

Nonperturbative renormalization group up to 6th order of the derivative expansion

Bertrand Delamotte ¹, Ivan Balog ², Hugues Chaté ³ & Léonie Canet ⁴

¹Université Pierre et Marie Curie, Paris, France

²Institute of Physics, Zagreb, Croatia

³CEA Saclay, Gif-sur-Yvette, France

⁴Université Joseph Fourier, Grenoble, France

ERG2016 - 19.9.2016

Question 1: Does the derivative expansion (DE) converge in the Nonperturbative RG (NPRG) calculations?

Question 2: How to remove/minimize the regulator dependence of the universal quantities?

Question 3: How to remove the dependence of the results on numerical details?

Ising model (ϕ^4 theory)

$$S_0[\phi] = \int_x \left\{ \frac{1}{2}(\nabla\phi_x)^2 + \frac{1}{2}\tau\phi_x^2 + \frac{g}{4!}\phi_x^4 - h_x\phi_x \right\}$$

Infrared regulator $\Delta S_k[\phi] = \frac{1}{2} \int_q R_k(q)\phi_q\phi_{-q}$ to separate fast from slow degrees of freedom

$$Z[h] = \int \mathcal{D}\phi(x) e^{-S_0[\phi] + \int h\phi}$$

$$\Rightarrow Z_k[h] = e^{W_k[h]} = \int \mathcal{D}\phi(x) e^{-S_0[\phi] - \Delta S_k[\phi] + \int h\phi}$$

Formulate the exact flow equation for

$$\Gamma_k[\varphi] + W_k[h] = \int h\varphi - \frac{1}{2} \int_q R_k(q)\varphi_q\varphi_{-q}$$

Effective average action approach ([Wetterich '93](#))

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \int_q \partial_k R_k[q] (\Gamma_k^{(2)}[\varphi] + R_k[q])^{-1}$$

Derivative expansion

A natural approximation for studying long distance behavior

- expand in momentum dependence $\frac{p_i}{k}$
- keep all $\Gamma_k^{(n)}$ vertex functions
- keep the full field dependence

LPA: $\Gamma_k = \int_x \{ U_k[\varphi] + \frac{1}{2}(\partial_\mu \varphi)^2 \}$

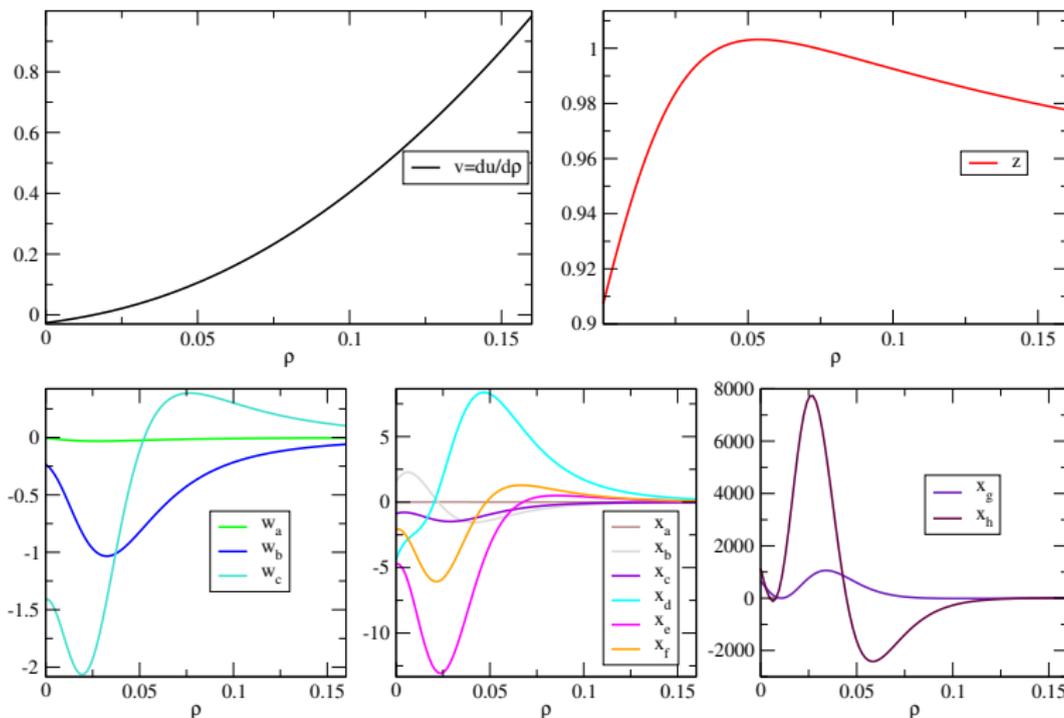
DE2: $\Gamma_k = \int_x \{ U_k[\varphi] + \frac{1}{2} Z_k[\varphi](\partial_\mu \varphi)^2 \}$

