Universal short-time dynamics: FRG for a temperature quench

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Quench dynamics

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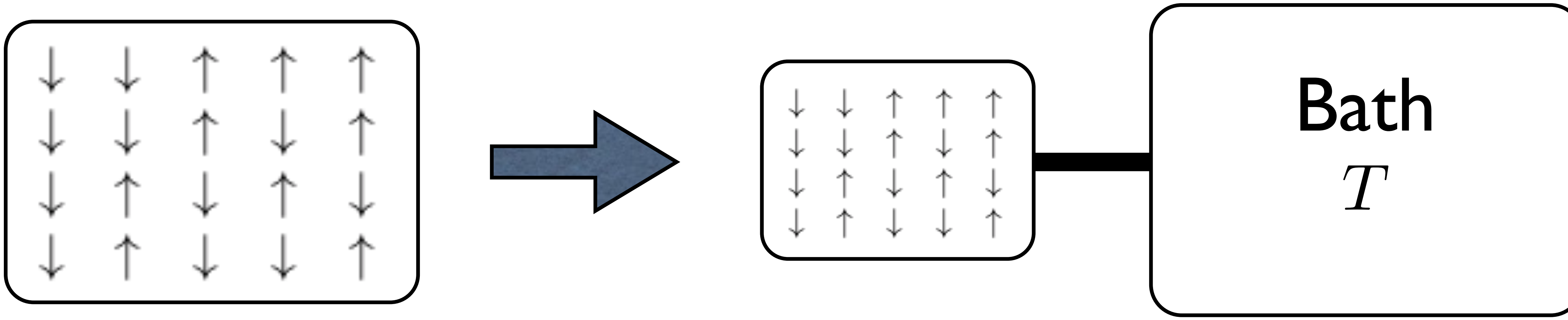
Quenching

From Wikipedia, the free encyclopedia

In materials science, **quenching**, a type of heat treating, is the rapid cooling of a workpiece to obtain certain material properties.

