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FQH-Based Topological Quantum Computer: Materials, Devices & Algorithms

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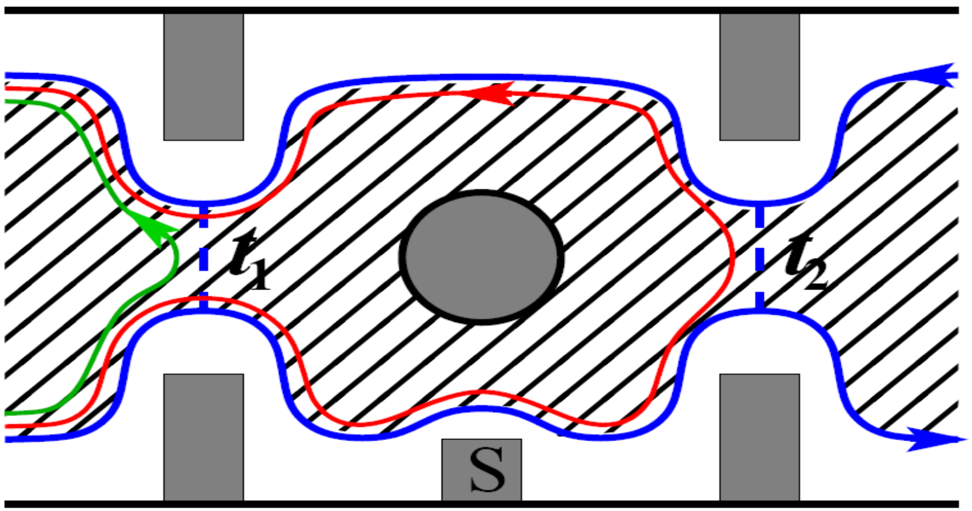
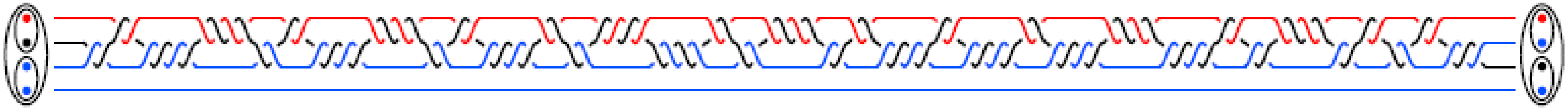
Motivation

- Goal: “Engineering the topological quantum processor”.
- Materials
  - 2DEG in GaAs/AlGaAs
  - Graphene



- Devices
  - Quantum point contact
  - Interferometer (double quantum point contact)
- Engineering quantum gates – algorithms only



Nobel Laureates Said …

- Technology evokes new physics

  "It is frequently said that having a more or less specific practical goal in mind will degrade the quality of research. I do not believe that this is necessarily the case and to make my point in this lecture I have chosen my examples of the new physics of semiconductors from research projects which were very definitely motivated by practical considerations."

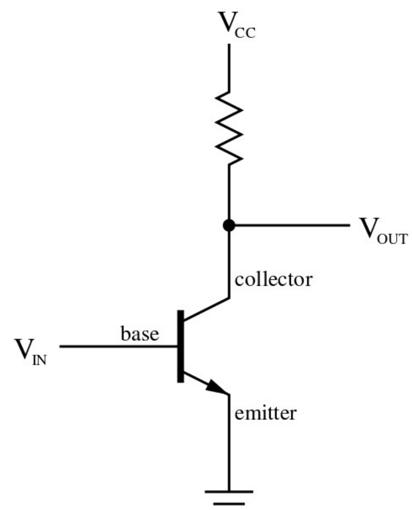
  -- William Shockley, Nobel Lecture, Dec. 11, 1956

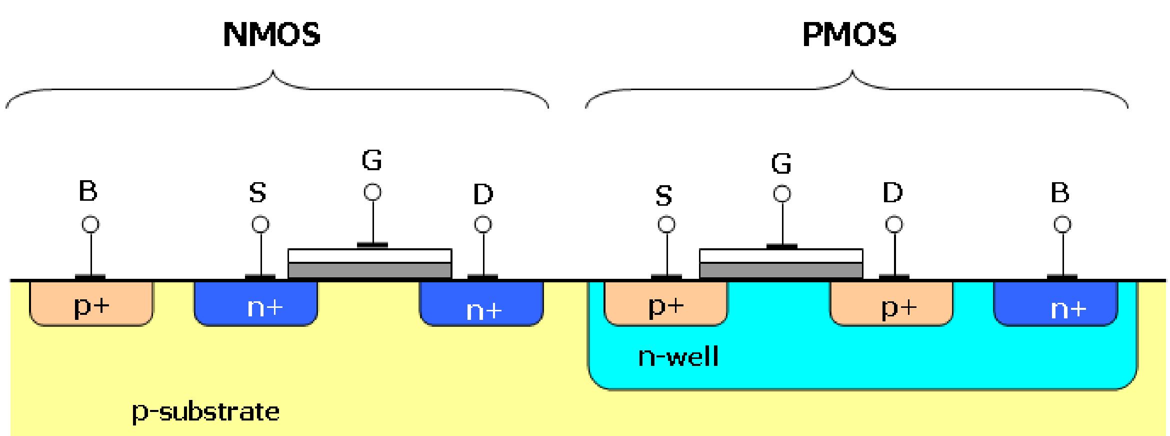
- Futuristic, but not crazy

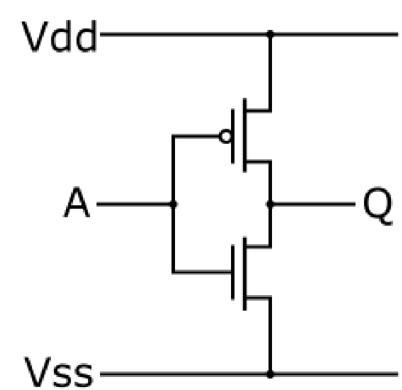
  [Frank] Wilczek also notes a number of new proposals to look for more exotic anyon states of FQH systems that could form the basis for quantum computers. Such ideas are "futuristic," he says, "but not as crazy as they used to be."

  -- Phys. Rev. Focus 16, 14 (2005)

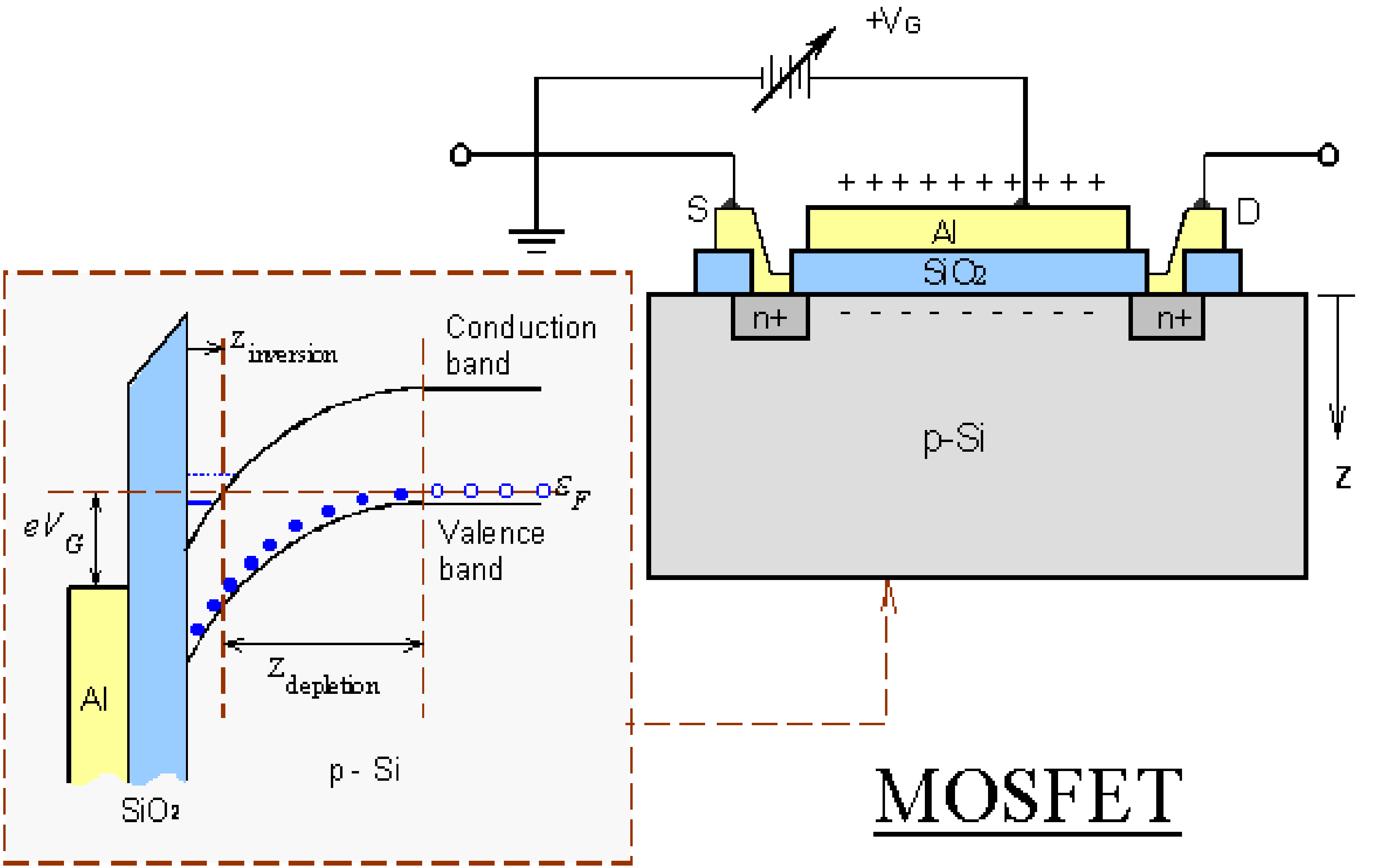
From Ge Transistor to Si CMOS



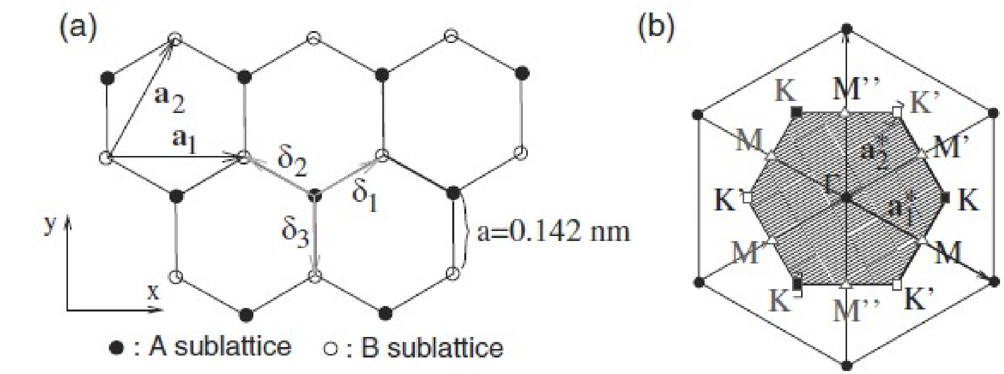


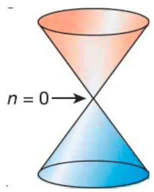


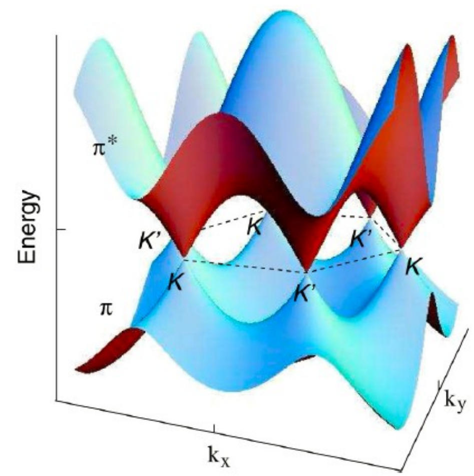
Metal-Oxide-Semiconductor Field Effect Transistor

