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The Analytic Hierarchy Process AHP

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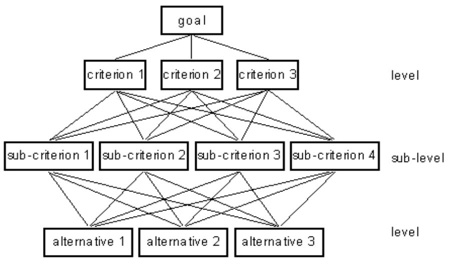
Analytic Hierarchy Process

- **Analytic**
  - The separating of any material or abstract entity into its constituent elements (opposed to synthesis)
- **Hierarchy**
  - Divide into units which are subdivided into smaller units, which are, in turn, subdivided and so on
  - Hierarchical subdivision is common to virtually all complex systems of which we have knowledge. ... The near universality of hierarchy in the composition of complex systems suggest that there is something fundamental in this structural principle that goes beyond the peculiarities of human organization
  - Hierarchy is the adaptive form for finite intelligence to assume in the face of complexity (after Simon)
- **Process**
  - As opposed to an event

A decision problem and the AHP

- The essence is decomposition of a problem into a hierarchy with goal (objective) at the top, criterions and sub-criterions at levels and sub-levels, and decision alternatives at the bottom of the hierarchy.



- The decision problem is to determine weights of alternatives with respect to goal by taking into consideration 'mediating' elements in between.

Hierarchies

- Hierarchy may be incomplete.
- Element in higher level does not have to be a criterion for all elements of lower level.
- Hierarchy can be divided into hierarchies that have only goal in common, but different criterions and even different alternatives.

Process goes on …

- After decomposing the problem into a hierarchy, elements at given hierarchy level are compared in pairs to assess their relative preference with respect to each of the elements at the next higher level.
- The verbal terms of the Saaty's scale are used to assess the intensity of preference between two elements. This way facilitated is the weighting of quantifiable and non-quantifiable elements.
- Once the verbal judgments are made, they are translated into numbers by means of the scale.
- The procedure is repeated for elements at each level in downward direction.

Saaty`s 9-point scale

Numerical values	Judgment Definition
1	Equal importance
3	Weak dominance
5	Strong dominance
7	Very strong dominance
9	Absolute dominance
2, 4, 6, 8	Intermediate values

Solution methodology General

- For created hierarchy, an assessment of mutual importance of elements is performed at each level.
- Pair wise comparisons of elements at given level are made with respect to elements of the upper level.
- This procedure is repeated for all levels downwards.
- In synthesis part of the process, matrix manipulations are performed with created judgmental matrices to obtain final performance ratings of alternatives.

Solution Methodology Step-by-Step

Example Problem: Choosing career

- The main areas of concern in a choice of career are payment and job satisfaction.
- Within payment, attributes are current salary and future prospects.
- Within job satisfaction, attributes are level of mental stimulation required by the job, the amount of social interaction the job involves, how stressful the job is and also how secure it is.
- Alternative careers are: accountant, lecturer and politician.

Stage 1: Set up a Decision Hierarchy

Break the decision problem down into a hierarchy of interrelated decision elements.

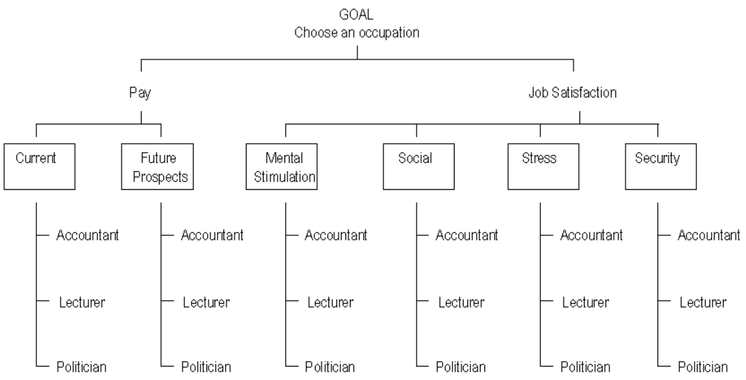


Figure 1 - First Stage: Set up a Decision Hierarchy

Stage 2: Make Pairwise Comparisons of Attributes and Alternatives

- Each attribute is compared in turn with every other one at the same level in the hierarchy (e.g. A with B, A with C, and B with C). Pair wise comparisons should be carried out using verbal responses.
- For example, a person might be asked to consider whether the attributes 'mental stimulation' and 'social interaction' are of equal importance or whether one is more important than the other. If one is more important, is it:

weakly more important	(3)
strongly more important	(5)
very strongly more important	(7)
absolutely (extremely) more important	(9) ?

