Chiral symmetry breaking in continuum QCD

Mario Mitter

Ruprecht-Karls-Universität Heidelberg

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fQCD collaboration - QCD (phase diagram) with FRG:

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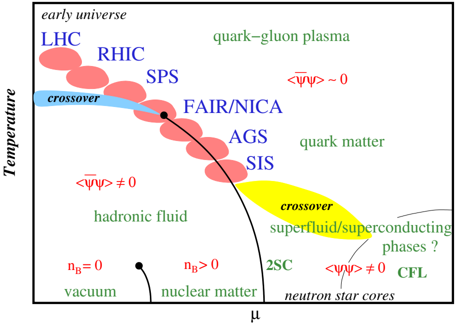

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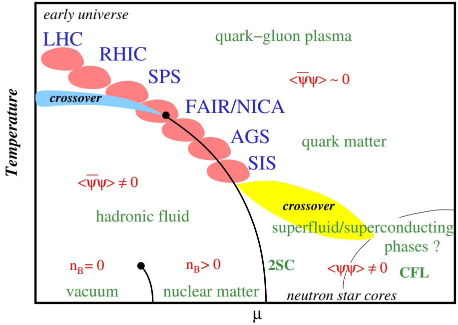

large part of this effort: vacuum YM-theory and QCD

QCD with the FRG

- $S[\Phi] = \Gamma_\Lambda[\Phi]$: use only perturbative QCD input
  - $\alpha_S(\Lambda = \mathcal{O}(10)$ GeV)
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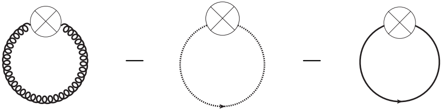
$$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2}$$



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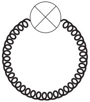
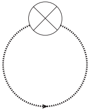
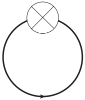
$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2}$  $-$  $-$ 

- vertex expansion:

$$\Gamma[\Phi] = \sum_n \int_{p_1,\ldots,p_{n-1}} \Gamma^{(n)}_{\Phi_1 \cdots \Phi_n}(p_1, \ldots, p_{n-1}) \Phi^1(p_1) \cdots \Phi^n(-p_1 - \cdots - p_{n-1})$$

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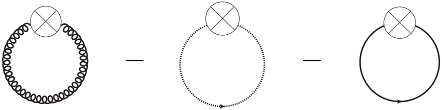

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- aim for "apparent convergence" of $\Gamma[\Phi] = \lim_{k\to 0} \Gamma_k[\Phi]$

Landau gauge QCD

- two crucial phenomena: SχSB and confinement
- similar scales - hard to disentangle see e.g. [Williams, Fischer, Heupel, 2015]
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- quenched matter part [MM, Strodthoff, Pawlowski, 2014]
- pure YM-theory [Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]
- outlook: [Cyrol, MM, Strodthoff, Pawlowski, in preparation]
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- use results from lattice QCD to gauge truncation

Chiral symmetry breaking

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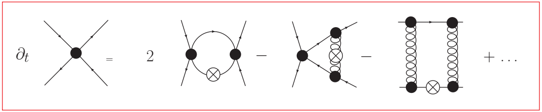
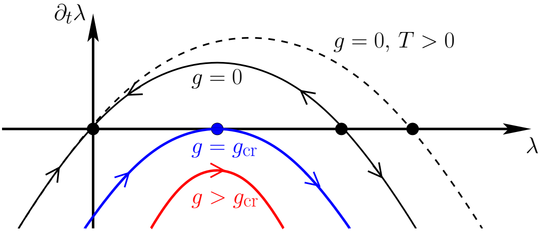
- resonance $\Rightarrow$ singularity without momentum dependency

$$\partial_t \lambda = a\,\lambda^2 + b\,\lambda\,\alpha + c\,\alpha^2, \quad b > 0,\ a, c \leq 0$$

