Equilibrium and out-of-equilibrium dynamics of disordered systems:

Avalanches, droplets and the Non-Perturbative Functional Renormalization Group

Gilles Tarjus (LPTMC, CNRS-Univ. PARIS 6)

Matthieu Tissier (LPTMC), Ivan Balog (Physics Institute, Zagreb)

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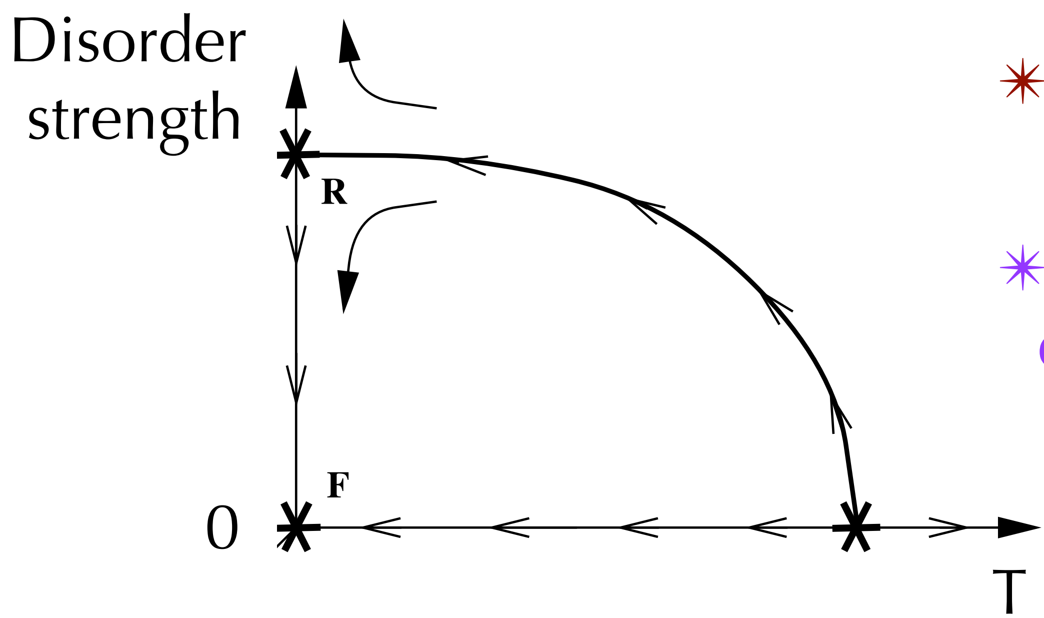
Equilibrium and out-of-equilibrium (classical) collective behavior of systems in the presence of quenched disorder

More specifically: Focus on a class of models in statistical physics and condensed-matter theory for which **the long-distance physics is dominated by disorder:**

- **Random field models**
- **Random anisotropy models**
- **Elastic manifolds (interfaces,...) in a random environment**

Dominance of quenched disorder

- Equilibrium critical behavior controlled by a **zero-temperature (*T=0*) fixed point**:



Phase diagram and RG flow of random field models

✳ **Additional exponent for the temperature flow:**

$$\theta > 0$$

✳**For *T>0*: very slow "activated" dynamics, with exponential dependence of time vs length:**

$$\tau \sim \xi^z \quad \text{-->} \quad \tau \sim \exp(\frac{c}{T}\,\xi^{\psi})$$

- Low-T nontrivial physics under driving/forcing:
  - ✳ **pinning/depinning of interfaces**
  - ✳ **hysteresis and out-of-equilibrium phase transitions**

Why does one need a nonperturbative functional RG (NP-FRG) ?

RG must be functional:

* Due to quenched disorder, presence of rare events, rare spatial regions or rare samples: at T=0, **avalanches** or **shocks**; at T>0, low-energy excitations (**droplets**)

  => How to keep their signature in the RG?

* Because of quenched disorder (*h*), one loses translational invariance:

  $W_h[J] = \ln Z_h[J]$ is then a random functional of the source *J*

  => One recovers translational invariance by considering the cumulants.

