Generation of structure in spin- and mass-imbalanced Fermi gases

Jens Braun

TU Darmstadt

8th International Conference on the Exact Renormalization Group

20/09/2016

Deutsche Forschungsgemeinschaft DFG

LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

HIC for FAIR Helmholtz International Center

SFB 1245

From balanced to imbalanced gases - why?

- Condensed-matter point of view:

Superconductors form
when spin-up and spin-down electrons pair up

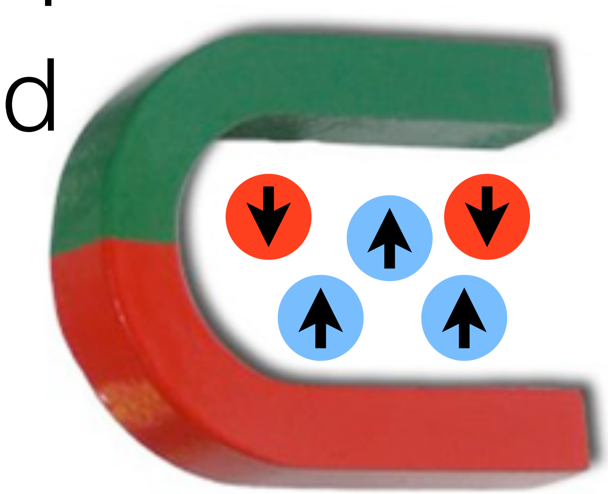
PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. Bardeen, L. N. Cooper,† and J. R. Schrieffer‡
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 8, 1957)

- What happens when a magnetic field is applied?

Superconductivity is destroyed
for a large enough field



A NOTE ON THE MAXIMUM CRITICAL FIELD OF HIGH-FIELD SUPERCONDUCTORS

B. S. Chandrasekhar
Westinghouse Research Laboratories, Pittsburgh, Pennsylvania
(Received July 23, 1962)

UPPER LIMIT FOR THE CRITICAL FIELD IN HARD SUPERCONDUCTORS

A. M. Clogston
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received August 9, 1962)

From balanced to imbalanced gases - why?

- *Related problem:*

Superfluids form

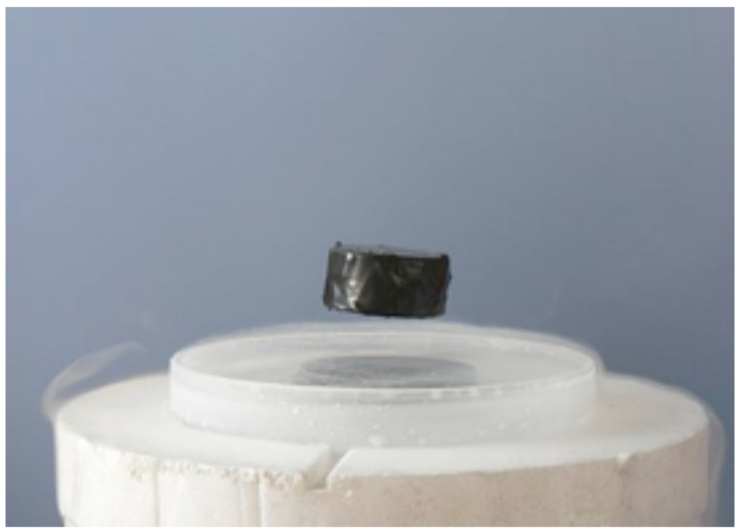
when spin-up and spin-down *fermions* pair up

- What happens when a *spin imbalance* is introduced?

Relevance for various research fields

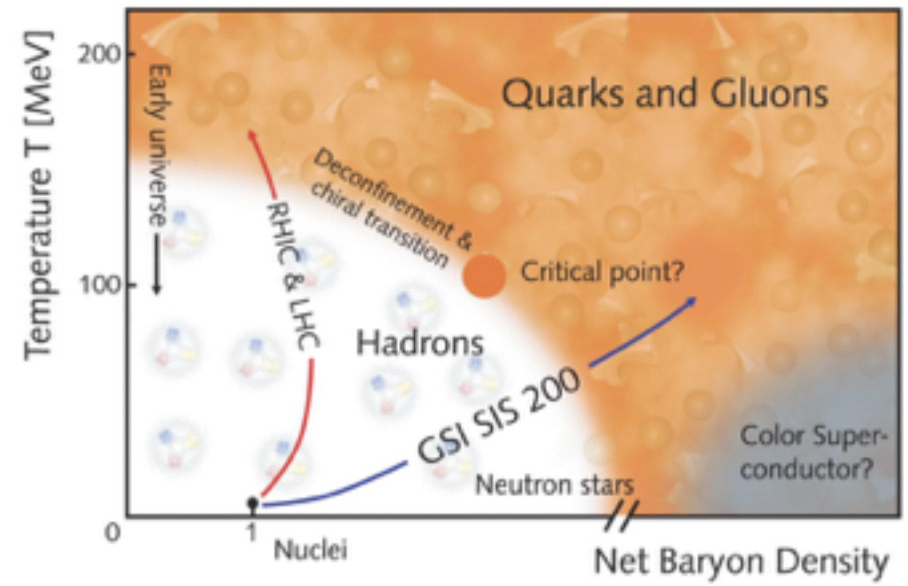
[clearly incomprehensive compilation, guided by the speaker's personal interests]

Superconductivity



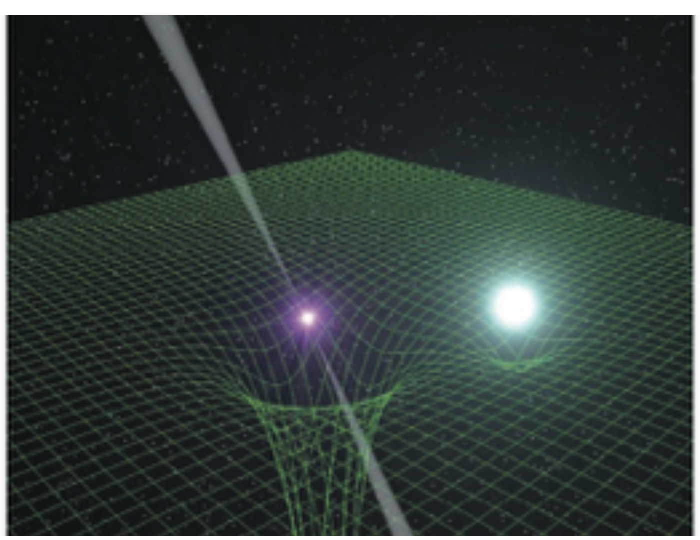
[high-T superconductor, wikipedia.org]

Quantum Chromo Dynamics



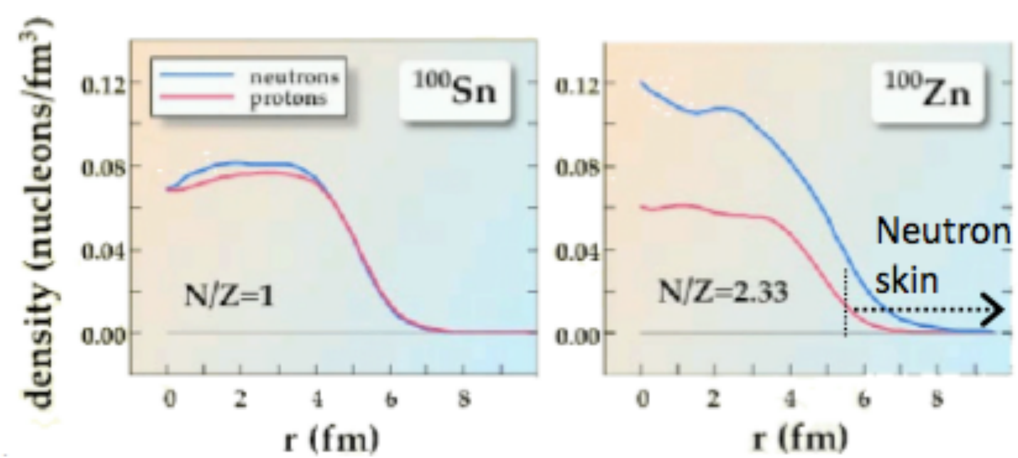
[FAIR, gsi.de]

Astrophysics



[Demorest *et al.*, Nature (2010); Antoniadis *et al.*, Science (2013)]

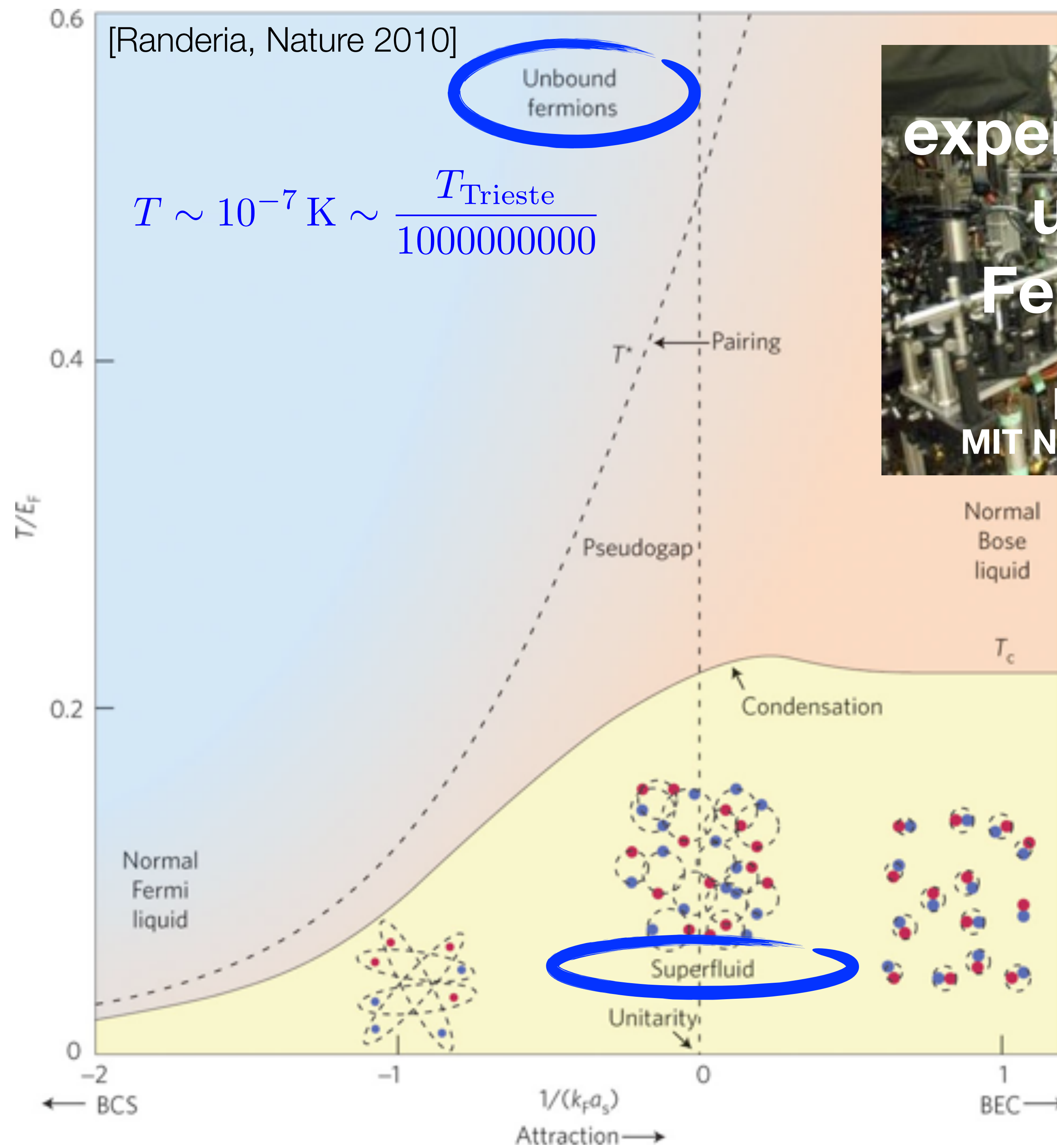
Nuclear Physics



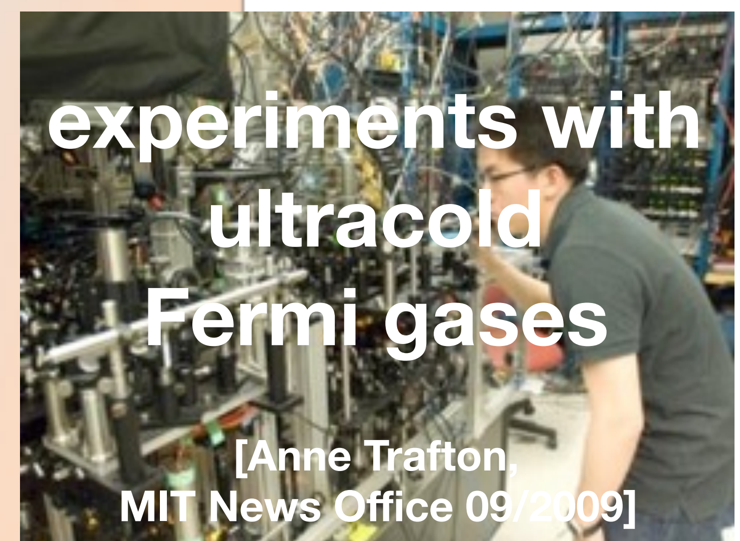
[Dobaczewski *et al.* '94]

Lab for controlled studies of imbalanced systems?

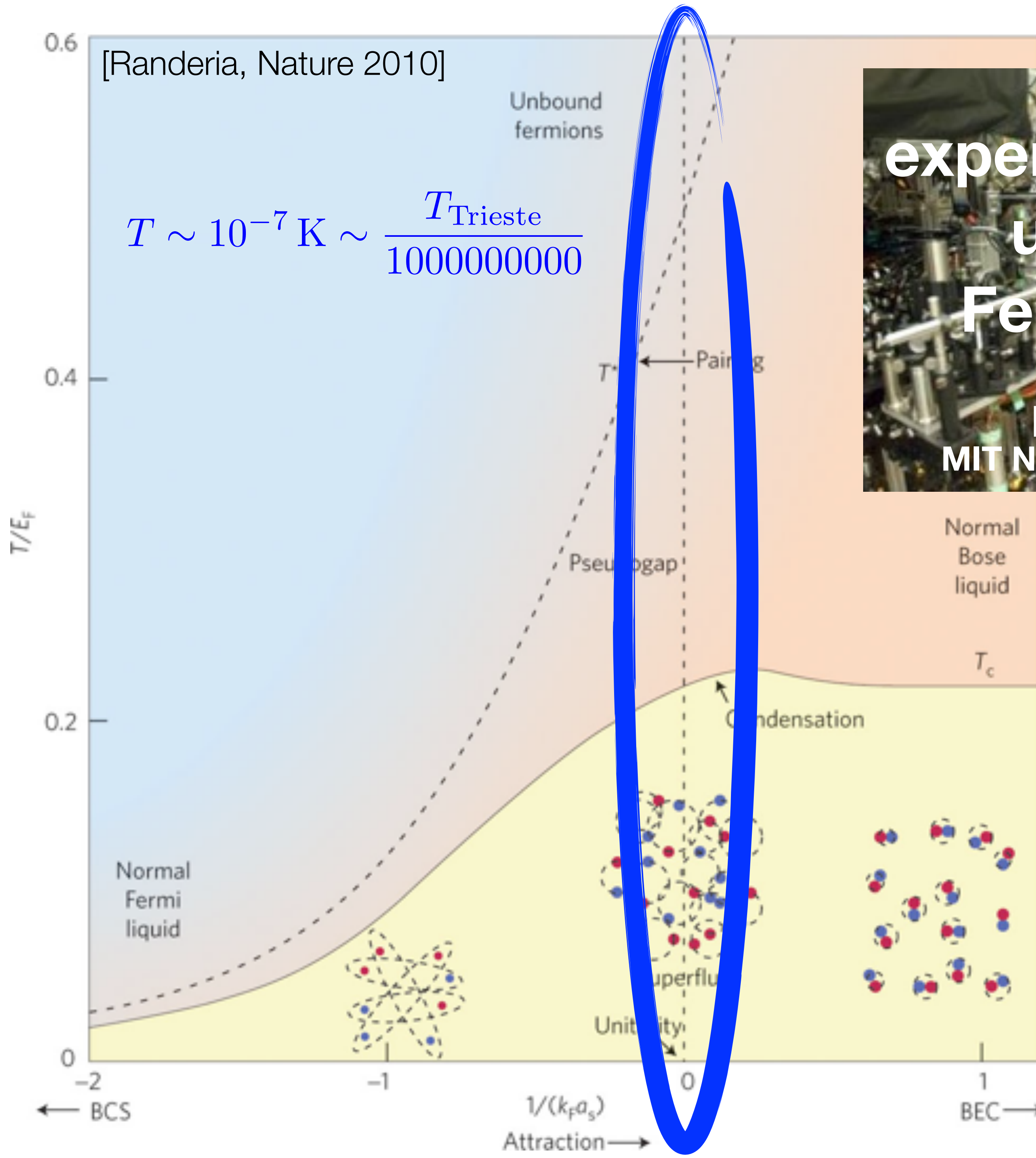
Lab for controlled studies of imbalanced systems?