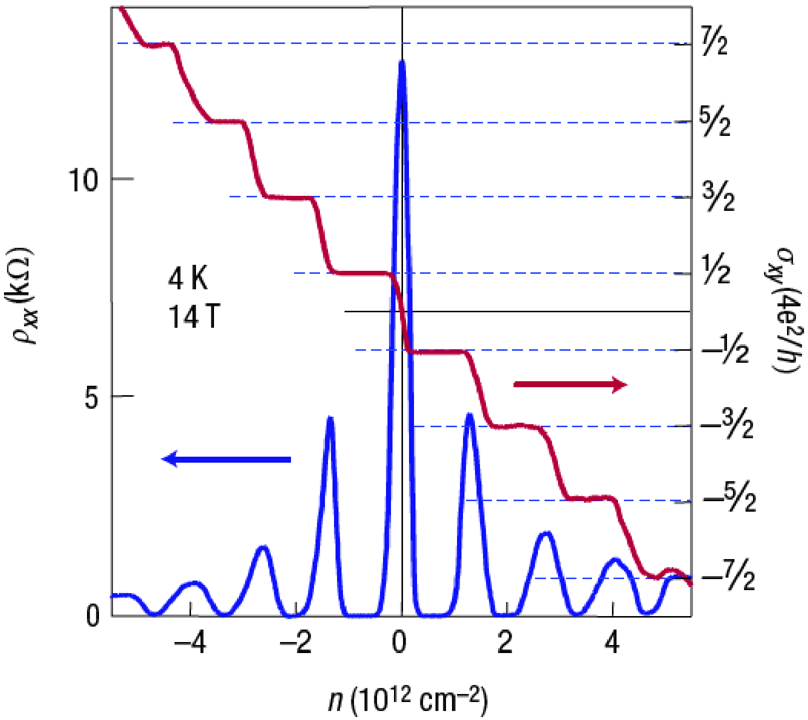


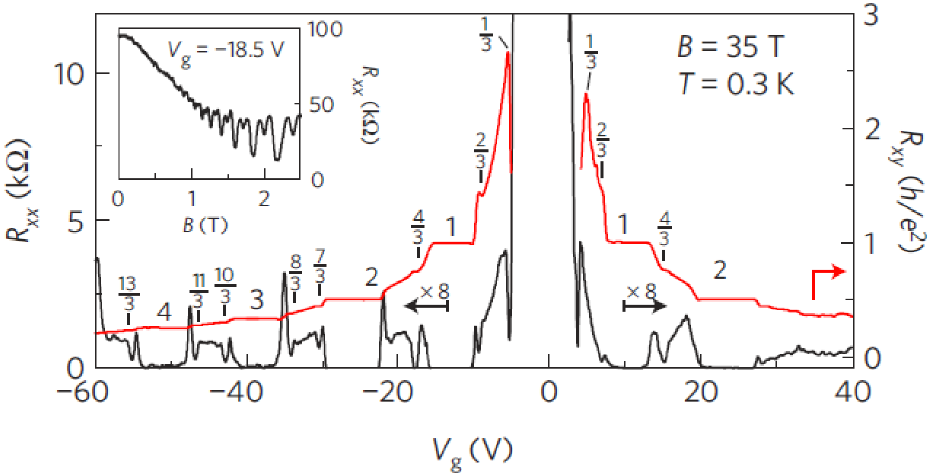
QHE in Graphene







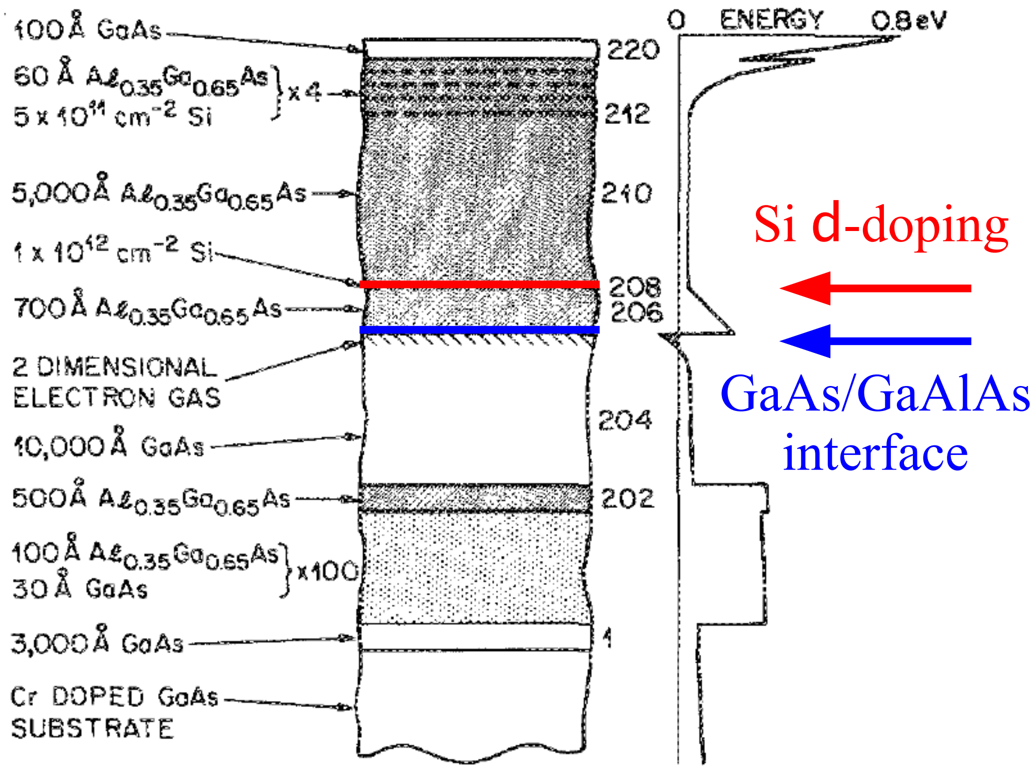


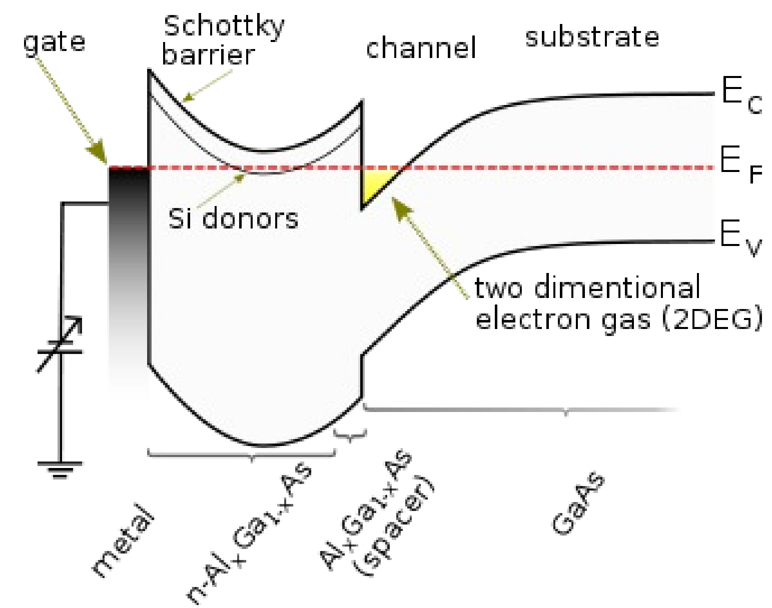


Novoselov *et al*., Nature (2005);
Zhang *et al*., Nature (2005)

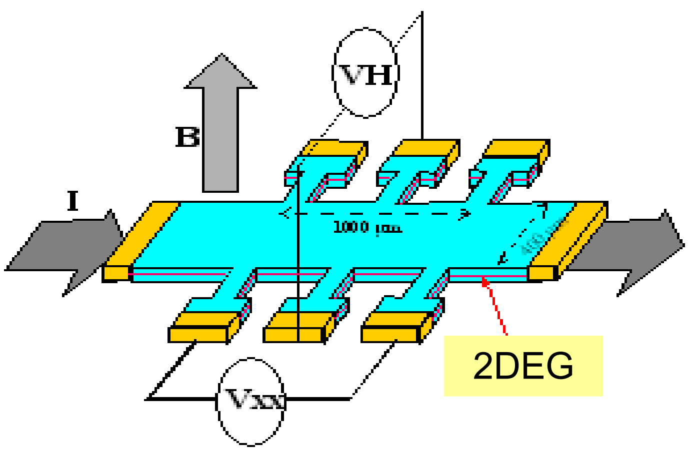
Dean et al., Nature Physics (2011);
Du et al. Nature (2009); Bolotin et al., ibid. (2009)

Two-Dimensional Electron Gas





Pfeiffer et al., Appl. Phys. Lett. **55**, 18 (1989)



Fractional Quantum Hall Effect (1982)

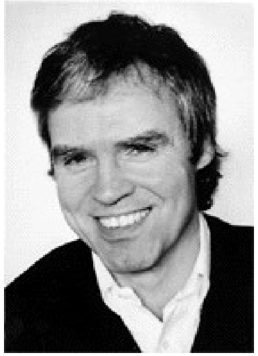
- High quality sample
- Low temperature
- High magnetic field

$$R_H = \frac{V_H}{I} = \frac{h}{\nu e^2}$$

$$R = \frac{V_{xx}}{I}$$

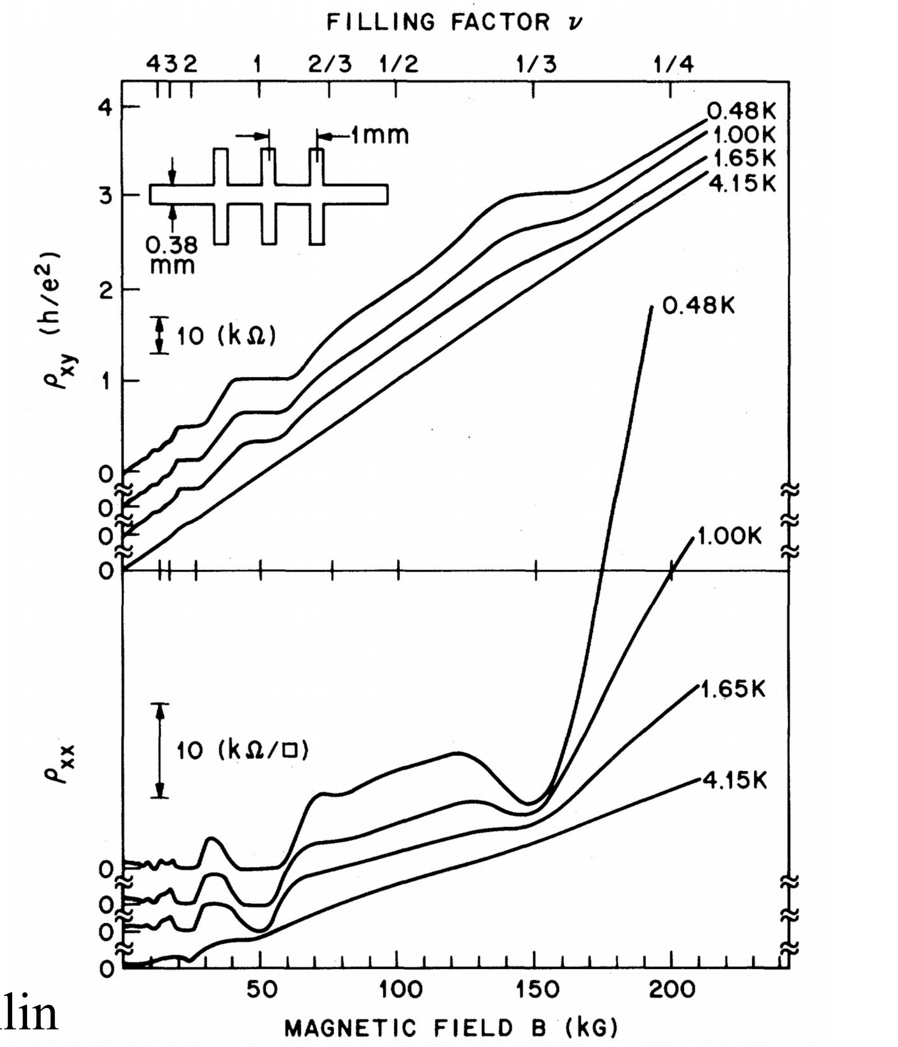
Fractional filling factor:
interaction important!

