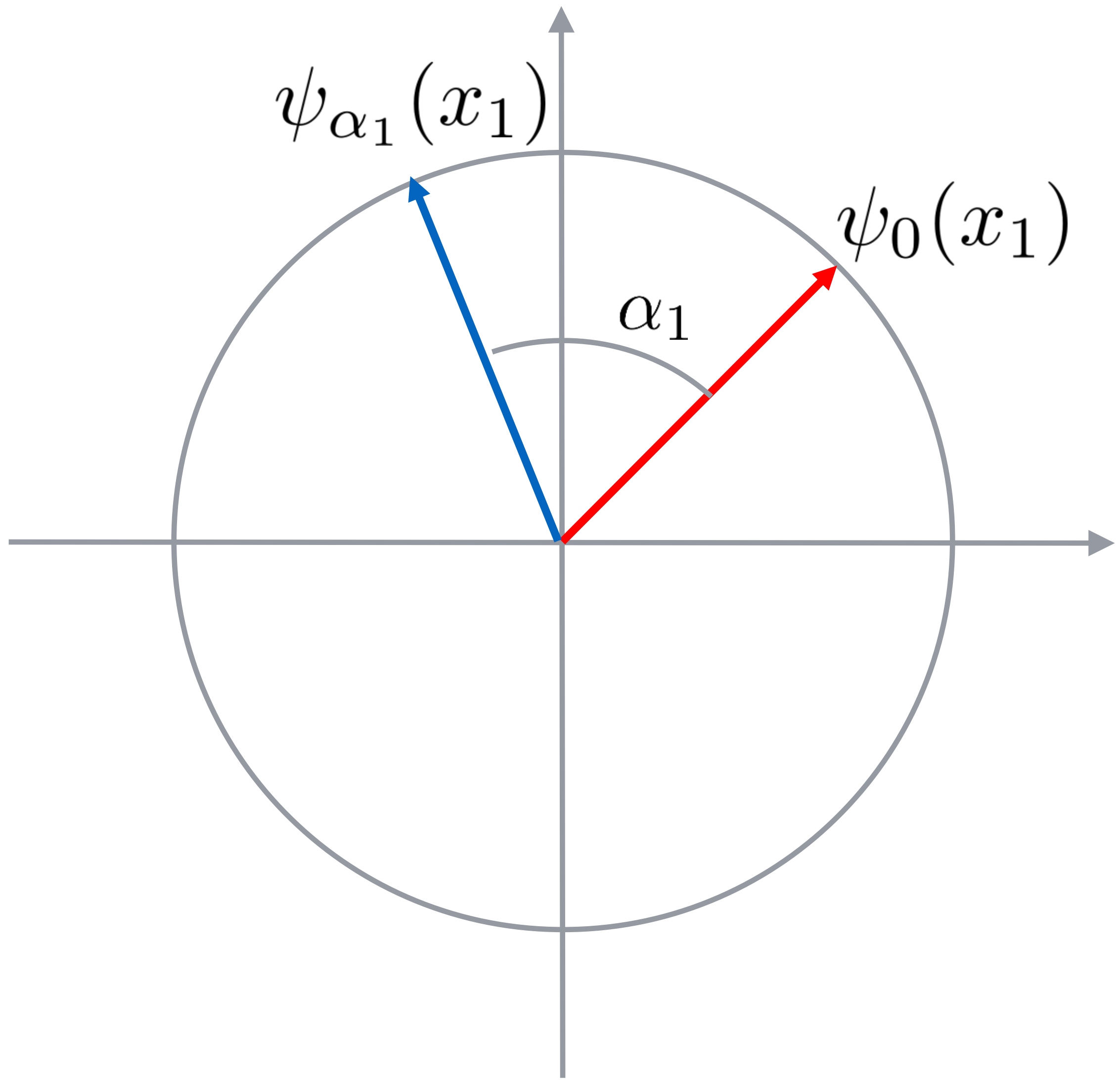


- A gauge field, as any other field, is a property of space that varies with time

