

Nonperturbative renormalization flow of the Higgs potential

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M. Warschinke

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seit 1558

Carl Zeiss Stiftung

LHC works quite well!

- search for new physics beyond the standard model



http://people.physics.tamu.edu/kamon/research/refColliders/LHC/LHC_is_back.html

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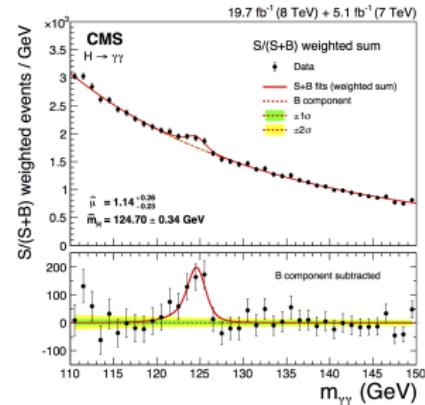
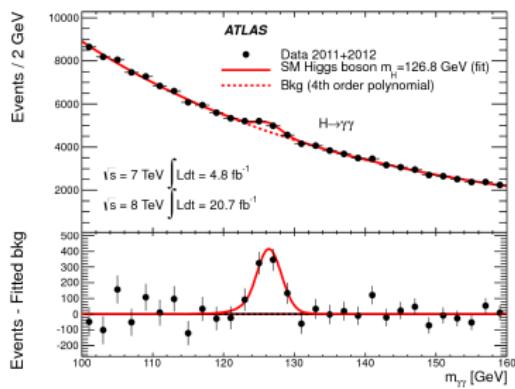
- search for new physics beyond the standard model
- detailed studies of the Higgs



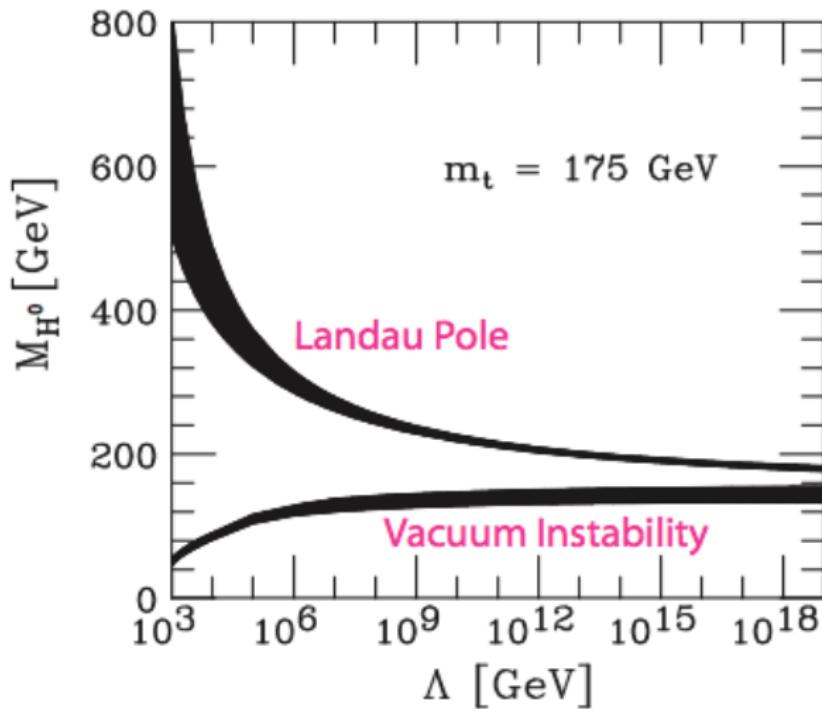
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Highlight LHC run I:

$$m_H = 125.09_{\pm 0.11(\text{sys})}^{\pm 0.21(\text{stat})} \text{ GeV}$$



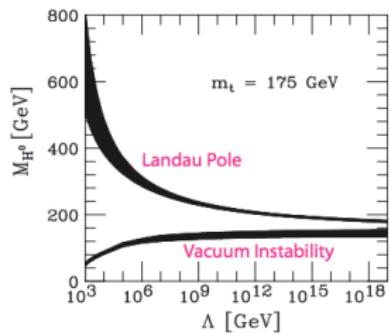
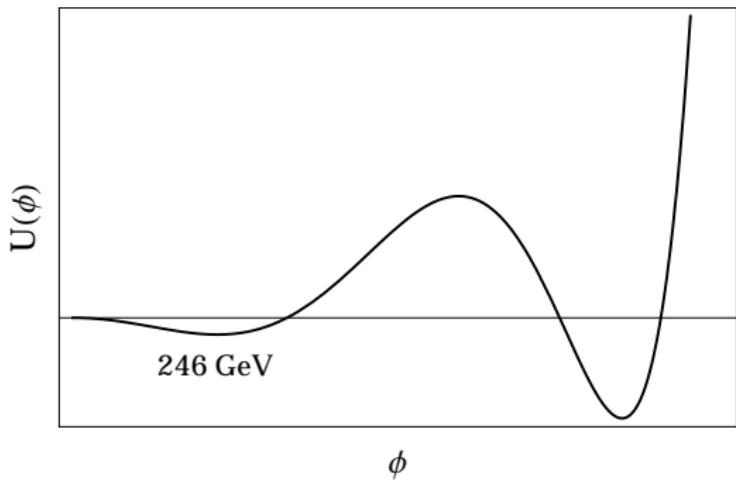
Higgs mass bounds & vacuum stability



Krige, Linde '76
Maiani et al '78
Krasnikov '78
Politzer, Wolfram '78
Cabibbo et al. '79
Hung '79
Linde '80
Lindner '85
Wetterich '87
Lindner et al '89
Sher '89
Ford et al '93
Altarelli, Isidori '94
Espinosa, Quiros '95
Schrempp, Wimmer '96
...