Daniel C. Tsui



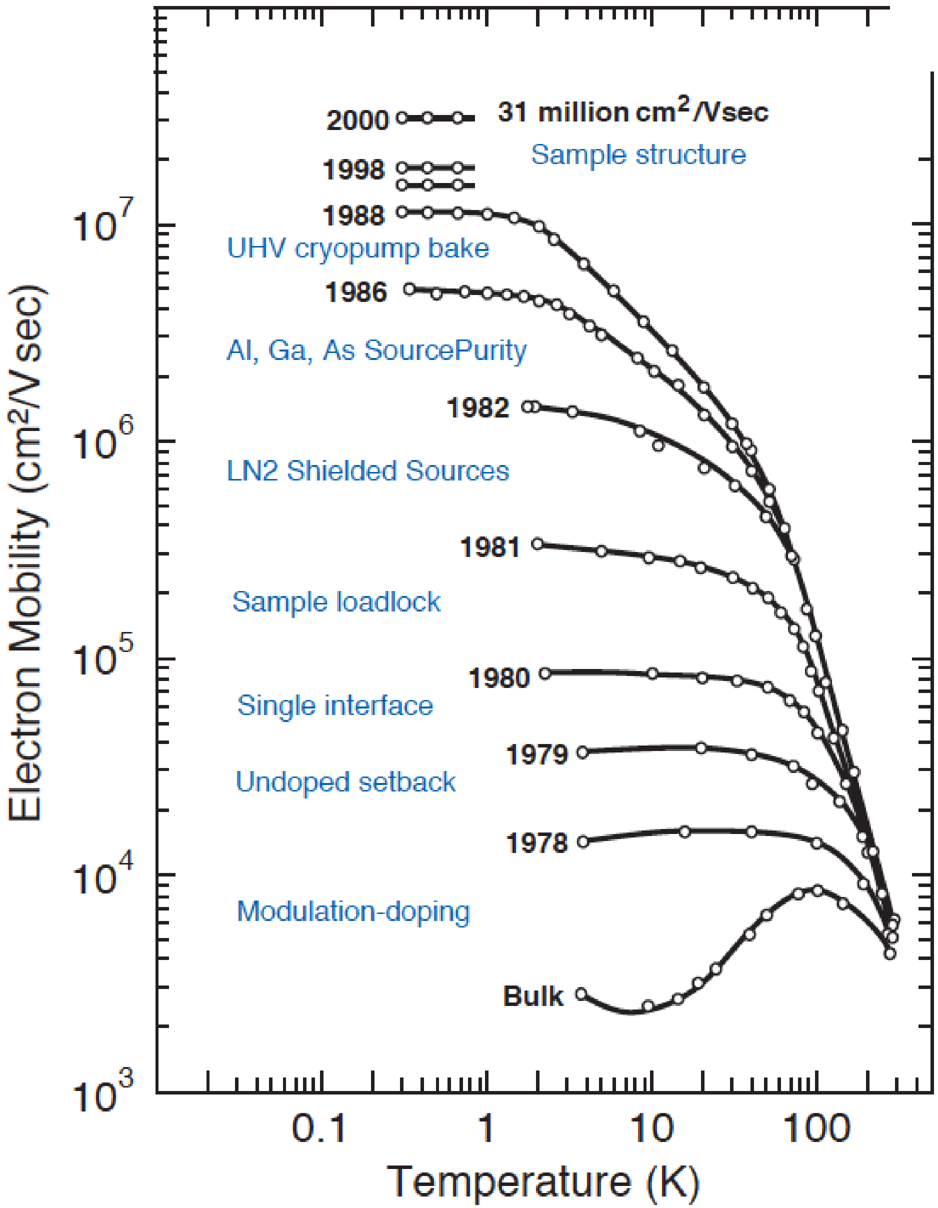
Horst L. Störmer

Robert B. Laughlin

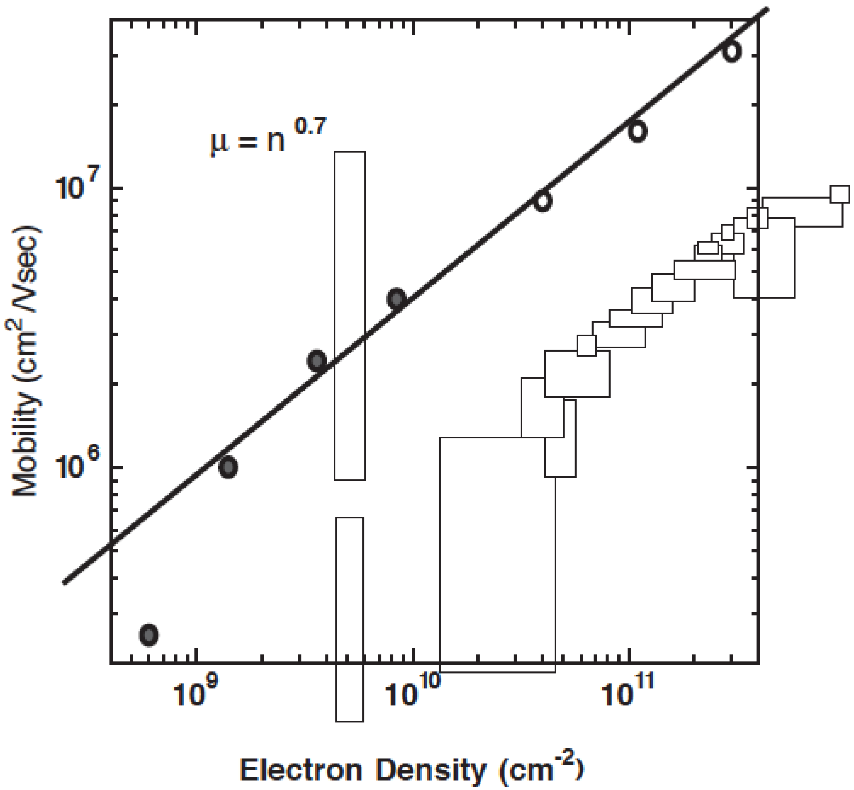


Nobel Prize 1998: "for their discovery of a new form of quantum fluid with fractionally charged excitations."

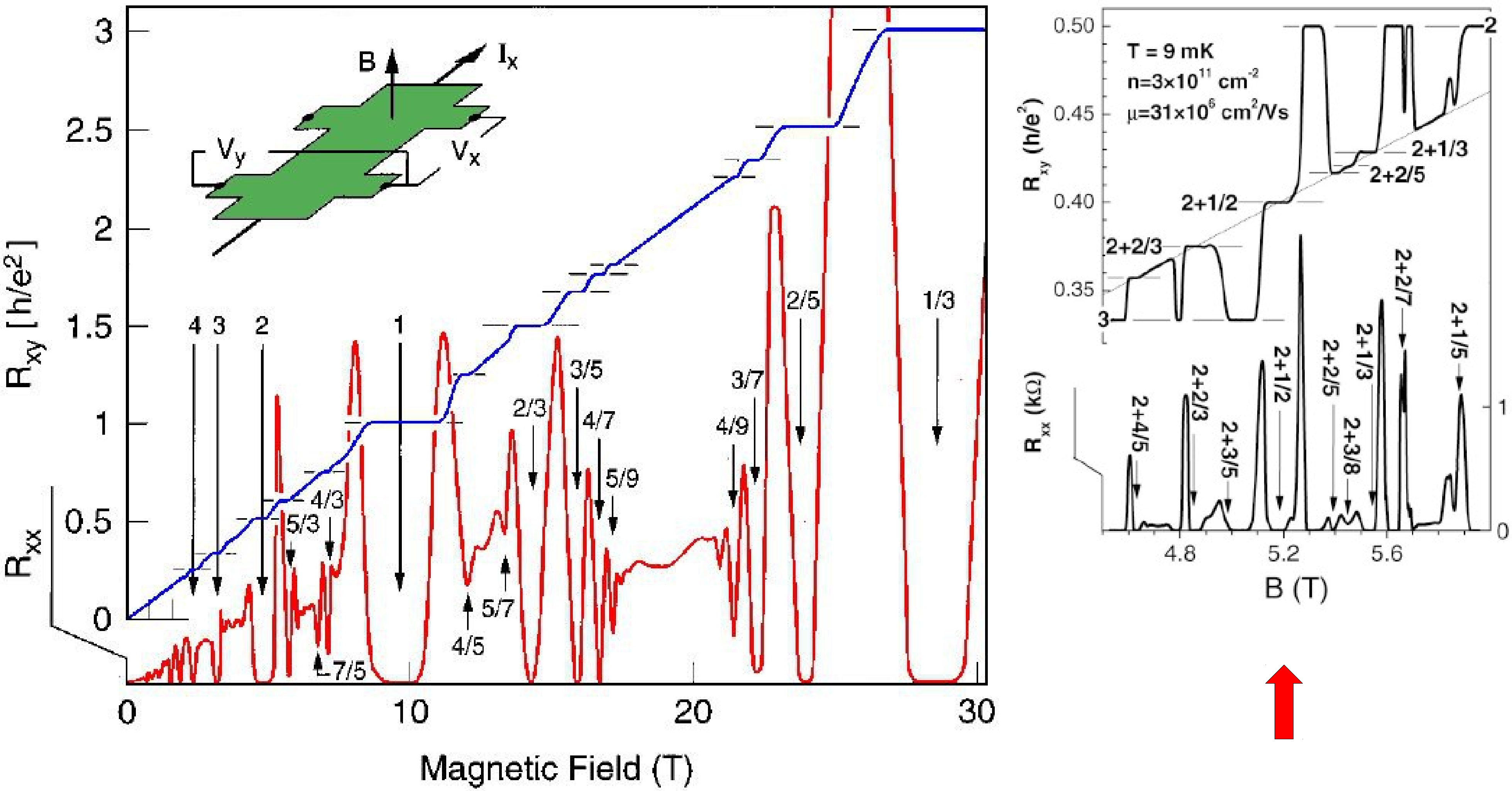
On Samples



L. Pfeiffer, K.W. West / Physica E 20 (2003) 57–64



FQHE: Distinct Topological Phases

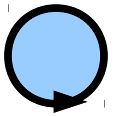


- Dominated by odd denominators, with notable exception at (5/2)
- Condensate of charge and flux composites

2DEGs: Algebraic Approach

- Coordination of electrons in a plane described by a complex $z = x + iy$
- Perpendicular magnetic field, choose symmetric gauge
- Hamiltonian (free spin-polarized electrons)

$$H_0 = \frac{1}{2m}(\vec{p} - e\vec{A})^2 \longrightarrow H_0 = \hbar\omega_c\left(a^+ a + \frac{1}{2}\right)$$



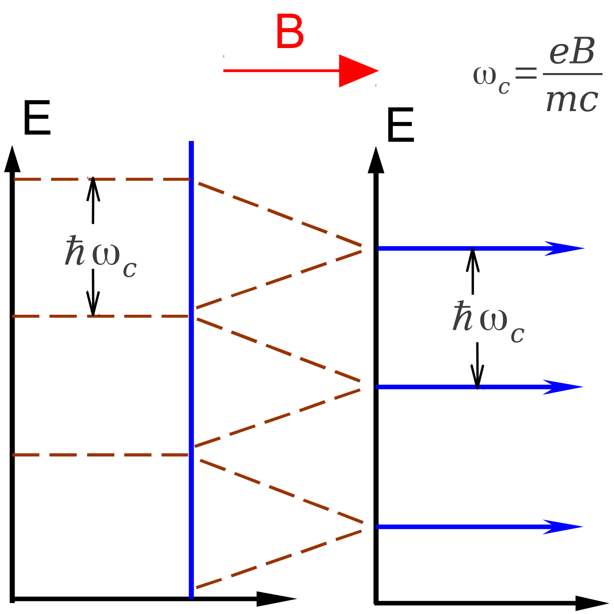
- Two sets of ladder operators

Inter-LL $\quad a = \sqrt{2}\left(l_B \partial_{\bar{z}} + \frac{1}{4 l_B} z\right)$ cyclotron motion

Intra-LL $\quad b = \sqrt{2}\left(l_B \partial_z + \frac{1}{4 l_B} \bar{z}\right)$ guiding center motion

$$l_B = \sqrt{\frac{\hbar c}{eB}} \qquad E_n = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

$$\omega_c = \frac{eB}{mc}$$



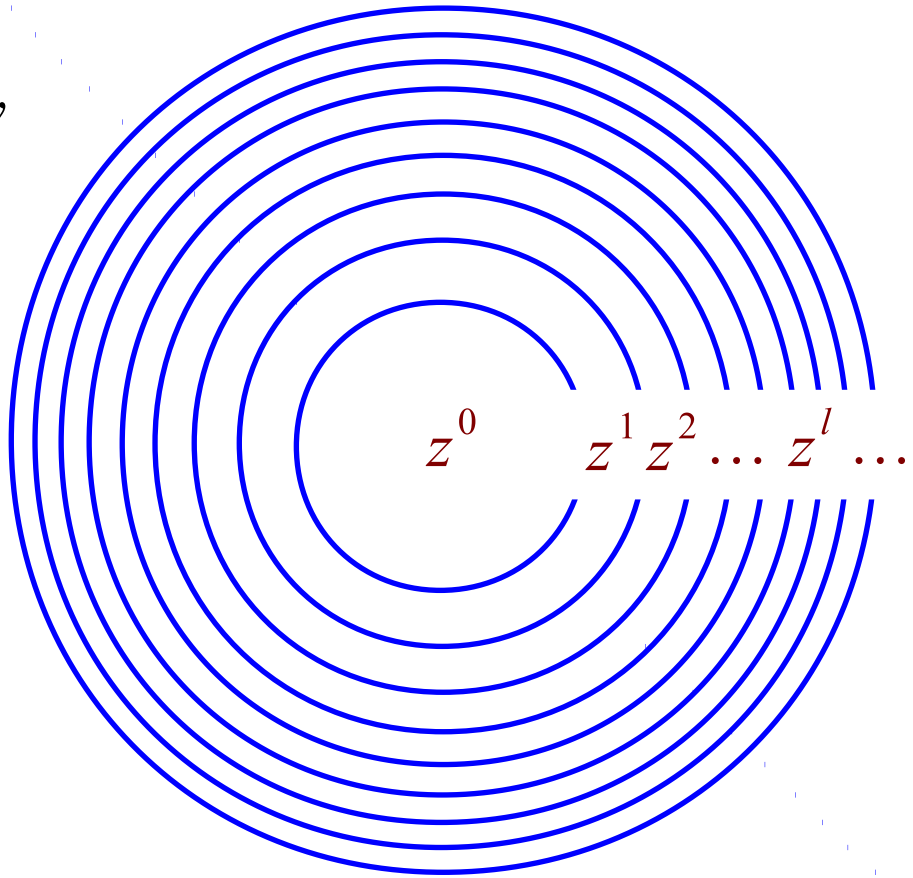
Density of States (DOS)

Laughlin State – Disk Geometry

- In the LLL, electron-electron interaction is not a perturbation. Nevertheless,

$$\phi_l(z) \sim z^l e^{-|z|^2/4} \qquad z = x + iy$$

- Basic requirement for an electron wave function in the LLL:
  - antisymmetric function
  - analytic function
  - a universal Gaussian factor



$$R_l = \sqrt{\langle l | r^2 | l \rangle} = \sqrt{2(l+1)}$$

- Laughlin state

$$\Psi_L = \prod_{i<j} (z_i - z_j)^m e^{-\sum_i |z_i|^2/4}$$

Model Hamiltonian for the Laughlin State

- Laughlin wavefunction is the ground state of

$$H_{hardcore} = \sum_{i<j}^{N} \partial_i^2 \delta^2 (z_i - z_j)$$

$$\Psi_L = \prod_{i<j} (z_i - z_j)^3 e^{-\sum_i |z_i|^2/4}$$

- Its LLL projection has a simple pseudopotential form

When N = 2 particles approach the same point, the wavefunction vanishes as q = 3 powers.

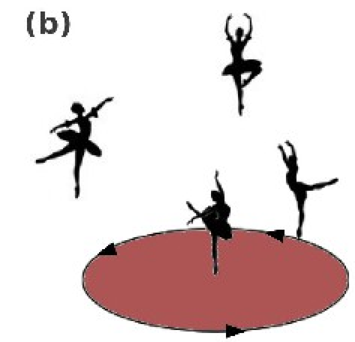
  - Two-particle wavefunction

$$(z_1 + z_2)^M (z_1 - z_2)^m$$

  - Interaction can be written, in general, as

$$H_i = \sum_m V_m P_m(1,2)$$

  - One produces the 1/3 Laughlin factor by $V_1 > 0$ only

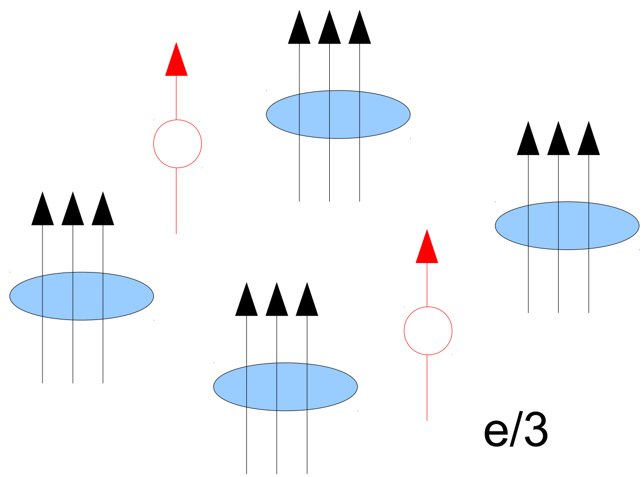


- In general, the Laughlin state is the zero-energy ground state of

$$H = \sum_{m=0}^{q-1} V_m \sum_{i<j} P_m(i,j)$$

Abelian Laughlin Quasiholes

- FQHE for electrons (ν = 1/3, 1/5, …)
  - Condensate of composite bosons



qps created away from 1/3 filling

$$\Psi_L = \prod_{i<j} (z_i - z_j)^3 e^{-\sum_i |z_i|^2/4}$$

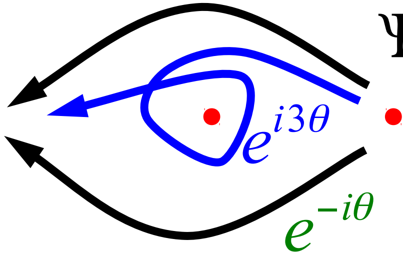
$$\Psi_\xi^{1\mathrm{qh}} = \prod_j (z_j - \xi) \prod_{i<j} (z_i - z_j)^3 e^{-\sum_i |z_i|^2/4}$$

$$\Psi_{\xi_1,\xi_2}^{2\mathrm{qh}} = \prod_j (z_j - \xi_1)(z_j - \xi_2) \prod_{i<j} (z_i - z_j)^3 e^{-\sum_i |z_i|^2/4}$$

Path equiv. in 3D; NOT equiv. in 2D:

Abelian anyons (i.e., different by a phase)

$\Psi \rightarrow \Psi\, e^{i\theta}$



Exercises on the Laughlin State

- Why is the filling fraction for the following Laughlin state?

$$\Psi_{Laughlin} = \prod_{1 \leqslant i < j \leqslant N} (z_i - z_j)^m e^{-\sum_i |z_i|^2/4}$$

  - m = 2: bosonic
  - m = 3: fermionic

- What is its total angular momentum?

