Fundamentals of Synchrotron Radiation and Free Electron Lasers

The birthplace: Ed Rowe and his Tantalus ring in Wisconsin, 1968

Giorgio Margaritondo
Ecole Polytechnique Fédérale de Lausanne (EPFL)

Outline:

- Building an excellent x-ray source :
  - 3.5 minute explanation
  - 9.5 minute explanation
- Essential details of synchrotron light
- Coherence
- Free electron lasers: the basic mechanism
- X-ray Free electron lasers: subtle points

This is how this presentation started:

$$P(\nu) = \frac{\sqrt{3}e^3 B \sin\alpha}{mc^2}\left(\frac{\nu}{\nu_c}\right)\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)d\eta$$

$$\nu_c = \frac{3}{2}\gamma^2 \nu_G \sin\alpha$$

Must synchrotron sources be so formal and complicated?

J. Synchrotron Rad. (1995). **2**, 148–154

A Primer in Synchrotron Radiation: Everything You Wanted to Know about SEX (Synchrotron Emission of X-rays) but Were Afraid to Ask

G. Margaritondo

NO!!!
What matters is the underlying physics

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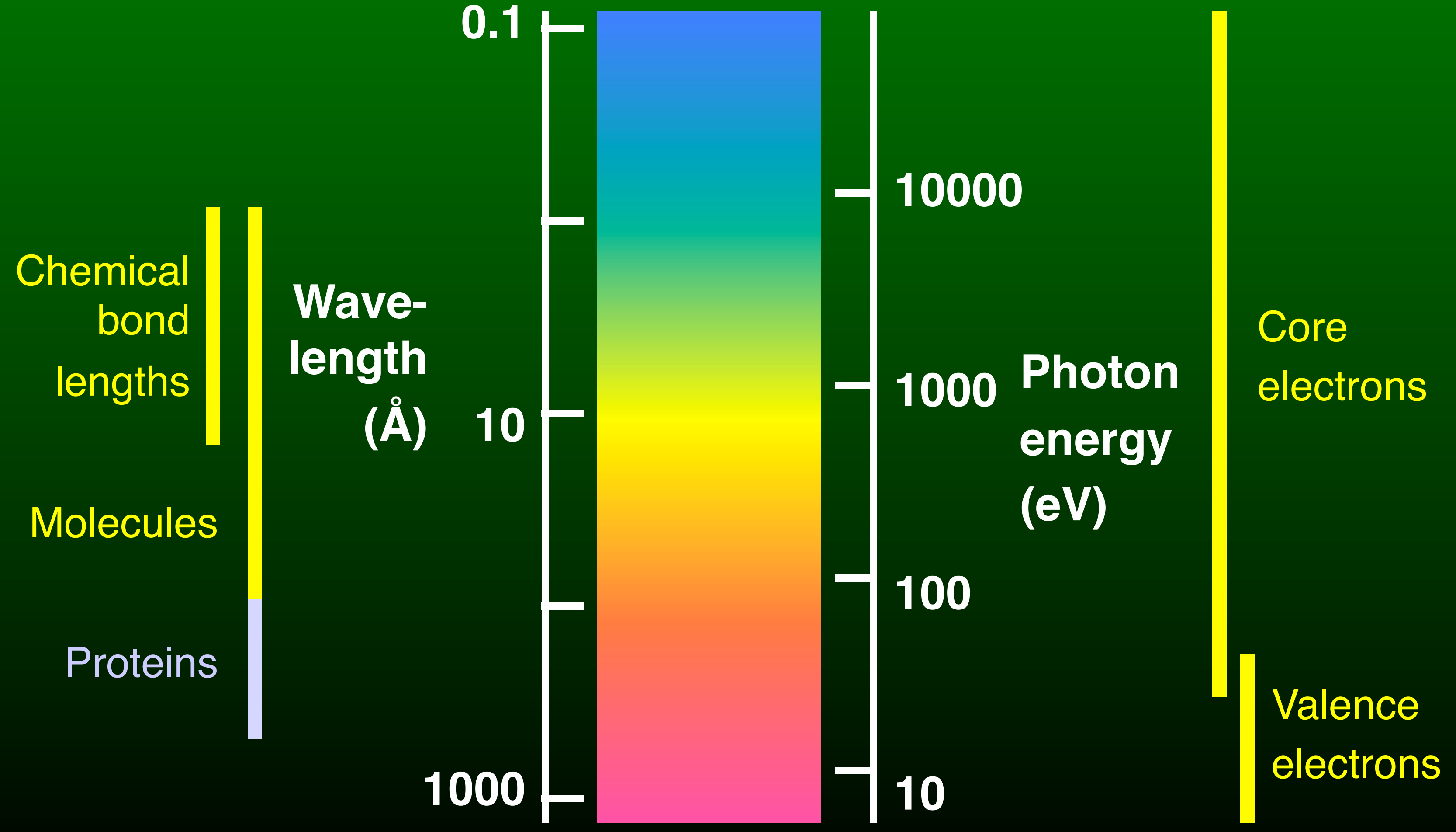
A simplified description of X-ray free-electron lasers

G. Margaritondo[a,*] and Primoz Rebernik Ribic[a]

Why x-rays and ultraviolet light?

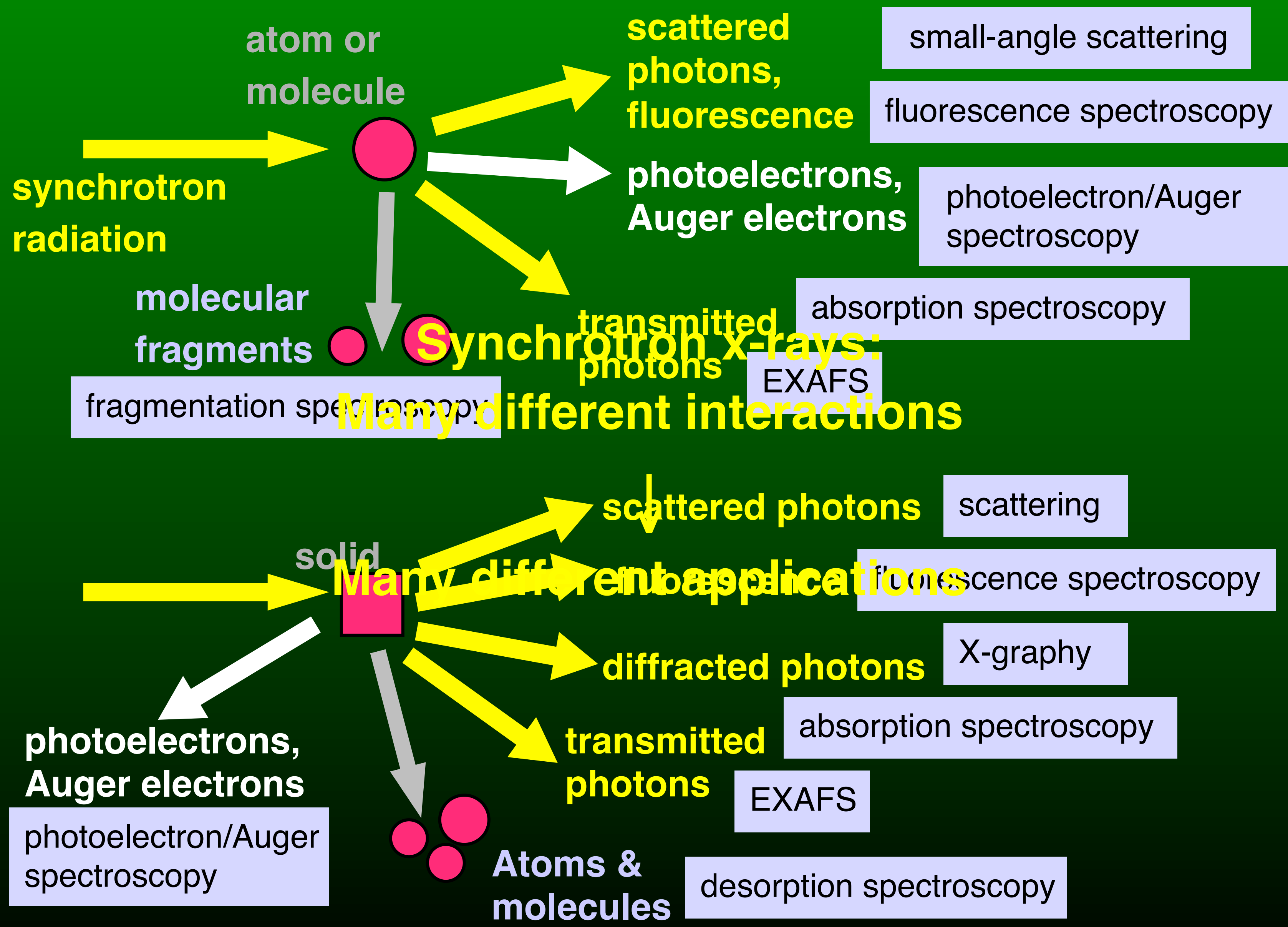
To study something, it is better to use a probe with similar magnitude (size and energy)



Synchrotron x-rays: Many different interactions

↓

Many different applications



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WE NEED X-RAYS AND SYNCHROTRONS GIVE THEM TO US: BUT HOW DO THEY WORK?

OUR "RELAXED APPROACH" TO UNDERSTANDING:

START! STEP A (3.5 minutes): why do synchrotrons emit x-rays?

OPTIONS: (1) relax for the day, or (2) go to step B

STEP B (9.5 minutes): Why synchrotron light is narrow like a laser? And, again, why do synchrotrons emit x-rays?

OPTIONS: (1) relax for the day, or (2) go to step C

STEP C (the rest of the time... maybe more): (almost) everything about synchrotrons and FELs

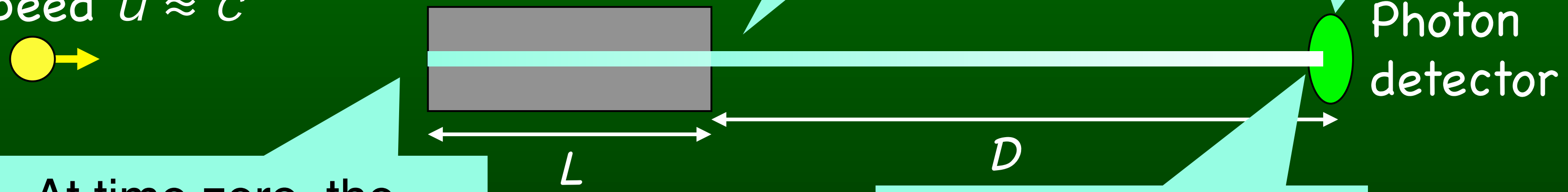
Synchrotron light in 3.5 minutes for lazy students (and teachers):

Electron: Speed $u \approx c$

Magnet: Lorentz force $\Rightarrow$ acceleration $\Rightarrow$ photon emission

At the time L/u, the electron leaves the magnet

The last photons arrive at the time $L/u + D/c$



At time zero, the electron enters the magnet, is accelerated and emits photons

The first photons arrive at the detector at the time $(L + D)/c$

Photon pulse duration: $\Delta t = L/u + D/c - (L/c + D/c) = (L/u)\,(1-u/c)$

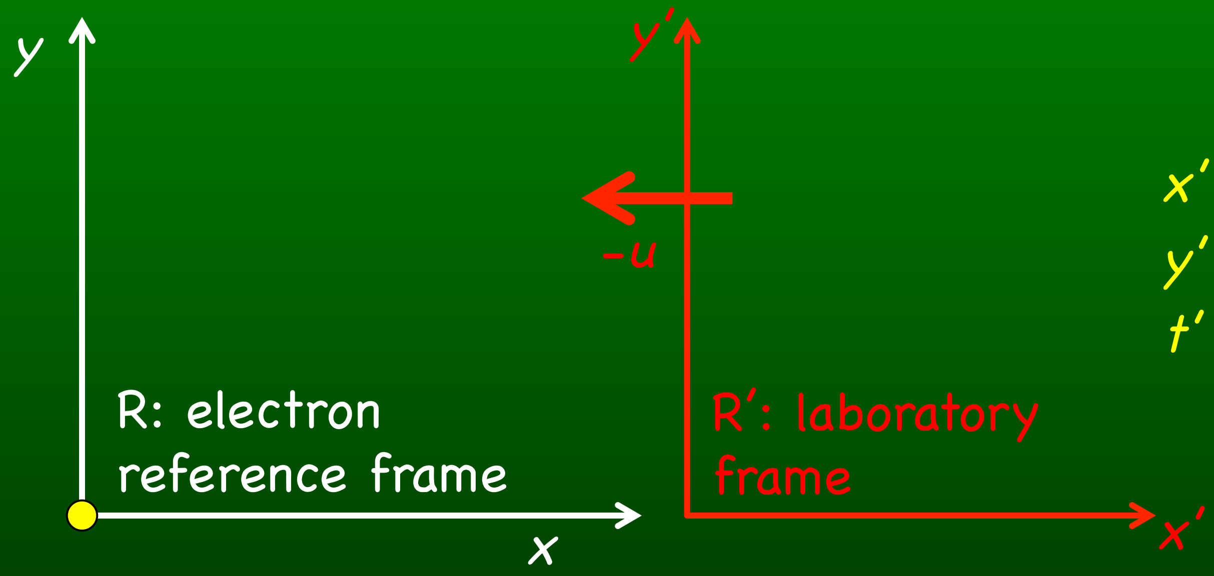
Characteristic frequency: $\nu = 1/\Delta t = u/[L(1-u/c)] = u\,\gamma^2\,(1+u/c)/L$

$\gamma^2 = 1/(1-u^2/c^2)$

For $u \approx c$, $(1+u/c) \approx 2$ and $\nu \approx 2c\gamma^2/L$

For L = 0.1 m and γ = 4000, $\nu \approx 10^{17}$ s^{-1} -- x-rays!

For the next step, you only need a bit of relativity:



$x' = \gamma (x + ut)$

$y' = y$

$t' = \gamma (t + ux/c^2)$

$v_x' = (v_x + u)/(1 + v_x u/c^2)$

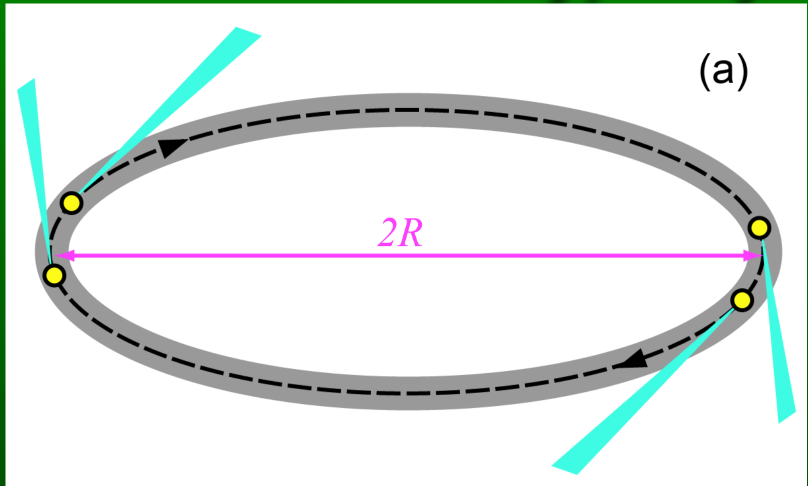
$v_y' = (v_y/\gamma)/(1 + v_x u/c^2)$

Lorentz contraction:

$L' = x_2' - x_1' = \gamma (x_2 - x_1) = \gamma L$

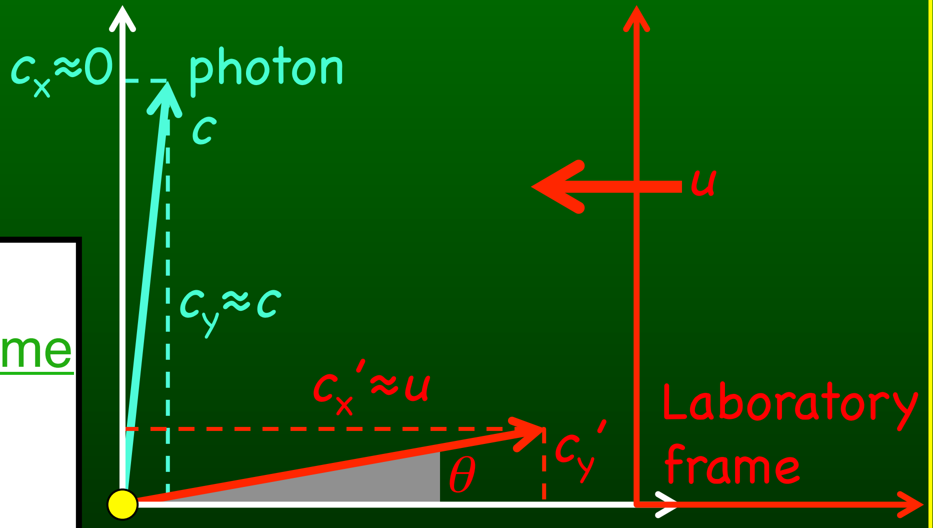
$L' = L/\gamma$

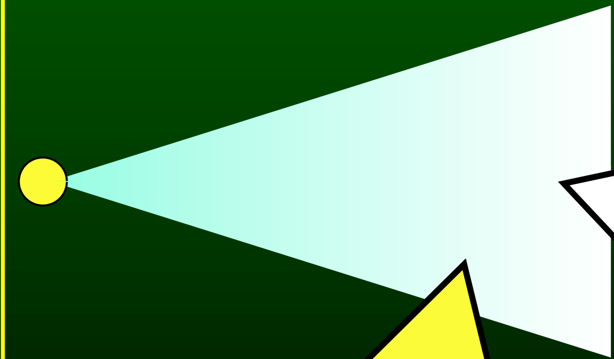
Synchrotron light in 9.5 minutes for (not entirely) lazy students (and teachers):



Electrons circulating at a speed $u \approx c$ in a storage ring emit photons in a narrow angular cone, <u>like a "flashlight"</u>: why?