- The AHP then converts the responses to the numbers shown above. For example if A is 'strongly more important' than B, this is assumed to imply that A is 5 times more important than B.
- Responses in between those are allowed, e.g. 'between strongly and very strongly more important', when pair numbers apply.
- Judgments lead to a matrix of comparisons, for example

	Ment. stim.	Social	Stress	Security
Ment. stim.	1	4	6	4
Social		1	4	3
Stress			1	1/3
Security				1

- The comparisons are expressed by comparing the attribute in the left hand column with attribute in the row above. E.g. mental stimulation is between weakly and strongly (4) more important than social interaction, while stress is only 1/3 as important as job security, i.e. job security is weakly more important than stress.

	Ment. stim.	Social	Stress	Security
Ment. stim.	1	4	6	4
Social		1	4	3
Stress			1	1/3
Security				1

Stage 3: Transform the Matrix of Comparisons into Weights

- Matrices of comparisons are converted into sets of weights. Weights show the relative importance of the attributes at the same level in the hierarchy.
- A number of methods are possible to do that. Saaty recommends a mathematical method based on eigenvalues.
- For the matrix shown above the eigenvalue method produces the following weights:

	Mental stimulation	Social	Stress	Security
Weights	0.572	0.240	0.062	0.127

Stage 4: Aggregate the weights to compare the alternative courses of action

- Decision hierarchy now contains computed weights of attributes and alternatives. The relative attraction of each career is found by identifying all the paths through the tree which end in that career.
- For each path all the weights are multiplied together and the resulting products are summed for all the paths involving the same career. For example, the score for accountancy will be:

$$
\begin{aligned}
& 0.167 \times 0.250 \times 0.649 \\
+ & 0.167 \times 0.750 \times 0.696 \\
+ & 0.833 \times 0.572 \times 0.058 \\
+ & 0.833 \times 0.240 \times 0.072 \\
+ & 0.833 \times 0.062 \times 0.696 \\
+ & 0.833 \times 0.127 \times 0.293 = 0.223
\end{aligned}
$$

- Similarly, for 'lecturer' the score is 0.455, while for 'politician' it is 0.322.

Stage 4: Aggregate the weights to compare the alternative courses of action

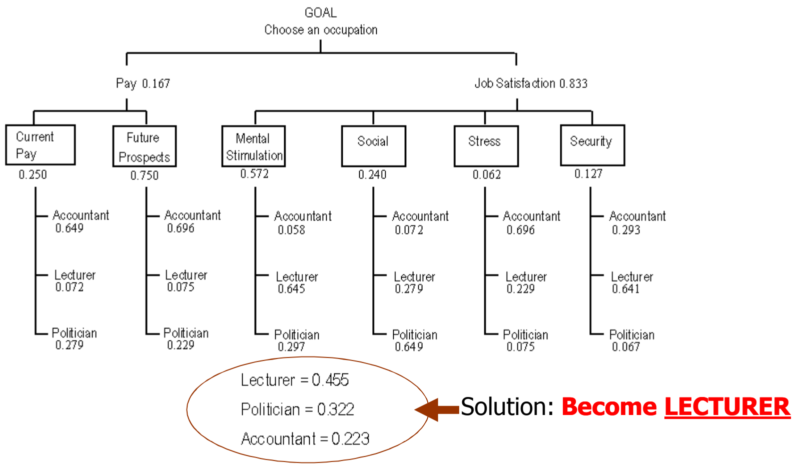


Figure 2 - Fourth Stage: Aggregate weights and Compare alternatives

Stage 5: Make DECISION

The LECTURER should be the career choice if one puts forward the responses which have been analyzed

Mathematical foundation

Assume that hierarchy of the decision problem consists only of a goal (*G*), a set of criterions C_j (j = 1, 2,…, M), and a set of alternatives A_i(i = 1, 2,…,N). This hierarchy may be called 3-level hierarchy, with levels counting from top to bottom.