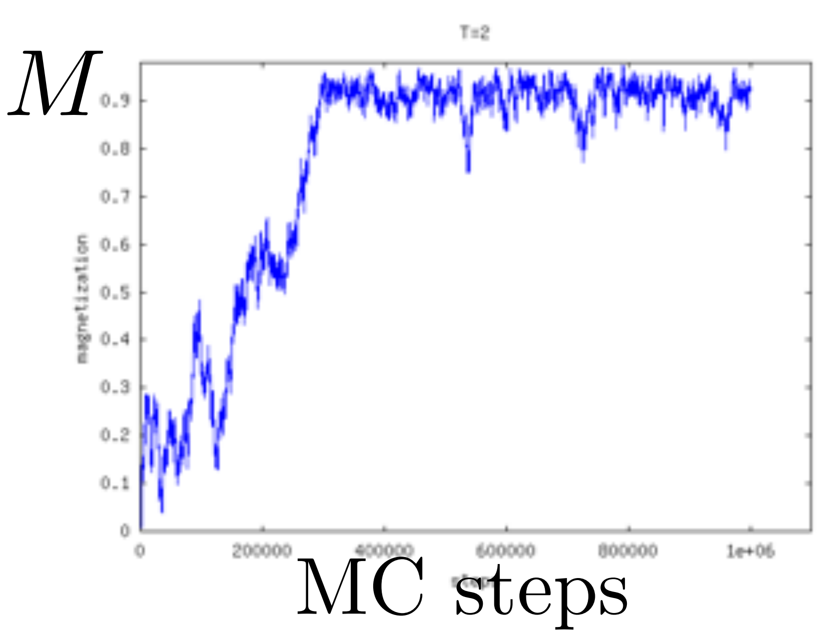
Quench dynamics

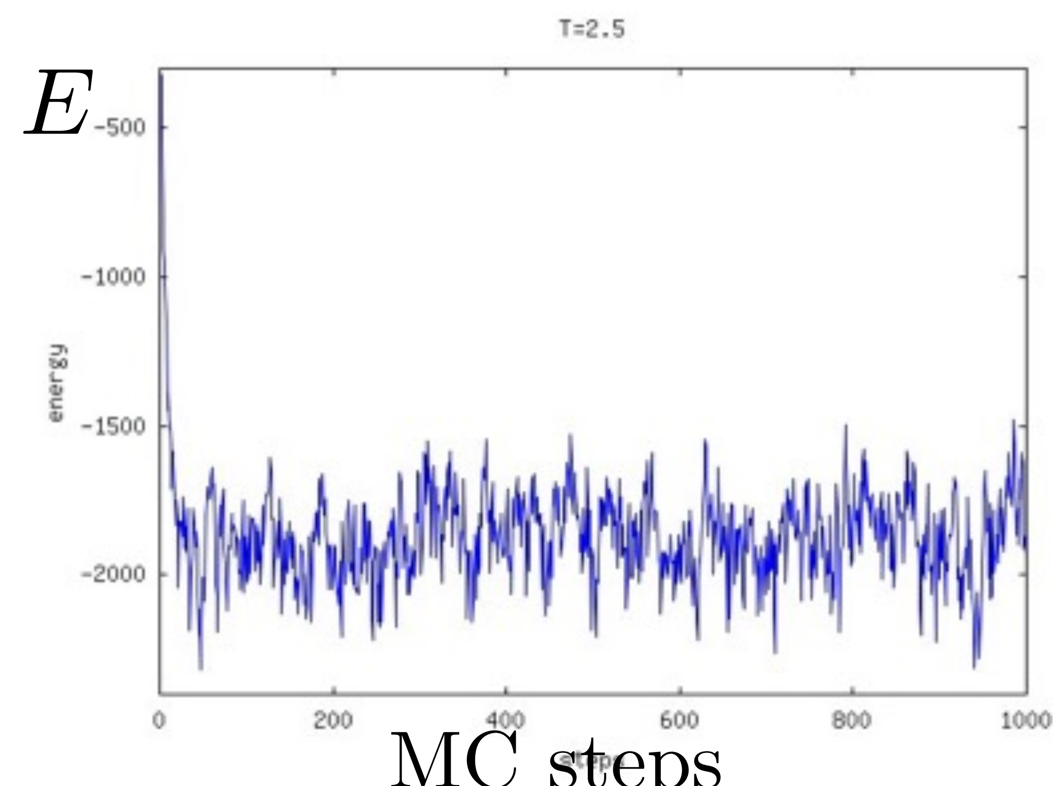


initial distribution $P_0(\{\sigma\})$

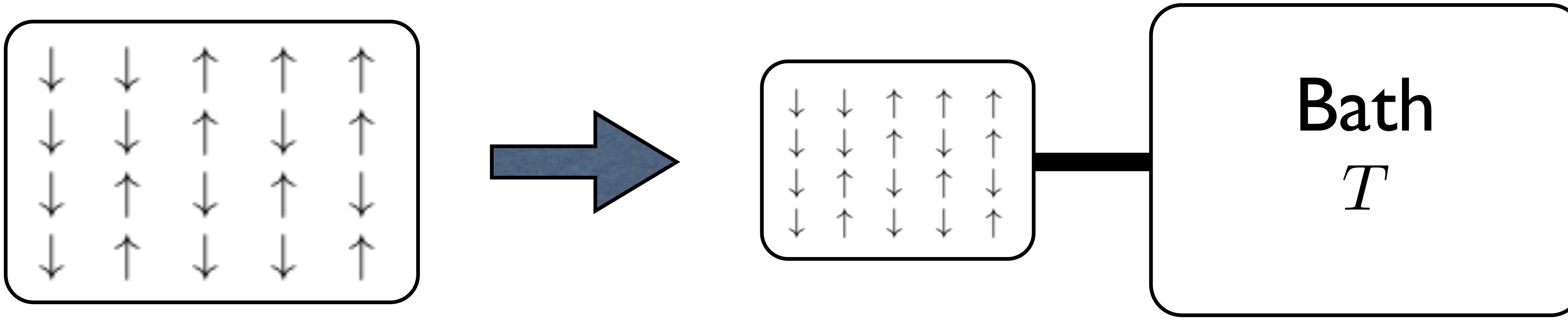
equilibrium driven by reservoir $P_{\text{eq}}(\{\sigma\}, T)$