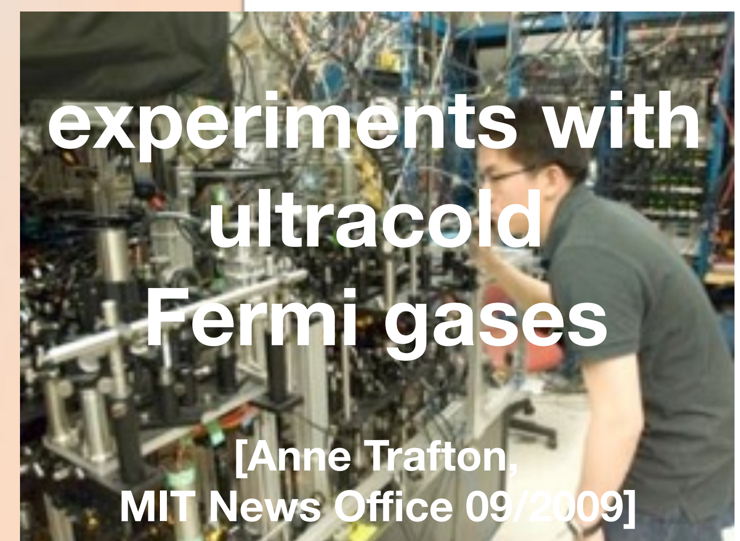
$$T \sim 10^{-7}\,\mathrm{K} \sim \frac{T_{\mathrm{Trieste}}}{1000000000}$$



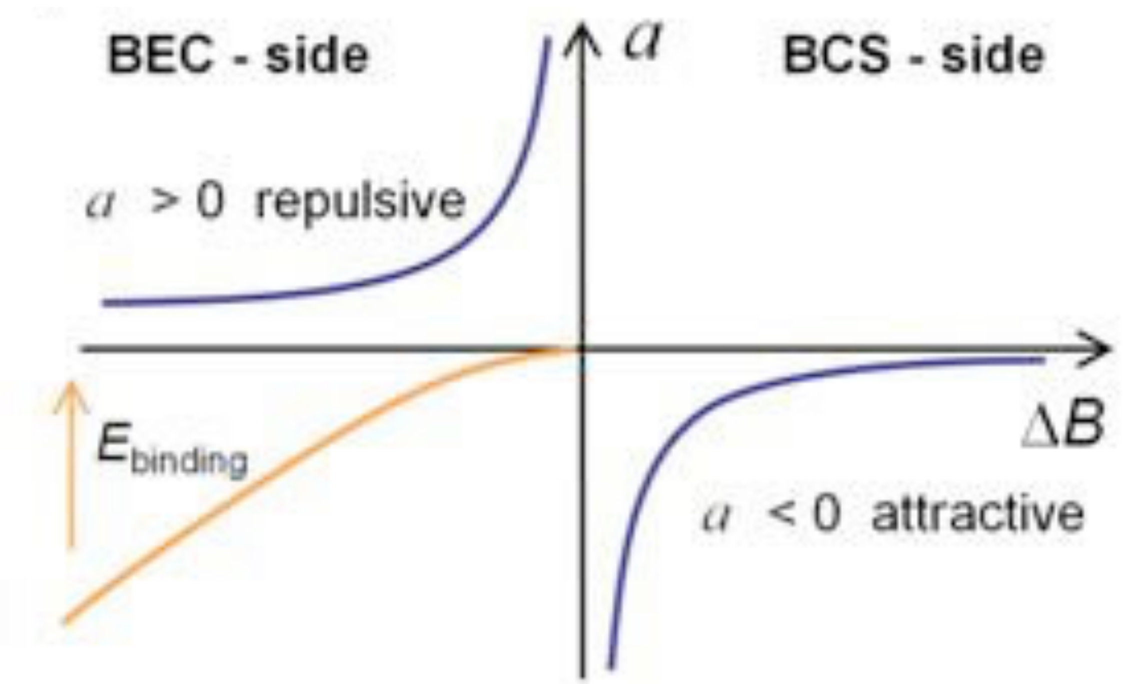
Lab for controlled studies of imbalanced systems?

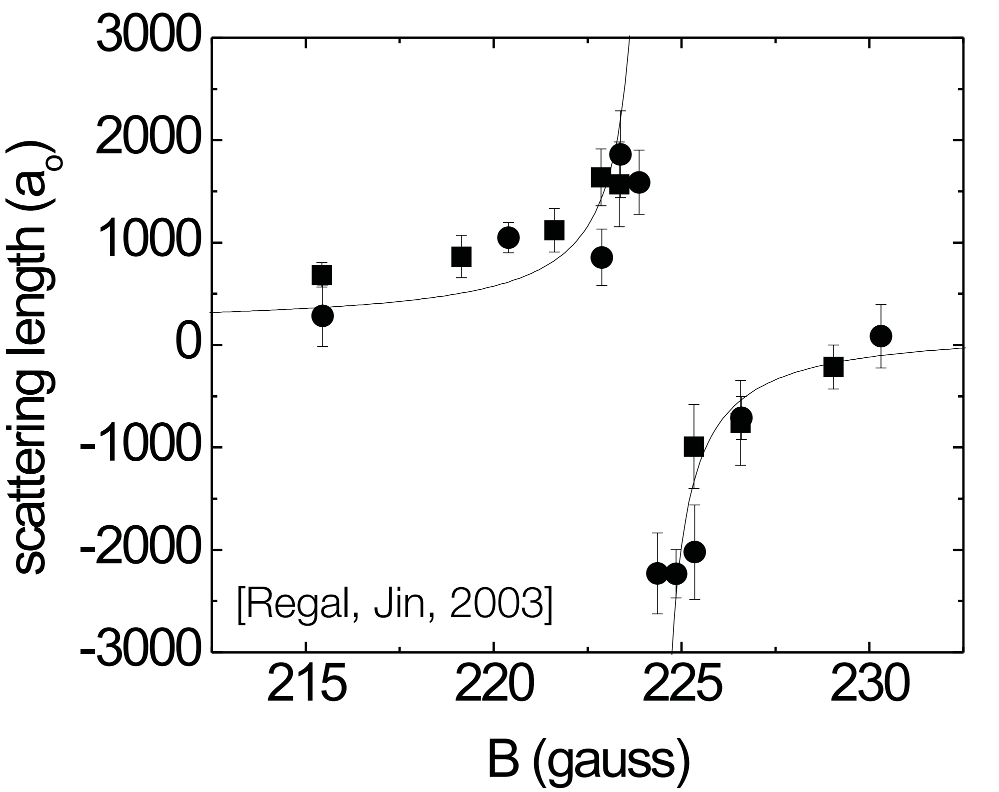


$$T \sim 10^{-7}\,\mathrm{K} \sim \frac{T_{\mathrm{Trieste}}}{1000000000}$$



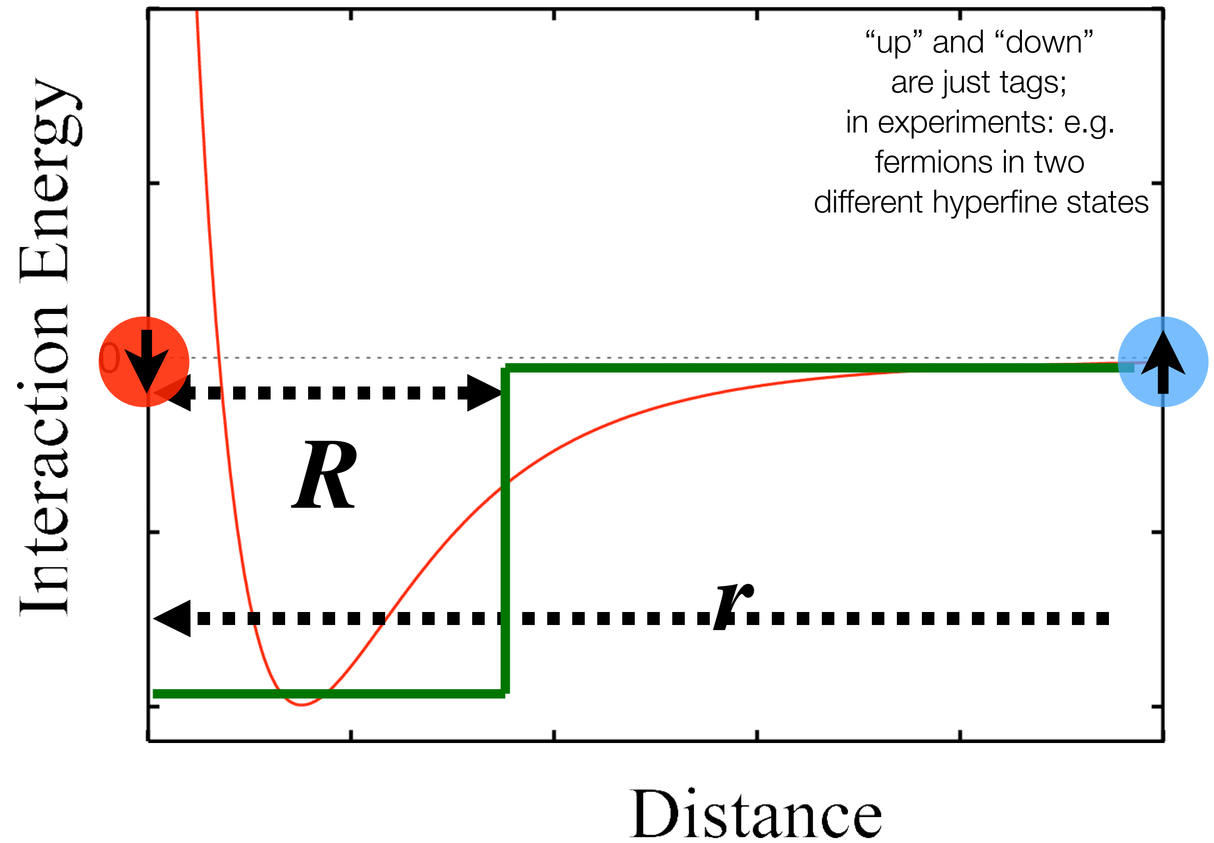
Unitary Regime





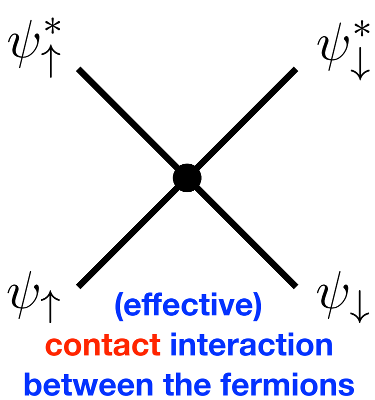
- s-wave scattering length is tunable by an external magnetic field (Feshbach resonance)
- interaction strength is directly related to the s-wave scattering length a_s

Unitary Regime



- limit of infinite scattering length a_s defines a universal regime:

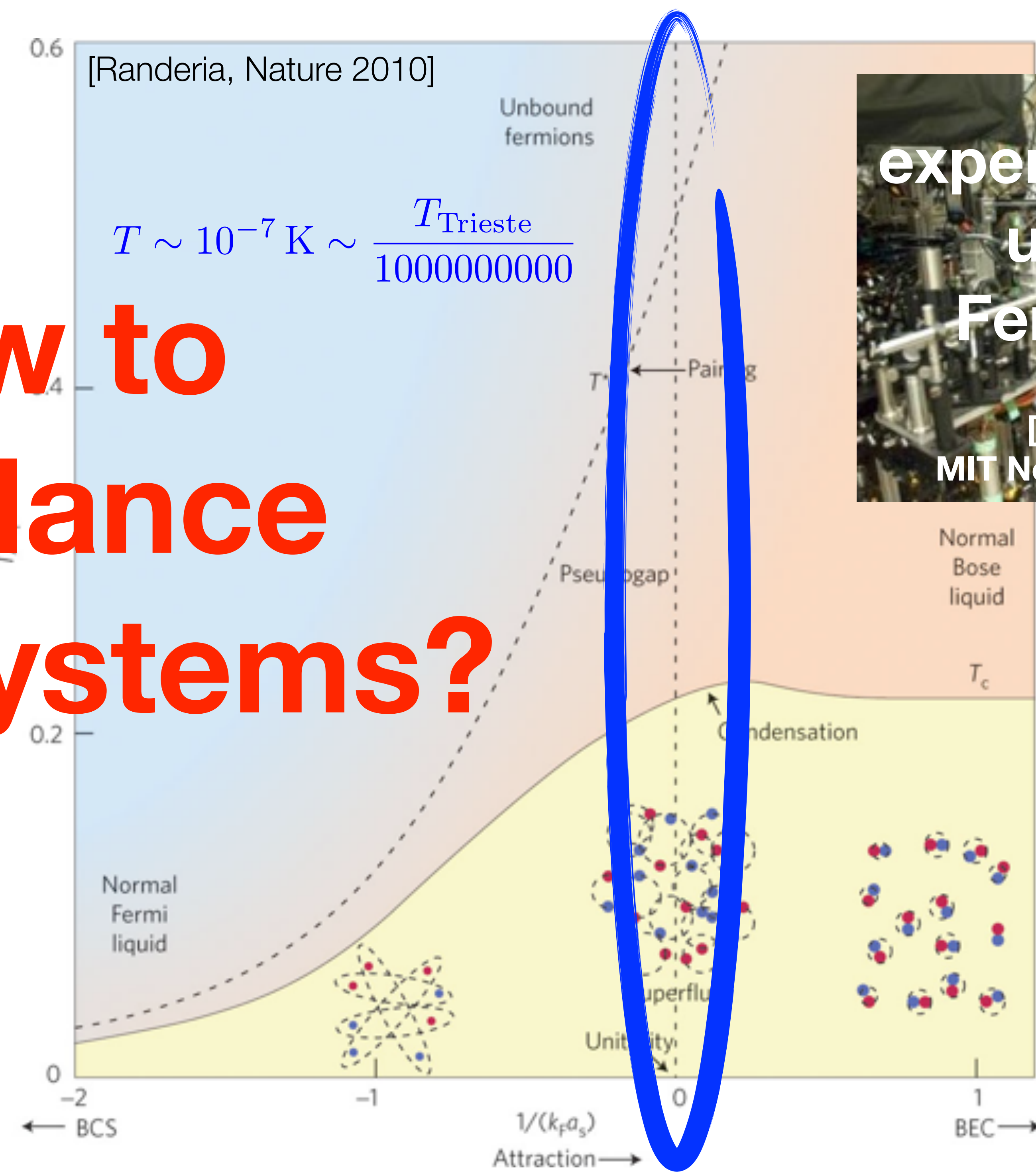
$$0 \approx \frac{1}{|a_s|} \ll \sqrt{\epsilon_{\mathrm{F}}} \sim k_{\mathrm{F}} \sim \frac{1}{r} \ll \frac{1}{R} \approx \infty$$



- universal properties:

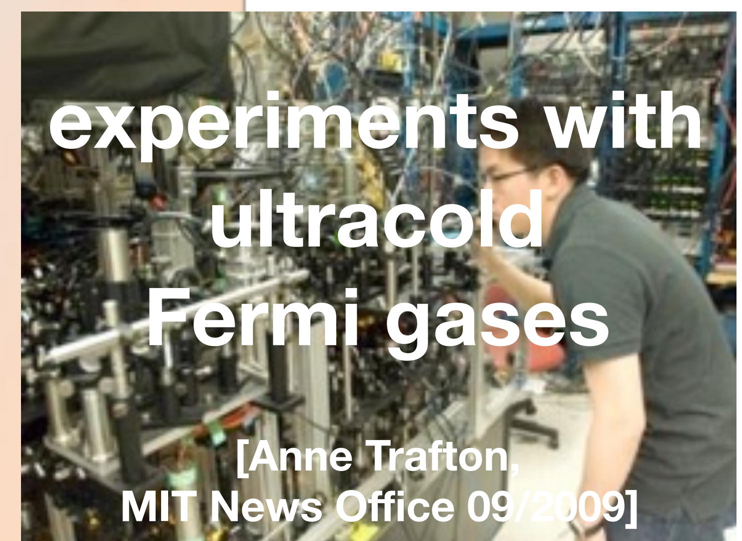
$$E/N, T_c, \cdots \propto \text{universal const(s).} \times \epsilon_{\mathrm{F}}$$

Lab for controlled studies of imbalanced systems?

