

Background independence, gauge/diffeomorphism invariance & locality

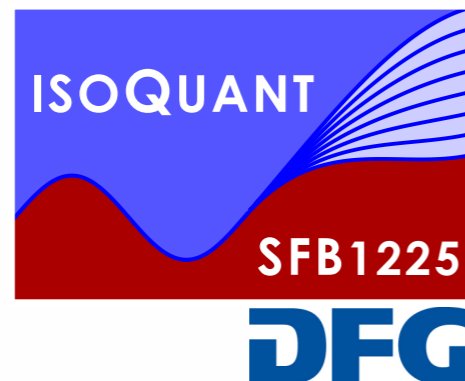
Jan M. Pawłowski
Universität Heidelberg & ExtreMe Matter Institute

Trieste, September 21st 2016



GEFÖRDERT VOM

Bundesministerium
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Related talks & posters

talks

Astrid Eichhorn (Thursday) *'Quantum effects on a Higgs–Yukawa model'*

Nicolai Christiansen (Thursday) *'Vertex functions in quantum gravity'*

Posters

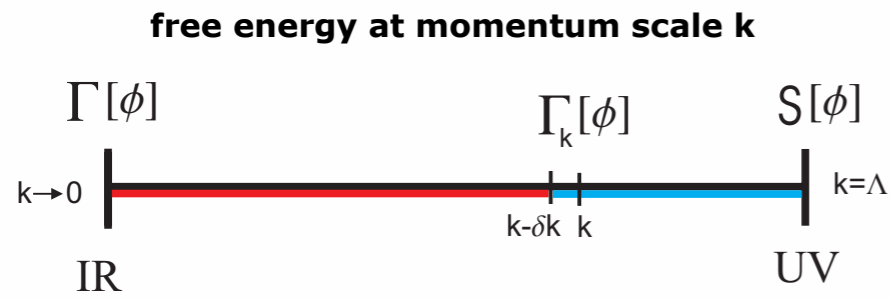
Tobias Denz *'Towards apparent convergence in asymptotically safe QG'*

Aaron Held *'QG-effects on a Higgs–Yukawa model'*

Manuel Reichert *'QG with a vertex expansion on curved backgrounds'*

Background independence in the FRG

Background (in)dependence in the FRG



RG-scale k : $t = \ln k$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{gravity/gauge field quantum fluctuations} - \text{fermionic quantum fluctuations} \right) + \frac{1}{2} \left(\text{bosonic quantum fluctuations} \right)$$

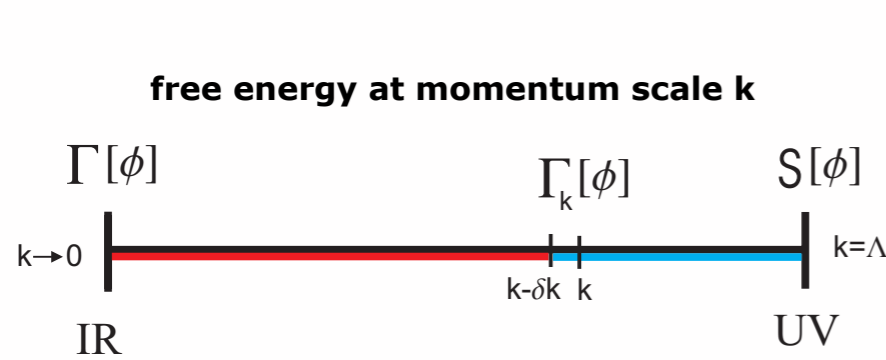
gravity/gauge field quantum fluctuations

fermionic quantum fluctuations

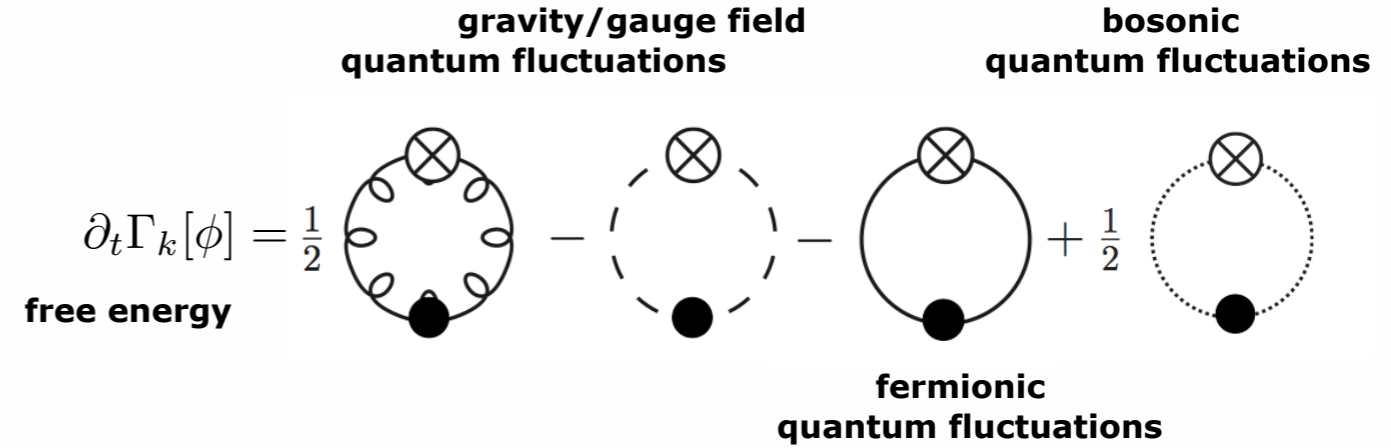
bosonic quantum fluctuations

$$\phi = \bar{\phi} + \varphi$$

Background (in)dependence in the FRG



$$\text{RG-scale } k: t = \ln k$$



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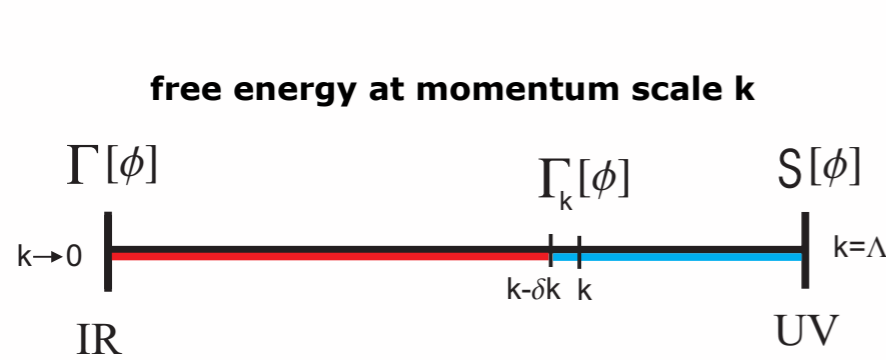
Expansion of the effective action

$$\Gamma_k[\bar{\phi} + \varphi] = \Gamma_k[\bar{\phi}] + \Gamma_k^{(0,1)}[\bar{\phi}] * \varphi + \frac{1}{2} \Gamma_k^{(0,2)}[\bar{\phi}] * \varphi^2 + \frac{1}{6} \Gamma_k^{(0,3)}[\bar{\phi}] * \varphi^3 + \dots$$