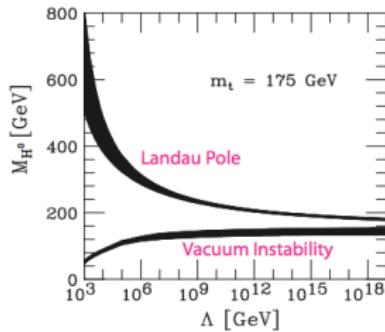
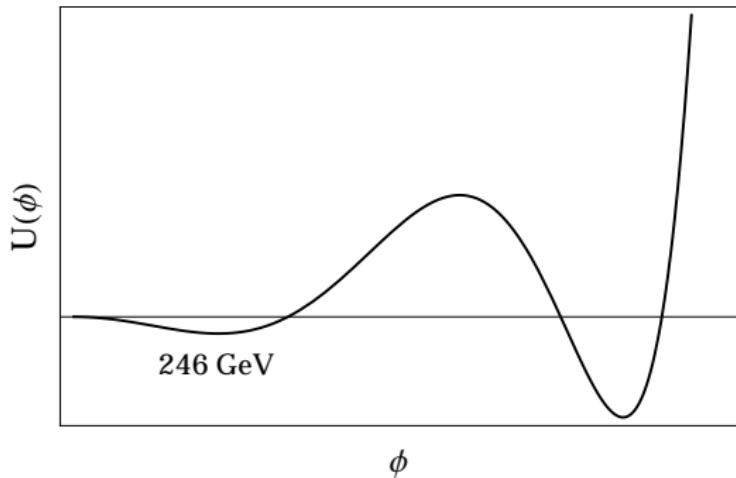
Hagiwara et al '02

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Higgs mass bounds & vacuum stability



Hagiwara et al '02

- second minimum occurs at a trans-Planckian scale [Gabrielli et al '13](#)
- discrepancy to lattice studies [Holland, Kuti '04; Fodor et al '13](#)
- implicit renormalization conditions in conflict with a well defined partition function [Branchina, Faivre '05](#)

Higgs-Top Model

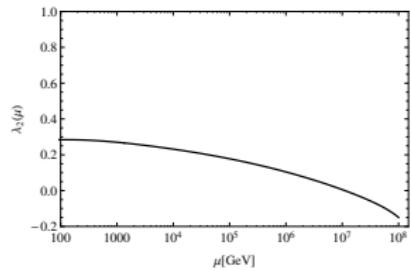
$$S = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + U(\phi^2) + \bar{\psi} i \not{\partial} \psi + i h \phi \bar{\psi} \psi \right]$$

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$$\mu \frac{d\lambda_2}{d\mu} = \frac{1}{4\pi^2} [3\lambda_2^2 + \lambda_2 h^2 - h^4]$$

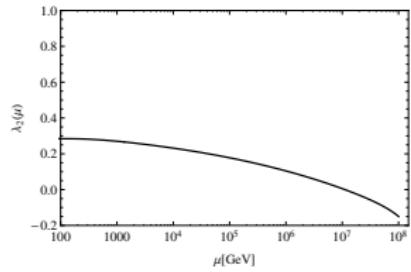


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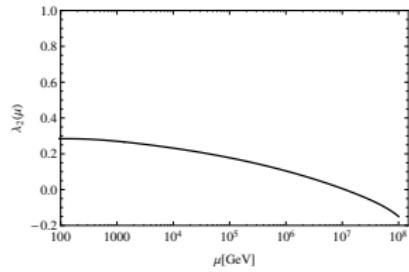
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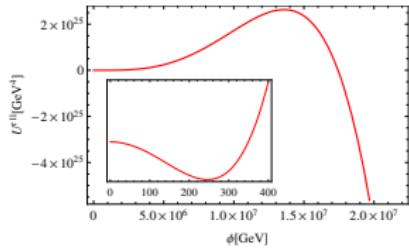
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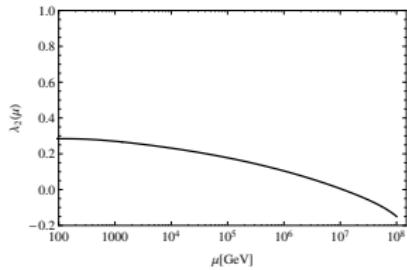


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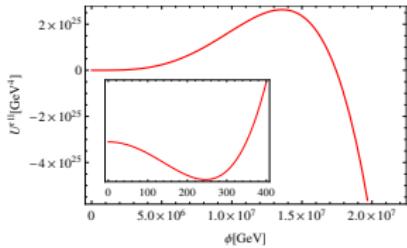
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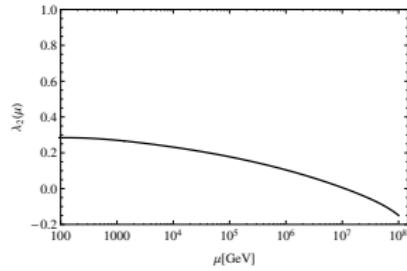
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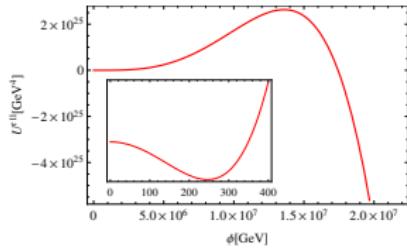
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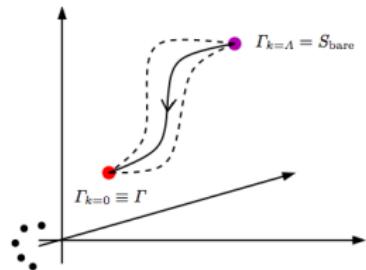
BUT:

- interaction part of the fermion determinant is strictly positive [Gies, RS '14](#)
- multi-scale problem $U(\mu; \phi)$

Functional renormalization group

$$\partial_k \Gamma_k = \frac{1}{2} S \text{Tr} \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Wetterich '93



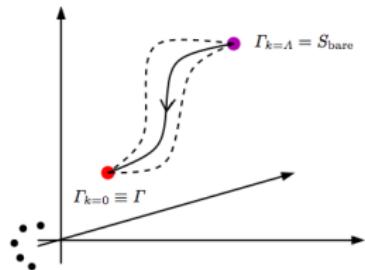
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$$\partial_k U(k, \phi) = \beta_U, \quad \partial_k h^2 = \beta_{h^2}$$



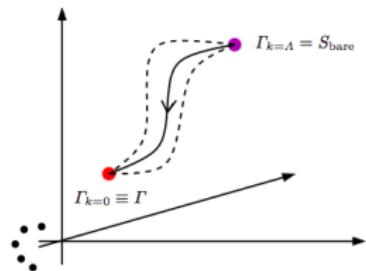
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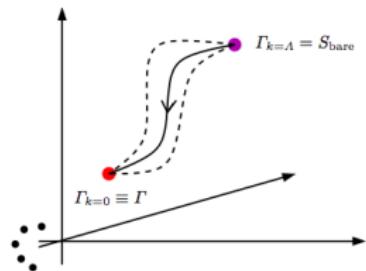
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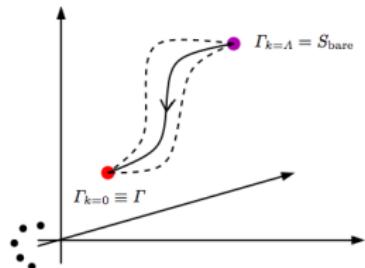
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$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2} \phi^2 + \frac{\lambda_{2\Lambda}}{8} \phi^4 \quad \text{or} \quad U_\Lambda = \frac{\lambda_{2\Lambda}}{8} (\phi^2 - v_\Lambda^2)^2$$



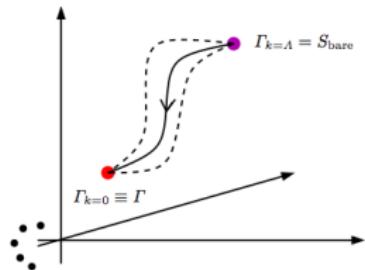
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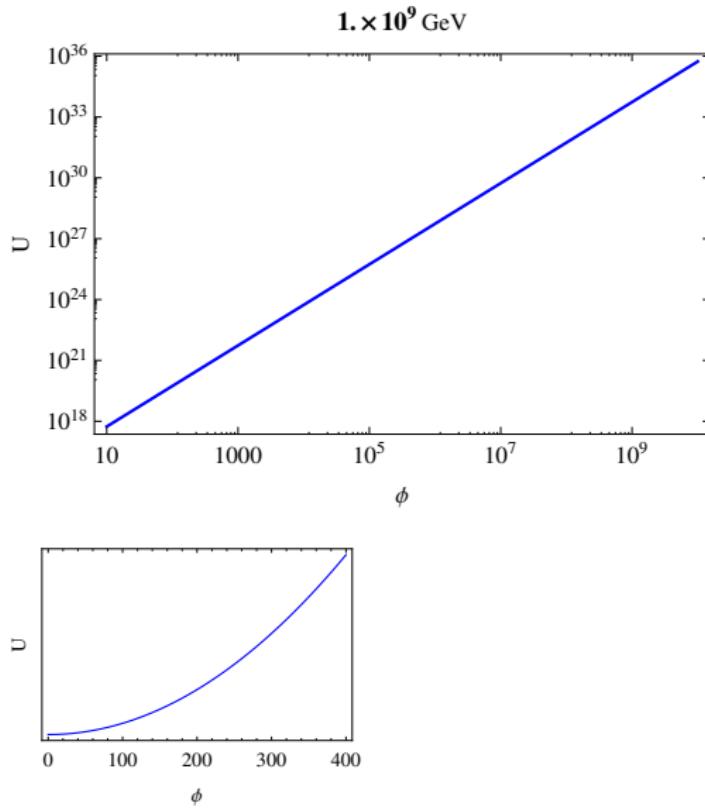
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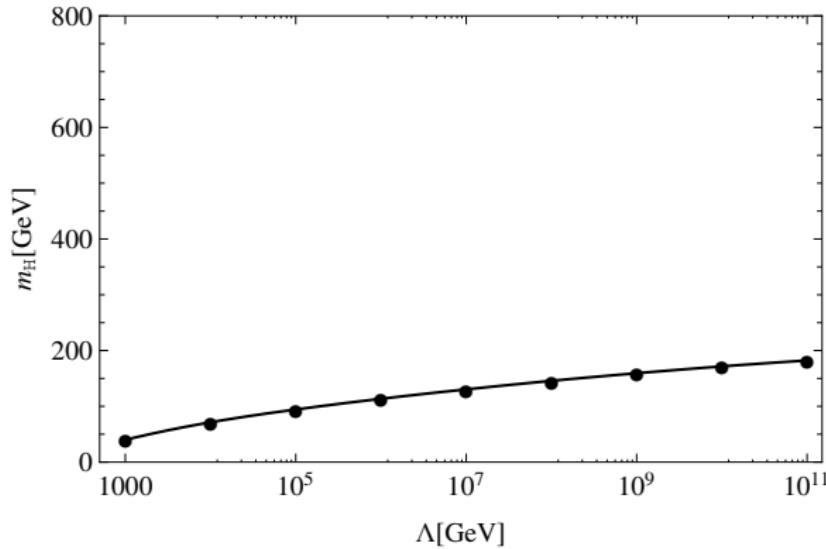
$$\begin{aligned} \lambda_{1\Lambda} \text{ (or } v_\Lambda) &\rightarrow v_0 = 246 \text{ GeV} \\ h_\Lambda &\rightarrow m_{\text{top}} = 173 \text{ GeV} \end{aligned}$$



