

Spontaneous breaking of Lorentz symmetry in QED_3

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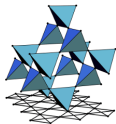
Technische Universität Dresden

LJ, arXiv:1604.06354 [hep-th];

J. Braun, H. Gies, **LJ**, D. Roscher, PRD **90**, 036002 (2014)



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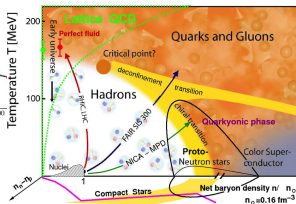
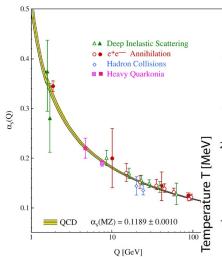
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QED₃ historically ...

[Pisarski '84, Appelquist *et al.* '88]

... toy model for QCD₄:

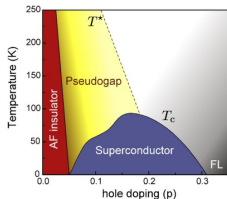
- asymptotic freedom
... due to dimensional fine structure constant
- chiral symmetry breaking
... or conformal phase
- confinement
... in its compact version



QED₃ today ...

... effective description of high- T_c 's:

[Franz, Tesanovic, Vafeek '02; Herbut '02; ...]



... with $N = 2$ four-component Dirac fermions

... field theory of the half-filled Landau level?

[Son '15]



... with $N = 1/2$

... emergent theory of fractionalized excitations in spin systems:

[Hermele *et al.* '04; Ran *et al.* '07; Xu '08; He *et al.* '15; Wang & Senthil '16; ...]

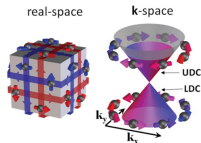
- U(1) Dirac spin liquids?



... with $N = 1, 2, 4, \dots$

... dual description of topological insulators?

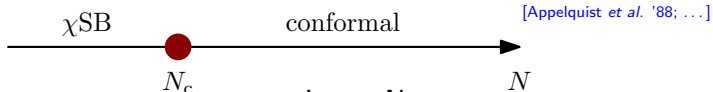
[Wang & Senthil '15; Metlitski & Vishwanath '15; Mross *et al.* '15]



... with $N = 1/2, 3/2, 5/2, \dots$

QED₃: phase diagram

(Expected) phase diagram of QED₃ with N massless Dirac fermions:



Low N :

- chiral symmetry breaking (χ SB)
- dynamical mass generation
- Mott insulator

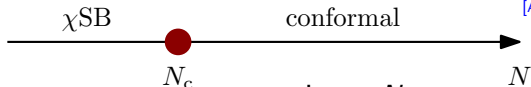
Large N :

- conformal symmetry
- interacting gapless fermions
- non-Fermi liquid

QED₃: phase diagram

(Expected) phase diagram of QED₃ with N massless Dirac fermions:

[Appelquist et al. '88; ...]



Low N :

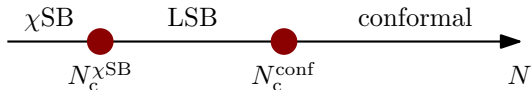
- chiral symmetry breaking (χSB)
- dynamical mass generation
- Mott insulator

Large N :

- conformal symmetry
- interacting gapless fermions
- non-Fermi liquid

We show:

[LJ, arXiv:1604.06354]



Intermediate N :

- spontaneous Lorentz symmetry breaking (LSB)
- gapless fermions, anisotropic propagation
- dynamical generation of charge density $\langle \psi^\dagger \psi \rangle$

Outline

(1) $2 + \epsilon$ expansion: $N_c^{\text{conf}} \nearrow \infty$ when $d \searrow 2$

(2) (generalized) F theorem: $N_c^{\chi\text{SB}} \leq 1 + \mathcal{O}(d - 2) < N_c^{\text{conf}}$

(3) Mean-field theory, F theorem, susceptibility analysis:

$$\langle v_\mu \rangle \neq 0 \quad \text{for} \quad N_c^{\chi\text{SB}} < N < N_c^{\text{conf}}$$

