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Higher Derivative Quantum Gravity and Vertex Functions

Trieste, ERG 2016

September 22, 2016

Outline

- (Introduction: Quantum Gravity and Asymptotic Safety)
- Quantum Gravity and Vertex Expansions
- Higher Derivative Interactions
- Fixed Points
- Outlook

Perturbative Quantization

- Einstein-Hilbert action: quadratic in derivatives

$$[G_{\mathrm{N}}] = \mathrm{energy}^{-2}$$

$$S_{EH} = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{g}\left(-\frac{1}{G}R + 2\frac{\Lambda}{G}\right)$$

- expansion parameter for n-point Greens functions:

dimensionless $\longrightarrow$ $g \equiv G_{\mathrm{N}} E^2 = \dfrac{E^2}{M_{\mathrm{Pl}}^2}$, $M_{\mathrm{Pl}} \approx 10^{19}\mathrm{GeV}$

⇒ higher loop orders require higher derivative counterterms !

⇒ full theory is either divergent or includes infinitely many free parameters

(perturbatively) <u>non-renormalizable</u>

Asymptotic Safety in a Nutshell

- Non-perturbative renormalization in Quantum Gravity

  (i) d.o.f. carried by the metric field
  (ii) diffeomorphism invariance
  (iii) quantum field theory of point particles

- Quantum Fluctuations $\longrightarrow$ scale dependent couplings

$$g_i \longrightarrow g_i(k)$$

k = energy scale

g dimensionless couplings

UV fixed point:

$$\lim_{k\to\infty} g_i(k) = g_{i,*} < \infty$$

+ finite number of free parameters (predictive)

Asymptotic Safety

$\Longleftrightarrow$

UV completion

S. Weinberg (1979)

- example: Asymptotic Freedom : $g_* = 0$ (perturbative)

A Challange for Asymptotic Safety

- Technical tool: Functional Renormalization (Wetterich equation)

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma_k[\phi] = \mathrm{STr}\left(\frac{\delta^2}{\delta\phi^2}\Gamma_k + R_k\right)^{-1}[\phi]\ k\frac{\mathrm{d}}{\mathrm{d}k}R_k$$

Wetterich (1993)

- Quantum Gravity: UV physics unknown!

→ systematic expansions and truncation enhancements!
Relevant subsets of theory space?

(Falls, Litim, Nikolakopoulos, Rahmede 2013)

→ Convergence?

→ ...find a deeper, underlying guiding principle...

Approaches to Asymptotic Safety with the FRG

- The background flow equation:

1

$$\dot{\Gamma}_k[\bar{g}, g]\Big|_{\bar{g}=g} = \mathrm{STr}\frac{\dot{R}_k}{\Gamma_k^{(2)} + R_k}\Bigg|_{\bar{g}=g}$$

- Drawbacks:
  - equation is not closed as it is evaluated at $\bar{g} = g$ but $\Gamma_k^{(2)} = \dfrac{\delta^2 \Gamma_k}{\delta g^2}$
  - no direct access to vertex functions

- Different approach: vertex expansion → write $g = \bar{g} + h$

2

→ $$\Gamma[\bar{g}, h] = \sum \frac{1}{n!} \frac{\delta^n \Gamma}{\delta h^n}\Bigg|_{h=0} h^n$$ (schematically)

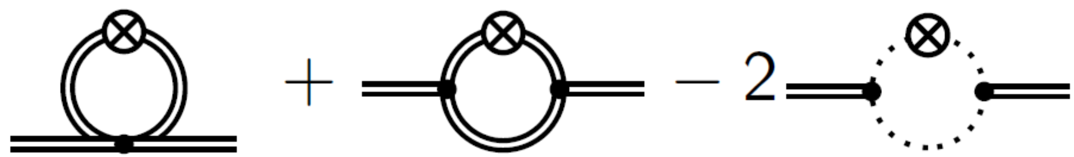
→ Flow equations for fluctuation field correlators

Vertex Expansions I

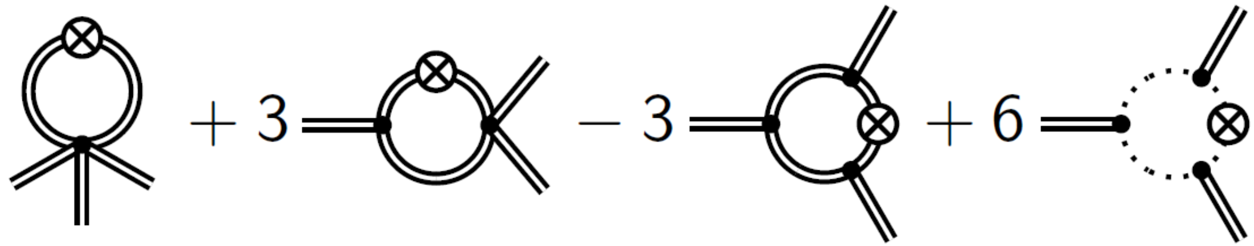
- Functional derivatives of Wetterich equation

