

# On the spectrum of light mesons in an SU(2) gauge theory with dynamical fermions

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8th International Conference on the Exact Renormalization Group,  
Trieste, Sept. 19, 2016



# Outline

- 1 Motivation: A Minimal Template for Extensions of the SM
- 2 Fermion—gauge-boson vertex
- 3 Numerical results for bound state masses
- 4 Conclusions and Outlook

# Motivation: A Minimal Test Case for Model Building

SU(2) gauge theory with two fundamentally charged Dirac fermions

- simplest field theoretical realization of unified theory of **Composite Goldstone Boson Higgs** and **Technicolor**
- bosonic mesons and “baryons” (Pauli-Gürsey symmetry)
- dynamically broken chiral symmetry  $SU(4) \rightarrow SP(4) \sim SO(5)$ :  
*five Goldstone bosons in chiral limit*
- breaking direction w.r.t. SM:
  - vanishing angle: Composite Goldstone Boson Higgs  
four GB: (massless) complex Higgs doublet; 5th: SM neutral
  - maximal angle: Technicolor Theory  
EW symmetry completely broken, 3 GB  $\rightarrow$  long. W/Z states,  
Higgs = lightest scalar meson, 2 GB: dark matter candidates
- angle determined dynamically:  
GB Higgs mixes with technicolor scalar meson  
 $\Rightarrow$  two scalars with lighter one = Higgs<sup>1</sup>

<sup>1</sup>Back-coupling to SM particles will lead to large corrections!



# Motivation

Spectrum of this theory has been calculated using lattice simulations

[R. Arthur *et al.* [CP<sup>3</sup>], arXiv:1602.06559 [hep-lat]; arXiv:1607.06654 [hep-lat]; and references therein]

with main conclusion:

Significantly different spectrum than in  $N_f = 2$  SU(3) gauge theory  
(i.e. QCD)!

Try to understand the spectra by using another non-perturb. approach!

# Motivation

Functional (continuum) methods include:

- Dyson-Schwinger — Bethe-Salpeter — cov. Faddeev eqs.
- $nPI$  effective actions
- scale-dependent bosonisation in ERG

see poster by Jordi Paris Lopez

- 😊 Suitable for multi-scale problems! (Proximity of conformal window)
- 😊 Bound-state formation in terms of microscopic d.o.f.!
- 😢 Truncation is necessary

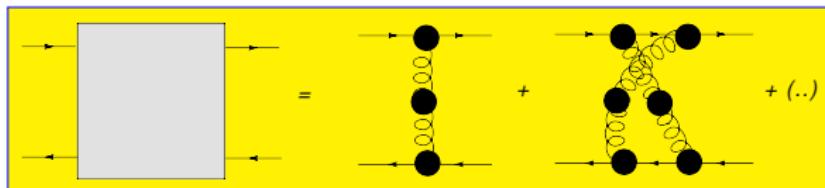


# Fermion—gauge-boson vertex

Truncation?

Refer to symmetries expressed via Ward ids. and/or nPI actions ...

Example: 3PI  $\rightarrow$  Kernel of the fermion-antifermion Bethe-Salpeter eq.



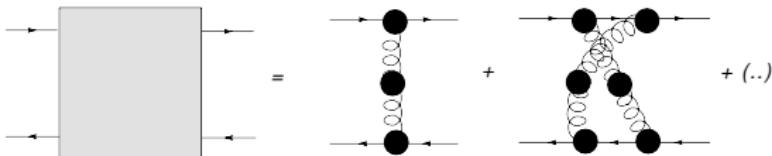
R.A., C.S. Fischer, F.J. Llanes-Estrada, Mod.Phys.Lett. **A23** (2008) 1105;  
H.Sanchis-Alepuz, R.Williams, J.Phys.Conf.Ser.631 (2015)012064 [arXiv:1503.05896]

**Fully dressed fermion—gauge-boson vertices!**



# Fermion—gauge-boson vertex

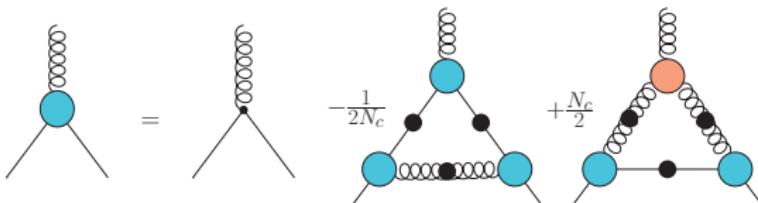
Kernel of  $\psi\bar{\psi}$  BS  
equation via 3PI action:



Needed input:

Gauge-boson propagator and **fermion—gauge-boson vertex!**

Eq. from 3PI action:



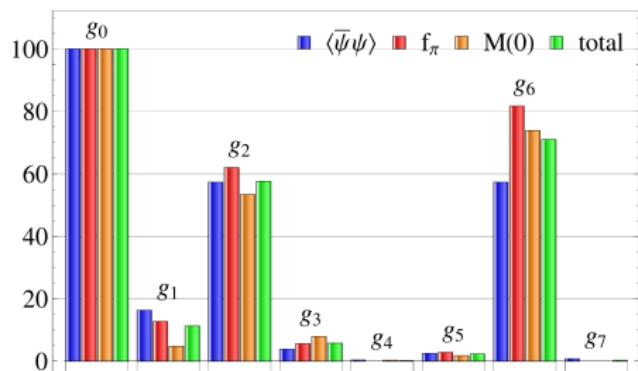
Needed input:

Fermion & gauge-boson propagator and **3-gauge-boson vertex!**

⚠ These 2- & 3-point functions are needed for complex arguments  
to describe bound states with physical masses (timelike momentum)!

# Fermion—gauge-boson vertex

Relative importance of the eight transverse tensor structures:



dynamically generated  $\chi$ SB coupling important!

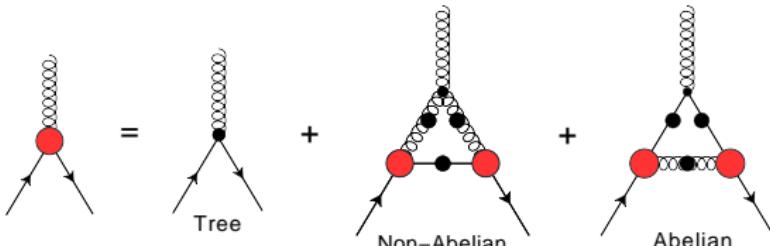
⚠ Technically required truncation of the fermion–gauge boson vertex has to be chosen very careful!