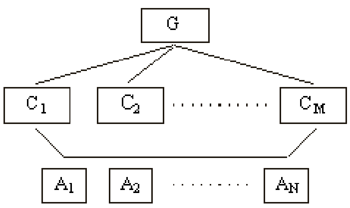


Fig. 4. Hierarchy of criterions and alternatives

The AHP starts by performing a sequence of $M x (M-1)/2$ pairwise comparisons of criterions with respect to a goal. 9-point scale is used. This way a judgment matrix A is created.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1M} \\ a_{21} & a_{22} & \dots & a_{2M} \\ \dots & \dots & \dots & \dots \\ a_{M1} & a_{M2} & \dots & a_{MM} \end{bmatrix}$$

Entries a_{ij} $(i,j=1,2,\dots,M)$ are values from 9-point scale; $a_{ij}=1$ for all $i=j$ $(i,j=1,2,\dots,M)$; $a_{ij}=1/a_{ji}$.

If by assumption the vector $w = (w_1, w_2, \dots, w_M)$ contains weighting coefficients of evaluated elements, the goal is to determine these values so that A matrix is as more as possible approximate of the X matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1M} \\ a_{21} & a_{22} & \dots & a_{2M} \\ \dots & \dots & \dots & \dots \\ a_{M1} & a_{M2} & \dots & a_{MM} \end{bmatrix} \rightarrow X = \begin{bmatrix} w_1/w_1 & w_1/w_2 & \dots & w_1/w_M \\ w_2/w_1 & w_2/w_2 & \dots & w_2/w_M \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ w_M/w_1 & w_M/w_2 & \dots & w_M/w_M \end{bmatrix}$$

Extraction of the vector $w = (w_1, w_2, ..., w_M)$ from pair wise comparison matrix A in a way **to account for inherent inconsistencies**, that is to minimize overall sum given by

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} (w_i - a_{ij} w_j)^2$$

subject to:

$$\sum_{i=1}^{n} w_i = 1$$

is called **Prioritization Method in AHP.**

Generally NOT equal

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1M} \\ a_{21} & a_{22} & \cdots & a_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ a_{M1} & a_{M2} & \cdots & a_{MM} \end{bmatrix} \rightarrow X = \begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_M \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_M \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ w_M/w_1 & w_M/w_2 & \cdots & w_M/w_M \end{bmatrix}$$

Prioritization methods in AHP

- There are 6 most commonly used techniques for deriving priority vector from pair wise comparison matrix A.
- The first two are matrix algebra related:
  - **Additive Normalization Method (AN)**
  - **Eigenvector Method (EV)**
- Remaining four methods are based on optimization:
  - **Weighted Least Squares (WLS)**
  - **Logarithmic Least Squares (LLS)**
  - **Logarithmic Goal Programming (LGP)**
  - **Fuzzy Preference Programming (FPP).**

Additive Normalization (AN)

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & a_{2n} \\ . & & & & . \\ . & & & & . \\ a_{n1} & a_{n2} & . & . & a_{nn} \end{bmatrix}$$

To obtain the priority vector *w*:

1. Divide the elements of each column of matrix *A* by the sum of that column (i.e. normalize the column)
2. Add the elements in each resulting row
3. Divide this sum by the number of elements in the row

$$a'_{ij} = a_{ij} / \sum_{i=1}^{n} a_{ij}\ , \quad i,j = 1, 2, \ldots, n$$

$$w_i = (1/n) \sum_{j=1}^{n} a'_{ij}\ , \ i = 1, 2 \ldots, n$$

- Popular and wide use in practice
- Extremely simple
- Although considered inferior, it may significantly outperform more sophisticated methods

Eigenvector Method (EV)

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & a_{2n} \\ . & & & & . \\ . & & & & . \\ a_{n1} & a_{n2} & . & . & a_{nn} \end{bmatrix}$$

1. The principal eigenvector of *A* is the desired priority vector *w*.
2. The linear system $A\mathbf{w} = \lambda\, \mathbf{w},\ \mathbf{e}^T\mathbf{w}=1$

   is solved where λ is the principal eigenvalue of matrix *A*.
3. If the DM is consistent then $\lambda = n$, otherwise $\lambda > n$.
4. A good estimate of the principal eigenvector for inconsistent matrix is obtainable by consecutive squaring the matrix, normalizing the row sums each time, and stopping the procedure when the difference between normalized sums in two consecutive calculations is smaller than a prescribed value.