Example: Metropolis algorithm





Quench dynamics



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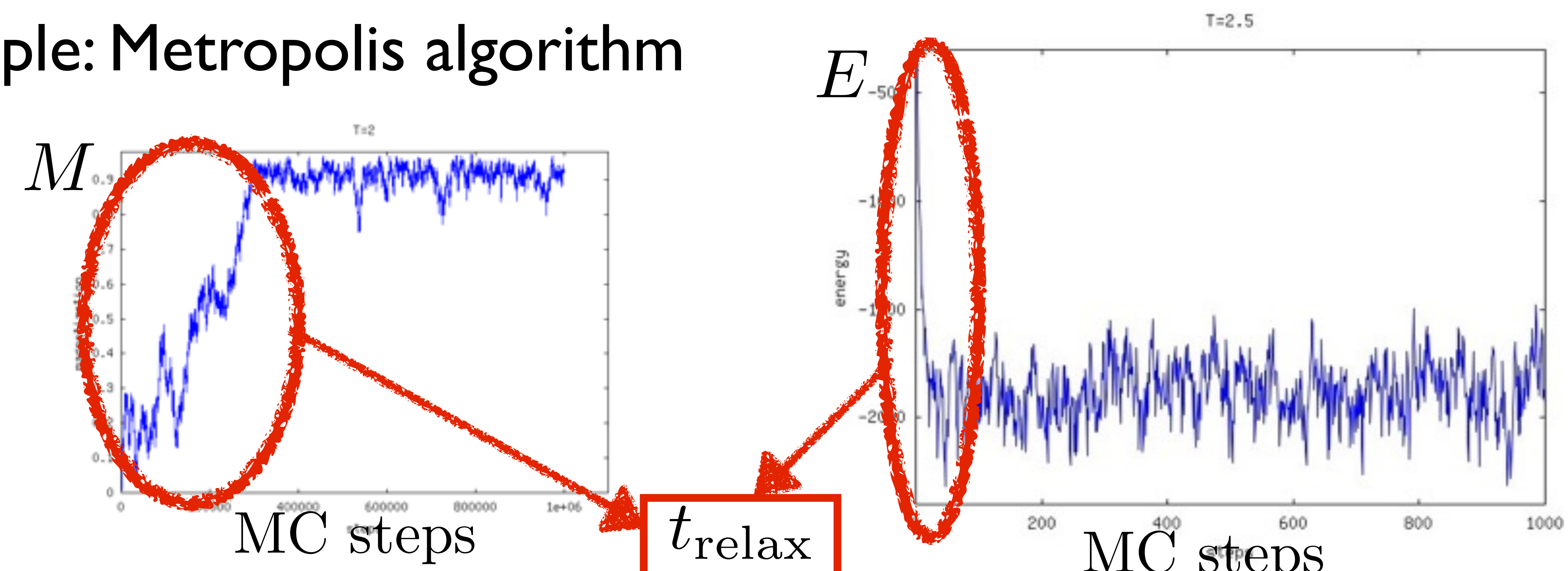
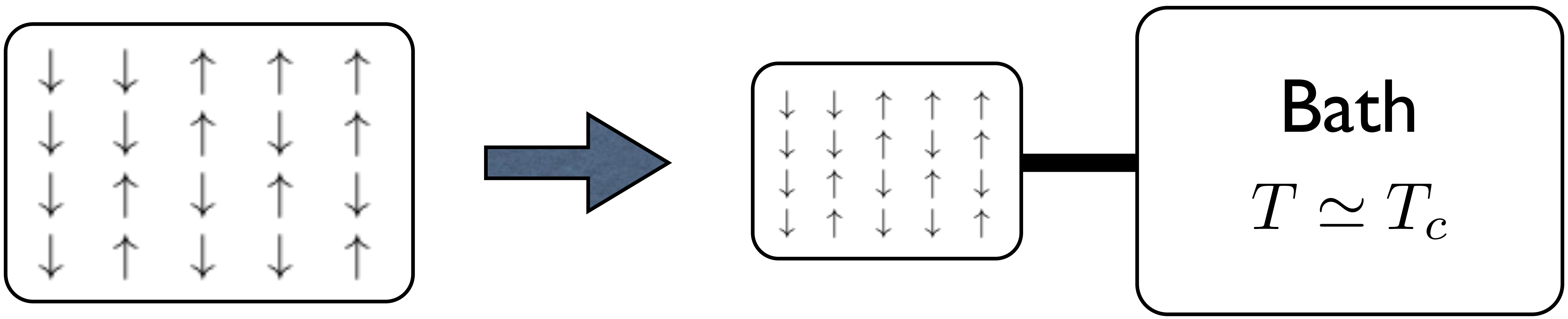