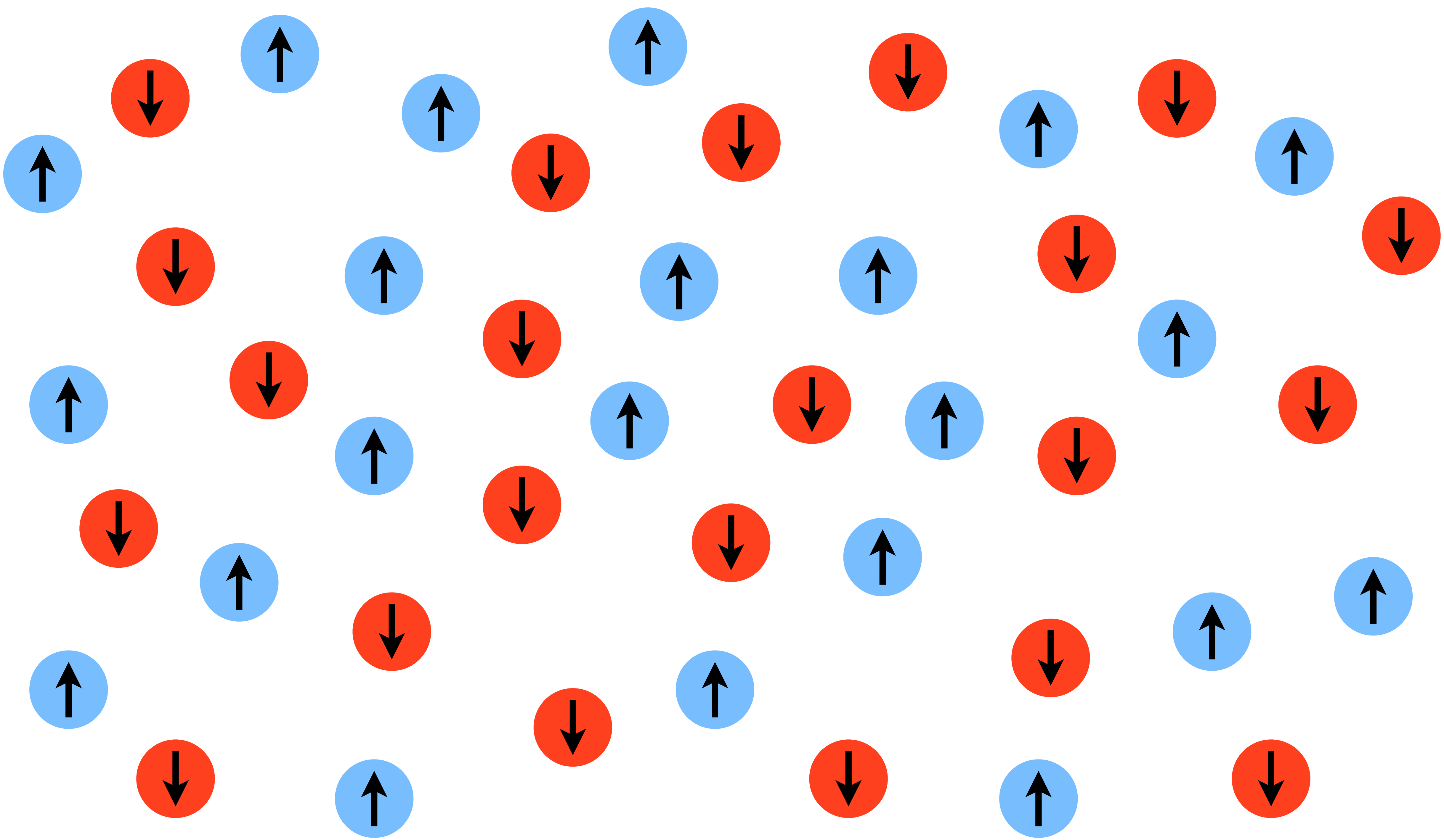


$$T \sim 10^{-7}\,\mathrm{K} \sim \frac{T_{\mathrm{Trieste}}}{1000000000}$$

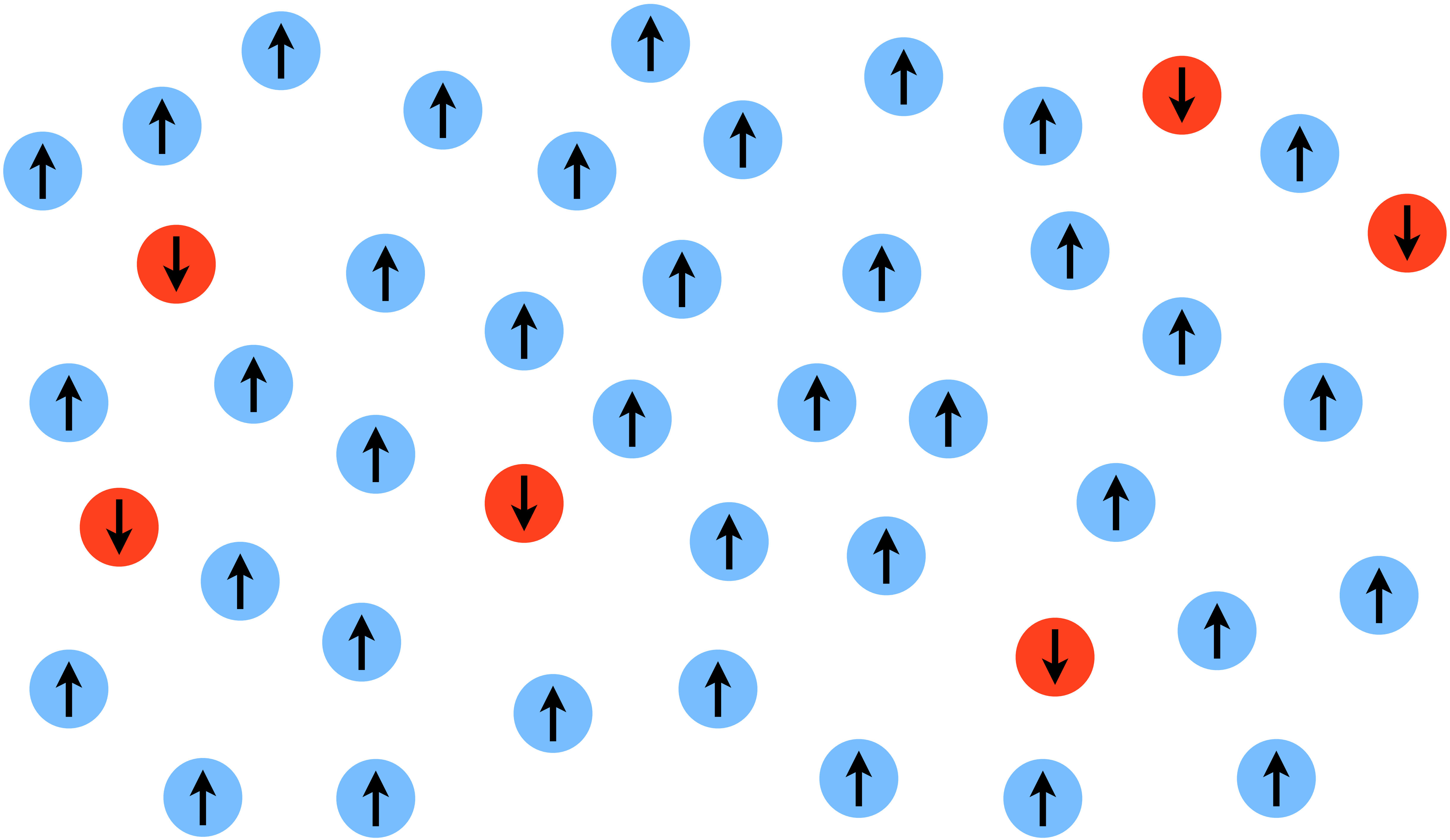
How to imbalance these systems?



