Perturbative study of the QCD phase diagram with heavy quarks

Julien Serreau – Univ. Paris Diderot

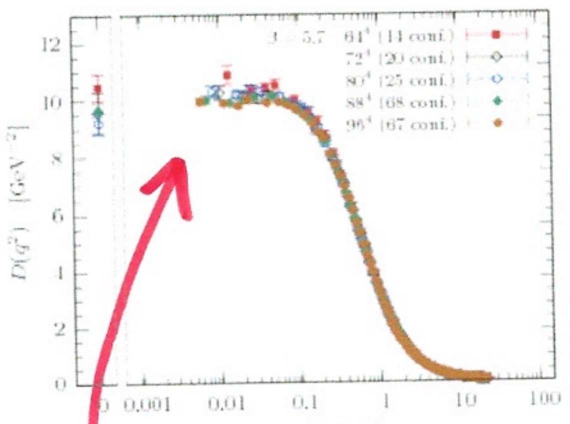
- U. Reinosa
- M. Tissier
- N. Wschebor

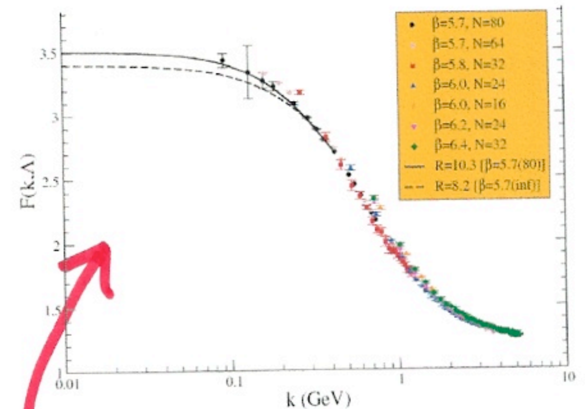
• M. Peláez
• A. Tresmontant

MOTIVATION

Lattice calculations of gluon and ghost correlators in the Landau gauge

⇒ "Decoupling" behavior





"Massive" gluon

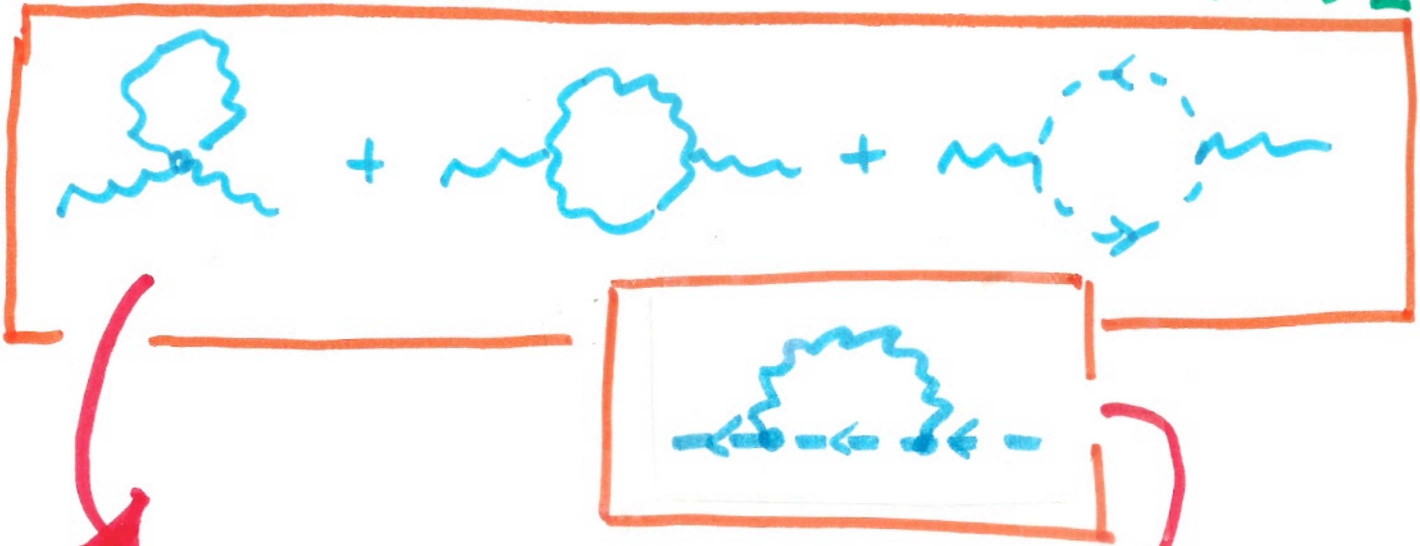
Massless ghost

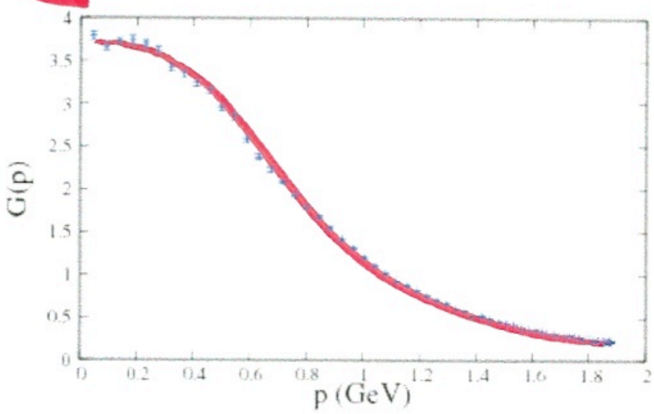
A massive model (Curci-Ferrari)

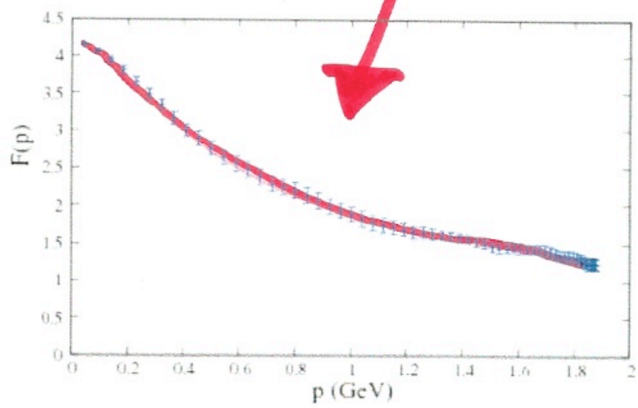
$$S=\int_x\left\{\underbrace{\frac{F^2}{4}+ih\partial A+\partial\bar{c}Dc}_{\text{Fadeev-Popov}}+\boxed{\frac{m^2}{2}A^2}\right\}$$

One-loop calculation

[Tissier, Wschebor ('10)]







- excellent agreement with lattice in $d=4$

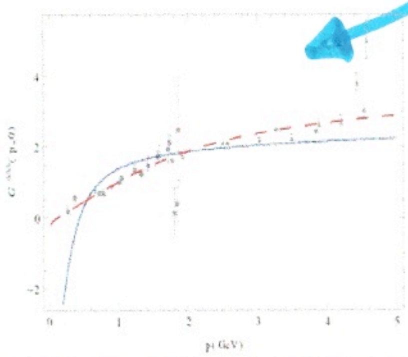
  $m = 680\,\text{MeV}$, $g = 7.5$

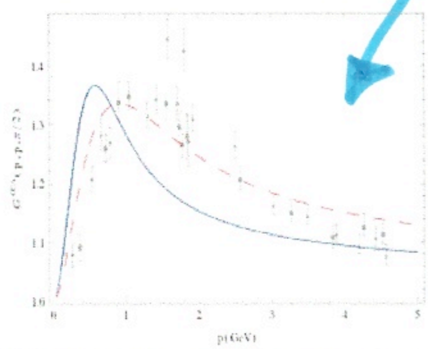
- qualitative agreement in $d=3$, $d=2$

Massive model : more

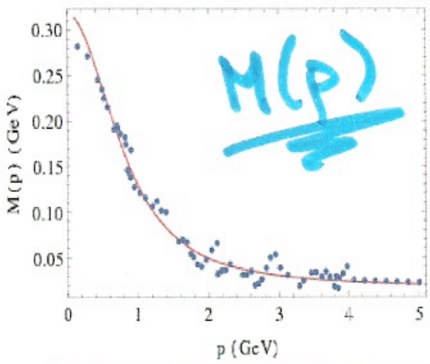
- higher correlators : $\langle AAA \rangle$, $\langle c\bar{c}A \rangle$

