Asymptotically Safe Quantum Gravity

En route to background independence

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ERG2016@ICTP Trieste, 21.Sep. 2016

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arXiv:1404.4537

outline

background field/geometry independence

i] special status of quantum gravity

ii] background field method & quantum gravity

background independence & the frg

i] bi-metric actions, constrained flow

ii] Ward identities of split-symmetry

a first test of background independence

i] a bi-metric Einstein-Hilbert truncation

ii] asymptotic safety <u>&</u> background independence?

conclusion & future tasks

background field independence

quantum field theory should be background (field) independent

i] . . . dynamical quantities are full predictions of the theory

ii] violation: justification for distinguished background choice?!

background field independence

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i] …dynamical quantities are full predictions of the theory

ii] violation: justification for distinguished background choice?!

general relativity is background (field & geometry) independent

i] part of geometry is fixed input: topology, b.c. …

ii] perturbation $g = \bar{g} + h$ depends on background field:

$$(\bar{\Delta} + \cdots)\, h_{\mu\nu} + \mathcal{O}(h^2) = -16\pi G_{\mathsf{N}} T_{\mu\nu} \quad \text{with } \bar{G}_{\mu\nu} = 0$$

background geometry independence

definition theory of quantum gravity:

"spacetime (the base space of qft's) is dynamical degree of freedom"

$$\left.\begin{array}{ll} \text{i]} & \text{background field independent} \implies \text{no distinguished field} \\ \text{ii]} & \text{background geometry independent} \implies \text{predicts geometry} \end{array}\right\} \text{dual role}$$

issue: standard quantum physics makes extensive use of preset geometry
(equal-time commutators, time-direction, foliation, scales, . . .)

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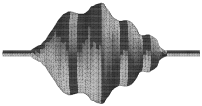
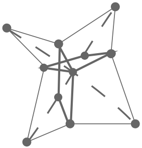
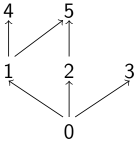
manifest background independent approaches:



causal dynamical triangulations



loop quantum gravity

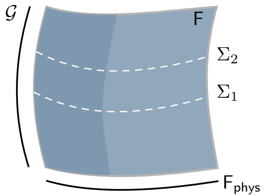


causal sets

input:
topology
causal structure

background field method & quantum gravity

the general idea:



$$\int_{\mathsf{F}_{\mathsf{phys}}} \mathrm{d}\mu \stackrel{!}{=} \int_{\mathsf{F}} \mathrm{d}\mu \, \delta(\Sigma) \cdots$$

gauge-invariance

$\mathsf{obs}(\Sigma_1) \equiv \mathsf{obs}(\Sigma_2) \equiv \cdots$

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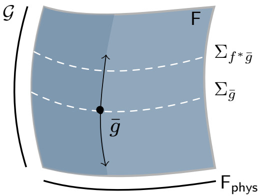
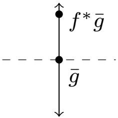


gauge-invariance

obs$(\Sigma_{\bar{g}}) \equiv$ obs$(\Sigma_{f^*\bar{g}})$



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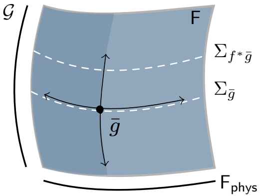

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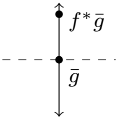
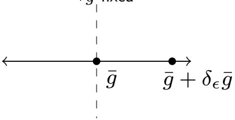


background independence

$$\delta_{\bar{g}}\, \mathsf{obs}(\Sigma_{\bar{g}})\Big|_{g\ \mathrm{fixed}} = 0$$



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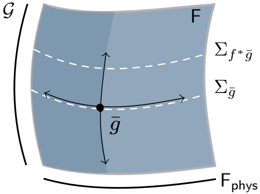

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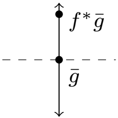

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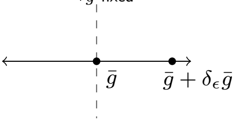

special status of quantum gravity:

i] background field sets also reference geometry
scales (as $k, \ldots$) defined by $\bar{g}$ (sets consistent notion for UV and IR)

ii] the measure (S, S_{gf}, and S_{gh}) is unknown (prediction in frg-approach)

functional renormalization group: a constrained flow

functional renormalization group equation (frge):

$$k\partial_k \Gamma_k[g,\bar{g}] = \frac{1}{2}\mathsf{STr}\left[\left(\Gamma_k^{(2)}[g,\bar{g}] + \mathcal{R}_k[\bar{g}]\right)^{-1} k\partial_k \mathcal{R}_k[\bar{g}]\right]$$

renormalizable & UV-complete:

$$\text{qft's} \equiv \left\{k \mapsto \Gamma_k[g,\bar{g}] \in \mathscr{S}_{\mathsf{UV}} \;\middle|\; \dim \mathscr{S}_{\mathsf{UV}} < \infty\right\}$$