  → Scale dependence of full vertex functions $\Gamma^{(n)} = \dfrac{\delta^n \Gamma[\bar{g}, h]}{\delta h^n}$

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma^{(2)} = -\frac{1}{2}$$



$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma^{(3)} = -\frac{1}{2}$$



$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma^{(n)} = \mathrm{Flow}[\Gamma^{(2)}, \ldots, \Gamma^{(n+2)}] \longrightarrow$$ infinite hierarchy of flow equations

- Direct access to fluctuation correlation functions/couplings
- Access to momentum dependence

Vertex Expansion II

- Essential ingredient: parameterization/truncation of vertices
- Vertex construction:

$$\Gamma^{(n)} = \prod_{i=1}^{n} \sqrt{Z(p_i)} G_n^{\frac{n}{2}-1} \mathcal{T}^n(p_1, ..., p_n; \lambda_j)$$

- Construction from „defining action" $S[\bar{g}, h]$

→ Linear split $g = \bar{g} + h = \mathbb{I} + h$

→ Expand in powers of h: i.e. Tensor structures from $\dfrac{\delta^n S}{\delta h^n}$

→ Rescaling: $h \longrightarrow \sqrt{GZ} h$

→ Dressing: $g_i \longrightarrow g_{i,k}$ more general: $g_{i,k}(p)$

Systematics in the Vertex Expansion

- Vertex expansions in quantum gravity:

$\Gamma^{(2)}$ $\Gamma^{(3)}$ $\Gamma^{(4)}$

Denz,Pawlowski,Reichert in prep

NC,Litim,Pawlowski,Rodigast 2012

NC,Knorr,Meibohm,Pawlowski,Reichert 2015

NC,Knorr,Pawlowski,Rodigast 2014

→ expansion around curved background with g(R) couplings:

NC,Falls,Pawlowski,Reichert in prep

→ expansion is consistent so far

→ support for Asymptotic Safety: Fixed Point

→ based on Einstein-Hilbert: $S = S_{\rm EH} + {\rm gauge}$

→ Higher order operators ???

Higher Derivative Gravity I

- General local action with curvature invariants up to 4-th order in derivatives: Stelle (1977)

$$S_{HD} = \frac{1}{16\pi} \int \mathrm{d}^4x \sqrt{g} \left(\underbrace{a\, R_{\mu\nu} R^{\mu\nu} + b\, R^2}_{\text{fourth order}} - \underbrace{\frac{1}{G} R + 2 \frac{\Lambda}{G}}_{\text{Einstein-Hilbert (2nd order)}} \right)$$

- No need for (Riemann-tensor)^2 term due to topological Gauss-Bonnet term in d=4
- (Ric)^2 can be traded for (Weyl)^2
- dimensionless couplings a,b
- Power-counting perturbatively renormalizable but:

non-unitary

Higher Derivative Gravity II

- **Higher Derivative Operators and Asymptotic Safety**

Background Field Flows for Γ_k

R + R^2 : (Lauscher, Reuter 2002)

R + R^2 + C^2: (Codello, Peracci 2006), (Benedetti, Machado, Saueressig 2009/10),

f(R): (Codello, Peracci, Rahmede 2008), (Machado, Saueressig 2008), (Falls, Litim, Nikolakopoulos, Rahmede 2013/2014), (Benedetti 2013), (Dietz, Morris 2013), (Eichhorn 2015)

R + C^3: (Gies, Knorr, Lippoldt,Saueressig 2016)

Flows for Vertex Functions/ Fluctuation correlators/ dynamical couplings

NC, in prep 2016

Higher Derivative Gravity III

- Start with general action quartic in derivatives

$$S_{HD} = \frac{1}{16\pi} \int \mathrm{d}^4x \sqrt{-\mathrm{det} g} \left(a\, R_{\mu\nu} R^{\mu\nu} + b\, R^2 - \frac{1}{G} R + 2\frac{\Lambda}{G} \right)$$