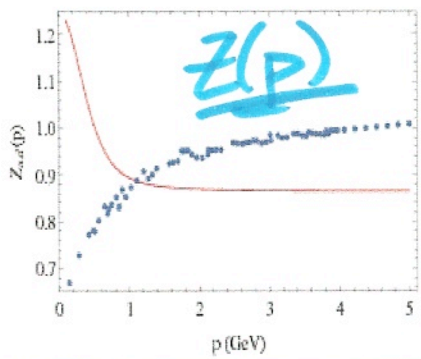
[Peláez, Tissier, Wschebor ('13)]



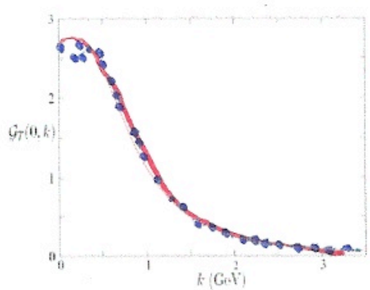


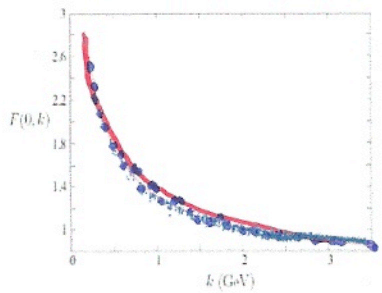
- Dynamical quarks [Peláez et al. ('14)]

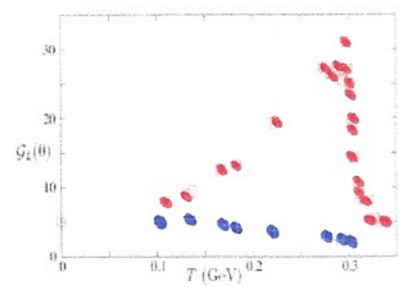




- Finite temperature







[Reinosa, Serreau, Tissier, Wschebor ('14)]

Massive model

- One-loop calculations compare well with lattice data (Landau gauge)

- Also compare well with other continuum approaches

e.g.
- (refined) Gribov-Zwanziger [Dudal et al. ('08)]
- Functional RG [Fisher, Maas, Pawlowski ('09); Fister Pawlowski ('11), Cyrol et al ('16) ...]
- Dyson-Schwinger Eqs. [Aguilar et al. ('08); Boucaud et al. ('08); Fisher et al. ('09); Huber et al. ('14) ...]

Deconfinement transition from (modified) pert. theory

[Reinosa, Serreau, Tissier, Wschebor, PLB 2015]

- (de) confinement of static quarks measured by Z_N sym. breaking

$\Rightarrow$ Polyakov loop : $\ell = \frac{1}{N} \mathrm{tr} \left\langle e^{ig\int A_0} \right\rangle$

- Background field methods
[Braun, Eichhorn, Gies, Pawlowski ('10)]

Landau-DeWitt gauge

$$\boxed{\begin{aligned} &\bar{D}a = 0 \\ \text{with} \quad &\bar{D} = \partial + ig\bar{A} \\ \text{and} \quad &a = A - \bar{A} \end{aligned}}$$

$$\boxed{\tilde{\Gamma}(\bar{A}) = \Gamma(\bar{A}, a=0)}$$

Background field gauge invariance

Massive Landau-DeWitt gauge

$$S = \int_x \Big\{ \underbrace{\frac{E^2}{2} + ih\bar{D}a + \bar{D}\bar{c}\,Dc}_{\text{Fadeev-Popov}} + \boxed{\frac{m^2}{2}a^2} \Big\}$$

Respects BF gauge invariance

"Lifting" procedure

Gribov copies:

$$F(\bar{A},a) = \int_x a^2$$

extrema : $\boxed{\bar{D}a = 0}$

[Cucchieri, Mendes ('12)]

Physical observables : Absolute min. of $\tilde{\Gamma}(\bar{A})$

Perturbative calculation of the background field potential

Static temporal background

$$\bar{A}_\mu(x) = \delta_{\mu 0}\,\bar{A}_0 \quad , \quad \bar{A}_0 \in \text{Cartan}$$

⚠ Not Polyakov gauge

$$\boxed{V(r) = \frac{1}{\beta\,\mathrm{Vol.}}\,\tilde{\Gamma}(\bar{A}) \qquad r = \beta g \bar{A}}$$

<u>One-loop :</u>

$$V(r) = \frac{1}{\beta\,\mathrm{Vol.}}\left\{\frac{1}{2}\,\mathrm{Tr}\,\mathrm{Ln}\,\underbrace{G^{-1}}_{\text{gluons}} - \mathrm{Tr}\,\mathrm{Ln}\,\underbrace{D^{-1}}_{\text{ghost}}\right\}$$

Results : SU(2)

$$V^{SU(2)}(r) = \frac{3}{2} F_m(r) - \frac{1}{2} F_0(r) \quad ; \quad r \in [0,\pi]$$

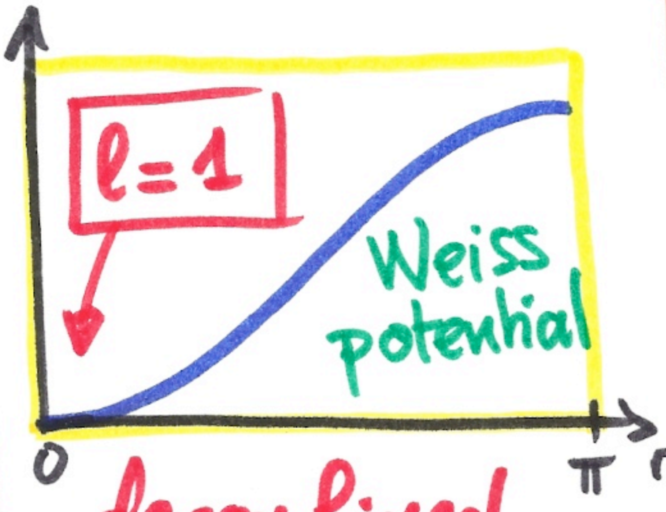
massive modes

massless modes

$$F_m(r) = \frac{T}{\pi^2} \int_0^\infty dq\, q^2 \Big\{ \ln\left(1 + e^{-2\beta\varepsilon_q} - 2 e^{-\beta\varepsilon_q} \cos r\right) + \ln\left(1 + e^{-\beta\varepsilon_q}\right) \Big\}$$