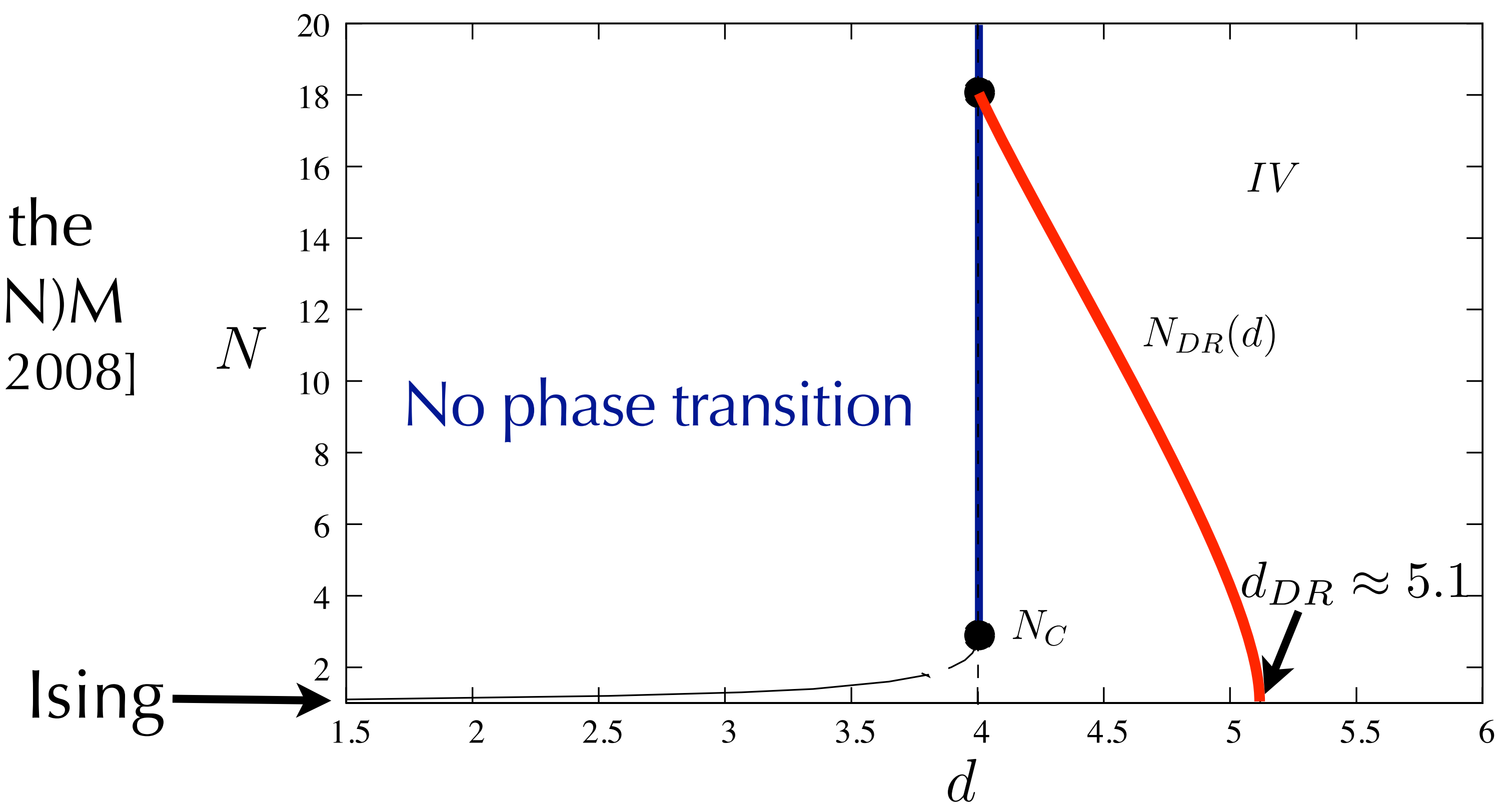
* The possible influence of avalanches and droplets can then be described only through a **singular dependence of the cumulants of the renormalized disorder** on their arguments.

Why does one need a NP-FRG ?

RG must be nonperturbative,

because standard perturbation theory completely fails (dimensional-reduction problem), and because for random fields (RF) and anisotropies (RA) the behavior changes at a **nontrivial critical dimension $d_{DR}(N)$**.

