‘Kaon Condensation: Functional RG approach.

B. Krippa

Nottingham Trent University

ERG2016

Generalities

- At low energy the QCD can be reduced to an effective theory containing Goldstone bosons as effective degrees of freedom. The extension of this effective theory to finite density requires taking into consideration nonzero isospin/strange chemical potential.

Generalities

- At low energy the QCD can be reduced to an effective theory containing Goldstone bosons as effective degrees of freedom. The extension of this effective theory to finite density requires taking into consideration nonzero isospin/strange chemical potential.
- At finite density meson being bosons may condense. For example, there is a strong possibility that a kaon condensation may exist in a core of the neutron stars so that a realistic analysis of such possibility as well as estimates of the value of the condensate may turn out to be important for establishing correct EoS.

Generalities

- At low energy the QCD can be reduced to an effective theory containing Goldstone bosons as effective degrees of freedom. The extension of this effective theory to finite density requires taking into consideration nonzero isospin/strange chemical potential.
- At finite density meson being bosons may condense. For example, there is a strong possibility that a kaon condensation may exist in a core of the neutron stars so that a realistic analysis of such possibility as well as estimates of the value of the condensate may turn out to be important for establishing correct EoS.
- We study a phenomena of kaon condensation using the framework of the functional renormalisation group (FRG).

The FRG approach makes use of the Legendre transformed effective action: $\Gamma[\phi_c] = W[J] - J \cdot \phi_c$, where W is the usual partition function in the presence of an external source J.

The action functional Γ generates the 1PI Green's functions and it reduces to the effective potential for homogeneous systems. In the FRG one introduces an artificial renormalisation group flow, generated by a momentum scale k and we define the effective action by integrating over components of the fields with $q \leq k$.

The RG trajectory then interpolates between the classical action of the underlying field theory (at large k) when the quantum fluctuation effects are excluded, and the full effective action (at $k = 0$) with all quantum fluctuations taken into account.

The flow evolution equation for Γ in the FRG has a one-loop structure and can be written as

$$\partial_k \Gamma = -\frac{i}{2}\, Tr\left[(\partial_k R)(\Gamma^{(2)} - R)^{-1}\right]$$

Generalities

The ERG equation being fully nonperturbative has one-loop structure.

Cutoff acts as an infrared regulator, goes to zero at vanishing scale, where physics is defined.

Initial conditions are defined at large scale, where theory is relatively "simple".

Convenient tool to provide a link between vacuum and in-medium physics.

Many uses: many-body and few-body physics, QCD, Gravity etc.

As always, real life requires approximations so we need physically motivated ansatz for the Effective action

The ansatz assumed is

$$\Gamma[\phi,\phi^\dagger] = \int d^4x \left[Z_\phi \,(\partial_0 + i\mu)\phi^\dagger \,(\partial_0 - i\mu)\phi - Z_m \,\partial_i\phi^\dagger\partial_i\phi - U(\phi,\phi^\dagger)\right],$$

where Z_ϕ and Z_m are the renormalisation factors depending on the running scale and ϕ is a complex doublet field defined as follows

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \bar{\phi}_1 + i\bar{\phi}_2 \end{pmatrix}$$

The first and second components of the doublet can be identified with the pair of (K^+, K^0) and $(K^-, \bar{K}^0)$ mesons correspondingly. The effective potential depends only on the combination $\rho = \phi^\dagger\phi$. We expand the effective potential $U(\rho)$ near its minima and keep terms up to order ρ^3.

$$U(\phi,\phi^\dagger) = u_1(\rho - \rho_0) + \frac{1}{2}\,u_2(\rho - \rho_0)^2 + \frac{1}{6}\,u_3(\rho - \rho_0)^3 + \bar{u}_1(\bar{\rho} - \bar{\rho}_0) + \frac{1}{2}\,\bar{u}_2(\bar{\rho} - \bar{\rho}_0)^2 + \frac{1}{6}\,\bar{u}_3(\bar{\rho} - \bar{\rho}_0)^3 \ldots,$$

The first three terms correspond to the expansion near the minimum with respect to the first doublet and the rest is the expansion near the minimum with respect to the second doublet. Note that the standard mass term is included in the definition of the u_1 coupling.