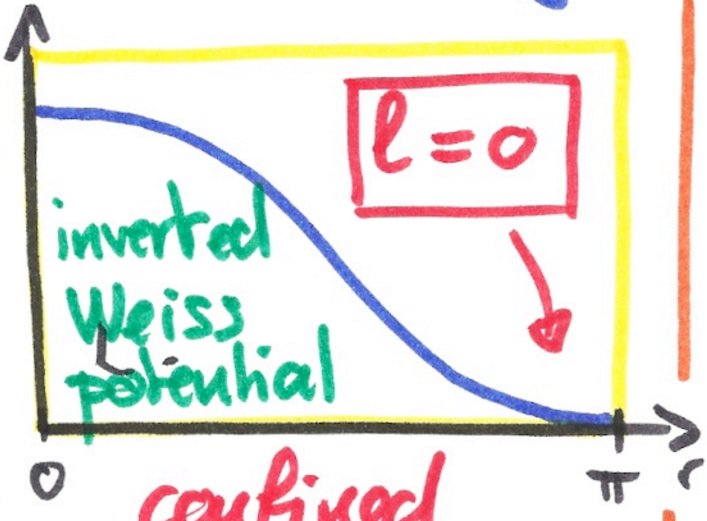
$$\varepsilon_q = \sqrt{q^2 + m^2}$$

$T \gg m : V \approx F_0(r)$



deconfined

$T \ll m : V \approx -\frac{1}{2} F_0(r)$

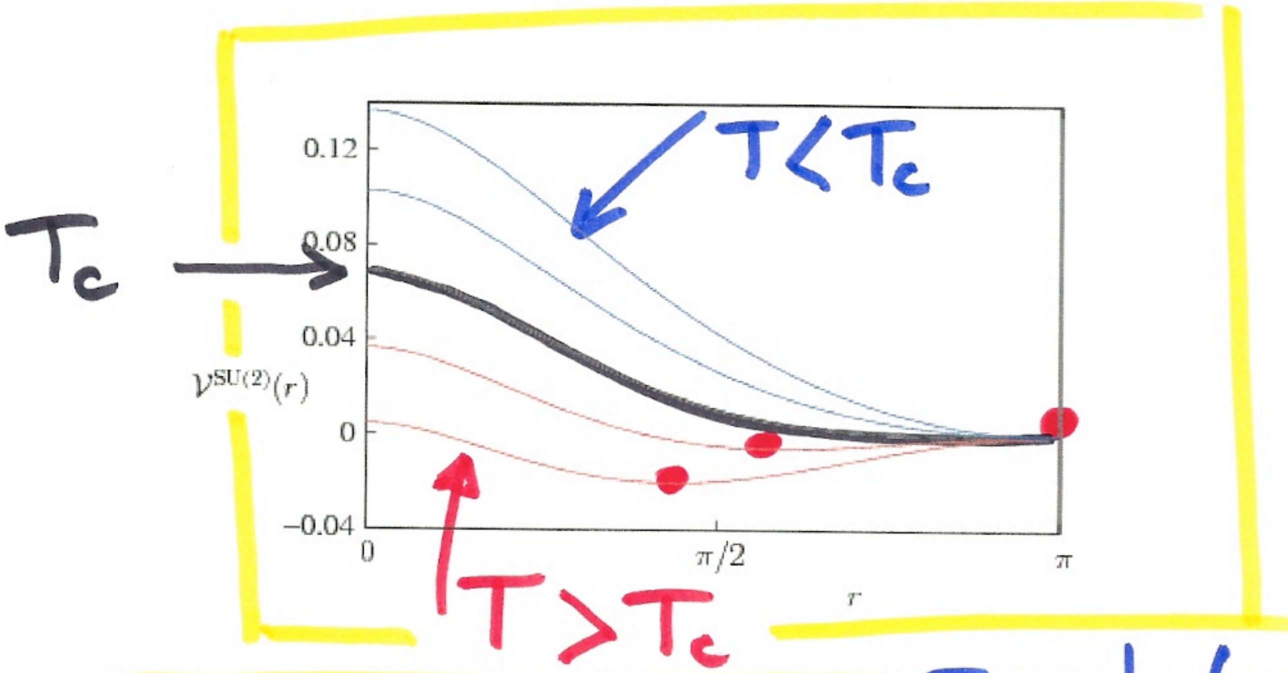


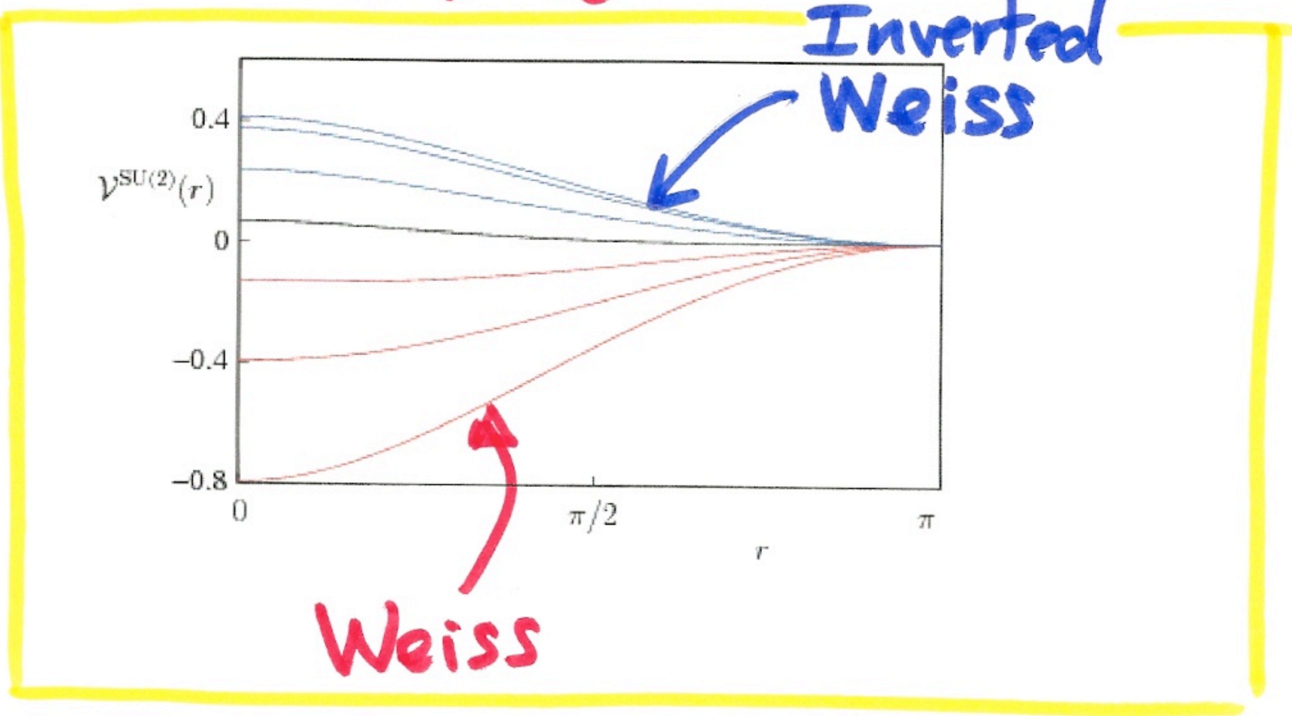
confined

see also [Braun, Gies, Pawlowski ('10)]

Results : SU(2)

2nd order phase transition

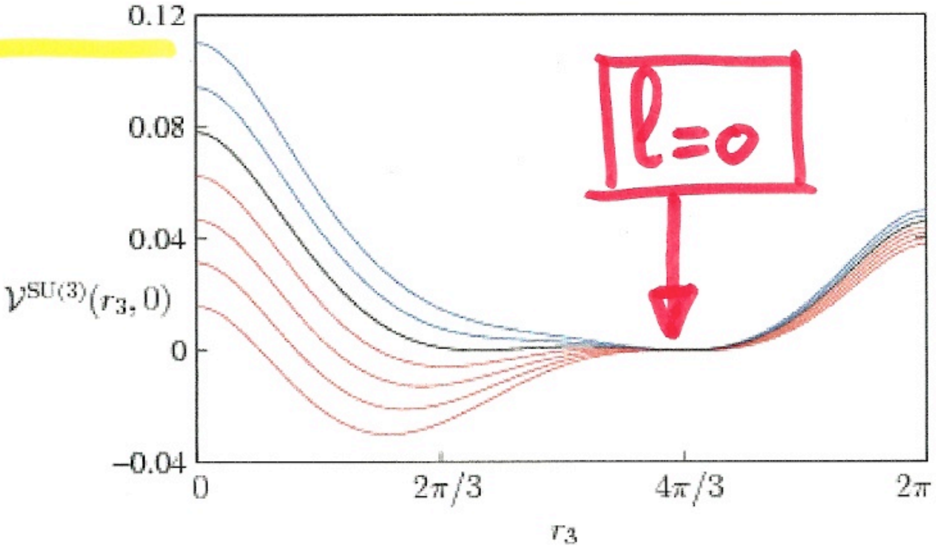


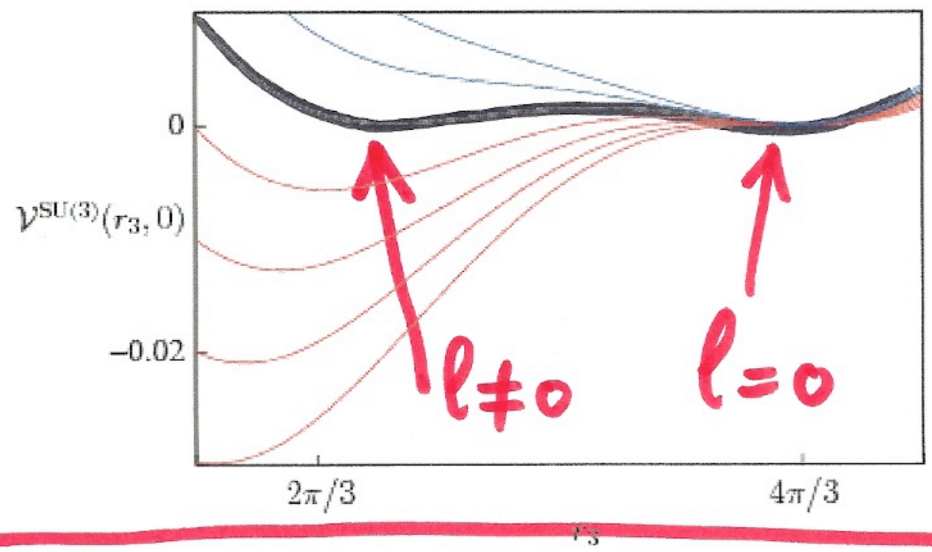


Results : SU(3)