Model

Action:

$$S_{\text{QED}} = \int d^d x \left(\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}_i \gamma_\mu D_\mu \psi_i \right) \quad \text{in } 2 < d < 4,$$

where $i = 1, \dots, N$ and $\mu, \nu = 0, \dots, d - 1$ and

- Clifford algebra: $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \mathbb{1}_4$ 4-dimensional
- Covariant derivative: $D_\mu = \partial_\mu + ieA_\mu$
- Charge: $[e^2] = 4 - d$ RG relevant in $d < 4$... QED₃ (super-)renormalizable & asymptotically free
- Symmetries: “chiral” SU(2N), parity, Lorentz, local U(1)
... “chirality” consequence of the reducible rep. [LJ & Gies '12; ...]
- Gauge fixing:

$$S_{\text{gf}} = -\frac{1}{2\xi} \int d^d x (\partial_\mu A_\mu)^2, \quad \xi \in \mathbb{R}$$

... allows to check gauge invariance via ξ

Renormalization group

Loop corrections can induce new operators:

... which are not present in S_{QED}

Most of them **irrelevant**, but

local **4-fermion terms** are marginal in $d = 2!$

... and can thus become relevant at interacting fixed points in $d = 2 + \epsilon$

Full basis:

[Gies & LJ '10; ...]

$$S_{4\text{-fermi}} = \int d^d x [g_1(\bar{\psi}_i \gamma_{35} \psi_i)^2 + g_2(\bar{\psi}_i \gamma_\mu \psi_i)^2]$$

with $\gamma_{35} \equiv i\gamma_3\gamma_5$

... and where γ_3 and γ_5 the two "left-over" gamma matrices not present in \mathcal{D}

RG flow for $S = S_{\text{QED}} + S_{\text{gf}} + S_{4\text{-fermi}}$

Flow of charge:

RG "time" $t \in [-\infty, 0]$

$$\partial_t e^2 = (d - 4 + \eta_A) e^2 \leq 0$$

... no vertex corrections \Leftrightarrow Ward identity

with anomalous dimension $\eta_A = \frac{4}{3} N e^2 + \mathcal{O}(e^4)$.

- e^2 flows to **strong coupling**
- charged fixed point:

$$\eta_A = 4 - d$$

... as in Abelian Higgs model: [Herbut & Tesanovic '96]

- photon propagator:

$$D_{\mu\nu}(q) \propto |q|^{\eta_A - 2} = |q|^{2-d} = \begin{cases} \text{const.}, & d = 2 & \text{[Schwinger '62]} \\ \frac{1}{|q|}, & d = 3 & \text{[Pisarski '84]} \end{cases}$$

[Gusynin, Hams, Reenders '01]

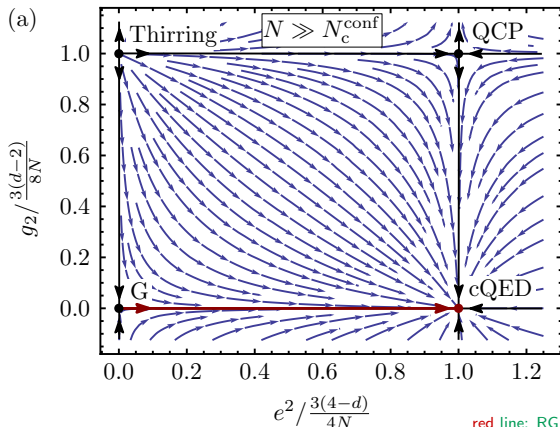
- strong e^2 **generates** g_1, g_2 :

$$\partial_t g_i = (d - 2) g_i + \text{diagrams} + \dots$$

$2 + \epsilon$ expansion

RG flow in e^2 - g_2 plane:

... with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



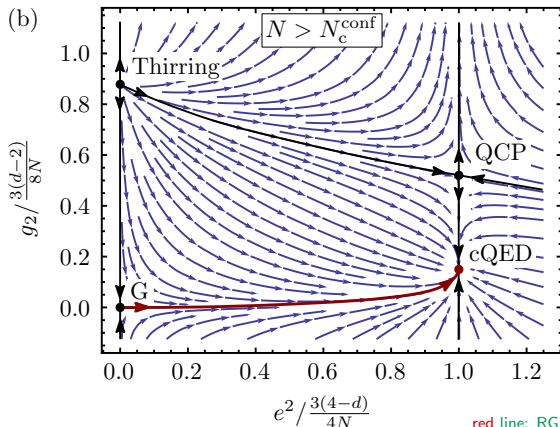
3 interacting fixed points:

Thirring [Gies & LJ, ... '10, '12, '15], conformal (cQED), quantum critical (QCP)