Quench dynamics



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equilibrium driven by reservoir $P_{\mathrm{eq}}(\{\sigma\}, T)$

$T \simeq T_c$ equilibrium: correlation length and time diverges
quench: t_{relax} too! Equilibrium is attained in infinite time

Is there some universal behaviour? $\rightarrow$ **AGING**

Janssen, Schaub, Schmittman '89
rev: Calabrese & Gambassi '05

Aging dynamics

New critical exponents? at criticality: $(\xi = \infty)$

Janssen, Schaub, Schmittman '89
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1) two-time correlation functions

(response function) $$G_R(q,t,t') = q^{-2+\eta+z}\mathcal{G}_R(q^z t, q^z t')$$

(correlation function) $$G_C(q,t,t') = q^{-2+\eta}\mathcal{G}_C(q^z t, q^z t')$$

η, z
equilibrium exponents

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if $t' \ll t$

$$G_C(q,t,t') = q^{-2+\eta}(t/t')^{\theta-1}\widetilde{\mathcal{G}}_C(q^z t, q^z t')$$

$$G_R(q,t,t') = q^{-2+\eta+z}(t/t')^{\theta}\widetilde{\mathcal{G}}_R(q^z t, q^z t')$$

new non-equilibrium exponent θ !

Aging dynamics

New critical exponents? at criticality: $(\xi = \infty)$

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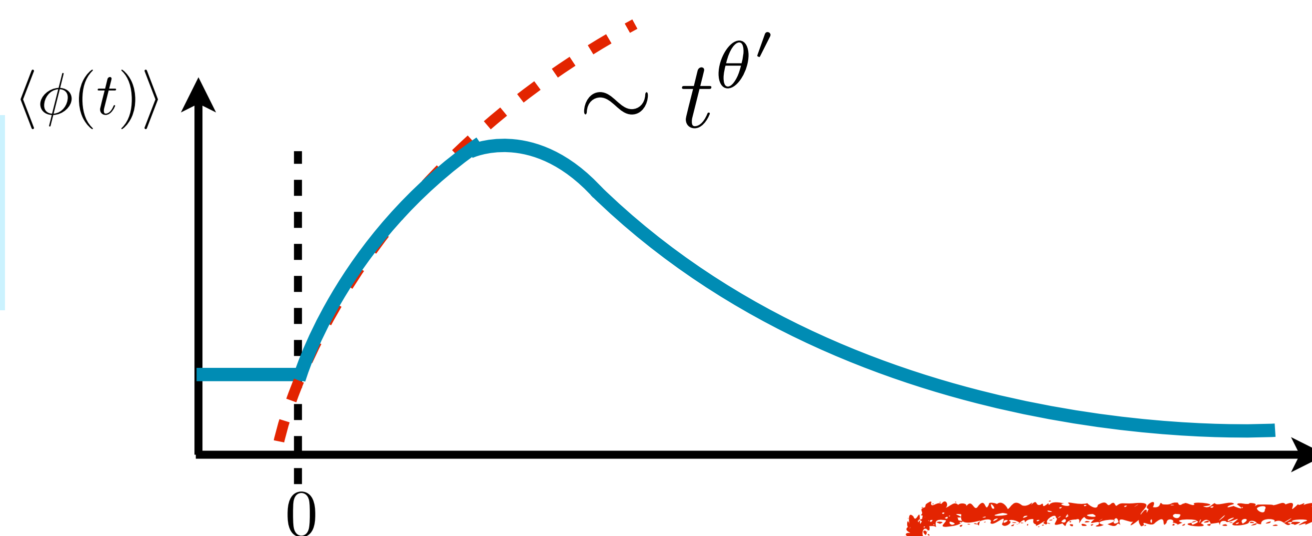
2) magnetization

