(Non) Running Higgs inflation from an ERG perspective

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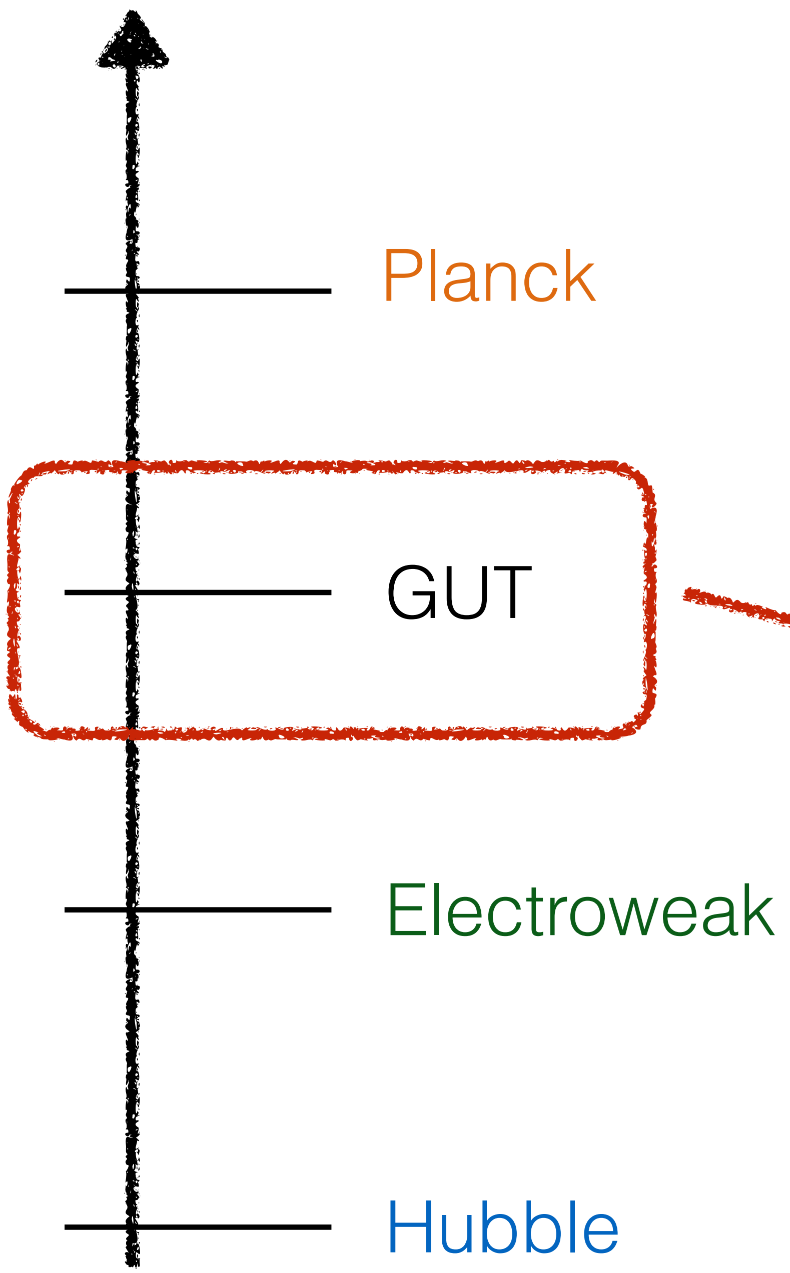
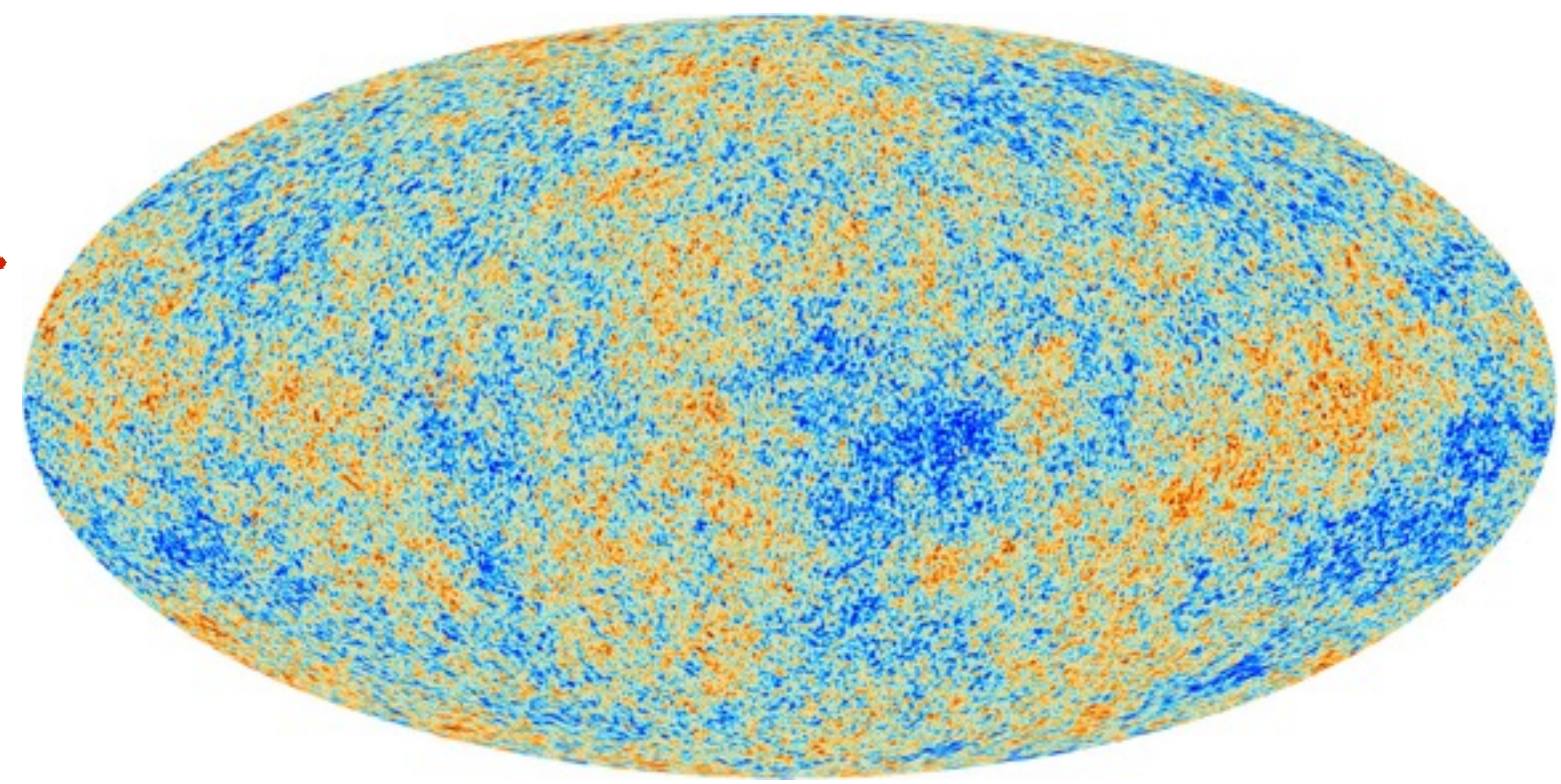
Probing new physics with the early universe