$2 + \epsilon$ expansion

RG flow in $e^2 - g_2$ plane:

... with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$

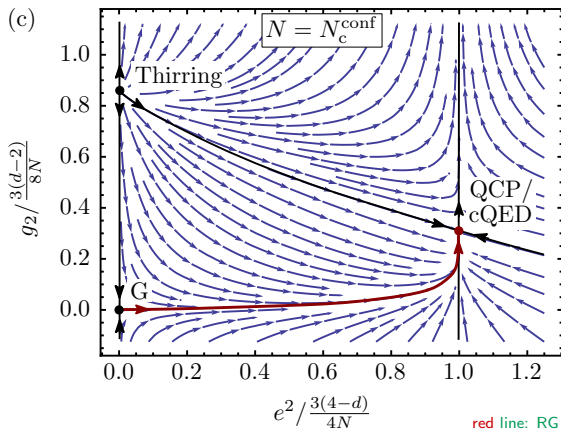


cQED and QCP **approach each other** when lowering N

$2 + \epsilon$ expansion

RG flow in $e^2 - g_2$ plane:

... with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$

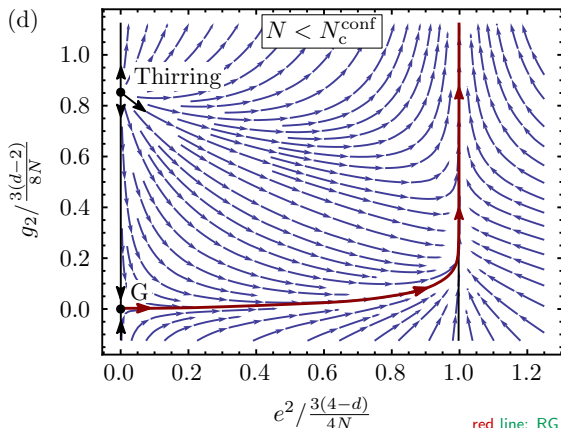


cQED and QCP **merge** when $N = N_c^{\text{conf}}$

$2 + \epsilon$ expansion

RG flow in $e^2 - g_2$ plane:

... with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



cQED and QCP disappear for $N < N_c^{\text{conf}}$...

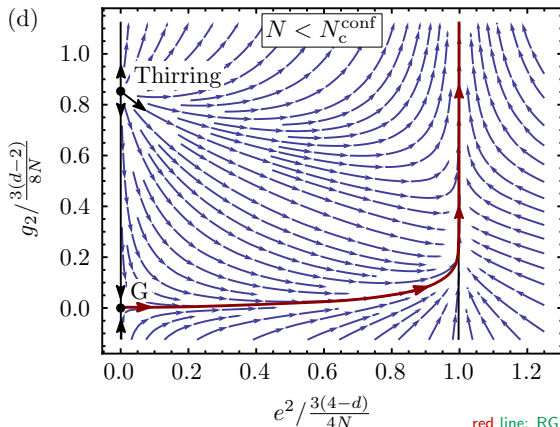
... leaving behind the runaway flow!

... i.e., conformal phase is unstable below N_c^{conf}

$2 + \epsilon$ expansion

RG flow in $e^2 - g_2$ plane:

... with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



red line: RG trajectory of pure QED

$$N_c^{\text{conf}} = \frac{8\sqrt[3]{2}}{\epsilon^{4/3}} \rightarrow \infty \quad \text{for} \quad \epsilon \rightarrow 0$$

... agrees with [Di Pietro *et al.* '16]
... perturbative expansion under control

RG monotonicity

Observation:

“effective” number of degrees of freedom decreases under RG

... e.g., N critical modes at Wilson-Fisher $O(N)$ fixed point $\rightarrow (N - 1)$ massless modes in the SSB phase
or 0 massless modes in the symmetric phase

Quantification?

1+1D: c theorem

[Zamolodchikov '86]

$$C_{UV} > C_{IR}$$

c central charge of conformal algebra



... RG flow goes “downhill”

RG monotonicity



Observation:

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... e.g., N critical modes at Wilson-Fisher $O(N)$ fixed point $\rightarrow (N - 1)$ massless modes in the SSB phase
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Quantification?