RG flow of the Higgs potential



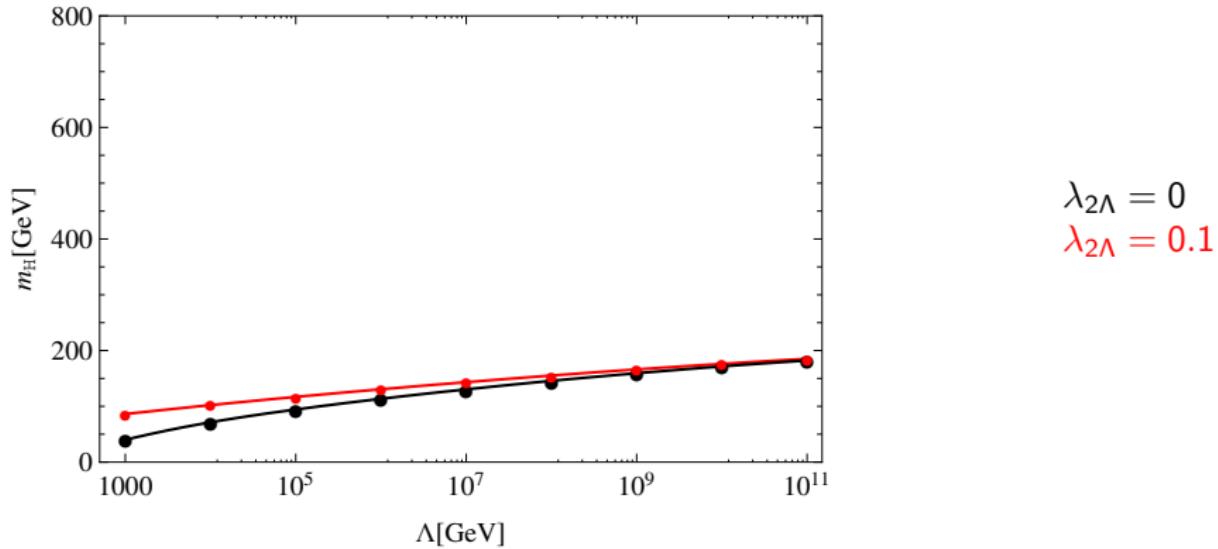
Nonperturbative Higgs mass bounds



$$\lambda_{2\Lambda} = 0$$

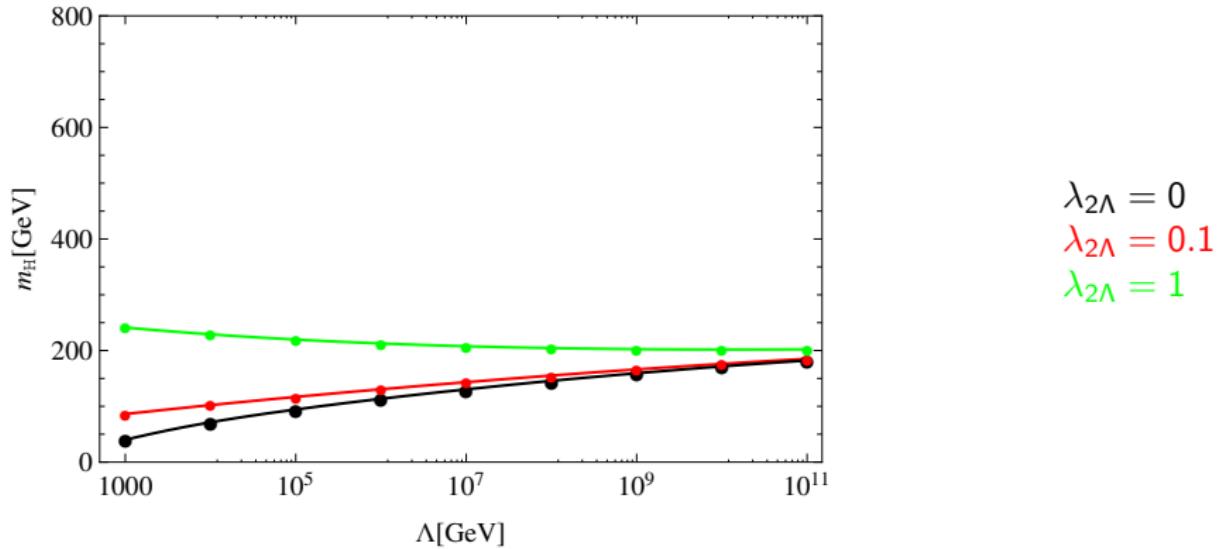
Gies, Gneiting, RS '13

Nonperturbative Higgs mass bounds



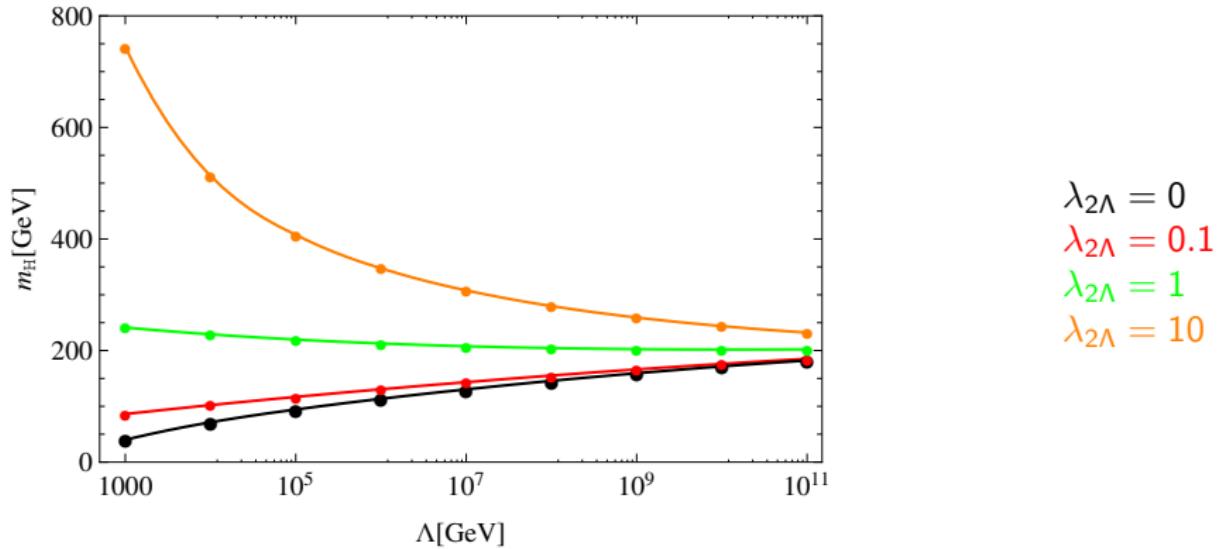
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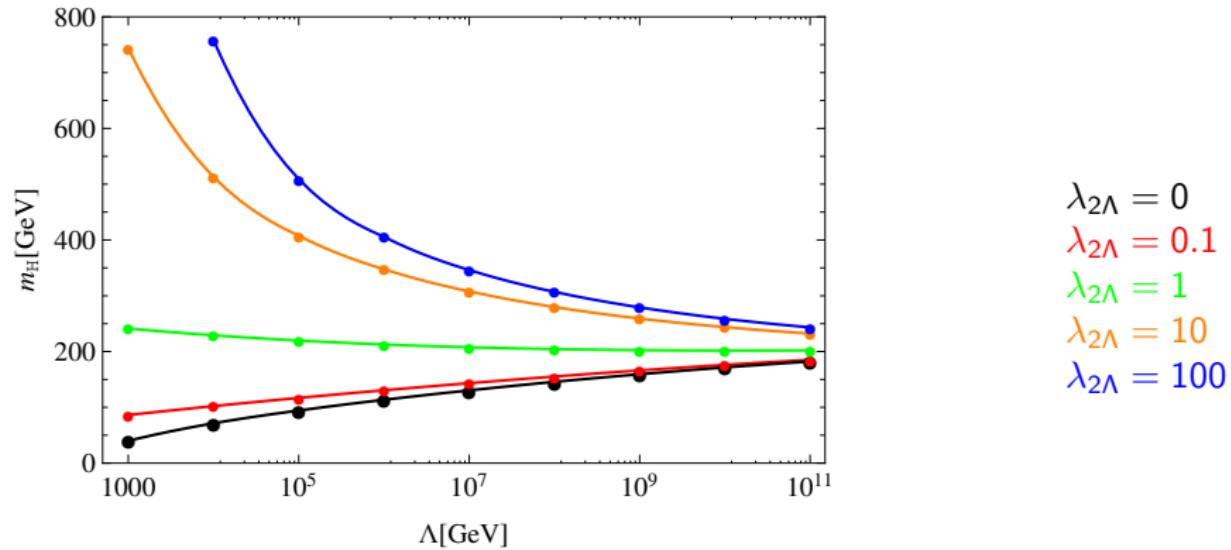
