Sine-Gordon models: from Conformal Field Theory to Higgs physics

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Why Sine-Gordon (SG) models?

$$S_{\mathrm{SG}}[\varphi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \varphi)^2 + u \cos(\beta\varphi) \right]$$

- BKT (Berezinski-Kosterlitz-Thouless) phase transition
  $\Rightarrow$ e.g. to study the effect of amplitude fluctuations
- Higgs physics – sine-Gordon type Higgs potential
  $\Rightarrow$ e.g. to solve stability problem?
- CFT (Conformal Field Theory) – Zamolodchikov c-function
  $\Rightarrow$ e.g. to determine the c-function in the RG flow

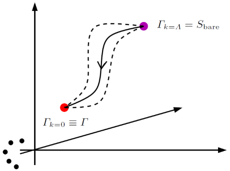
$\Rightarrow$ Functional Renormalization Group (FRG)

Functional Renormalization Group

- Wetterich RG equation

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\frac{k\partial_k R_k}{\Gamma_k^{(2)} + R_k}\right),$$

$$R_k(p) \equiv p^2\, r(y), \quad y = \frac{p^2}{k^2}$$

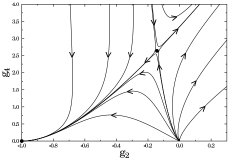


regulator: $R_{k\to 0}(p) = 0,\ R_{k\to\Lambda}(p) = \infty,\ R_k(p\to 0) > 0$

- Approximations

$$\Gamma_k[\varphi] = \int_x \left[V_k(\varphi) + Z_k(\varphi)\frac{1}{2}(\partial_\mu\varphi)^2 + \ldots \right]$$

$$V_k = \sum_{n=1}^{N_{\mathrm{cut}}} \frac{g_n(k)}{(2n)!}\varphi^{2n}$$



- Sine-Gordon, Z_2 + periodic $\longrightarrow$ topological $\beta_c^2 = 8\pi$

$$S_{\rm SG}[\varphi] = \int d^2x \left[\frac{1}{2}(\partial_\mu\varphi)^2 + u\cos(\beta\varphi)\right]$$

$\Longrightarrow$ Coulomb-gas, 2D XY spin model, 2D superfluid, bosonised Thirring model

- Massive Sine-Gordon, Z_2 $\longrightarrow$ Ising-like ($\beta_c^2 = \infty$)

$$S_{\rm MSG}[\varphi] = \int d^2x \left[\frac{1}{2}(\partial_\mu\varphi)^2 + \frac{1}{2}M^2\varphi^2 + u\cos(\beta\varphi)\right]$$

$\Longrightarrow$ Yukawa-gas, 2D XY spin model + external field, 2D charged superfluid, bosonised QED_2

- Layered Sine-Gordon, Z_2 + (periodic) $\longrightarrow$ $\beta_c^2(N) = 8\pi\frac{N}{N-1}$

$$S_{\rm LSG}[\varphi] = \int {\rm d}^2x \left[\sum_{n=1}^{N}\frac{1}{2}(\partial_\mu\varphi_n)^2 + \frac{1}{2}M^2\left(\sum_{n=1}^{N}\varphi_n\right)^2 + \sum_{n=1}^{N}u_n\cos(\beta\varphi_n)\right]$$

$\Longrightarrow$ Layered vortex-gas, layered 2D XY spin model, layered superconductor, bosonised multiflavour QED_2

- Sine-Gordon, Z_2 + periodic $\longrightarrow$ topological $\beta_c^2 = 8\pi$

$$S_{\rm SG}[\varphi] = \int d^2x \left[\frac{1}{2}(\partial_\mu\varphi)^2 + u\cos(\beta\varphi)\right]$$

$\Longrightarrow$ Conformal Field Theory, characterised by the central charge $C = 1$

- Sinh-Gordon, Z_2 $\longrightarrow$ phase-structure?

$$S_{\rm ShG}[\varphi] = \int d^2x \left[\frac{1}{2}(\partial_\mu\varphi)^2 + u\cos(i\beta\varphi)\right] = \int d^2x \left[\frac{1}{2}(\partial_\mu\varphi)^2 + u\cosh(\beta\varphi)\right]$$