1+1D: c theorem

[Zamolodchikov '86]

$$c_{UV} > c_{IR}$$

c central charge of conformal algebra

3+1D: a theorem

[Cardy '88; Komargodski & Schwimmer '11]

$$a_{UV} > a_{IR}$$

anomaly coefficient $a \sim \int_{S^4} \langle T_{\mu}^{\mu} \rangle$

2+1D: F theorem

[Jafferis '10; Jafferis *et al.* '11; Casini & Huerta '12; ...]

$$F_{UV} > F_{IR}$$

“sphere free energy” $F = -\log Z_{S^3}$

RG monotonicity: continuous dimension?

Generalized F for $d \in \mathbb{R}$:

[Giombi & Klebanov '14]

$$\tilde{F} = \sin\left(\frac{\pi d}{2}\right) \log Z_{S^d}$$

with:

$$\tilde{F}|_{d=2} = \frac{\pi}{6} c \quad c \text{ theorem} \quad \checkmark$$

$$\tilde{F}|_{d=3} = F \quad F \text{ theorem} \quad \checkmark$$

$$\tilde{F}|_{d=4} = \frac{\pi}{2} a \quad a \text{ theorem} \quad \checkmark$$

Thus:

$$\boxed{\tilde{F}_{UV} > \tilde{F}_{IR}} \quad \text{for all integer } d$$

... and various **evidence for continuous d** :

$$d = 2 + \epsilon, \quad d = 3 - \epsilon, \quad d = 4 - \epsilon, \quad d = 6 - \epsilon, \dots$$

[Giombi, Klebanov, Tarnopolsky, Fei, ... '14, '15, '16]

... though no rigorous proof yet

RG monotonicity: entanglement

Reduced density matrix:

$$\rho_A = \text{Tr}_{\bar{A}} |\psi\rangle\langle\psi|$$

Entanglement entropy:

$$S_A = -\text{Tr}_A \rho_A \log \rho_A$$

E.g., $A = \bar{A} = \{|\uparrow\rangle, |\downarrow\rangle\}$:
$$S_A = \begin{cases} 0 & \text{for } |\psi\rangle = |\uparrow\uparrow\rangle \\ \log 2 & \text{for } |\psi\rangle = \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \end{cases}$$

... measures entanglement between A and \bar{A}

Scaling with subsystem size $\sim R$?

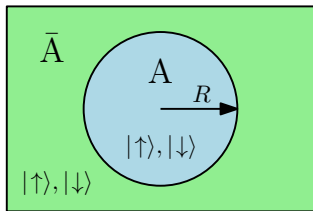
- (trivial) gapped states: $S_A \propto R^{(d-1)-1}$
- conformal phases (or points):

... "area law"

| |
|---|
| $1+1\text{D}: S_A \propto c \log R + \mathcal{O}(1/R)$ |
| $2+1\text{D}: S_A \propto R - \gamma + \mathcal{O}(1/R), \quad \text{with } \gamma = F$ |
| $3+1\text{D}: S_A \propto R^2 + a \log R + \mathcal{O}(1/R)$ |

[Holzhey, Larsen, Wilczek '94]
[Calabrese & Cardy '04]
[Kitaev & Preskill '06]
[Casini, Huerta, Myers '11]
...

\Rightarrow Universal coefficients of S_A scaling are **monotonic** under RG!



QED₃: breaking of $U(2N) \rightarrow U(N) \times U(N)$?

Compute \tilde{F} in $d = 2 + \epsilon$:

[Giombi, Klebanov, Tarnopolsky '16]

$$\text{UV: } \tilde{F}_{\text{UV}} = \underbrace{N\tilde{F}_f}_{\text{fermions}} + \underbrace{(d-2)\tilde{F}_b}_{\text{photon}} = N\frac{\pi}{3} + \mathcal{O}(\epsilon)$$

\tilde{F}_f : free fermion
 \tilde{F}_b : free boson

$$\text{cQED: } \tilde{F}_{\text{conf}} = (N - \frac{1}{2})\tilde{F}_f = \underbrace{(2N-1)}_{\text{cSchwinger } \checkmark} \frac{\pi}{6} + \mathcal{O}(\epsilon)$$

$$\chi\text{SB: } \tilde{F}_{\chi\text{SB}} = (2N^2 + (d-2))\tilde{F}_b = N^2\frac{\pi}{3} + \mathcal{O}(\epsilon)$$

$$\Rightarrow \tilde{F}_{\chi\text{SB}} > \tilde{F}_{\text{UV}} > \tilde{F}_{\text{conf}} \quad \text{for all } N > 1$$

Hence:

$$N_c^{\chi\text{SB}} \leq 1 + \mathcal{O}(d-2) < N_c^{\text{conf}}$$

... the phase below N_c^{conf} cannot exhibit χSB !