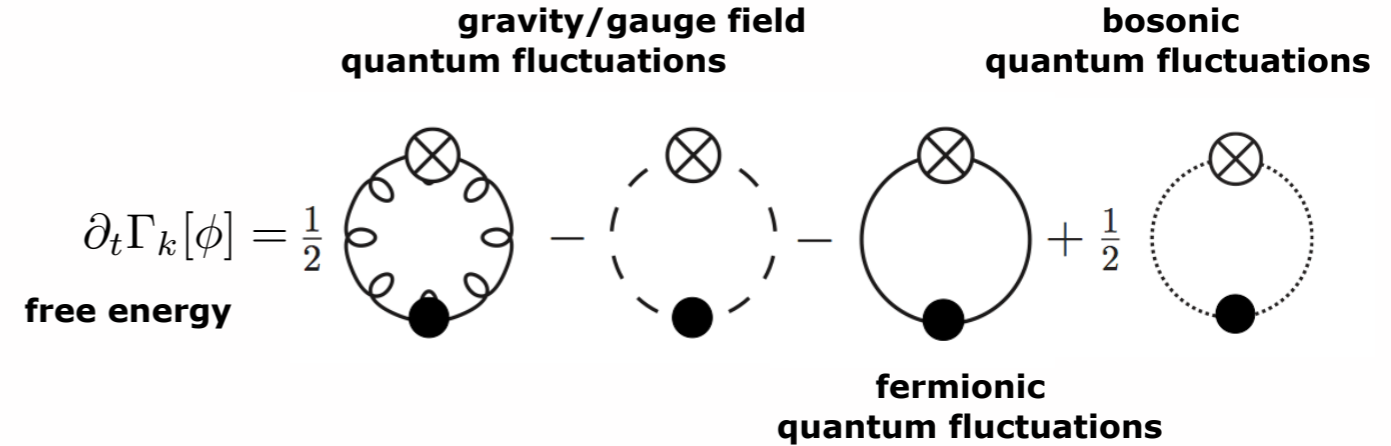
Appropriately chosen backgrounds may facilitate convergence

$$\Gamma_k^{(n,m)} = \frac{\delta^{n+m} \Gamma_k}{\delta \bar{\phi}^n \delta \varphi^m}$$

Background (in)dependence in the FRG



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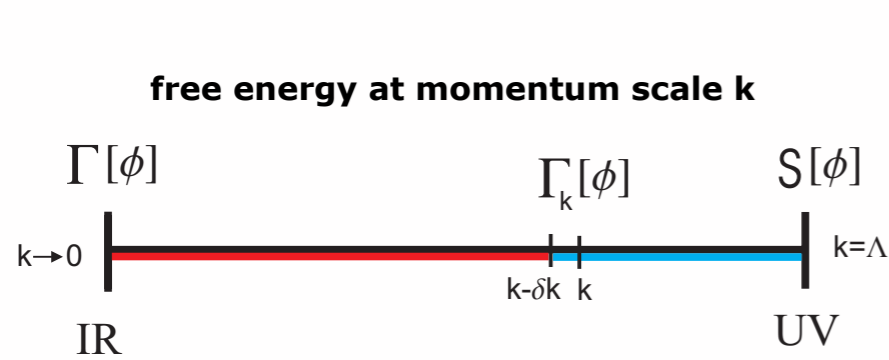
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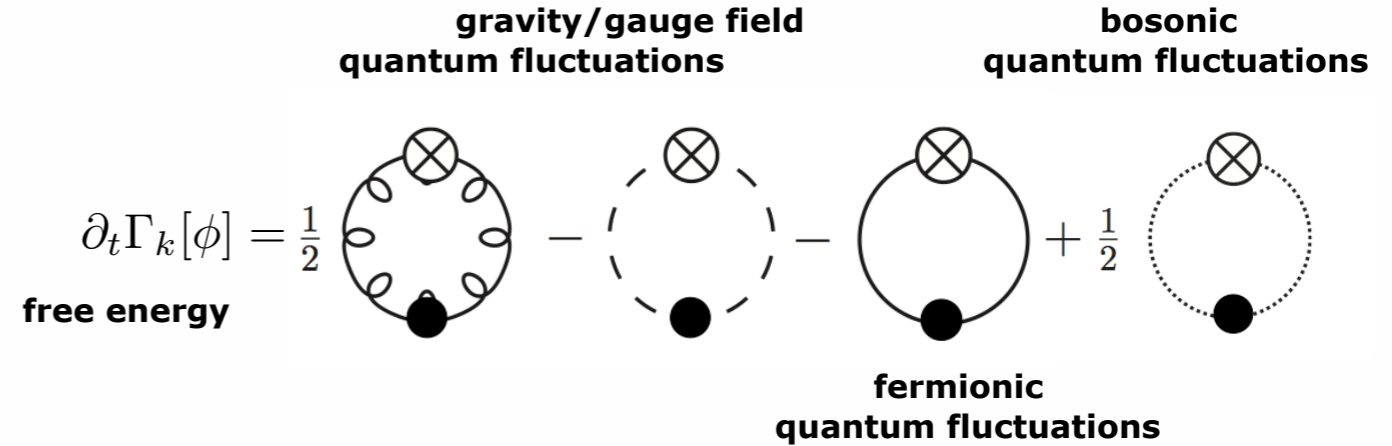
Examples:

- **solution of the equation of motion**
- **topological configuration or defect**
- **simplification of numerics**

Background (in)dependence in the FRG



$$\text{RG-scale } k: t = \ln k$$



$$\phi = \bar{\phi} + \varphi$$

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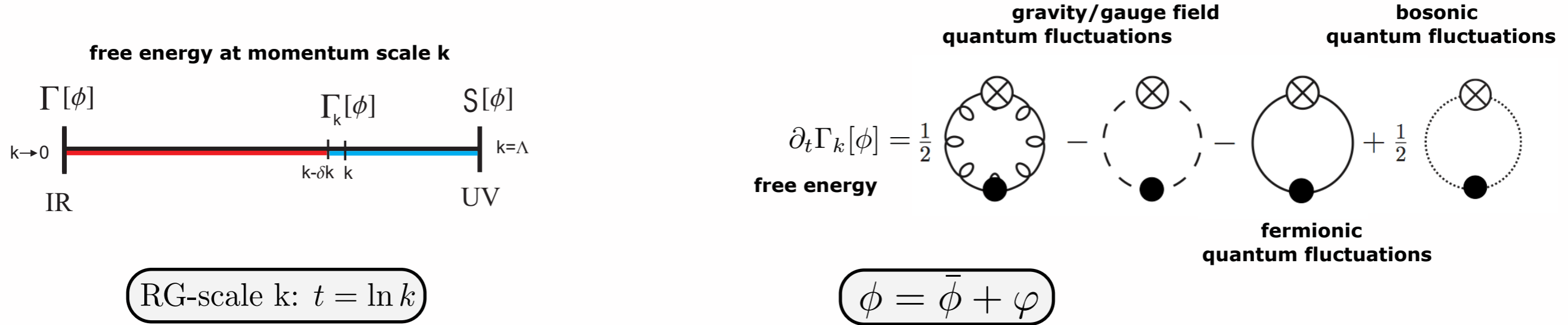
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Translation invariance

$$\frac{\delta \Gamma_k}{\delta \bar{\phi}} = \frac{\delta \Gamma_k}{\delta \varphi}$$