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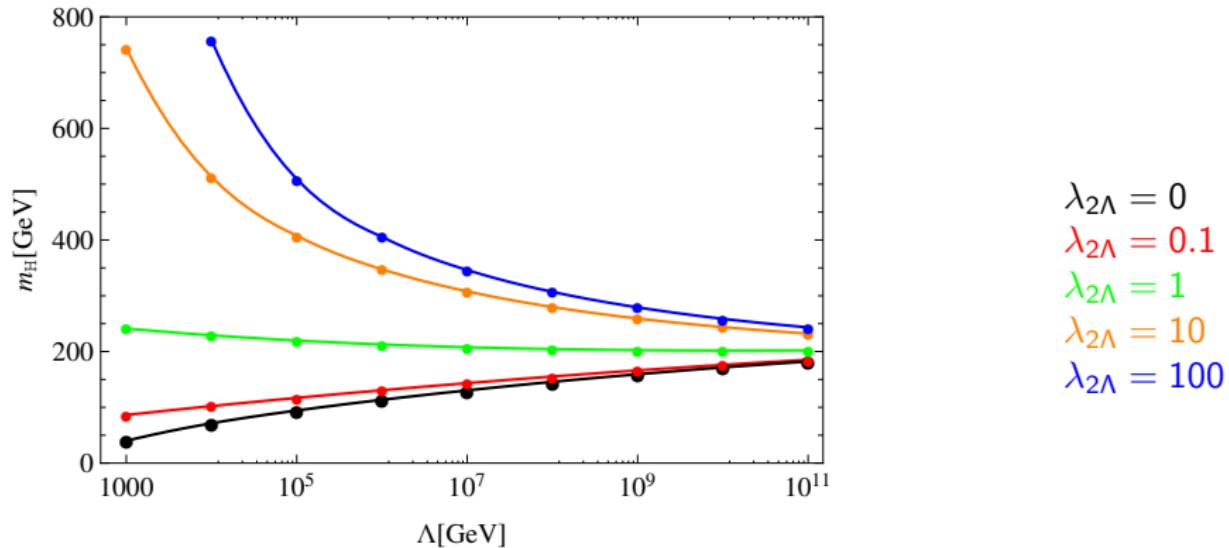
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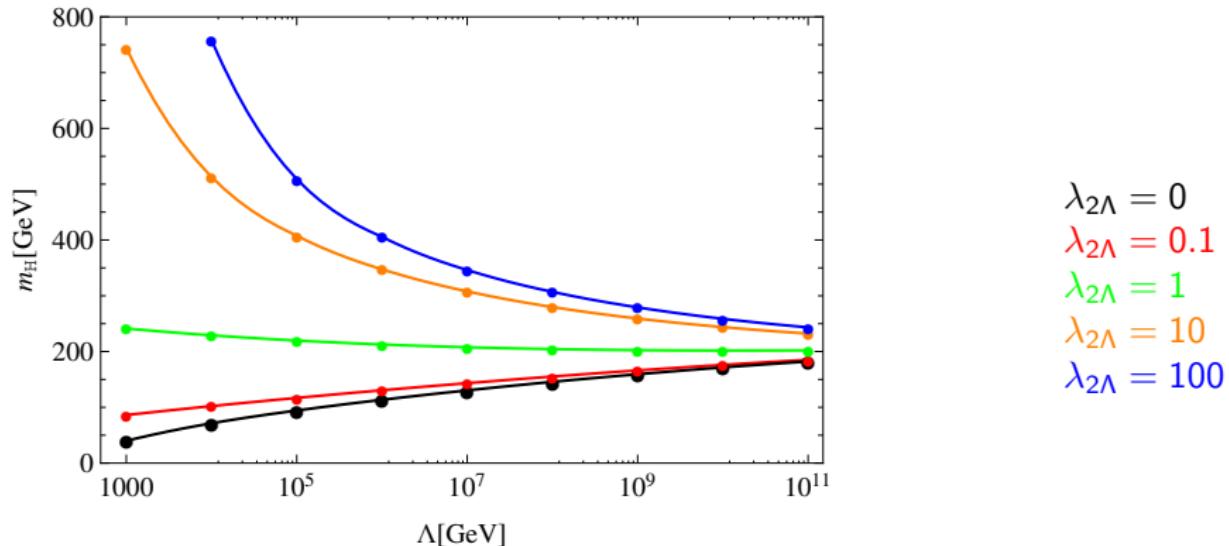
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- $m_H(\lambda_{2\Lambda})$ is monotonically increasing