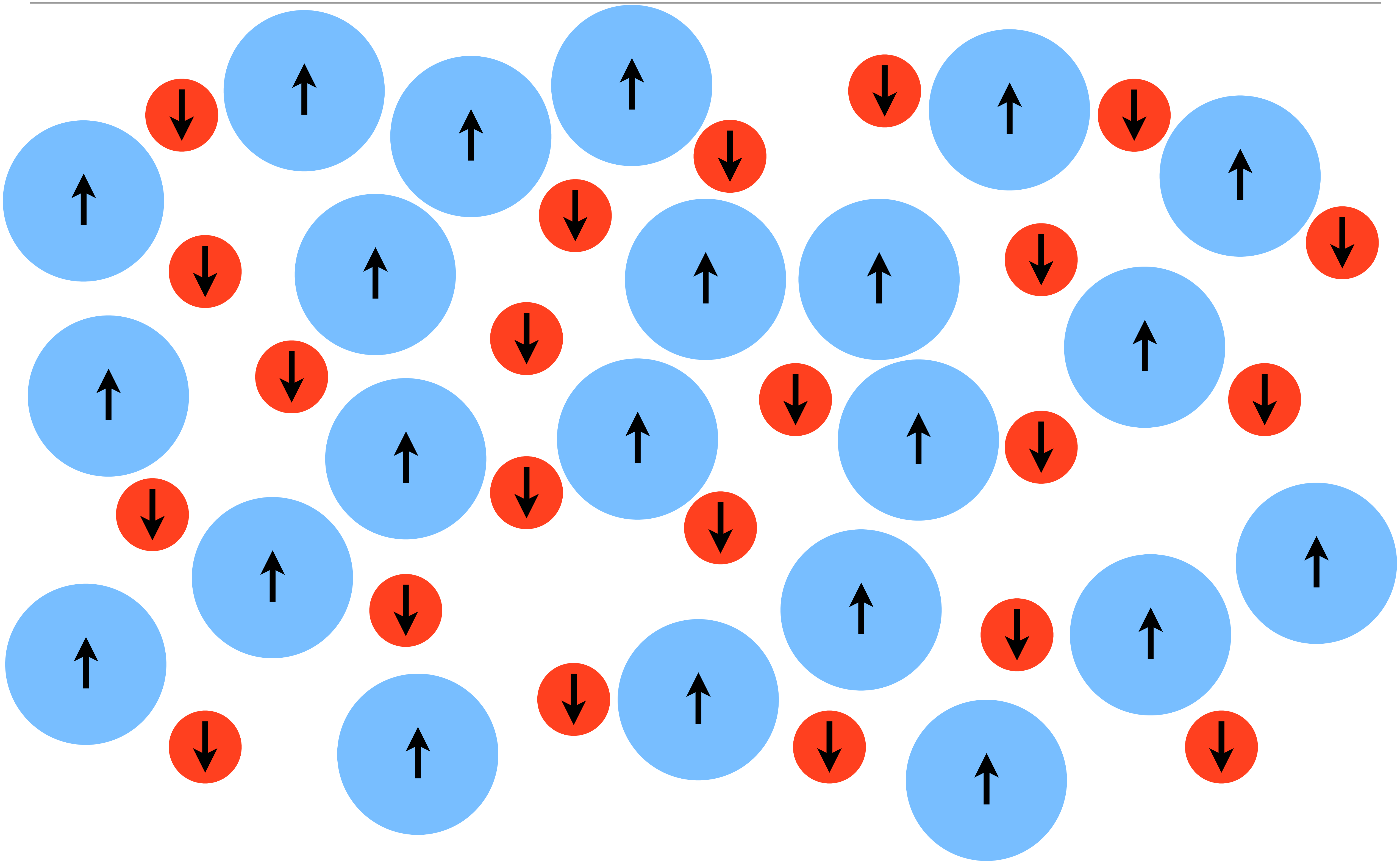
“Balanced” unitary Fermi gas



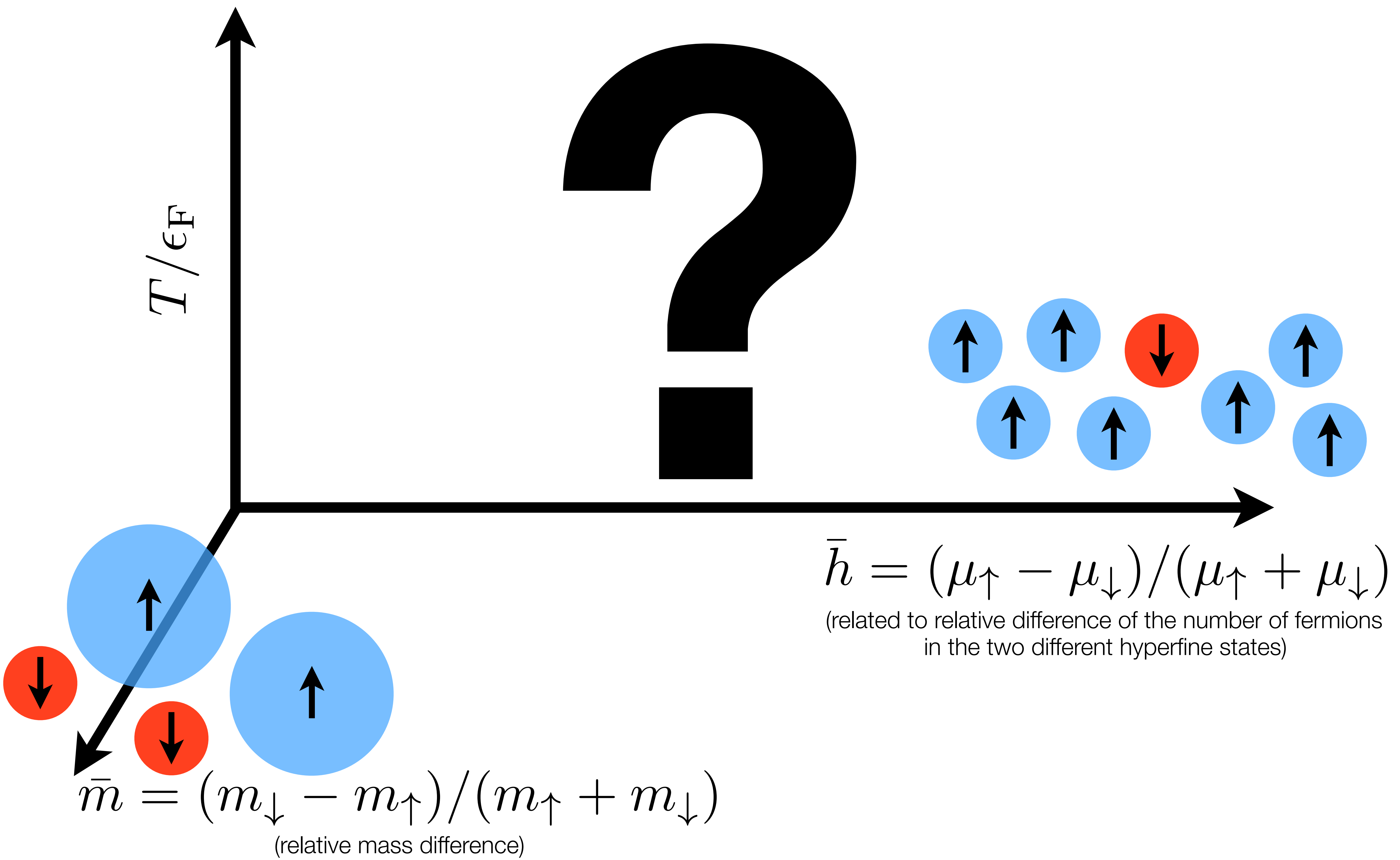
Deforming Fermi gases: spin imbalance



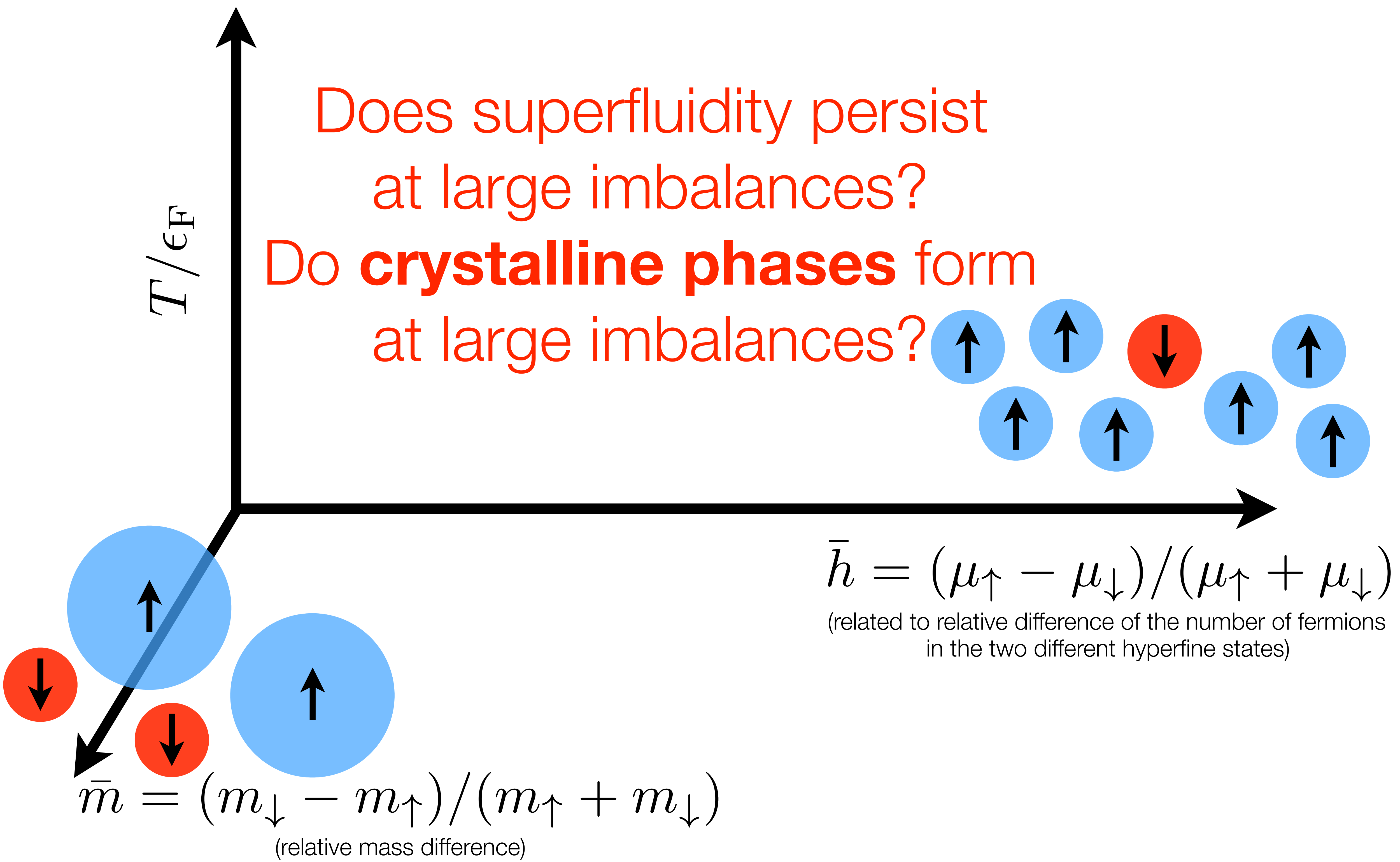
Deforming Fermi gases: mass imbalance



Phase diagram of imbalanced unitary Fermi gases



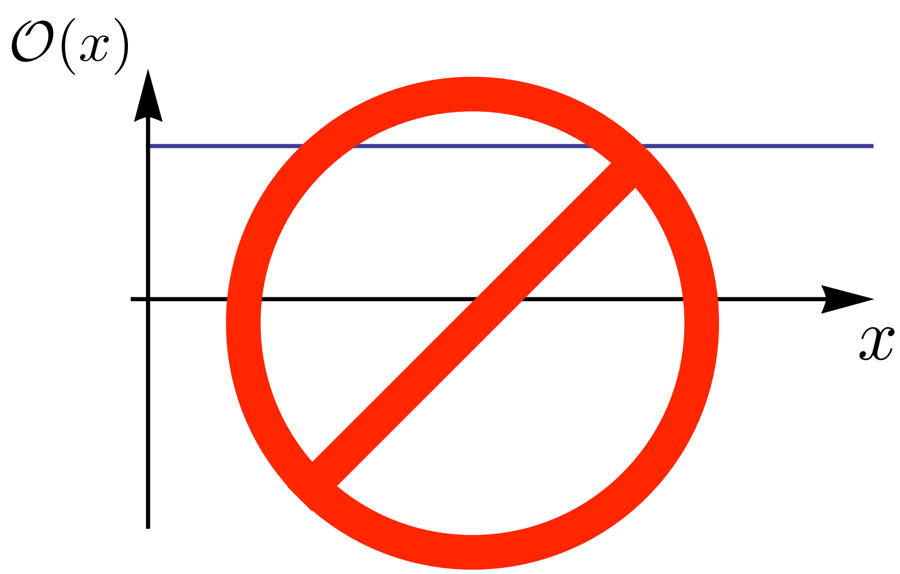
Phase diagram of imbalanced unitary Fermi gases

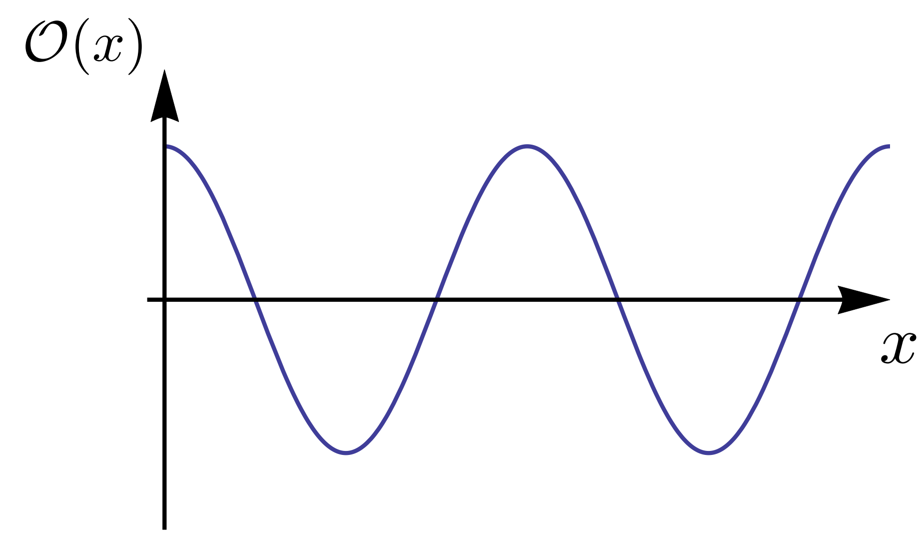


“Crystal”-type phases in Fermi gases

- what does “crystal” mean in this case?

Ground-state configuration (and observables) are **not** homogeneous but, e.g., periodic functions of the space-like coordinates





How to study imbalanced Fermi gases?

Schrödinger equation

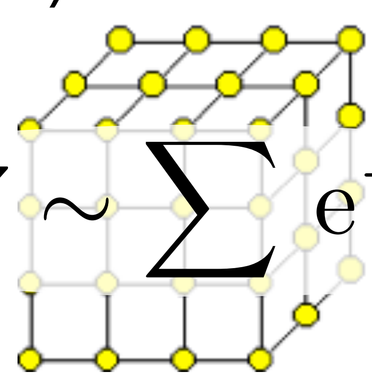
$$\hat{H}|\psi\rangle = \mathrm{i}\partial_t|\psi\rangle$$

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

[Wetterich '92]

(lattice) Monte Carlo



$$Z \sim \sum \mathrm{e}^{-S}$$

Density Functional Theory

$$\psi_{\mathrm{gs}}(\vec{x}_1, \ldots, \vec{x}_N) \Leftrightarrow n_{\mathrm{gs}}(\vec{x})$$

[Hohenberg & Kohn '65]

How to study imbalanced Fermi gases?

Schrödinger equation

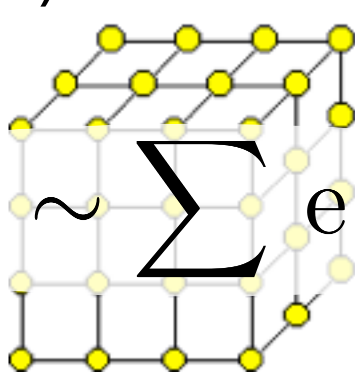
$$\hat{H}|\psi\rangle = \mathrm{i}\partial_t|\psi\rangle$$

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

[Wetterich '92]

(lattice) Monte Carlo



$$Z \sim \sum \mathrm{e}^{-S}$$

Density Functional Theory

$$\psi_{\mathrm{gs}}(\vec{x}_1, \ldots, \vec{x}_N) \Leftrightarrow n_{\mathrm{gs}}(\vec{x})$$

[Hohenberg & Kohn '65]

How to study imbalanced Fermi gases?

Schrödinger equation

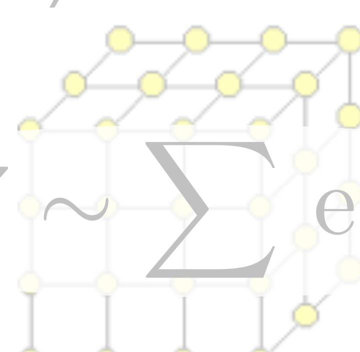
$$\hat{H}|\psi\rangle = \mathrm{i}\partial_t|\psi\rangle$$

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

[Wetterich '92]

(lattice) Monte Carlo



$$Z \sim \sum e^{-S}$$

Density Functional Theory

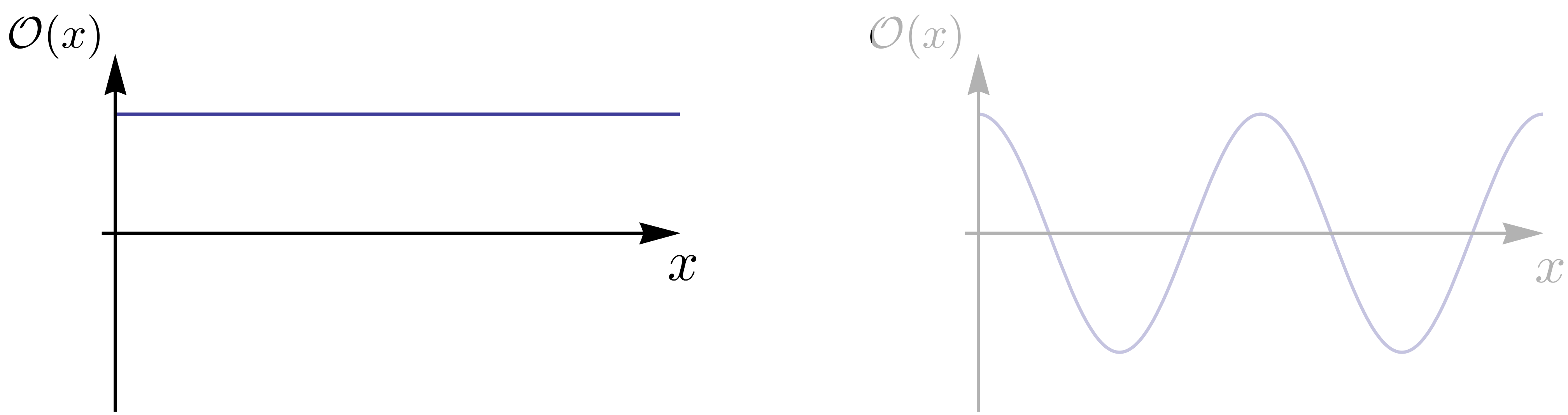
$$\psi_{\mathrm{gs}}(\vec{x}_1, \ldots, \vec{x}_N) \Leftrightarrow n_{\mathrm{gs}}(\vec{x})$$

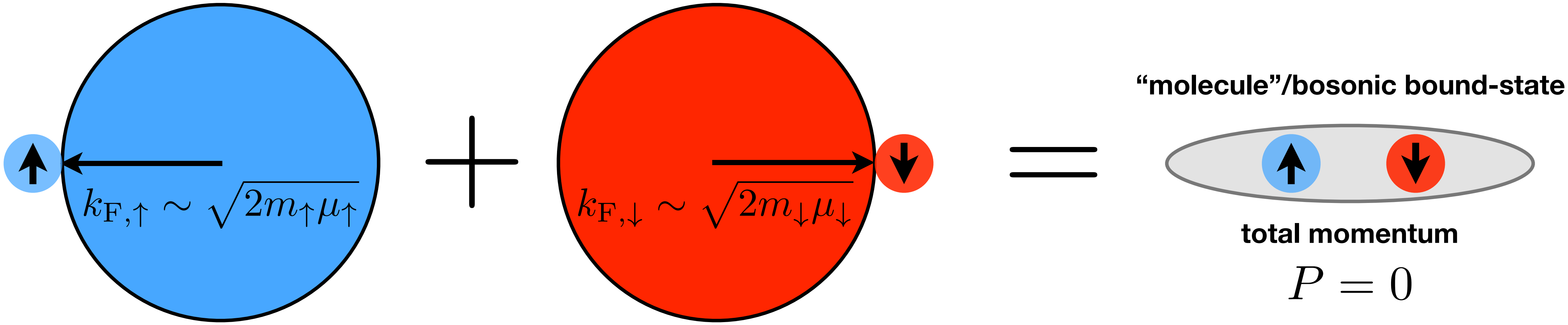
[Hohenberg & Kohn '65]

“Crystal”-type phases in Fermi gases

- pairing in a nutshell (**BCS**):

[Bardeen, Cooper, Schrieffer ’57]