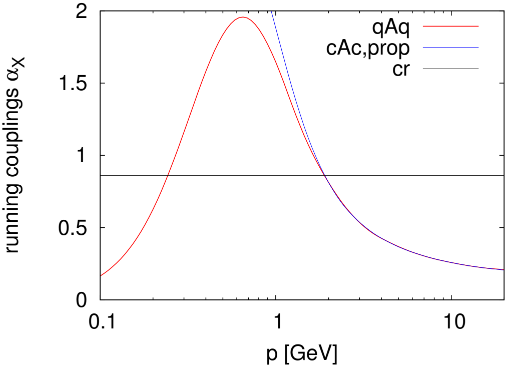




[Braun, 2011]

(transverse) running couplings

[Cyrol, MM, Pawlowski, Strodthoff, in preparation]

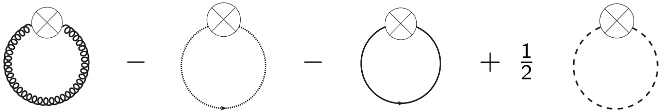


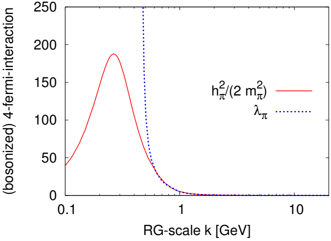
- agreement in perturbative regime required by gauge symmetry
- non-degenerate in nonperturbative regime: reflects gluon mass gap
- $\alpha_{\bar{q}Aq} > \alpha_{cr}$: necessary for chiral symmetry breaking
- area above α_{cr} very sensitive to errors

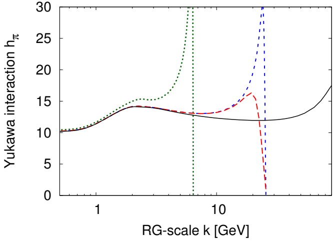
4-Fermi vertex via dynamical hadronization

[Gies, Wetterich, 2002]

- change of variables: particular 4-Fermi channels → meson exchange
- efficient inclusion of momentum dependence ⇒ no singularities
- calculation of model parameters from QCD

$\partial_k \Gamma_k = \frac{1}{2}$ 





[MM, Strodthoff, Pawlowski, 2014]

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Vertex Expansion

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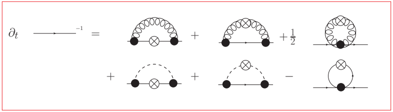

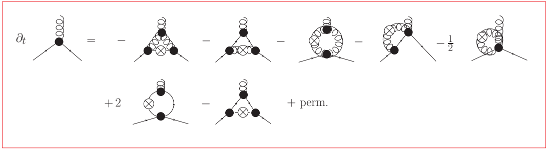

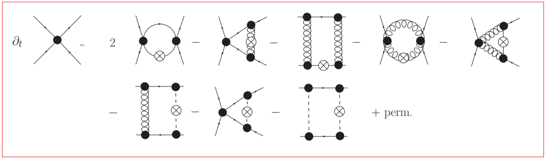

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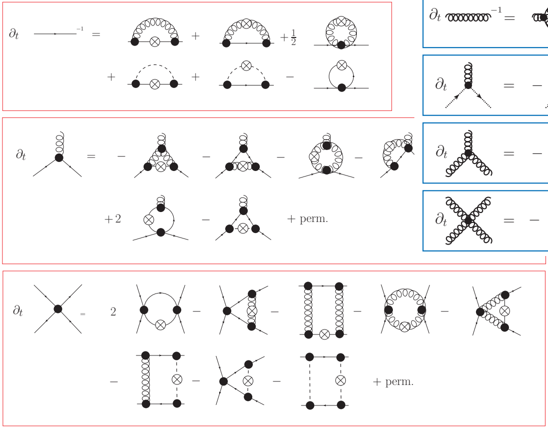