$r \in \{r_3, r_8\}$

$$V^{SU(3)}(r_3, r_8=0) = V^{SU(2)}(r_3) + 2V^{SU(2)}(r_3/2) + \text{const.}$$

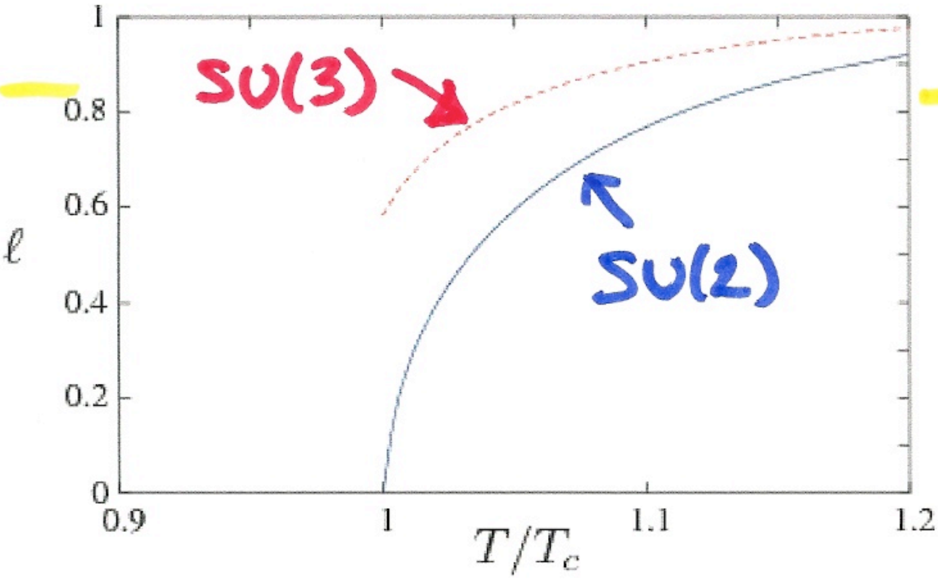




$\Rightarrow$ $\boxed{1^{st} \text{ order phase transition}}$

Results : Summary

$$\ell(r) = \frac{1}{N}\mathrm{tr}\left[e^{ir^a t^a}\right] + \mathcal{O}(g^2)$$



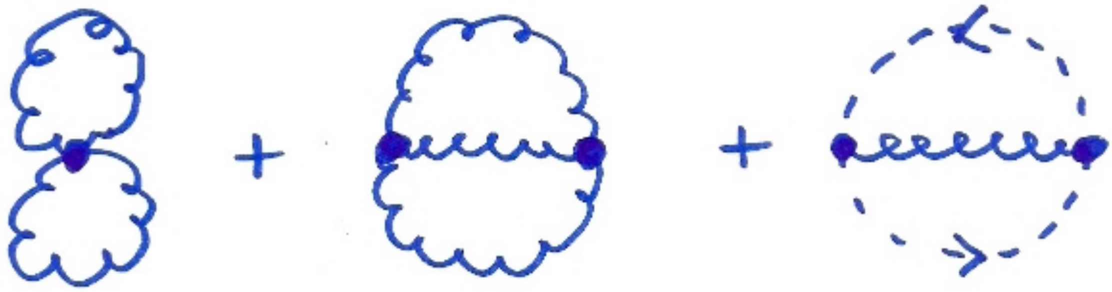
	T_c/m	m	T_c	$T_c^{\text{latt.}}$ *	T_c^{FRG} **
SU(2)	0.33	710	238	295	230
SU(3)	0.36	510	185	270	275

(All in MeV)

* [Lucini, Panero ('13)]
** [Fister, Pawlowski ('13)]

Perturbative results : Two loop

[Reinosa, Serreau, Tissier, Wschebor, PRD 2015 - 2016]



+ Values of m and g from vacuum propagators

	$T_c^{1\,\text{loop}}$	$T_c^{2\,\text{loop}}$	$T_c^{\text{latt.}}$ *	T_c^{FRG} **
SU(2)	238	284	295	230 (300)
SU(3)	185	254	270	275

(All in MeV)

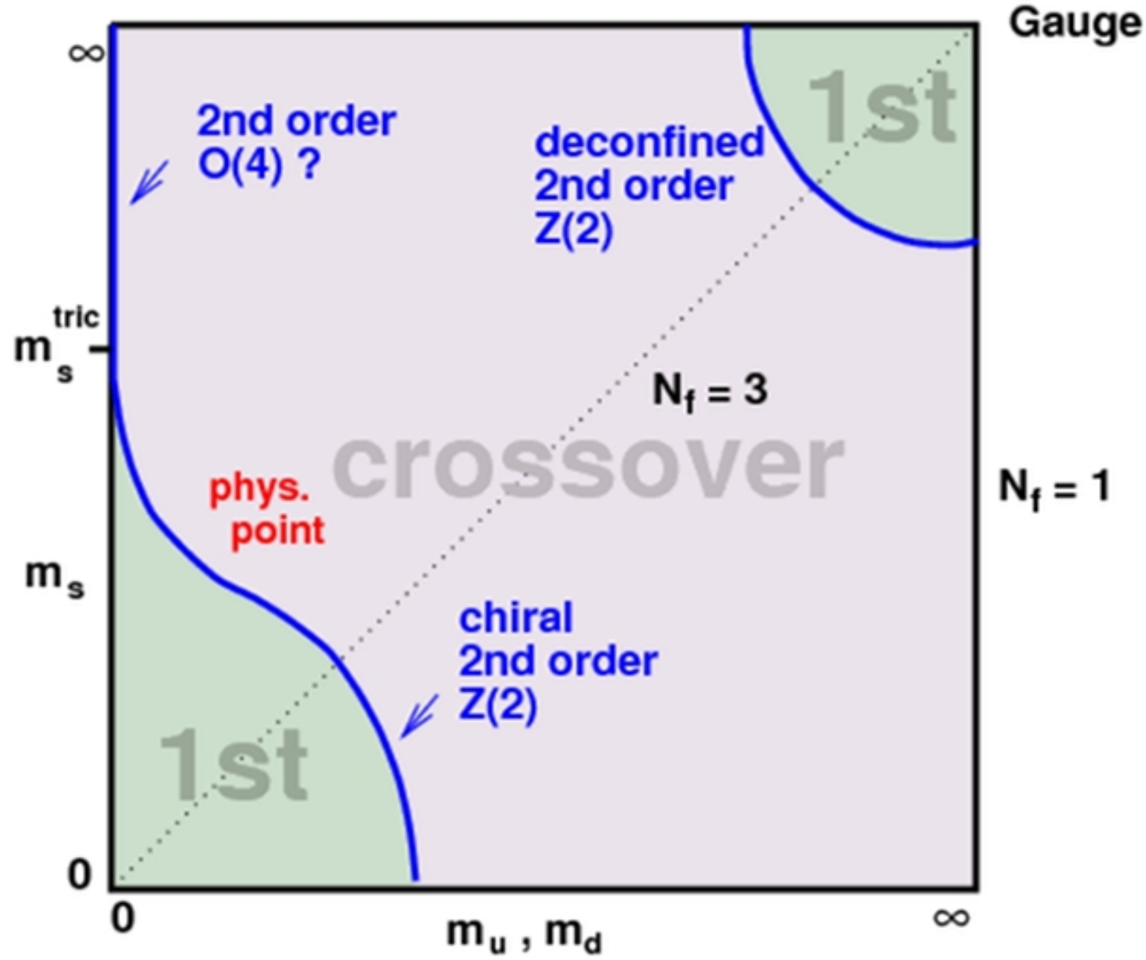
QCD with heavy quarks: finite T and μ

[Reinosa, Serreau, Tissier, PRD 2015]

$$S = S_{gauge} + \int_x \bar{\Psi}_i (\not{D} + M + \mu \gamma_0)_{ij} \Psi_j$$

Background field gauge + $\underline{\underline{m}}^2 a_\mu a_\mu$

N_f flavors + nonzero μ.