Gauge field

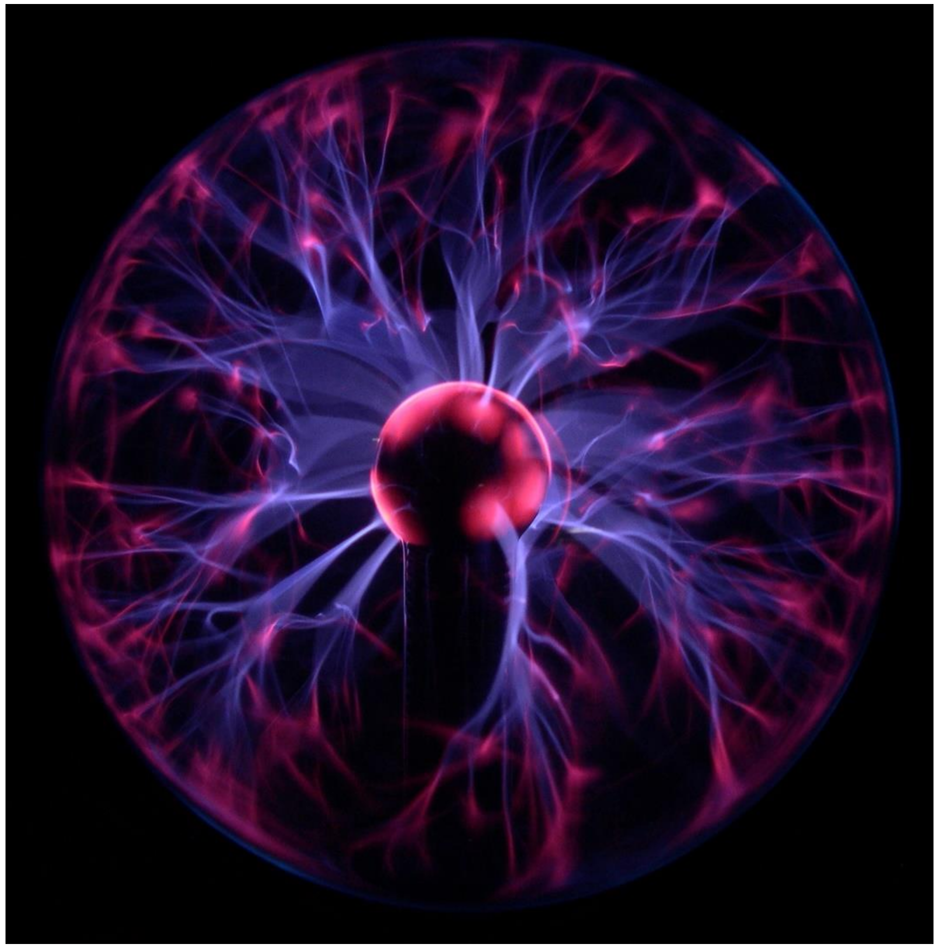
- The gauge field corresponding to local (a.k.a "gauged") phase symmetry is nothing but the well-known electromagnetic field