Background (in)dependence in the FRG



Expansion of the effective action

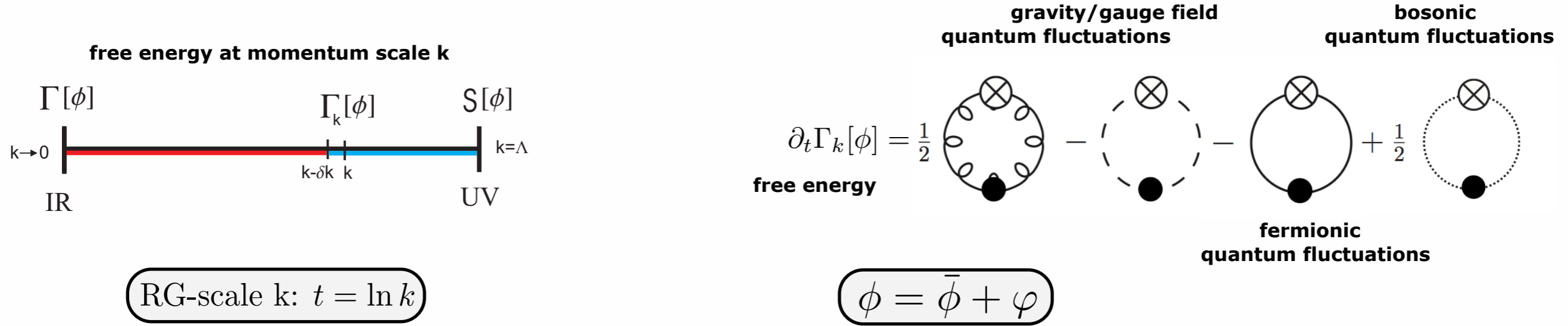
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Translation invariance

$$\frac{\delta \Gamma_k}{\delta \bar{\phi}} = \frac{\delta \Gamma_k}{\delta \varphi} \longrightarrow \frac{\delta \Gamma_k^{(0,n)}}{\delta \bar{\phi}} = \Gamma_k^{(0,n+1)}$$

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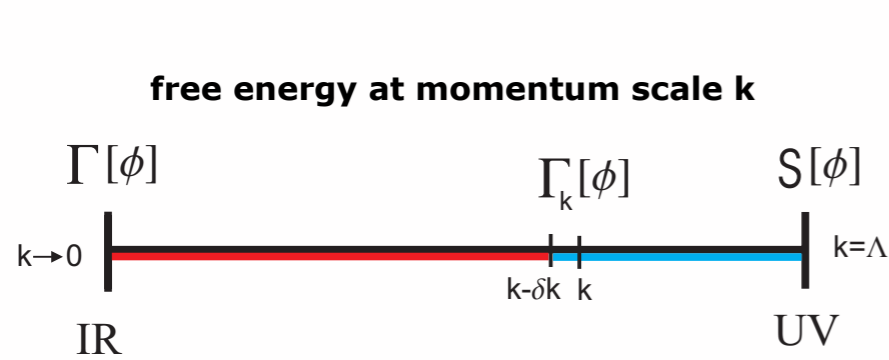
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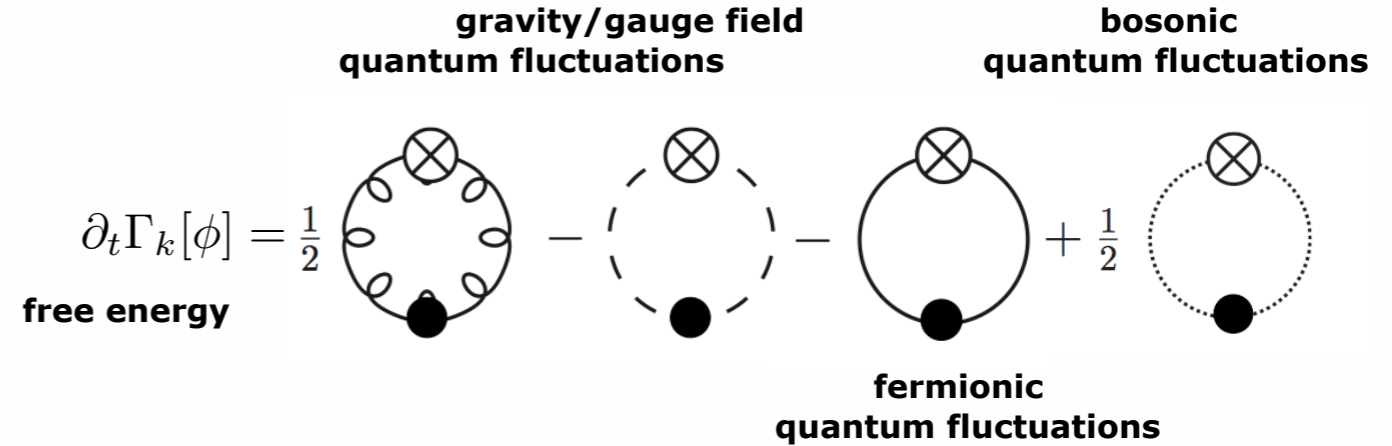
'Split WI/Nielsen ID'

$$\frac{\delta \Gamma_k^{(0,n)}}{\delta \bar{\phi}} = \Gamma_k^{(0,n+1)}$$

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Translation invariance

'Split WI/Nielsen ID'

broken by regulators

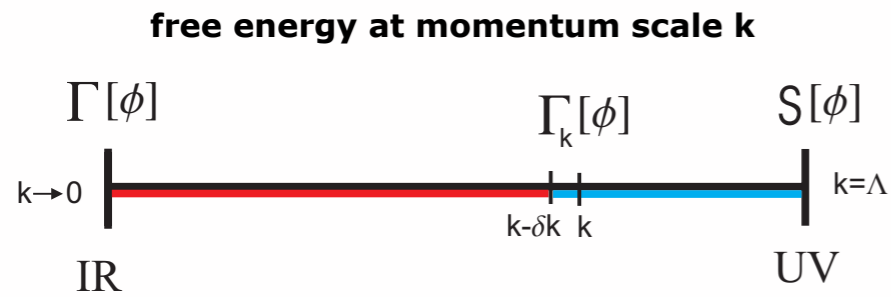
$R_k[\bar{\phi}]$

$$\frac{\delta \Gamma_k}{\delta \bar{\phi}} = \frac{\delta \Gamma_k}{\delta \varphi}$$



$$\frac{\delta \Gamma_k^{(0,n)}}{\delta \bar{\phi}} = \Gamma_k^{(0,n+1)}$$

Background dependence in gravity



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$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{gravity quantum fluctuations} - \text{fermionic quantum fluctuations} + \frac{1}{2} \text{bosonic quantum fluctuations}$$