Order parameters

Polyakov loops

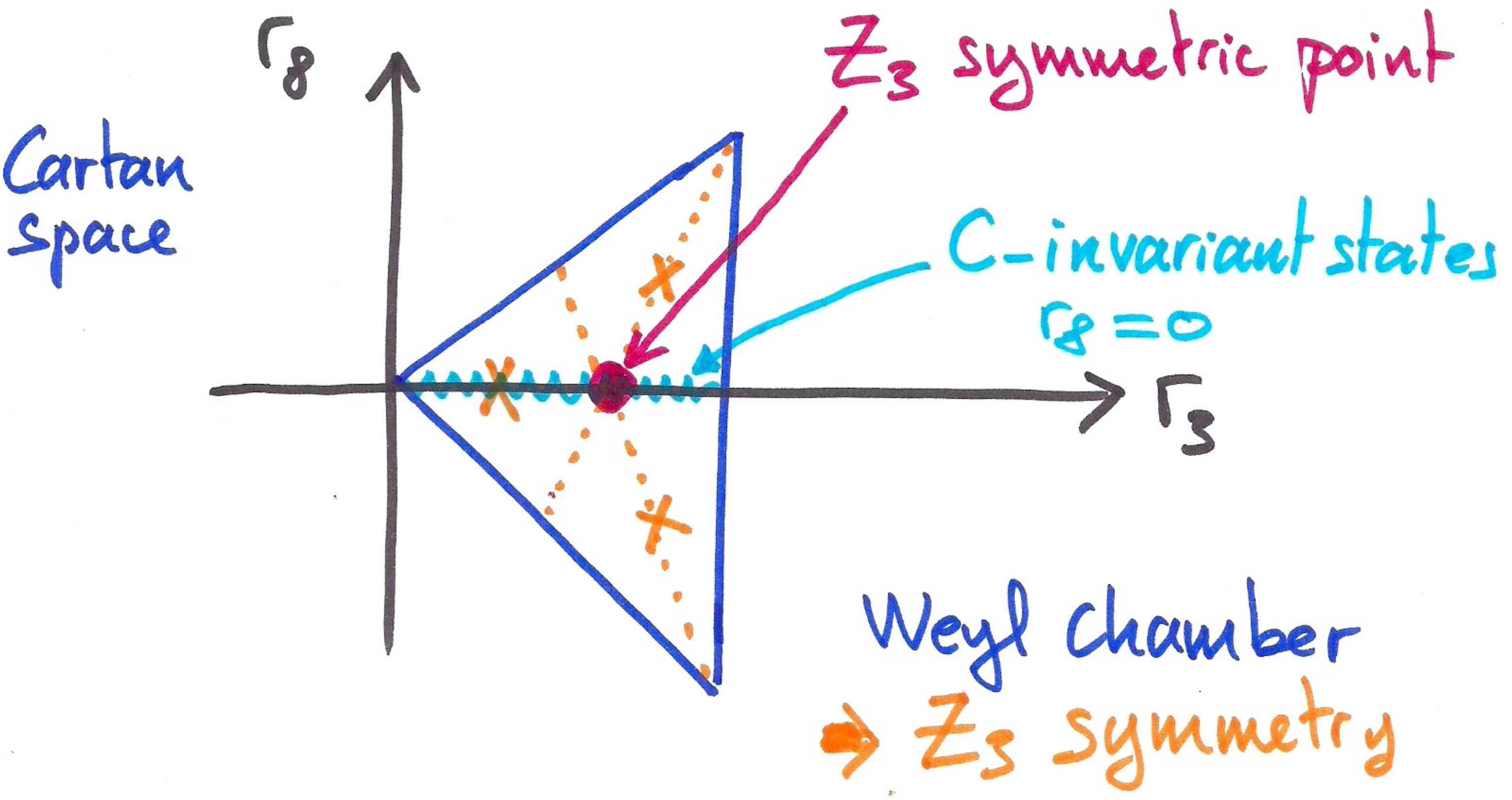
$$\ell = \frac{1}{3}\,\mathrm{tr}\left\langle P e^{ig\int_0^\beta A_0}\right\rangle \; ; \; \bar{\ell} = \frac{1}{3}\,\mathrm{tr}\left\langle \bar{P} e^{-ig\int_0^\beta A_0}\right\rangle$$

Equivalently :

$$\boxed{r = \beta g \bar{A}_{min}}$$

: Minimum of the background field potential

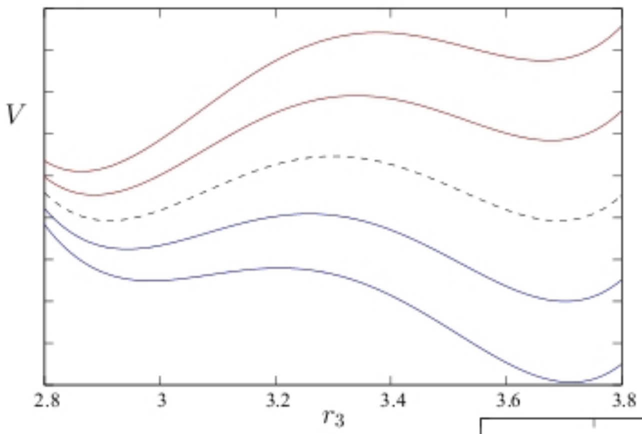
[Reinosa et al. ('16) ; Braun et al. ('10)]



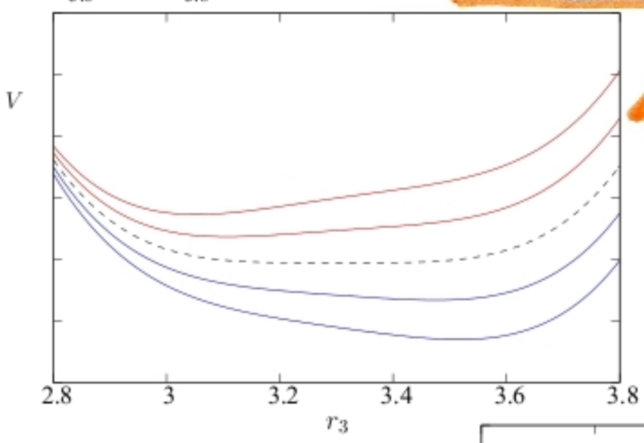
Background field potential at one-loop order

$$V(\underline{r}) = V_{glue}(\underline{r}) + \mathrm{Tr}\,\mathrm{Ln}\left(\underline{\underline{\not{D}}} + M + \mu\gamma_0\right)^{-1}$$