Modify quench protocol: ext. magnetic field for t<0 → no ext. magnetic field for t>0

$$M(t) = M_0\, t^{\theta'}\, \mathcal{M}(M_0 t^{\theta' + \beta/z\nu})$$

$$\theta' = \theta + (2 - z - \eta)/z$$



Take home message: breaking of time-translational invariance (or fluctuation-dissipation theorem) generates new exponents

Sieberer et al. '13, '14
Marino & Diehl '16

cf. equilibrium systems with spatial boundaries

Theoretical description

criticality → coarse graining: Hohenberg-Halperin models

Hohenberg & Halperin '77

e.g., Ising model:

lattice → continuum

spin variables σ_i → field $\phi(\mathbf{x})$

microscopic Hamiltonian → effective Hamiltonian

Glauber dynamics of Ising model: **Model A**

$$\dot{\phi} = \left[\nabla^2 - r - \frac{g}{6}\phi^2\right]\phi + \zeta$$

Gaussian noise

$$\langle \zeta(\mathbf{r},t)\zeta(\mathbf{r}',t')\rangle = 2T\delta^{(d)}(\mathbf{r}-\mathbf{r}')\delta(t-t')$$

Initial condition ϕ_0 is stochastic: $P[\phi_0] \propto \mathrm{e}^{-\int \tau_0 \phi_0^2/2}$

Initial correlation length: $\tau_0^{-1/2} \to 0$

Response functional

Martin, Siggia, Rose '73
Janssen '76
De Dominicis '76

$$\langle O(t)\rangle = \int O(\phi)\, \langle \delta[\phi - \phi(t)]\rangle_{\zeta,\phi_0} = \int O(\phi) \mathrm{e}^{-S[\phi,\widetilde{\phi}]}$$

- New field: response field $\widetilde{\phi}$
- $S[\phi,\widetilde{\phi}]$ contain info also on initial conditions
- This action is viable for renormalization

Renormalization and critical exponents

Equilibrium: some divergencies, renormalization required

renormalization $r,\ \phi,\ \tilde{\phi}$	$\rightarrow$	critical exps. $\nu,\ \eta,\ z$

Quench: new divergences, additional renormalization required

renormalization $\tilde{\phi}_0$	$\rightarrow$	initial slip exp. θ

value of θ:
- second order in ϵ expansion
- large-N limit of O(N)
- Monte Carlo

rev: Calabrese & Gambassi '05

Functional renormalization group

Modified action: $S_k[\Psi] = S[\Psi] + \Delta S_k[\Psi]$ $\qquad \Psi = (\varphi, \widetilde{\varphi})$

$\Delta S_k[\Psi]$ imposes a IR cutoff parametrized by k

$S_k[\Psi] \Rightarrow \Gamma_k[\Phi]$ effective action (coarse-grained over volume $\sim k^{-d}$)

$$\boxed{\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\int \mathrm{tr}\left[\left(\frac{\delta^2\Gamma_k}{\delta\Phi^{\mathrm{t}}\delta\Phi} + R_k\right)^{-1} \frac{\mathrm{d}R_k}{\mathrm{d}k}\right]}$$