- What is the fractional charge of the m = 2 state?

Hint: Two-Electron Laughlin State

- Laughlin state for electrons (n = 1/3)

$$\Psi_L = \prod_{i<j} (z_i - z_j)^3 e^{-\sum_i z_i^2/4}$$

$$(z_1 - z_2)^3 \;=\; 1 \cdot (z_1^3 - z_2^3) + (-3) \cdot (z_1^2 z_2 - z_1 z_2^2)$$

1 ●□□● + (- 3) □●●□ Orbitals: 0,1,2,3

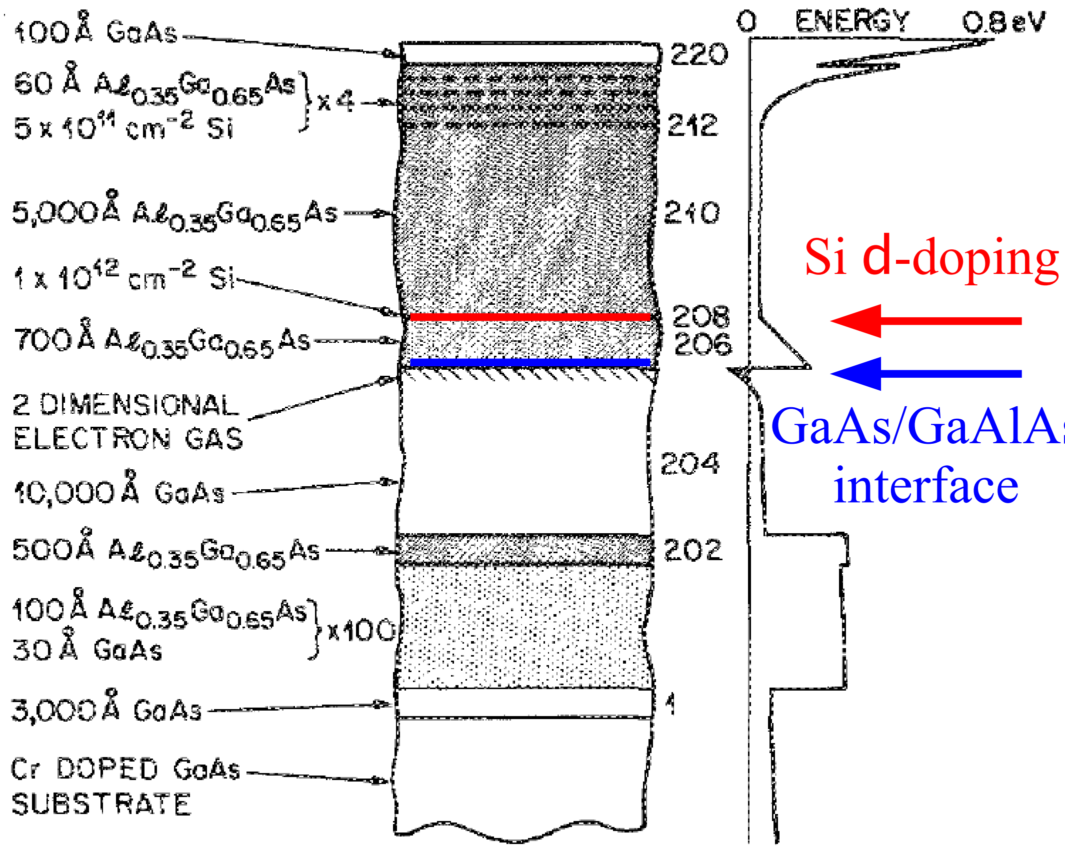
Generalize it for 3 electrons. Use Mathematica for N electrons.

- For N electrons,

$$\Psi_L = Sym\left(z_1^{3(N-1)} z_2^{3(N-2)} \cdots z_N^0 + \cdots \right) e^{-\sum_i |z_i|^2/4}$$

$$\nu \;=\; \lim_{N \to \infty} \frac{N}{3(N-1)+1} \;=\; \frac{1}{3}$$

Realistic Model

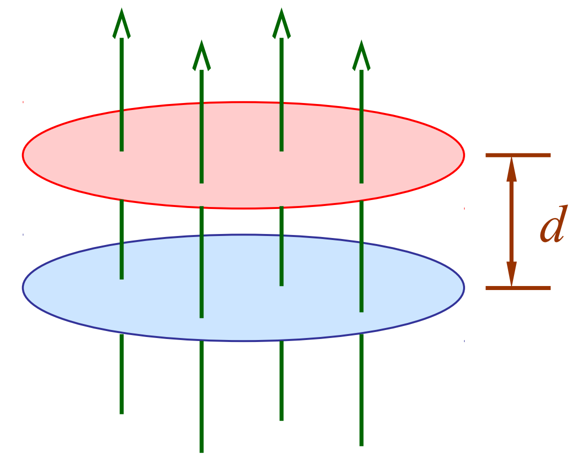


Si d-doping

GaAs/GaAlAs interface

Background charge (+Ne)

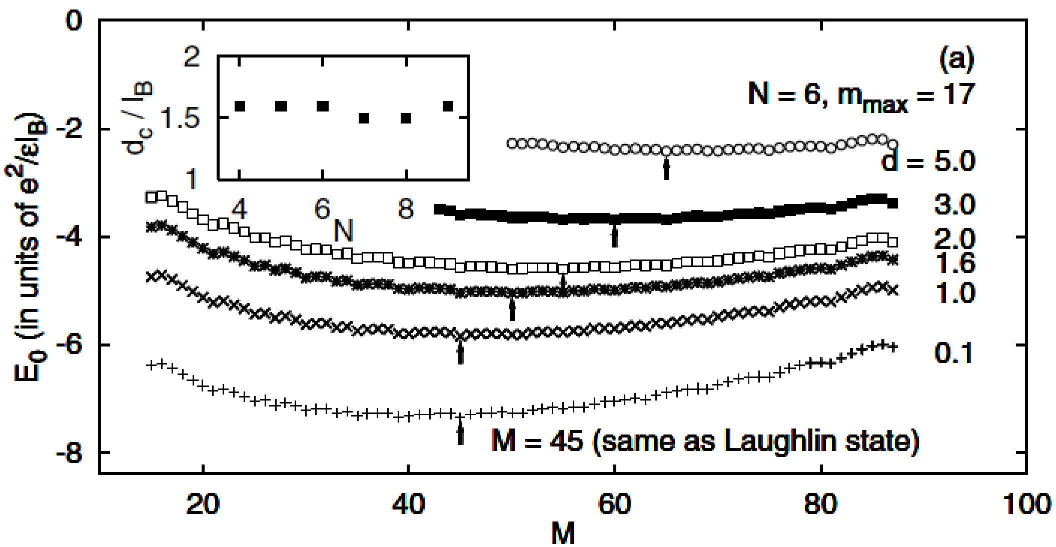
Electron layer (-Ne)

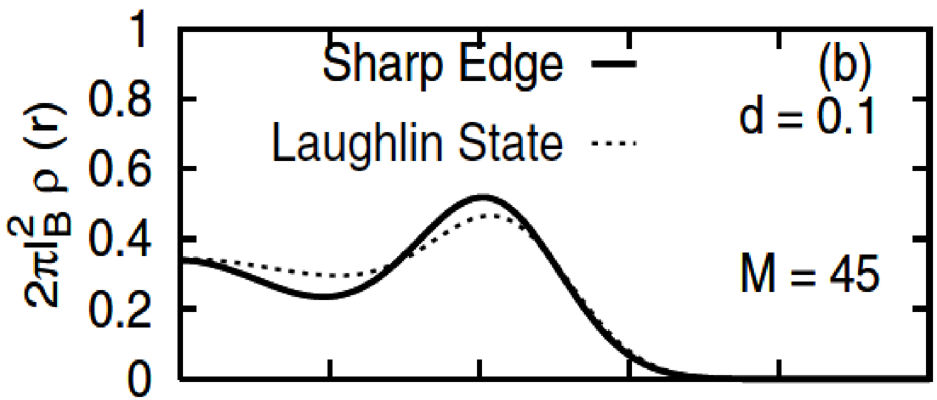


$$H = \frac{1}{2}\sum_{mnl} V^{l}_{mn} c^{+}_{m+l} c^{+}_{n} c_{n+l} c_{m} + \sum_{m} U_{m} c^{+}_{m} c_{m}$$