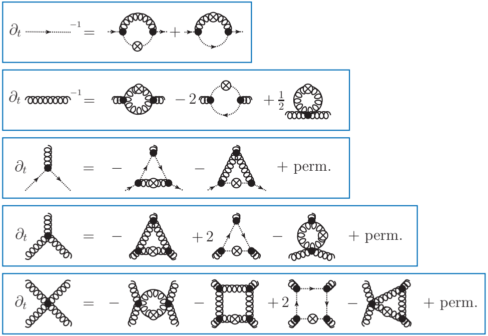

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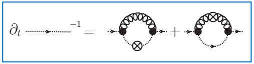
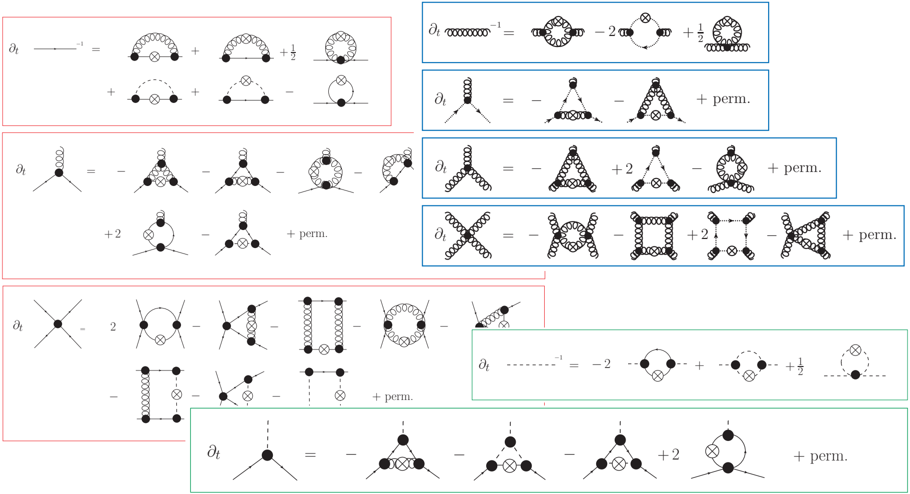
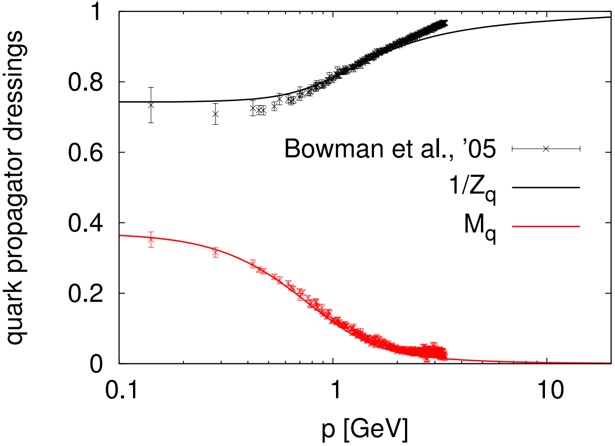




Quark propagator

[MM, Pawlowski, Strodthoff, 2014]

- $\Gamma^{(2)}_{\bar{q}q}(p) \propto Z_q(p)\,(\not{p} + M(p))$



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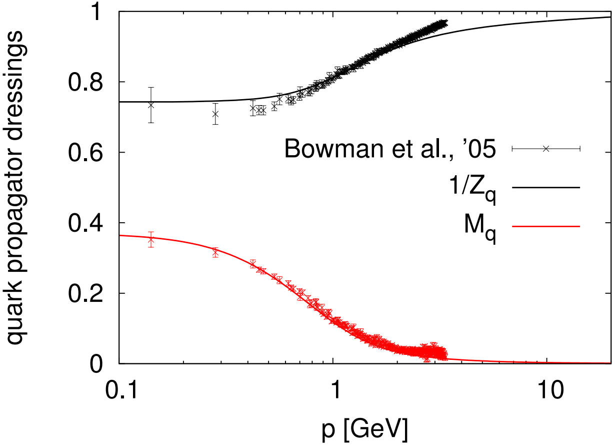
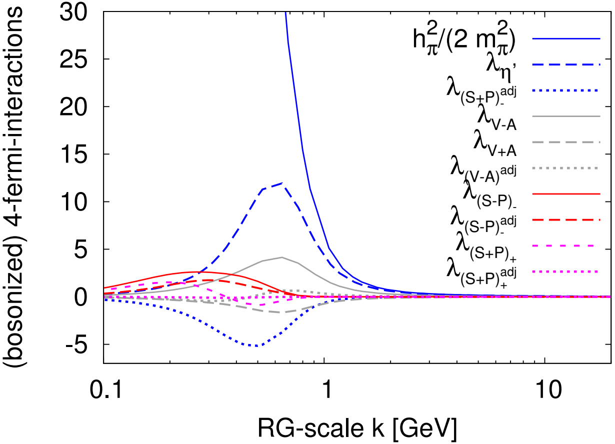