FRG equation

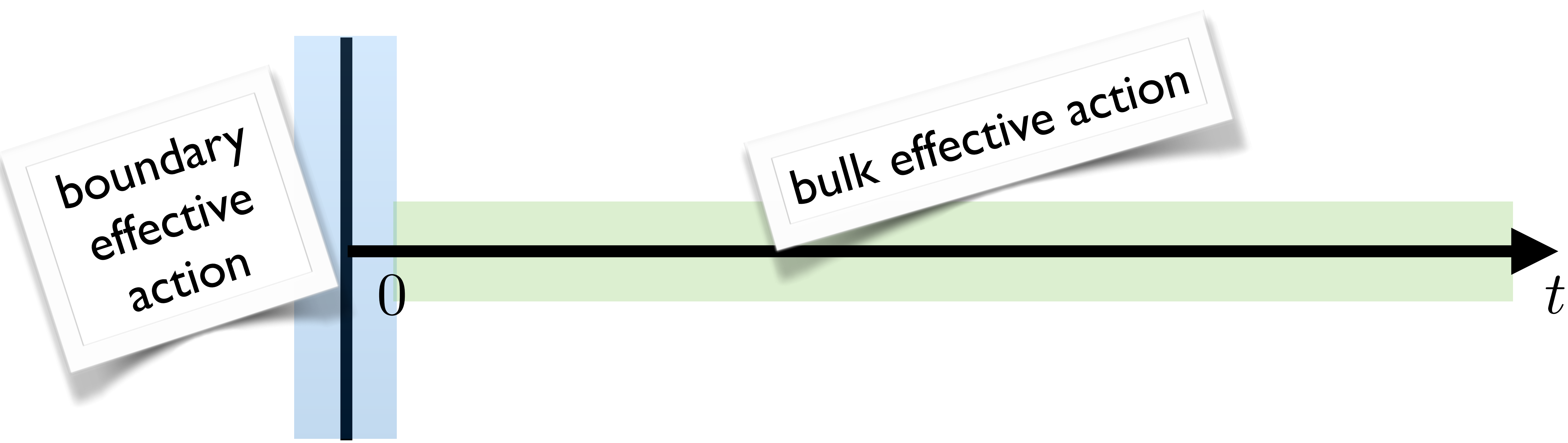
Wetterich '93

Functional renormalization group

How to include the quench? An ansatz for $\Gamma_k[\Phi]$ has to be used.

$$\Gamma[\phi,\widetilde{\phi}] = \underbrace{\Gamma_0[\phi_0,\widetilde{\phi}_0]}_{\text{initial conditions}} + \int_x \underbrace{\vartheta(t-t_0)}_{\text{quench}}\widetilde{\phi}\left(Z\dot{\phi} + K\nabla^2\phi + \underbrace{\frac{\partial\mathcal{U}}{\partial\phi}}_{\text{potential}} - D\widetilde{\phi}\right)$$

Formally similar to a boundary problem:



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$$\Gamma_0 = \int\left(-\frac{Z_0^2}{2\tau_0}\widetilde{\phi}_0^2 + Z_0\widetilde{\phi}_0\phi_0\right)$$

determined from $P[\phi_0]$ and causality arguments

How to perform FRG with a boundary

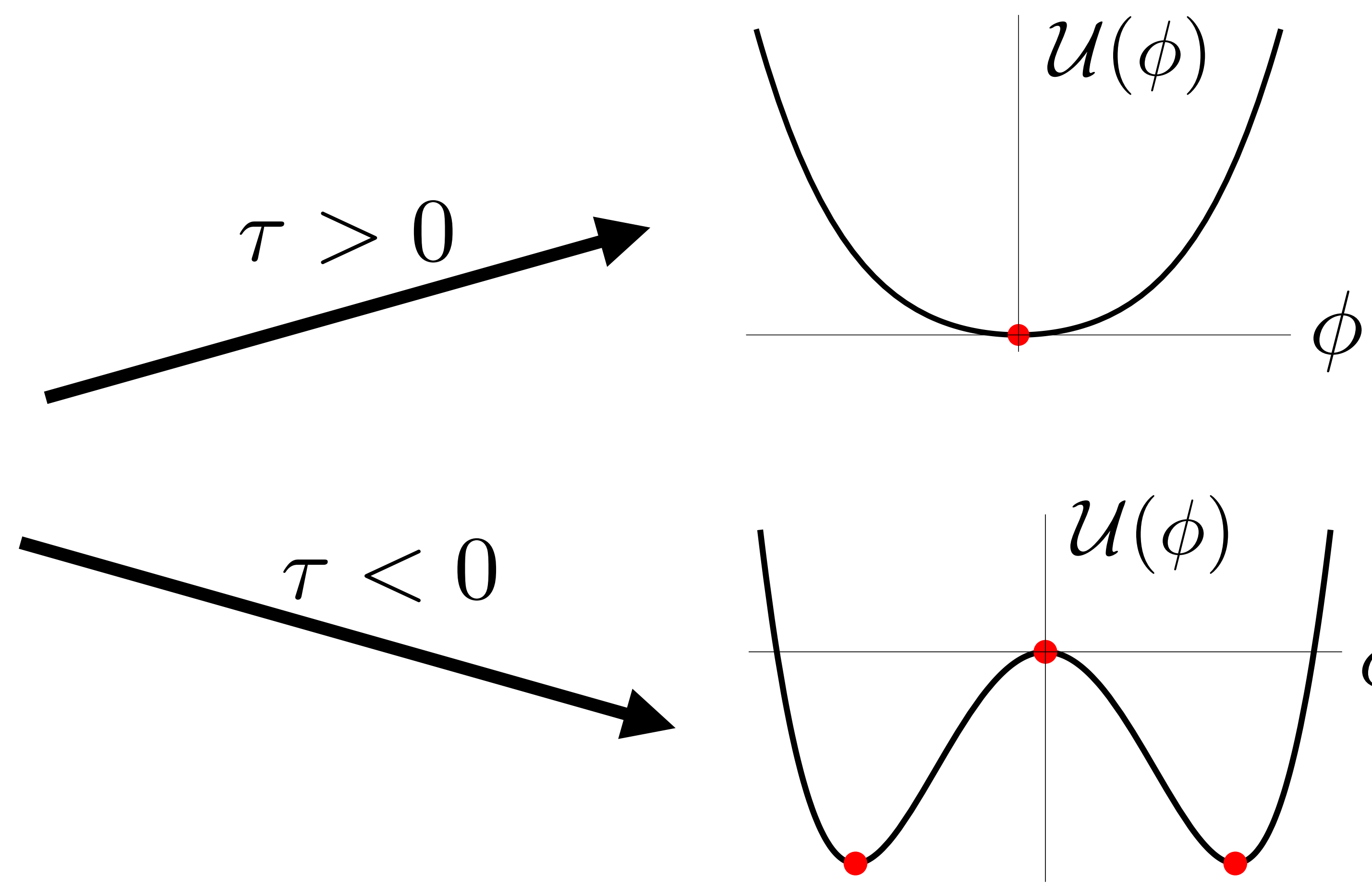
Breaking of TTI → Fourier transform not useful