Primordial inflation opens an observational window to physics beyond the SM



CMB map: The Planck collaboration

Why (not) Higgs inflation?

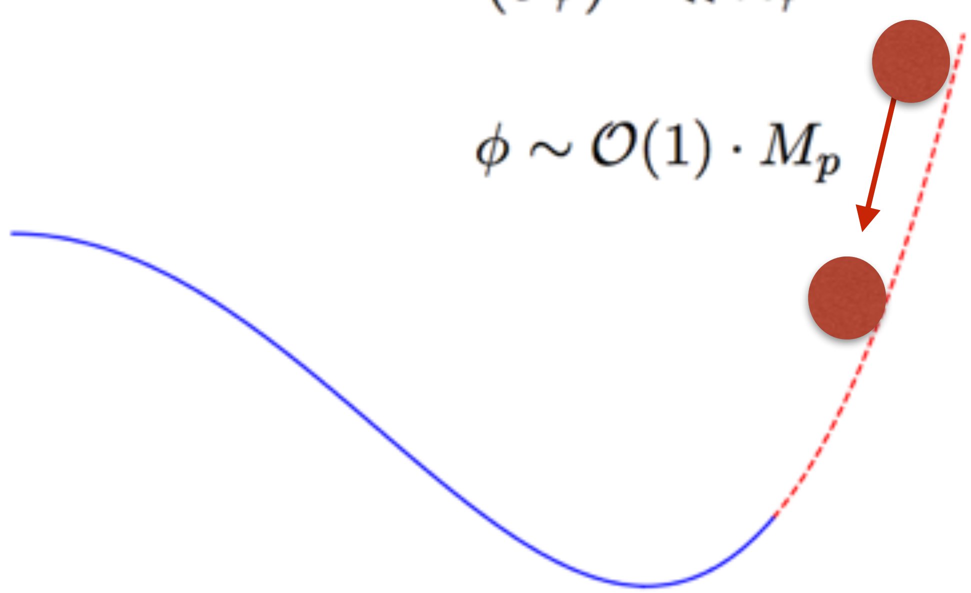
Idea: The Higgs boson inflates the Universe at very early times

The model is economical and could fit within the SM (or not?)

$$\frac{M_p^2}{2}R - \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{4}\lambda(\phi^2 - v^2)^2$$

$(\partial\phi)^2 \ll \lambda\phi^4$

$\phi \sim \mathcal{O}(1) \cdot M_p$



$\lambda(k = k_{\text{infl.}})$

$\partial_t \lambda = \beta_\lambda(\lambda, y_t, \cdots)$

$\lambda(k = k_{\text{EW}})$

No-go: Primordial fluctuations

$$\boxed{\mathcal{A}_{GW} \sim \frac{V(\phi)}{M_p^4} \sim 10^{-1} \cdot \lambda_{\text{infl.}} < 10^{-10}}$$