- The chemical potential is provided by the external conditions (we assume $\mu > 0$.

The first three terms correspond to the expansion near the minimum with respect to the first doublet and the rest is the expansion near the minimum with respect to the second doublet. Note that the standard mass term is included in the definition of the u_1 coupling.

- The chemical potential is provided by the external conditions (we assume $\mu > 0$.
- The action is invariant under the global $SU(2) \times U(1)$ group, where $SU(2)$ is the isospin group and $U(1)$ is related to hypercharge. It essentually captures the mean features of the kaon condensation phenomena.

The first three terms correspond to the expansion near the minimum with respect to the first doublet and the rest is the expansion near the minimum with respect to the second doublet. Note that the standard mass term is included in the definition of the u_1 coupling.

- The chemical potential is provided by the external conditions (we assume $\mu > 0$.
- The action is invariant under the global $SU(2) \times U(1)$ group, where $SU(2)$ is the isospin group and $U(1)$ is related to hypercharge. It essentually captures the mean features of the kaon condensation phenomena.
- Substituting the ansatz for Γ into flow equation and performing the contour integration one can get the evolution equation for the effective potential which acts as a driving term generating the flow of the couplings. At large scale we expect symmetric state with the trivial minimum of the effective potential whereas at lower scale $k \simeq \mu$ a formation of the condensate is expected

The resulting equation for the effective potential (first dublet) is

$$\partial_k U = \frac{1}{4}\int \frac{d^3\vec{q}}{(2\pi)^3} \frac{(2Z_\phi Q_1^2 - \alpha - \beta - 2R)\partial_k R}{4Z_\phi^2 Q_1^3 - 2Z_\phi Q_1(\alpha + \beta + 2R + 4\mu^2 Z_\phi)} + \frac{1}{4}\int \frac{d^3\vec{q}}{(2\pi)^3} \frac{(2Z_\phi Q_2^2 - \alpha - \beta - 2R)\partial_k R}{4Z_\phi^2 Q_2^3 - 2Z_\phi Q_2(\alpha + \beta + 2R + 4\mu^2 Z_\phi)},$$

here

$$\alpha = Z_m q^2 + u_1 + u_2(3\rho_1 + \rho_2 - \rho_0) + \frac{u_3}{2}(4\rho_1(\rho_1 + \rho_2 - \rho_0) + (\rho_1 + \rho_2 - \rho_0)^2),$$

$$\beta = Z_m q^2 + u_1 + u_2(\rho_1 + 3\rho_2 - \rho_0) + \frac{u_3}{2}(4\rho_2(\rho_1 + \rho_2 - \rho_0) + (\rho_1 + \rho_2 - \rho_0)^2),$$

$\rho_0(k)$ is the scale dependent minimum of the effective potential and Q_1 and Q_2 are the pole positions of the propagator. The pole position defines the corresponding dispersion relations in the general case of nonzero regulator $R \neq 0$.

Taking $R \to 0$, $Z_\phi \to 1$ and $u_1 \to 0$ one can recover the dispersion relations in the broken phase derived Miransky/Gusinin and Son/Stephanov.

$$Q_{1,2} = \sqrt{3\mu^2 - m^2 + q^2) \pm \sqrt{(3\mu^2 - m^2)^2) + 4\mu^2 q^2}}.$$

and

$$\bar{Q}_{1,2} = \sqrt{\mu^2 + q^2} \pm \mu.$$

- Two of the dispersion relations describe Goldstone bosons carrying the quantum numbers of (K^+, K^0) dublet. This is a nontrivial as the number of the broken generators for the $SU(2) \times U(1) \to U(1)$ breaking pattern is not equal to the number of the massless modes

Taking $R \to 0$, $Z_\phi \to 1$ and $u_1 \to 0$ one can recover the dispersion relations in the broken phase derived Miransky/Gusinin and Son/Stephanov.

$$Q_{1,2} = \sqrt{3\mu^2 - m^2 + q^2) \pm \sqrt{(3\mu^2 - m^2)^2) + 4\mu^2 q^2}}.$$

and

$$\bar{Q}_{1,2} = \sqrt{\mu^2 + q^2} \pm \mu.$$

- Two of the dispersion relations describe Goldstone bosons carrying the quantum numbers of (K^+, K^0) dublet. This is a nontrivial as the number of the broken generators for the $SU(2) \times U(1) \to U(1)$ breaking pattern is not equal to the number of the massless modes
- The physical reason is the presence of the chemical potential which induces a mass splitting between the doublets (K^+, K^0) and $(K^-, \bar{K}^0)$ so that the first one asquire the effective mass $m - \mu$ whereas the effective mass for the second one becomes $m + \mu$.