DE4 (Canet et al. '03): $\Gamma_k = \int_x \{ U_k[\varphi] + \frac{1}{2} Z_k[\varphi](\partial_\mu \varphi)^2 + \frac{1}{2} W_{a;k}[\varphi](\partial_\mu \partial_\nu \varphi)^2 + \frac{1}{2} W_{b;k}[\varphi] \phi \partial^2 \phi (\partial_\mu \varphi)^2 + \frac{1}{2} W_{c;k}[\varphi](\partial_\mu \varphi)^4 \}$

DE6: additional 8 functions necessary (13 in all)

Methodology

- Problem involving N coupled PDE - solved by directly looking for the fixed point solutions (by adapted Newton-Raphson method)



- Things to be careful about from numerical side:
 - grid range
 - grid resolution
 - numerical integration procedures
 - evaluation of derivatives from discretized functions
 - addition errors (DE6 !)
- compactifying expressions as much as possible
- doing as much partial integrations in q as possible
- ⇒ results independent on numerical details!

Arbitrariness in DE

- choice of the regulator
- choice of the renormalization point

Arbitrariness in DE

- choice of the regulator
- choice of the renormalization point

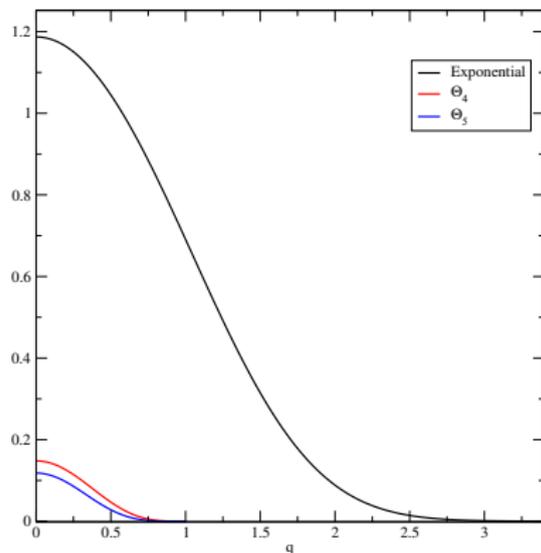
Regulator dependence

- Exponential regulator

$$R_k[q^2] = Z_{0,k} \frac{\alpha q^2}{e^{\frac{q^2}{k^2}} - 1}$$

- “ Θ_n ” regulator:

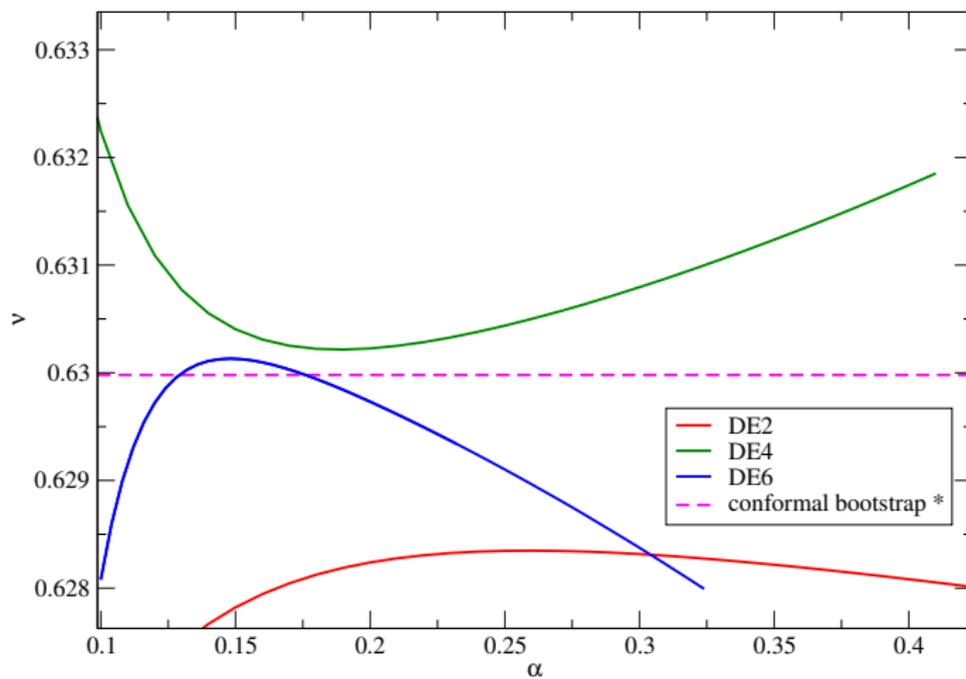
$$R_k[q^2] = Z_{0,k} k^2 \alpha \left(1 - \frac{q^2}{k^2}\right)^n \Theta(q^2 - k^2)$$



How to minimize regulator dependence within an approximation scheme?

Optimize (Canet '05) α !

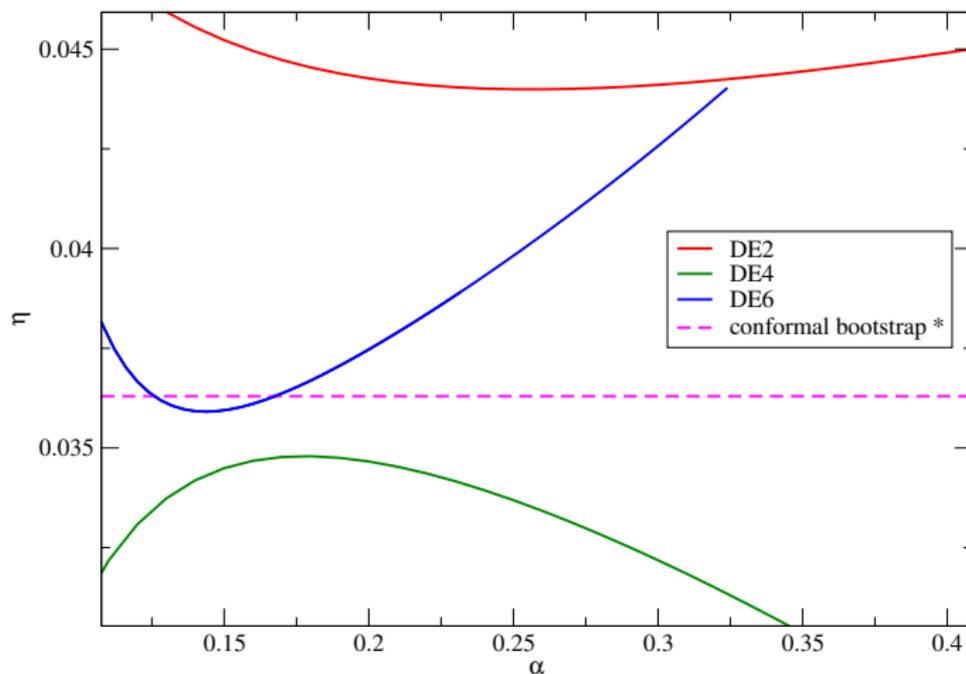
Θ_4 regulator



* - El-Showk et al. J. Stat. Phys. 157, 869 (2014)