free energy

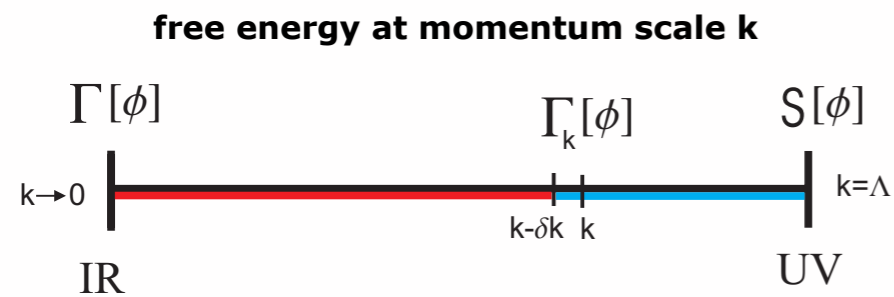
gravity quantum fluctuations

fermionic quantum fluctuations

bosonic quantum fluctuations

$\phi = (g, \dots)$

Background dependence in gravity



RG-scale k : $t = \ln k$

Wetterich equation (pure gravity)

$$\partial_t \Gamma_k[g] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[g] + R_k} \partial_t R_k$$

Background dependence in gravity

free energy at momentum scale k



RG-scale k : $t = \ln k$

Gauge fixing

$$S_{\text{gf}}[\bar{g}, g] = \frac{1}{2\alpha} \int d^4x \sqrt{\bar{g}} (L[\bar{g}] * g)^2$$

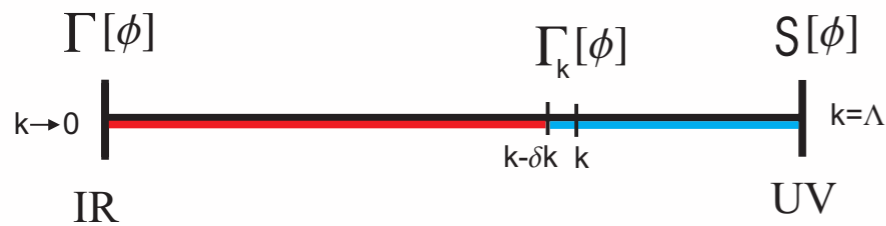
e.g.: $(L[\bar{g}] * g)_\mu = \bar{\nabla}^\nu g_{\mu\nu} - \frac{1}{2} \nabla^\mu g^\nu{}_\nu$

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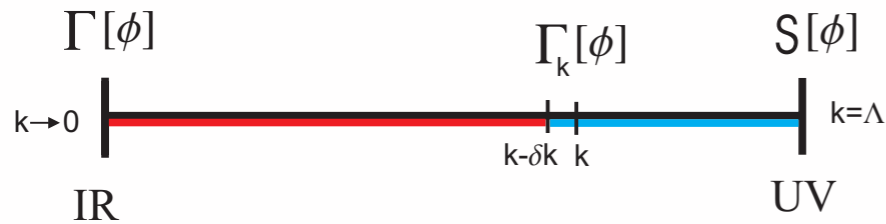
$$\partial_t \Gamma_k[\bar{g}, h] = \frac{1}{2} \text{Tr} \frac{1}{\frac{\delta^2 \Gamma_k}{\delta h^2} + R_k} \partial_t R_k + \text{ghosts}$$

linear split

$$g = \bar{g} + h$$

Background dependence in gravity

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Gauge/diffeomorphism invariance
mSTIs/Nielsen IDs (split mWI)

$$\frac{\delta \Gamma_k}{\delta h} = \frac{\delta \Gamma_k}{\delta \bar{g}} + L \& R_k - \text{terms}$$

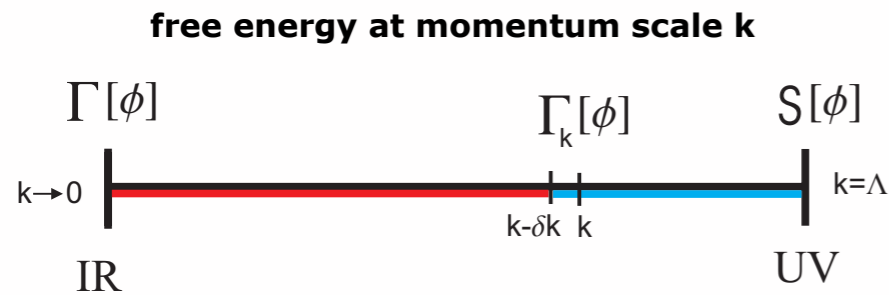
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Background dependence in gravity



Only one field!

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Background approximation I

mSTIs/Nielsen IDs (split mWI)

$$\frac{\delta\Gamma_k}{\delta h} \approx \frac{\delta\Gamma_k}{\delta\bar{g}}$$

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Wetterich equation in background approximation

$$\partial_t\Gamma_k[\bar{\phi}] = \frac{1}{2}\text{Tr} \frac{1}{\Gamma_k^{(2)} + R_k[\bar{\phi}]} \partial_t R_k[\bar{\phi}]$$

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Regulator bootstrap I

$$R_k[\bar{\phi}] = \Gamma_k^{(2)}[\bar{\phi}] r$$

$$\partial_t\Gamma_k[\bar{\phi}] = \frac{1}{2}\text{Tr} \frac{1}{1+r} \partial_t r + \frac{1}{2}\text{Tr} \partial_t\Gamma_k^{(2)} \frac{1}{\Gamma_k^{(2)}} \frac{r}{1+r}$$

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Regulator bootstrap II

Given $F[\bar{\phi}]$

Welches Schweinderl hätten's denn gerne?

Solve

$$F[\bar{\phi}] = \frac{1}{2}\text{Tr} \frac{1}{\Gamma_k^{(2)} + R_k[\bar{\phi}]} \partial_t R_k[\bar{\phi}]$$

Litim, JMP, PLB 546 (2002) 279, PRD 66 (2002) 025030

Gravity Folkerts, Litim, JMP, PLB 709 (2012) 234

Background approximation II

mSTIs/Nielsen IDs (split mWI)

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Testing universal beta-functions: A simple one loop example in Yang-Mills theory

Litim, JMP, JHEP 0209 (2002) 049

mSTIs/Nielsen IDs (split mWI)

