On de Sitter solutions in asymptotically safe $f(R)$-theories

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RICMASS

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Motivation

- Evidence for asymptotic safety of gravity from the ERG
- Impact on cosmology (at early and maybe also at late times)
- Inflation possibly originating from the fixed point regime
- Assess reliability of approximation
- Reliable determination of stationary solutions

f(R)-approximation

- Use $f(R)$-approximation
- Flow equation for arbitrary functional form
- Truncation well-studied to very high polynomial order (up to 35 couplings)
- Fixed point with only three relevant directions
- Critical exponents with small deviations from Gaussian values

A. Codello, R. Percacci, C. R. (2007,2008)
P. Machado, F. Saueressig (2007)
A. Buonanno, A. Contillo, R. Percacci (2011)
D. Benedetti, F. Caravelli (2012)
K. Falls, D. Litim, K. Nikolakopoulos, C. R. (2013, 2014)
J. Dietz, T. Morris (2013)
N. Ohta, R. Percacci, G. Vacca (2015, 2016)

Ansatz

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k$$

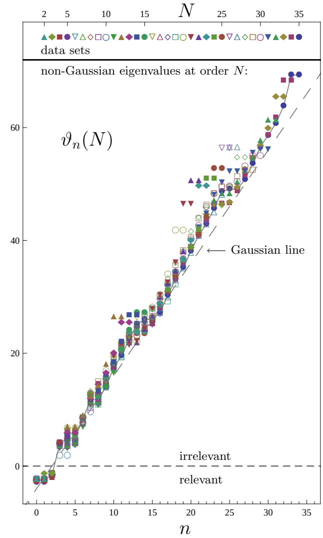
- Ansatz:

$$\Gamma_k = \int d^4x \sqrt{|\det g_{\mu\nu}|}\, F_k(\bar{R})\,.$$

$$f(R) = 16\pi\, F(\bar{R})/k^4 \text{ with } R = \bar{R}/k^2$$

$$\begin{aligned}
\frac{df''(R)}{dR} \quad = \quad & \frac{\frac{37}{756}R^4 + \frac{29}{10}R^3 + \frac{121}{5}R^2 + 12R - 216}{\frac{181}{1680}R^4 + \frac{29}{15}R^3 + \frac{91}{10}R^2 - 54} \frac{f''(R)}{R} - \frac{\frac{37}{756}R^3 + \frac{29}{15}R^2 + 18R + 48}{\frac{181}{1680}R^4 + \frac{29}{15}R^3 + \frac{91}{10}R^2 - 54} \frac{f'(R)}{R} \\
+ \quad & \frac{(R-3)^2 f''(R) + (3-2R)f'(R) + 2f(R)}{R\left(\frac{181}{1680}R^4 + \frac{29}{15}R^3 + \frac{91}{10}R^2 - 54\right)} \times \\
\times \quad & \left[\frac{R\left(-\frac{311}{756}R^3 + \frac{1}{6}R^2 + 30R - 60\right) f''(R) + \left(\frac{311}{756}R^3 - \frac{1}{3}R^2 - 90R + 240\right) f'(R)}{3f(R) - (R-3)f'(R)} \right. \\
& \left. - \frac{607R^2 - 360R - 2160}{15(R-4)} - \frac{511R^2 - 360R - 1080}{30(R-3)} + 48\pi\left(Rf'(R) - 2f(R)\right) \right]
\end{aligned}$$

Eigenvalues to order $N = 35$



K. Falls, D. Litim, K. Nikolakopoulos, C. R. (2013)

Convergence

- Singularities

$$R = -9.99\,855\ , 0\ ,\ 2.00\,648\cdots\ ,\ 3\ ,\ 4$$

- Polynomial ansatz

$$f(R) = \sum_{n=0}^{N-1} \lambda_n R^n\,.$$

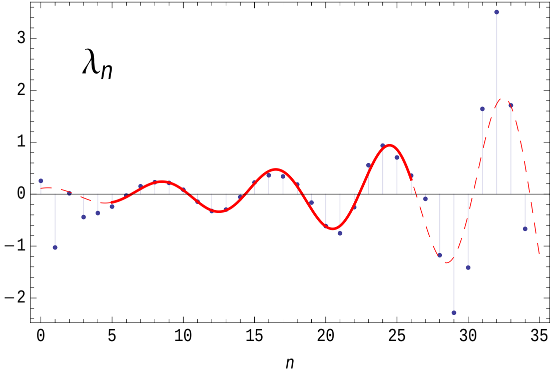
- Radius of convergence:

$$R_c \leq R_{\rm max} = 2.00648\cdots$$

- Eight-fold periodicity pattern

$$(+ + + + - - - -)$$

Convergence pattern



$N = 35$ and four-parameter fit

$$\lambda_n = A \, \frac{\cos(n\phi + \Delta)}{(R_c)^n}$$

$A = \ 0.1172$, $R_c = 0.9182$, $\phi = 0.7863$, $\Delta = -0.2919$
Dashed lines: extrapolations of the fit.