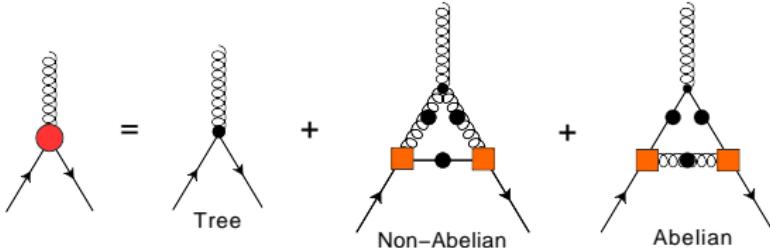
# Fermion—gauge-boson vertex

## Truncation I:

1PI-based

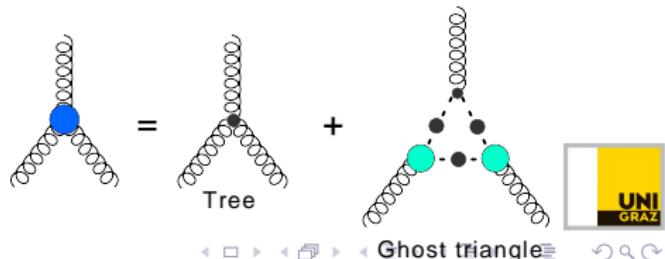


but not fully  
self-consistent

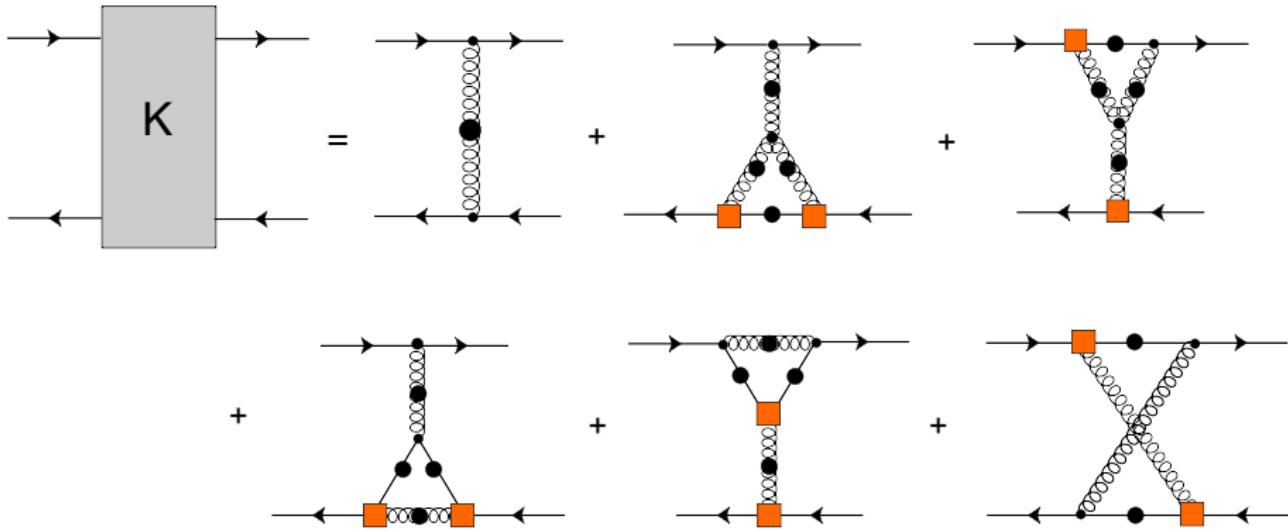


## Truncation II:

“3PI”, partially self-consistent,  
simplified 3-gauge-boson vertex



# Bound state equation

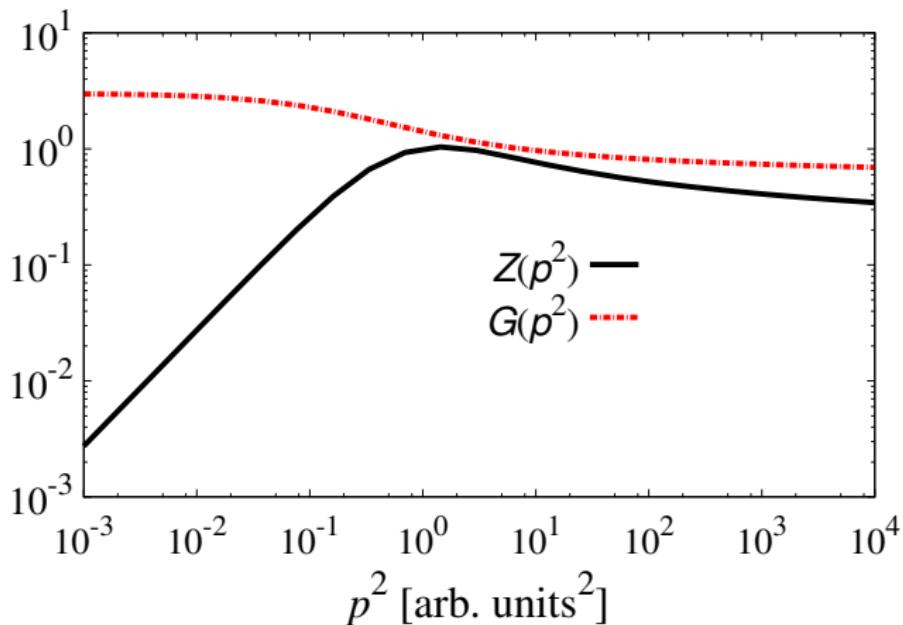


In truncation II (*i.e.*, 3PI-type):

Additional dressings for the 3-gauge-boson and the fermion–gauge-boson vertices.