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Regulators

$$R_k(\bar{\Delta}) = \bar{\Delta} r(\bar{\Delta}/k^2)$$

IR-behaviour: $r(x \rightarrow 0) \propto \frac{1}{x^n}$

$$\bar{\Delta} = -\bar{D}^2 + \text{spin} - \text{contr.}$$

Background approximation II

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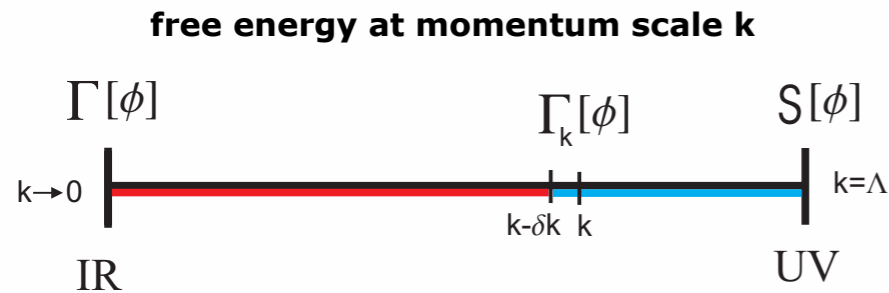
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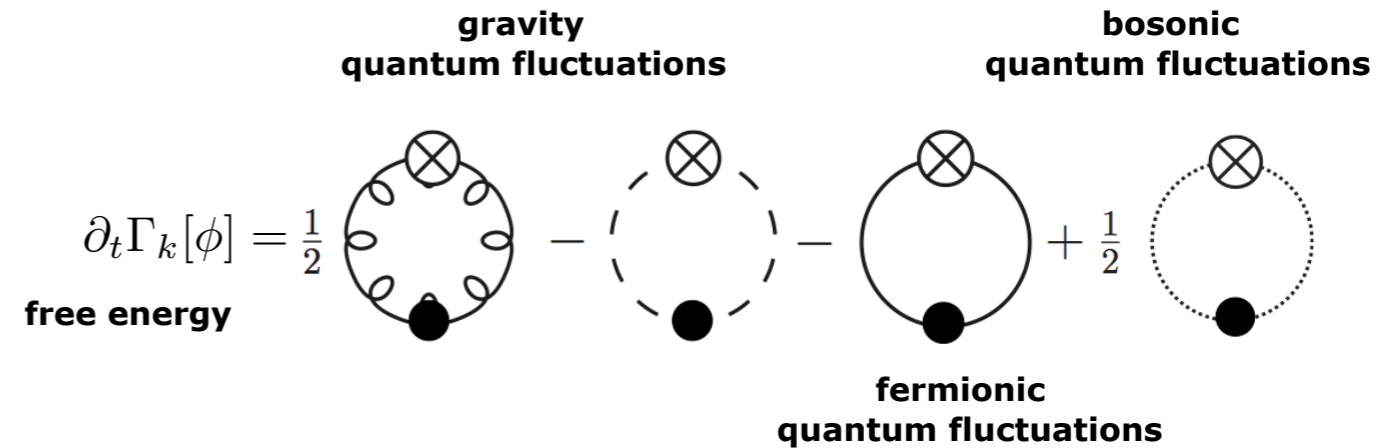
1-loop beta-function

$$\beta_{\text{background-approx}} = \frac{1}{11} [10n_a + n_c] \beta_{1\text{-loop}}$$

Background (in)dependence in gravity



RG-scale k : $t = \ln k$



Geometrical approach: fully diffeomorphism invariant

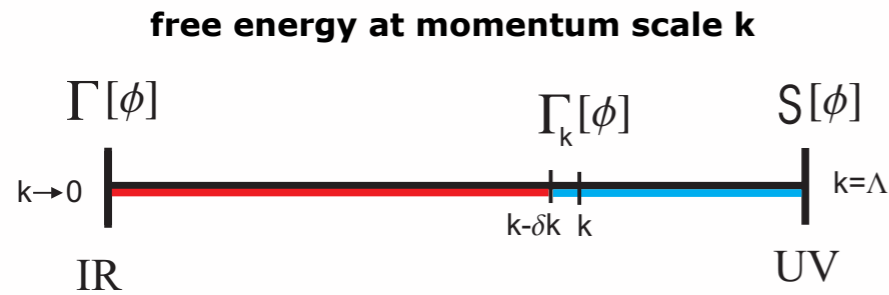
$$g = \bar{g} + h + O(h^2)$$

Effective action

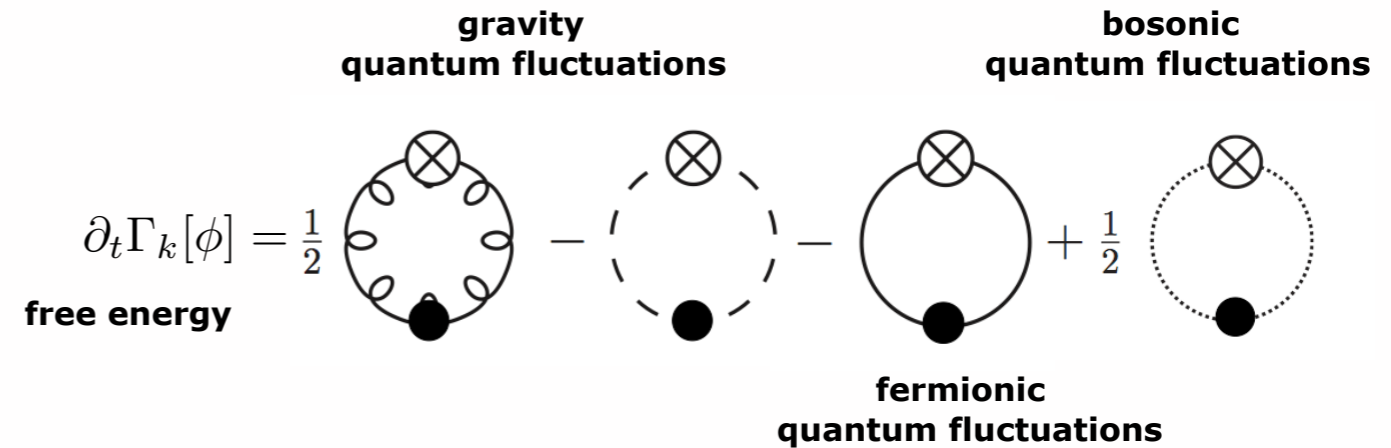
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$$\bar{h} = \langle h \rangle$$