The couplings may in general depend not only on running scale but also on the magnitude of the condensate so that we define the total derivative as

$$d_k = \partial_k + (d_k\rho)\,\frac{\partial}{\partial\rho},$$

where $d_k\rho = d\rho/dk$. Applying this to effective potential gives the set of the flow equations

$$-u_2\,d_k\rho_1 = \frac{\partial}{\partial\rho_1}\Big(\partial_k U\Big)\Big|_{\rho_1=\rho_0},$$

$$\mathrm{d}_k u_2 - u_3 d_k\rho_1 = \frac{\partial^2}{\partial\rho_1^2}\Big(\partial_k U\Big)\Big|_{\rho_1=\rho_0},$$

$$\mathrm{d}_k Z_\phi = -\,\frac{1}{2}\,\frac{\partial^2}{\partial^2\mu\partial\rho_1}\Big(\partial_k U\Big)\Big|_{\rho_1=\rho_0},$$

$$\mathrm{d}_k u_3 = \frac{\partial^3}{\partial\rho_1^3}\Big(\partial_k U\Big)\Big|_{\rho_1=\rho_0}$$

The actual flow is determined by the choice of the cut-off function R. We have chosen the cut-off in the form $\mathbf{R}(\mathbf{k}, \mathbf{q}) = (\mathbf{k}^2 - \mathbf{q}^2)\boldsymbol{\Theta}(\mathbf{k} - \mathbf{q})$.
The advantage of this form of cut-off is that it simplifies some algebra, allowing part of the calculations to be carried out analytically.

To solve the system of the flow equations one needs to fix a set of initial conditions. One starts from the effective action in vacuum where $\mathbf{u_1^v}(\mathbf{k} = \mathbf{0}) = \mathbf{m_K^2}, \mathbf{Z_v}(\mathbf{k} = \mathbf{0}) = \mathbf{1}, \mathbf{u_3^v}(\mathbf{k} = \mathbf{0}) = \mathbf{0}$ and $\mathbf{u_2^v}(\mathbf{k} = \mathbf{0})$ is determined by the value of kaon-kaon scattering length in vacuum. The flow equations in vacuum can be obtained from the general expression for the $\partial_k U$ by differentiating it with respect to ρ and μ and putting $\rho = 0$, $\mu = 0$ afterwards. For example, the flow equation for the couplings u_1^v takes the form

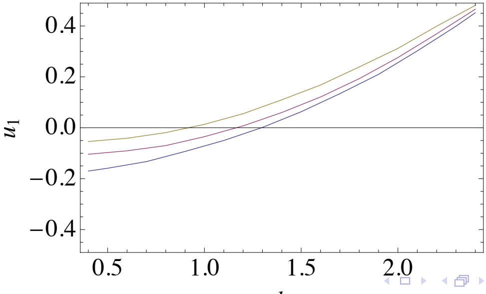
$$\partial_k u_1^v(k) = \frac{k^4 u_2^v}{8\pi^2(k^2 + u_1^v(k))^{3/2}}$$

The equations in vacuum are solved using the values of couplings at $k = 0$ as initial conditions.

From the vacuum flow the the values of the couplings at the starting scale for the general flow $k = \Lambda$ are extracted. The value of the scale Λ is chosen to be much larger then any other mass scale involved in the problem.

We have chosen the value $\Lambda = 50$ GeV as the starting scale. It is large enough to provide practically independent results for the couplings at the physical scale $k = 0$. At the scale $k \simeq M_k$ the mass term approaches zero thus signalling the onset of spontaneous symmetry breaking (SSB). The corresponding vacuum expectation value of the field ϕ becomes nonzero and the system undergoes phase transition.