[LJ, arXiv:1604.06354]

Intermediate phase

Vafa-Witten theorem: QED_3 should have

[Vafa & Witten '84]

- (a) unbroken $U(N) \times U(N)$ symmetry, and
... rules out other chiral breaking patterns
- (b) gapless spectrum
... rules out plain parity breaking

... at least

What else can it be?

Mean-field theory

Recall flow ...

... towards divergent g_2 (and finite e^2 , g_1)!

Effective description: **Thirring** model!

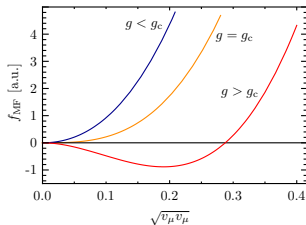
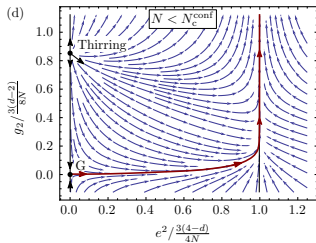
$$S_{\text{Thirring}} = \int d^d x \left[\bar{\psi}_i \gamma_\mu \partial_\mu \psi_i + g_2 (\bar{\psi}_i \gamma_\mu \psi_i)^2 \right]$$

Large- N (saddle-point) solution:

$$f_{\text{MF}}(v_\mu) = \frac{1}{2} \left(\frac{1}{g_2} - \frac{40N}{3} \right) v_\mu^2 + \sqrt{6}\pi N (v_\mu^2)^{3/2} + \mathcal{O}(v^4)$$

for **vector** order parameter $v_\mu \propto \langle \bar{\psi}_i \gamma_\mu \psi_i \rangle$

\Rightarrow transition towards $v_\mu \neq 0$: spontaneous **Lorentz** symmetry breaking!



... spontaneous formation of charge and/or current
... consistent with Vafa-Witten and F theorem

Susceptibility analysis in $d = 2 + \epsilon$: bilinear terms

Add small symmetry-breaking “seeds”:

... à la magnetic field

$$S_{\Delta} = \int d^d x \bar{\psi}_i [\Delta_{\chi\text{SB}} \mathbb{1}_4 + \Delta_{\text{PSB}} \gamma_{35} + i\Delta_{\text{Kek}} (\gamma_3 \cos \varphi + \gamma_5 \sin \varphi) + i\Delta_{\text{LSB}}^{\mu} \gamma_{\mu}] \psi_i$$

Flow of bilinears:

$$\partial_t \Delta_{\alpha} = (-1 + \eta_{\psi}) \Delta_{\alpha} + \times^{\Delta_{\alpha}} \circlearrowleft_{g1} + \times^{\Delta_{\alpha}} \circlearrowleft_{g2} + \times^{\Delta_{\alpha}} \text{wavy}$$
$$\equiv -x_{\alpha} \Delta_{\alpha} + \mathcal{O}(\Delta^2)$$

... with $\Delta_{\alpha} \in \{\Delta_{\chi\text{SB}}, \Delta_{\text{PSB}}, \Delta_{\text{Kek}}, \Delta_{\text{LSB}}^{\mu}\}$

Critical exponents in $d = 2 + \epsilon$

Scaling form of dimensionless free energy:

$$f(\delta\vec{g}, \Delta_\alpha) = b^{-d} f(b^y \delta\vec{g}, b^{x_\alpha} \Delta_\alpha)$$

$$\Rightarrow \boxed{\gamma_\alpha = \frac{2x_\alpha - d}{y}} \quad \& \quad \boxed{\nu = \frac{1}{y}}$$

... where $b \equiv \frac{\Lambda}{k} = e^{-t}$
 $\partial_t \delta\vec{g} = -y \delta\vec{g} + \mathcal{O}(\delta g^2)$
 $\partial_t \Delta_\alpha = -x_\alpha \Delta_\alpha$

$N \lesssim N_c^{\text{conf}}$: Flow “hovers” over complex pair of fixed points QCP/cQED:

\Rightarrow dominant order $\hat{=}$ largest γ at QCP

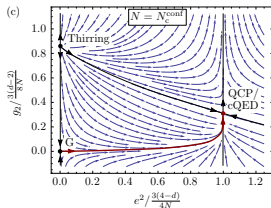
$$1/\nu = \epsilon \sqrt{1 - (N_c^{\text{conf}}/N)^{3/2}} + \mathcal{O}(\epsilon^2),$$

$$\gamma_{\chi\text{SB}} < 0, \quad \gamma_{\text{PSB}} < 0, \quad \gamma_{\text{Kek}} < 0,$$

... superconducting gaps also lead to $\gamma_{\text{SC}} < 0$

$$\boxed{\gamma_{\text{LSB}} = 1 + \mathcal{O}(\epsilon) > 0 \quad \color{red}{!}}$$

\Rightarrow instability towards spontaneous Lorentz symmetry breaking!