- FRG vs. lattice: bare mass, quenched, scale set via gluon propagator
- agreement not sufficient: need apparent convergence at $\mu \neq 0$

lattice data: Bowman, Heller, Leinweber, Parappilly, Williams, Zhang , Phys. Rev. D71, 054507 (2005).

other 4-Fermi channels

[MM, Pawlowski, Strodthoff, 2014]



- bosonized only σ-π-channel momentum dependently $\Rightarrow$ sufficient
- other channels: quantitatively not important in loops

Quark-gluon interactions

[MM, Pawlowski, Strodthoff, 2014]

- quark-gluon interaction most crucial for chiral symmetry breaking
- full tensor basis, e.g. $\gamma^\mu,\ \mathrm{i}\left(\not{p}+\not{q}\right)\gamma^\mu,\ \frac{1}{2}\left[\not{p},\not{q}\right]\gamma^\mu$

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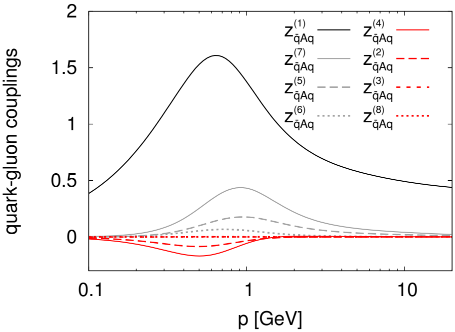

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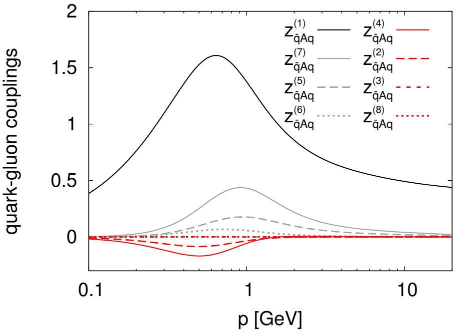

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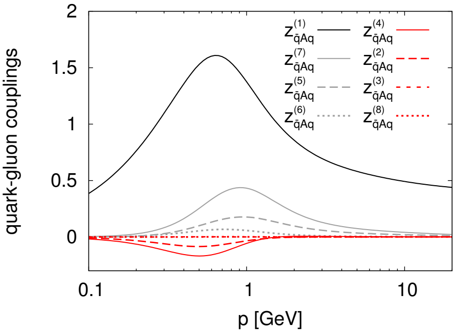

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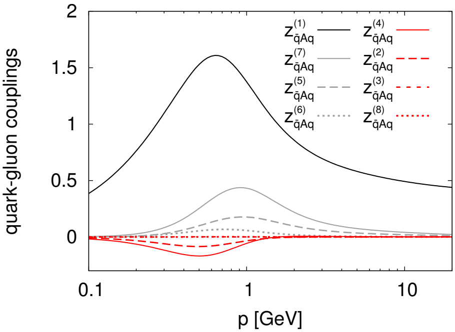

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$\Rightarrow$ expansion in BRST-invariant operators improves convergence?

YM theory: modified STIs (mSTIs)

- regulator breaks BRST-symmetry of gauge-fixed action

  $\Rightarrow$ modification to Slavnov-Taylor identities $\propto R_k$ ($\to 0$ for $k \to 0$)

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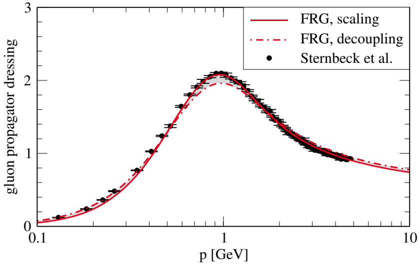
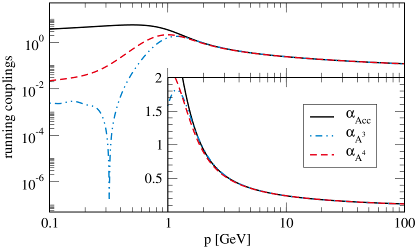
- lattice simulations: no scaling (decoupling solutions) [Cucchieri, Mendes, 2008]
- non-perturbative gauge-fixing on the lattice? [Maas, 2009]