Answer: RELATIVITY





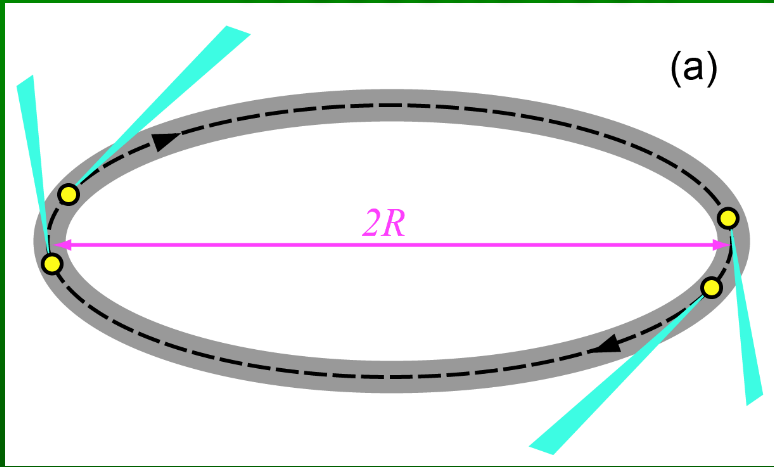
But in the <u>laboratory frame</u> the emission shrinks to a narrow cone

Seen in the <u>electron reference frame</u>, the photon are emitted in a wide angular range

Take a photon emitted in a near-transverse direction in the electron frame. In the (green) laboratory frame its velocity components become $c_x' \approx u$ and c_y'. But c, the speed of light, cannot change, so $c_y' \approx (c^2 - u^2)^{1/2} = c\,(1 - u^2/c^2)^{1/2} = c/\gamma$.

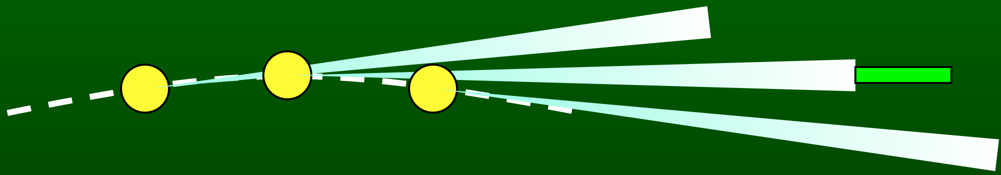
The angle θ' is $\approx c_y'/c = 1/\gamma$ -- narrow!!!

A second look -- the emission is x-rays: why?

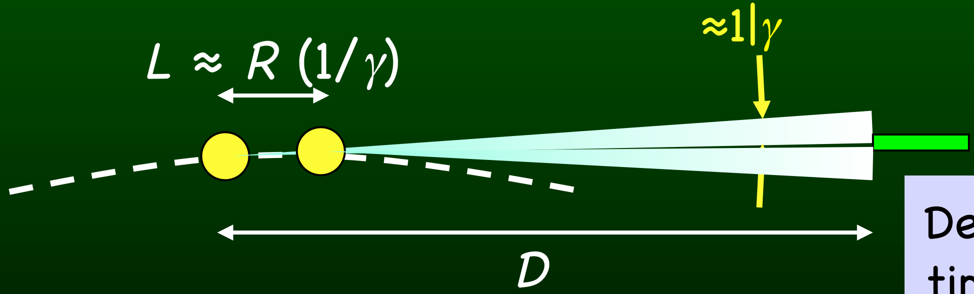


Seen from the side of the ring, each electron looks like an oscillating charge in an antenna, emitting photons with a frequency $2\pi R/c$ -- in the radio wave range.

What shifts the emission to the x-ray range? RELATIVITY AGAIN!



A torchlight-electron illuminates a small-area detector once per turn around the ring for a short time Δt



Photons start to be detected at the time D/c

Detection ends at the time $L/u + (D - L)/c$

Photon pulse duration:

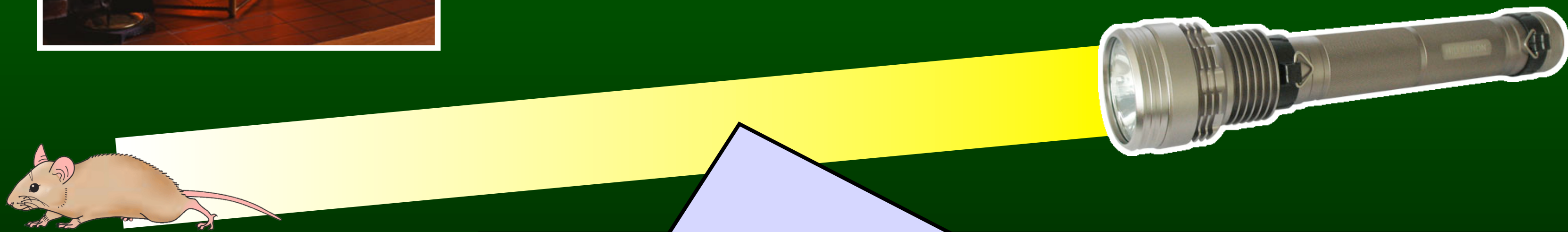
$\Delta t = L/u + (D - L)/c - D/c = L/u - L/c = (L/u)\,(1-u/c) = (L/u)\gamma^2/(1+u/c)$

For $u \approx c$, $(1+u/c) \approx 2$ and $\Delta t \approx L/(2c\gamma^2) \approx R/(2c\gamma^3)$.

Characteristic frequency $\nu = 1/\Delta t \approx 2c\gamma^3/R$ -- again, x-rays

So, synchrotrons emit x-rays: but why is this interesting? Consider fireplaces and torchlights:

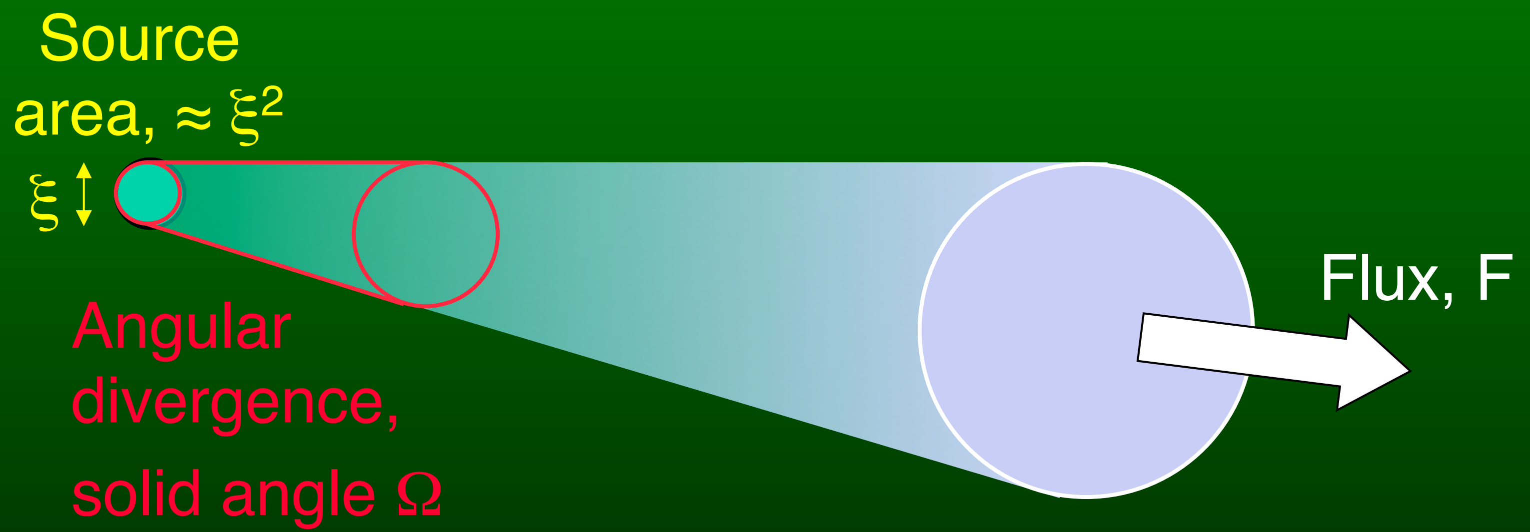
A fireplace is not very effective in "illuminating" a specific target: its emitted power is spread in all directions



A torchlight is much more effective: it is a small-size source with emission concentrated within a narrow angular spread

This can be expressed using the “brightness”

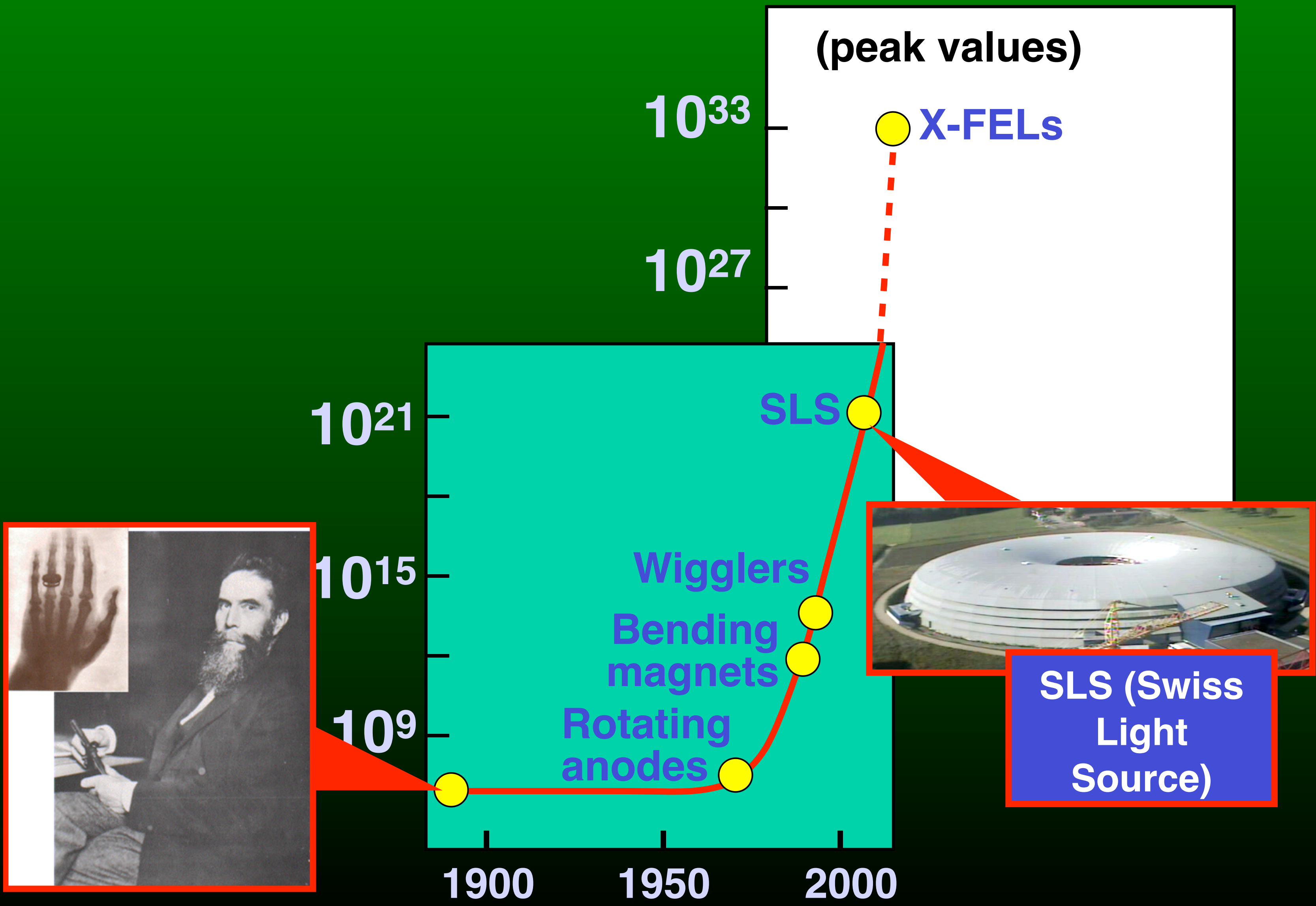
The "brightness" (or brilliance) of a source of light :



$$\text{Brightness} = \text{constant}\ \frac{F}{\xi^2\ \Omega}$$

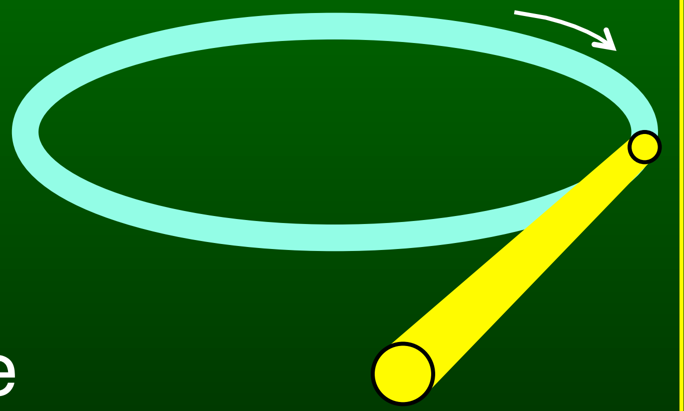
Historical growth in X-ray brightness

(units: photons/mm2/s/mrad2, 0.1% bandwidth)



What allows synchrotron light to be so bright? Three factors:

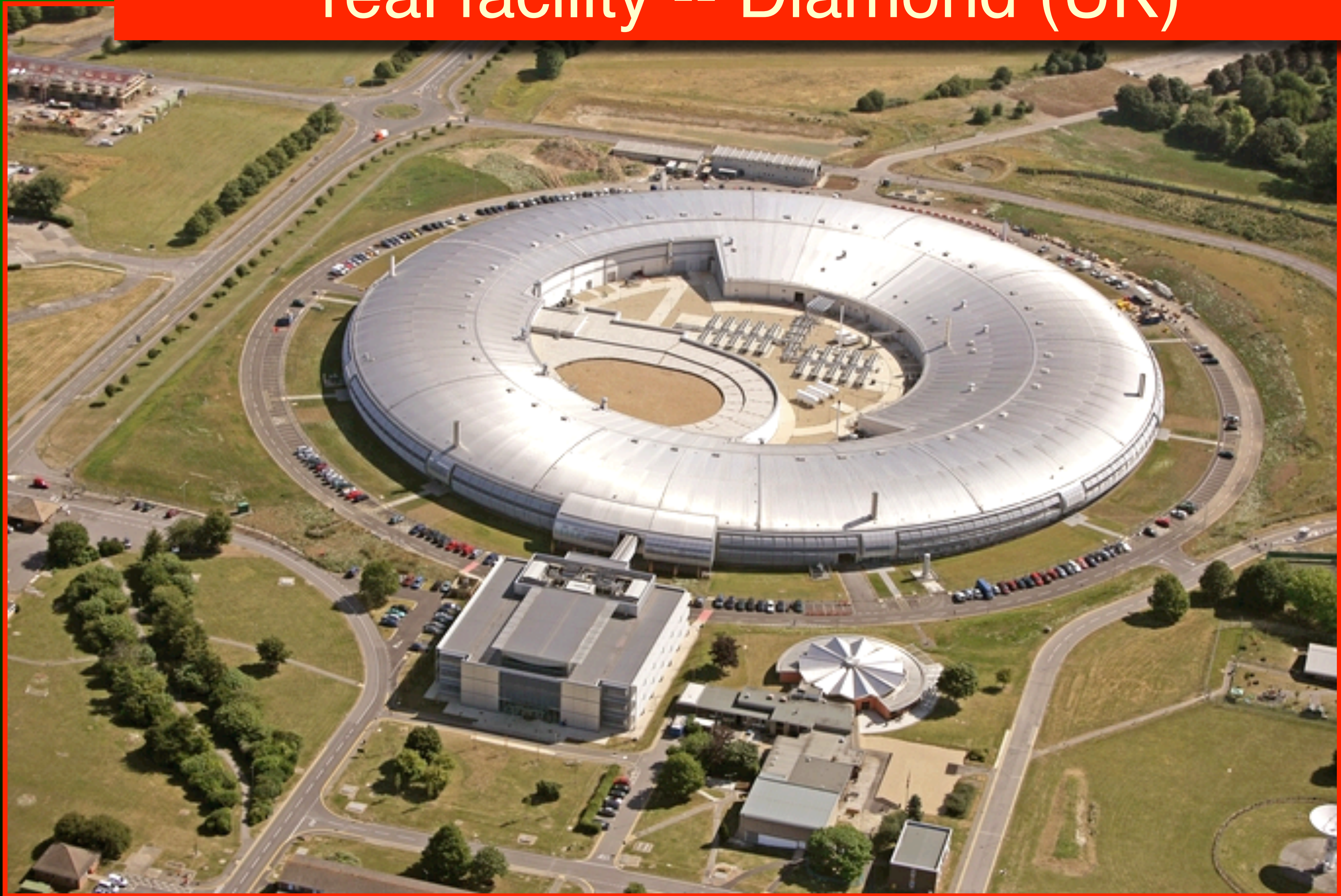
1. Electrons in vacuum can emit more power than electrons in a solid because the power does not damage their environment ⇒ high flux



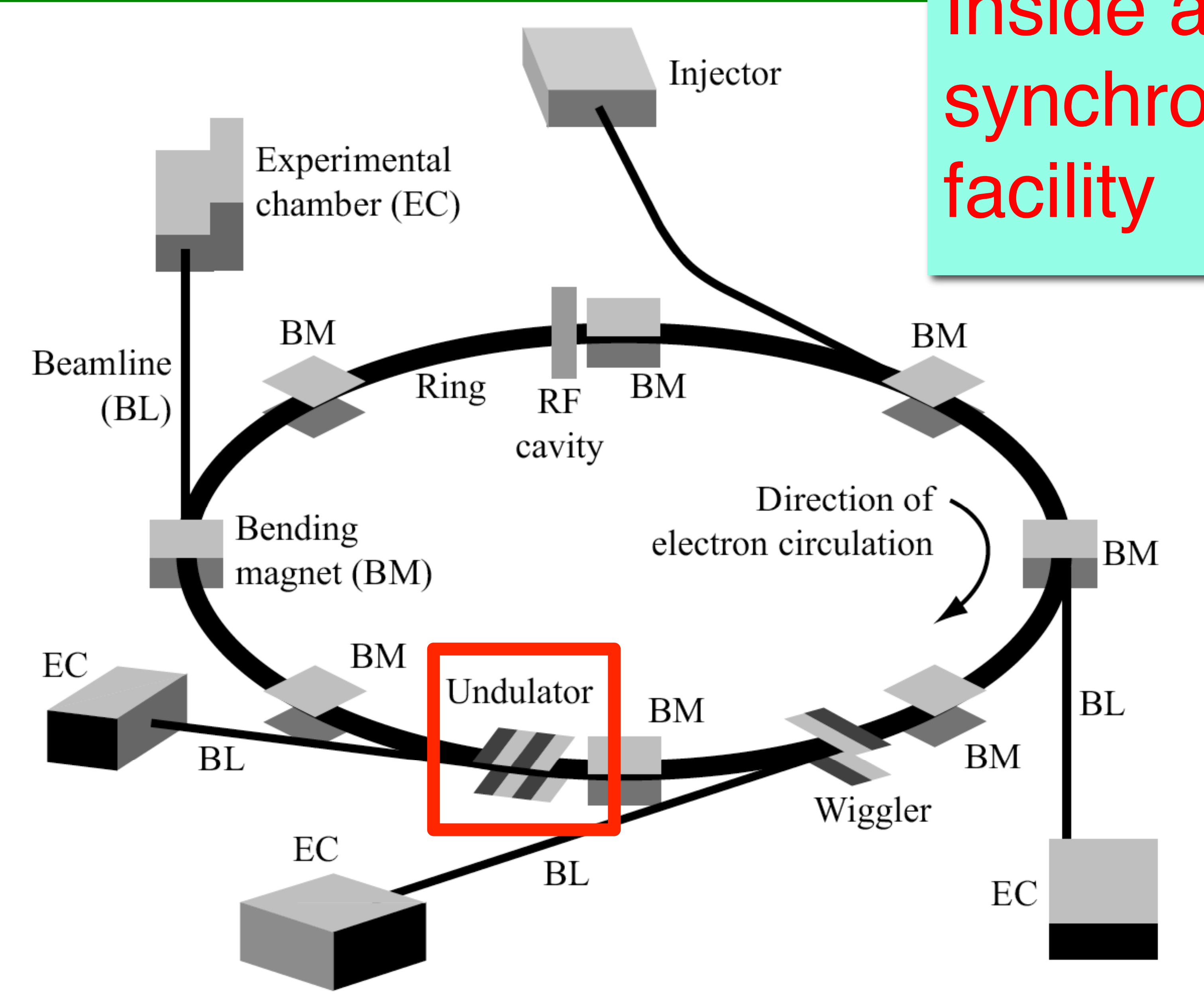
2. The source size is not a single electron but the transverse cross section of the electron beam. The sophisticated electron beam controls make it very small

3. Relativity drastically reduces the angular divergence of the emission

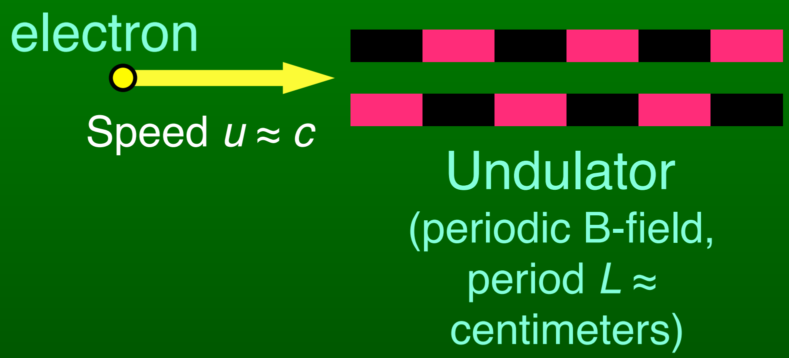
Let's discover synchrotron light: this is a real facility -- Diamond (UK)



Inside a synchrotron facility

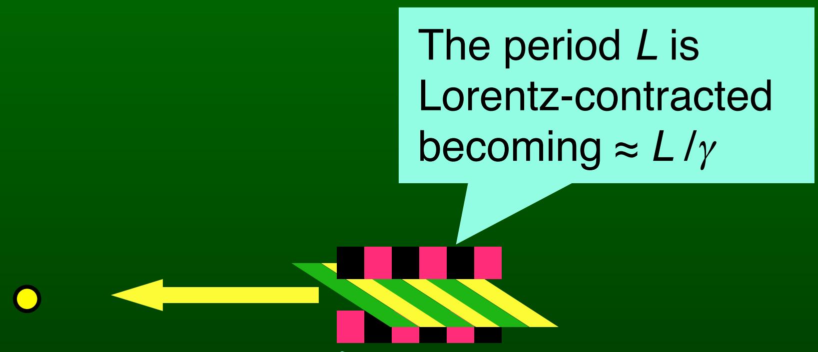


Objective: building a very bright x-ray source using an "undulator" and relativity



In the undulator (laboratory) frame, the electron moves at speed ≈c

In the electron frame:

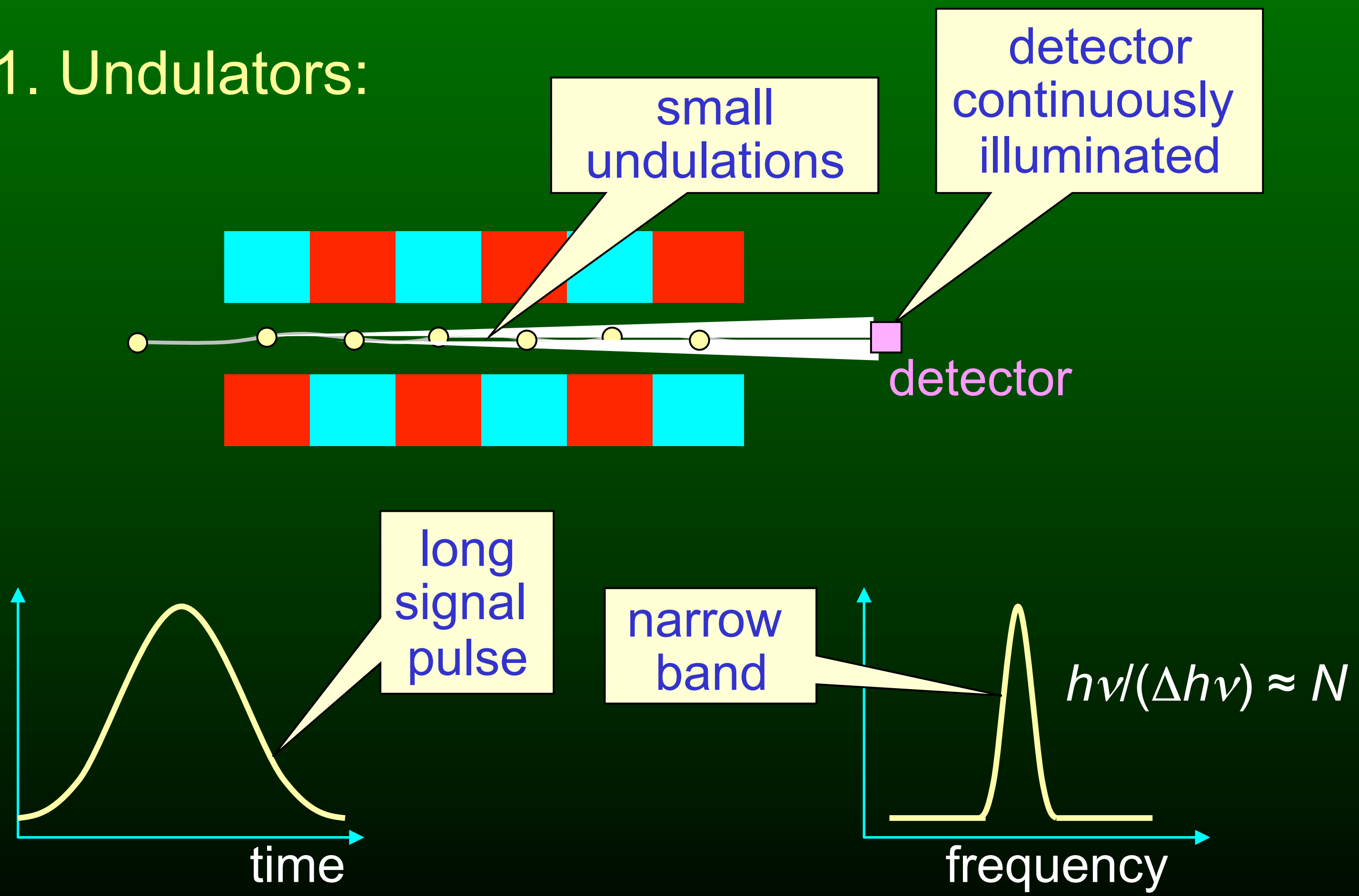


The periodic B-field is accompanied by a perpendicular periodic E-field. Moving at a speed ≈c towards the electron, the undulator looks like an electromagnetic wave with wavelength L/γ. Synchrotron radiation is produced by the elastic scattering of this wave by the electron.

Back to the laboratory frame, the wavelength L/γ emitted by the moving electron is Doppler-shifted by a factor $\approx 2\gamma$, becoming $L/(2\gamma^2)$. The "macroscopic" undulator period is transformed into x-ray wavelengths!

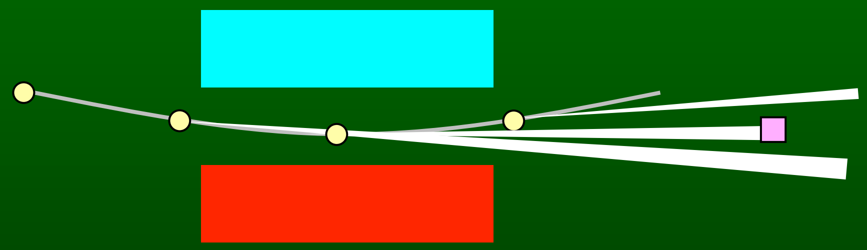
Undulators are one of the 3 types of synchrotron light sources:

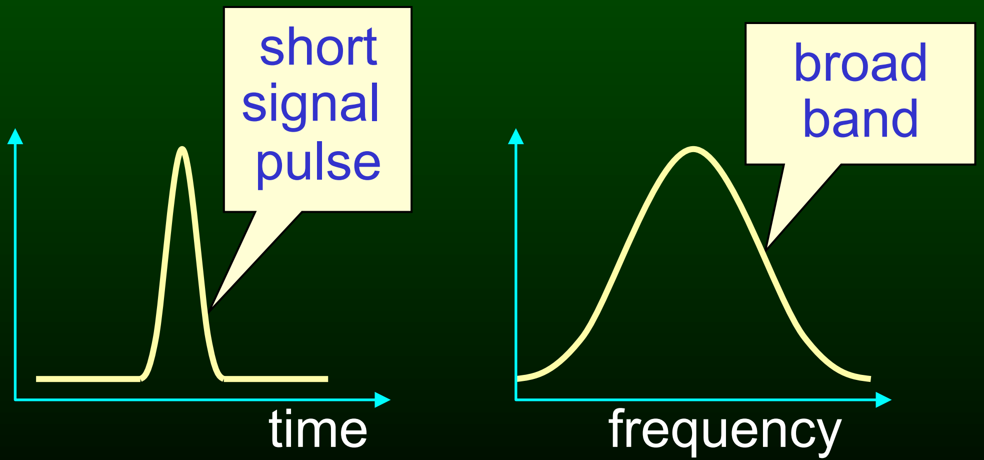
1. Undulators:



Three types of sources:

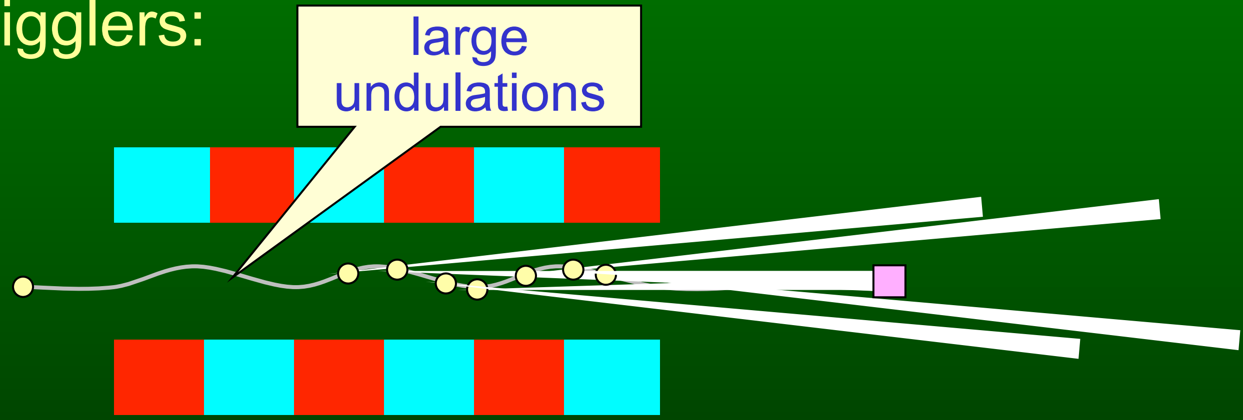
2. Bending magnets:

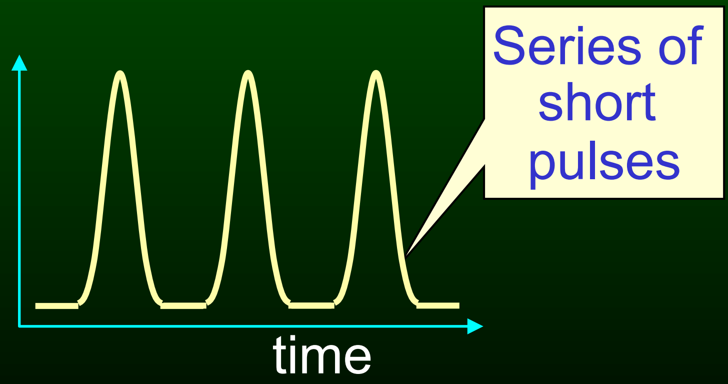


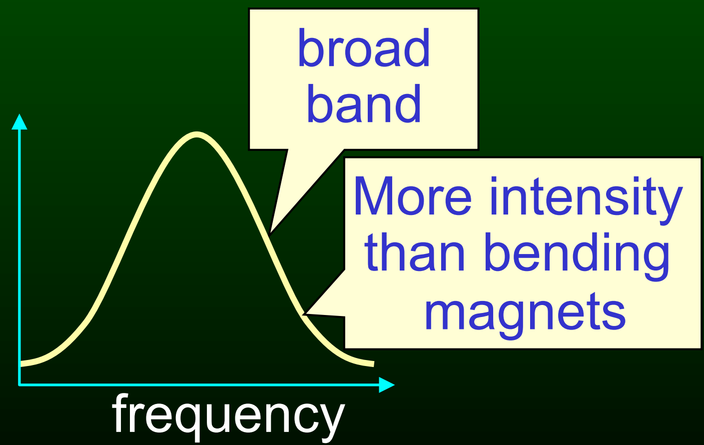


Three types of sources:

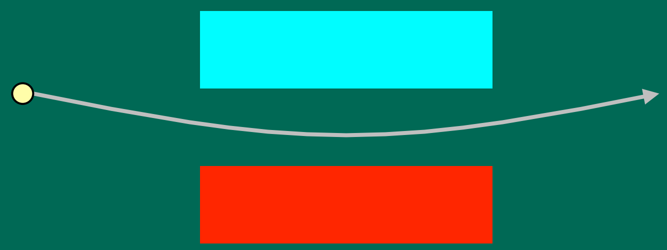
3. Wigglers:







Bending magnet emission spectrum:

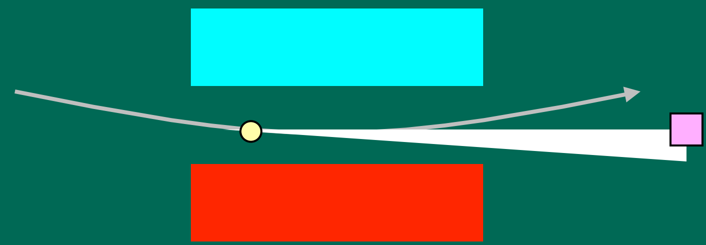


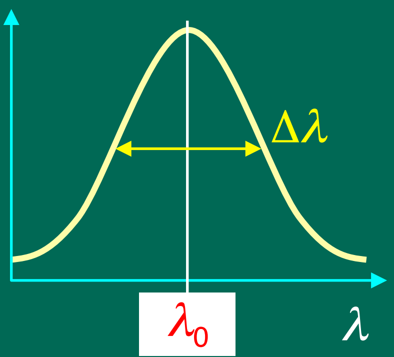
The (relativistic) rotation frequency of the electron determines the (Doppler-shifted) central wavelength:

$\lambda_o = (1/2\gamma^2)(2\pi c m_o/e)(1/B)$

The "sweep time" δt of the emitted light cone determines the frequency spread $\delta\nu$ and the wavelength bandwidth:

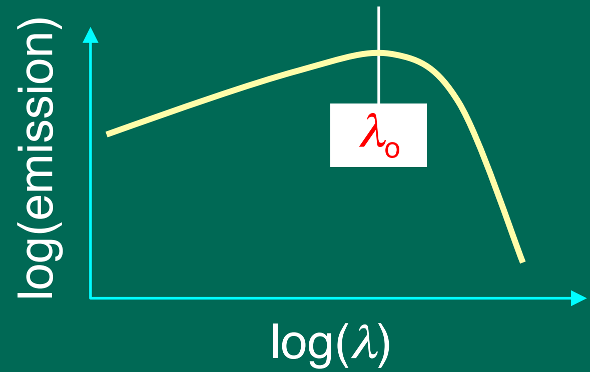
$\Delta\lambda / \lambda_o = 1$



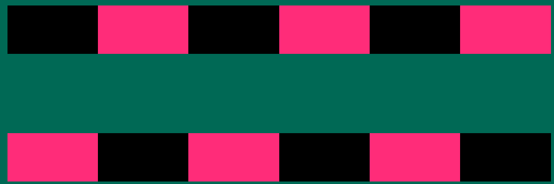


A peak centered at λ_c with width $\Delta\lambda$: is this really the well-known synchrotron spectrum?

YES -- see the log-log plot:



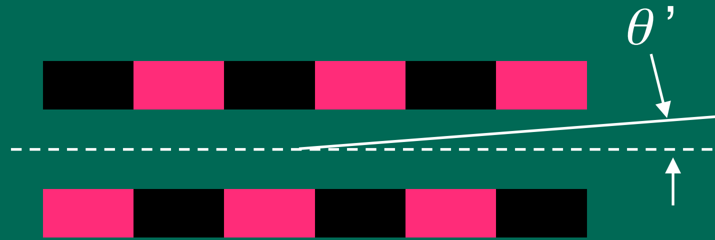
Undulator emission spectrum:



L = period

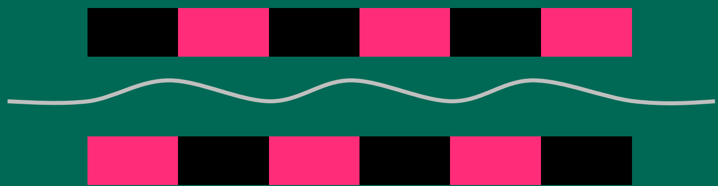
Central wavelength: $L/(2\gamma^2)$

First correction: out of axis, the Doppler factor is not $2\gamma^2$ but $2\gamma^2(1+2\gamma^2\theta'^2)$



Central wavelength: $(L/2\gamma^2)/(1+2\gamma^2\theta'^2)$

(changes with θ' !)

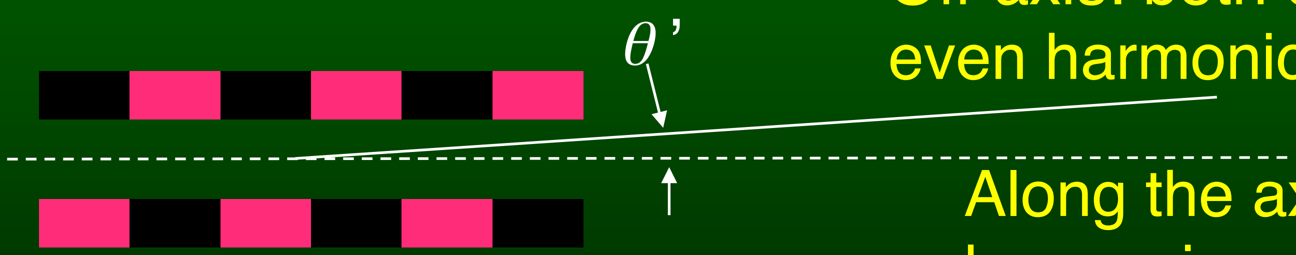


Second correction: stronger B-field means stronger undulations and less on-axis electron speed. This changes γ so that:

Central wavelength: $[L/(2\gamma^2)]/(1+aB^2)$

…furthermore, an undulator also emits the <u>harmonics</u> of the central wavelength:

$[L/(2\gamma^2)]/\mu$, μ = integer number



Off-axis: both odd and even harmonics

Along the axis: only <u>odd</u> harmonics:

$L/(2\mu\gamma^2)$, μ = 1, 3, 5,…

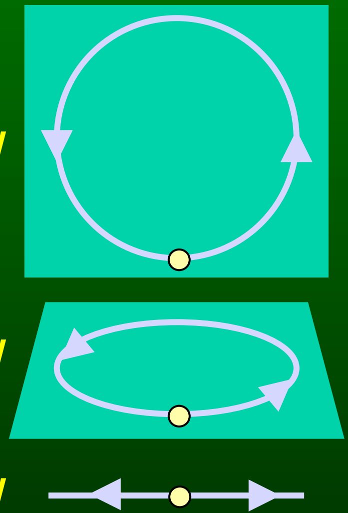
Synchrotron light polarization:

Electron in a storage ring:

TOP VIEW

TILTED VIEW

SIDE VIEW



Polarization:

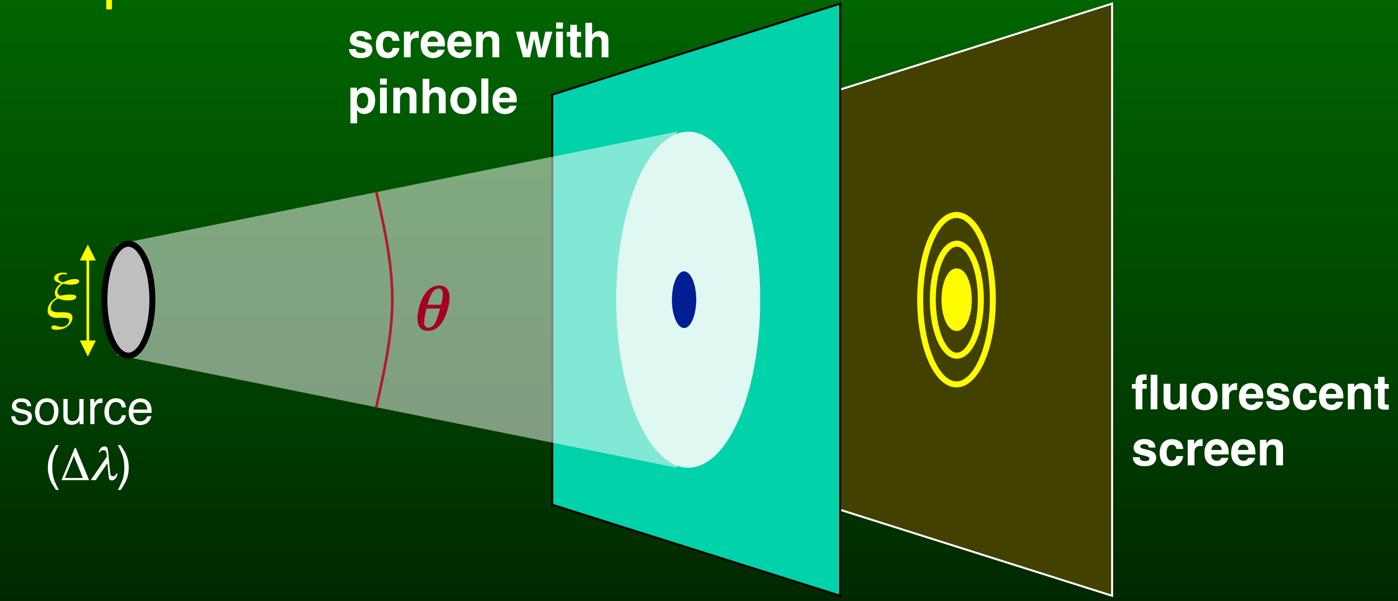
Linear in the plane of the ring,

elliptical out of the plane (weak intensity)

Special (elliptical) wigglers and undulators provide elliptically polarized light with high intensity

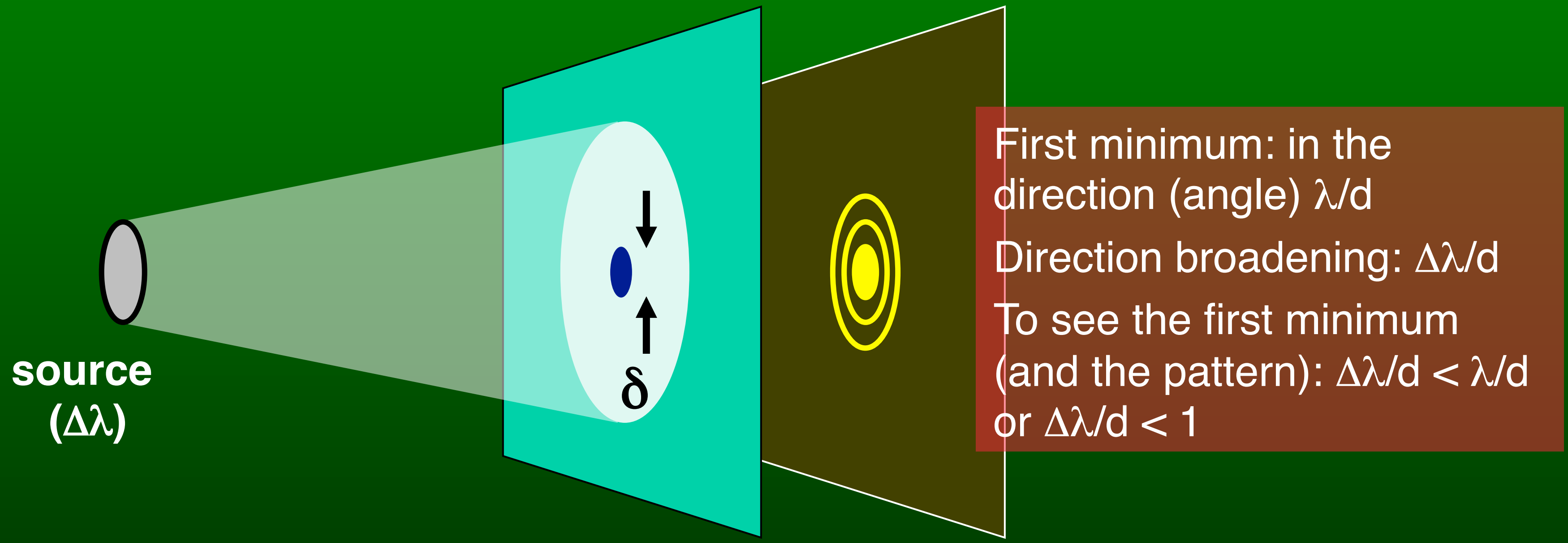
Coherence: "the property that enables a wave to produce visible diffraction and interference effects"

Example:



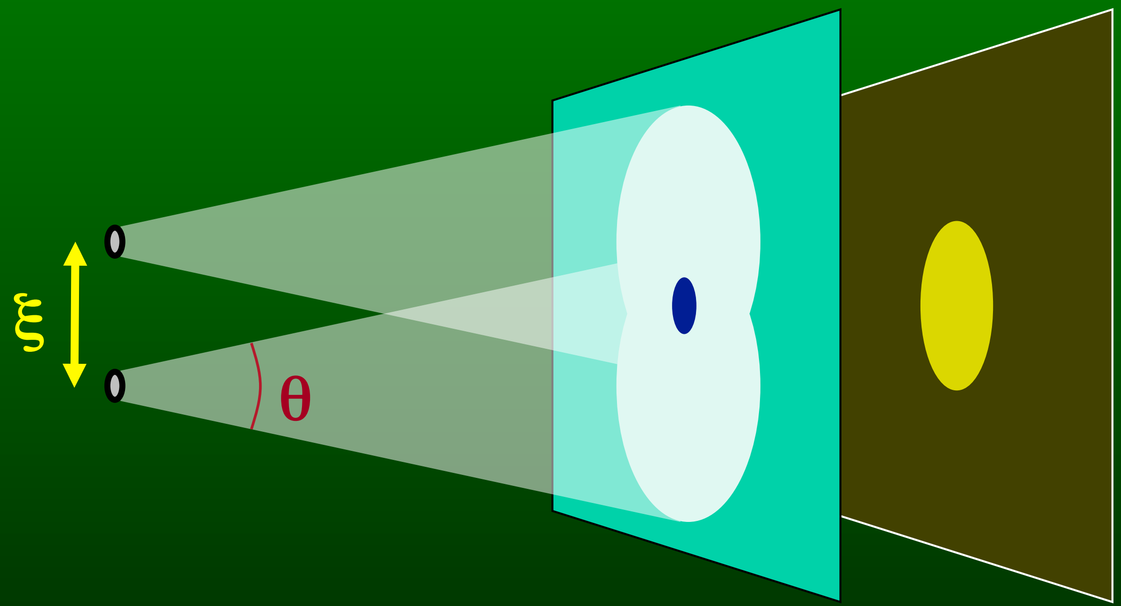
The diffraction pattern may or may not be visible on the fluorescent screen depending on the source size ξ, on its angular divergence θ and on its wavelength bandwidth $\Delta\lambda$

Longitudinal (time) coherence:



- Condition to see the pattern: $\Delta\lambda/\lambda < 1$
- Parameter characterizing the longitudinal coherence: "coherence length": $L_c = \lambda^2/\Delta\lambda$
- Condition of longitudinal coherence: $L_c > \lambda$

Lateral (space) coherence — analyzed with an extended source formed by <u>two point sources</u>:



- Two point sources produce overlapping patterns: diffraction effects are no longer visible.
- However, if the two source are close to each other an overall diffraction pattern may still be visible: the condition is to have a large "coherent power" $[2\lambda/(\xi\theta)]^2$

Coherence — summary:

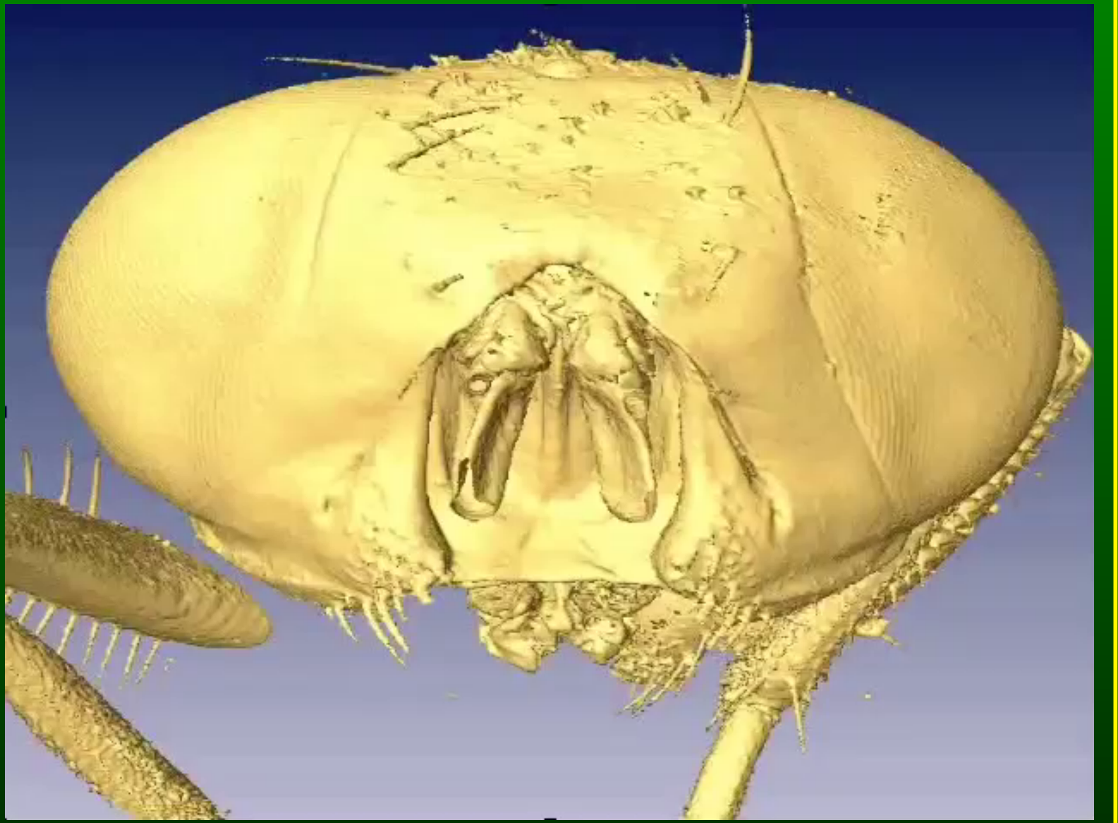
- Large coherence length $L_c = \lambda^2/\Delta\lambda$
- Large coherent power $[2\lambda/(\xi\theta)]^2$
- Both difficult to achieve for small wavelengths (x-rays)
- The conditions for large coherent power are equivalent to the geometric conditions for high brightness

Early example of coherence-based imaging with synchrotron radiation

Conventional radiology

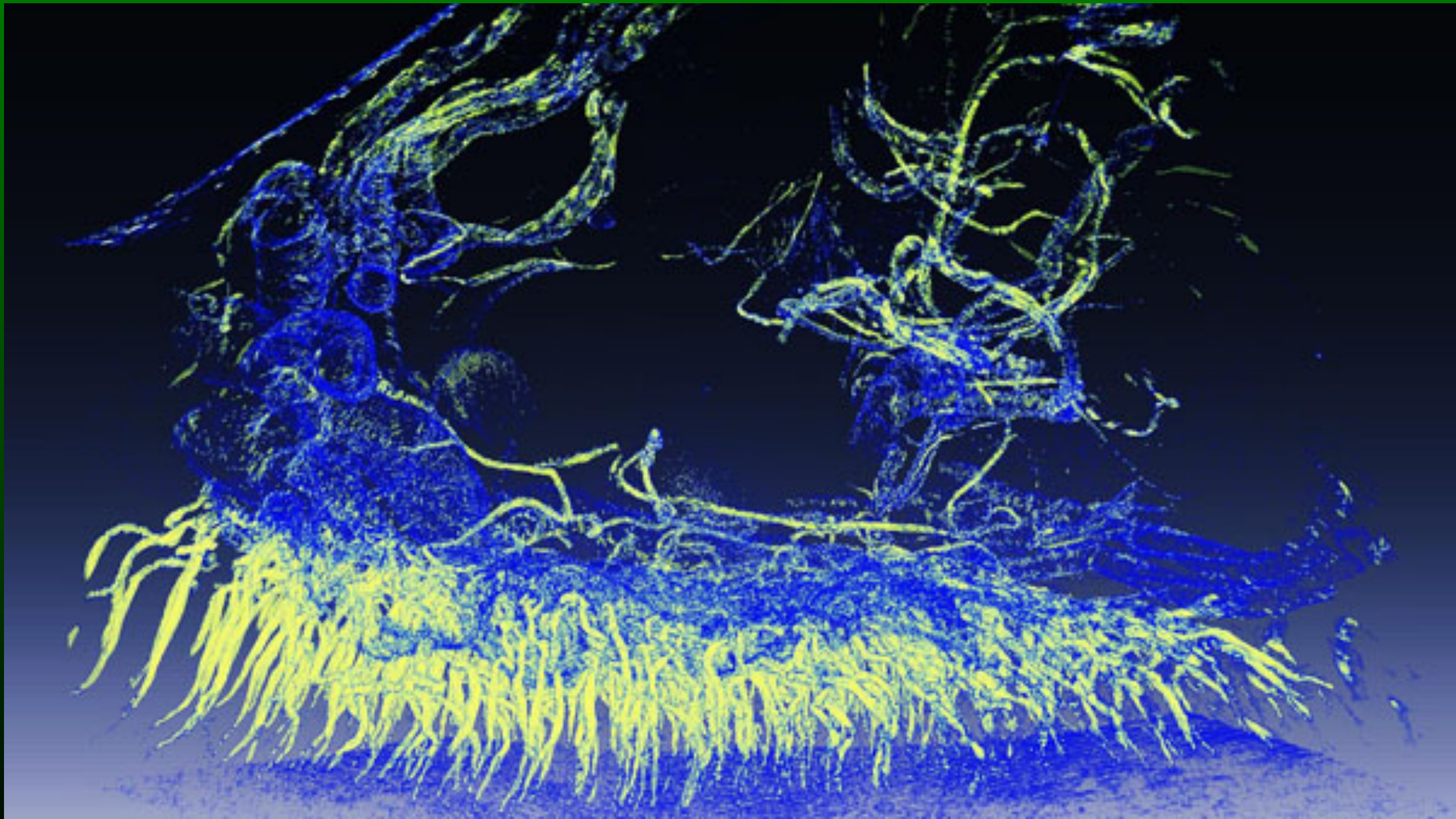
Refractive-index radiology (Giuliana Tromba, Trieste)

Coherence-based phase contrast micro-tomography: housefly



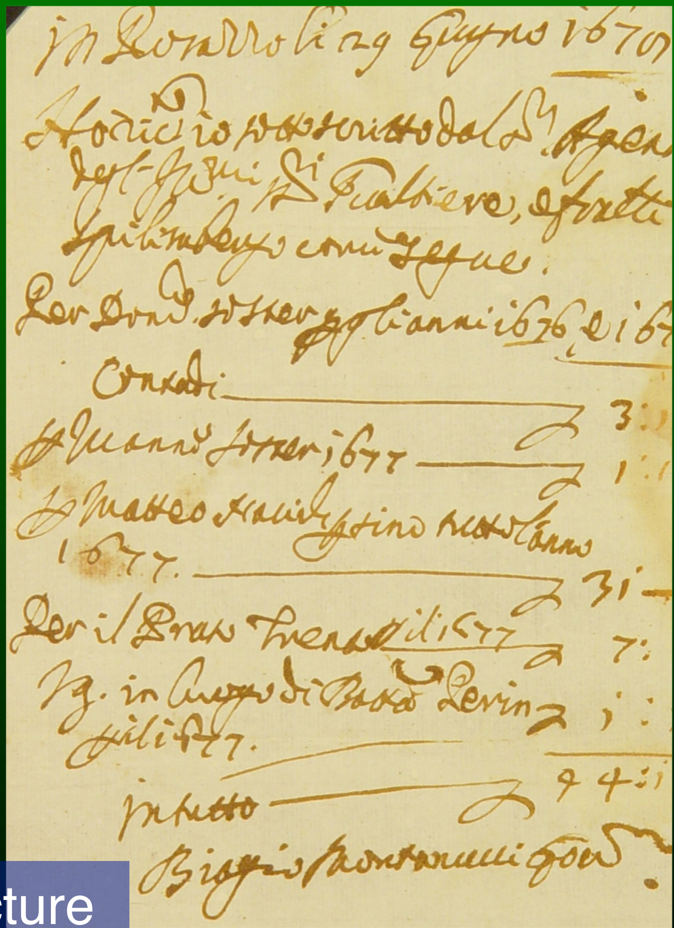
Yeukuang Hwu, Jung Ho Je et al.

Coherence-based imaging of a firefly lantern: understanding how light is emitted

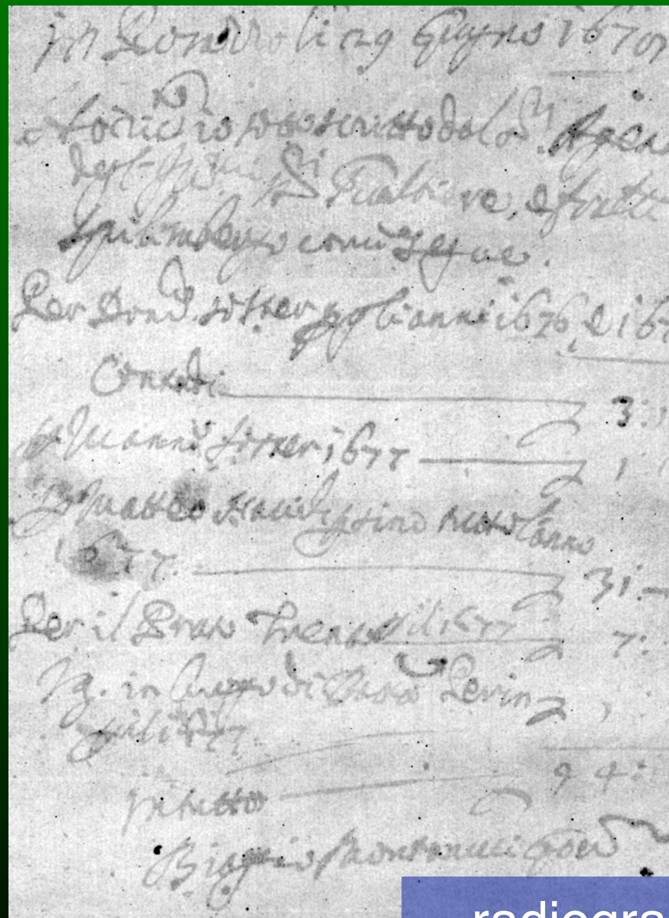


Yeukuang Hwu, et al.

Coherence-based imaging "reads" ancient manuscripts without opening them:



visible picture



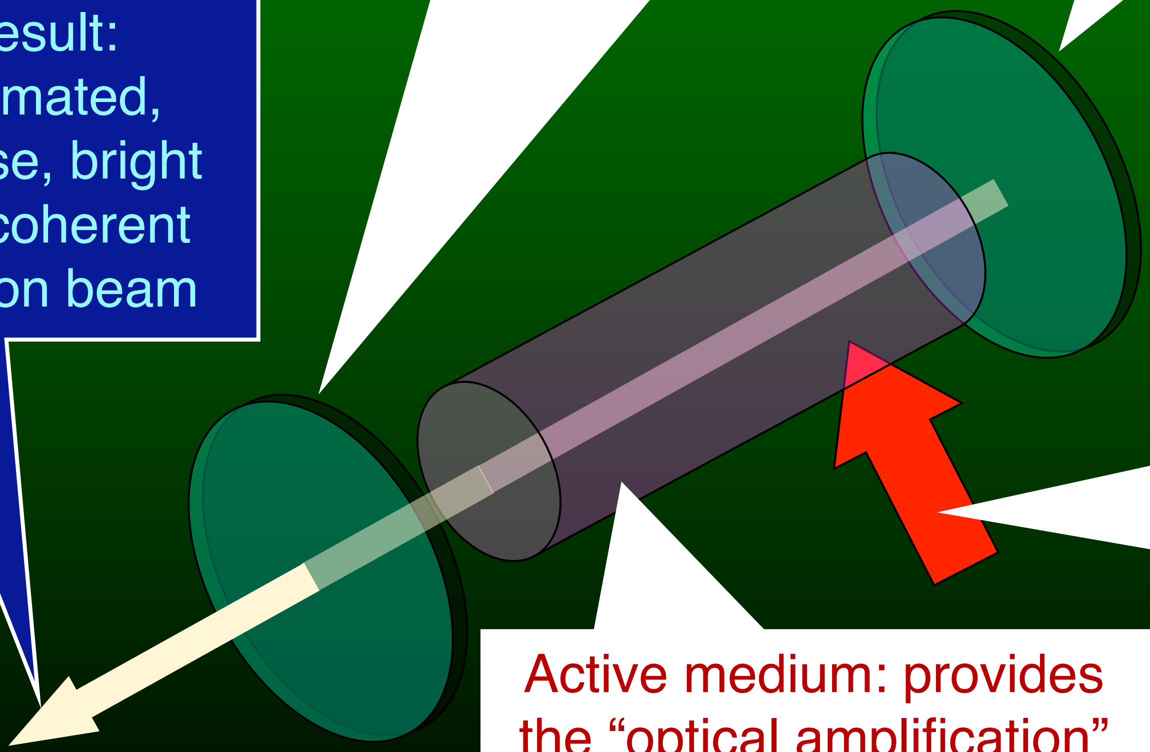
radiograph

Fauzia Albertin, et al.