- For small deviations around the consistent ratios w_i / w_j, EV method gives reasonably good approximation of the priorities vector.
- However, when the inconsistencies are large, it is generally accepted that solutions are not so satisfactory.
- Used in commercial ExpertChoice products that support AHP method

Weighted Least Squares (WLS)

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & a_{2n} \\ . & & & & . \\ . & & & & . \\ a_{n1} & a_{n2} & . & . & a_{nn} \end{bmatrix}$$

1. The WLS is a modification of the Direct Least Squares Method (DLS).
2. Method minimizes Euclidean (*L*2) distance defined for elements of the unknown priority vector *w* and known judgment ratios $a_{ij} = w_i/w_j$ by solving constrained non-linear optimization problem:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} (w_i - a_{ij} w_j)^2$$

subject to:

$$\sum_{i=1}^{n} w_i = 1$$

- The optimization problem is transformed into a system of linear equations by differentiating the Lagrangian of second equation and equalizing it to zero.
- It is shown that the WLS provides a unique and strictly positive solution ($wi > 0$, $i = 1,2 , \dots , n$).

Logarithmic Least Squares (LLS)

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & a_{2n} \\ . & & & & . \\ . & & & & . \\ a_{n1} & a_{n2} & . & . & a_{nn} \end{bmatrix}$$

1. Method also uses Euclidean (*L*2) metrics in defining objective function of the following optimiization problem:

$$\min \sum_{i=1}^{n} \sum_{j>i}^{n} [\ln a_{ij} - (\ln w_i - \ln w_j)]^2$$

subject to:

$$\prod_{i=1}^{n} w_i = 1, \quad w_i > 0, \ i = 1, 2, \dots, n.$$

2. It was shown that the solution for the problem is unique and can be found simply as the geometric means of the rows of matrix *A*:

$$w_i = \prod_{j=1}^{n} a_{ij}^{1/n}, \qquad i=1,2,\dots,n.$$

- Simple and easy to use
- Competitive results

Logarithmic Goal Programming (LGP)

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & a_{2n} \\ . & & & & . \\ . & & & & . \\ a_{n1} & a_{n2} & . & . & a_{nn} \end{bmatrix}$$

Priorities are desired to satisfy the equalities

a_{ij} - (w_i/w_j) $(\delta_{ij}^{+} / \delta_{ij}^{-}) = 0$, $i,j = 1,2, ..., n, j>i$

where $\delta_{ij}^{+} \geq 1$ and $\delta_{ij}^{-} \geq 1$ are deviation variables, which cannot be greater than 1 in the same priorities w_i, $i = 1, 2, ..., n$ are obtained as solutions of the following linear goal programming problem:

$$\min \sum_{i=1}^{n} \sum_{j>i}^{n} (\ln \delta_{ij}^{+} + \ln \delta_{ij}^{-})$$

subject to

$\ln w_i - \ln w_j + \ln \delta_{ij}^{+} - \ln \delta_{ij}^{-} = \ln a_{ij}$, $i, j = 1,2, ..., n, \; j > i$

The method generally produces non-satisfactory priority vector.

where all $\ln \delta_{ij}^{+}$ and $\ln \delta_{ij}^{-}$ are non-negative.

Fuzzy Preference Programming (FPP)

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & a_{2n} \\ . & & & & . \\ . & & & & . \\ a_{n1} & a_{n2} & . & . & a_{nn} \end{bmatrix}$$

The FPP firstly states that if reciprocal matrix A is consistent, then $a_{ij} w_j - w_i = 0$ for all $i,j = 1, 2, ..., n, \; j > i$, which can be represented as a system of $m = n(n-1)/2$ linear equations:

$R\mathbf{w}=0.$

If A is inconsistent, it is desirable to find such values of $\mathbf{w}$, so that $a_{ij} w_j - w_i = 0$ is approximately satisfied, i.e. $R\mathbf{w} \approx 0$.