YM theory: results

[Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]

- $\Gamma^{(2)}_{AA}(p) \propto Z_A(p)\, p^2 \, \left(\delta^{\mu\nu} - p^\mu p^\nu / p^2\right)$
- running couplings

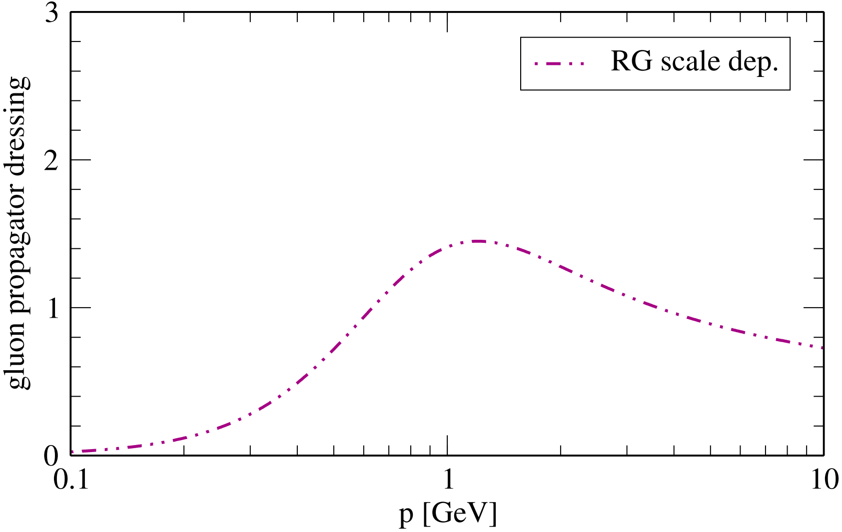




- decoupling solutions:
  - vary m^2_Λ slightly away from scaling value
  - dashed-dot line: largest variation of m^2_Λ that leads to “back-bending”

lattice data: A. Sternbeck, E. M. Ilgenfritz, M. Muller-Preussker, A. Schiller, and I. L. Bogolubsky, PoS LAT2006, 076.

YM-theory: Truncation dependence [Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]



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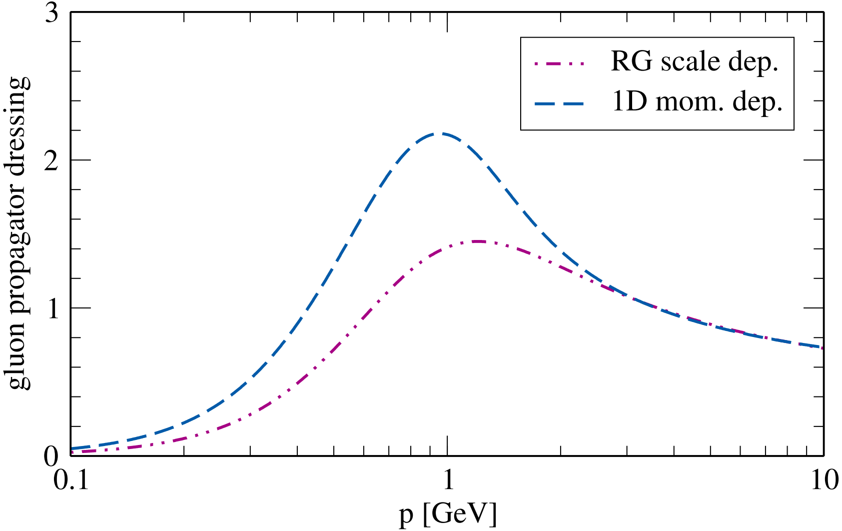