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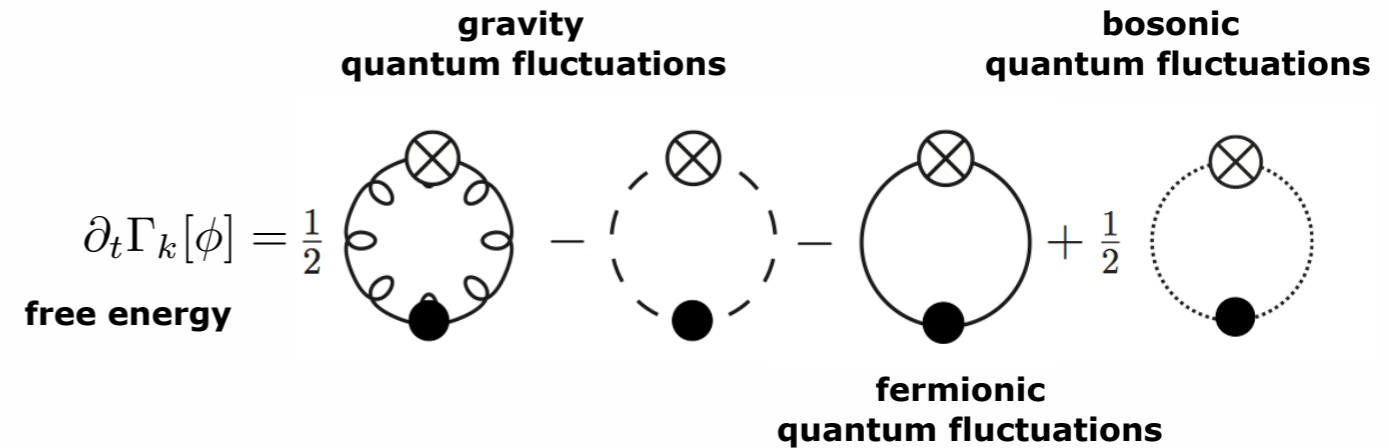
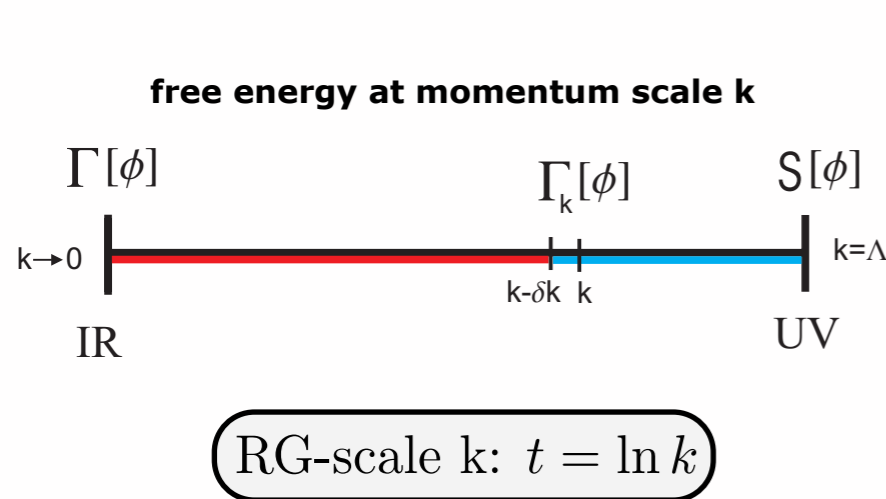
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$$\left\{ \Gamma_k[\bar{g}], \Gamma_k^{(0,1)}[\bar{g}], \Gamma_k^{(0,2)}[\bar{g}], \Gamma_k^{(0,3)}[\bar{g}], \dots \right\}$$

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*Se vogliamo che tutto rimanga come è,
bisogna che tutto cambi.*

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Il Gattopardo

Background (in)dependence in gravity

expansion schemes

Effective action

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- **no diffeomorphism-invariant expansion scheme**

mSTIs/Nielsen IDs

Litim, JMP '02

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Litim, JMP '02

- **what is at stake?**

at vanishing cutoff: loss of the confining property of the order parameter potential in QCD

$$\frac{\delta^2 \Gamma}{\delta \bar{A}^2}(p \rightarrow 0) \propto p^2$$

$$\frac{\delta^2 \Gamma}{\delta \bar{a}^2}(p \rightarrow 0) \propto \text{mass gap}$$

Braun, Gies, JMP '07

Braun, Eichhorn, Gies, JMP '10

Fister, JMP '13

Background (in)dependence in gravity

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- **what is at stake?**

qualitative difference

semi-qualitative/quantitative difference

cosmological constant \neq graviton mass parameter

Newton constant ren. \neq graviton wave function

\neq const. part of vertex $\Gamma^{(3)}$

⋮

⋮

Background (in)dependence in gravity

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- **what is at stake? Does it matter?**

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- **what is at stake? Does it matter?**

Examples in gravity-matter systems:

$$\text{sign} \left[\partial_t \Gamma_{\text{back-approx}}^{(n)} \right] \neq \text{sign} \left[\partial_t \Gamma_{\text{fluc}}^{(n)} \right]$$

Functional approach to quantum gravity

expansion schemes

Effective action

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- **no diffeomorphism-invariant expansion scheme**

mSTIs/Nielsen IDs
Litim, JMP '02

Fluctuation flows determine the fixed point structure!

$$\begin{aligned} \partial_t g_{i,\text{fluc}} &= \text{Flow}_{g_{i,\text{fluc}}}(\vec{g}_{\text{fluc}}) \\ \partial_t g_{i,\text{back}} &= \text{Flow}_{g_{i,\text{back}}}(\vec{g}_{\text{fluc}}, \vec{g}_{\text{back}}) \end{aligned}$$

dynamical flow

background flow

Nielsen ID

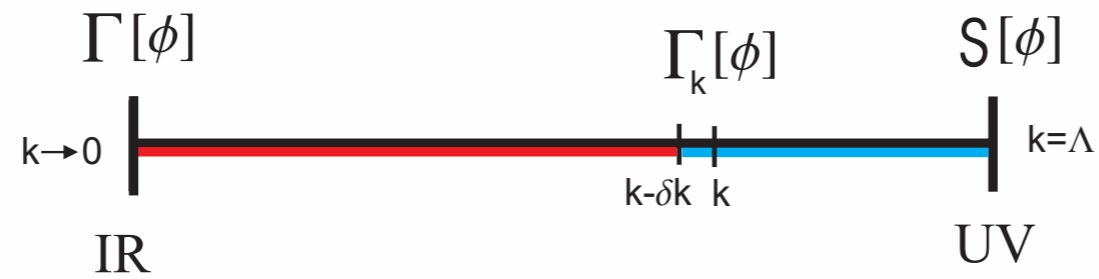
JMP '03

Donkin, JMP '12

Momentum locality of the RG

Christiansen, Knorr, Meibohm, JMP, Reichert, PRD 92 (2015) 12, 121501

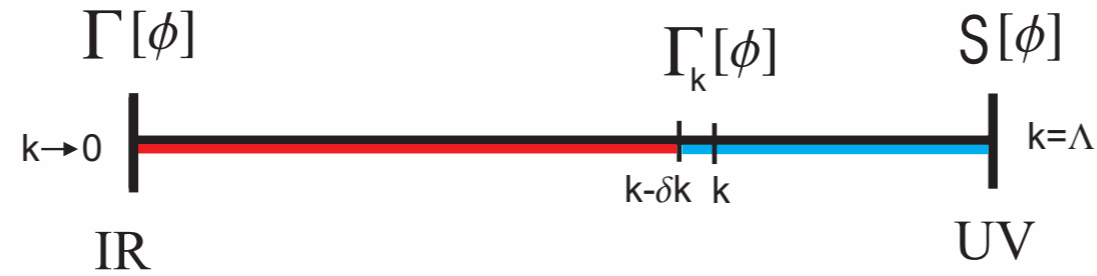
free energy at momentum scale k



Momentum locality of the RG

Christiansen, Knorr, Meibohm, JMP, Reichert, PRD 92 (2015) 12, 121501

free energy at momentum scale k



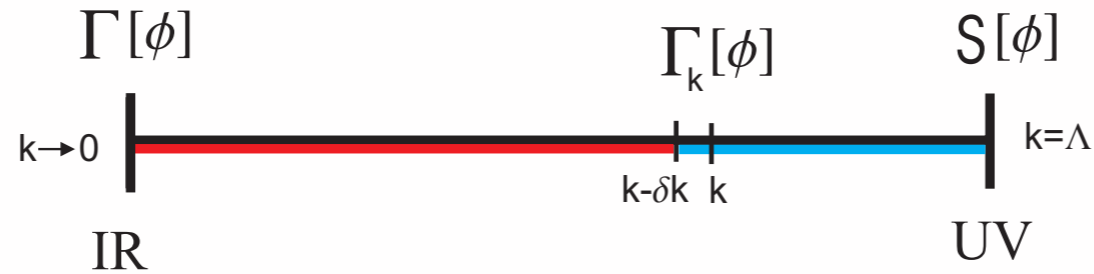
Locality

$$\lim_{p_1, \dots, p_n \rightarrow \infty} \frac{|\partial_t \mathcal{O}_k(p_1, \dots, p_n)|}{|\mathcal{O}_k(p_1, \dots, p_n)|} \rightarrow 0$$