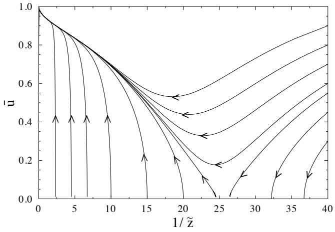
$\Longrightarrow$ Conformal Field Theory, characterised by the central charge $C = 1$ Ising: $C = 1/2$

- Shine-Gordon, Z_2 + (periodic $\beta_2 = 0$) $\longrightarrow$ phase-structure?

$$S_{\rm Shine}[\varphi] = \int d^2x \left[\frac{1}{2}(\partial_\mu\varphi)^2 + u\,\mathrm{Re}\cos[(\beta_1 + i\beta_2)\phi]\right] = \int d^2x \left[\frac{1}{2}(\partial_\mu\varphi)^2 + u\cos(\beta_1\phi)\cosh(\beta_2\phi)\right]$$

$\Longrightarrow$ Conformal Field Theory

- Global dilatation symmetry ($a \to \lambda a$)
  $\Rightarrow$ symmetry under the dilatation of the length scale
  $\Rightarrow$ scale invariance
  $\Rightarrow$ phase transition point
  $\Rightarrow$ fixed point of the FRG
  For example: SG model $\Rightarrow$



- Local dilatation symmetry ($a \to \lambda(x)a$)
  $\Rightarrow$ conformal transformations: relative angles unchanged
  $\Rightarrow$ conformal group: finite dimensional for $d \neq 2$
- $d = 2$ dimensions:
  $\Rightarrow$ conformal group: infinite dimensional
  $\Rightarrow$ central charge (C) of the Virasoro algebra
  conformal invariance $\Leftrightarrow$ scale invariance
- fixed points of FRG $\Rightarrow$ central charges (C)

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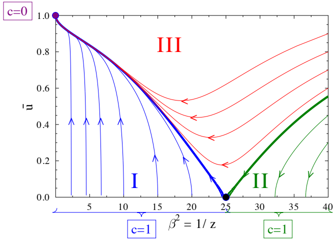

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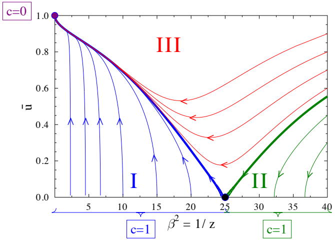
Out of the fixed points? → C-theorem in $d = 2$!

- C-theorem → c-function A. B. Zamolodchikov, JETP Lett. **43**, 730 (1986)
  ⇒ function of the couplings: c(g)
  ⇒ decreasing from UV to IR
  ⇒ at fixed points: $c(g^\star) = C$



- C-function in FRG A. Codello, G. D'Odorico, and C. Pagani, JHEP **1407**, 040 (2014)
  ⇒ expression in LPA

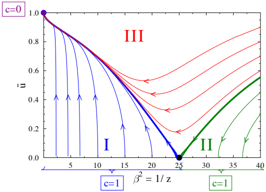
$$k\partial_k c_k = \frac{[k\partial_k \tilde{V}_k''(\varphi_0)]^2}{[1 + \tilde{V}_k''(\varphi_0)]^3},$$

- Questions:
  ⇒ c-function for the SG?
  ⇒ phase diagram, c-function for the ShG?
  ⇒ (interpolating models?)

sine-Gordon, LPA + z

- Rescaling $z = 1/\beta^2$, $\tilde{u} = uk^{-2}$

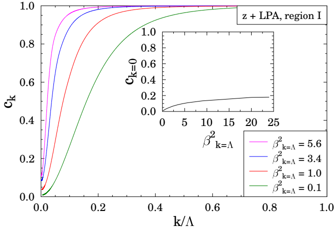
$$S_{\rm SG}[\varphi] = \int d^2x \left[\frac{1}{2} z(\partial_\mu \varphi)^2 - (\tilde{u}k^2)\cos(\varphi)\right]$$