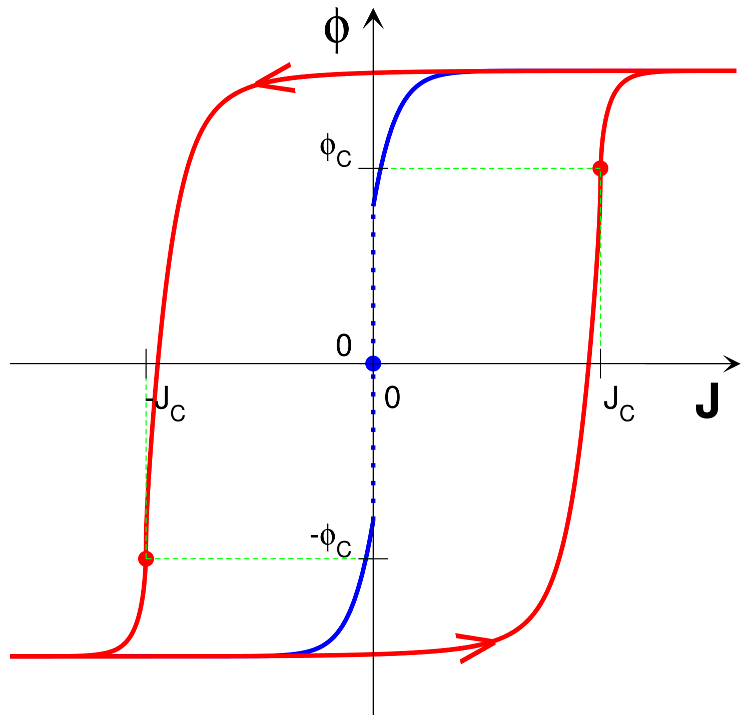
N-d diagram for the equilibrium RFO(N)M
[M. Tissier, G.T.,2006,2008]



Below $d_{DR}(N)$: strong nonanalyticity in the renormalized cumulants, dominance of avalanches, dimensional-reduction breakdown, and SUSY breaking.

From now on, focus on **RFIM** (Ising, *N=1*)

RFIM in and out of equilibrium



Magnetization Φ vs applied source J at T=0

Blue: **Equilibrium** curves (ground state)
Red: Ascending and descending branches of the hysteresis loop for the **driven** RFIM

- From numerical studies of the *d=3* **slowly driven RFIM at *T=0***:
  The out-of equilibrium critical point(s) along the hysteresis curve and the equilibrium critical point have very similar exponents and scaling functions.
  [PerezReche-Vives 04, Colaiori et al 04, Liu-Dahmen 07,09]

However: Not the same value of the critical disorder, not the same symmetry...
Are the two critical phenomena controlled by the same fixed point of the RG flow?

- **Different effect of temperature:** Equilibrium phase transition persists for *T>0*, not the out-of-equilibrium one (**equilibrium vs metastability**).

RFIM dynamics: field-theoretical formalism

- **Langevin equation**: $\partial_t \varphi_{xt} = \dfrac{\delta S_B[\varphi]}{\delta \varphi_{xt}} - h(x) - J(t) + \eta_{xt}$

  * ***S_B*** -> standard (Z_2 symmetric) phi^4 action
  * ***h(x)*** -> random field with $\overline{h(x)h(x')} = \Delta_B \, \delta^{(d)}(x-x')$
  * **η*(xt)*** -> stochastic noise with $< \eta_{xt}\eta_{x't'} >= 2T \, \delta^{(d)}(x-x')\delta(t-t')$
  * ***J(t)*** -> applied source:

  $J(t) = J \; (= 0 \text{ at criticality})$ at equilibrium for *T* ≥ *0*

  $J(t) = J + \Omega t$ for the quasi-statically driven system at *T*=*0*

  $(\Omega \to 0^+ \text{ or } 0^- \text{ for ascending/descending hysteresis branch})$

- Introduce **copies/replicas** of the system coupled to distinct sources.
- Use the Janssen-deDominicis-MSR formalism (response fields $\widehat{\varphi}$).