Regulator dependence

Θ_4 regulator



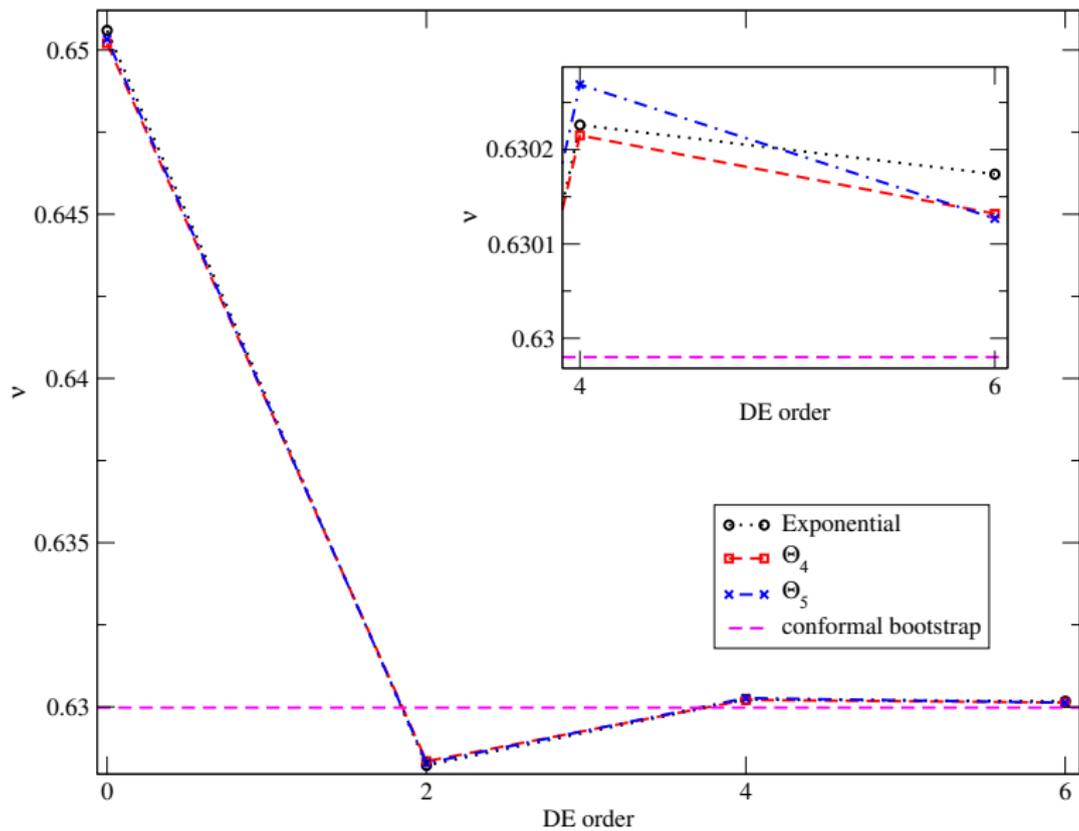
* - El-Showk et al. J. Stat. Phys. 157, 869 (2014)