Coulomb interaction Confining potential

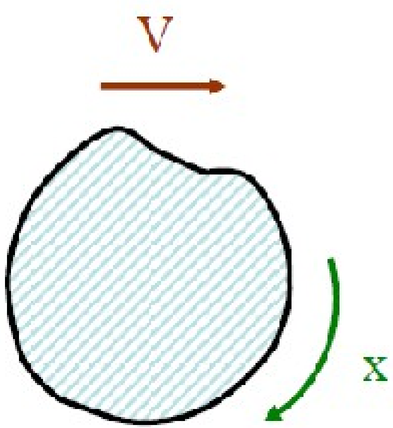
Ground State and Edge Spectrum



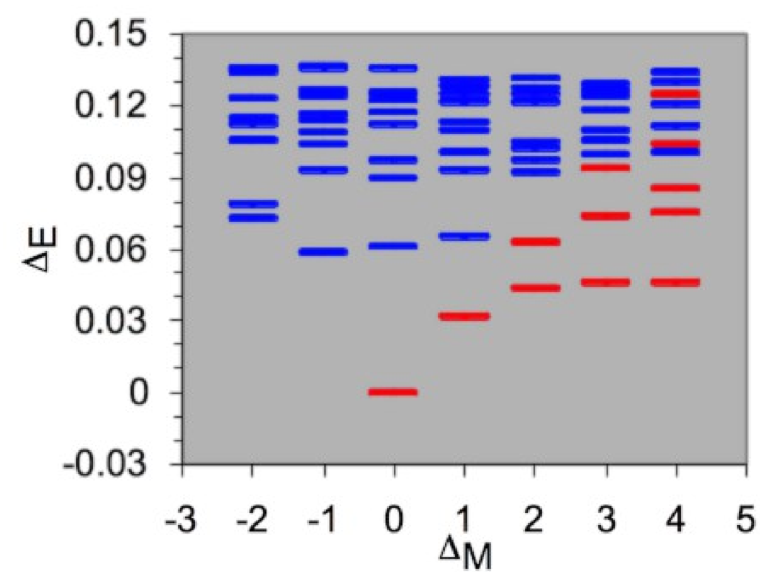


- Edge excitations generated by symmetric polynomials

$$P(\{z_i\})\prod_{i<i}(z_i-z_j)^m e^{-\sum_i |z_i|^2/4}$$

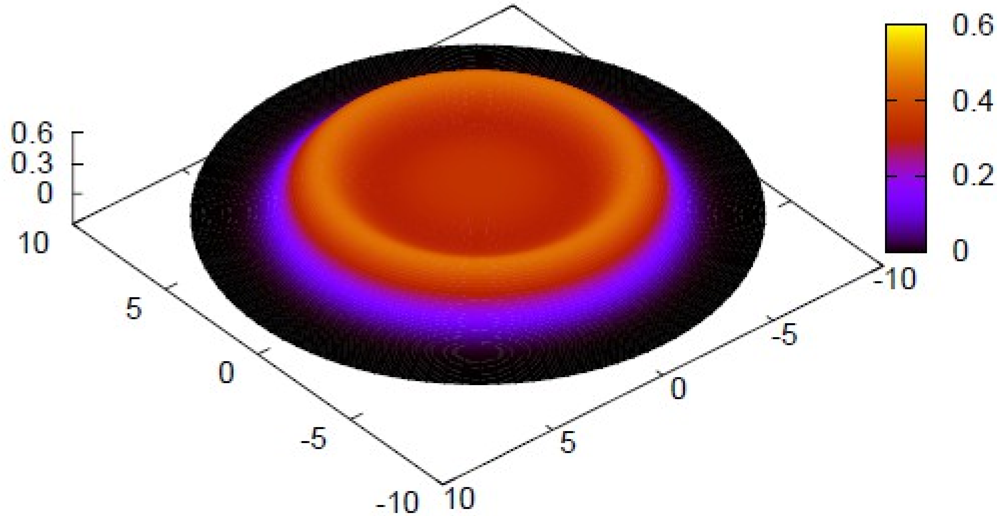


Gapless chiral bosonic charge mode

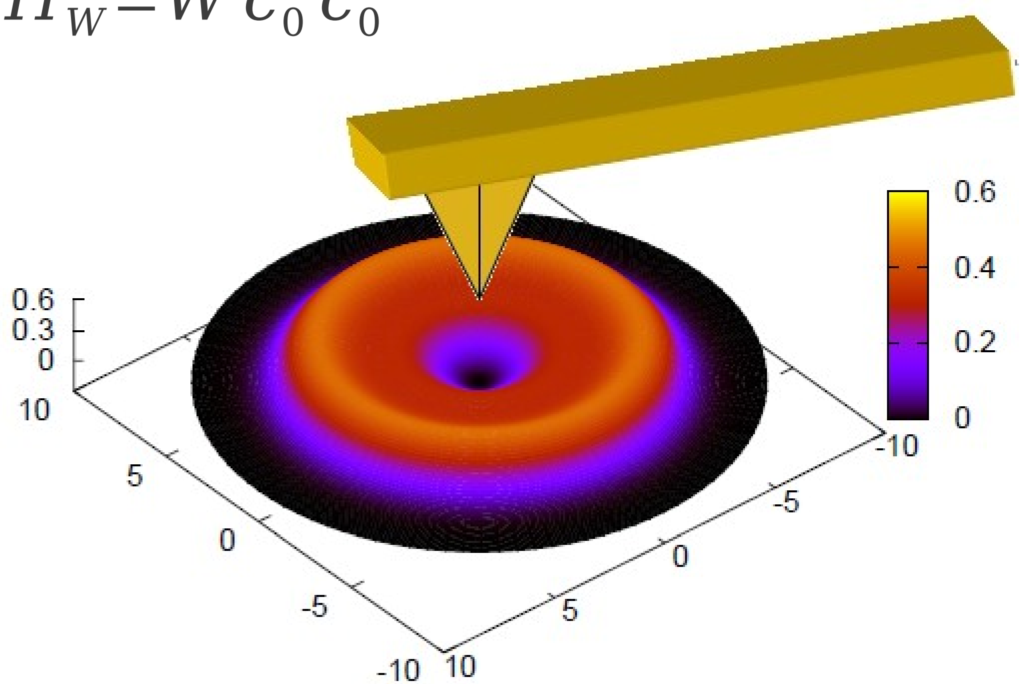


Introducing Quasihole

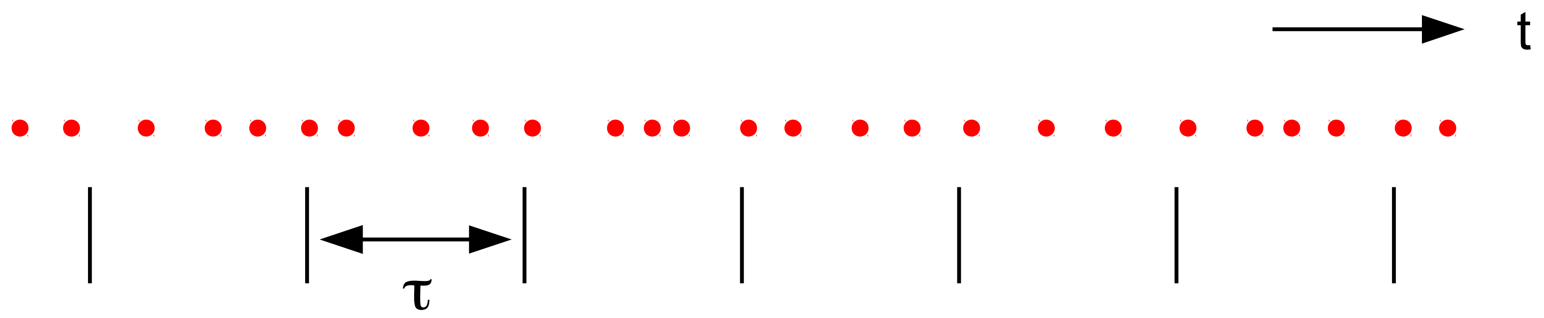
9-electron
Laughlin state



$$H_W = W c_0^+ c_0$$



How to Measure the Charge Experimentally?

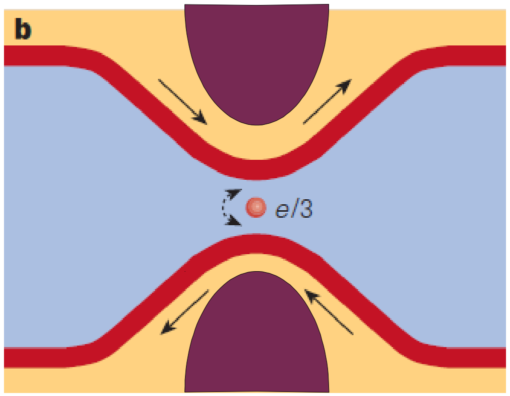


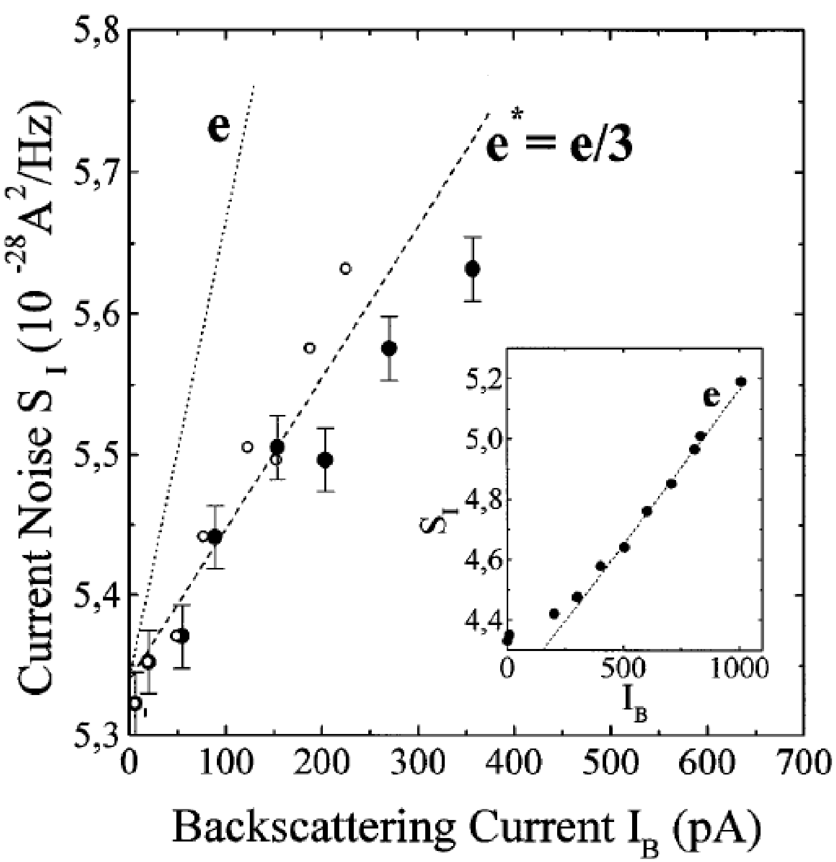
$$I = \frac{\text{average \# of events in } \tau}{\tau} \times \text{charge contribution per event}$$

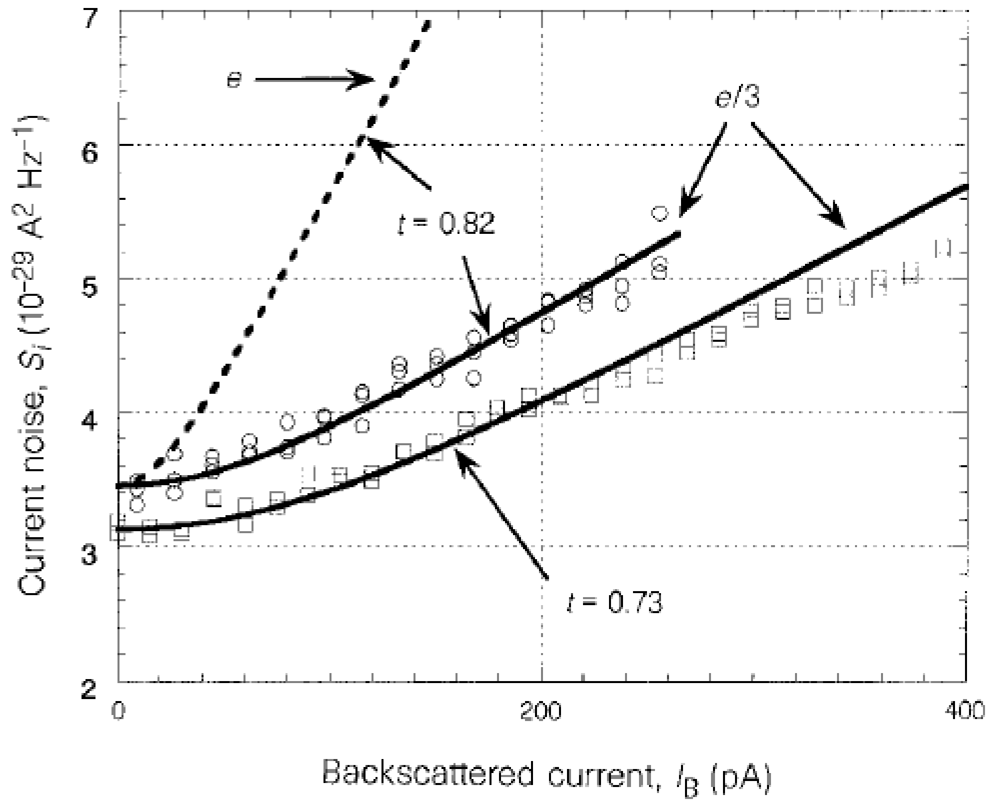
$$I_n = \frac{\text{rms fluctuation of \# in } \tau}{\tau} \times \text{charge contribution per event}$$

$$S_I \propto I_n^2 \propto 2e^* I$$

Fractional Charge in Shot Noise



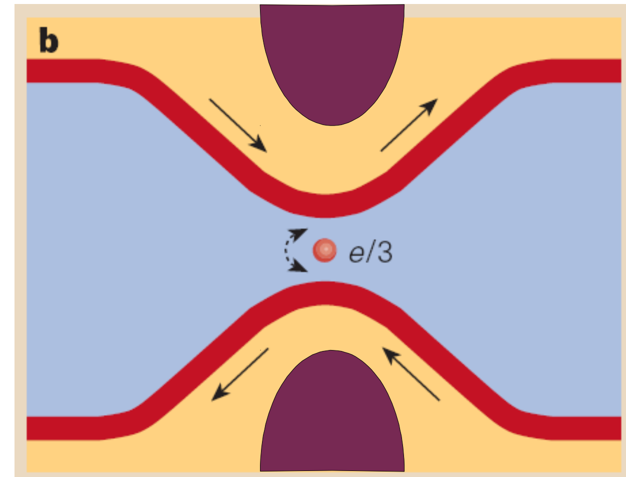


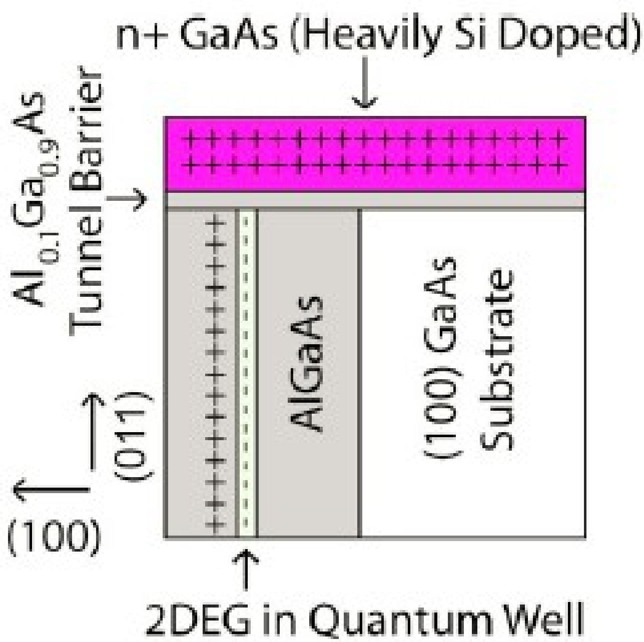


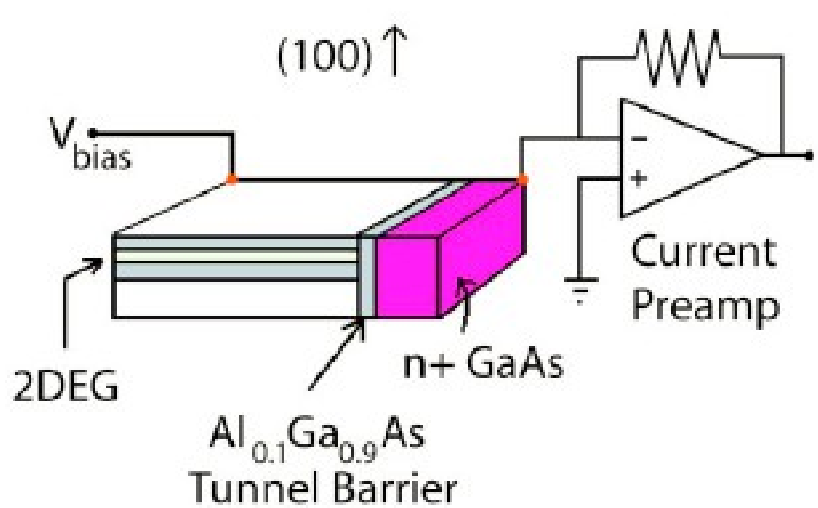
- De-Picciotto et al., Nature 389, 162 (1997)
- Saminadayar et al., PRL 79, 2526 (1997)

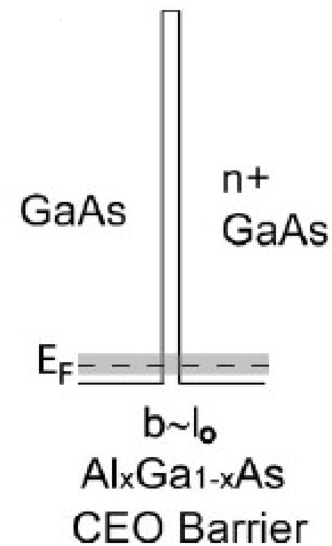
Devices for Edge Physics

- Quantum point contact
  - Smooth potential, tunable
- Cleaved-edge overgrowth
  - Broad energy range









#1: Messages So Far

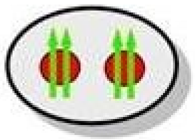
- FQH effect can be routinely observed in two-dimensional electron systems in GaAs quantum wells or in high-mobility graphene.
- Mobility is an important quantity to determine which fractions can be observed. Higher mobility means smaller disorder.
- A model wave function can be thought of as the fixed point for the corresponding topological phase, which is stable under long-range interaction and disorder.
- Laughlin states support (gapped) Abelian quasiparticle excitations which carry a fraction of an electron charge. The fractional charge has been detected by shot noise measurement.
- Laughlin states support gapless chiral edge excitations. Quasiparticles can propagate along the edge.
- Other odd-denominator FQH states can be thought of as the descendants of the Laughlin states.