Dynamical NP-FRG formalism

- Average over the Gaussian noise and random field => **dynamical action with replica fields** (*a=1,...,n*):

$$S_{dyn}[\{\varphi_a,\widehat{\varphi}_a\}]=\sum_a\int_{x,t}\widehat{\varphi}_{a,xt}[\frac{\delta S_B[\{\varphi_a\}]}{\delta\varphi_{a,xt}}-T\widehat{\varphi}_{a,xt}]$$

$$-\frac{1}{2}\sum_{ab}\Delta_B\int_x\int_{t_1t_2}\widehat{\varphi}_{a,xt_1}\widehat{\varphi}_{a,xt_2}$$

- Add an IR regulator through a *k*-dependent quadratic term $\Delta S_k[\{\varphi_a,\widehat{\varphi}_a\}]$

- **Exact RG equation for the effective average action** $\Gamma_k[\{\phi_a,\widehat{\phi}_a\}]$:

$$\partial_k\Gamma_k[\{\phi_a,\widehat{\phi}_a\}]=\frac{1}{2}\mathrm{Tr}\;\partial_\mathrm{k}\mathcal{R}_\mathrm{k}\left(\Gamma_\mathrm{k}^{(2)}[\{\phi_\mathrm{a},\widehat{\phi}_\mathrm{a}\}]+\mathcal{R}_\mathrm{k}\right)^{-1}$$

Dynamical NP-FRG formalism

- **Nonperturbative truncation** (cumulant expansion + space and time derivative expansion)**:**

$$\Gamma_k[\{\phi_a,\widehat{\phi}_a\}] = \sum_a \int_{x,t} \widehat{\phi}_{a,xt}[U_k'(\phi_{a,xt}) + \frac{\delta}{\delta\phi_{a,xt}}[Z_k(\phi_{a,xt})(\partial_x\phi_{a,xt})^2]$$
$$+ X_k(\phi_{a,xt})(\partial_t\phi_{a,xt} - T\widehat{\phi}_{a,xt})]$$
$$-\frac{1}{2}\sum_{ab}\int_x\int_{t_1t_2} \widehat{\varphi}_{a,xt_1}\widehat{\varphi}_{a,xt_2}\Delta_k(\phi_{a,xt_1},\phi_{b,xt_2})$$

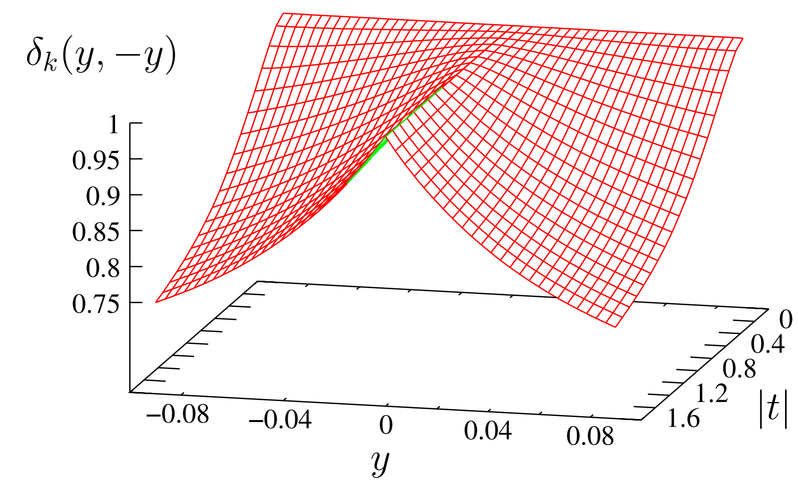
- From the ERGE, obtain the flow equations for the static functions ***U'***

- By using the scaling dimensions for **zero-temperature fixed points**, cast the flow equations in a dimensionless form for $u_k'(\varphi)$, $z_k(\varphi)$, $\delta_k(\varphi_1,\varphi_2)$

Beware the cusp!

- For $d < d_{DR} \simeq 5.1$ and at $T=0$, there is an avalanche-induced **cusp** in $\delta_k(\varphi_1, \varphi_2)$:

$$\delta_k(\varphi_1,\varphi_2) = \delta_{k0}(x = \frac{\varphi_1+\varphi_2}{2}) +$$
$$\delta_{k,cusp}(x)|\varphi_1-\varphi_2| + \mathrm{O}((\varphi_1-\varphi_2)^2)$$



Equil. flow in d=4

- The flow equations involve derivatives of $\delta_k(\varphi_1, \varphi_2)$ evaluated for the same replica => **ambiguity!**