Our approach: **3 steps**

Step 1: write an ansatz for the potential $\mathcal{U}(\phi)$

Simplest ansatz:

$$\mathcal{U}(\phi) = \frac{\tau}{2}\phi^2 + \frac{g}{4!}\phi^4$$



How to perform FRG with a boundary

Breaking of TTI $\Longrightarrow$ Fourier transform not useful

Our approach: **3 steps**

Step 2: rewrite FRG eq. as follows:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}k} = \frac{1}{2}\int_x \mathrm{tr}\left[\vartheta(t-t_0)G(x,x)\frac{\mathrm{d}R}{\mathrm{d}k}\sigma\right]$$

$$G = \left(\Gamma^{(2)} + R\right)^{-1} = (\underbrace{G_0^{-1}}_{\text{field independent}} - \underbrace{V}_{\text{field dependent}})^{-1}$$

How to perform FRG with a boundary

Breaking of TTI $\Rightarrow$ Fourier transform not useful

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$$G(x,x') = G_0(x,x') + \int_y G_0(x,y)\,V(y)G(y,x')$$

Dyson-like equation

$$= G_0(x,x') + \sum_{n=1}^{+\infty} \underbrace{G_n(x,x')}_{\sim\,\phi^{2n}}$$

Each term can be evaluated!

How to perform FRG with a boundary

Breaking of TTI $\Rightarrow$ Fourier transform not useful

Our approach: **3 steps**

Step 3: replace these terms into FRG eq.

constant value: renormalizes **bulk**

$$\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} \propto \int_{\mathbf{r}} \int_0^\infty \mathrm{d}t\, \widetilde{\phi}\phi\, [1 + f(t)]$$

vanishing function

$$\int_0^\infty \mathrm{d}t\, \widetilde{\phi}(t)\phi(t) f(t) = \sum_{n=0}^\infty c_n \frac{\mathrm{d}^n}{\mathrm{d}t^n} \left[\widetilde{\phi}(t)\phi(t)\right]\bigg|_{t=0}$$

Bulk renormalizes the boundary!