Nonperturbative Higgs mass bounds



Gies, Gneiting, RS '13

- $m_H(\lambda_{2\Lambda})$ is monotonically increasing
- natural lower bound for quartic bare potentials $\lambda_{2\Lambda} \phi^4$
(cf. lattice simulations Gerhold et al. '07)

Generalized UV potentials

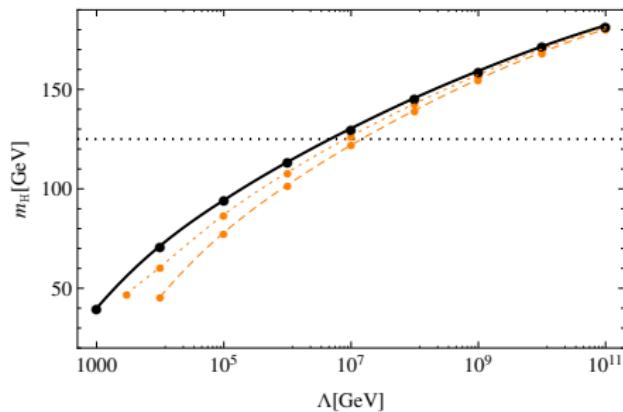
$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 + \frac{\lambda_{3\Lambda}}{48\Lambda^2}\phi^6$$

- we can choose $\lambda_{2\Lambda} < 0$, if the potential is stabilized by $\lambda_{3\Lambda} > 0$