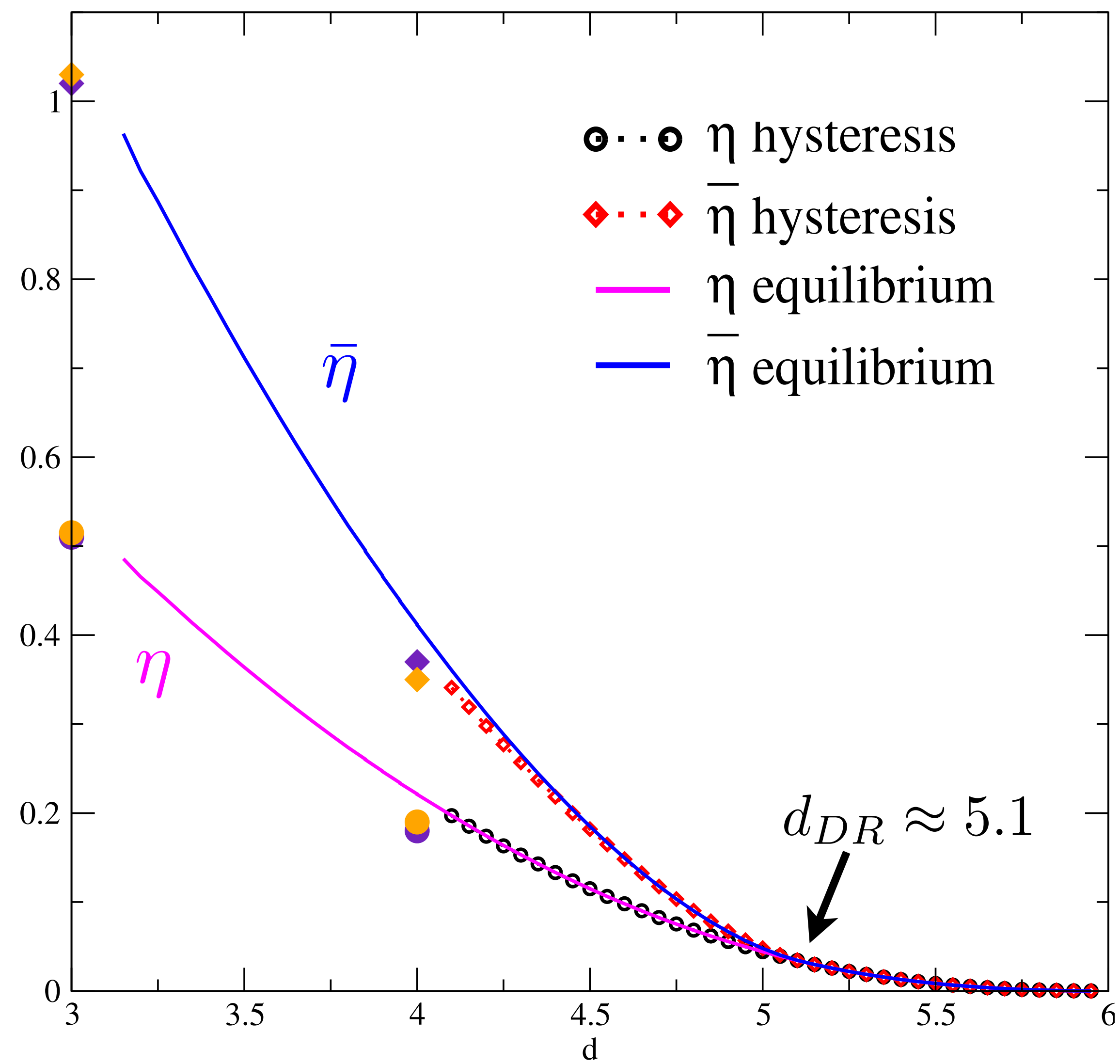
- **In equilibrium for T>0**, the cusp is rounded in a **thermal boundary layer** of width $T_k \sim k^\theta T$ => lifts the ambiguity & leads to a zero-T fixed point and activated dynamic slowing down.

- **For hysteresis at T=0**, no rounding => use **an infinitesimal velocity** ***v*** so that the time dependence chooses one side of the cusp (due to causality):

$$\varphi_a(t) = \varphi_a + vt,\ \text{with v} \to 0^+\ \textbf{or}\ 0^-\ \text{(ascending \textbf{or} descending)}$$

Results: in and out-of equilibrium fixed points are different for $d<d_{DR}$

Dimension-dependence of the two anomalous dimensions (related by the temperature exponent θ) for equilibrium and hysteresis critical points: **different fixed points but small differences in the exponents**