In the FPP, $R\mathbf{w} \approx 0$ is represented geometrically as an intersection of fuzzy hyper lines, and the prioritization problem is transformed to optimization one, determining the values of the priorities that correspond to the point with the highest measure of intersection.

Fuzzy Preference Programming (FPP)

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & a_{2n} \\ . & & & & . \\ . & & & & . \\ a_{n1} & a_{n2} & . & . & a_{nn} \end{bmatrix}$$

This way the prioritization problem is reduced to a fuzzy programming problem that can easily be solved as a standard linear program:

maximize μ

subject to:

$$\mu \, d_j^{+} + R_j w \leq d_j^{+}$$

- The measure of intersection μ is considered as natural consistency index of the FPP.
- Its value however depends on the tolerance parameters and it was also suggested as reasonable that in practical implementations all these parameters should be set equal.
- Method is not so easy to use in practice.

Described prioritization methods can be combined to minimize total Euclidean distance for non-consistent matrices !

This is what is called Combined AHP Synthesis.

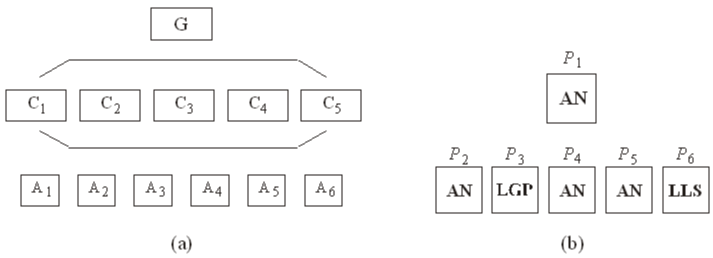
(after Srdjevic B. in Comp. & Op. Res. 32 (2005) 1897-1919, Elsevier)

Combined AHP Synthesis

AN EXAMPLE: ALLOCATING THE SURFACE WATER RESERVOIR STORAGE TO MULTIPLE USES

- **A global economical goal is defined as to find the most profitable use of reservoir**
- **6 purposes are considered as decision alternatives:**
  - **(A1) electric power generation**
  - **(A2) irrigation**
  - **(A3) flood protection**
  - **(A4) water supply**
  - **(A5) tourism and recreation**
  - **(A6) river traffic**
- **Alternatives are evaluated across 5 economical criteria of different metrics:**
  - **(C1) gain in national income**
  - **(C2) earning foreign exchange**
  - **(C3) improvement of the balance of payment**
  - **(C4) import substitution (self-sufficiency)**
  - **(C5) gain in regional income**

All described methods were applied to select their best combination for the AHP final synthesis.



Hierarchy and prioritization methods selected for AHP synthesis

Priority vectors for criteria

Criteria	Priority vectors					
	AN	EV	WLS	LLS	FPP	LGP
C1	0.352	0.358	0.411	0.356	0.391	0.356
C2	0.300	0.306	0.291	0.313	0.283	0.356
C3	0.043	0.041	0.047	0.042	0.065	0.051
C4	0.172	0.171	0.140	0.166	0.152	0.119
C5	0.133	0.123	0.111	0.122	0.109	0.119