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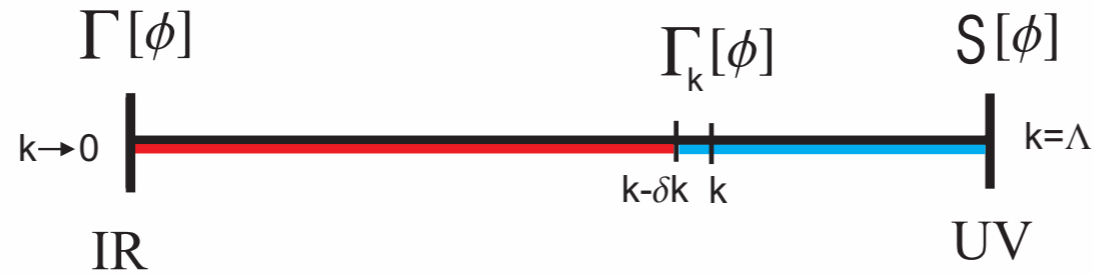
- **local momentum space RG steps** \longleftrightarrow **local quantum field theory**
- **gravity-matter systems: locality** \longleftarrow **diffeomorphism invariance**

does not work in e.g. $\phi^2 \Delta \phi^2$ -theories

Momentum locality of the RG

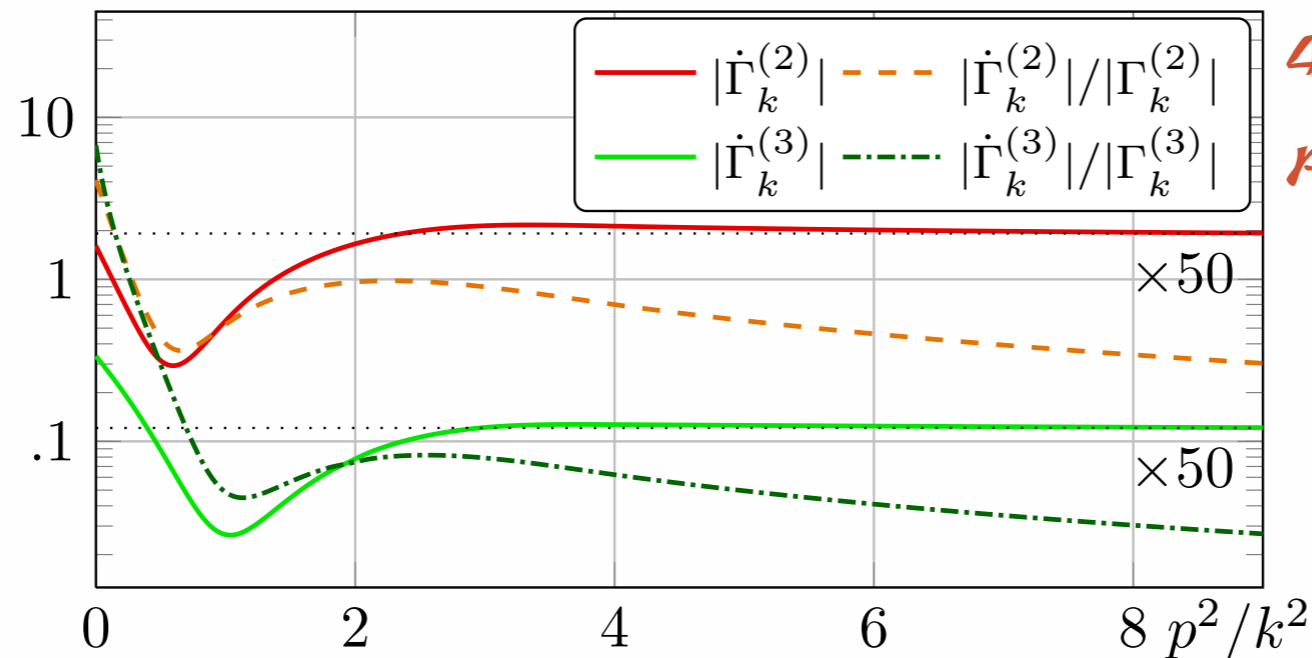
Christiansen, Knorr, Meibohm, JMP, Reichert, PRD 92 (2015) 12, 121501

free energy at momentum scale k



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*4-graviton vertex: see
Poster of Tobias Denz*

symmetric point

Approximation scheme

Christiansen, Knorr, Meibohm, JMP, Reichert, PRD 92 (2015) 12, 121501

Denz, JMP, Reichert, in preparation

Christiansen, Falls, JMP, Reichert, in preparation

Propagators

graviton

$$k\partial_k \text{ (wavy line with circle) }^{-1} = -\frac{1}{2} \text{ (loop with 4 wavy lines) } + 2 \text{ (loop with 2 wavy and 2 dashed lines) } + \text{ (loop with 2 wavy and 2 dashed lines, different topology) } - 2 \text{ (loop with 2 wavy and 2 dashed lines, another topology) }$$

ghost

$$k\partial_k \text{ (dashed line with circle) }^{-1} = -\frac{1}{2} \text{ (loop with 4 dashed lines) } + \text{ (loop with 2 dashed and 2 wavy lines) } + \text{ (loop with 2 dashed and 2 wavy lines, different topology) } + \text{ (loop with 2 dashed and 2 wavy lines, another topology) }$$

full momentum dependence


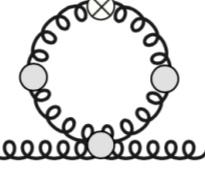
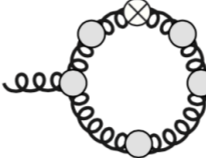
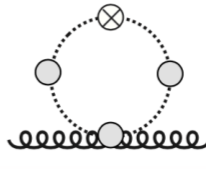
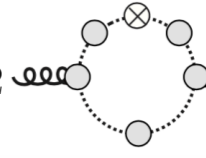
Approximation scheme

Christiansen, Knorr, Meibohm, JMP, Reichert, PRD 92 (2015) 12, 121501


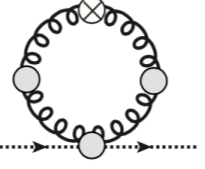
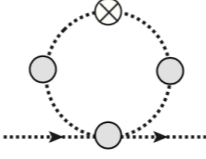
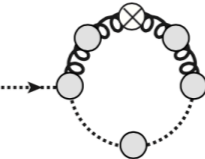
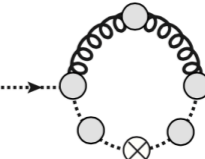
Denz, JMP, Reichert, in preparation