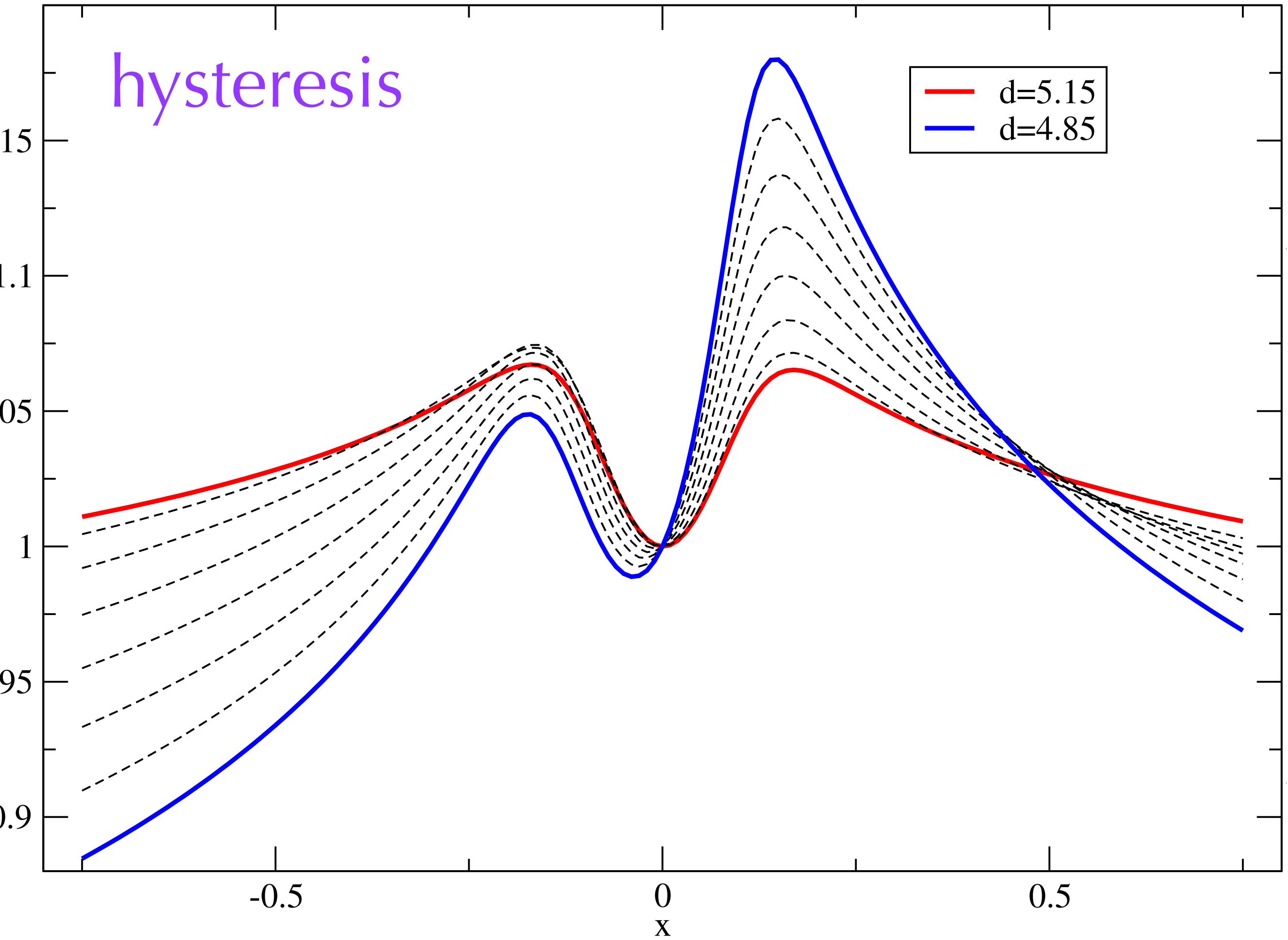


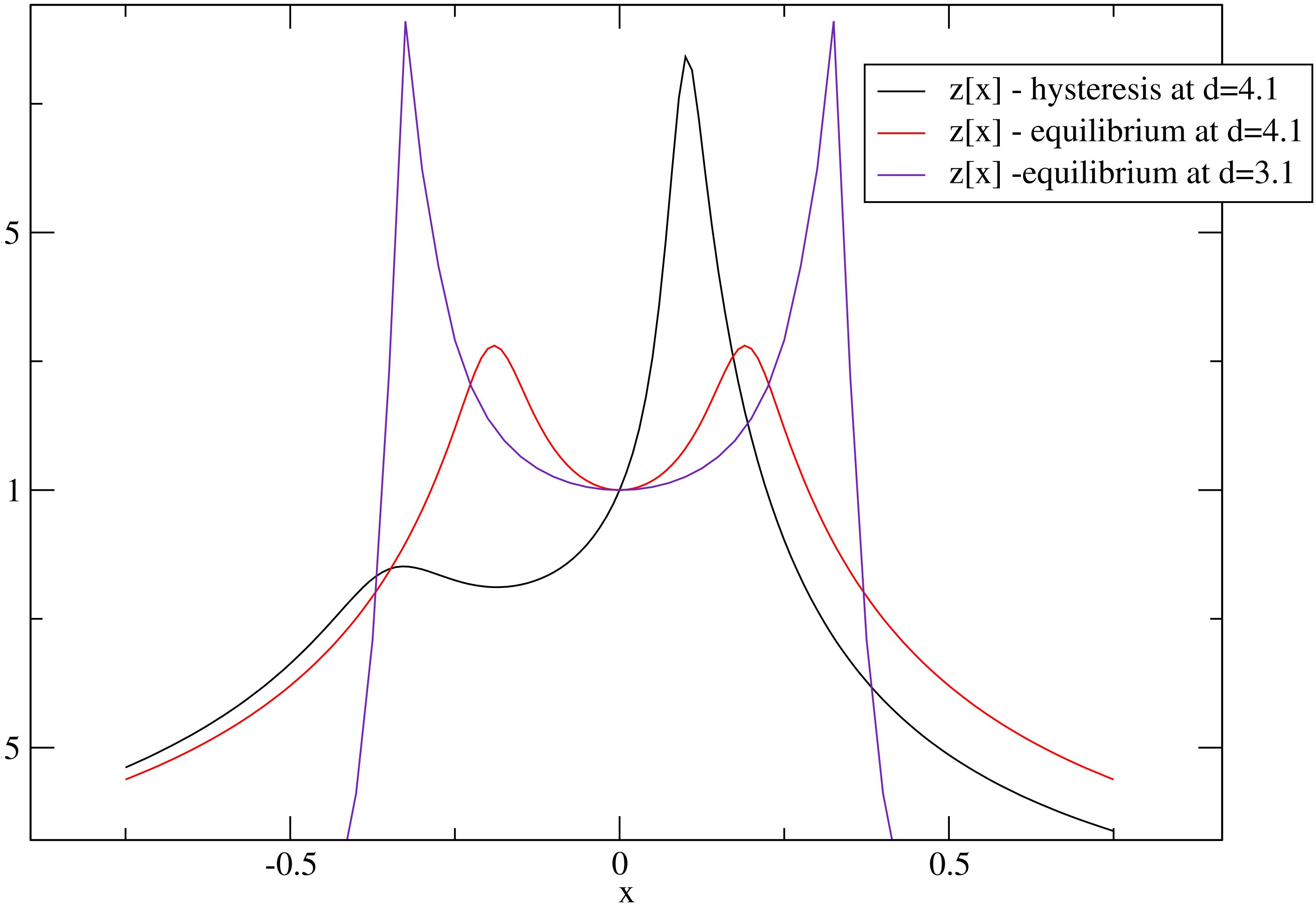
$$\bar{\eta} = \eta + (2 - \theta)$$

In *d=3,4* for equilibrium: good agreement with simulation results (big symbols)

Different fixed points for equilibrium and hysteresis

Field renormalization function z(x=φ) for the critical fixed points:
Note the absence of inversion symmetry for hysteresis below d_{DR}





Conclusion

- For a theoretical description of scale-free avalanches and droplets that are important for a class of disordered systems, one needs a **functional and nonperturbative RG** (NP-FRG)
- One can formulate a **dynamical version** of the NP-FRG (already used by us for equilibrium problems) to describe both equilibrium and driven dynamics.
- The equilibrium and out-of-equilibrium (hysteresis) critical points of the RFIM are in **different universality classes**.
- The same dynamical NP-FRG treatment applied to an elastic manifold in a random environment gives good results compared to simulations/experiments and perturbative FRG

Hysteresis critical fixed point

Cusp amplitude in the second cumulant of the renormalized random field at the **hysteresis critical fixed point** in d=4.85 [blue] and d=5.15 [red]