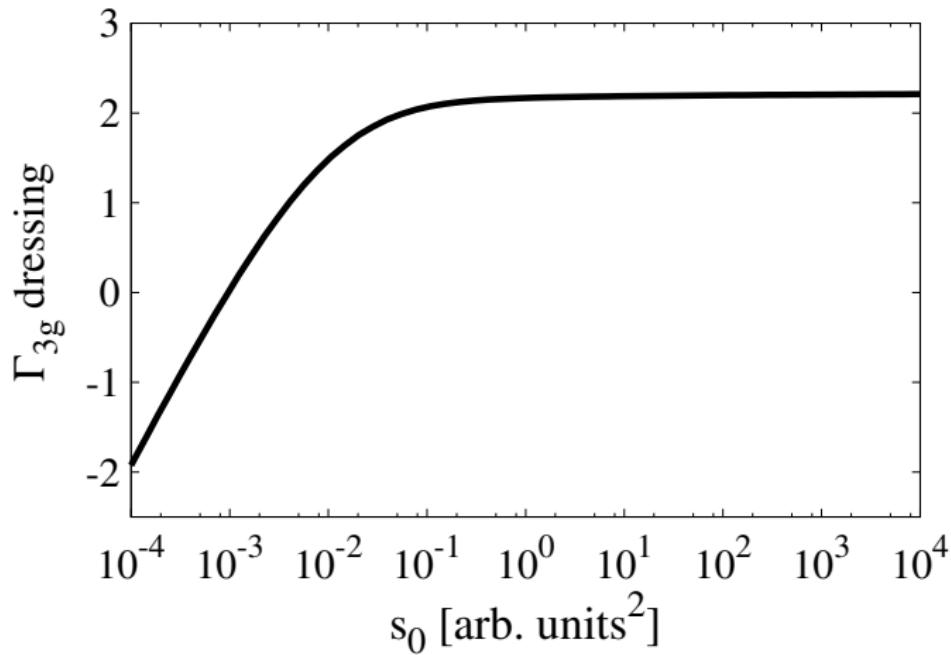
# Numerical results for input

Gauge-boson and ghost renormalization function



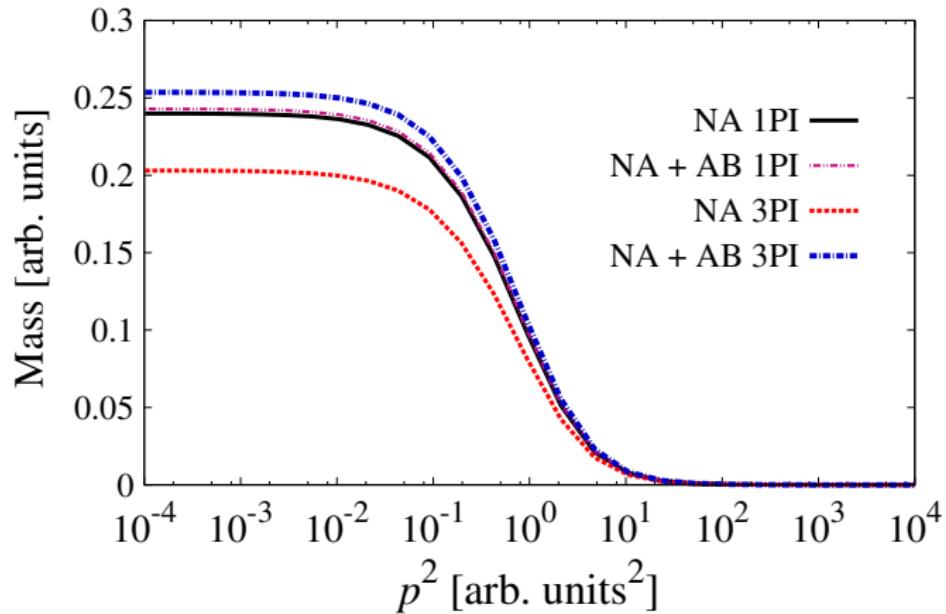
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3-gauge-boson vertex as function of  $s_0 = \frac{1}{6}(p_1^2 + p_2^2 + p_3^2)$ :



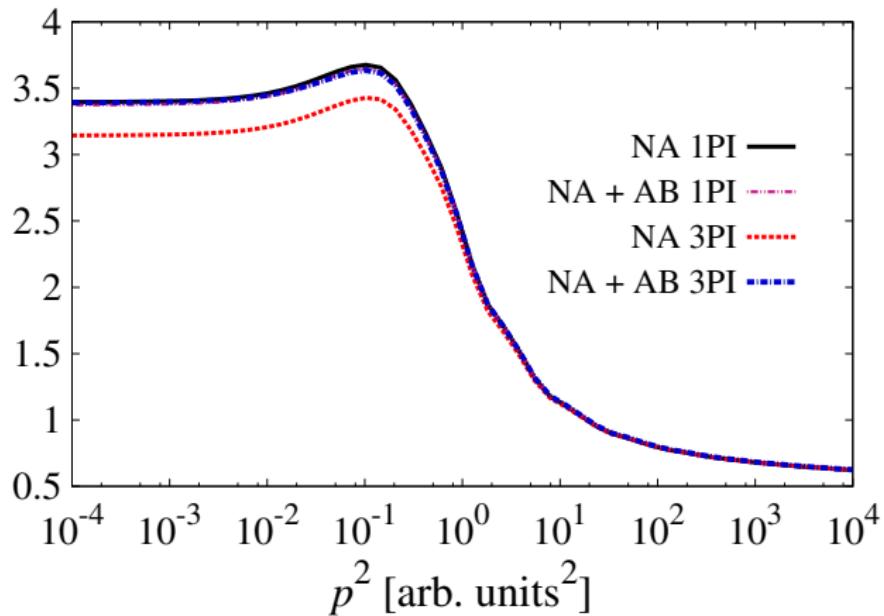
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## Quark mass function