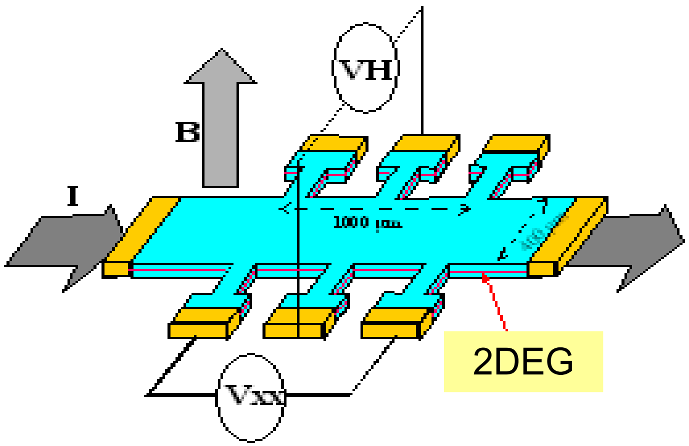
Next: non-Abelian state at ν = 5/2

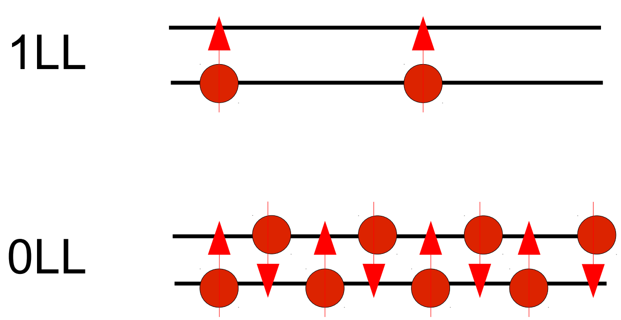
FQH at the First Excited Landau Level

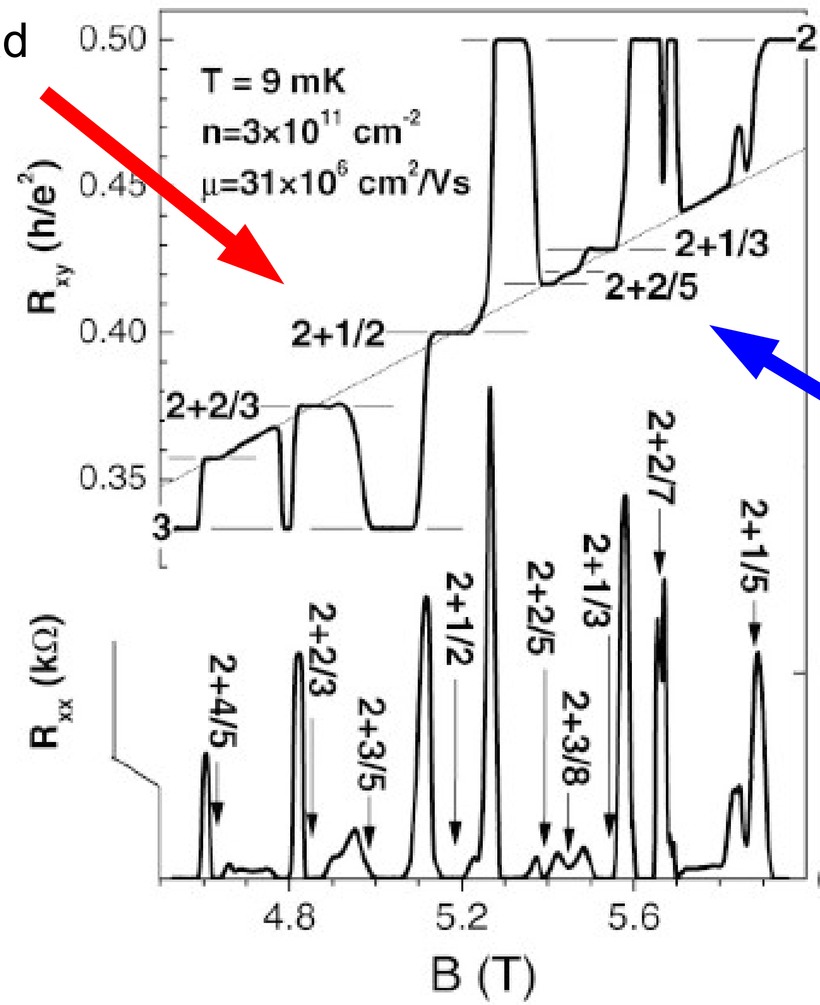
Ising anyon / **Majorana** fermion mode

Moore-Read









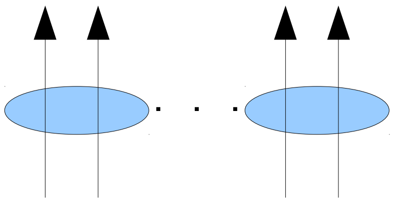
Read-Rezayi?

Fibonacci anyon

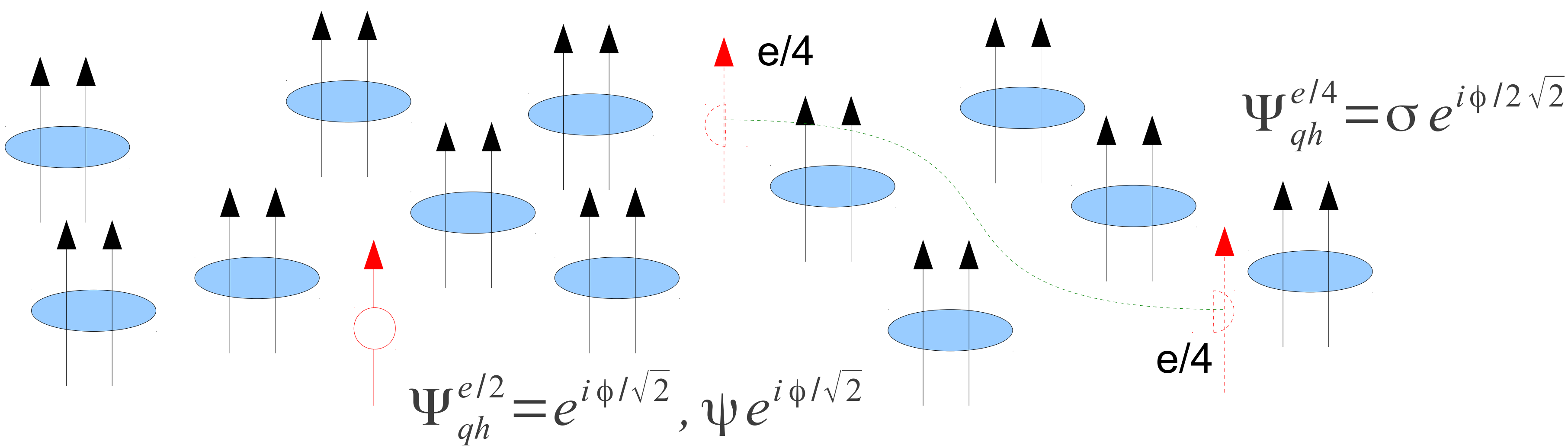
Xia et al., PRL (04)

A Cartoon of the Moore-Read State

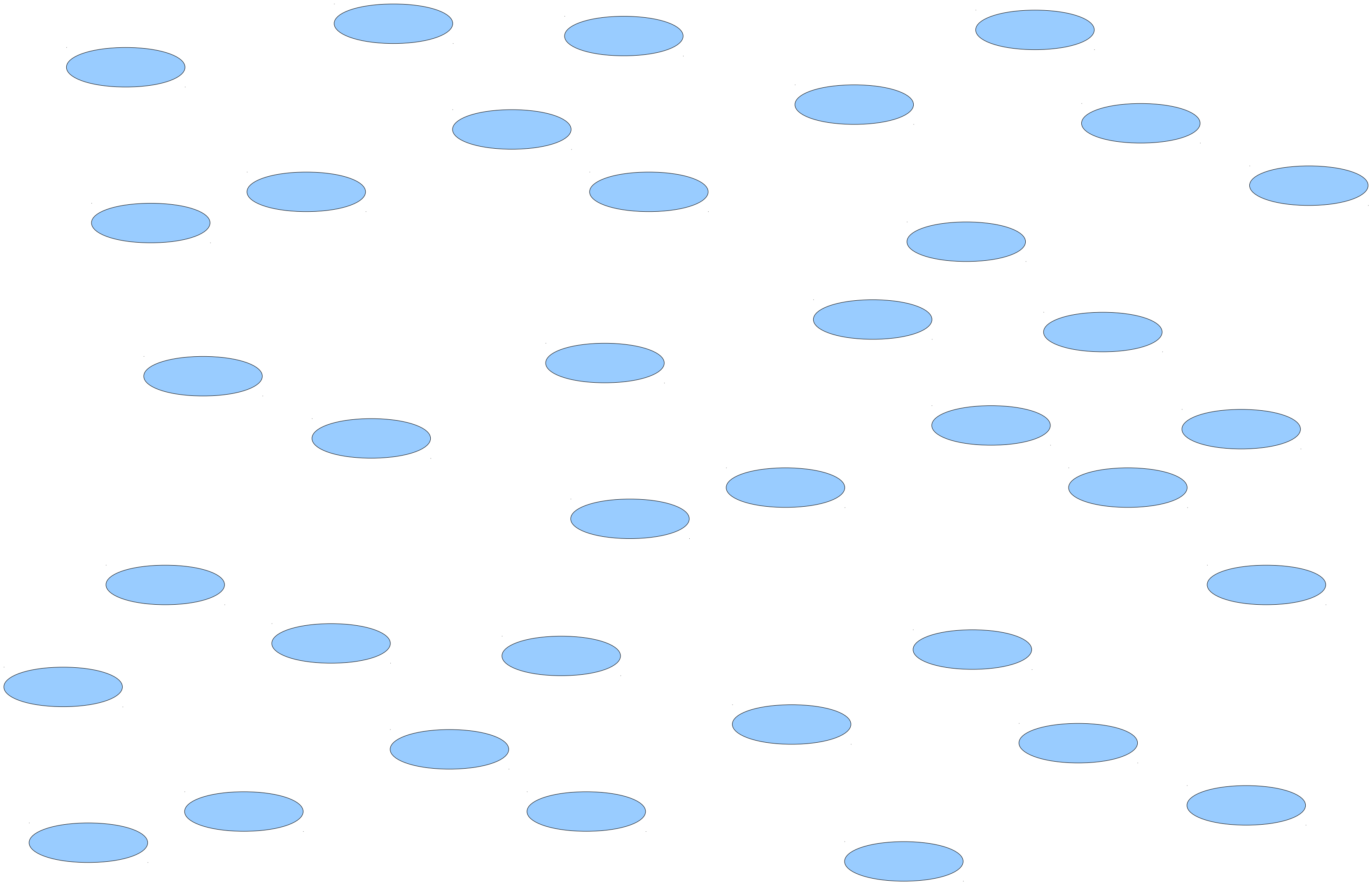
- Half-filling $\nu = 1/2$: CF at zero effective field ($B^* = 0$)
  - 0LL (or LLL): Fermi sea of composite fermions
  - 1LL: Superfluid of Cooper pairs of composite fermions
  - 2+LL: Charge density wave