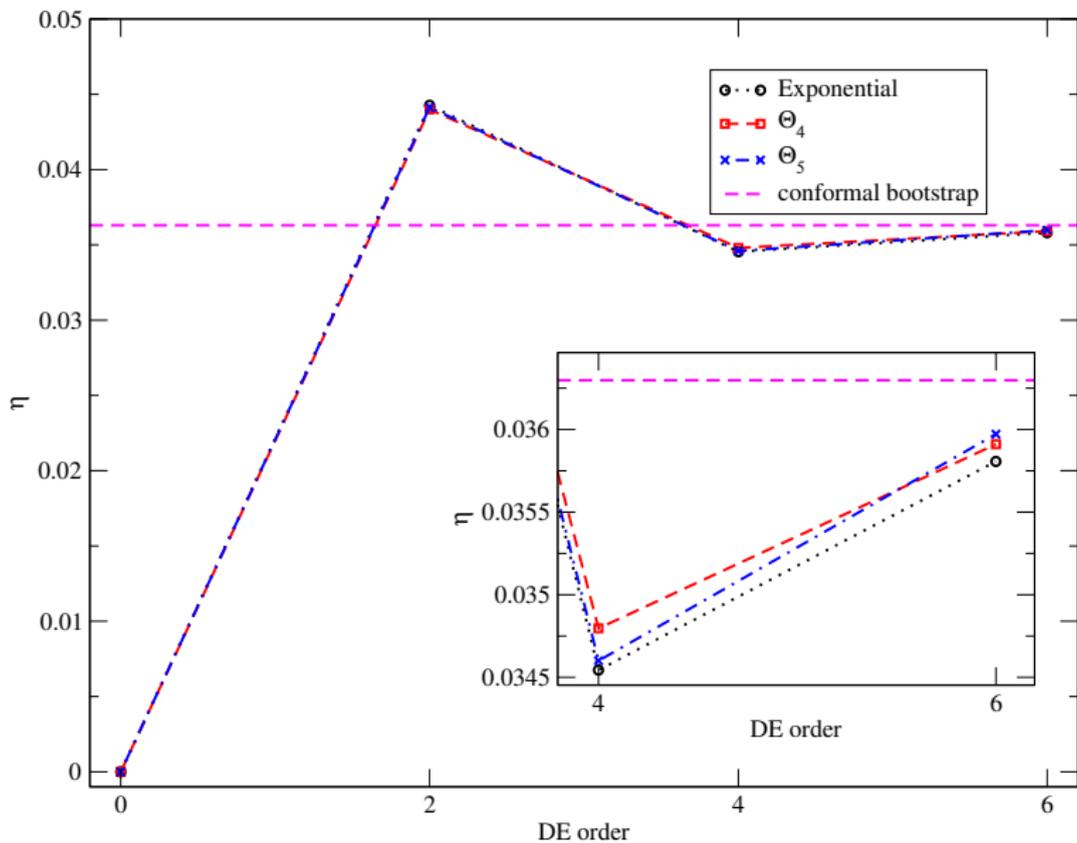
Table: Optimal critical exponent ν in $d = 3$.

	LPA	DE2	DE4	DE6
Θ_4	0.65020	0.62835	0.63022	0.63013
Θ_5	0.65033	0.62829	0.63027	0.63013
Exponential	0.65060	0.62822	0.63027	0.63017

Table: Optimal critical exponent η in $d = 3$.

	LPA	DE2	DE4	DE6
Θ_4	0	0.04399	0.03480	0.03591
Θ_5	0	0.04413	0.03460	0.03597
Exponential	0	0.04426	0.03454	0.03581