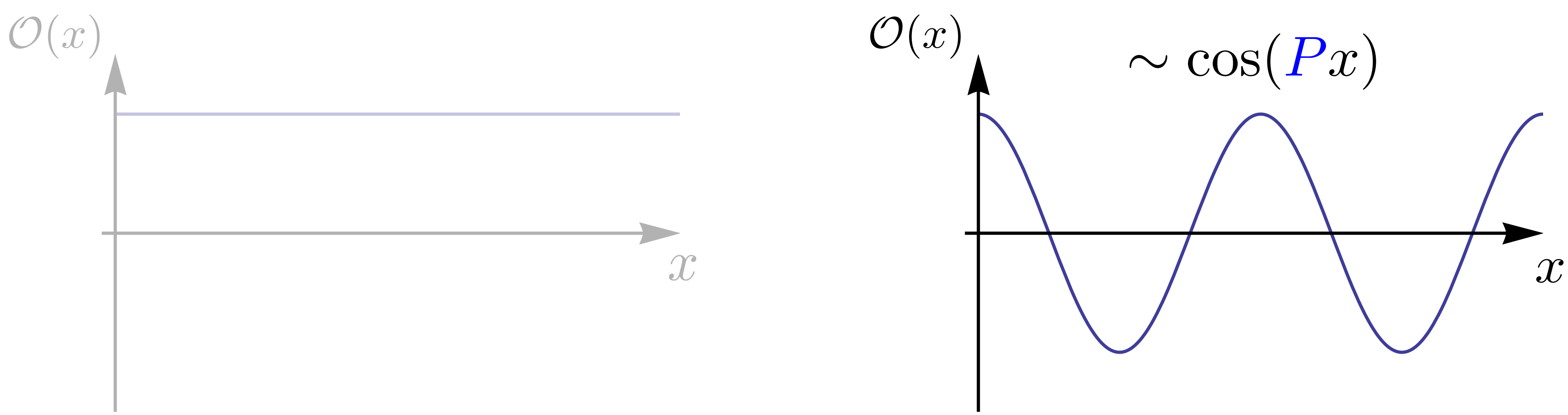


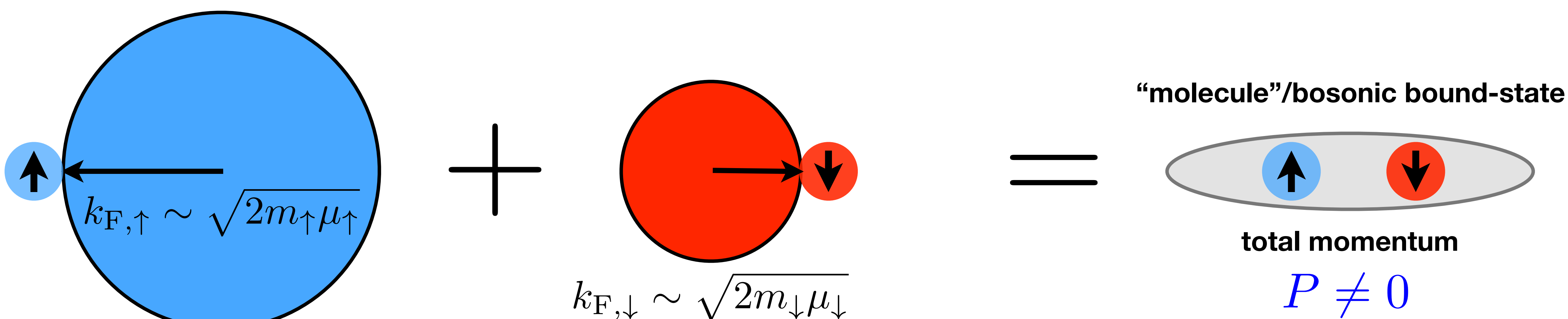


“Crystal”-type phases in Fermi gases

- pairing in a nutshell (**FFLO**):

[Fulde, Ferrell ’64; Larkin, Ovchinnikov ’64]

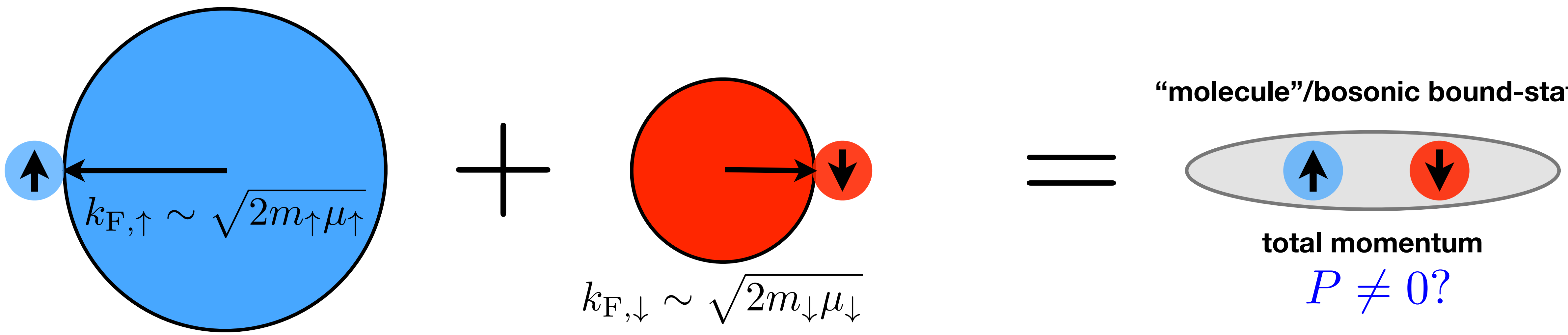




“In-medium” two-body Schrödinger equation

[Roscher, JB, Drut ’14, ‘15]

- study pairing in the presence of “inert” Fermi spheres:



- Schrödinger equation:

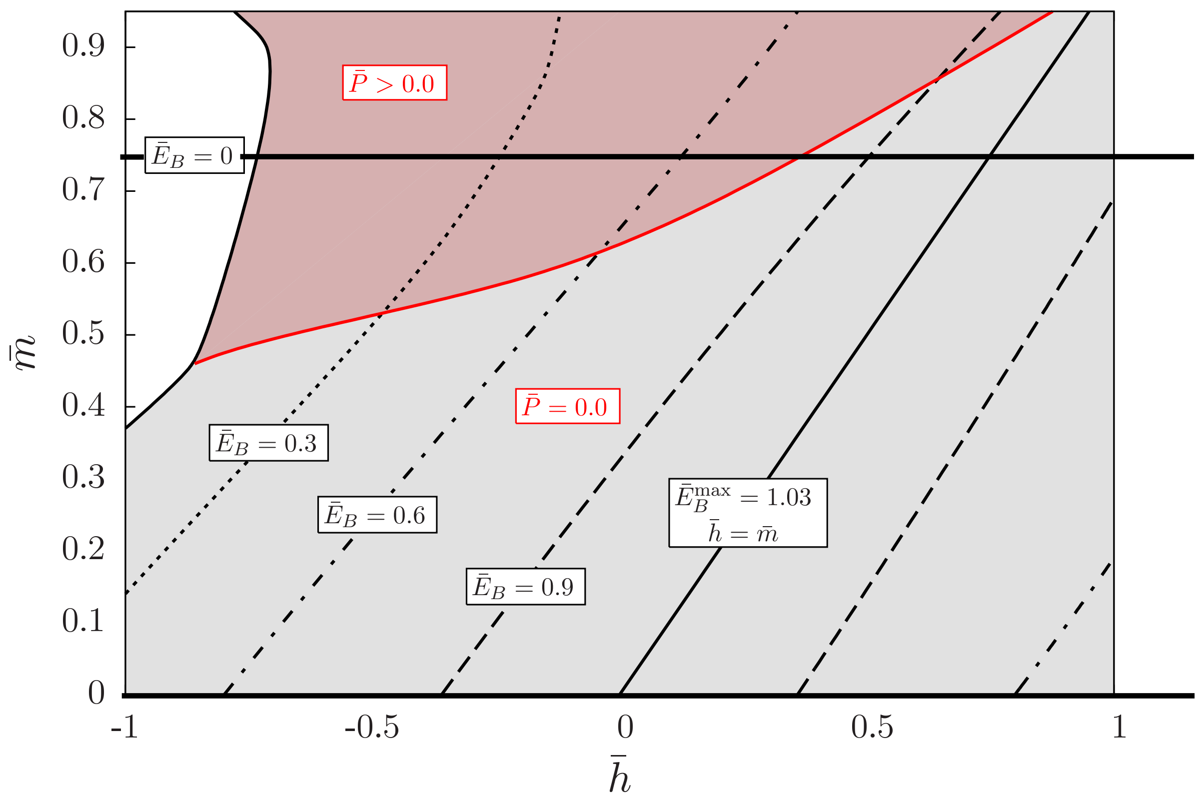
$$\left[\sum_{\sigma=\uparrow,\downarrow} \epsilon_\sigma(\partial_{x_\sigma}) - g\delta(x_\uparrow - x_\downarrow) + E_B\right]\Psi(x_\uparrow, x_\downarrow) = 0$$

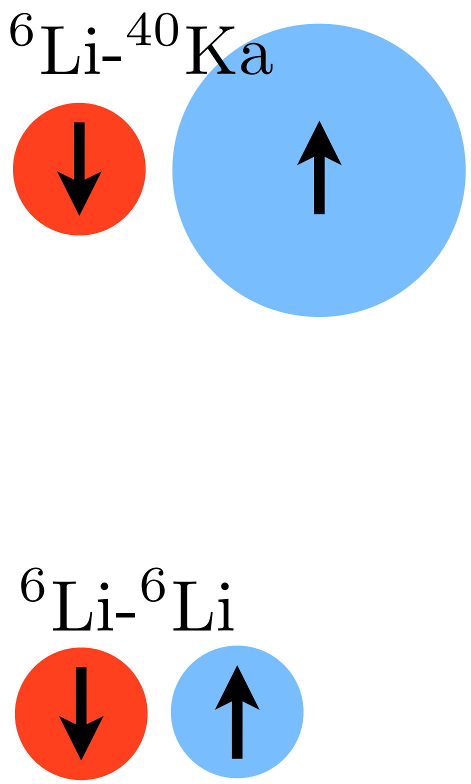
- disperion relation:

$$\epsilon_\sigma(\partial_{x_\sigma}) = |-(2m_\sigma)^{-1}\partial^2_{x_\sigma} - \epsilon_{\mathrm{F},\sigma}|$$

Phase diagram of imbalanced Fermi gases I

[Roscher, JB, Drut '15]

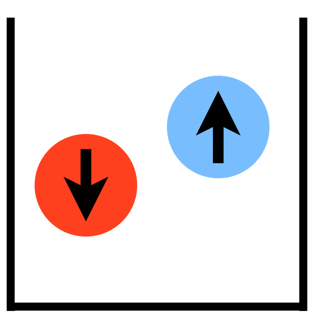




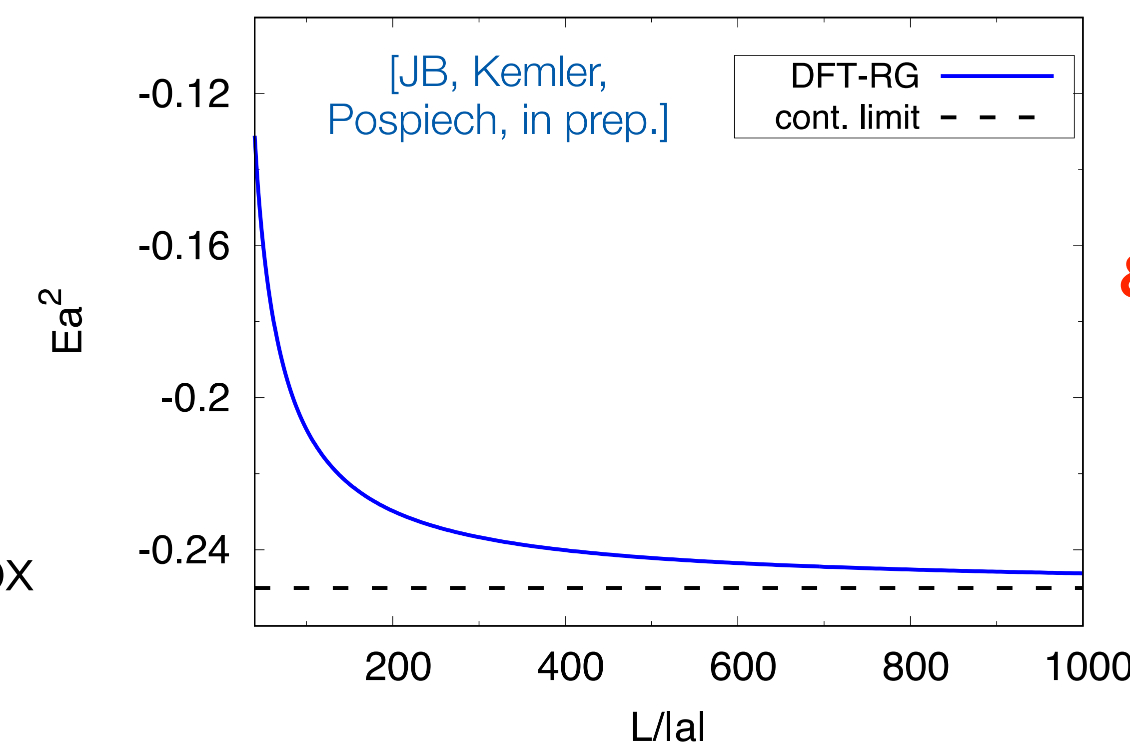
- bound-state formation in the presence of inert Fermi spheres
- necessary condition for macroscopic occupation of the ground state (~condensation): bound-state formation

From few to many?

- approach via Schrödinger equation becomes essentially impossible for large fermion numbers to study the fully interacting problem [“exponential barrier”, Kohn, Nobel lecture ’99]
- functional methods allow to bridge the gap from “few to many” efficiently, such as an **RG approach to Density Functional Theory** [Polonyi & Sailer ’02, Polonyi & Schwenk ’04; JB ’11; Kemler & JB ’13; Kemler, Pospiech, JB ’16; JB, Kemler, Pospiech, in prep.]



two fermions
in a periodic 1D box
with extent L



cf. talk by Sandra Kemler & poster by Martin Pospiech on DFT-RG study of a nuclear physics model

How to study imbalanced Fermi gases?