New types of sources:

- Ultrabright storage rings (SLS, new ESRF source) reaching the diffraction limit in a large part of the emitted spectrum
- Inverse-Compton-scattering table-top sources
- Energy-recovery machines
- VUV free electron lasers (FEL's) (such as CLIO)
- X-ray FEL's

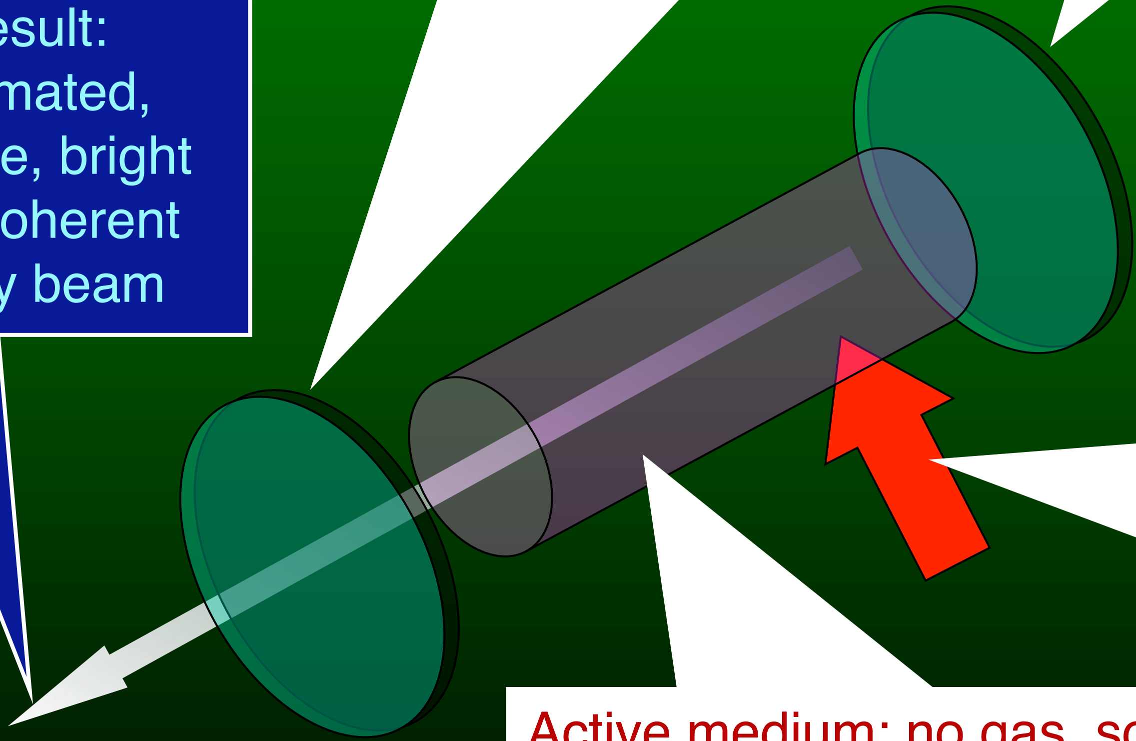
Towards FEL’s – let’s start from a normal laser:



from a normal laser to an x-ray FEL:

No x-ray mirrors ⇒ no optical cavity ⇒ enough amplification needed for <u>one-pass</u> lasing

Result: collimated, intense, bright and coherent x-ray beam

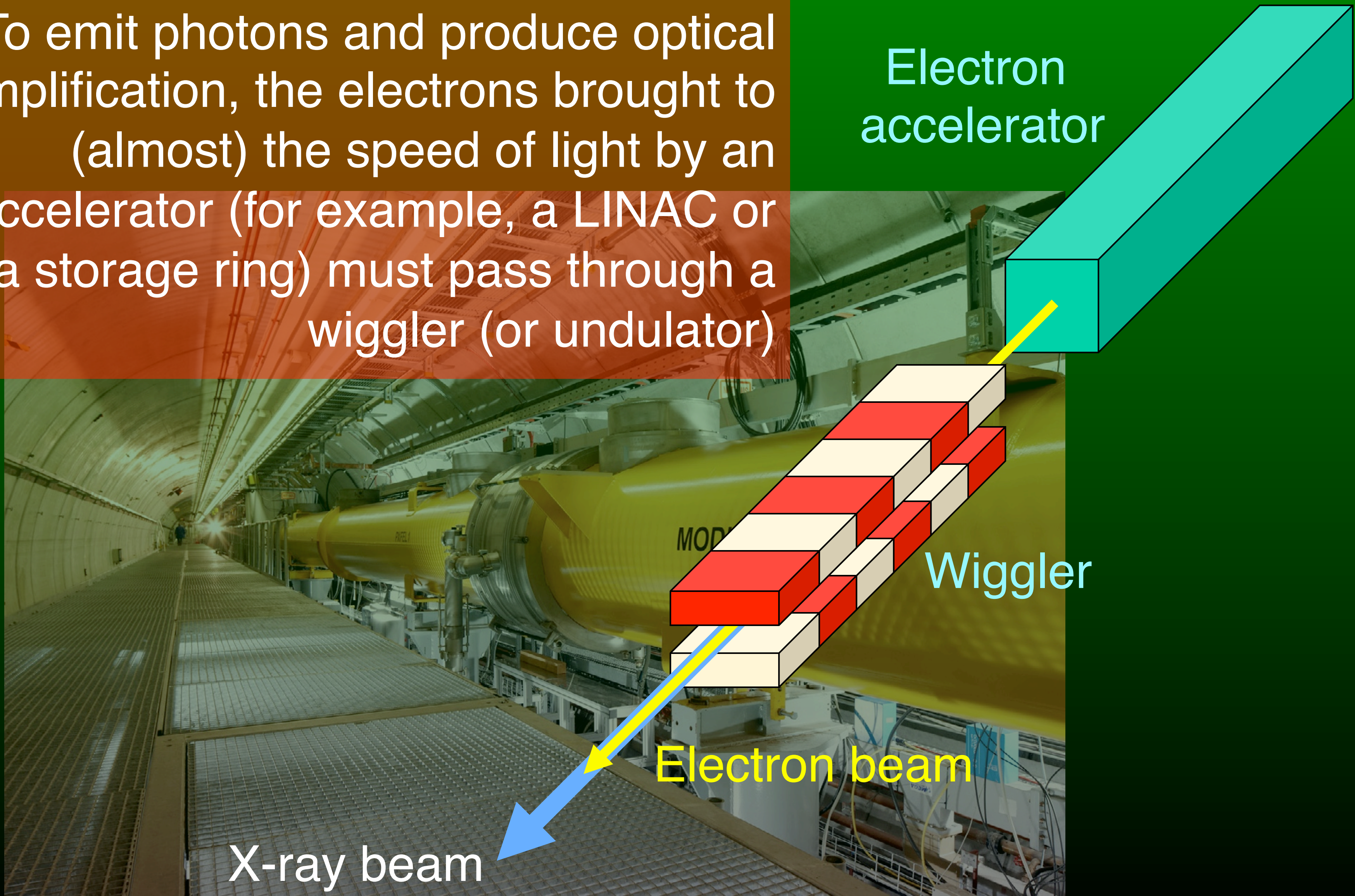


Optical pump: the free electrons provide the energy and transfer it to the photons

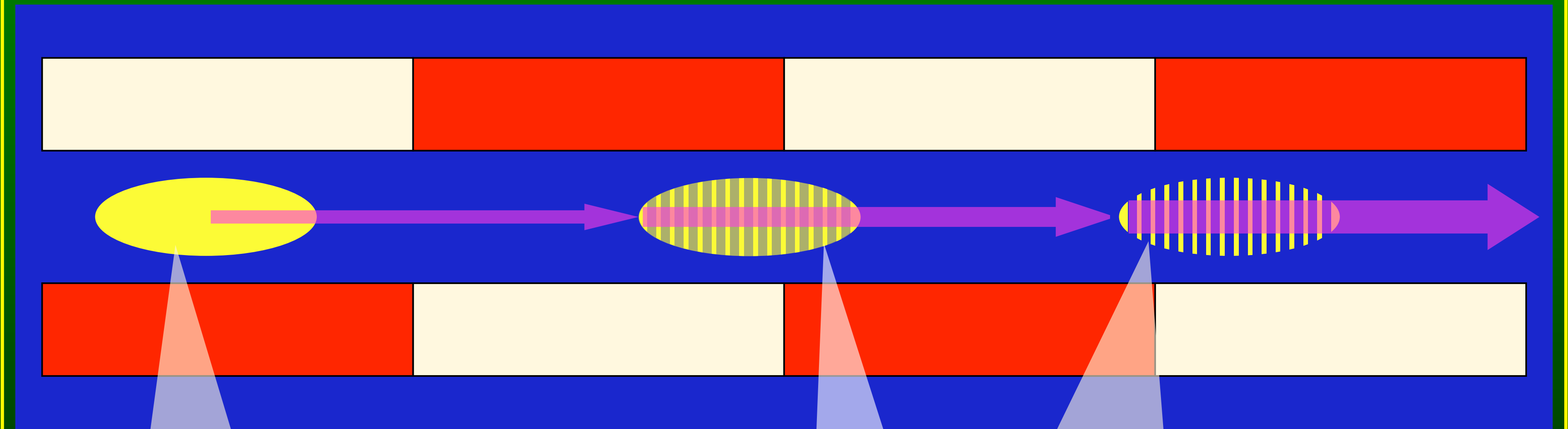
Active medium: no gas, solid or liquid but “free electrons” in an accelerator: hight power possible without damage

FEL's: general scheme

To emit photons and produce optical amplification, the electrons brought to (almost) the speed of light by an accelerator (for example, a LINAC or a storage ring) must pass through a wiggler (or undulator)



This is what happens in detail:



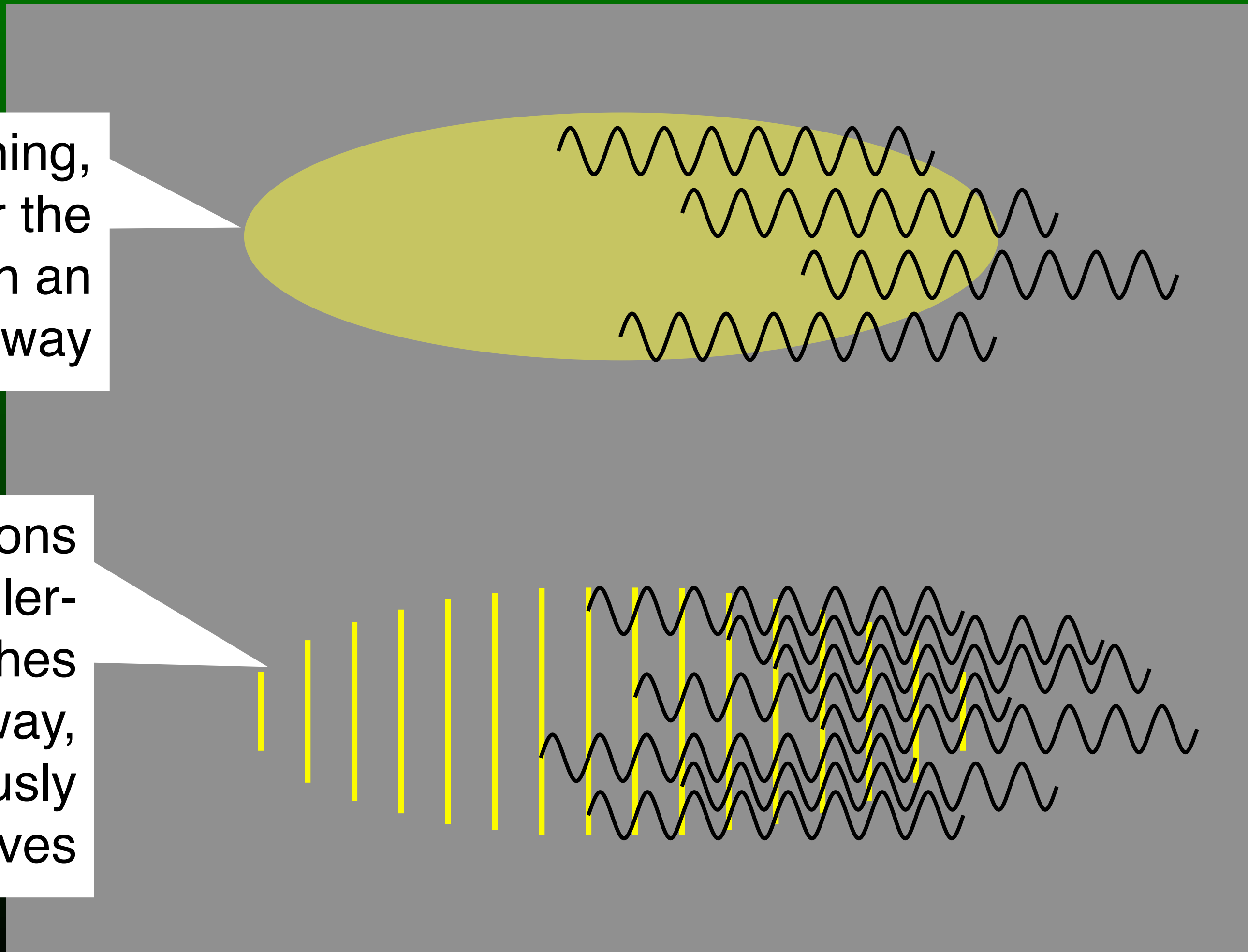
A bunch of electrons enters the wiggler: some of them stochastically start emitting waves

The combined wiggler+wave action progressively microbunches the electrons. The emission of microbunched electrons enhances the previously emitted waves

Correlated emission from microbunched electrons:

With no microbunching, as electrons enter the wiggler, they emit in an uncorrelated way

Instead, the electrons confined to the wiggler-induced microbunches emit in a correlated way, enhancing previously emitted waves



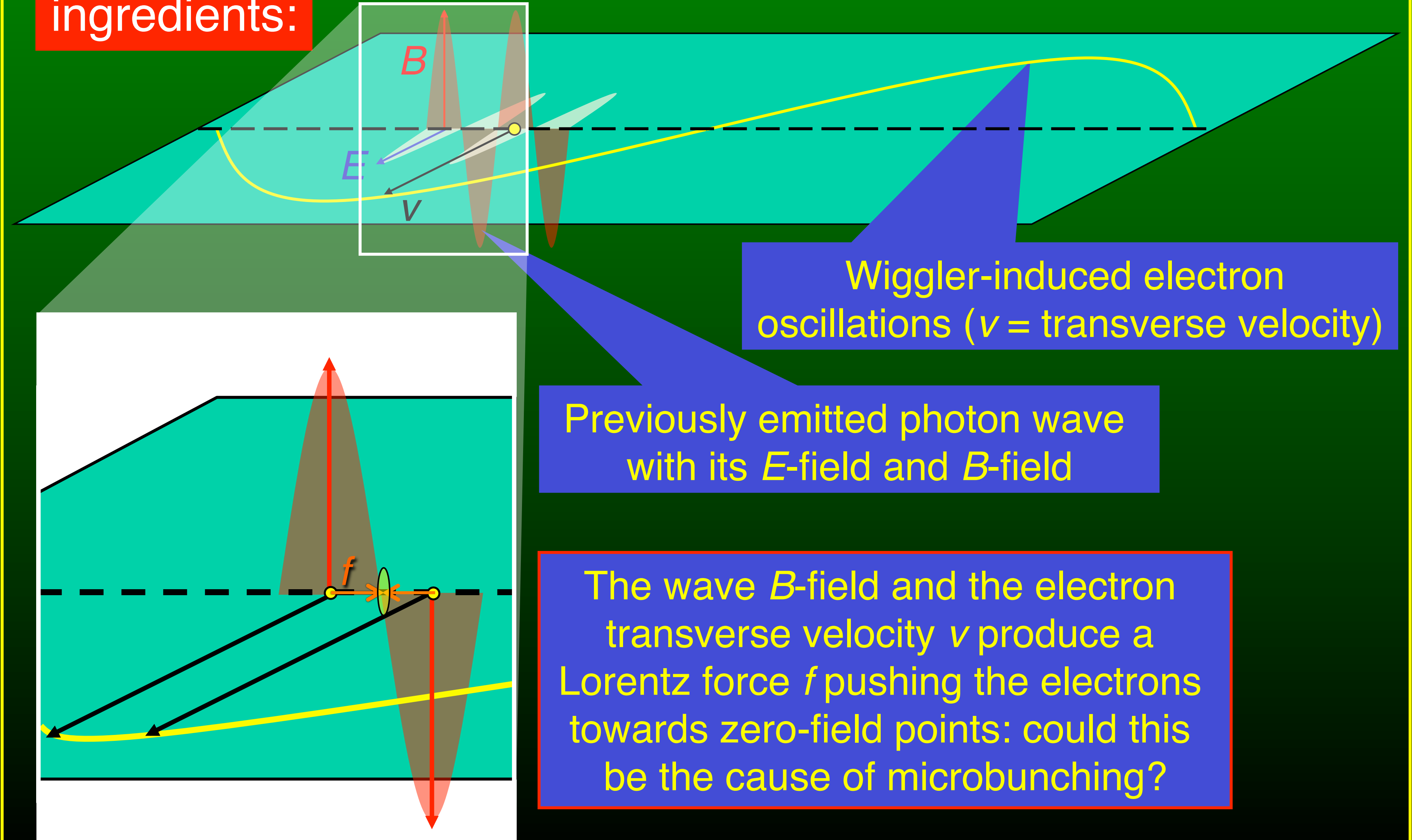
In summary, the wiggler induces transverse electron oscillations that:

1. Accelerate the electron charges enabling them to emit photon waves
2. In collaboration with previously emitted waves, cause the microbunching of the electrons

Note: without microbunching, the wave intensity is proportional to the number of electrons, *N*. With microbunching, the electrons in each microbunch emit in a correlated way: the wave amplitude is proportional to *N*. The wave intensity is proportional to the square of the amplitude and therefore proportional to N^2.