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constraints:

i] Ward identities of BRS symmetry: gauge invariance $^{\star}\checkmark$

ii] Ward identities of split-symmetry: background field independence $^{\star}\checkmark$

iii] unitarity*, classical limit, ...

$\star$ reconstruction problem: S_{bare} or S_{gf} needed

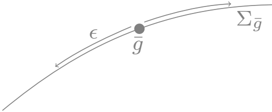
$\checkmark$ compatible with frge (on exact level!)

Ward identities for split-symmetry (msWI)

underlying split-symmetry $g(\bar{g}, h) \equiv g(\bar{g} + \delta_\epsilon g, h + \delta_\epsilon h)$:

for linear parametrization $g = \bar{g} + h$

$$\delta_{\bar{g}}\Gamma_k[g,\bar{g}] = \frac{1}{2}\mathsf{STr}\left[\left(\Gamma_k^{(2)}[g,\bar{g}] + \mathcal{R}_k[\bar{g}]\right)^{-1} \delta_{\bar{g}} S_{\mathsf{tot}}^{(2)}[g,\bar{g}]\right]$$



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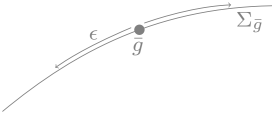

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i] $\Gamma_k[g, \bar{g}]$ is by definition a bi-metric functional

distinguish background and dynamical couplings ($\Gamma[g, \bar{g}]$ vs. $\delta_g^2 \Gamma_k[g, \bar{g}]$)

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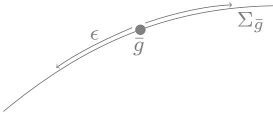

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special limits/approximation of msWI:

i] background independence tested at $k = 0$: $\lim_{k \to 0} \delta_{\bar{g}} \Gamma_k[g, \bar{g}]$

ii] split-symmetry in "tree-level" approximation: $\delta_{\bar{g}} \Gamma'_k[g, \bar{g}] \approx 0$

a bi-metric truncation

investigate split-symmetry at "tree-level" for bi-metric EH-truncation

$$\Gamma_k^{\mathsf{grav}}[g,\bar{g}] = \frac{1}{16\pi G_k^{\mathsf{B}}}\int_{\mathsf{M}}\sqrt{\bar{g}}\,\left(2\bar{\lambda}_k^{\mathsf{B}} - \bar{R}\right) + \frac{1}{16\pi G_k^{\mathsf{D}}}\int_{\mathsf{M}}\sqrt{g}\,\left(2\bar{\lambda}_k^{\mathsf{D}} - R\right)$$

for two different gauge choices

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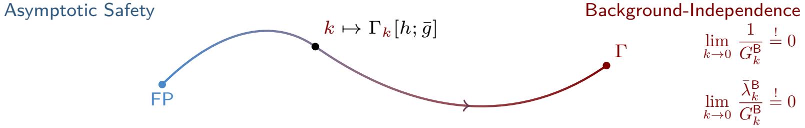
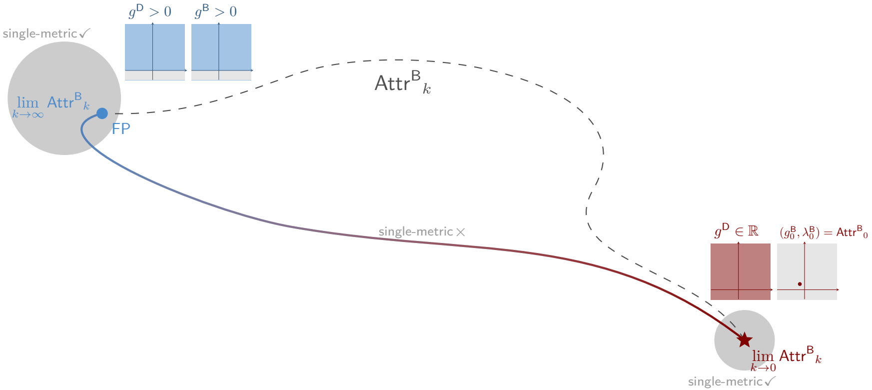
the global question of asymptotic safety & background independence



asymptotic safety ✓   background independence: 1] all trajectories 2] subclass 3] none ?