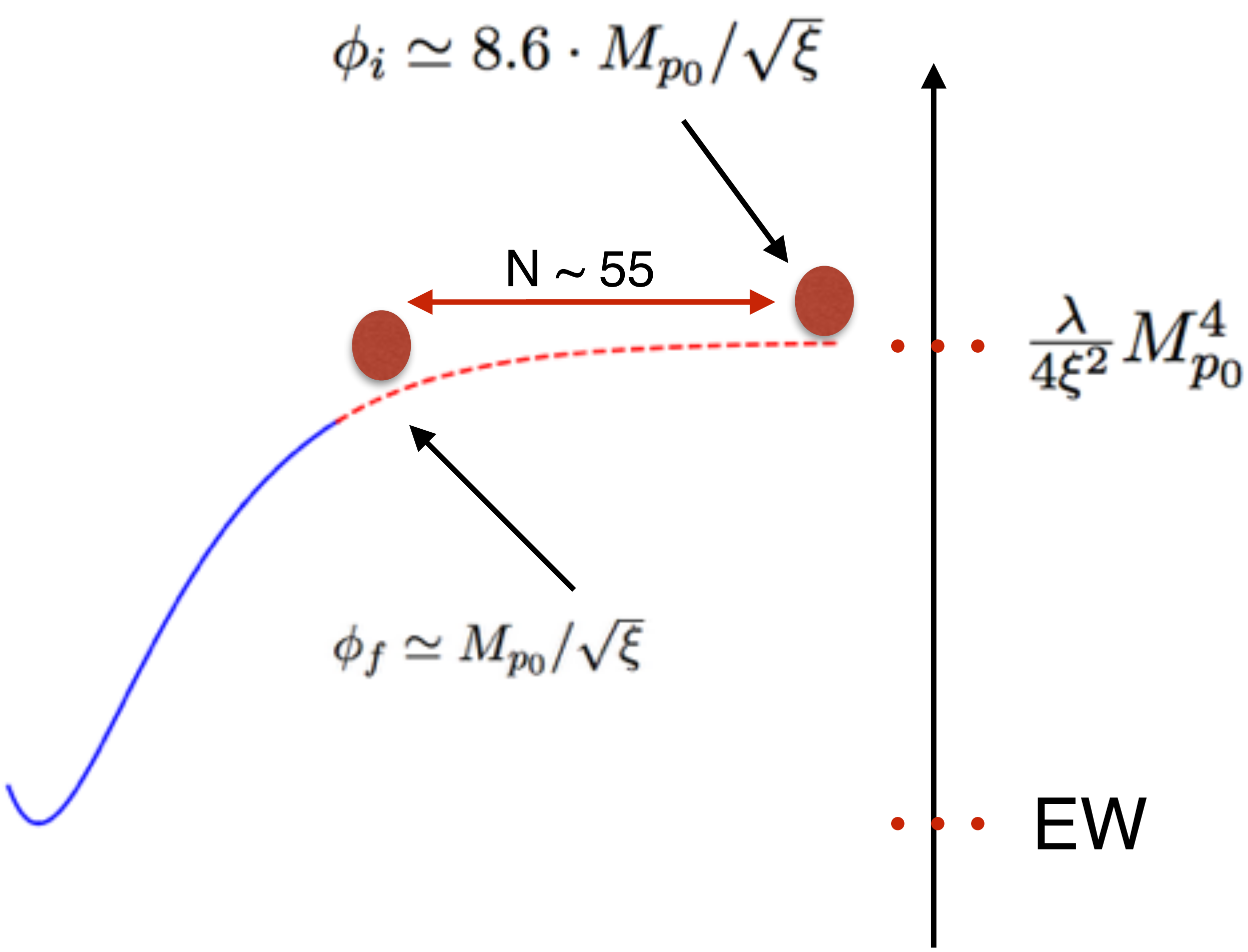
A. Linde 1985

The twist: Non-minimal coupling to gravity

$$\frac{1}{2}(M_p^2 + \xi\phi^2)R - \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{4}\lambda(\phi^2 - v^2)^2$$

Diagonalise kinetic terms

$$\frac{M_{p0}^2}{2}\widetilde{R} - \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) - M_{p0}^4 \cdot \frac{\lambda}{4\xi^2} \cdot \left(1 - e^{-\sqrt{2/3}\cdot\frac{\chi}{M_{p0}}}\right)^2$$



$$\boxed{\xi_{\text{inflation}} \equiv \xi[N_0, \mathcal{A}_{\text{Scalar}};\ \lambda_{(\text{EW})}, M_{t(\text{EW})}, \cdots]}$$

$$\boxed{\xi_{\text{inflation}} \simeq 10^4 \cdot \sqrt{\lambda_{\text{inflation}}}}$$

F. Bezrukov & M. Shaposhnikov 2008

The ERG approach

$$\boxed{\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left(\Gamma^{(2)} + R_k \right)^{-1} \cdot \partial_t R_k}$$

$$\Gamma = - \int \sqrt{-\bar{g}} \left(f_k(\phi) R - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V_k(\phi) \right) + S_{ghost} + S_{GF}$$

Background-field method

$$\phi = \bar{\phi} + \delta\phi \qquad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$\hat{h}_{\mu\nu} + \frac{1}{4} \bar{g}_{\mu\nu} h$$

Eucledian sphere, R, ϕ = constant

$$\Gamma^{(2)}_{\Phi_A \cdot \Phi_B} = Z_{\Phi_A \Phi_B}(\bar{\phi}, \bar{R}) \cdot (-\Box) + U_{\Phi_A \Phi_B}(\bar{\phi}, \bar{R})$$

$$\Gamma^{(2)}_{h_{\mu\nu} h_{\gamma\delta}} \quad \Gamma^{(2)}_{h_{\mu\nu} \phi} \quad \Gamma^{(2)}_{\phi\phi}$$

C. Wetterich 1993 , T. R. Morris 1994

The ERG approach

$$\Gamma = -\int \sqrt{-\bar{g}}\, \left(f_k(\phi) R - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V_k(\phi) \right) + S_{ghost} + S_{GF}$$

Type 1 cut—off & optimised (Litim's) regulator

$$\Gamma^{(2)}_{k\,\Phi_A \Phi_B}(-\Box) \to \Gamma^{(2)}_{k\,\Phi_A \Phi_B}(-\Box) + R_{k\,\Phi_A \Phi_B}(-\Box)$$