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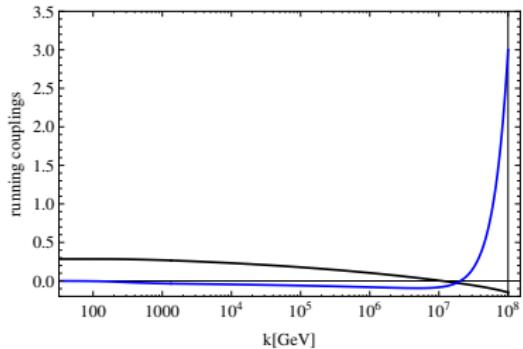
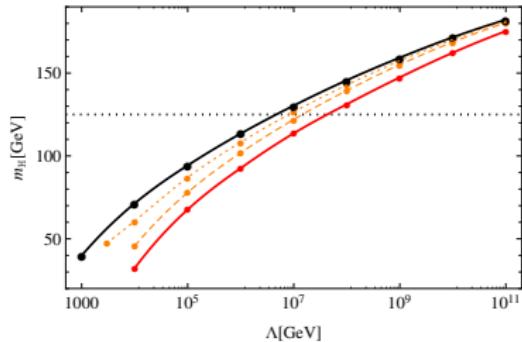
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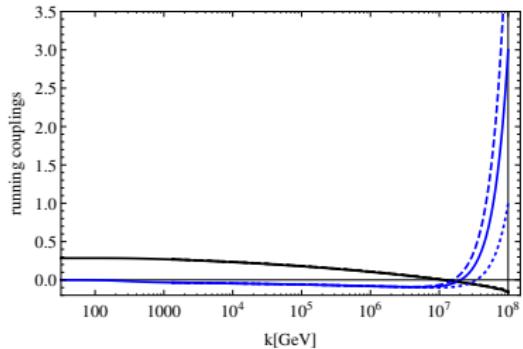
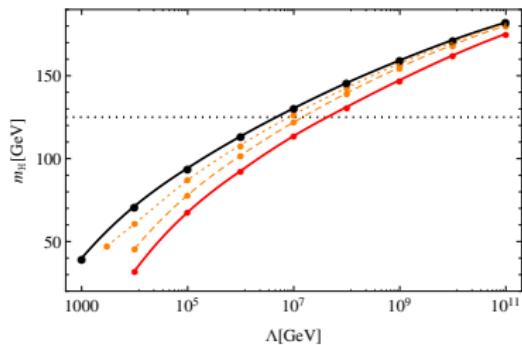


$$\begin{aligned}\lambda_{2\Lambda} &= 0, & \lambda_{3,\Lambda} &= 0 \\ \lambda_{2\Lambda} &= -0.05, & \lambda_{3,\Lambda} &= 3 \\ \lambda_{2\Lambda} &= -0.08, & \lambda_{3,\Lambda} &= 3\end{aligned}$$

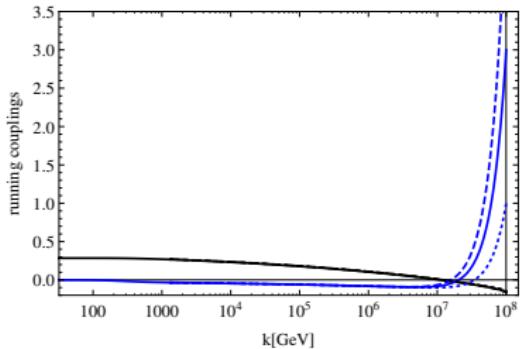
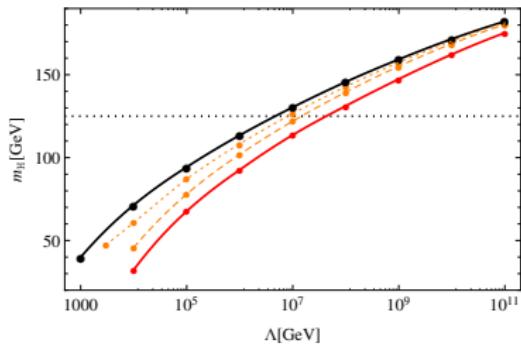
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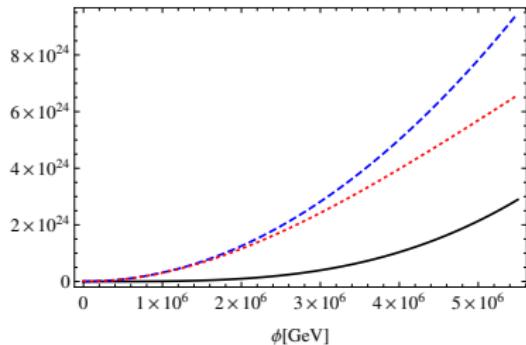
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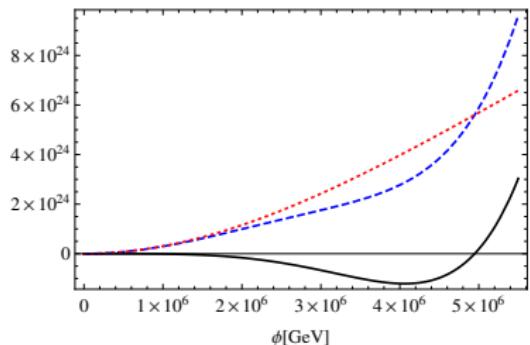
Generalized UV potentials



ϕ^4 -type bare potential



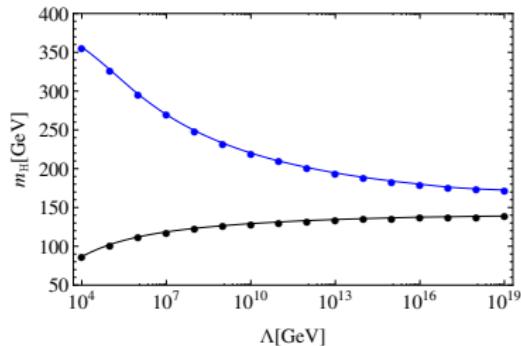
ϕ^6 -type UV potential (below red mass bound)