a study in violet & red



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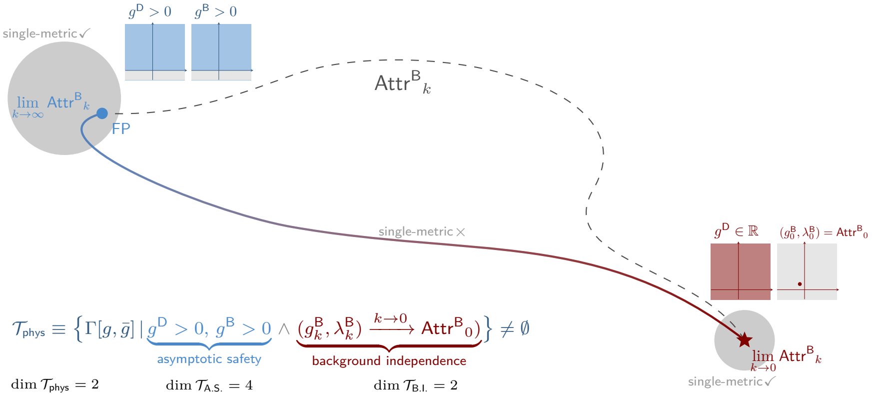

$$\mathcal{T}_{\mathsf{phys}} \equiv \Big\{ \Gamma[g,\bar{g}] \,\big|\, \underbrace{g^{\mathsf{D}} > 0,\ g^{\mathsf{B}} > 0}_{\text{asymptotic safety}} \wedge \underbrace{(g^{\mathsf{B}}_k, \lambda^{\mathsf{B}}_k) \xrightarrow{k\to0} \mathsf{Attr}^{\mathsf{B}}{}_0}_{\text{background independence}} \Big\} \neq \emptyset$$

$\dim \mathcal{T}_{\mathsf{phys}} = 2$ $\quad \dim \mathcal{T}_{\mathsf{A.S.}} = 4$ $\quad \dim \mathcal{T}_{\mathsf{B.I.}} = 2$

background independence and asymptotic safety can be simultaneously satisfied!
predictivity increased; reduction from $4 \to 2$ free parameters!

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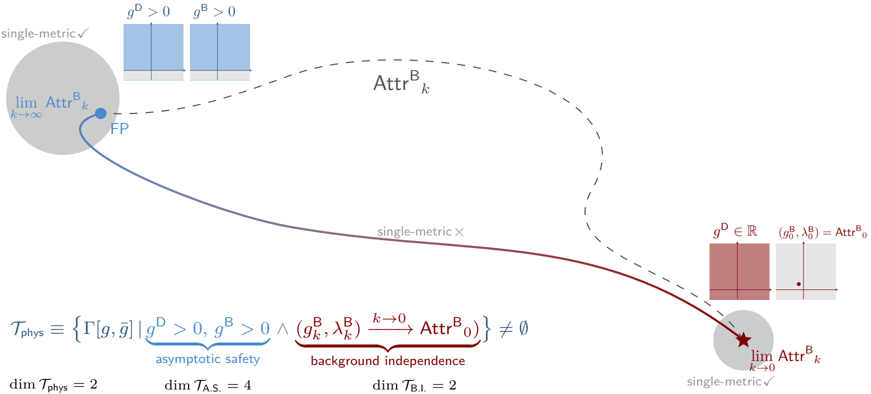

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remark: mathematical feature of the flow ensures background independence
conceptual property or coincidence in this special case $\Rightarrow$ further studies needed

conlusion & outlook

summary

i] background independence is essential for a theory of quantum gravity
no preset geometry; spacetime dynamical degree of freedom

ii] background field: gauge-invariant Γ_k & sets reference geometry
Ward identities of split-symmetry control "extra" $\bar{g}$-dependence

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tested for bi-metric EH-truncations on "tree-level"; UV-attractor mechanism

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related future tasks:

i] establish error estimates for truncations
design truncation to suit certain exact property may be on cost of others
(consistent failure or force non-failure of exact properties?)

ii] reconstruction problem
complete msWI needs the full UV-theory $S_{\text{tot}} = S + S_{\text{gf}} + \dots$

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thank you for your attention!