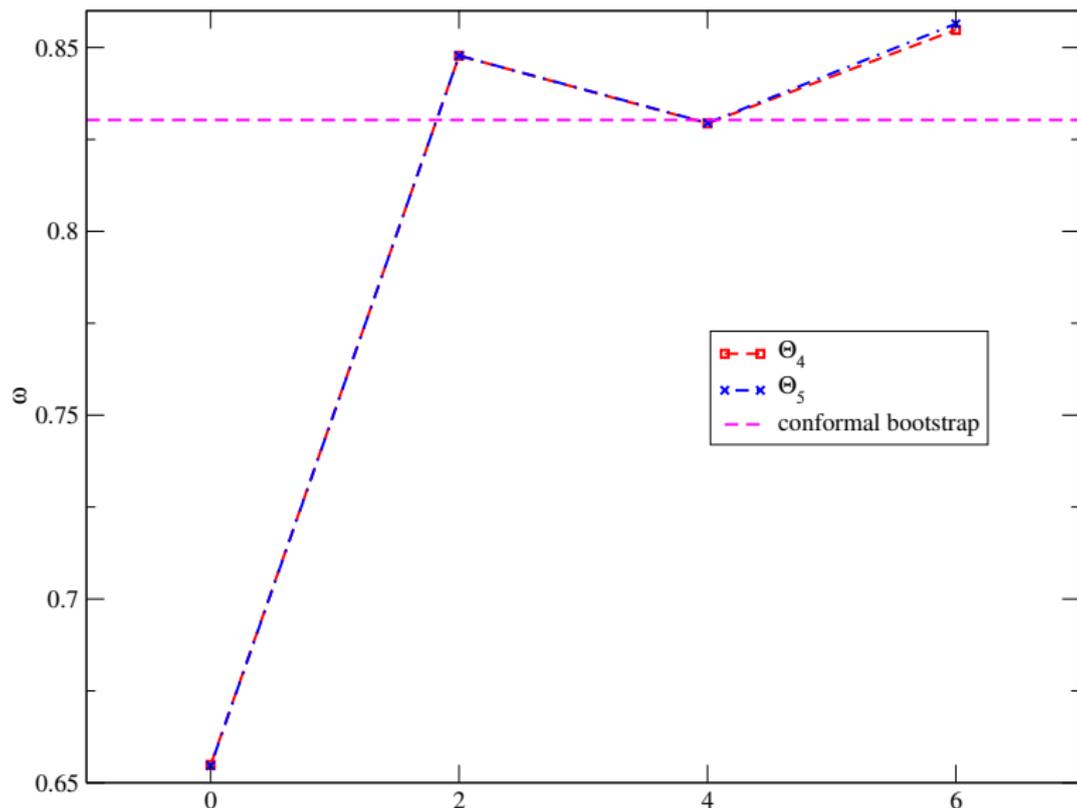
Weak convergence of DE

Lessons learned:

- increasing DE order with fixed regulator parameters gives nonsense
- exponents ν and η always show a minimum sensitivity point in α
- the convexity of $\nu(\alpha)$ and $\eta(\alpha)$ curves alternates with increasing order
- optimizing the regulator at any order of DE gives very consistent values of exponents and they are closer and closer as the order of DE increases
- the difference between $\alpha_{opt,\nu}$ and $\alpha_{opt,\eta}$ diminishes as the order of DE increases

Exponent ω

Problematic!



Reasons and clues why ω is bad

- the convexity of $\omega(\alpha)$ curve does not alternate with increasing order
- truncating terms with high order q^{2n} integrations where $2n > \text{order}$, has **NO** influence on ν or η , **BUT** seems to be crucial for ω
- studying truncating schemes on “to do list”

- DE is in a worse situation a priori
- BMW scheme (Blaizot, Mendes-Galain and Wscherbor '06) much more suited
- preliminary results well off from the exact ones ($\nu = 1.06$, $\eta = 0.238$)
- numerically much more demanding
- in progress..

Table: Comparison with previous results for $d = 3$ Ising model.

	ν	η	ω
present (DE6)	0.63013(1)	0.03592(6)	0.855(3)
MC (Hasenbusch '10)	0.63002(10)	0.03627(10)	0.832(6)
HT (Campostini et al. '02)	0.63012(16)	0.03639(15)	0.825(50)
ϵ^5 (Guida and Zinn-Justin '98)	0.63050(250)	0.03650(500)	0.814(18)
conf. boot. (El-Showk et al. '14)	0.62999(5)	0.03631(3)	0.8303(18)