Priority vectors for alternatives

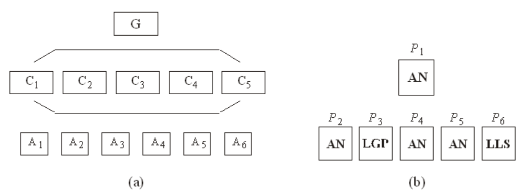
Cr./Al.	AN							EV					
	A1	A2	A3	A4	A5	A6		A1	A2	A3	A4	A5	A6
C1	0.432	0.082	0.281	0.066	0.081	0.057		0.440	0.077	0.286	0.065	0.078	0.054
C2	0.364	0.108	0.079	0.040	0.206	0.203		0.365	0.110	0.078	0.040	0.211	0.197
C3	0.403	0.178	0.034	0.107	0.159	0.119		0.409	0.180	0.032	0.106	0.153	0.120
C4	0.413	0.151	0.036	0.043	0.129	0.228		0.416	0.150	0.036	0.040	0.125	0.233
C5	0.048	0.223	0.192	0.325	0.166	0.045		0.046	0.209	0.187	0.352	0.162	0.043
	WLS							LLS					
C1	0.498	0.060	0.233	0.057	0.075	0.077		0.448	0.073	0.282	0.063	0.079	0.056
C2	0.394	0.099	0.056	0.043	0.205	0.203		0.384	0.108	0.071	0.041	0.204	0.192
C3	0.469	0.160	0.040	0.087	0.132	0.112		0.432	0.167	0.034	0.106	0.154	0.108
C4	0.464	0.131	0.046	0.039	0.117	0.202		0.423	0.147	0.036	0.039	0.125	0.231
C5	0.057	0.126	0.220	0.369	0.177	0.051		0.053	0.212	0.193	0.326	0.167	0.049

	FPP							LGP					
C1	0.453	0.121	0.201	0.050	0.086	0.088		0.504	0.101	0.216	0.054	0.072	0.054
C2	0.389	0.117	0.078	0.067	0.194	0.156		0.391	0.098	0.065	0.056	0.195	0.195
C3	0.442	0.184	0.063	0.095	0.104	0.112		0.462	0.154	0.038	0.077	0.154	0.115
C4	0.423	0.165	0.059	0.045	0.132	0.176		0.461	0.154	0.051	0.066	0.115	0.154
C5	0.170	0.188	0.160	0.279	0.146	0.056		0.056	0.337	0.169	0.236	0.169	0.034

Selecting vectors derived by different methods for the Final AHP Synthesis

Method	P_1		P_2		P_3	
	ED	MV	ED	MV	ED	MV
AN	4.583	0.00	6.209	0.06	5.305	0.08
EV	4.961	0.00	6.255	0.08	5.359	0.08
V						0.08
L						0.08
L						0.08
L						0.06
						LGP

M						
						MV
						0.08
						0.08
V						0.14
L						0.08
FPP	7.318	0.08	6.634	0.03	8.740	0.14
LGP	7.005	0.08	7.643	0.03	8.162	0.14
	Selected: AN		Selected: AN		Selected: LLS	



Hierarchy and prioritization methods selected for AHP synthesis

(Selected methods for $\alpha_{ED} = 0.8$ and $\alpha_{MV} = 0.2$)

Final priority vectors for alternatives derived by standard AHP synthesis for 6 prioritization methods and by Synthesis

Alternatives		Prioritization methods							Rank
		AN	EV	WLS	LLS	FPP	LGP	Synthesis	
A1		0.356	0.363	0.412	0.375	0.399	0.403	0.365	1
A2		0.125	0.120	0.093	0.117	0.138	0.137	0.120	5
A3		0.156	0.157	0.145	0.154	0.131	0.128	0.152	2
A4		0.091	0.090	0.087	0.086	0.082	0.079	0.095	6
A5		0.142	0.140	0.133	0.140	0.131	0.137	0.138	3
A6		0.131	0.130	0.130	0.129	0.119	0.117	0.129	4
Total	ED	33.785	35.463	37.148	34.816	43.069	41.244	33.945	
	MV	0.05	0.05	0.08	0.05	0.07	0.07	0.05	

Performance of prioritization methods with regard to different deviation measures

Measure	Performance 'quality'
Euclidean distance (ED)	AN ≻ LLS ≻ EV ≻ WLS ≻ LGP ≻ FPP
Minimum violation (MV)	AN = EV = LLS ≻ FPP = LGP ≻ WLS
Conformity (C)	EV ≻ LLS ≻ AN ≻ WLS ≻ FPP ≻ LGP

- **Criteria ranking**

(C1) gain in national income (0.352)
(C2) earning foreign exchange (0.300)
(C4) import substitution (self-sufficiency) (0.172)
(C5) gain in regional income (0.133)
(C3) improvement of the balance of payment (0.043)