- Tensor structures of $\Gamma^{(n)}$ from $\frac{\delta^n S_{\mathrm{HD}}}{\delta h^n}$
- Rescaling: $h \longrightarrow \sqrt{GZ} h$
- Dressing: $(Z \longrightarrow Z_k,\, G \longrightarrow G_k,\, a \longrightarrow a_k,\, b \longrightarrow b_k,\, -2\Lambda \longrightarrow M_k^2)$

$\longrightarrow$ Flow equation for the inverse propagator:

$$k\partial_k \Gamma^{(2)}(Z_k, a_k, b_k, M_k^2) = \mathrm{Flow}^{(2)}(G_k, Z_k, a_k, b_k, M_k^2)$$

Vertex-coupling: free parameter or from EH-flows

Higher Derivative Gravity IV

- Gauge fixing: 2nd order in derivatives!

$$S_{\mathrm{GF}} = \frac{1}{2\alpha}\int \mathrm{d}^4x\sqrt{g}\,\bar{g}^{\mu\nu}F_\mu F_\nu \quad , \quad \text{in general:} \quad F_\mu = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar{\nabla}_\mu h^\nu_\nu$$

This work: $\alpha = 0 \;,\; \beta = -1$ → See also: (C. Wetterich 2016)

- Propagator: Projector representation $\left(\Gamma^{(2)}\right)^{-1} = \sum_i \left(\Gamma^{(2)}\right)^{-1}_i \mathcal{P}_i$

TT component

$$\left(\Gamma^{(2)}\right)^{-1} = \frac{16\pi}{Z_k}\begin{pmatrix} \left(p^2 + a_k G_k p^4 + M_k^2\right)^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\left(4p^2 - 8p^4(3a_k G_k + b_k G_k) + M_k^2\right)^{-1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Tr component

Higher Derivative Gravity V

- Projection proecdure:

$$\lim_{p\to 0} \mathcal{P}_{TT}\dot{\Gamma}^{(2)} \quad \longleftrightarrow \quad k\partial_k M_k^2$$

$$\lim_{p\to 0} \frac{\partial^2}{\partial p^2}\mathcal{P}_{TT}\dot{\Gamma}^{(2)} \quad \longleftrightarrow \quad k\partial_k Z_k$$

$$\lim_{p\to 0} \frac{\partial^4}{\partial p^4}\mathcal{P}_{TT}\dot{\Gamma}^{(2)} \quad \longleftrightarrow \quad k\partial_k a_k$$

$$\lim_{p\to 0} \frac{\partial^4}{\partial p^4}\mathcal{P}_{Tr}\dot{\Gamma}^{(2)} \quad \longleftrightarrow \quad k\partial_k b_k$$

$\longrightarrow$ clean separation of the four-derivative couplings via tensor projection

Higher Derivative Gravity VI

- **Fixed point solutions**

1 Solve $(\beta_M \ , \ \eta \ , \ \beta_a \ , \ \beta_b)$ and assume fixed point in g:

→ parametric dependence on Newtons coupling, e.g. $g_* = 0.6$

$$(\mu_* = -0.17 \ , \ \eta_* = 0.26 \ , \ a_* = -0.94 \ , \ b_* = 0.4)$$

Critical exponents: $(\theta_1 = -5.8 \ , \ \theta_2 = -21.5 \ , \ \theta_3 = 6.3)$

Irrelevant direction

2 Solve $(\beta_M \ , \ \eta \ , \ \beta_a \ , \ \beta_b \ , \ \beta_g)$, β_g from EH calculation

$$(g_* = 0.98 \ , \ \mu_* = -0.2 \ , \ \eta_* = 1.06 \ , \ a_* = -0.56 \ , \ b_* = 0.25)$$

$$(\theta_1 = -1.67 \ , \ \theta_2 = -14.5 \ , \ \theta_3 = -18.7 \ , \ \theta_4 = 11.1)$$

Higher Derivative Gravity VII

- **Discussion:**

→ system seems to be stable („simulation" of changes in truncation)

→ reasonable fixed point values,
shares features with EH vertex expansion: e.g. $\eta > 0$

Asymptotic Safety: ☑

→ Three relevant, one irrelevant direction

...in agreement (Benedetti, Machado, Saueressig 2009)

→ critical exponents a bit too large....

→ stabilization by higher order operators?

(Falls, Litim, Nikolakopoulos, Rahmede 2013/2014)

Summary and Outlook

- Vertex expansion and flow of the propagator with all four-derivative operators
- Fixed point with three relvant and one irrelavant direction

→ Asymptotic Safety

Outlook

- Including more momentum dependence
- Including R^3 operators
- Three-point/four-point function with higher-derivative operators
- Unitarity

Thank You!!!