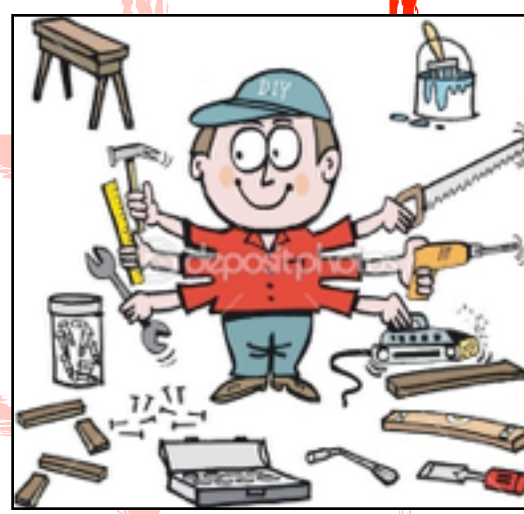
Schrödinger equation

$$\hat{H}|\psi\rangle = \mathrm{i}\partial_t|\psi\rangle$$

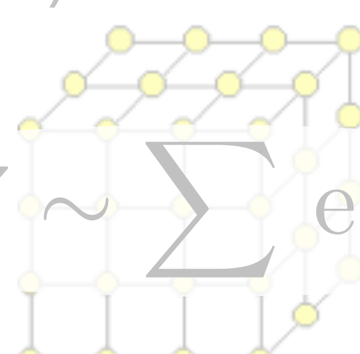
Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

[Wetterich '92]



(lattice) Monte Carlo



$$Z \sim \sum e^{-S}$$

Density Functional Theory

$$\psi_{\mathrm{gs}}(\vec{x}_1, \ldots, \vec{x}_N) \Leftrightarrow n_{\mathrm{gs}}(\vec{x})$$

[Hohenberg & Kohn '65]

Imbalanced Fermi gases & Wetterich equation

[Roscher, JB, Drut ‘15]

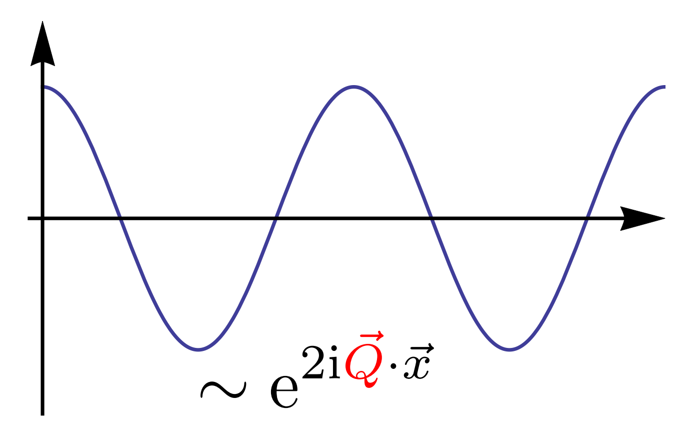
- classical action (partially bosonized version):

$$S[\{\psi_\sigma\},\bar{\varphi}] = \int_{\tau,\vec{x}} \Big[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* \Big(\partial_\tau - \frac{\nabla^2}{2m_\sigma} - \mu_\sigma\Big)\psi_\sigma + \bar{m}_\varphi^2 \bar{\varphi}^* \bar{\varphi} - \bar{h}_\varphi \left(\bar{\varphi}^* \psi_\uparrow \psi_\downarrow - \bar{\varphi}\psi_\uparrow^* \psi_\downarrow^*\right) \Big].$$

- consider **space-dependent** background/mean field:

$$\bar{\varphi}(\vec{x}) = \bar{\varphi}_0\, \mathrm{e}^{2\mathrm{i}\vec{Q}\cdot\vec{x}}$$

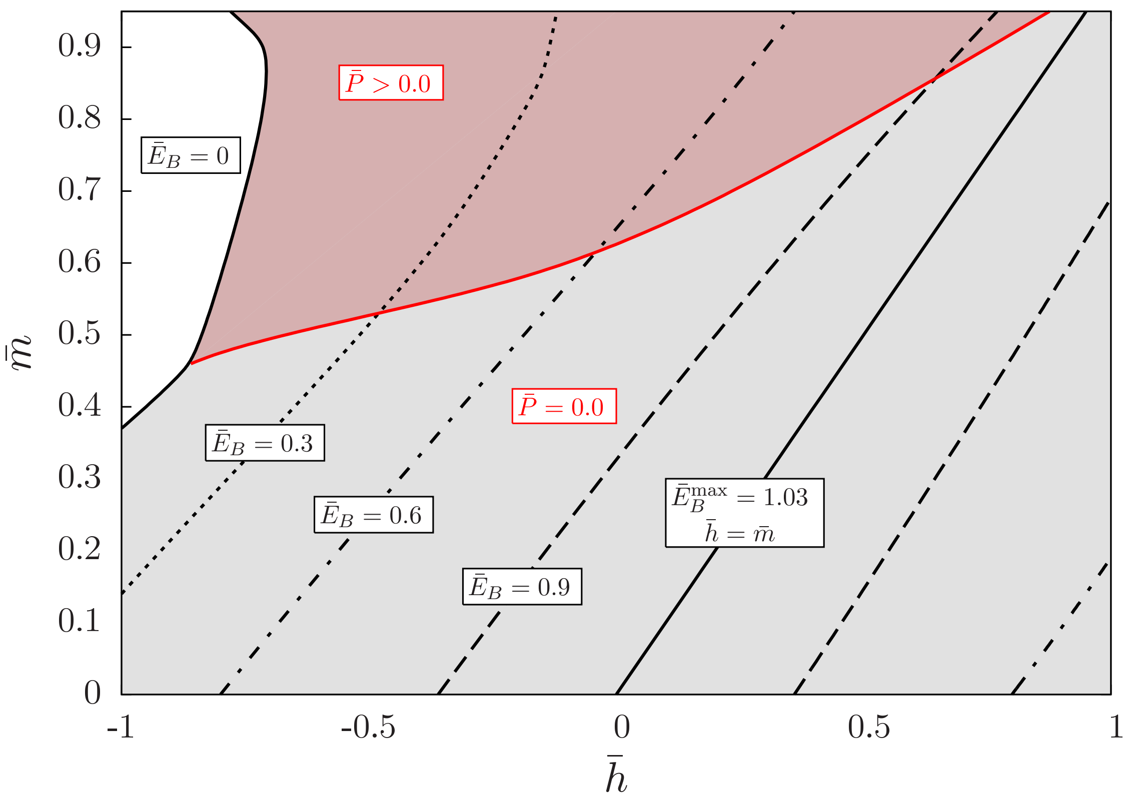
[Fulde-Ferrell ansatz]

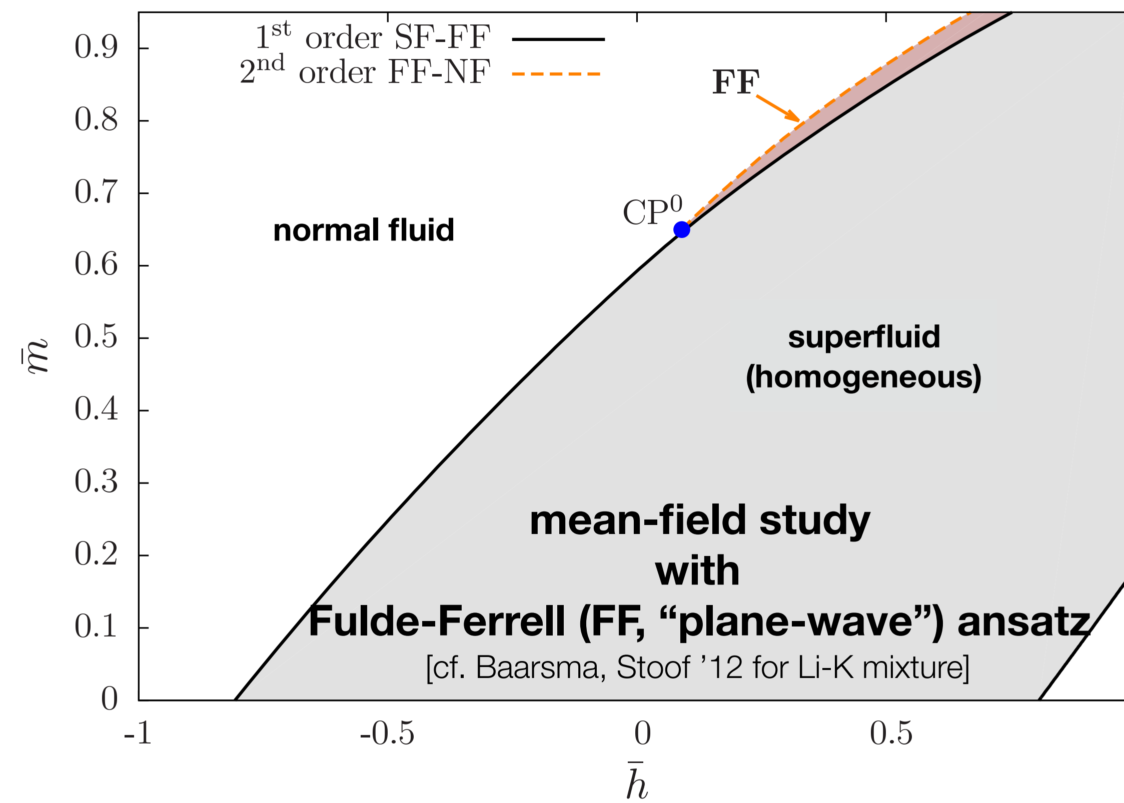


- lowest order: **mean-field truncation** (loops only with internal fermion lines); minimize with respect to $\bar{\varphi}_0$ & $\vec{Q}$

Phase diagram of imbalanced Fermi gases II

[Roscher, JB, Drut '15]

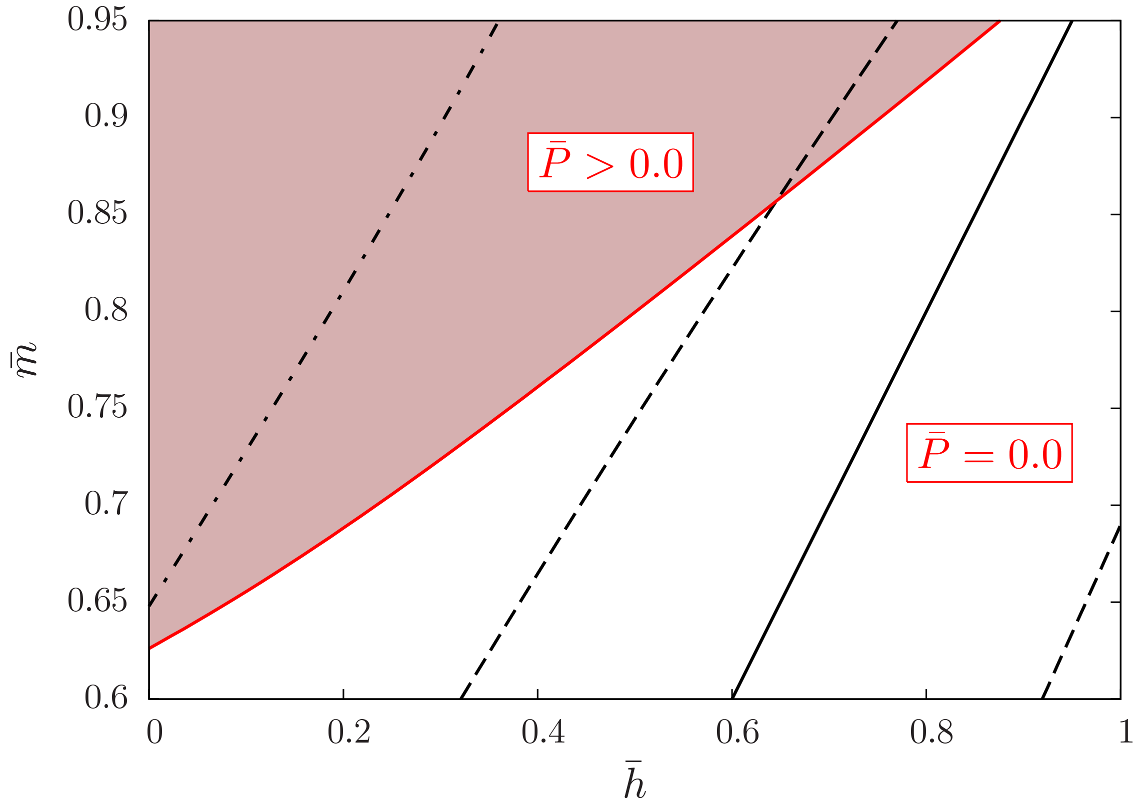


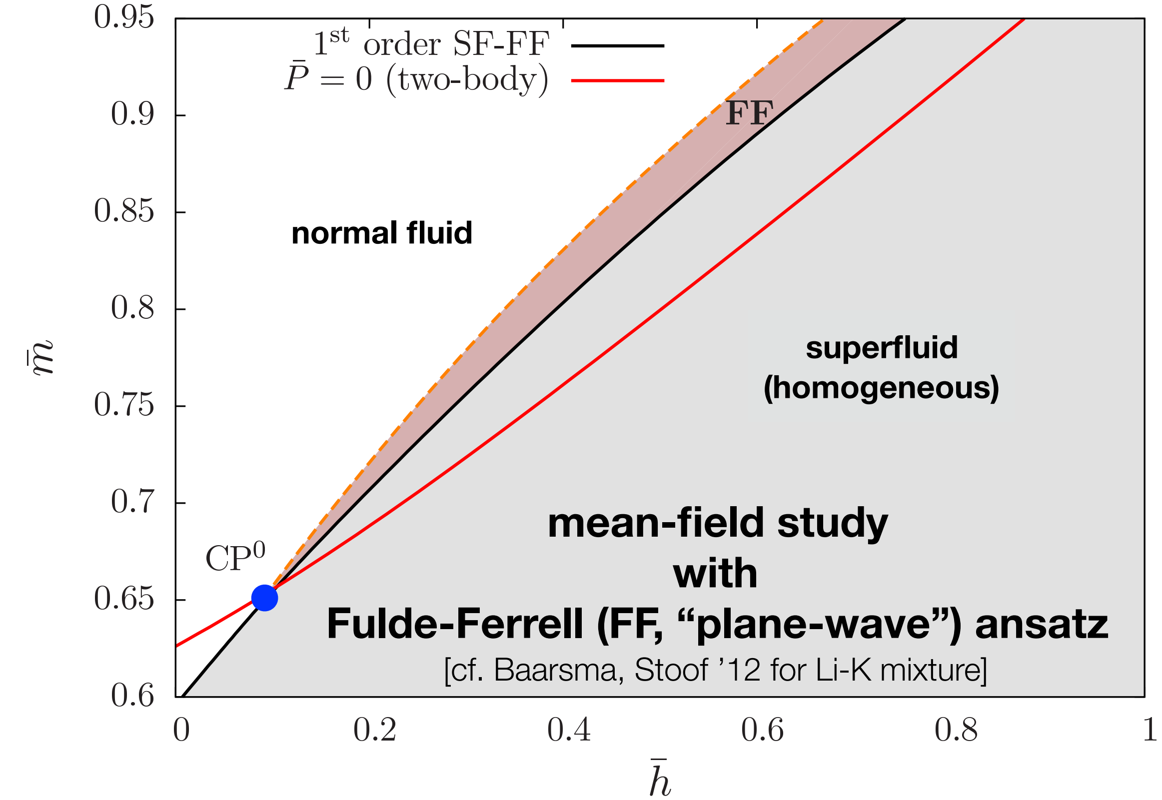


- bound-state formation in the presence of inert Fermi spheres [left panel]
- necessary condition for macroscopic occupation of the ground state (~condensation): bound-state formation
- many-body phase diagram [right panel]

Phase diagram of imbalanced Fermi gases II

[Roscher, JB, Drut '15]





- bound-state formation in the presence of inert Fermi spheres [left panel]
- necessary condition for macroscopic occupation of the ground state (~condensation): bound-state formation
- many-body phase diagram [right panel]

Mean-field study “beyond” the Fulde-Ferrell ansatz

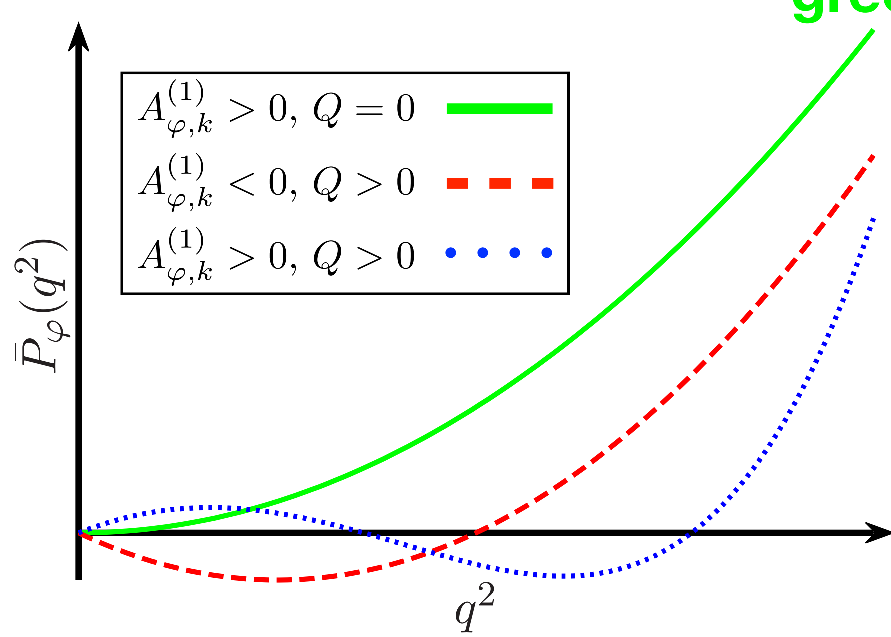
[Roscher, JB, Drut ‘15]

- use **spatially constant** background/mean field
- inverse propagator of the order-parameter field φ:

$$\bar{P}_{\varphi}(q_0,\vec{q}^{\,2}) = \left(iZ_{\varphi,k}q_0 + \frac{1-\bar{m}^2}{2}A_{\varphi,k}\vec{q}^{\,2} + \frac{\partial^2 U}{\partial\bar{\varphi}\partial\bar{\varphi}^*}\Big|_{\bar{\varphi}_0}\right)$$