								Synthesis	Rank
								0.365	1
								0.120	5
A3		0.156	0.157	0.145	0.154	0.131	0.128	0.152	2
A4		0.091	0.090	0.087	0.086	0.082	0.079	0.095	6
A5		0.142	0.140	0.133	0.140	0.131	0.137	0.138	3
A6		0.131	0.130	0.130	0.129	0.119	0.117	0.129	4
Total	ED	33.785	35.463	37.148	34.816	43.069	41.244	33.945	
	MV	0.05	0.05	0.08	0.05	0.07	0.07	0.05	

- **Reservoir uses ranking**

(A1) electric power generation (0.365)
(A3) flood protection (0.152)
(A5) tourism and recreation (0.138)
(A6) river traffic (0.129)
(A2) irrigation (0.120)
(A4) water supply (0.095)

THE FINAL RESULT

Brief repetition of the AHP methodology

- Firstly hierarchy is created.
- Then, *M* criteria (level 2 of hierarchy) is compared in pairs with respect to a goal (level 1 of hierarchy) to create *MxM* comparison matrix and derive it`s eigenvector. This vector contains approximates of the weights of criterions with respect to a goal.
- Then, *N* alternatives at the level 3 are compared in pairs with respect to each criterion at level 2. A set of *M* comparison matices of size *NxN* is elicited.
- By applying the same method on each matrix (but now with *N* instead of *M*), a set of *M* 'partial' eigenvectors is obtained.
- Finally, synthesis is performed by multiplying each partial eigenvector by the weight of respective criterion and by summing obtained vectors.
- The result is the vector containing the final weights of alternatives. The weghts sum to 1. The highest value is associated with the ´best´, and the lowest with the ´worst´ alternative.
- This way AHP is concluded.

Strengths of the AHP

- Helps decision maker to cope with a problem complexity by decomposing problem into a hierarchical structure.
- Only two elements are compared at the time. Both qualitative and quantitative elements are compared with ease.
- Verbal terms, numeric scale or graphic bars may be used to express the intensity of preference of one element over the other (interactive seanse at computer).
- Calculates inconsistency index as a ratio of the decision maker's inconsistency and randomly generated index.
- Its simplicity and intuitive logic facilitate the participation of various decision makers and even stimulate their involvement in brainstorming sessions which ultimately may improve collective thinking, reasoning, and the efficiency of group decision.

Criticisms of the AHP

- If verbal judgements are used, then a quantitative scale is imposed on the decision maker; e.g. one may think that A is weakly more important than B. The AHP assumes that this implies that A is 3 times more important than B.
- The method for obtaining weights (there are many) will not always be transparent to most decision makers.
- The failure to distinguish options and attributes reduces clarity.
- The addition of a new option to a decision problem can lead to a reversal of the rankings of the original options.
- The number of comparisons can make the method extremely time consuming; e.g. 5 options compared with respect to 5 attributes would need 60 pairwise comparisons.

Applications

Analytic Hierarchy Process has been used in almost all areas of practice and

- **IBM Benchmarking for Baldridge Award**
- **IBM A/S 400 Configuration**
- **Boeing Supercomputer Selection**
- **Digital Outsourcing Vendor Selection**
- **U.S. Army Base Closings**
- **U.S. Navy CVX Platform**
- **Israeli Air Force Fighter Selection**
- **THOUSANDS of other examples**

More applications

- Environmental Impact Evaluations (EIE)
- Decisions in Offshore Development Projects
- Measuring and Interpreting Information
- Budget Allocation at Woods Hole Fisheries
- Selecting Working Fluids - An Engineering Application
- Managing National Park Service Resources
- Formulating Policies for the Sea of Japan
- Internal Control Judgments by EDP Auditors
- Multicriteria Decision Making with SAS Software
- Application of AHP to Quality Management
- Budget Allocation
- Ranking Architecture Alternatives for the FAA
- Using the AHP to Develop and Disseminate Medical Guidelines
- Optimizing Quality Costs Through Expert Choice
- Selection of Water-Supply Projects Under Drought
- Investment Analysis Application
- New Bridge to Environmental Application
- Downsizing Military Facilities
- Patient versus Physician Preferences
- Selection of Flood Control Projects
- Management Reorganization
- Productivity in Software Development
- Tactical R&D Project Prioritization
- Assessing the Risks of an FAA Emergency Communications
- Accounting Research using the AHP
- Strategic Planning in the Military

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