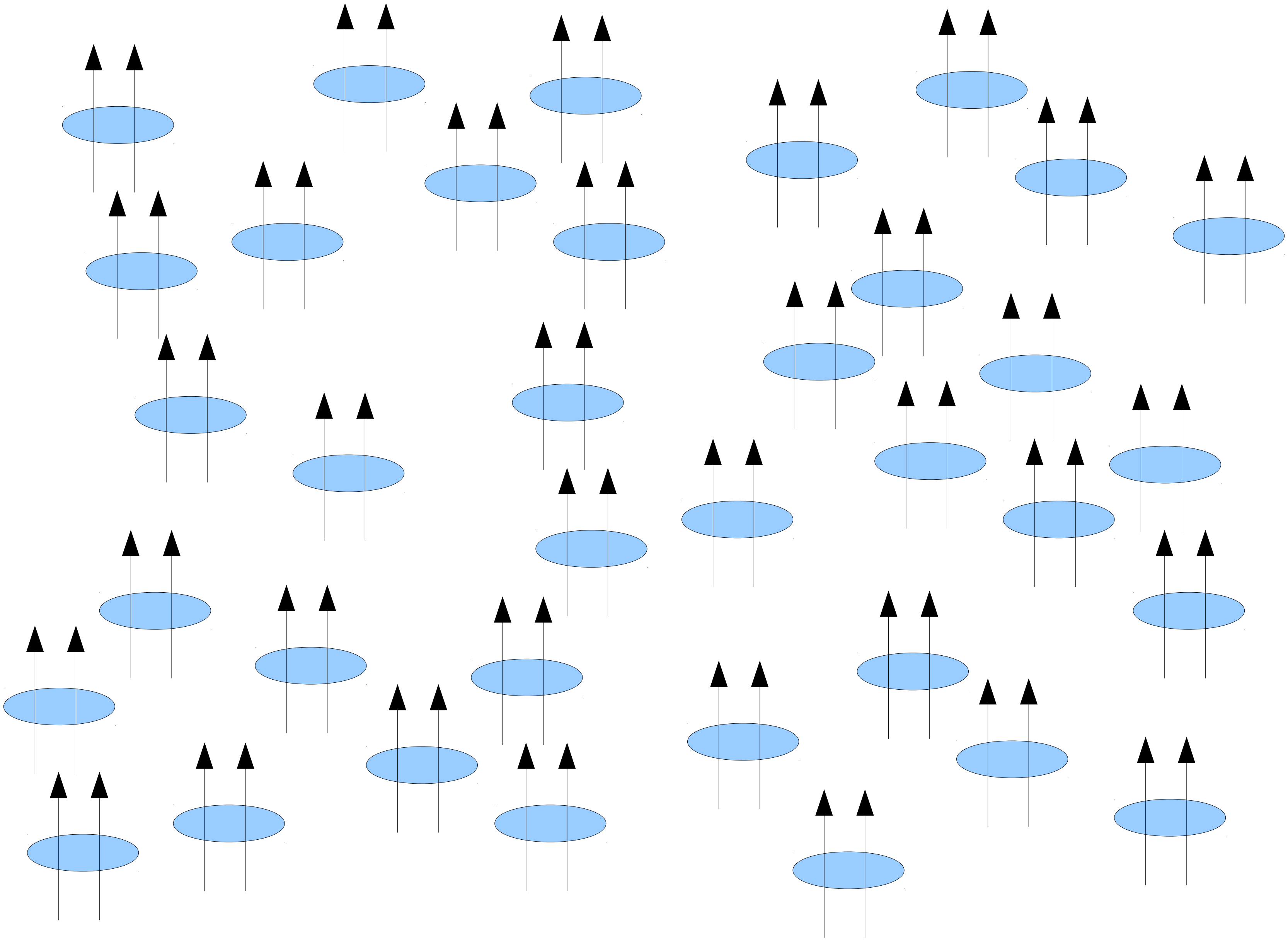


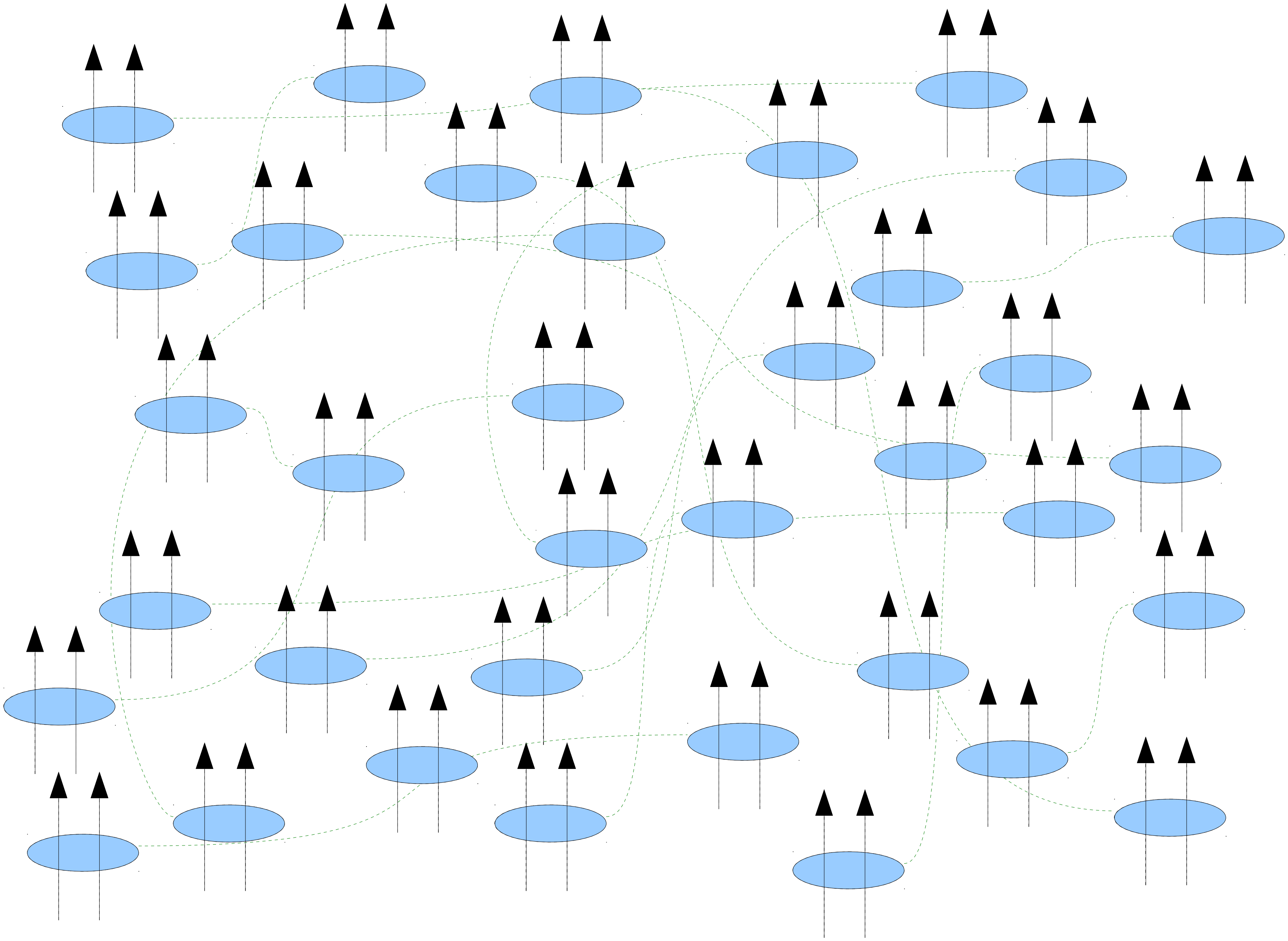
- Condensate of composite fermions ($\nu = 5/2 = 2 + 1/2$)

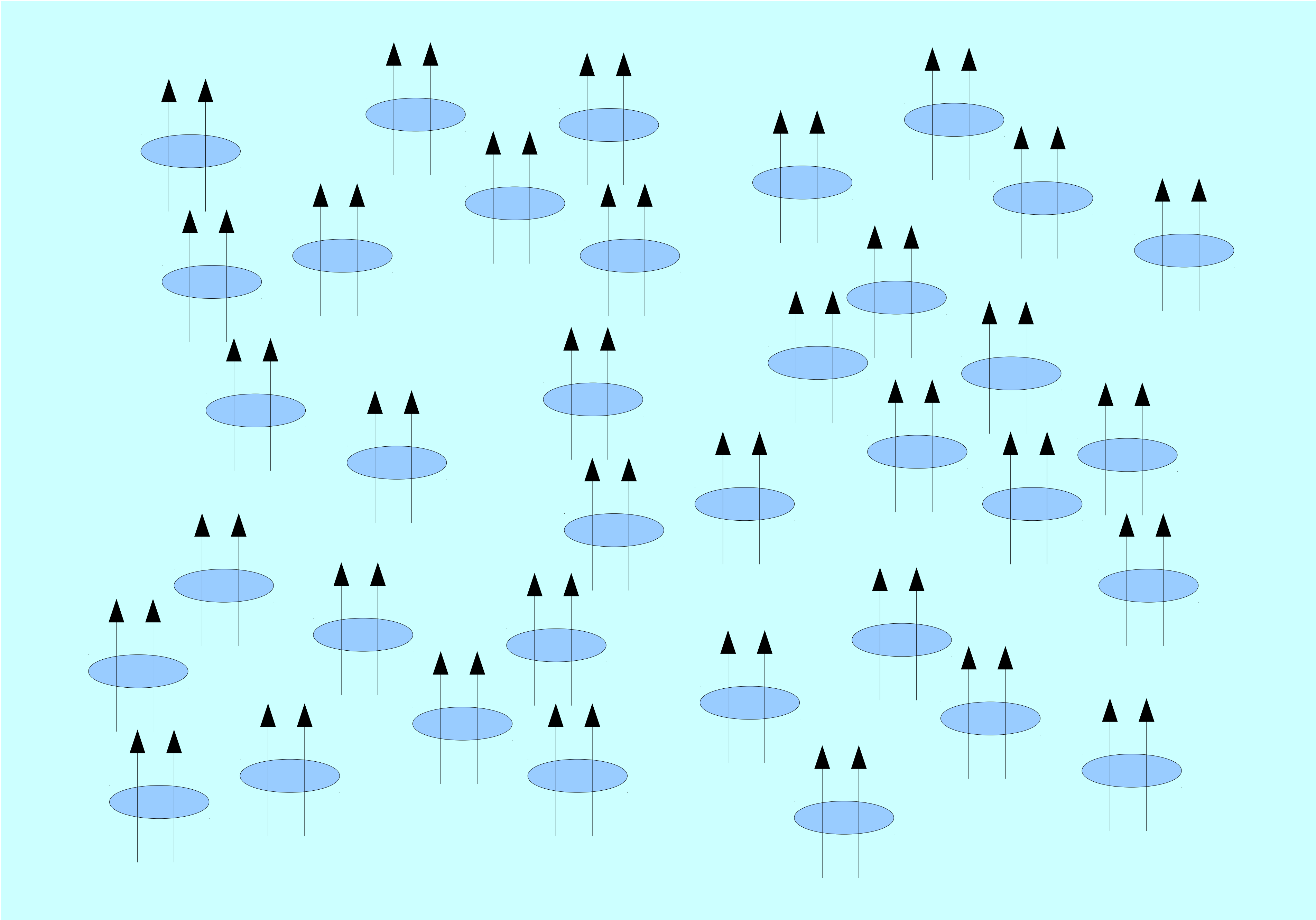


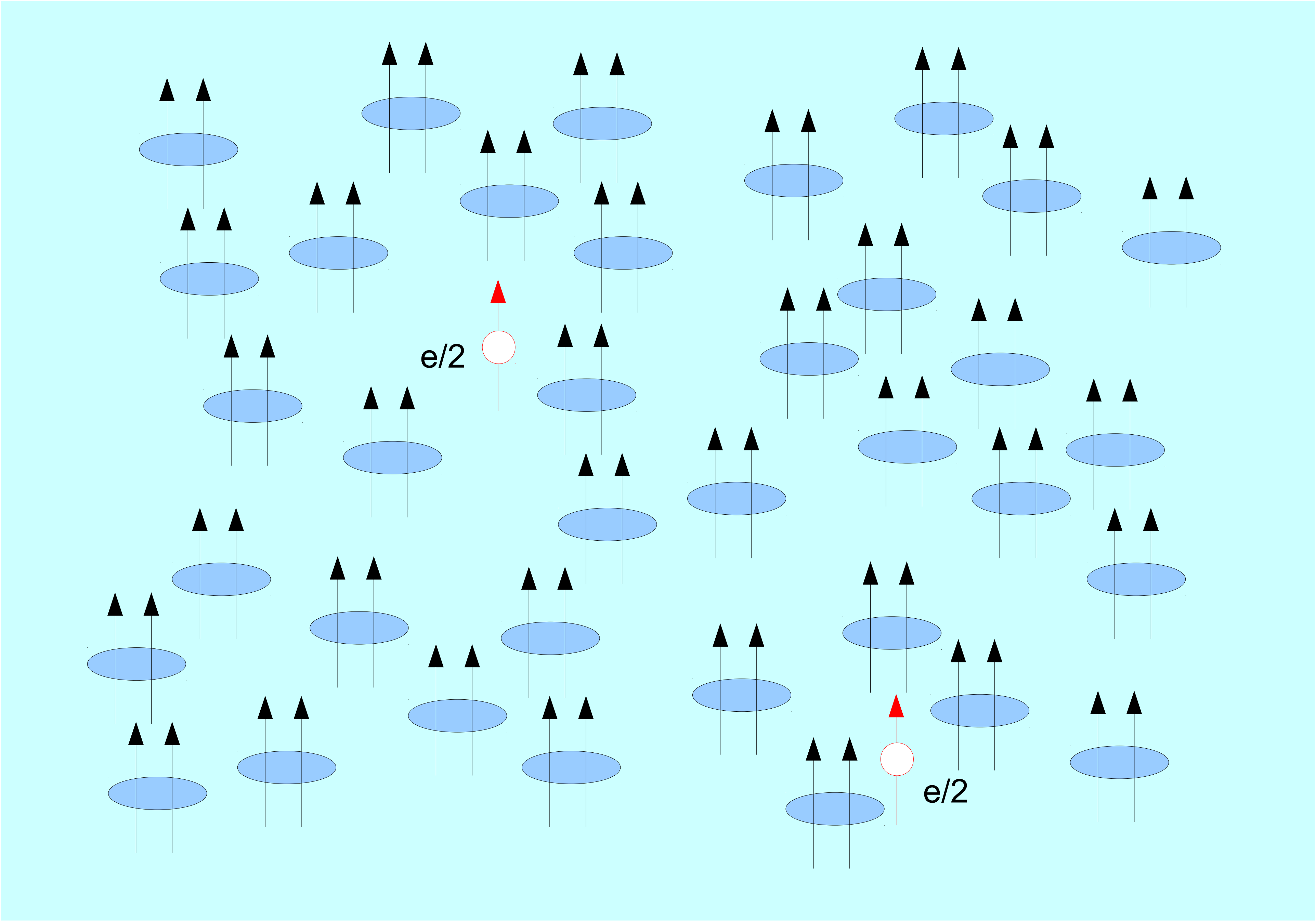
e/4 quasihole = charge-e/4 boson + neutral Majorana fermion mode