Towards $d = 3$

In $d = 3$ and finite N : strong-coupling problem

... difficult ...

$N_c^{\chi\text{SB}}$ controlled by F theorem:

$$F_{\text{conf}} = NF_f + \frac{1}{2} \log \left(\frac{\pi N}{4} \right) + \mathcal{O}(N^{-1})$$

[Klebanov, Pufu, Sachdev, Safdi '12]

$$F_{\chi\text{SB}} = (2N^2 + 1)F_b$$

$$\Rightarrow \boxed{N_c^{\chi\text{SB}} \leq 4.422}$$

in $< 1\%$ agreement with bound from $4 - \epsilon$ expansion of \tilde{F}_{conf} :
[Giombi, Klebanov, Tarnopolsky '16]

N_c^{conf} by RG flow:

naive extrapolation to $\epsilon \rightarrow 1$: $N_c^{\text{conf}} \simeq 8\sqrt[3]{2} \approx 10.1$

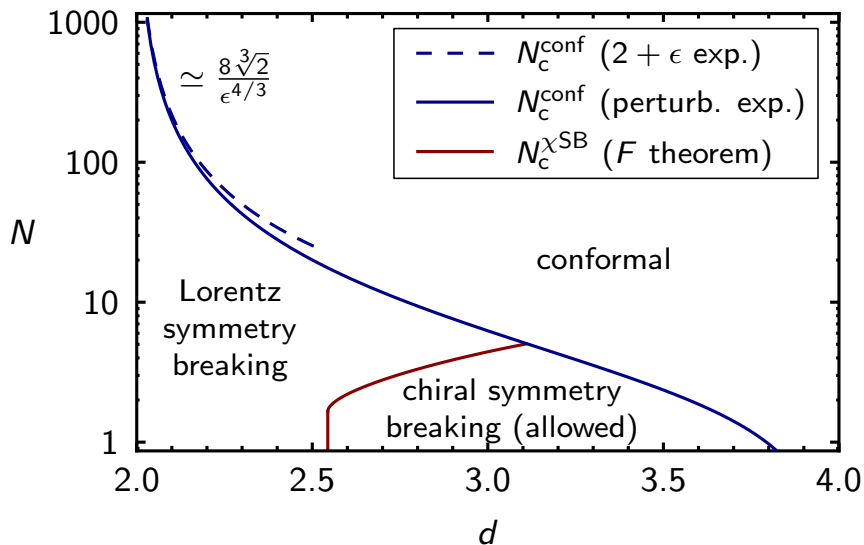
one-loop in fixed $d = 3$: $N_c^{\text{conf}} \approx 6.24$

functional RG: $N_c^{\text{conf}} \approx 4.1 \dots 10.0$ [Braun, Gies, LJ, Roscher '14]

$$\boxed{N_c^{\chi\text{SB}} < N_c^{\text{conf}} \text{ possibly even in } d = 3!}$$

... higher-order calculations necessary

Phase diagram of $\text{QED}_{2 < d < 4}$



Conclusions

- (1) QED₃ has **conformal** ground state at large $N > N_c^{\text{conf}} = \frac{8\sqrt[3]{2}}{\epsilon^{4/3}}$
... in $d = 2 + \epsilon$
- (2) Common scenario with direct transition towards χ^{SB} **inconsistent** with (generalized) F theorem
... at least in $d = 2 + \epsilon$
... there are just too many Goldstones!
- (3) Only possibility for $N \lesssim N_c^{\text{conf}}$:
spontaneous **Lorentz** symmetry breaking (LSB)!
... compatible with F theorem & Vafa-Witten
- (4) Mean-field & susceptibility analyses:
confirm existence of LSB phase
... independently
- (5) Extrapolation $\epsilon \rightarrow 1$ & perturbative expansion in fixed $d = 3$:
suggest finite window of LSB phase also in physical dimension