But: what really creates the electron microbunches?

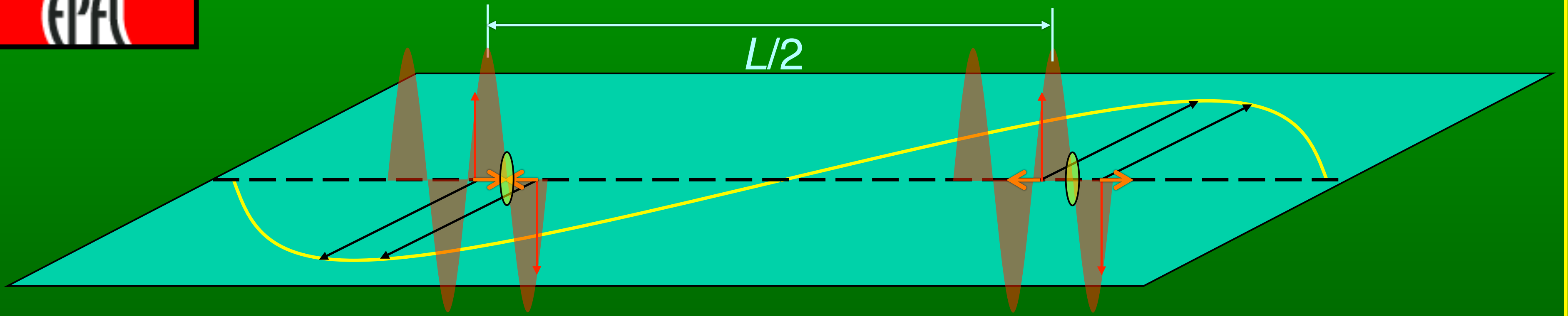
Two key ingredients:



Wiggler-induced electron oscillations (*v* = transverse velocity)

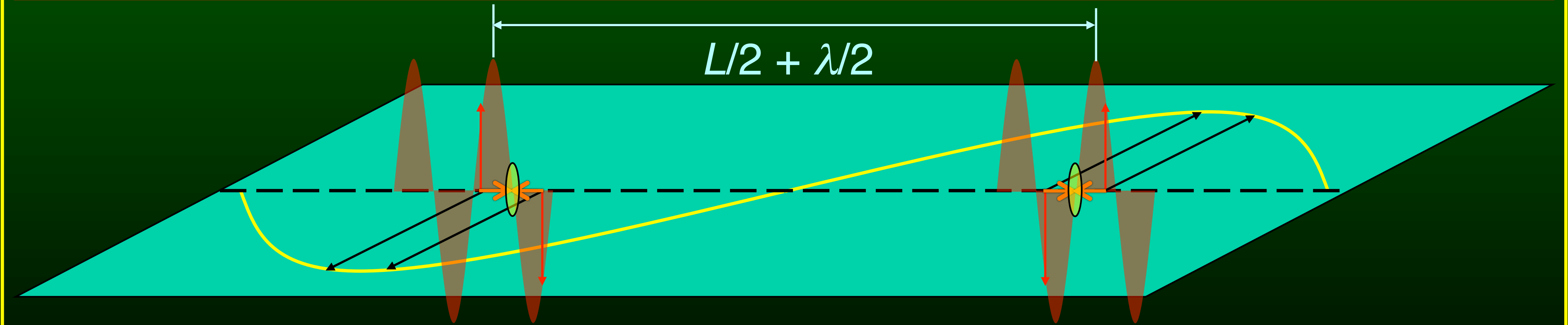
Previously emitted photon wave with its *E*-field and *B*-field

The wave *B*-field and the electron transverse velocity *v* produce a Lorentz force *f* pushing the electrons towards zero-field points: could this be the cause of microbunching?



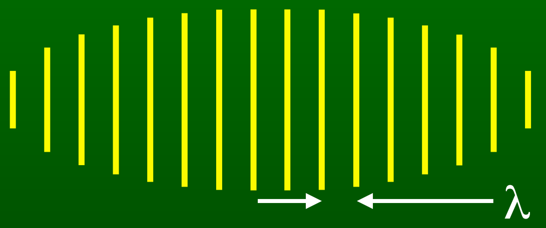
…maybe, but something seems wrong: after 1/2 wiggler period, the electron transverse velocity is reversed. If the wave travels together with the electron, the *B*-field stays the same. Are the forces and the microbunching reversed?

No! Electron and wave do not travel together: the electron speed is $u < c$. As the electron travels over $L/2$ in a time $L/(2u)$, the wave travels over $[L/(2u)]c$. The difference is $(L/2)(c/u - 1) \approx L/(4\gamma^2)$ = half wavelength



B-fields, velocities are all reversed: the forces are not, and keep microbunching the electrons

Why is microbunching (and lasing) more difficut for x-rays than for infrared photon?

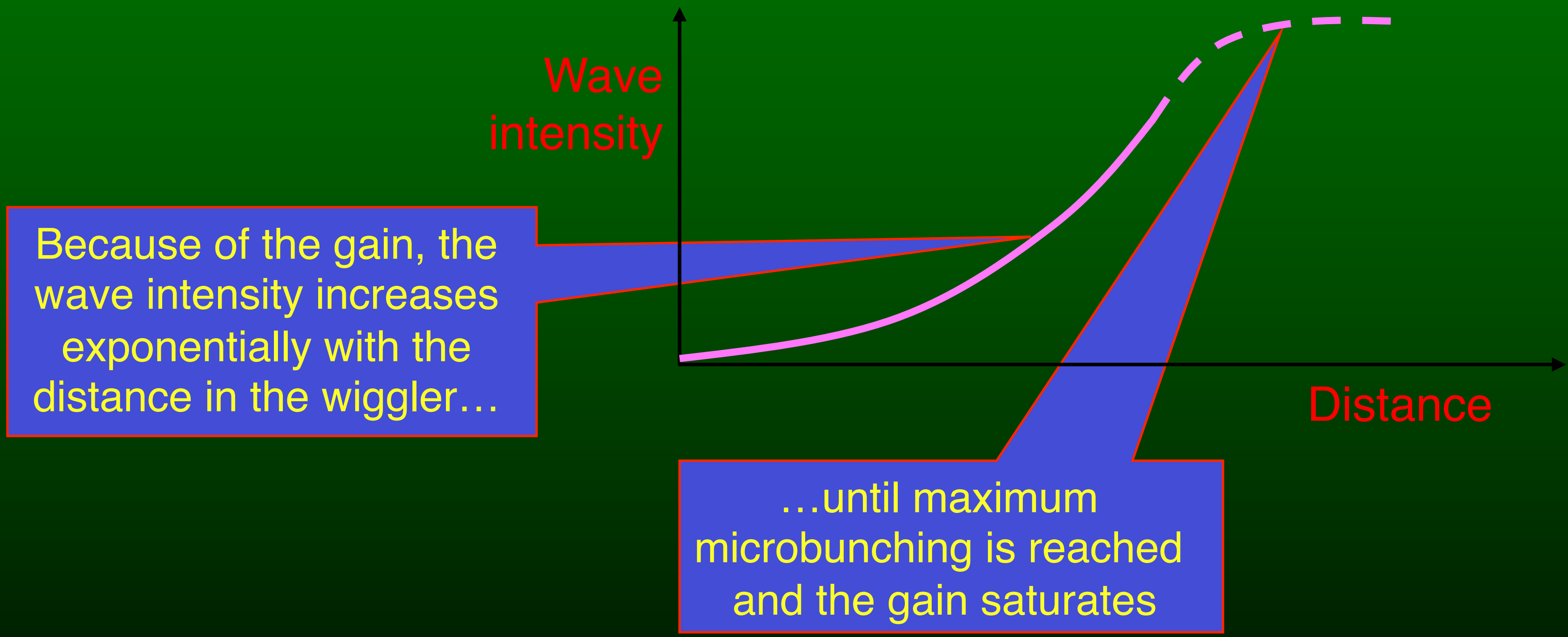


At short wavelengths the microbunches are closer to each other, and this should facilitate the microbunching

But:

- Short wavelengths require a high electron energy corresponding to a large γ - factor
- The large γ makes the electrons "heavy" and therefore difficult to move towards the microbunches: their longitudinal relativistic mass (that governs microbunching) is $\gamma^3 m_o$
- This offsets the advantage of closer microbunches, making microbunching very difficult

Microbunching produces correlated emission and a progressive gain in the wave intensity

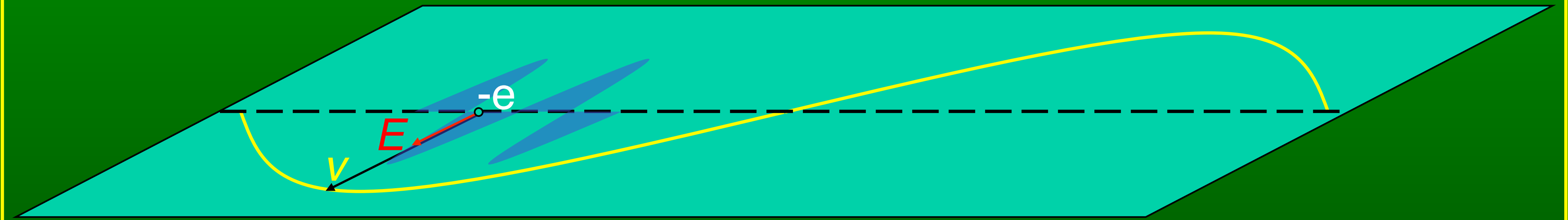


As mentioned, for an x-ray FEL (no 2-mirror cavity), gain saturation must be reached before the end of the (very long) wiggler, in a single pass

Why the FEL exponential intensity increase?

- The energy transfer rate from the electron beam to a pre-existing wave of intensity I is determined by: (1) the transfer rate for each single electron (2) the effects of microbunching
- The one-electron transfer rate is given by the (negative) work proportional to $E\,v$, where E = wave (transverse) E-field and v = electron transverse velocity.
- E is proportional to $I^{1/2}$ so the energy transfer rate for one electron is proportional to $I^{1/2}$
- The effects of microbunching are proportional to the Lorentz force that causes it, which is produced by v_T and by the B-field B of the pre-existing wave. Since B is proportional to $I^{1/2}$, this corresponds to another factor proportional to $I^{1/2}$
- Overall, dI/dt is proportional to $I^{1/2}\,I^{1/2} = I$
- This corresponds to an exponential increase as a function of t -- and therefore of the distance = ut

Why does the intensity increase saturate?

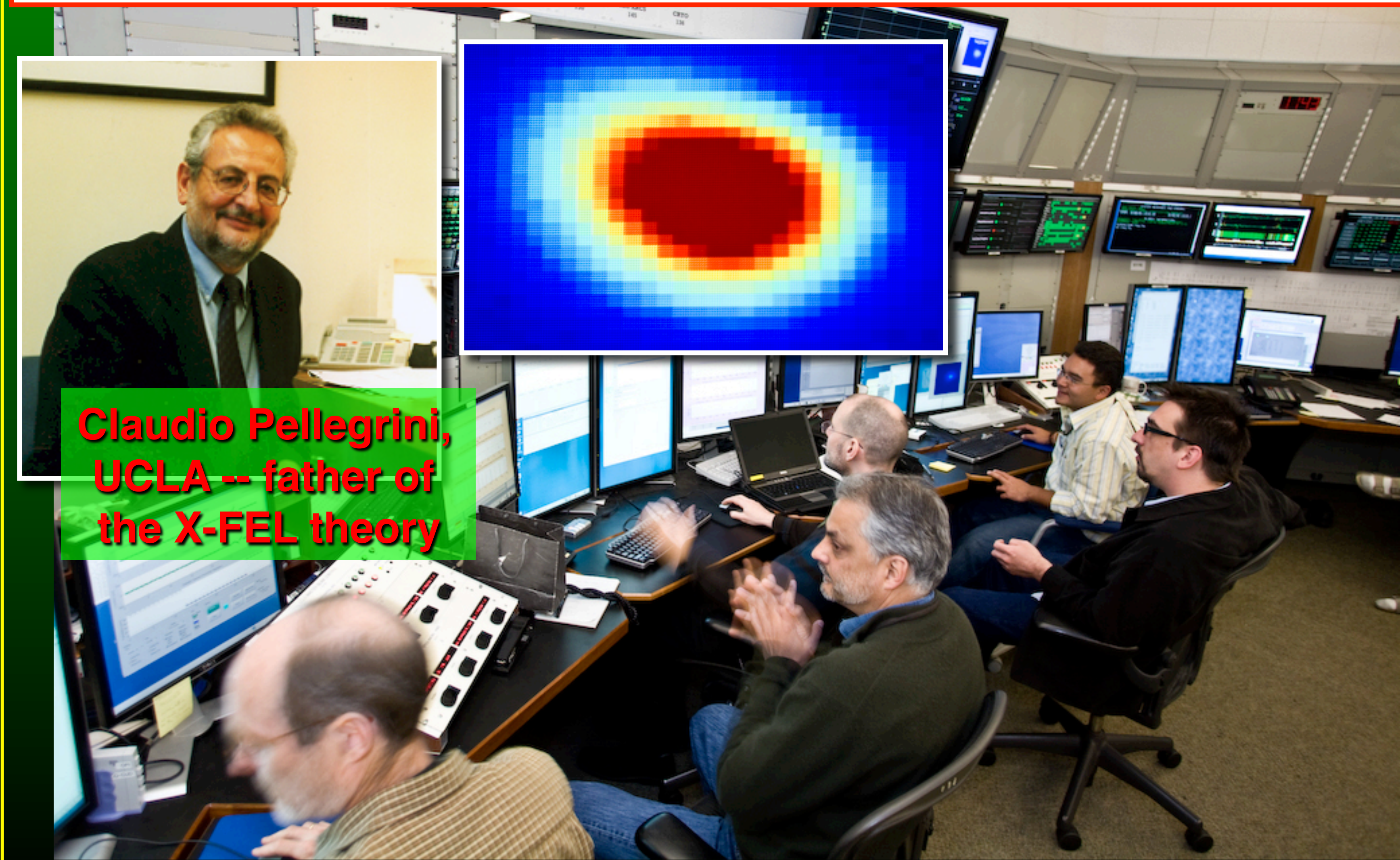


For the electron→wave energy transfer, the directions of the electron transverse velocity *v* and of the wave *E*-field must produce negative work. In this case, the phase difference between *v* and *E* fullfills that condition

- But as the electron gives energy to the wave, it slows down and the phase of the transverse velocity relative to the wave changes.
- Eventually, the conditions are reversed leading to wave→ electron energy transfer
- This accelerates the electrons until the conditions are restored for electron→wave energy transfer
- The mechanism goes on and on, producing an energy oscillation between electrons and wave rather than a continuing increase of the wave intensity: hence, saturation