Local phase symmetry



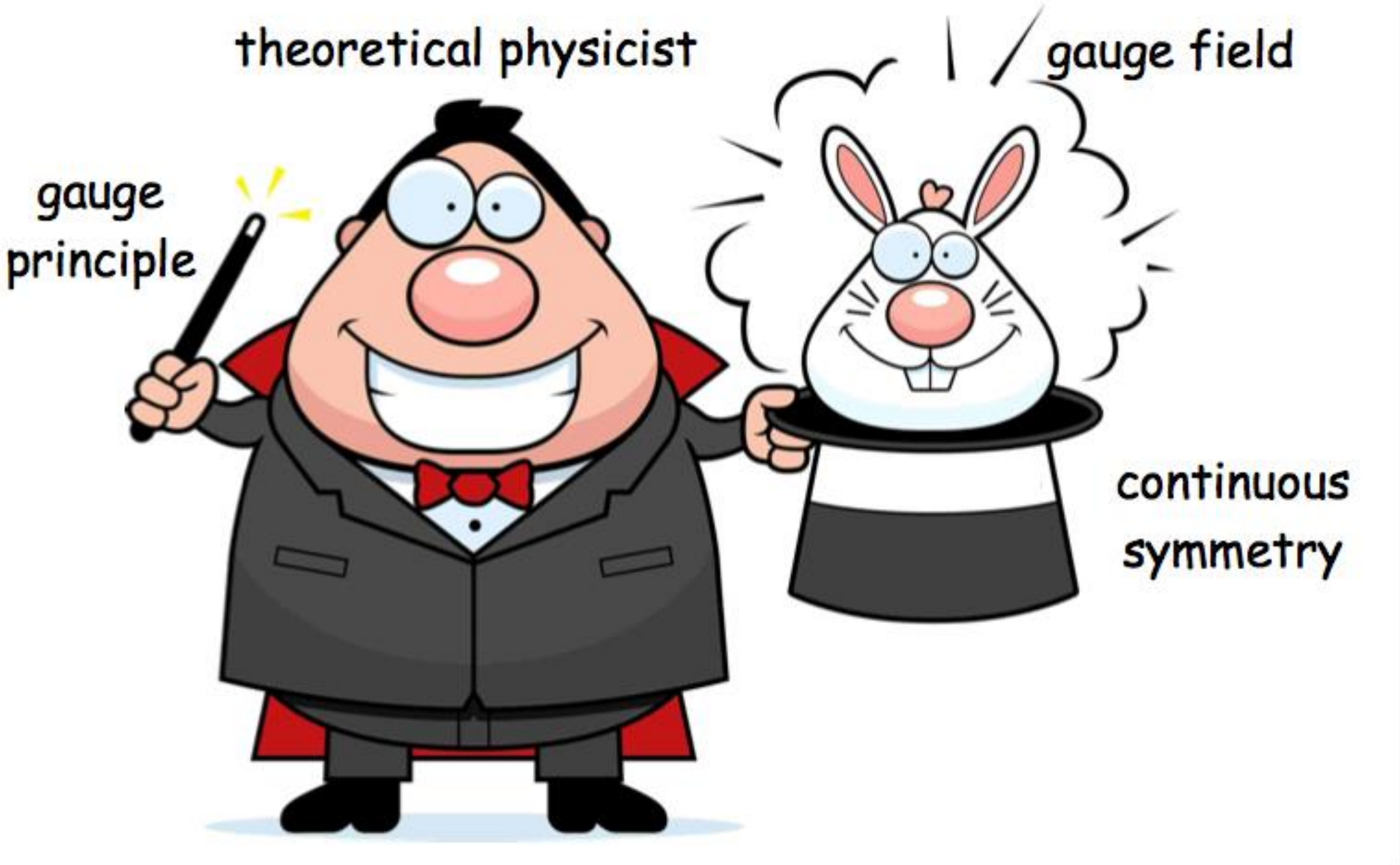
$\Rightarrow$

Electromagnetism



Gauge principle

- In modern physics, interactions are introduced via the gauge principle



Gauge principle

- Start with a theory with an internal continuous symmetry ⇒ conservation of some charge
- Impose that the physics is invariant when the corresponding symmetry transformation is different in every point ⇒ gauge symmetry
- This requires the introduction of a new field, the gauge field
- The invariance under the new gauge symmetry (a.k.a. gauge invariance) gives the forces ("interactions") between the charged particles and the gauge field

Level
3

Lecture 2: learning outcomes

In this lecture we have learnt

- For any continuous symmetry there exists a corresponding conservation law
- Conservation of charge corresponds to invariance of Schrödinger's equation with respect to a change of phase in the wave function
- "Gauging" phase invariance leads to a theory that corresponds to electromagnetism