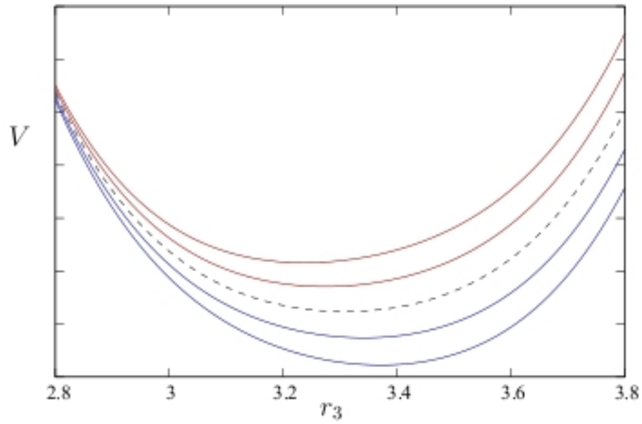
$M > M_c$: 1^{st} order



$M = M_c$: 2^{nd} order



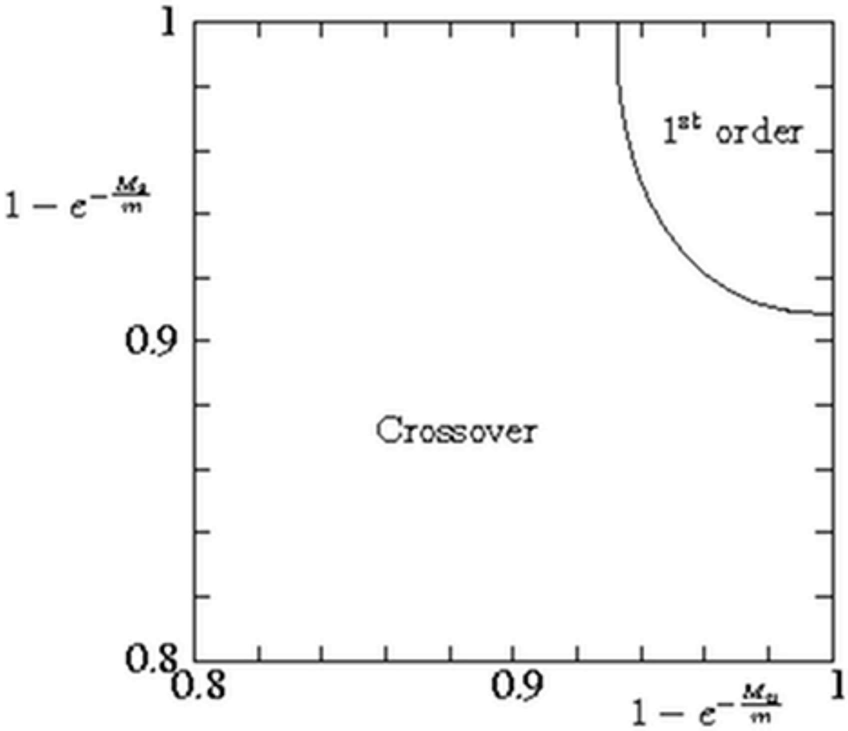
$\mu = 0$



$M < M_c$: crossover

One-loop results : $\mu = 0$

1+2 flavors

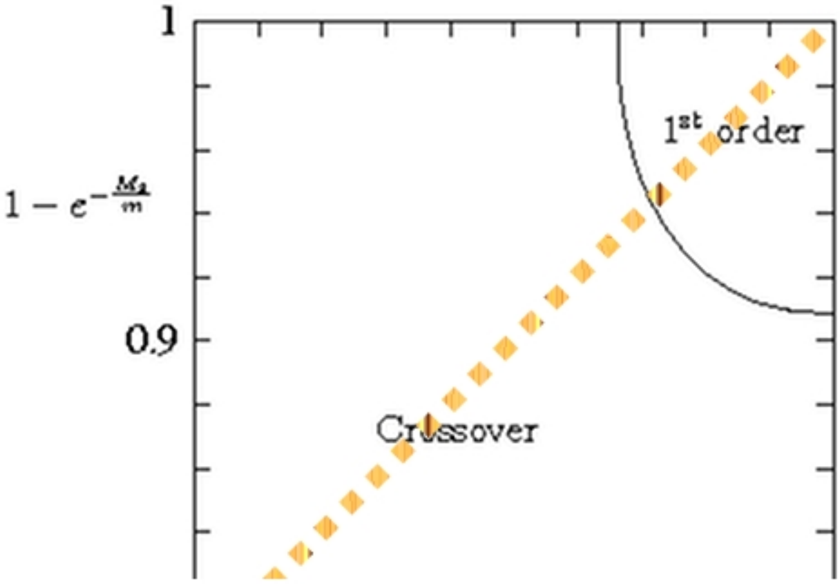


N_f	M_c/m	M_c/T_c	latt.*	matrix**	DSE***
1	2.39	6.74	7.22	8.04	1.42
2	2.69	7.59	7.91	8.85	1.83
3	2.86	8.07	8.32	9.33	2.04

* Fromm et al ('12) ; ** Kashiwa et al. ('12) ;
*** Fisher et al. ('15)

$$\boxed{\text{One-loop results : } \mu = 0}$$

$1+2$ flavors



For N_f degenerate flavors

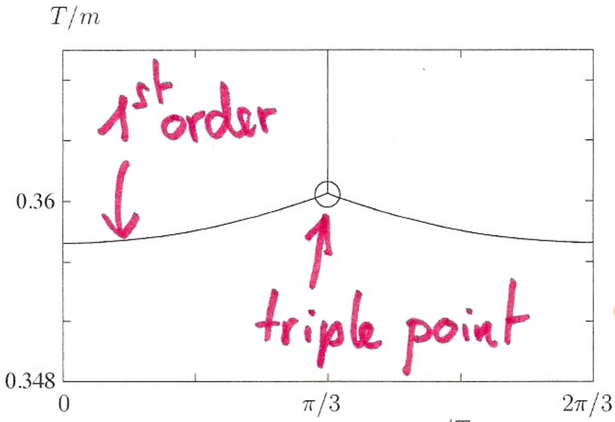
N_f	M_c/m	M_c/T_c	latt.*	matrix**	DSE***
1	2.39	6.74	7.22	8.04	1.42
2	2.69	7.59	7.91	8.85	1.83
3	2.86	8.07	8.32	9.33	2.04

* Fromm et al ('12) ; ** Kashiwa et al. ('12) ;
*** Fisher et al. ('15)

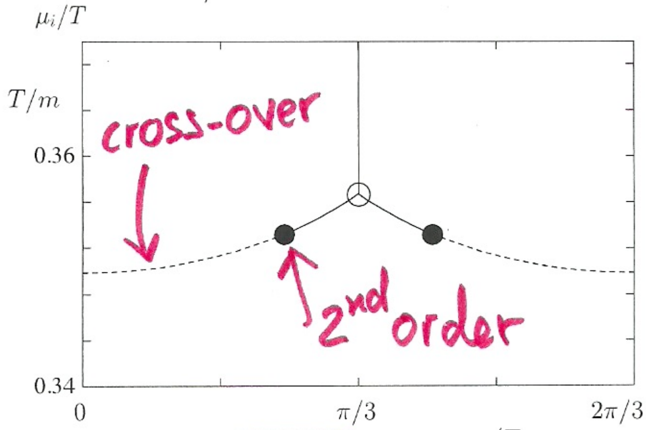
Imaginary μ : Roberge-Weiss transition

$\mu \in i\mathbb{R}$: no sign problem

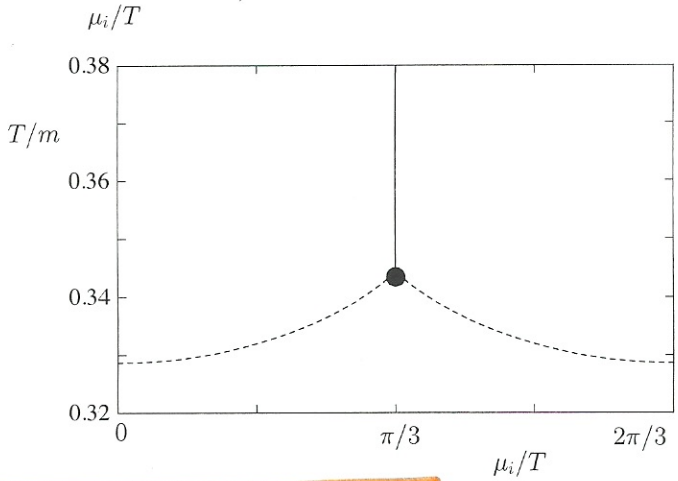
$r_3, r_8 \in \mathbb{R} \Rightarrow \bar{\ell} = \ell^*$



$\boxed{M > M_c(\mu=0)}$



$\boxed{M_c\left(i\frac{\pi T}{3}\right) < M < M_c(\mu=0)}$

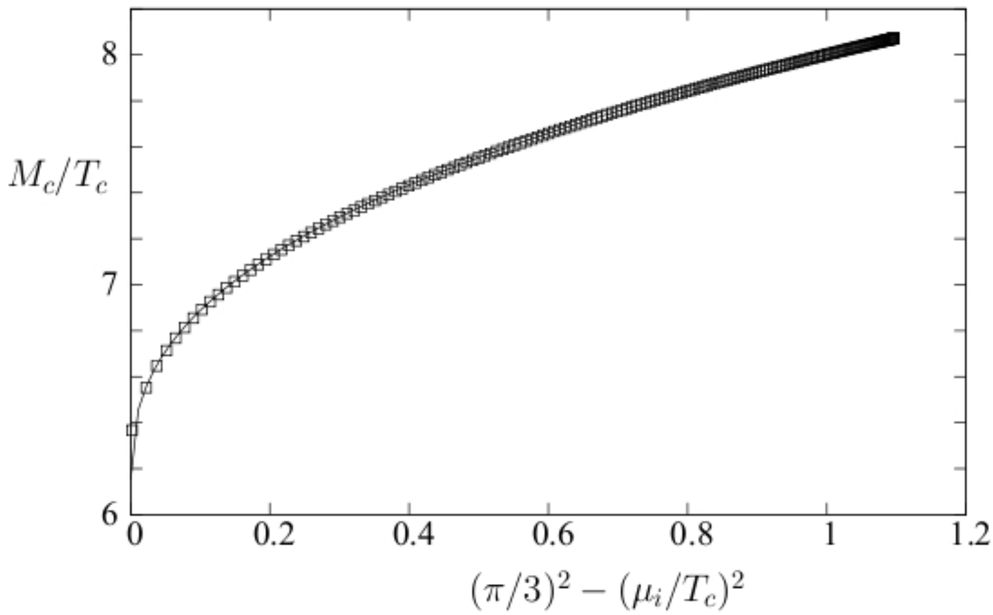


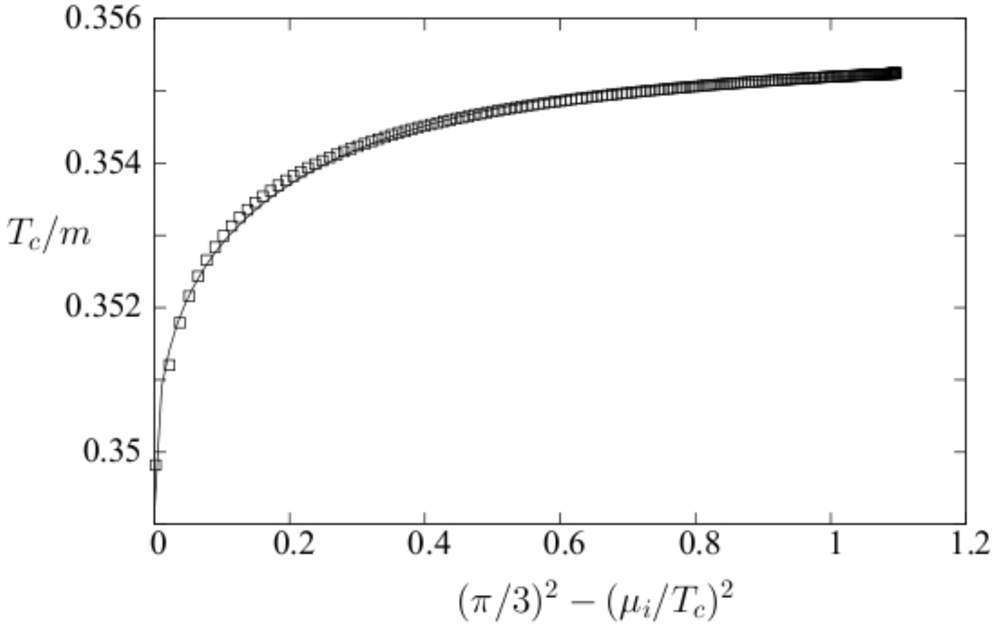
$\boxed{M < M_c\left(i\frac{\pi T}{3}\right)}$