Functional renormalization group

more refined ansatz: $$\mathcal{U} = \frac{g}{4!}(\phi^2 - \phi_m^2)^2 + \frac{\lambda}{6!}(\phi^2 - \phi_m^2)^3$$

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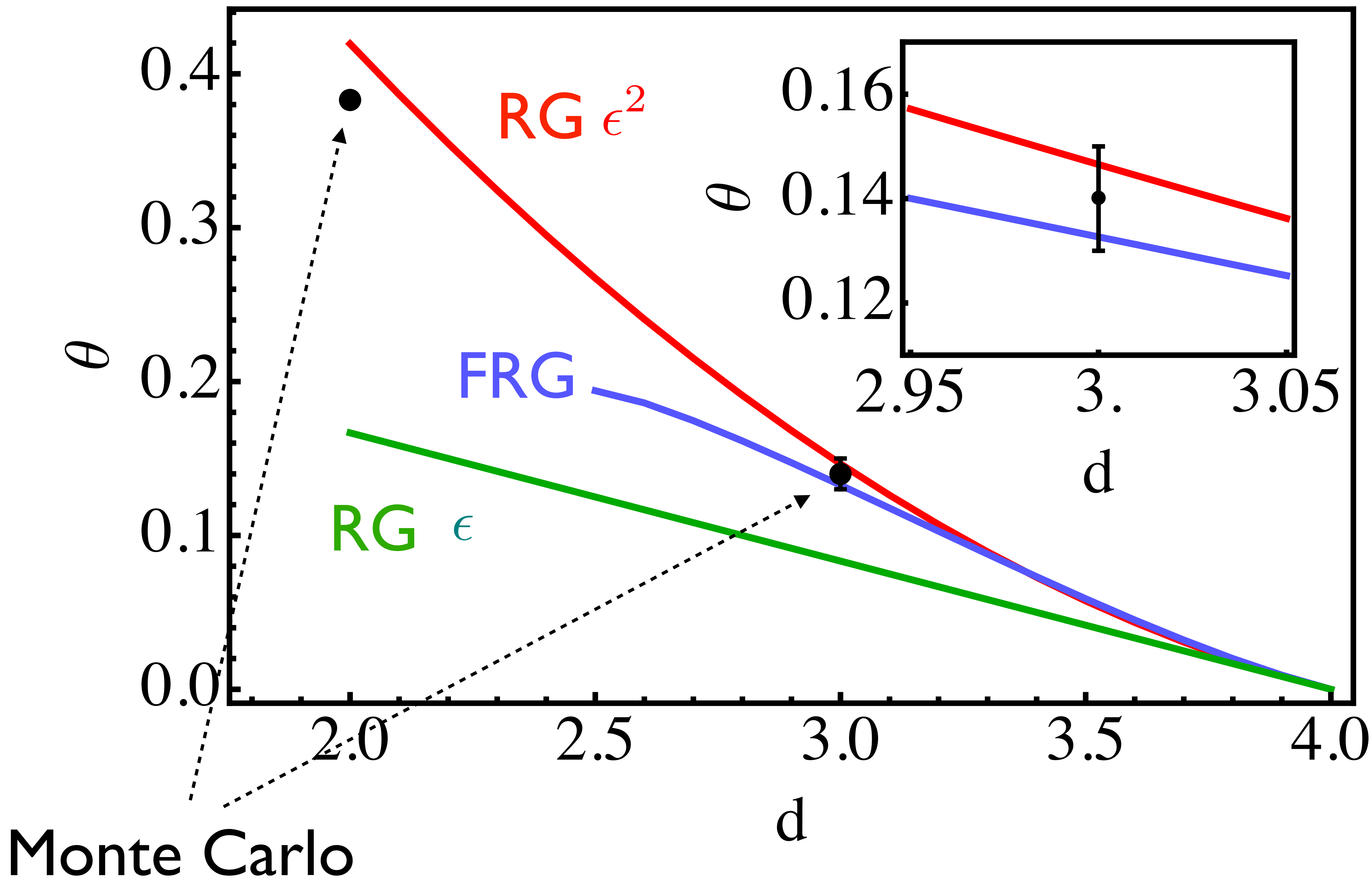

Conclusion & outlook

Summary

- Critical quench generates new critical exponents
- FRG can be used to compute non-equilibrium exponents
- Good agreement with numerics and perturbative RG

Near future...

- Full potential
- Application to quenches in isolated quantum systems
- O(N) and Potts models
- Other HH models (conserved dynamics)

see Jamir Marino's talk!

... far future

- non-thermal fixed points?
- coarsening dynamics
- towards new exp. platforms (cold atoms, exciton-polaritons…)

Thank you for your attention!