The behaviour of the mass term in the vicinity of the critical scale at several values of the chemical potential. The critical value for such a transition to occur is $\mu = M_k$. At $k \simeq 3$ GeV the curves merge and follow this pattern up to starting point. The yellow, orange and blue curves correspond to $\mu = 510$ MeV, 550 MeV and 595 MeV correspondingly.

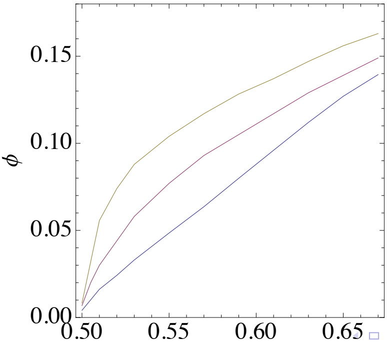


- The change of the transition point as a function of the chemical potential is such that it grows with the increase of the chemical potential but, in any case the system undergoes the transition to the broken phase at the scale $k \simeq \mu$. At this scale the mass term vanishes the and the condensate develops.

- The change of the transition point as a function of the chemical potential is such that it grows with the increase of the chemical potential but, in any case the system undergoes the transition to the broken phase at the scale $k \simeq \mu$. At this scale the mass term vanishes the and the condensate develops.
- On the other hand the character of the change of the mass term for the other fields in $\bar{\phi}$ is such that it stays positive for any scale so that SSB never happens. One may, therefore conclude that, whereas K^0 and K^+ mesons condense, the pair of the $\bar{K}^0$ and K^- mesons does not in agreement with the earlier results (Kaplan, Bedaque, Reddy etc)

- The change of the transition point as a function of the chemical potential is such that it grows with the increase of the chemical potential but, in any case the system undergoes the transition to the broken phase at the scale $k \simeq \mu$. At this scale the mass term vanishes the and the condensate develops.
- On the other hand the character of the change of the mass term for the other fields in $\bar{\phi}$ is such that it stays positive for any scale so that SSB never happens. One may, therefore conclude that, whereas K^0 and K^+ mesons condense, the pair of the $\bar{K}^0$ and K^- mesons does not in agreement with the earlier results (Kaplan, Bedaque, Reddy etc)
- One notes that the mechanism of the condensation considered here in is related to an instabilities in the colour - flavour locked phase of QCD and is therefore quite different from the conventional condensation of K^- mesons.

The value of the condensate depends on the running coupling u_2, which is related to the KK scattering length. Below is the behaviour of the condensate as the function of chemical potential μ for three values of the u_2 coupling. The condensate grows with the increase of μ in all three cases. This growth is relatively fast at the threshold $\mu \simeq \mu_{crit}$ and then slows down for the larger values of the chemical potential.



The important element of the approach is the running coupling u_2, which is related to the kaon-kaon scattering length.

- The upper curve corresponds the value for the scattering length obtained in the (rather old) lattice calculations a_{lat}= -0.310 m_K^{-1}.
- The middle curve corresponds to the use of the lowest order chiral perturbation theory result $a = \frac{m_K}{16\pi f_\pi}$.
- The **lower** one is obtained using the phenomenological one meson exchange model.

The important element of the approach is the running coupling u_2, which is related to the kaon-kaon scattering length.

- The upper curve corresponds the value for the scattering length obtained in the (rather old) lattice calculations a_{lat}= -0.310 m_K^{-1}.
- The middle curve corresponds to the use of the lowest order chiral perturbation theory result $a = \frac{m_K}{16\pi f_\pi}$.
- The **lower** one is obtained using the phenomenological one meson exchange model.
- The difference between the results is quite noticeable, especially at the threshold. It implies that for the quantitative description of the kaon condensation the kaon-kaon scattering lengh should be obtained from either improved lattice calculations or from the experiment.

Conclusion

- Functional RG provides a realistic description of the phenomena of kaon condensation

Conclusion

- Functional RG provides a realistic description of the phenomena of kaon condensation
- The results depend rather strongly on kaon-kaon scattering lengh which is not very well known at present.

Conclusion

- Functional RG provides a realistic description of the phenomena of kaon condensation
- The results depend rather strongly on kaon-kaon scattering lengh which is not very well known at present.
- Things for the future include: More realistic effective action based on chiral lagrangians, taking into account a mixture of kaon/pion/chiral and color condensates and finite temperature effects.