Christiansen, Falls, JMP, Reichert, in preparation

Propagators

graviton $k\partial_k$  ⁻¹ = $-\frac{1}{2}$  + 2  +  - 2 

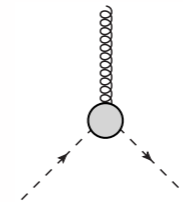
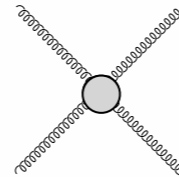
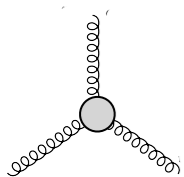
full momentum dependence

ghost $k\partial_k$  ⁻¹ = $-\frac{1}{2}$  +  +  + 

Vertices

flow

consistent momentum-dependent RG-dressing



a la Fischer, JMP '09
Donkin, JMP '12

similar: Codello, D'Odorico, Pagani '13

$$Z_{\text{graviton}} \neq Z_{g_N}$$

$$M_{\text{graviton}}^2 \neq -2\Lambda$$

Approximation scheme

Christiansen, Knorr, Meibohm, JMP, Reichert, PRD 92 (2015) 12, 121501

Denz, JMP, Reichert, in preparation

Christiansen, Falls, JMP, Reichert, in preparation

Propagators

graviton

$$k\partial_k \text{ (graviton line) }^{-1} = -\frac{1}{2} \text{ (loop 1)} + 2 \text{ (loop 2)} + \text{ (loop 3)} - 2 \text{ (loop 4)}$$

full momentum dependence

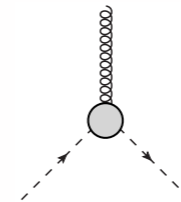
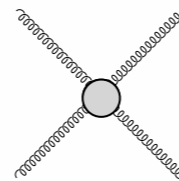
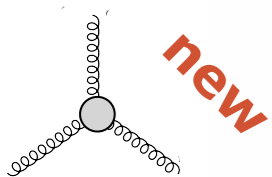
ghost

$$k\partial_k \text{ (ghost line) }^{-1} = -\frac{1}{2} \text{ (loop 1)} + \text{ (loop 2)} + \text{ (loop 3)} + \text{ (loop 4)}$$

Vertices

flow

consistent momentum-dependent RG-dressing



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Donkin, JMP '12

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Approximation scheme

Christiansen, Knorr, Meibohm, JMP, Reichert, PRD 92 (2015) 12, 121501

Denz, JMP, Reichert, in preparation

Christiansen, Falls, JMP, Reichert, in preparation

Propagators

graviton

$$k \partial_k \text{ (wavy line with circle) }^{-1} = -\frac{1}{2} \text{ (loop with 4 wavy lines) } + 2 \text{ (loop with 2 wavy and 2 dashed lines) } + \text{ (loop with 2 wavy and 2 dashed lines, different topology) } - 2 \text{ (loop with 4 dashed lines) }$$

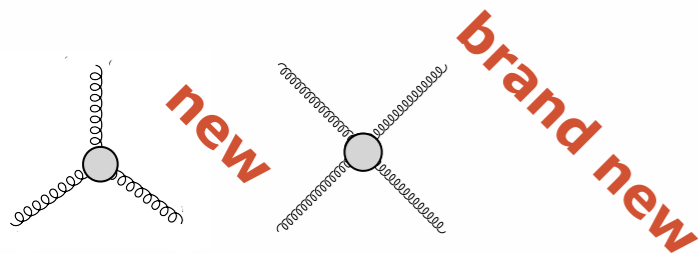
full momentum dependence

ghost

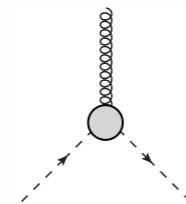
$$k \partial_k \text{ (dashed line with circle) }^{-1} = -\frac{1}{2} \text{ (loop with 4 wavy lines) } + \text{ (loop with 4 dashed lines) } + \text{ (loop with 2 wavy and 2 dashed lines) } + \text{ (loop with 2 wavy and 2 dashed lines, different topology) }$$

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Christiansen, Knorr, Meibohm, JMP, Reichert, PRD 92 (2015) 12, 121501

Denz, JMP, Reichert, in preparation

Christiansen, Falls, JMP, Reichert, in preparation

Propagators

graviton

$$k\partial_k \text{ (graviton propagator) }^{-1} = -\frac{1}{2} \text{ (loop 1)} + 2 \text{ (loop 2)} + \text{ (loop 3)} - 2 \text{ (loop 4)}$$

full momentum dependence

ghost

$$k\partial_k \text{ (ghost propagator) }^{-1} = -\frac{1}{2} \text{ (loop 1)} + \text{ (loop 2)} + \text{ (loop 3)} + \text{ (loop 4)}$$

Flows & scalings

propagators

$Z_{\text{graviton}}(p^2)$

M_{graviton}^2

$Z_{\text{ghost}}(p^2)$

vertices

$\Gamma_{hhh}^{(3)}(p_1, p_2)$

$G_3 \quad \Lambda_3$

$\Gamma_{hhh}^{(4)}(p_1, p_2, p_3)$

$G_4 \quad \Lambda_4$

background observables

Λ

cosmological constant

\bar{G}_N

Newton constant

Approximation scheme

Christiansen, Knorr, Meibohm, JMP, Reichert, PRD 92 (2015) 12, 121501

Denz, JMP, Reichert, in preparation

Christiansen, Falls, JMP, Reichert, in preparation

Propagators

graviton

$$k\partial_k \text{ (graviton line) }^{-1} = -\frac{1}{2} \text{ (loop 1)} + 2 \text{ (loop 2)} + \text{ (loop 3)} - 2 \text{ (loop 4)}$$

full momentum dependence

ghost

$$k\partial_k \text{ (ghost line) }^{-1} = -\frac{1}{2} \text{ (loop 1)} + \text{ (loop 2)} + \text{ (loop 3)} + \text{ (loop 4)}$$

Flows & scalings

propagators

$Z_{\text{graviton}}(p^2)$

M_{graviton}^2

$Z_{\text{ghost}}(p^2)$

background observables

$\Gamma_{hhh}^{(3)}(p_1, p_2)$

$G_3 \quad \Lambda_3$

Λ

\bar{G}_N

cosmological constant

Newton constant

$\Gamma_{hhh}^{(4)}(p_1, p_2, p_3)$

$G_4 \quad \Lambda_4$

brand brand new: $G_n(R), \Lambda_n(R)$ *see Poster of Manuel Reichert*

Summary & outlook

- **Background independence & Locality**

- **background independence from background dependence**

- **closed fluctuation flows**

- **momentum locality from diffeomorphism invariance**

- **Applications**

- **fully-coupled matter-gauge-gravity systems in the UV**

- **long & short distance physics**