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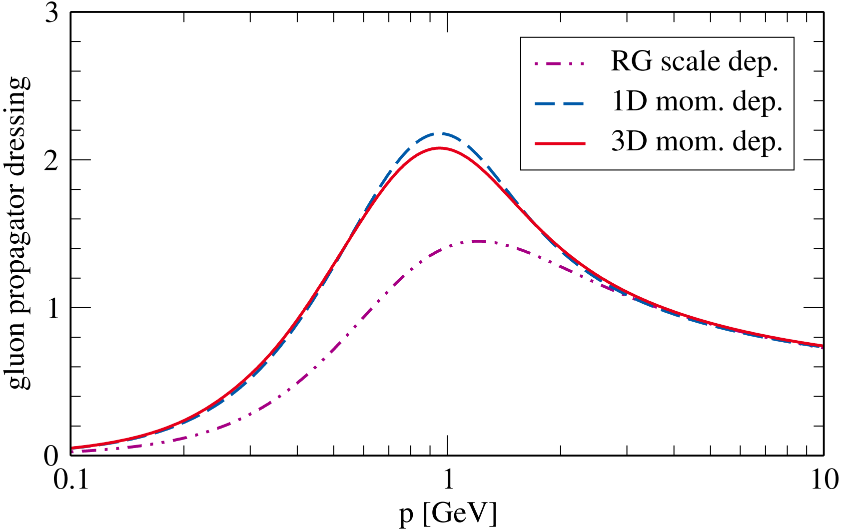

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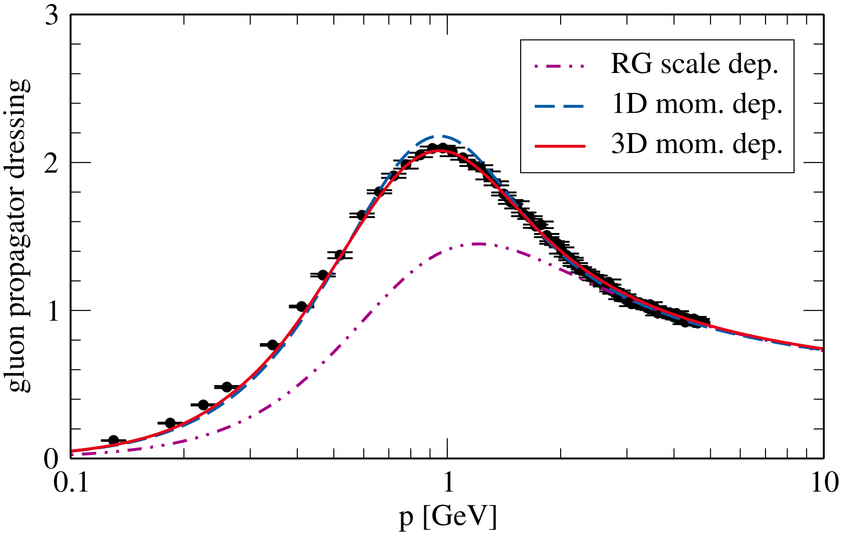

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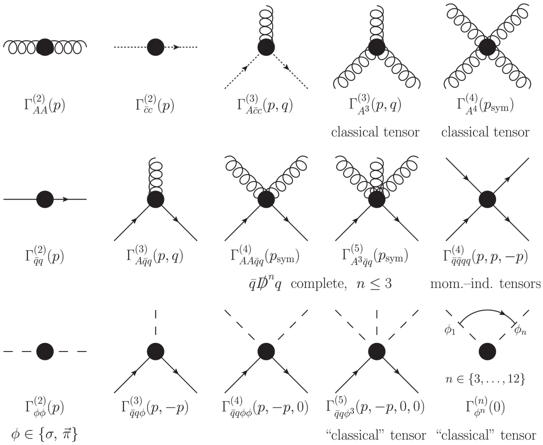
Outlook: unquenching

[Cyrol, MM, Pawlowski, Strodthoff, in prep.]

extended truncation:



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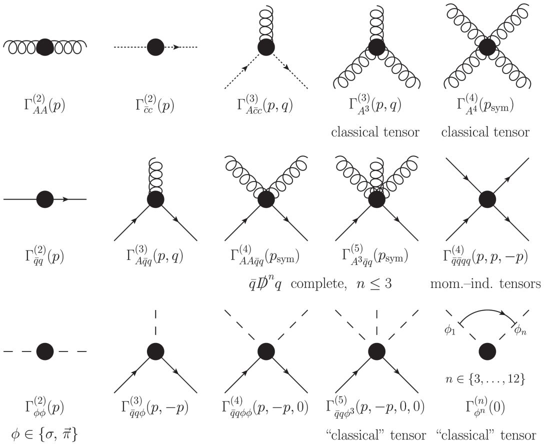


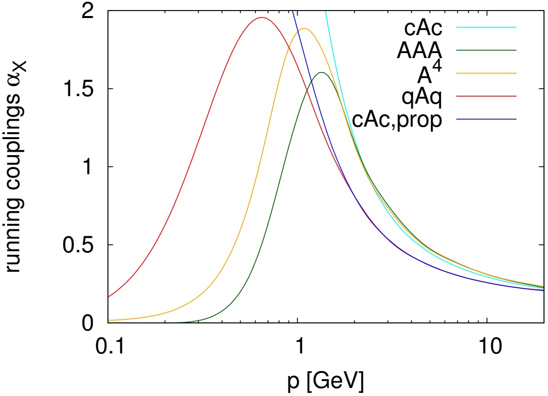
systematics of improving the truncation?

$\Rightarrow$ BRST-invariant operators, e.g. $\bar{\psi}\not{D}^n\psi$

Outlook: running couplings

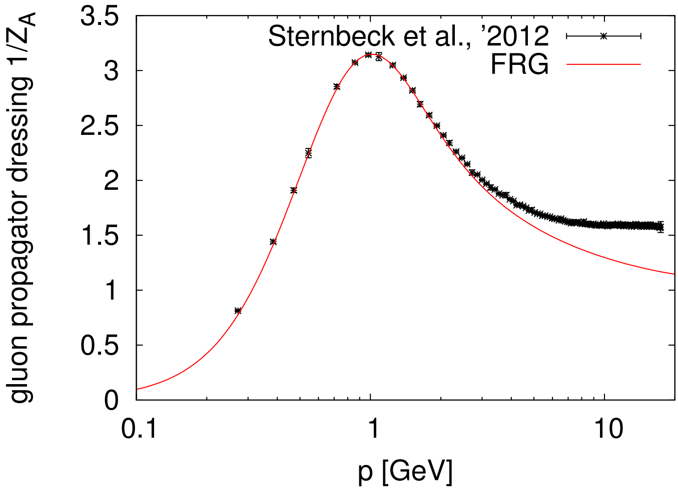
[Cyrol, MM, Pawlowski, Strodthoff, in preparation]

- $\alpha_{cAc} = \dfrac{\left(\Gamma^{(3)}_{cAc}(p)\right)^2}{4\pi\, Z_A(p) Z_c(p)^2}$
- $\alpha_{AAA} = \dfrac{\left(\Gamma^{(3)}_{AAA}(p)\right)^2}{4\pi\, Z_A(p)^3}$
- $\alpha_{A^4} = \dfrac{\left(\Gamma^{(4)}_{A^4}(p)\right)}{4\pi\, Z_A(p)^2}$
- $\alpha_{qAq} = \dfrac{\left(\Gamma^{(3)}_{qAq}(p)\right)^2}{4\pi\, Z_A(p) Z_q(p)^2}$
- $\alpha_{cAc,\mathrm{prop}} = \dfrac{1}{4\pi\, Z_A(p) Z_c(p)^2}$



Outlook: gluon propagator

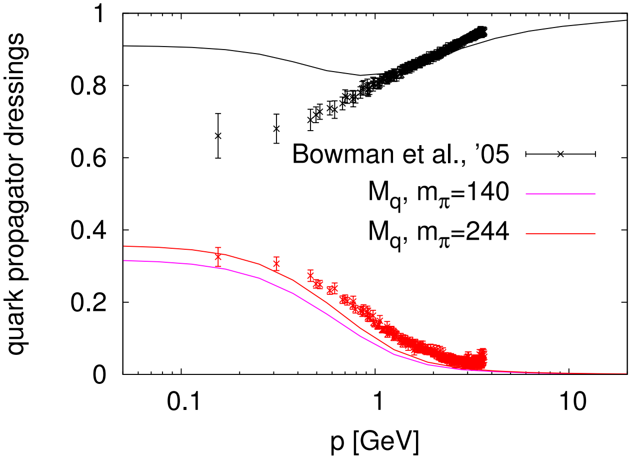
[Cyrol, MM, Pawlowski, Strodthoff, in preparation]



lattice data: A. Sternbeck, K. Maltman, M. Müller-Preussker, L. von Smekal, PoS LATTICE2012 (2012) 243.