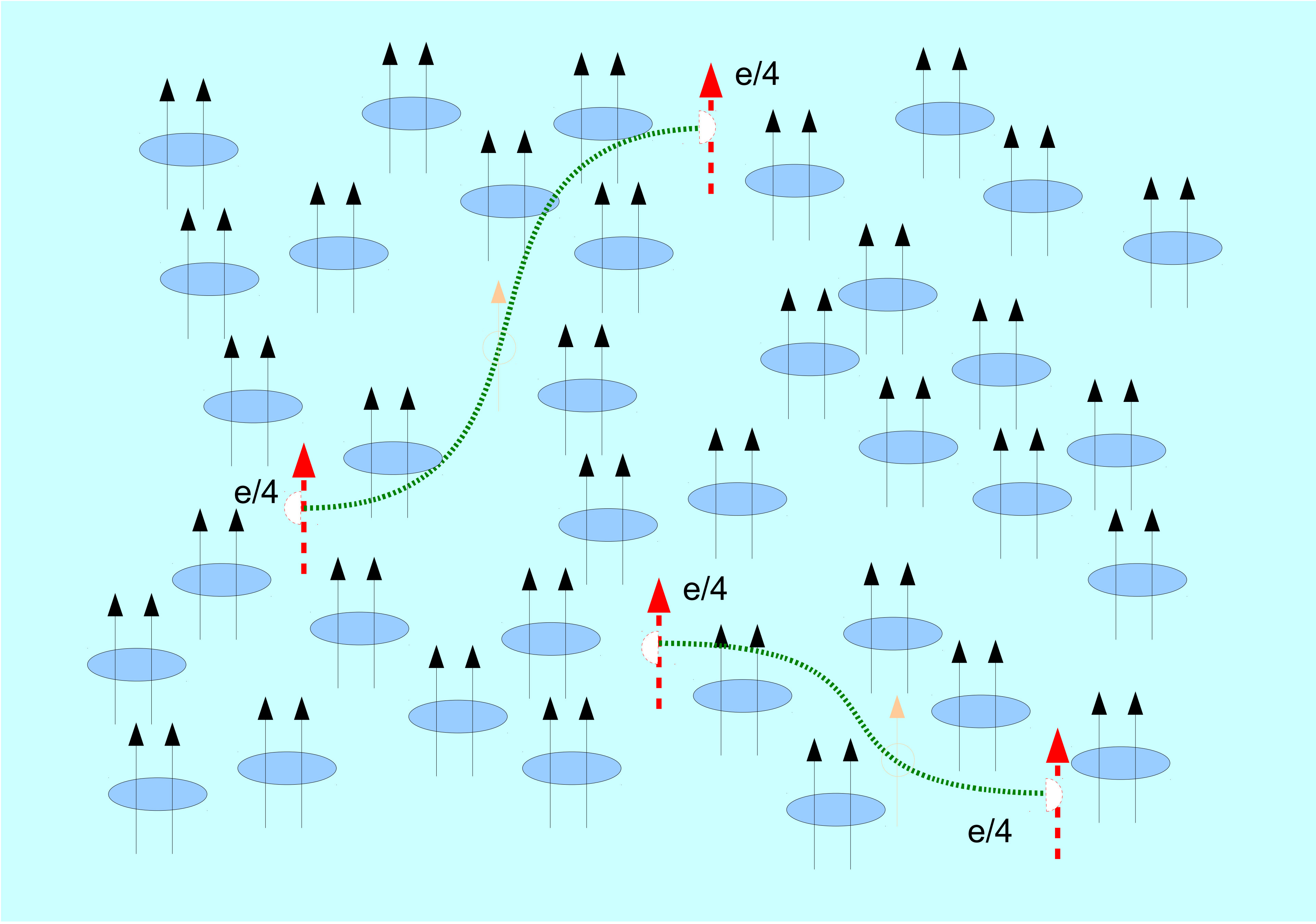






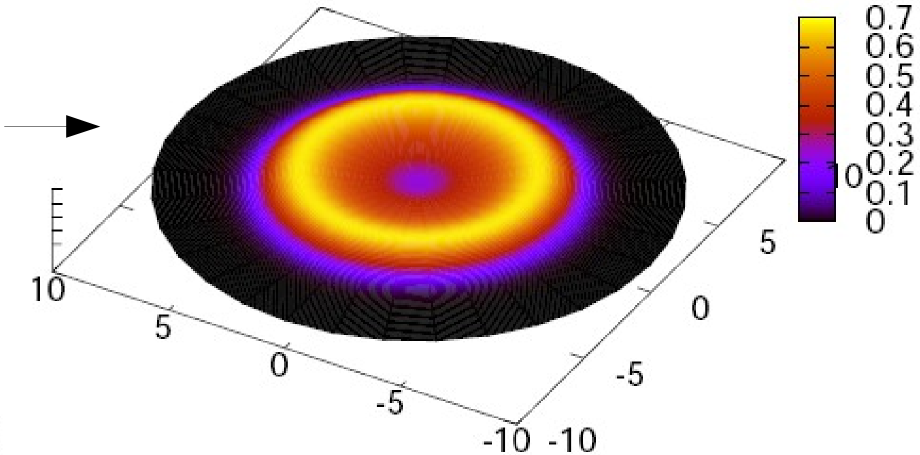


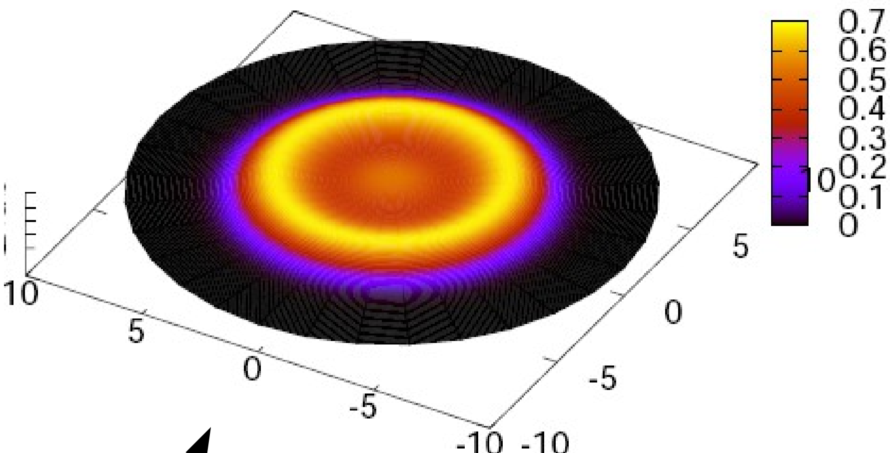

e/2
e/2


e/4
e/4
e/4
e/4

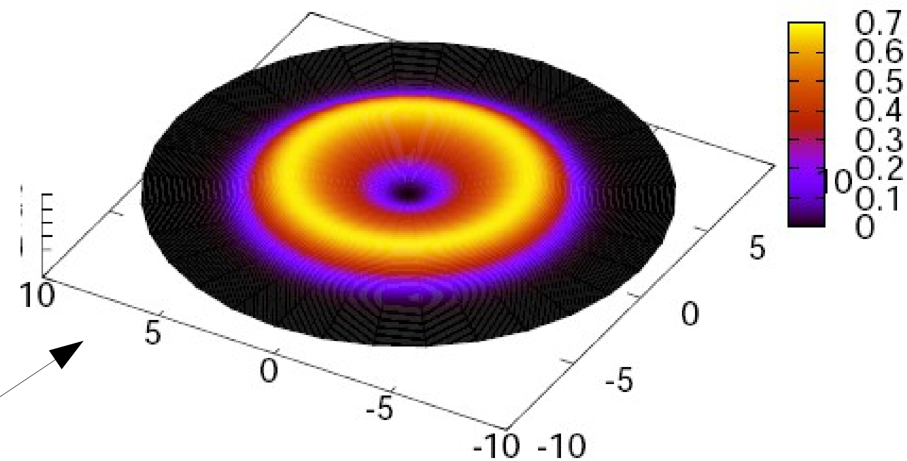
Density Profiles for a 12-electron Droplet (ED)

M-R + e/4 quasihole





M-R



M-R + e/2 quasihole

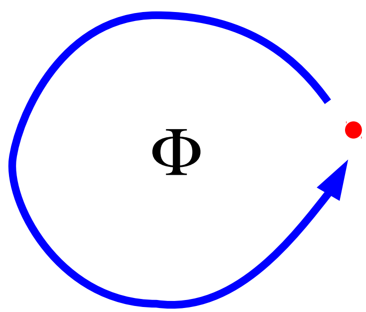
Appendix:

How to compute the charge of a quasihole in the Laughlin state?

How to compute the exchange statistics?

How to Measure the Charge in Theory?

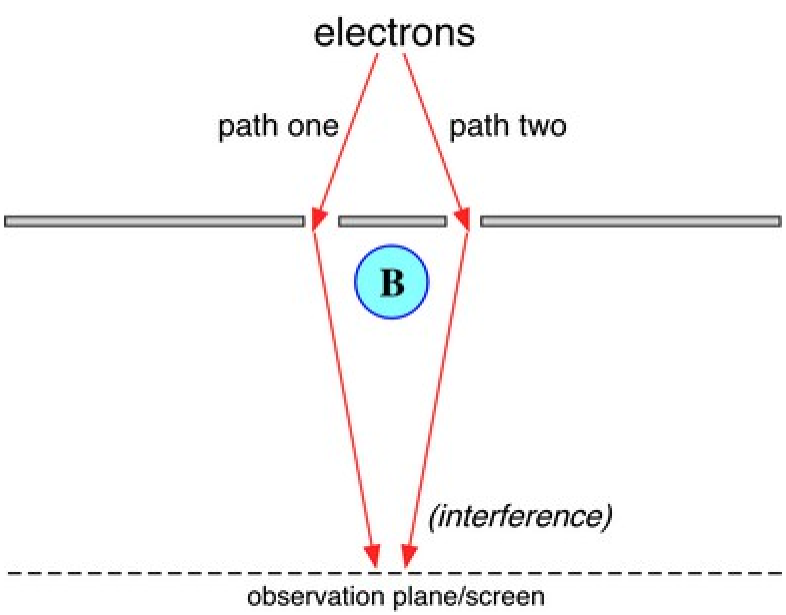
- Adiabatically move a charged particle/quasiparticle around a loop, the wavefunction will pick up a geometric phase (Berry phase) that is proportional to the magnetic flux through the loop, with the prefactor being essentially the effective charge.



$$\Delta\varphi = 2\pi(\Phi/\Phi_0)$$

$$\Phi_0 = \frac{hc}{e^*}$$

Aharonov–Bohm effect

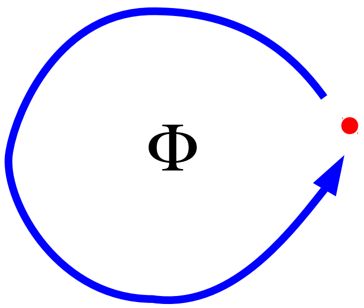


Berry Phase

See, e.g., J. J. Sakurai, *Modern QM*, Supplement I

$$H(R(t))|\Psi(R)\rangle = E(R)|\Psi(R)\rangle$$

$$|\Psi(t)\rangle = e^{i\varphi(t)}|\Psi(R(t))\rangle$$



$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = H(R(t))|\Psi(t)\rangle$$

$$\left[-\hbar\dot{\varphi} + i\hbar\frac{\partial}{\partial t}\right]|\Psi(R(t))\rangle = E(R)|\Psi(R(t))\rangle$$

$$\dot{\varphi} = -\frac{E(R)}{\hbar} - i\left\langle\Psi(R(t))\left|\frac{\partial}{\partial t}\right|\Psi(R(t))\right\rangle$$

$$\varphi_f - \varphi_i = -\int_{t_i}^{t_f} dt\, E(R)/\hbar - i\oint d\vec{R}\,\langle\Psi(\vec{R})|\nabla_{\vec{R}}|\Psi(\vec{R})\rangle$$

dynamical geometric + topological (path indep.)

Berry Phase

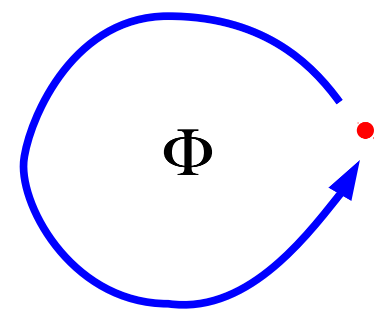
- [Berry] When a system evolves in the parameter space, its ground state picks up dynamical and geometric (including topological) phases.

$$\Delta\varphi = -\int_{t_i}^{t_f} dt \frac{E(R(t))}{\hbar} - i\oint d\vec{R}\cdot\langle\Psi(\vec{R})|\nabla_R|\Psi(\vec{R})\rangle$$

- Suppose we use real normalization constant and move quasihole at w.

$$\Psi_w = |N(w, w_1, w_2, \cdots, w_n)| \prod_j (z_j - w) \prod_{j,\alpha} (z_j - w_\alpha) \prod_{i<j} (z_i - z_j)^3 e^{-\sum_i |z_i|^2/4}$$

$$\nabla_w \Psi_w = \left[\nabla_w \ln|N| + \nabla_w \sum_j \ln(z_j - w)\right]\Psi_w$$



- Use trick $\rho(z) = \sum_i \delta^{(2)}(z - z_i)$

$$\langle\Psi_w|\nabla_w|\Psi_w\rangle = \nabla_w \ln|N| \langle\Psi_w|\Psi_w\rangle + \int d^2z \nabla_w \ln(z-w) \langle\Psi_w|\rho|\Psi_w\rangle$$

$$-i\oint dw \langle\Psi_w|\nabla_w|\Psi_w\rangle = 0 + \int d^2z \overbrace{\oint dw \nabla_w \ln(z-w)}^{2\pi i\text{, if z inside w loop; 0, otherwise.}} \langle\Psi_w|\rho|\Psi_w\rangle$$

Berry Phase (continued)

- Therefore, we find

$$\Delta\varphi = 2\pi \underbrace{\int d^2 z \langle \rho(z) \rangle}_{\text{charge enclosed inside the loop}}$$

- Without quasiholes inside the loop

$$\Delta\varphi = 2\pi \frac{\Phi}{hc/(e\nu)}$$

A-B effect for e* = eν around Φ

- With quasiholes inside, charge → charge + quasihole charge

$$\Delta\varphi \rightarrow \Delta\varphi + \underbrace{\left(\frac{2\pi}{m}\right)}_{\text{topological}} n_{quasihole}$$

$$\Psi \rightarrow \Psi e^{2i\theta}$$