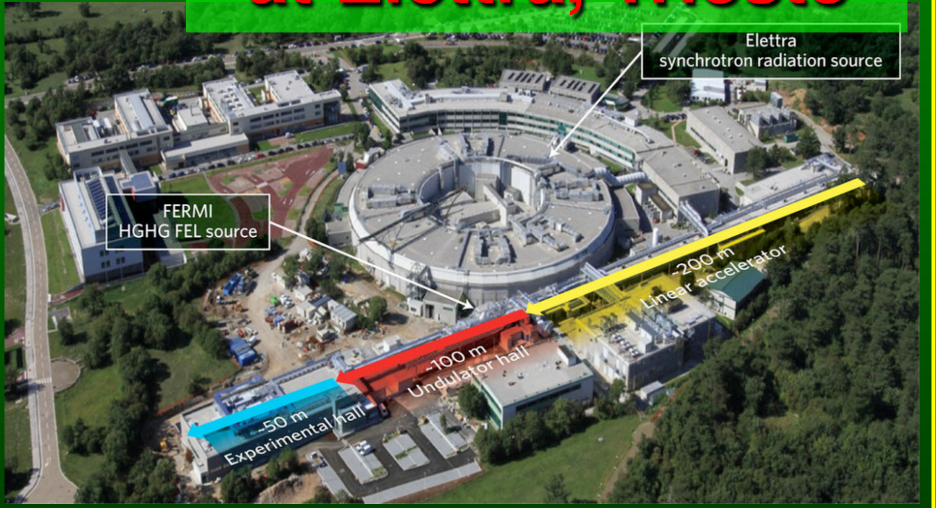
April 21, 2009 - New Era of Research Begins as World's First Hard X-ray Laser Achieves "First Light"

X-ray laser pulses of unprecedented energy and brilliance produced at SLAC



The European X-FEL in Hamburg

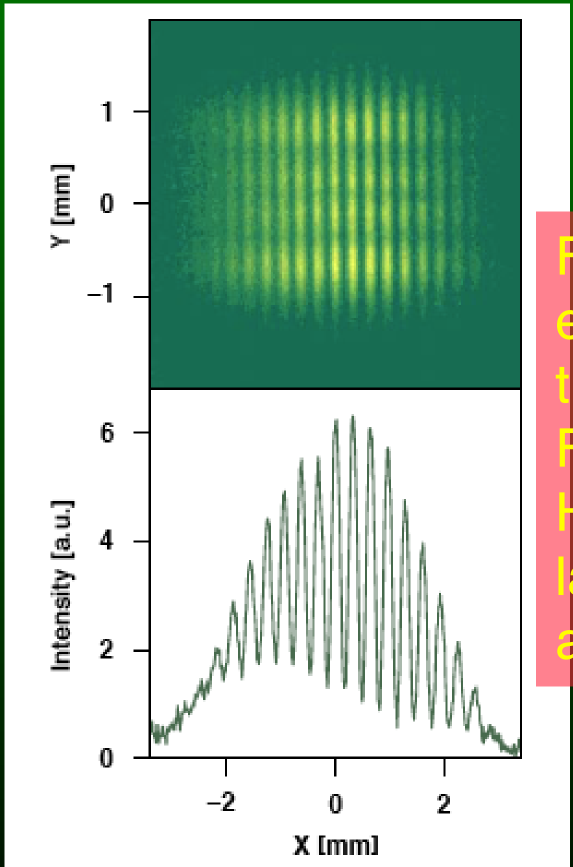
The FERMI X-FEL at Elettra, Trieste



The Swiss X-FEL at the Paul-Scherrer Institut

X-ray FEL coherence:

Full lateral (space) coherence all the way to the hard x-rays

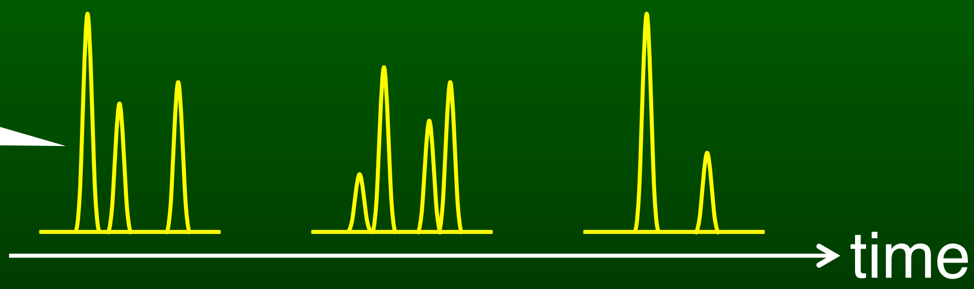


First coherence experiments on the Tesla Test Facility in Hamburg: full lateral coherence at $\lambda = 95$ nm

For the X-FEL longitudinal (time) coherence, a critical problem:

Amplification starts with the first waves stochastically emitted when the electron bunch enters the wiggler

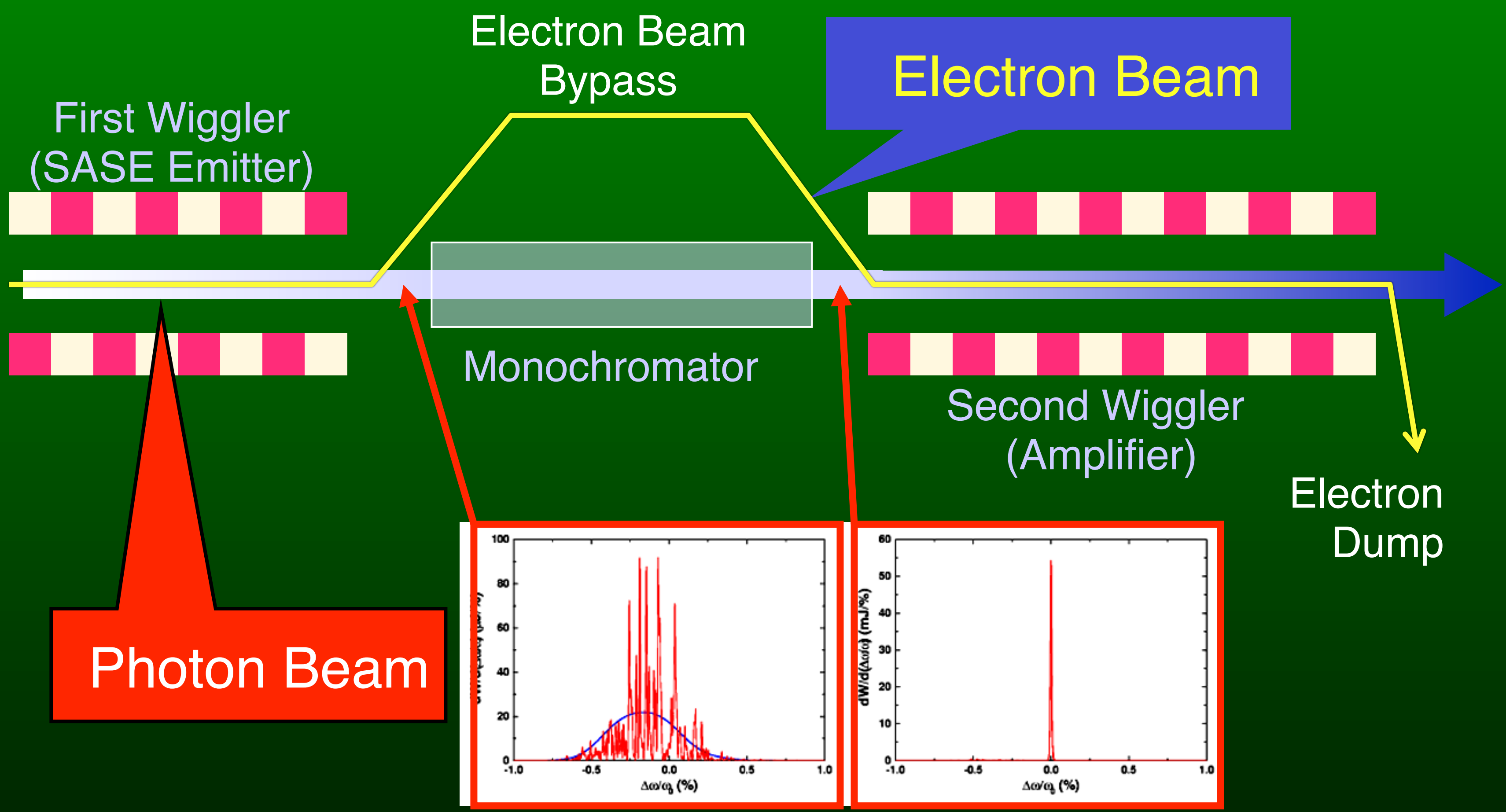
The pulse time structure changes with each bunch, limiting the time coherence



Solution: "seeding" – the process is triggered by an artificially injected wave

A complicated technology, recently implemented

Example of seeded X-FEL:



Note: seeding is also possible using radiation from an external laser source

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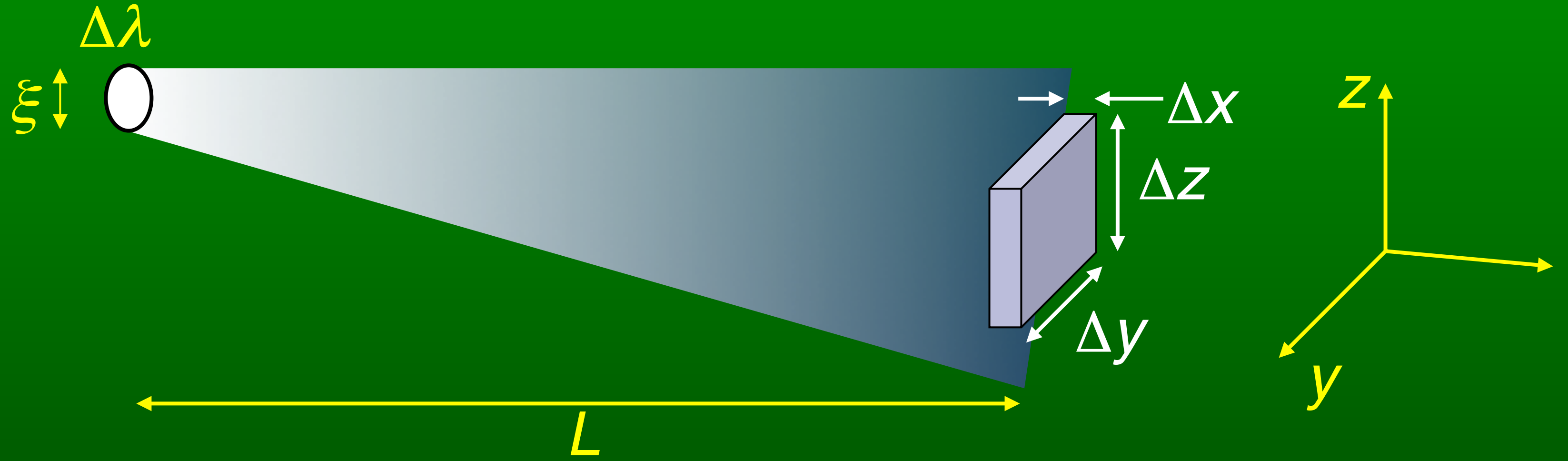
Refinements: A different look at coherence:



A source of size ξ and bandwidth $\Delta\lambda$ can illuminate coherently a volume $\Delta x \Delta y \Delta z$ at the distance L. This is this coherence volume.

Along x: if two waves of wavelength λ and $\lambda+\Delta\lambda$ are in phase ar a certain time, they will be out of phase after Δt such that $\Delta\omega\Delta t = 2\pi$ or $\Delta t = 2\pi/\Delta\omega = \lambda^2/(c\Delta\lambda)$.

Thus, $\Delta x = c\Delta t = \lambda^2/\Delta\lambda = L_c$.

Along y: the spread in k-vector is $\Delta k = k\xi/L = 2\pi\xi/(L\lambda)$.

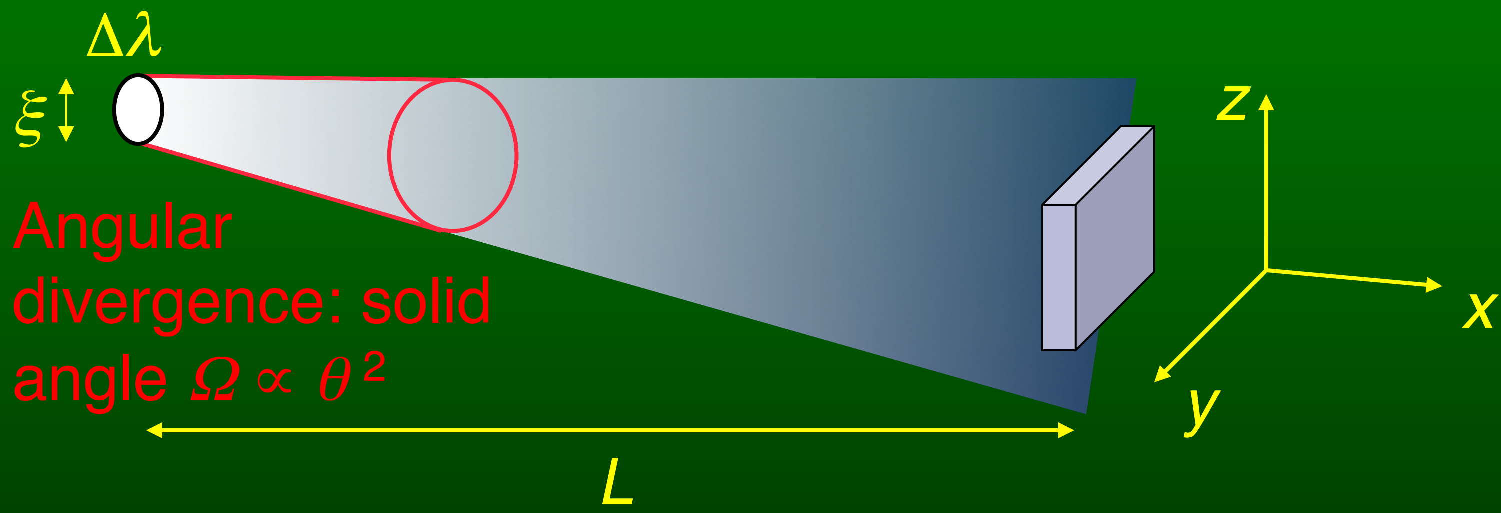
If two waves with k-vectors 0 and Δk along y are in phase at a certain point, they will be out of phase at a distance Δy such that $\Delta k \Delta y = 2\pi$; thus, $\Delta y = L\lambda/\xi$.

Along z: same as along y.

Coherence volume: $\Delta x \Delta y \Delta z = L^2\lambda^4/(\xi^2\Delta\lambda)$

Behind this: Heisenberg! Photons in the coherence volume cannot be distinguished from each other

The notion of "coherent power":



The solid angle corresponding to the area $\Delta y \Delta z$ is $\Delta y \Delta z / L^2$.

If the solid angle of the emitted light is $\approx \theta^2$, then only a portion $(\Delta y \Delta z / L^2)/\theta^2$ of the total emitted power illuminates the coherence volume.

This is the coherent power.

Since $\Delta y \Delta z = (L\lambda/\xi)^2$, the coherent power is $\approx [\lambda/(\xi\theta)]^2$.

What is the number n_c of photons in the "coherence volume" for an X-FEL with full transverse coherence?

Full transverse coherence means that all the emitted photons are within the "coherence volume". Thus, their number n_c is given by the flux F times $L_c/c = \lambda^2/(c\Delta\lambda)$.

The brightness B is proportional to $F/(\xi\theta)$; for full transverse coherence, $F/(\xi\theta) \approx F/(\lambda^2)$ and F is proportional to $\lambda^2 B$.

The F-B proportionality factor contains the relative bandwidth $\Delta\lambda/\lambda$.

Thus, $n_c = F\lambda^2/(c\Delta\lambda)$ is proportional to $(\lambda^2 B)[\lambda^2/(c\Delta\lambda)](\Delta\lambda/\lambda)$:

Overall, the number of photons in the "coherence volume" is proportional to $B\lambda^3$: high peak brightness helps, but short wavelengths are a problem!