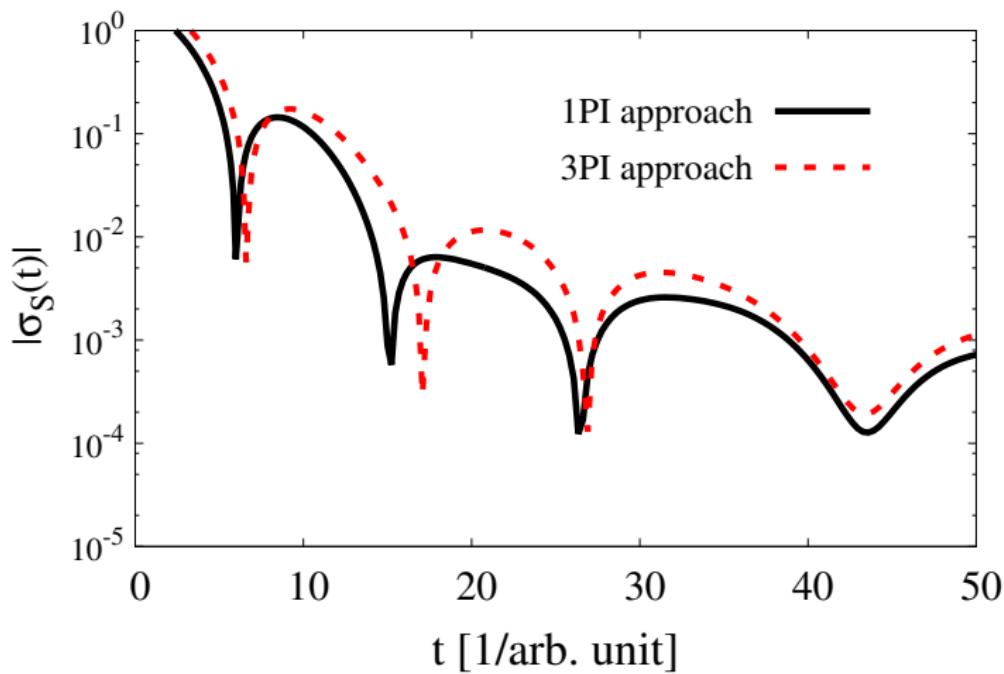
# Numerical results for input

## Ledaing tensor structure fermion–gauge-boson vertex



# Numerical results for input

## Positivity violation for quark propagator



# Numerical results for bound state masses

Scale set by  $f_{PS} = 0.246$  TeV

$J^{PC}$	NA, 1PI	NA + AB, 1PI	NA, "3PI "	NA + AB, "3PI"	Lattice
$0^{-+}$	0	0	0	0	–
$0^{++}$	1.39(3)	1.22(2)	1.33(3)	1.25(2)	4.7 (2.7)
$1^{--}$	2.27(5)	2.00(4)	2.37(5)	1.99(4)	3.2(5)
$1^{++}$	2.87(5)	2.65(5)	3.09(6)	2.67(5)	3.6(9)

Lattice:

R. Arthur *et al.* [CP<sup>3</sup>], arXiv:1602.06559 [hep-lat]; arXiv:1607.06654 [hep-lat];  
and references therein



# Numerical results for bound state masses

Light scalar meson in (unquenched) QCD from the 3PI effective action:

	RL	2PI - 3L	3PI - 3L		$m_{1--}/f_\pi$	$m_{1++}/f_\pi$
$m_{0^{++}}/f_\pi$	6.96	5.05	$10.5 \pm 1.0$			
				3PI-3L	7.0	$12.4 \pm 1.0$

R. Williams, C.S. Fischer, W. Heupel, Phys.Rev. D93 (2016) 034026

Light scalar meson in SU(2) gauge theory with two light flavours:

	1PI	3PI-type		$m_{1--}/f_\pi$	$m_{1++}/f_\pi$
$m_{0^{++}}/f_{PS}$	$5.0 \pm 0.1$	$5.1 \pm 0.1$			
			3PI-type	$8.1 \pm 0.2$	$10.9 \pm 0.2$

R.A., M. Vujinovic, to be published

<sup>†</sup> with non-Abelian diagram only:  $5.7 \pm 0.1$  and  $5.4 \pm 0.1$



# Conclusions and Outlook

- ☺ QCD(-like) gauge theories in Landau gauge:  
Established results for propagators &  
recent investigations of three-point functions
- ☹ No reliable quantitative results for bound states without  
self-consistent fermion–gauge-boson vertex!
- ☺☺ Unexpected results for SU(2) bound state spectrum ...  
Significantly lower masses than corresponding lattice values!  
Significantly different from SU(3) / QCD ?!
- ☺ Approach merging different functional methods  
to be applied soon to different gauge theories / matter content:  
Dynamical electroweak symmetry breaking  
& strongly-interacting dark matter.

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