$$\boxed{M = M_c(i\pi T/3) \quad : \text{ tricritical scaling}}$$

$$\frac{M_c(\mu)}{T_c(\mu)} = \frac{M_{tric.}}{T_{tric.}} + K\left[\left(\frac{\pi}{3}\right)^2 + \left(\frac{\mu}{T_c}\right)^2\right]^{2/5}$$

[de Forcrand, Philipsen ('10)]





Real μ : Columbia plot

An echo of the sign problem :

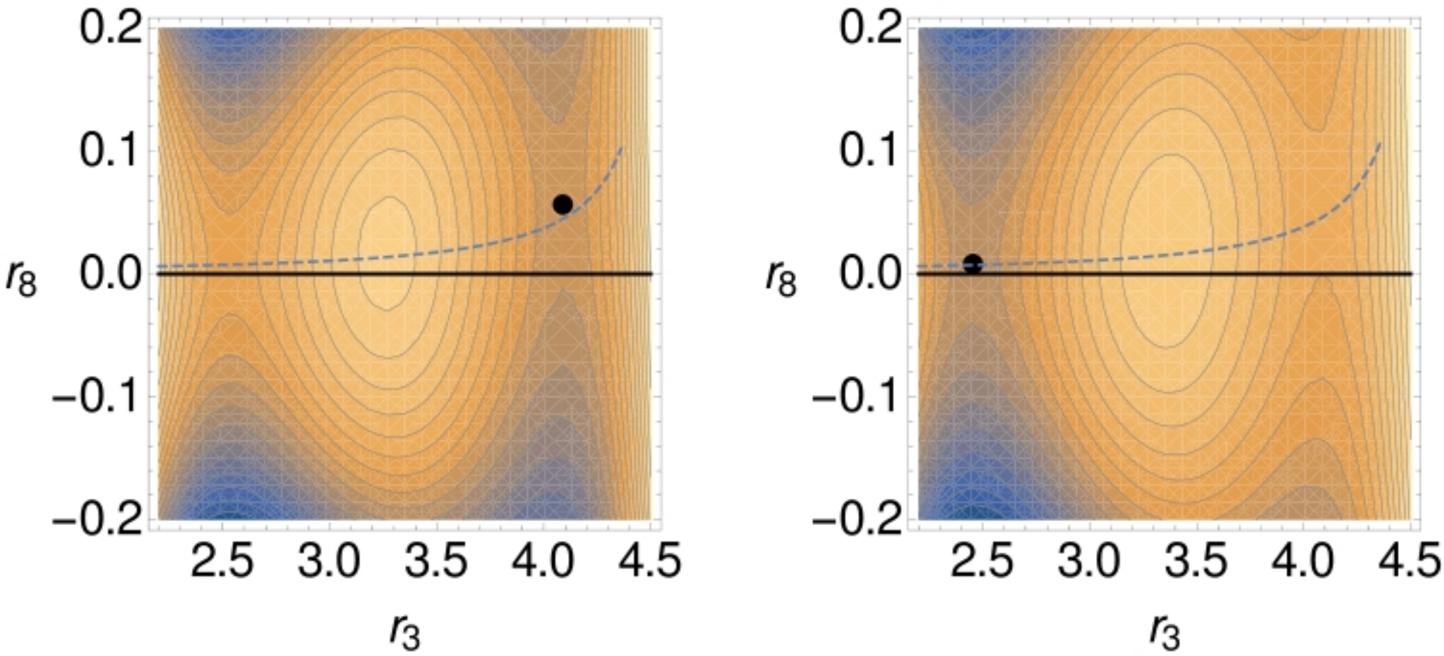
<u>$V(r)$ is not real in general</u>

<u>A solution</u> : $r_3 \in \mathbb{R}$ and $r_8 \in i\mathbb{R}$

$\Rightarrow$ $V(r_3, r_8)$ is real

$\Rightarrow$ $\ell \in \mathbb{R}$ and $\bar{\ell} \in \mathbb{R}$ are indep.

[see also Dumitru, Pisarski, Zschiesche ('08)]



BUT : Saddle points (not min!)

[see also Nishimura, Ogilvie, Pangeni ('15)]

Real μ : Columbia plot

An echo of the sign problem :

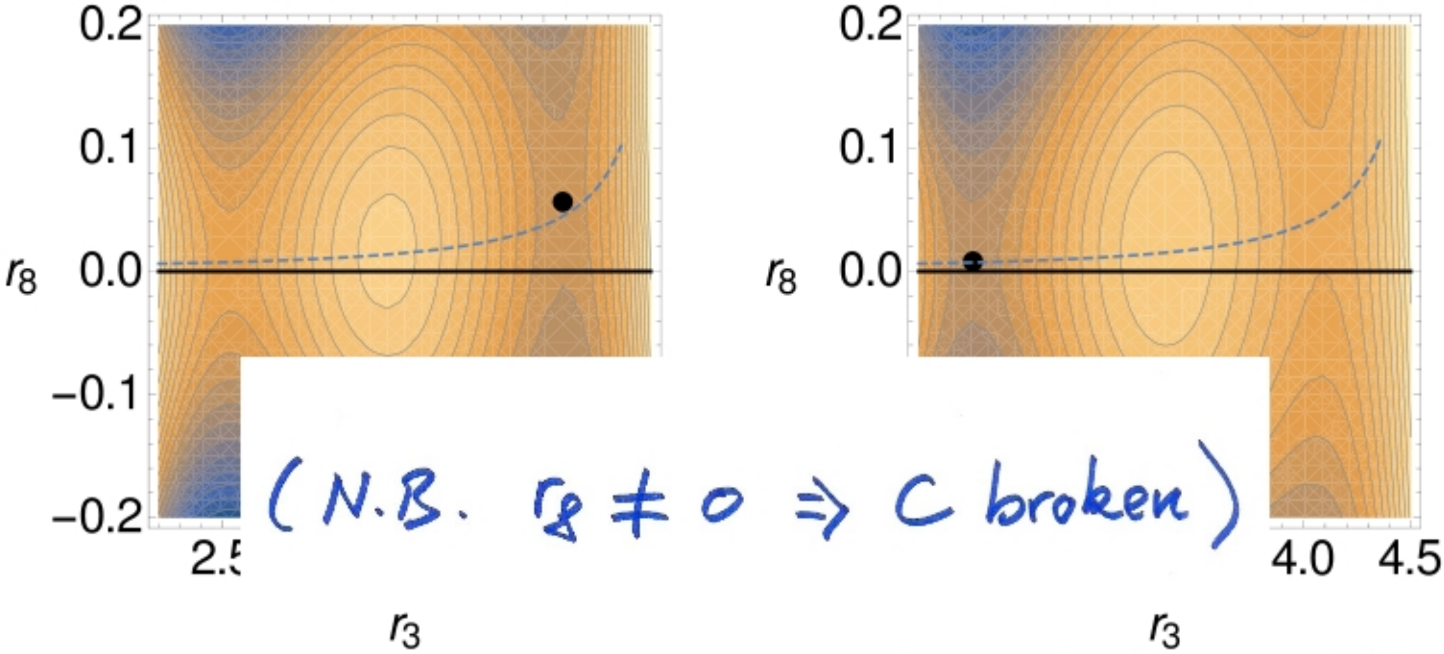
<u>$V(r)$ is not real in general</u>

A solution : $r_3 \in \mathbb{R}$ and $r_8 \in i\mathbb{R}$

$\Rightarrow$ $V(r_3, r_8)$ is real

$\Rightarrow$ $\ell \in \mathbb{R}$ and $\bar{\ell} \in \mathbb{R}$ are indep.