- derivative expansion: $A_{\varphi,k}(\vec{q}^{\,2}) = A^{(1)}_{\varphi,k} + A^{(2)}_{\varphi,k}\vec{q}^{\,2} + \dots$
- different scenarios:

use **sign change of** $A^{(1)}_{\varphi,k}$ at LO in the derivative expansion as **indicator for** the onset of **“crystalline” condensation**

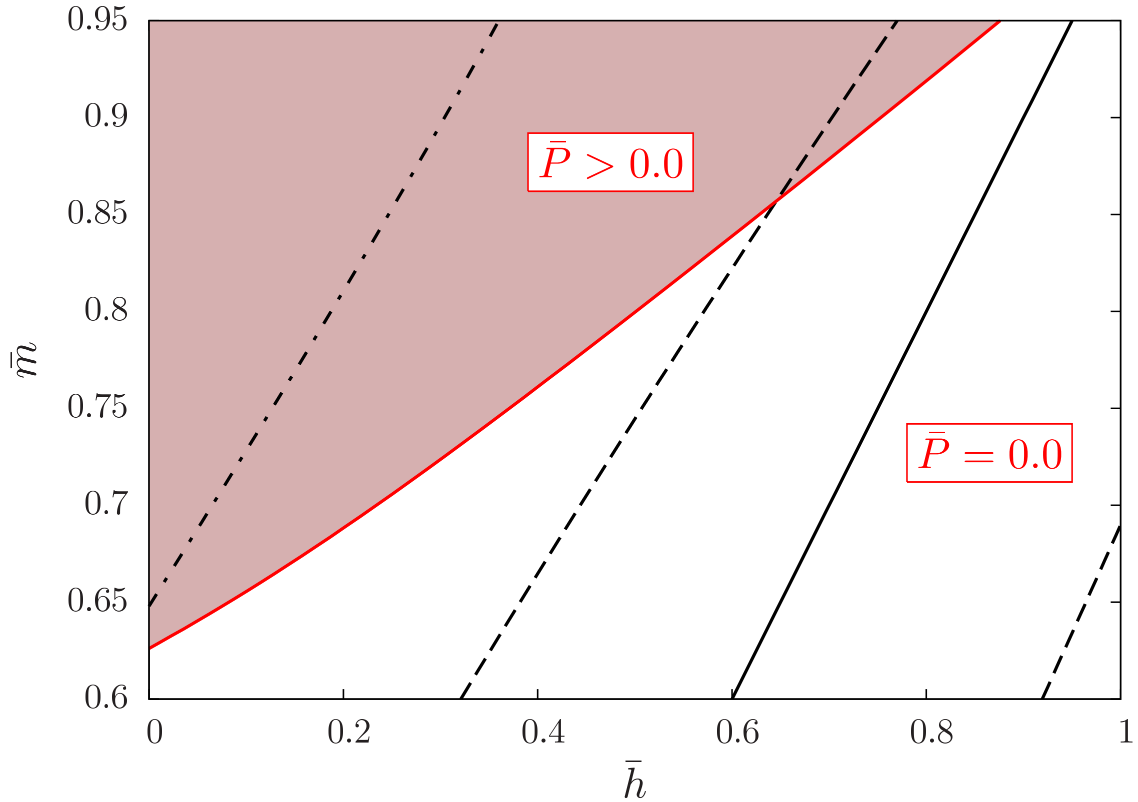


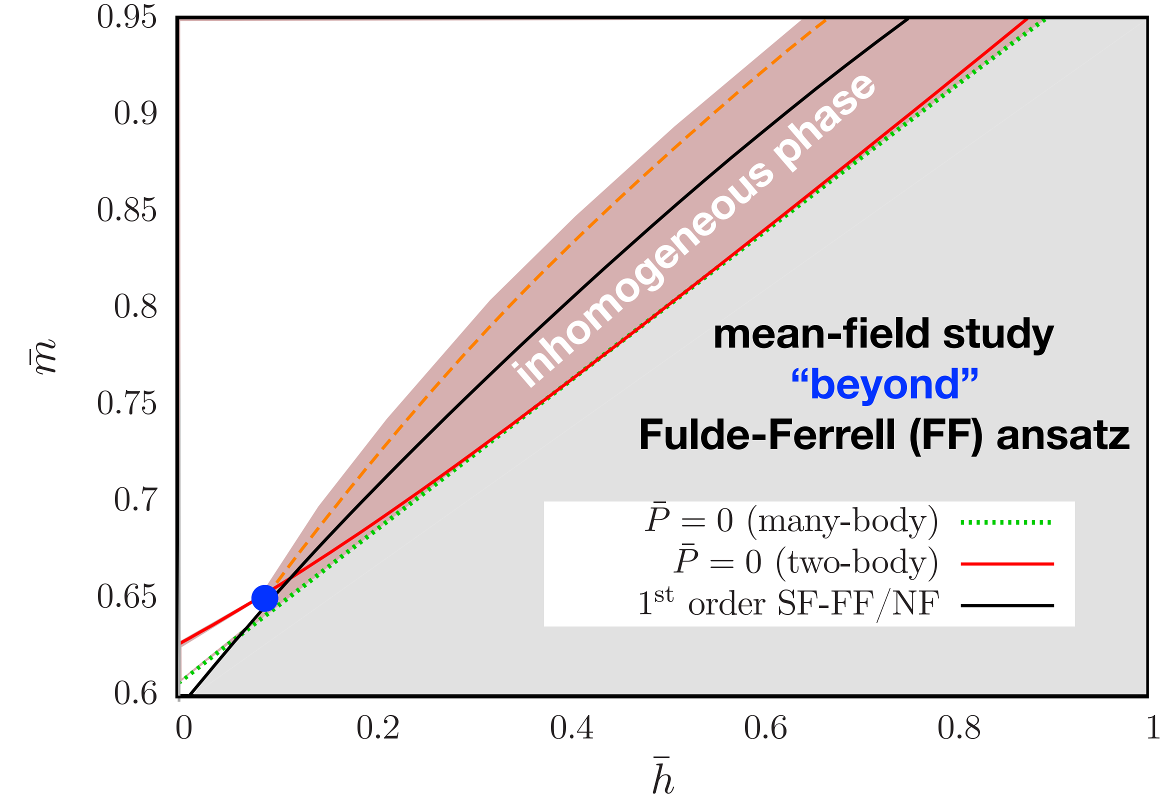
green: homogenous ground state

red/**blue**: “dominance” of modes favoring a “crystalline” phase

Phase diagram of imbalanced Fermi gases II

[Roscher, JB, Drut '15]





- bound-state formation in the presence of inert Fermi spheres [left panel]
- necessary condition for macroscopic occupation of the ground state (~condensation): bound-state formation
- many-body phase diagram [right panel]

Inclusion of order-parameter fluctuations

[Roscher, JB, Drut '15]

- ansatz for the effective action:

$$\Gamma_k[\{\psi_\sigma\},\varphi] = \int_{\tau,\vec{x}} \Big[\sum_{\sigma=\uparrow,\downarrow} \psi^*_\sigma \Big(\partial_\tau - \frac{\nabla^2}{2m_\sigma} - \mu_\sigma\Big)\psi_\sigma$$

$$+\varphi^*\Big(\frac{Z_{\varphi,k}}{A_{\varphi,k}}\partial_\tau - \frac{1-\bar{m}^2}{2}\nabla^2\Big)\varphi + U_k(\rho) - h_\varphi\Big(\varphi^*\psi_\uparrow\psi_\downarrow - \varphi\psi^*_\uparrow\psi^*_\downarrow\Big)\Big]$$

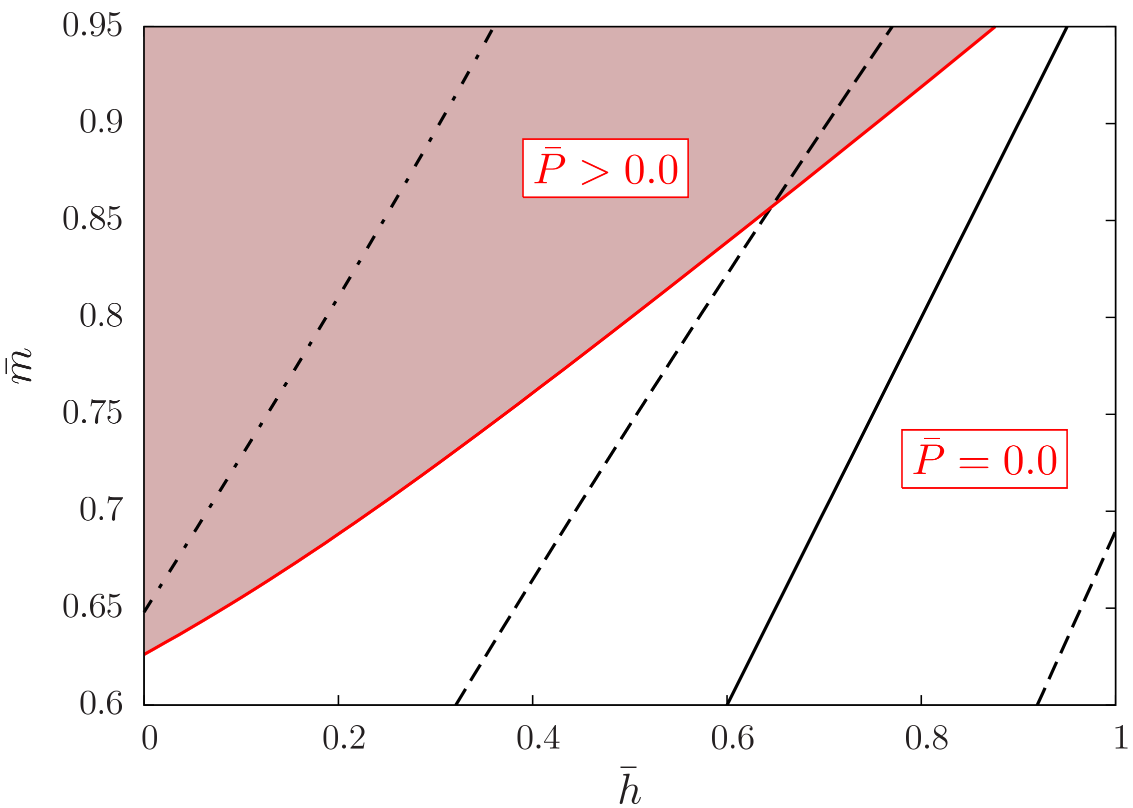
- use **spatially constant** background/mean field
- inverse propagator of the order-parameter field φ:

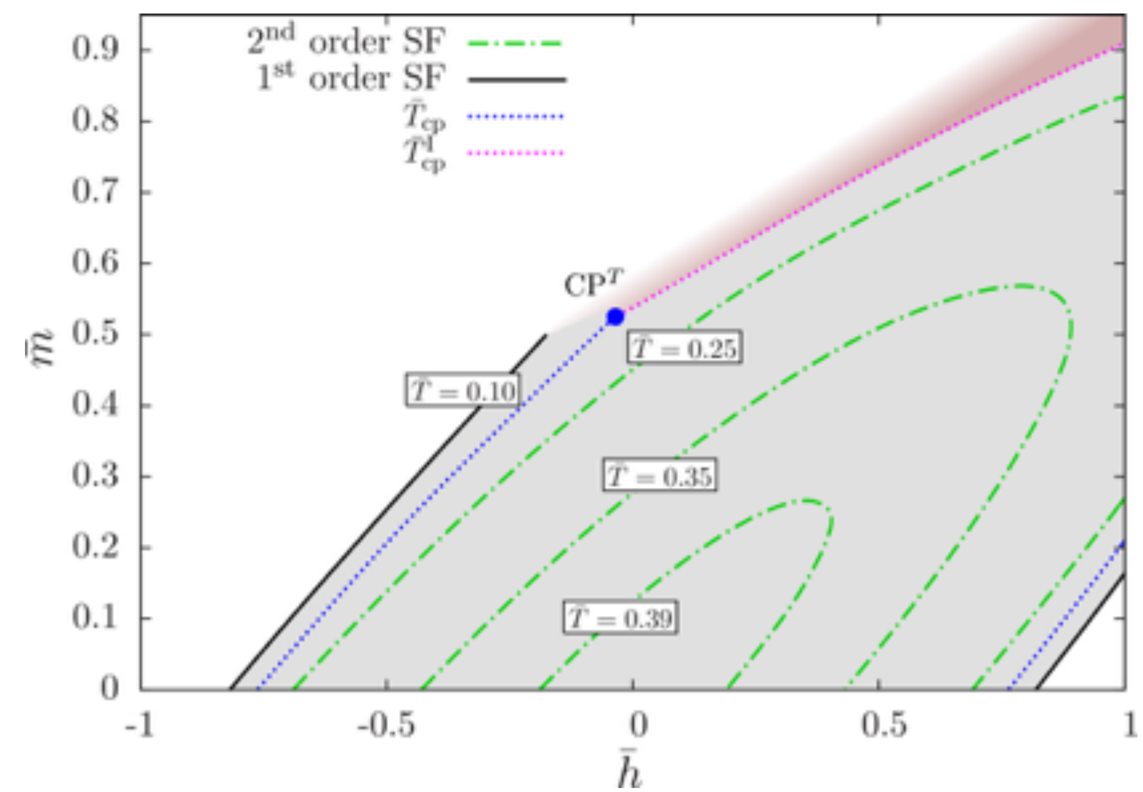
$$\bar{P}_\varphi(q_0,\vec{q}^{\,2}) = \Big(iZ_{\varphi,k}q_0 + \frac{1-\bar{m}^2}{2}A_{\varphi,k}\vec{q}^{\,2} + \frac{\partial^2 U}{\partial\bar{\varphi}\partial\bar{\varphi}^*}\Big|_{\bar{\varphi}_0}\Big)$$

- discretization of the effective potential U_k on a grid

Phase diagram of imbalanced Fermi gases III

[Roscher, JB, Drut '15]





- bound-state formation in the presence of inert Fermi spheres [left panel]
- necessary condition for macroscopic occupation of the ground state (~condensation): bound-state formation
- many-body phase diagram [right panel]

How to study imbalanced Fermi gases?