Outlook: quark propagator

[Cyrol, MM, Pawlowski, Strodthoff, in preparation]

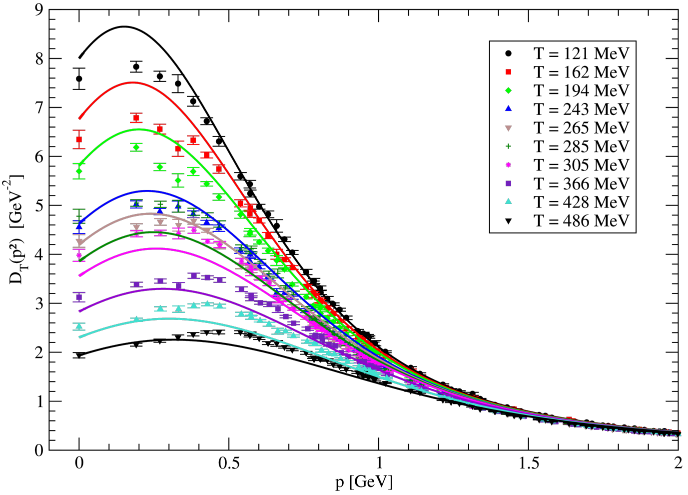


- comparison FRG with lattice: bare mass and scale setting

lattice data: Bowman, Heller, Leinweber, Parappilly, Williams, Zhang , Phys. Rev. D71, 054507 (2005).

Outlook: (averaged) gluon propagator at $T \neq 0$

[Cyrol, MM, Pawlowski, Strodthoff, in preparation]



lattice data: magnetic component of gluon propagator from Silva et. al. Phys.Rev. D89 (2014)

Summary and Outlook

QCD with functional RG

- vertex expansion
- sole input $\alpha_S(\Lambda = \mathcal{O}(10)\ \text{GeV})$ and $m_q(\Lambda = \mathcal{O}(10)\ \text{GeV})$
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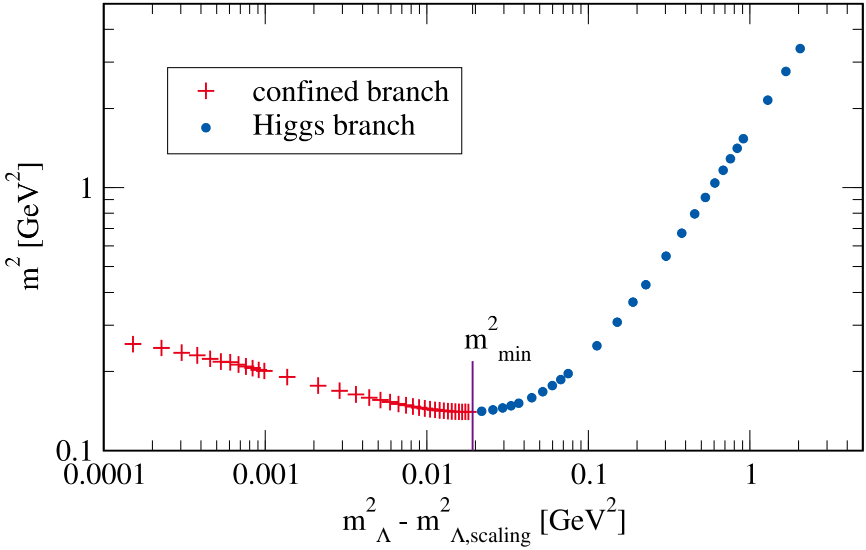
Poster by Anton K. Cyrol: "fQCD: QCD with the Functional RG"

Dynamical mass generation



Gluon propagator maximum over UV mass parameter