[see also Dumitru, Pisarski, Zschiesche ('08)]

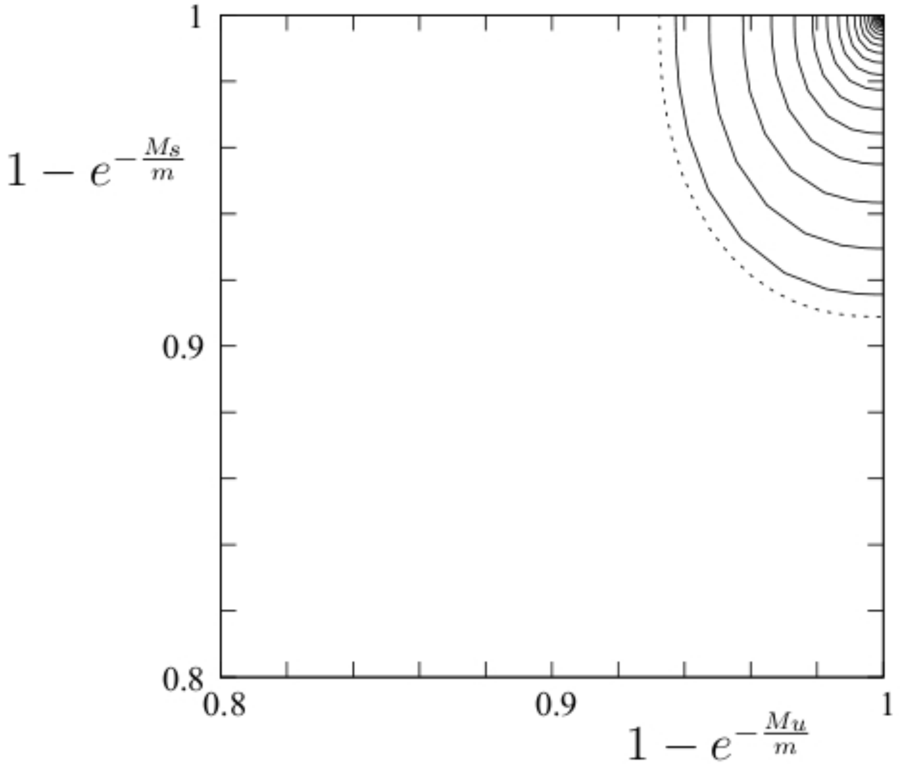


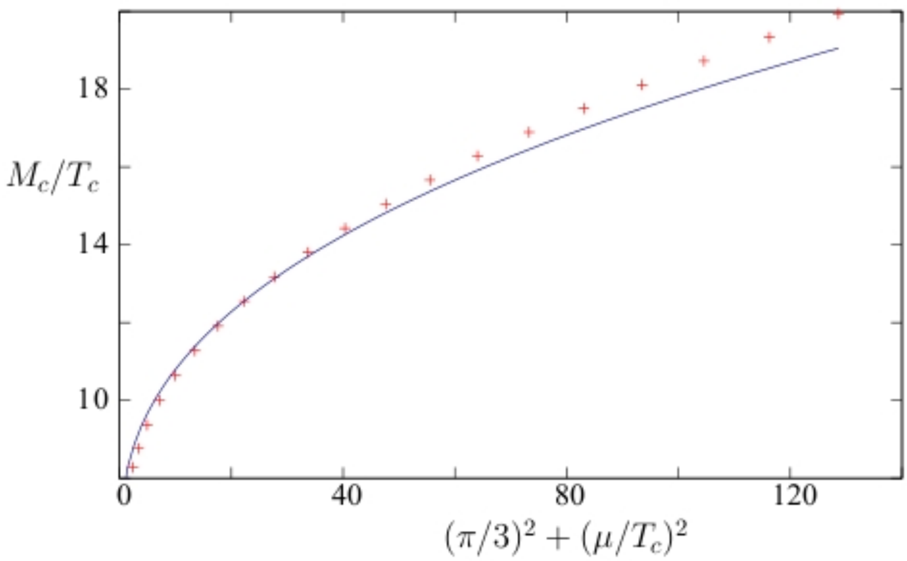
(N.B. $r_8 \neq 0 \Rightarrow C$ broken)

BUT : Saddle points (not min!)

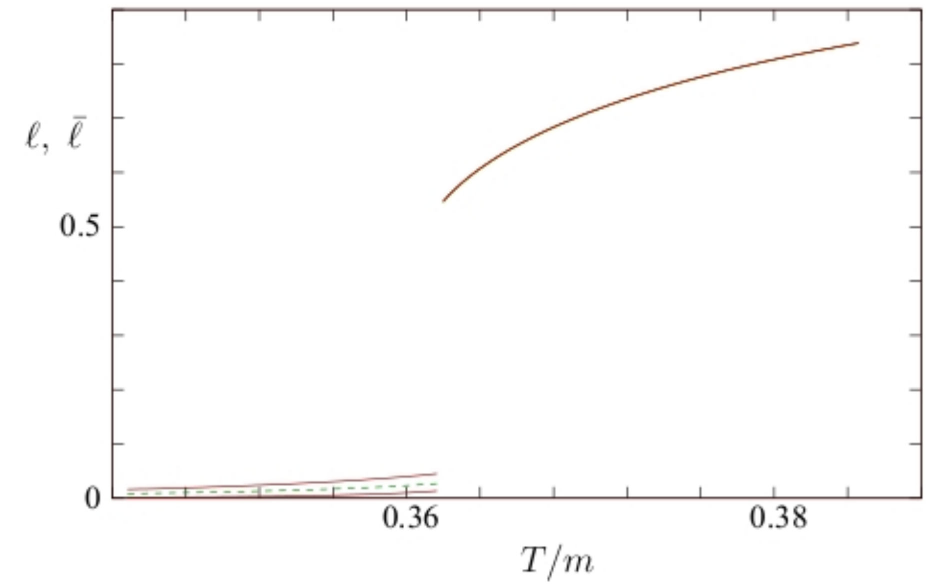
[see also Nishimura, Ogilvie, Pangeni ('15)]

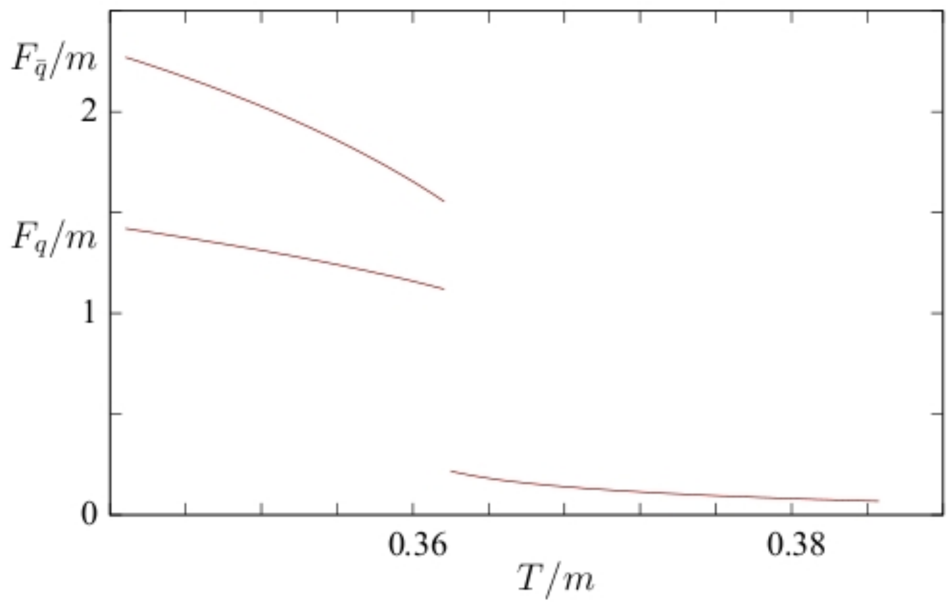
Real μ: Columbia plot and tricritical scaling





Real μ: Polyakov loops and q-$\bar{q}$ free energies





SUMMARY / CONCLUSIONS and PERSPECTIVES

Massive extension of standard Faddeev-Popov action (LDW gauges)

- Motivated by lattice (Landau gauge)
- provides a better starting point for perturbative calculations
- compares well with lattice and competes with other (nonperturbative) continuum approches
- quantitative description of the phase diagrams of YM theories and QCD_{heavy}

→ $\boxed{\text{Two-loop calculation of the phase diag. and thermodynamics of } QCD_{heavy}}$

→ "True" QCD (light quarks)
⇒ chiral sym. breaking