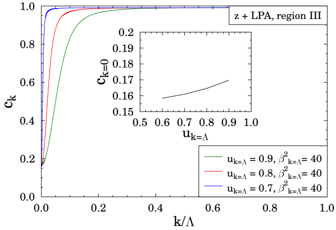


- FRG equations (power-law regulator $b = 1$)

$$(2 + k\partial_k)\tilde{u}_k = \frac{1}{2\pi z_k \tilde{u}_k}\left[1 - \sqrt{1 - \tilde{u}_k^2}\right], \qquad k\partial_k z_k = -\frac{1}{24\pi}\frac{\tilde{u}_k^2}{[1 - \tilde{u}_k^2]^{\frac{3}{2}}}, \qquad k\partial_k c_k = \frac{(k\partial_k \tilde{u}_k)^2}{(1 + \tilde{u}_k)^3}$$

- c-function results (power-law regulator $b = 2$)





V. Bacso, N. Defenu, A. Trombettoni, I. Nandori, Nucl. Phys. B **901** (2015) 444.

sinh-Gordon, LPA

- Linearised FRG equation, LPA

$$(2 + k\partial_k)\tilde{V}_k(\varphi) = -\frac{1}{4\pi}\tilde{V}_k''(\varphi) + \mathcal{O}(\tilde{V}_k''^2)$$

- SG model $\tilde{V}_{\rm SG}(\varphi) = \tilde{u}_k \cos(\beta\varphi)$

$$k\partial_k \tilde{u}_k = \tilde{u}_k\left(-2 + \frac{1}{4\pi}\beta^2\right) \qquad \rightarrow \qquad \beta_c^2 = 8\pi$$

$\Rightarrow$ topological phase transition

- ShG model $\tilde{V}_{\rm ShG}(\varphi) = \tilde{u}_k \cos(i\beta\varphi)$

$$k\partial_k \tilde{u}_k = \tilde{u}_k\left(-2 - \frac{1}{4\pi}\beta^2\right) \qquad \rightarrow \qquad \text{No} \ \ \beta_c^2$$

$\Rightarrow$ No topological phase transition

N. Defenu, V. Bacso, I. G. Márián, I. Nandori, A. Trombettoni, in preparation

sinh-Gordon, LPA + z

- Taylor expansion (Z_2 symmetry)

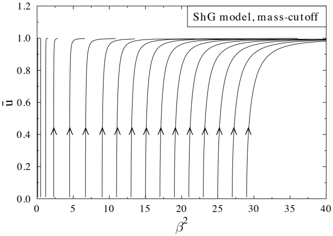
$$\tilde{V}_{\mathrm{ShG}}(\varphi) = \tilde{u}_k \cos(i\beta\varphi) \cong \tilde{u}_k \left[1 + \frac{1}{2}\beta^2\varphi^2 + \frac{1}{4!}\beta^4\varphi^4 + \ldots\right]$$

initial values: symmetric phase of the Ising ⇒ single phase

- FRG equations (power-law regulator $b = 1$) $\beta \to i\beta$

$$(2 + k\partial_k)\tilde{u}_k = -\frac{\beta^2}{2\pi\tilde{u}_k}\left[1 - \sqrt{1 - \tilde{u}_k^2}\right]$$

$$k\partial_k\beta_k^2 = -\frac{1}{24\pi}\frac{\beta_k^4\tilde{u}_k^2}{[1-\tilde{u}_k^2]^{\frac{3}{2}}}$$



⇒ single phase! N. Defenu, V. Bacso, I. G. Márián, I. Nandori, A. Trombettoni, in preparation

Summary I

- C-function for the SG model obtained in FRG
- No topological-type phase transition for the ShG model
- No Ising-type phase transition for the ShG model
- Phase structure of the ShG model ($\beta \to i\beta$)
- C-function for the ShG model ($\beta \to 0$)

Outlook

Interpolating models $\rightarrow$ Poster (No. 2)

- Standard model Brout-Englert-Higgs (BEH) mechanism

$$\mathcal{L} = (D_\mu\phi)^\star(D^\mu\phi) - V(\phi) - \frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right),$$

$$V = \mu^2\phi^\star\phi + \lambda(\phi^\star\phi)^2, \; (D_\mu = \partial_\mu + ig\mathbf{T}\cdot\mathbf{W}_\mu + ig' y_j B_\mu)$$

Two phases:

- VEV=0 for $\mu^2 > 0$
- VEV=$\sqrt{\phi^\star\phi} = \sqrt{-\mu^2/(2\lambda)} = v/\sqrt{2}$ for $\mu^2 < 0$.

