Asymptotically Safe Quantum Gravity

En route to background independence

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outline

background field/geometry independence
 i] special status of quantum gravity
 ii] background field method & quantum gravity

background independence & the FRG
 i] bi-metric actions, constrained flow
 ii] Ward identities of split-symmetry

a first test of background independence
 i] a bi-metric Einstein-Hilbert truncation
 ii] asymptotic safety & background independence?

conclusion & future tasks
background field independence

quantum field theory should be background (field) independent

i] …dynamical quantities are full predictions of the theory

ii] violation: justification for distinguished background choice?!
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general relativity is background (field & geometry) independent

i] part of geometry is fixed input: topology, b.c. . . .

ii] perturbation $g = \bar{g} + h$ depends on background field:

$$(\bar{\Delta} + \cdots) h_{\mu\nu} + \mathcal{O}(h^2) = -16\pi G_N T_{\mu\nu} \quad \text{with} \quad \bar{G}_{\mu\nu} = 0$$
background geometry independence

definition theory of quantum gravity:

“spacetime (the base space of qft’s) is dynamical degree of freedom”

i] background field independent \(\implies\) no distinguished field

ii] background geometry independent \(\implies\) predicts geometry

\{\text{dual role}\}

issue: standard quantum physics makes extensive use of preset geometry
(equal-time commutators, time-direction, foliation, scales, \ldots\)
background geometry independence

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(equal-time commutators, time-direction, foliation, scales, ...)

manifest background independent approaches:

causal dynamical triangulations

loop quantum gravity

causal sets

input:
topology
causal structure
the general idea:

\[ \int_{F_{\text{phys}}} d\mu = \int_{F} d\mu \delta(\Sigma) \cdots \]

\text{gauge-invariance}

\text{obs}(\Sigma_1) \equiv \text{obs}(\Sigma_2) \equiv \cdots
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gauge-invariance

\[ \text{obs}(\Sigma \bar{g}) \equiv \text{obs}(\Sigma f^* \bar{g}) \]

- background independence

\[ \delta \bar{g} \left|_{g \text{ fixed}} \right. = 0 \]

- special status of quantum gravity:
  1. background field sets also reference geometry
     scales (as \( k, \ldots \)) defined by \( \bar{g} \)
     (sets consistent notion for UV and IR)
  2. the measure (\( S, S_{gf}, \text{and } S_{gh} \)) is unknown (prediction in frg-approach)
background field method & quantum gravity

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the general idea:

\[ \int_{F_{\text{phys}}} \frac{1}{F} \, d\mu \equiv \int_{F} d\mu \delta(\Sigma) \cdots \]

\[ \text{gauge-invariance} \quad \text{obs}(\Sigma \bar{g}) \equiv \text{obs}(\Sigma f^{*} \bar{g}) \]

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functional renormalization group equation (frge):

\[ k \partial_k \Gamma_k[g, \bar{g}] = \frac{1}{2} STr \left[ \left( \Gamma_k^{(2)}[g, \bar{g}] + \mathcal{R}_k[\bar{g}] \right)^{-1} k \partial_k \mathcal{R}_k[\bar{g}] \right] \]

renormalizable & UV-complete:

qft’s \equiv \{ k \mapsto \Gamma_k[g, \bar{g}] \in \mathcal{I}_{\text{UV}} \mid \dim \mathcal{I}_{\text{UV}} < \infty \}
functional renormalization group: a constrained flow

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constraints:

i] Ward identities of BRS symmetry: gauge invariance *✓

ii] Ward identities of split-symmetry: background field independence *✓

iii] unitarity*, classical limit, . . .

* reconstruction problem: \( S_{\text{bare}} \) or \( S_{\text{gf}} \) needed ✓ compatible with frge (on exact level!)
Ward identities for split-symmetry (msWI)

underlying split-symmetry $g(\bar{g}, h) \equiv g(\bar{g} + \delta_\epsilon g, h + \delta_\epsilon h)$:

for linear parametrization $g = \bar{g} + h$

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distinguish background and dynamical couplings ($\Gamma[g, \bar{g}]$ vs. $\delta^2 g \Gamma_k[g, \bar{g}]$)

ii] reconstruction of total action $S_{\text{tot}}$ required
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special limits/approximation of msWI:

i] background independence tested at $k = 0$: $\lim_{k \to 0} \delta_\bar{g} \Gamma_k[g, \bar{g}]$

ii] split-symmetry in “tree-level” approximation: $\delta_\bar{g} \Gamma'_k[g, \bar{g}] \approx 0$
a bi-metric truncation

investigate split-symmetry at “tree-level” for bi-metric EH-truncation

\[
\Gamma_{k}^{\text{grav}}[g, \bar{g}] = \frac{1}{16\pi G_{k}^{B}} \int_{M} \sqrt{\bar{g}} \left( 2\bar{\lambda}_{k}^{B} - \bar{R} \right) + \frac{1}{16\pi G_{k}^{D}} \int_{M} \sqrt{g} \left( 2\bar{\lambda}_{k}^{D} - R \right)
\]

for two different gauge choices
a bi-metric truncation

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\[ \Gamma_{\text{grav}}^{k}[g, \bar{g}] = \frac{1}{16\pi G_{k}^{B}} \int_{M} \sqrt{g} \left( 2\tilde{\lambda}_{k}^{B} - \bar{R} \right) + \frac{1}{16\pi G_{k}^{D}} \int_{M} \sqrt{g} \left( 2\tilde{\lambda}_{k}^{D} - R \right) \]

for two different gauge choices

the global question of asymptotic safety & background independence

Asymptotic Safety

Background-Independence

\[ \lim_{k \to 0} \frac{\tilde{\lambda}_{k}^{B}}{G_{k}^{B}} = 0 \]

\[ \lim_{k \to 0} \frac{1}{G_{k}^{B}} = 0 \]

asymptotic safety ✓

background independence: 1] all trajectories 2] subclass 3] none ?
A study in violet & red
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background independence and asymptotic safety can be simultaneously satisfied!
predictivity increased; reduction from 4 → 2 free parameters!
background independence and asymptotic safety can be simultaneously satisfied! predictivity increased; reduction from $4 \rightarrow 2$ free parameters!

**remark:** mathematical feature of the flow ensures background independence conceptual property or coincidence in this special case $\Rightarrow$ further studies needed
conclusion & outlook

summary

i] background independence is essential for a theory of quantum gravity
   no preset geometry; spacetime dynamical degree of freedom

ii] background field: gauge-invariant $\Gamma_k$ & sets reference geometry
   Ward identities of split-symmetry control “extra” $\bar{g}$-dependence

iii] coexistence of asymptotic safety and background independence
   tested for bi-metric EH-truncations on “tree-level”; UV-attractor mechanism
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related future tasks:

i] establish error estimates for truncations
   design truncation to suit certain exact property may be on cost of others
   (consistent failure or force non-failure of exact properties?)

ii] reconstruction problem
   complete msWI needs the full UV-theory $S_{\text{tot}} = S + S_{gf} + \ldots$

iii] establish new techniques to simplify necessary bi-metric truncations

thank you for your attention!
conlusion & outlook

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