De Donder gauge

$$\partial_t \Gamma_k = \mathcal{F}_0 + \mathcal{F}_1 \cdot \frac{\partial_t f}{f} + \mathcal{F}_2 \cdot \frac{\partial_t f'}{f'}$$

$$\mathcal{F}_i = \mathcal{F}_i[f, f', f'', V, V', V'']$$

$$\boxed{\partial_t f(\phi) = \mathcal{F}_f[\tilde{\phi}_*; g_j, \partial_t g_j] \quad \partial_t V(\phi) = \mathcal{F}_V[\tilde{\phi}_*; g_j, \partial_t g_j]}$$

Structure of the 1—loop beta functions at leading order in G

$$k\partial_k \tilde{g}_i = \beta_{\tilde{g}_i}(\tilde{g}_j) \equiv (-n + \eta_{g_i}(g_j)) \cdot \tilde{g}_i \qquad \tilde{g}_i \equiv g_i(k)/k^n$$

$$\beta_\lambda = \frac{21}{16\pi^2} \cdot \frac{\lambda^2}{(1+8\pi\xi\tilde{G}\tilde{\phi}_*^2)\boxed{\cdot\,\Omega^3}} + \frac{1}{192\pi^2} \cdot \frac{\lambda \cdot \tilde{G}}{(1+8\pi\xi\tilde{G}\tilde{\phi}^2)\boxed{\cdot\,\Omega^3}} \cdot g(\mu,\xi) + \mathcal{O}\left(\tilde{G}^2\right)$$

$$\Omega \simeq 1 + 72\pi\xi^2\tilde{G}\tilde{\phi}_*^2$$

$$\mu \equiv \lambda\tilde{\phi}_*^2 \equiv \lambda\tilde{\phi}_*^2/k^2$$

Suppression effect for large ξ and field value, ϕ

$$\beta_\xi = \frac{1}{64\pi^2} \cdot \frac{\lambda\,(28\xi + 10\mu + 5)}{\Omega^3} + \frac{1}{384\pi^2} \cdot \frac{\xi \cdot \tilde{G}}{\Omega^3} \cdot q(\mu,\xi) + \mathcal{O}\left(\tilde{G}^2\right)$$

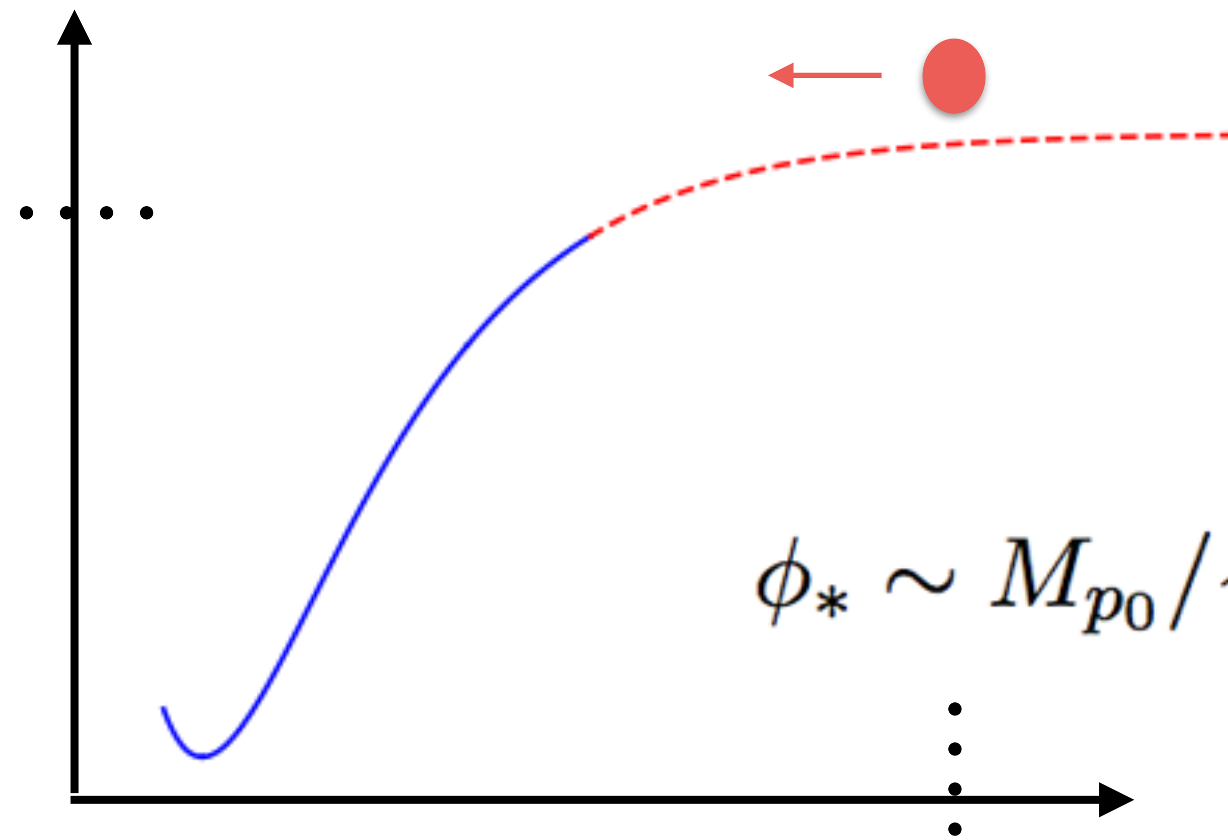
$$\beta_{\tilde{G}} = 2\tilde{G} + \frac{1}{24\pi} \cdot \frac{14\xi + 240\mu^2 + 230\mu - 55}{(1+8\pi\xi\tilde{G}\tilde{\phi}_*^2)^2 \cdot \Omega^2} \cdot \tilde{G}^2 + \mathcal{O}\left(\tilde{G}^3\right)$$