- field is parametrized as (with $v = 246$ GeV)

$$\phi(x) = \frac{1}{\sqrt{2}}\exp\left(i\frac{\mathbf{T}\cdot\xi(x)}{v}\right)\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- Complete Lagrangian for the Higgs sector

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu h\partial^\mu h - \frac{1}{2}M_h^2 h^2 - \frac{M_h^2}{2v}h^3 - \frac{M_h^2}{8v^2}h^4 \\ &+ \left(M_W^2 W_\mu^+ W^{-\,\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu\right)\left(1 + 2\frac{h}{v} + \frac{h^2}{v^2}\right)\end{aligned}$$

$$M_h = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2} = 125.6\,\mathrm{GeV} \rightarrow \lambda = 0.13.$$

SM Higgs potential is not a priori fixed. One can investigate

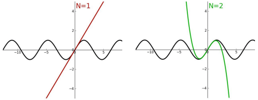
- developing more minima C.D. Froggatt and H.B. Nielsen, PLB **368** (1996) 96
- Higgs inflation G. Isidori, V. S. Rychkov, A. Strumia, N. Tetradis, PRD **77** (2008) 025034

$\Longrightarrow$ no drastic change in the RG running (polynomial potential).

Periodic potential $\rightarrow$ drastic change in the phase structure!

- Infinitely many minima: NO truncation in Taylor series!

$$V = u[\cos(\beta\sqrt{\phi^\star\phi}) - 1]$$



- by standard parametrisation the Higgs sector reads

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu h \partial^\mu h - u\left[\cos\left(\frac{1}{\sqrt{2}}\beta|v + h(x)|\right) - 1\right] \\ &+ \left(M_W^2 W_\mu^+ W^{-\,\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu\right)\left(1 + 2\frac{h}{v} + \frac{h^2}{v^2}\right).\end{aligned}$$

$$\beta = \sqrt{2}\pi/v \qquad \Rightarrow \qquad \text{VEV}_{\text{quadratic}} = \text{VEV}_{\text{periodic}}$$

Sine-Gordon Higgs potential II

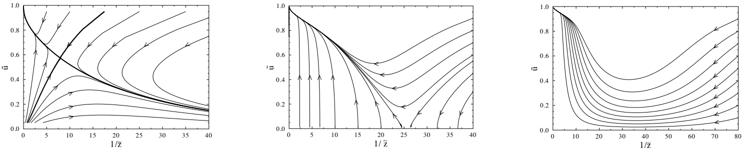
- Simplest realisation: SG model in LPA

$$\text{LPA}: \quad \Gamma_k = \int d^d x \left[\frac{1}{2} (\partial_\mu \varphi_x)^2 + u_k \cos(\beta \varphi_x) \right]$$

- beyond LPA requires rescaling $\tilde{\varphi} \equiv \beta\varphi$ and $z_k \equiv 1/\beta_k^2$

$$\text{LPA}': \quad \Gamma_k = \int d^d x \left[\frac{1}{2} z_k (\partial_\mu \tilde{\varphi}_x)^2 + u_k \cos(\tilde{\varphi}_x) \right]$$

- FRG results (LPA') I. Nandori, arXiv:1108.4643 [hep-th]



$d < 2$ $d = 2$ $d > 2$

$\Rightarrow$ SG model has a single phase in $d = 4$!

Summary II

- Periodic Higgs potential is proposed.
- Simplest realisation is the sine-Gordon (SG) model.
- SG model has a single phase in $d = 4$ but bounded.

Outlook

- New idea: massive sine-Gordon (MSG) model.
- MSG has two phases and bounded from below in $d = 2$.
- What happens in $d = 4$? Stability? $\rightarrow$ Poster (No. 14)

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