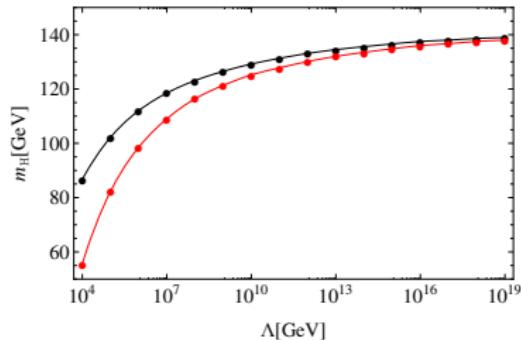
Borchardt, Gies, RS '16

Higgs mass bounds for the Standard-Model Higgs sector

- exactly the same physical mechanisms are at work



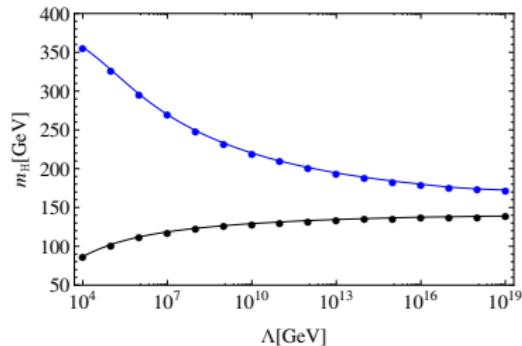
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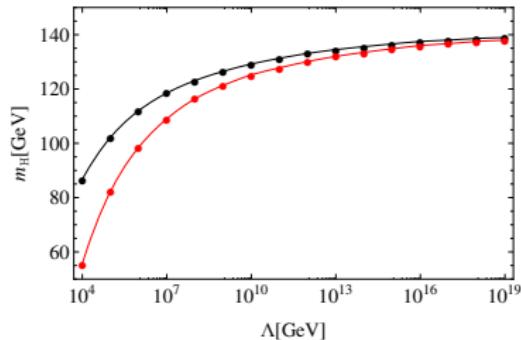
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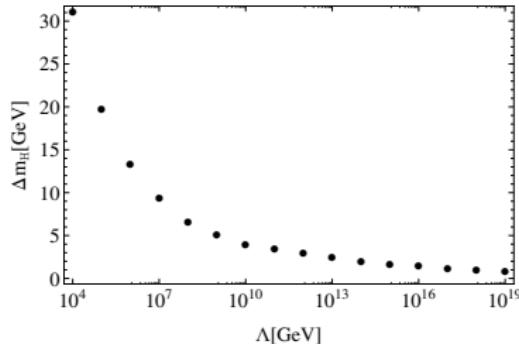


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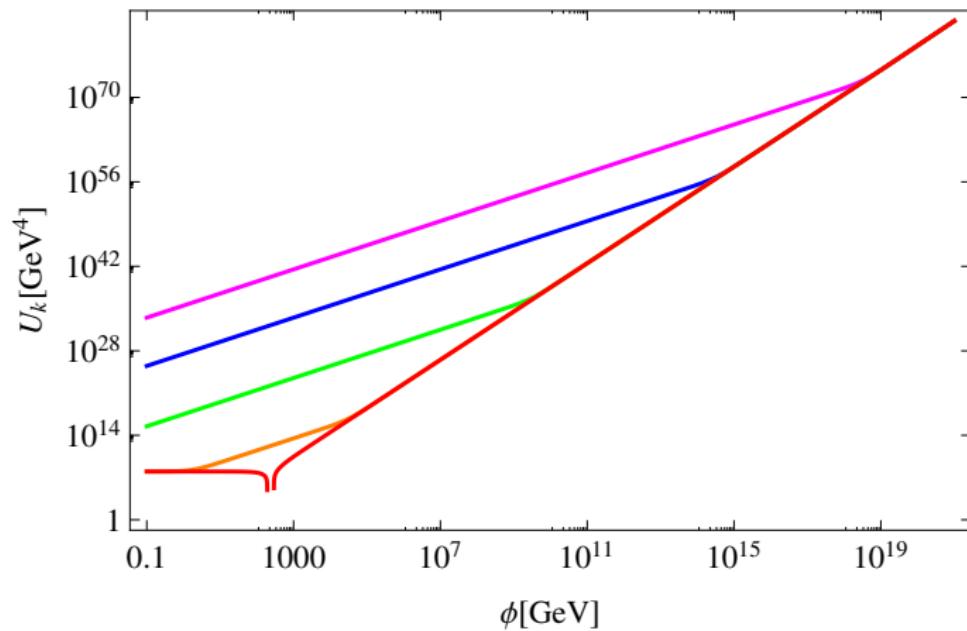


(extended bare potentials)

- difference between the ϕ^4 and ϕ^6 mass bound



Beyond polynomial bare actions



Conclusions & Outlook

- We found natural bounds for the Higgs mass in the framework of the functional RG for quartic UV potentials.
- The form of the UV potential can exert a significant influence on the mass bounds.

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- We found natural bounds for the Higgs mass in the framework of the functional RG for quartic UV potentials.
- The form of the UV potential can exert a significant influence on the mass bounds.
- A lot of improvements can be done!

Thanks for your attention!