See also: G. Narain & R. Percacci 2009/ G. Narain & C. Rahmede 2009

Estimating the cut-off during inflation

During slow-roll, the field and curvature are approximately constant

$$\bar{R} \simeq U/M_{p_0}^2 \simeq \lambda/\xi^2$$



$$\phi_* \sim M_{p_0}/\sqrt{\xi}$$

Slow-roll dynamics

$$\bar{R}/k^2 \lesssim \mathcal{O}(1)$$

$$\boxed{\left.\frac{k^2}{M_{p_0}^2}\right|_{\text{inflation}} \gtrsim \frac{\lambda}{\xi^2}}$$

$$\boxed{\tilde{\phi}_{*\text{inflation}} \equiv \left.\frac{\phi_*}{k}\right|_{\text{inflation}} \lesssim \sqrt{\frac{\xi}{\lambda}}}$$

Running during inflation

We are interested in the running of Newton's G, the scalar's quartic λ, and non-minimal coupling ξ

$$\beta_{\widetilde{G}} = 2\widetilde{G} + \boxed{\frac{1}{24\pi}\cdot\frac{14\xi+240\mu^2+230\mu-55}{(1+8\pi\xi\widetilde{G}\tilde{\phi}_*^2)^2\cdot\Omega^2}\cdot\widetilde{G}^2} + \mathcal{O}\left(\widetilde{G}^3\right)$$

$$\sim 10^{-7}\cdot\widetilde{G}^2$$

$$\Omega \simeq 72\pi\xi^2 G\phi_*^2 \sim 72\pi\xi$$

$$\mu \equiv \lambda\tilde{\phi}_*^2 \equiv \lambda\tilde{\phi}_*^2/k^2$$

$$\beta_\lambda = \frac{21}{16\pi^2}\cdot\frac{\lambda^2}{(1+8\pi\xi\widetilde{G}\tilde{\phi}_*^2)\cdot\Omega^3} + \boxed{\frac{1}{192\pi^2}\cdot\frac{\lambda\cdot\widetilde{G}}{(1+8\pi\xi\widetilde{G}\tilde{\phi}^2)\cdot\Omega^3}\cdot g(\mu,\xi)+\mathcal{O}\left(\widetilde{G}^2\right)}$$