Convergence radius estimate

Generalised ratio test

$$R_c = \lim_{n\to\infty} \left| \frac{\lambda_n}{\lambda_{n+m}} \right|^{1/m}$$

- Average over m for $(8 \leq m \leq N - m)$
- $N = 11$: $R_c \approx 1.0 \pm 20\%$, $N = 35$: $R_c \approx 0.91 \pm 5\%$

A. Buonanno, A. Contillo, R. Percacci (2011)

Lower bound

$$R_L(m) \equiv \min_n \left| \frac{\lambda_n}{\lambda_{n+m}} \right|^{1/m}$$

- For $N = 35$ average over m for $(8 \leq m \leq N - m)$

$$R_L \approx 0.82 \pm 5\%$$

Improved solutions

- Improve on polynomial approximation by resummation:

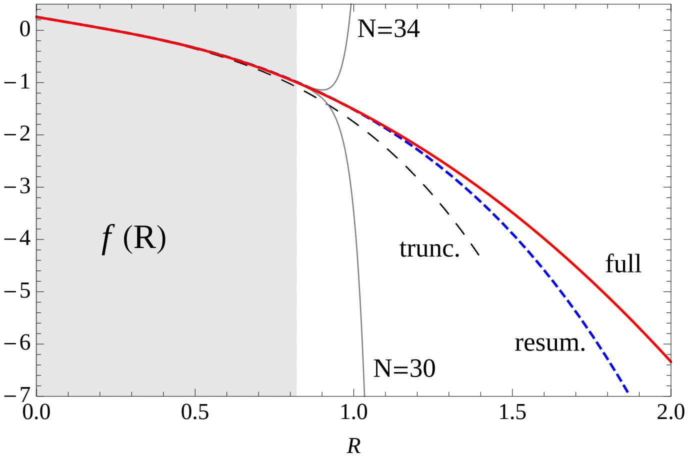
$$f_{\text{resum.}}(R) = \sum_{n=0}^{m-1} \lambda_n R^n + \sum_{n=m}^{\infty} \lambda_n R^n \,.$$

  - E.g. reduction to four parameters

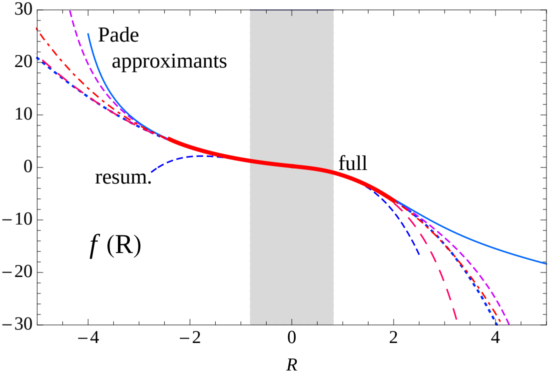
$$\sum_{n=5}^{\infty} \lambda_n R^n = A \frac{R^4}{R_c^4} \frac{R R_c \cos(\Delta + 5\phi) - R^2 \cos(\Delta + 4\phi)}{R^2 - 2 R R_c \cos\phi + R_c^2}$$

- Improve on polynomial approximation by Padé approximants:
  - Rational functions of polynomials in R of degree $M \geq 0$ and $K \geq 1$
  - Take the first $M + K + 1$ Taylor coefficients of $f(R)$ as input
  - No reduction of parameter number

Numerical, polynomial, truncated, resummed solutions



Numerical, resummed, Padé approximant solutions



Thick red line: full numerical integration; resum./dashed: resummation; Padé approximants:

dotted: [20/16], dashed-dotted: [17/14], long-dashed: [15/13], thin full: [16/15], short-dashed: [16/17]

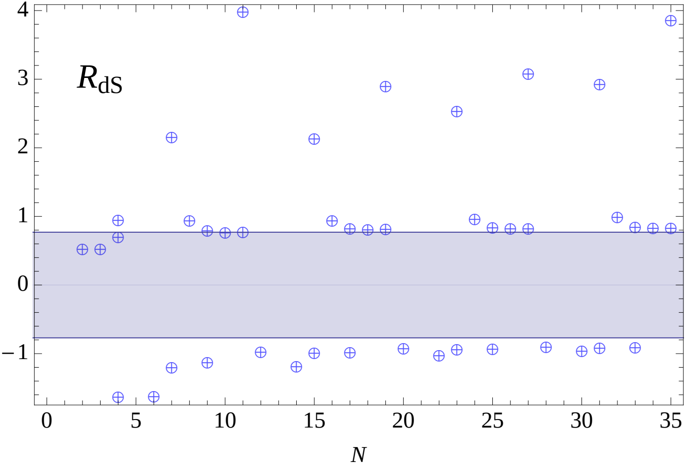
Constant curvature solutions

- Stationarity condition:

$$Rf'(R) - 2f(R) = 0$$

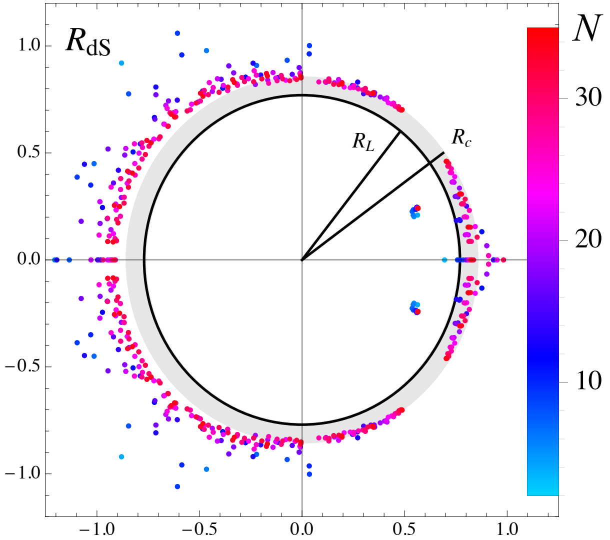
- Constant curvature solutions:
  - Always for $f(R) \approx R_{\text{infl}}^2$
  - Algebraic solutions for specific R_{dS}, e.g. Einstein-Hilbert with $R_{\text{dS}} = 4\Lambda$

Real (anti-)de Sitter solutions

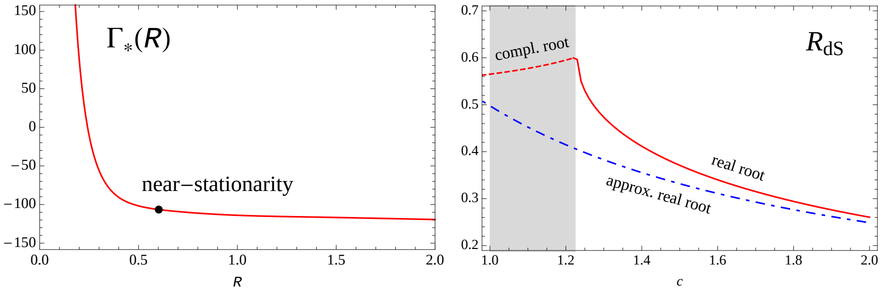


- Absence of real de Sitter or anti-de Sitter solutions for polynomial approximation order N and small curvature scalar within the radius of convergence of the polynomial expansion

Complex de Sitter solutions

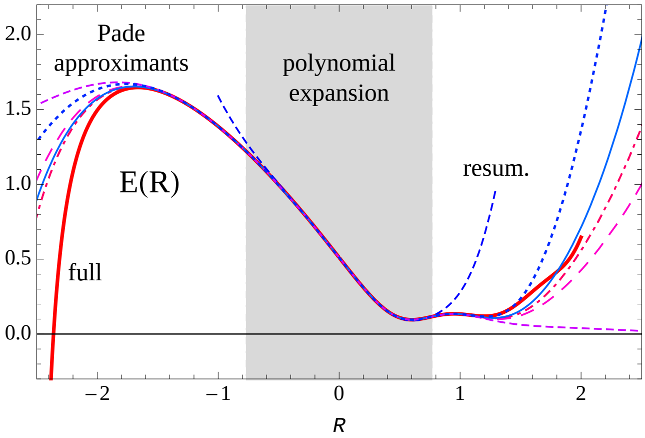


Near-stationarity



Left: the fixed point effective action as a function of scalar curvature
Right: solutions $R_{\rm dS}$ to the stationarity condition modifying λ_* by a fudge factor $c > 1$, all other fixed point couplings unchanged, becomes $R_{\rm dS} \approx -2\lambda_0/\lambda_1 = 4\lambda_*$ (blue line)

Stationarity condition in various approximations



$E(R) = 2f(R) - R\,f'(R)$; thick red line: full numerical integration; Padé approximants: ([20/16]: dotted line, [17/14]: dashed-dotted, [14/12]: long-dashed, [16/15]: thin full, [16/17]: short dashed): resummation (resum)

Summary

- Estimate for convergence radius of polynomial fixed point solution of f(R)-gravity
- Slow convergence requires going to high orders
- Numerical integration, resummation and Padé approximants give good fits
- No stable (real) de Sitter solutions within the radius of convergence
- Near-stationarity indicates that the inclusion of further interaction terms might lead to stationary solutions