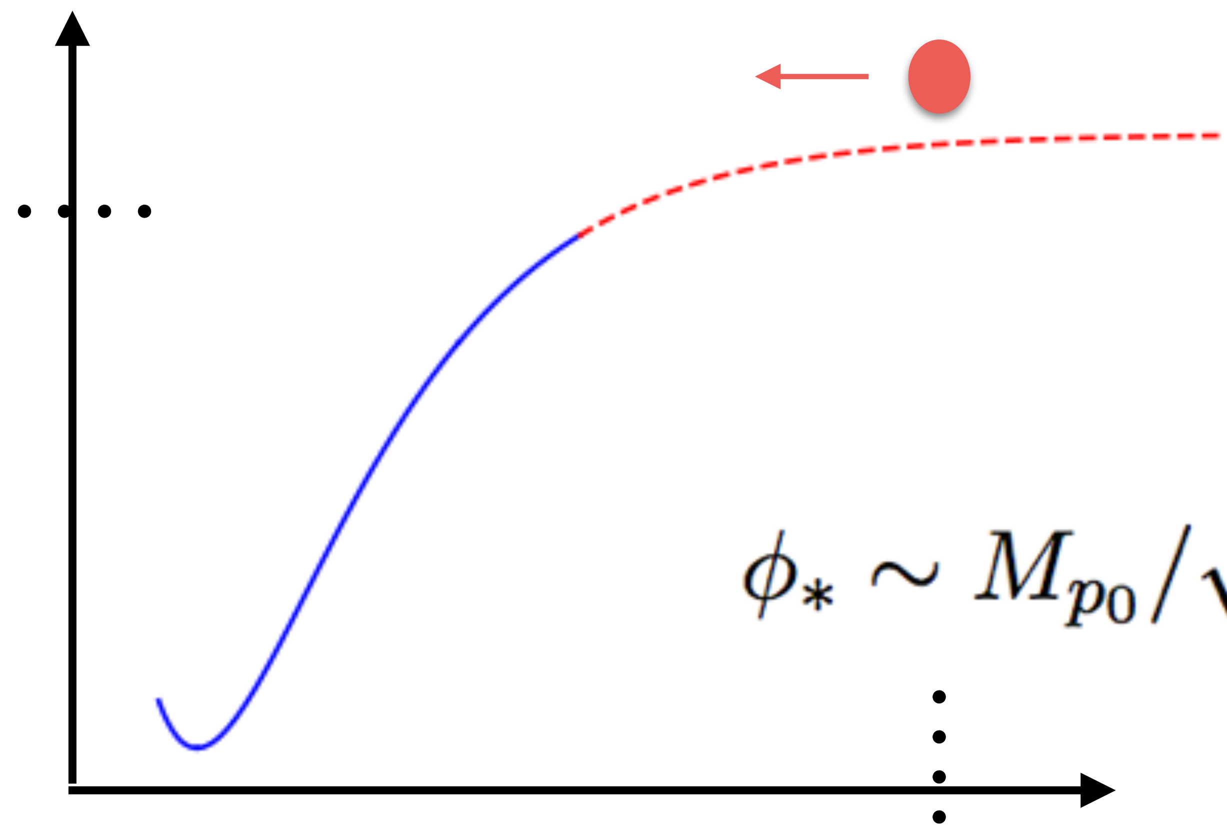
$$\sim (\lambda/\xi)^n$$

$$\beta_\xi = \frac{1}{64\pi^2}\cdot\frac{\lambda\,(28\xi+10\mu+5)}{\Omega^3} + \boxed{\frac{1}{384\pi^2}\cdot\frac{\xi\cdot\widetilde{G}}{\Omega^3}\cdot q(\mu,\xi)+\mathcal{O}\left(\widetilde{G}^2\right)}$$

Running during inflation

The flatness of the potential during inflation is crucial

$\bar{R} \simeq U/M_{p_0}^2 \simeq \lambda/\xi^2$

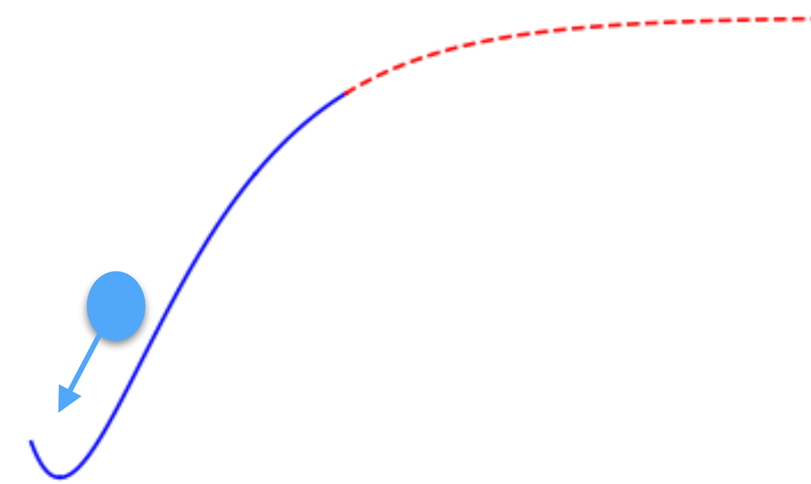


$\phi_* \sim M_{p0}/\sqrt{\xi}$

$$\left(\frac{\lambda}{\xi^2}\right)^{-1}\partial_t\left(\frac{\lambda}{\xi^2}\right) = \left(\frac{\beta_\lambda}{\lambda} - \frac{2\beta_\xi}{\xi}\right) \simeq \frac{1}{16\pi^2}\cdot\frac{\lambda}{\Omega^3} \sim \boxed{\frac{1}{16\pi^2}\cdot\frac{\lambda}{\xi^3} \ll 1}$$

The post-inflationary era

After inflation the scalar rolls down to its true vacuum ϕ/k <<1 followed by a graceful exit



$$\beta_{\tilde{G}} = \boxed{2\tilde{G} + \frac{48\pi}{55 - 14\xi}\cdot \tilde{G}^2} + \mathcal{O}(\tilde{G}^3)$$

$$\beta_{\xi} = \boxed{\frac{\lambda(28\xi + 5)}{64\pi^2}} + \frac{\xi^2(48\xi + 31)}{8\pi}\cdot \tilde{G}$$

$$\beta_{\lambda} = \boxed{\frac{21\lambda^2}{16\pi^2}} + \frac{\lambda}{\pi}\left(36\xi^2 + 14\xi + 5\right)\cdot \tilde{G}$$

Equations acquire their more familiar form at leading order

Gauge/Regulator dependence?

Asymptotic Safety ?

Asymptotic Safety could not only provide a UV completion to the model, but also explain its initial conditions

$$\tilde{G}_{\rm fp} = 0.527, \quad \xi_{\rm fp} = 0, \quad \lambda_{\rm fp} = 0$$

Gaussian Matter Fixed Point

$$\beta_\xi = \frac{\lambda(28\xi + 5)}{64\pi^2} + \frac{\xi^2(48\xi + 31)}{8\pi} \cdot \tilde{G}$$

$$\beta_{\tilde{G}} = (2 + \eta_G) \cdot \tilde{G} \qquad \eta_G = \frac{14\xi - 55}{2\left(12\pi - \tilde{G}\right)} \cdot \tilde{G}$$

$$\xi \sim 10^4, \lambda \sim 10^{-2}$$

$$\xi < \frac{55}{14} \simeq 3.93$$

$$k_{\rm pole} = \sqrt{\frac{12\pi}{7}} \cdot \frac{M_{p_0}}{\sqrt{\xi_0}}$$

See also: G. Narain & R. Percacci 2009/ G. Narain & C. Rahmede 2009

Open questions

The stability of the electroweak vacuum is crucial

Top-quark mass, Role of higher-order operators, …

Naturalness and initial conditions

Non-minimal-coupling scale, Connection with EW physics, Asymptotic Safety

Inclusion of fermionic/Yukawa sector*

Frame dependence: Jordan- and Einstein-frame calculations

* See e.g K. Oda & M. Yamada 2016/ A. Eichhorn, A. Held, J. M. Pawlowski 2016

Final remarks

Higgs inflation could provide a neat explanation of the inflationary paradigm within the SM

The ERG provides an elegant description of the quantum dynamics of the model

Running due to Higgs and gravitons during inflation appears to be sufficiently suppressed and flatness of the potential preserved

Thank you!