Schrödinger equation

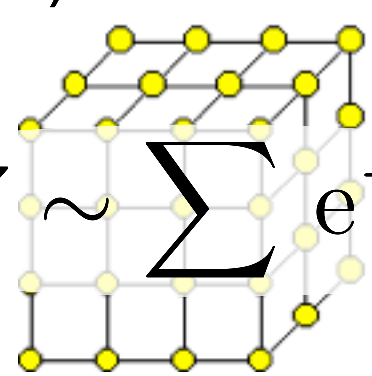
$$\hat{H}|\psi\rangle = \mathrm{i}\partial_t|\psi\rangle$$

Wetterich equation

$$\partial_t\Gamma_k = \frac{1}{2}\mathrm{STr}\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

[Wetterich '92]

(lattice) Monte Carlo



$$Z \sim \sum e^{-S}$$

Density Functional Theory

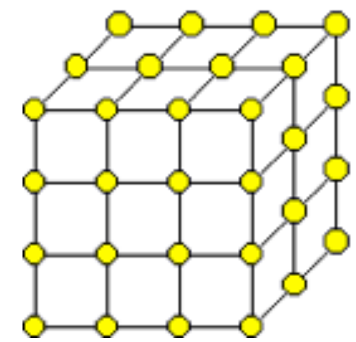
$$\psi_{\mathrm{gs}}(\vec{x}_1, \ldots, \vec{x}_N) \Leftrightarrow n_{\mathrm{gs}}(\vec{x})$$

[Hohenberg & Kohn '65]

Fermi gases and lattice MC simulations

- Monte Carlo (MC) simulations:

partition sum $Z \sim \sum_{\text{conf.}} \mathrm{e}^{-S}$



(discretization of space-time)

- physical observables:

$$\langle \mathcal{O} \rangle \sim \frac{1}{Z} \sum_{\text{conf.}} \mathcal{O}\, \mathrm{e}^{-S} \equiv \frac{1}{Z} \sum_{i} \mathcal{O}_i\, \rho_i$$

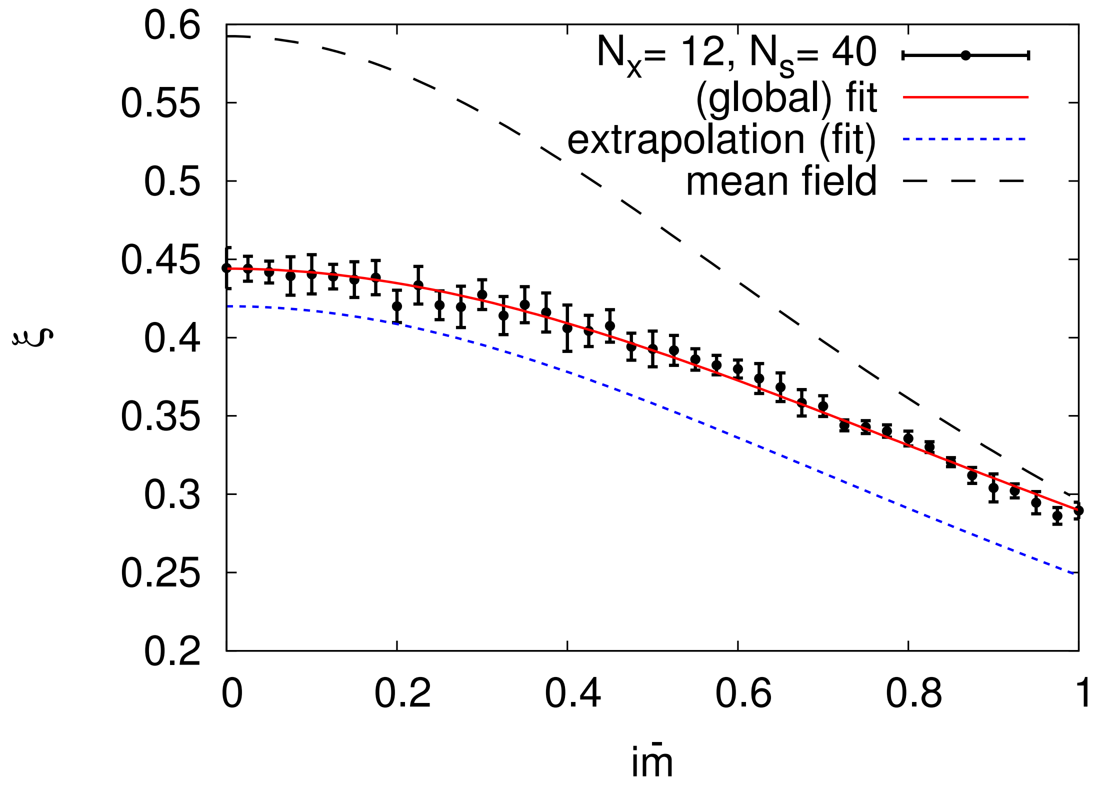
(ρ_i : probability measure)

Imbalanced Fermi gases and lattice MC

- sum/integral can be evaluated straightforwardly with Monte Carlo techniques for $\rho_i > 0$
- "**sign**" problem: the probability measure ρ_i is negative for "some" choices for the physical parameters
- strategy for ultracold gases:
  - Taylor expansion [JB, Drut, work in progress]
  - probability measure is rendered positive semi-definite by using imaginary-valued polarizations and imaginary mass-imbalance [JB et al. '13; Roscher, JB, Chen, Drut '14]

Equation of state (EOS) of ultracold Fermi gases

[JB, Drut, Roscher '14]



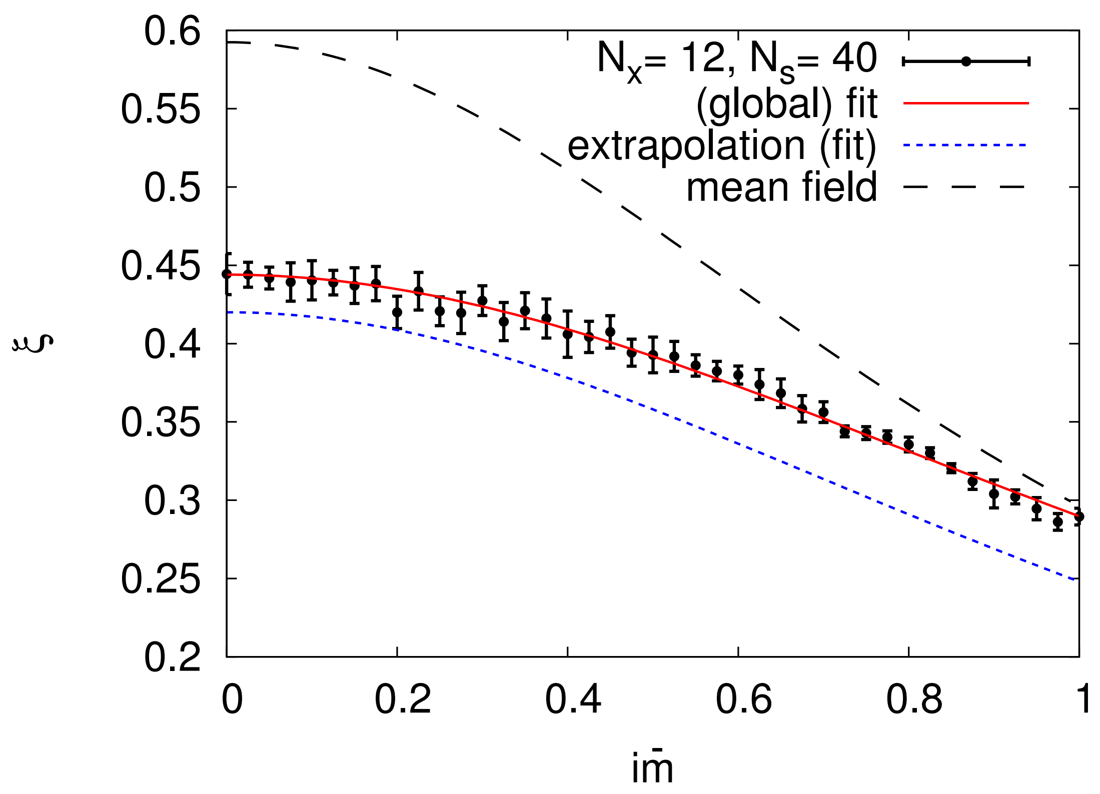
- *Bertsch* parameter & EOS (spin-balanced):

$$E_N(\bar{m}) = \xi E_N^{\text{free}}(\bar{m} = 0)$$

- Monte Carlo simulations for imaginary mass imbalances

Equation of state (EOS) of ultracold Fermi gases

[JB, Drut, Roscher '14]



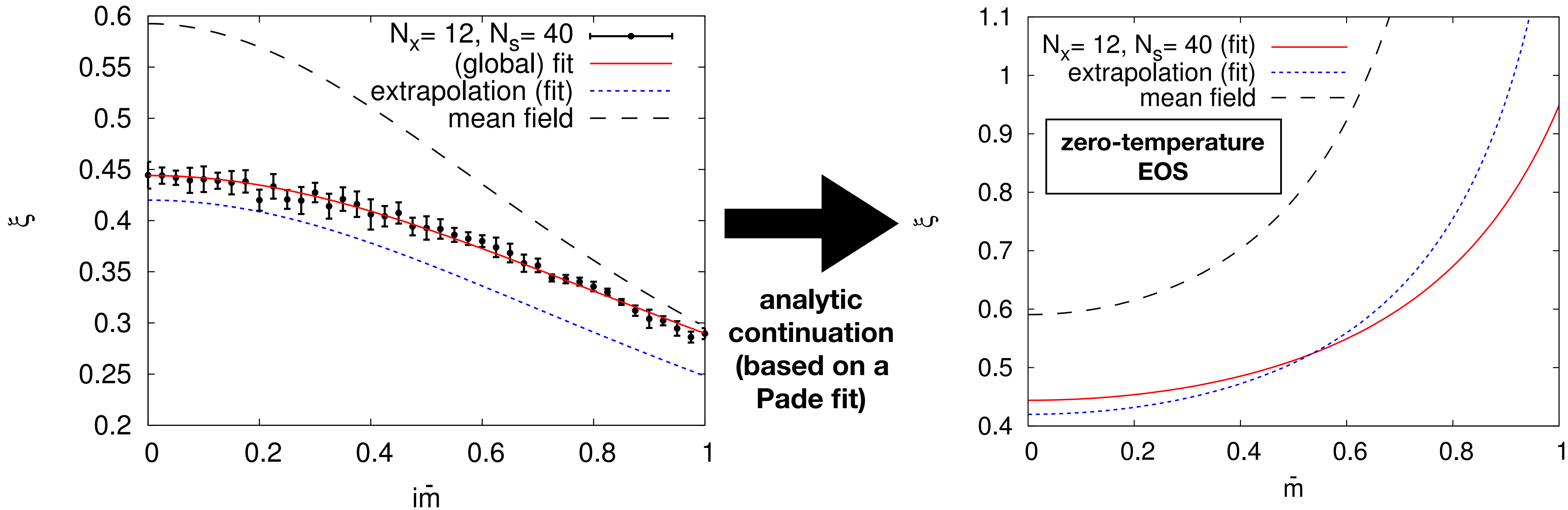
- Hellmann-Feynman theorem:

$$\left.\frac{d\xi(\bar{m})}{d\bar{m}^2}\right|_{\bar{m}=0} = \xi(\bar{m}=0)$$

- mean-field theory: $\xi(\bar{m}) = \dfrac{\xi(\bar{m}=0)}{1+(\mathrm{i}\bar{m})^2}$

Equation of state (EOS) of ultracold Fermi gases

[JB, Drut, Roscher '14]



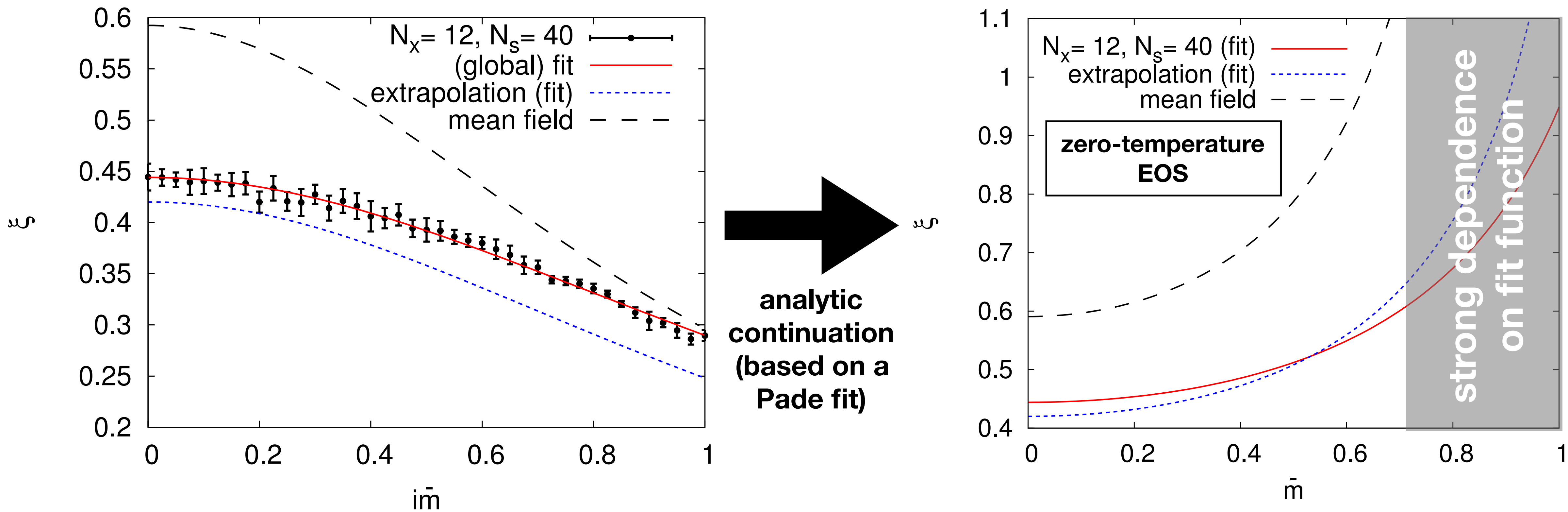
- *Bertsch* parameter & EOS (spin-balanced):

$$E_N(\bar{m}) = \xi E_N^{\text{free}}(\bar{m} = 0)$$

- Monte Carlo simulations for imaginary mass imbalances yield physical equation of state via analytic continuation

Equation of state (EOS) of ultracold Fermi gases

[JB, Drut, Roscher '14]



- *Bertsch* parameter & EOS (spin-balanced):

$$E_N(\bar{m}) = \xi E_N^{\text{free}}(\bar{m} = 0)$$

- **Phase transition**/emergence of "crystalline" phase at large imbalances**?** More involved studies are required …

How to study imbalanced Fermi gases?

Schrödinger equation

$\hat{H}|\psi\rangle$

Wetterich equation

$\frac{1}{2}\mathrm{Tr}\,\frac{\partial_t R_k}{\Gamma^{(2)} + R_k}$

(lattice) Mo...

$Z \sim \sum e^{-S}$

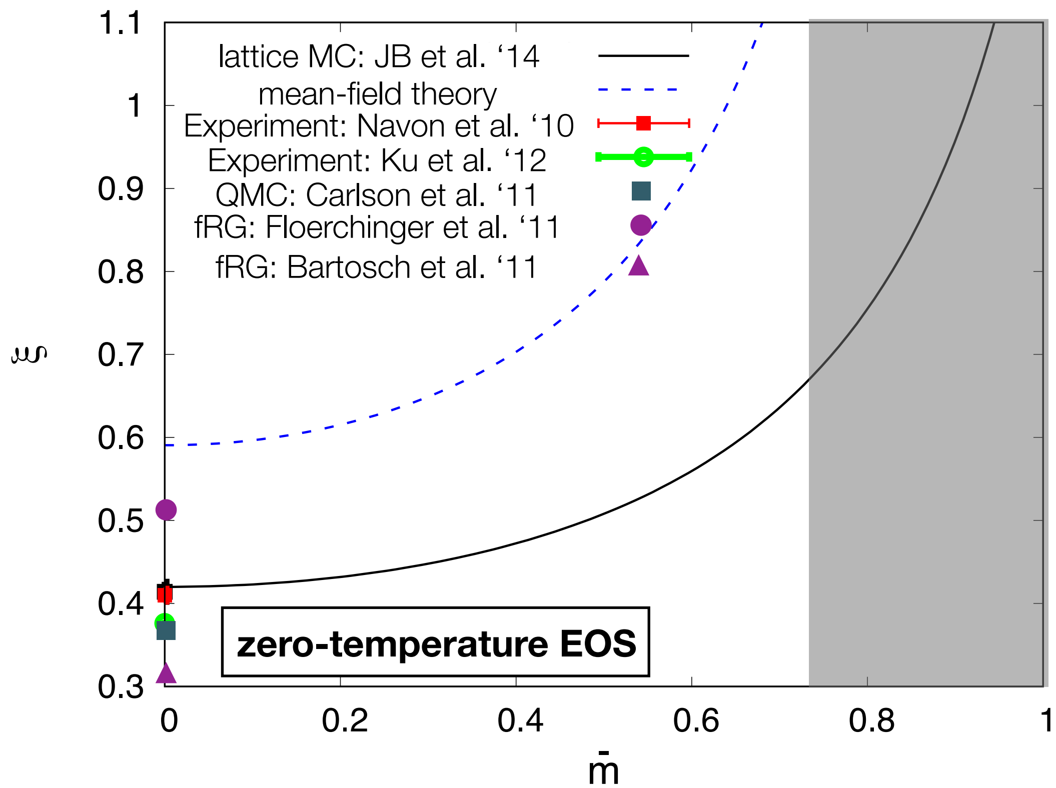
...Functional Theory

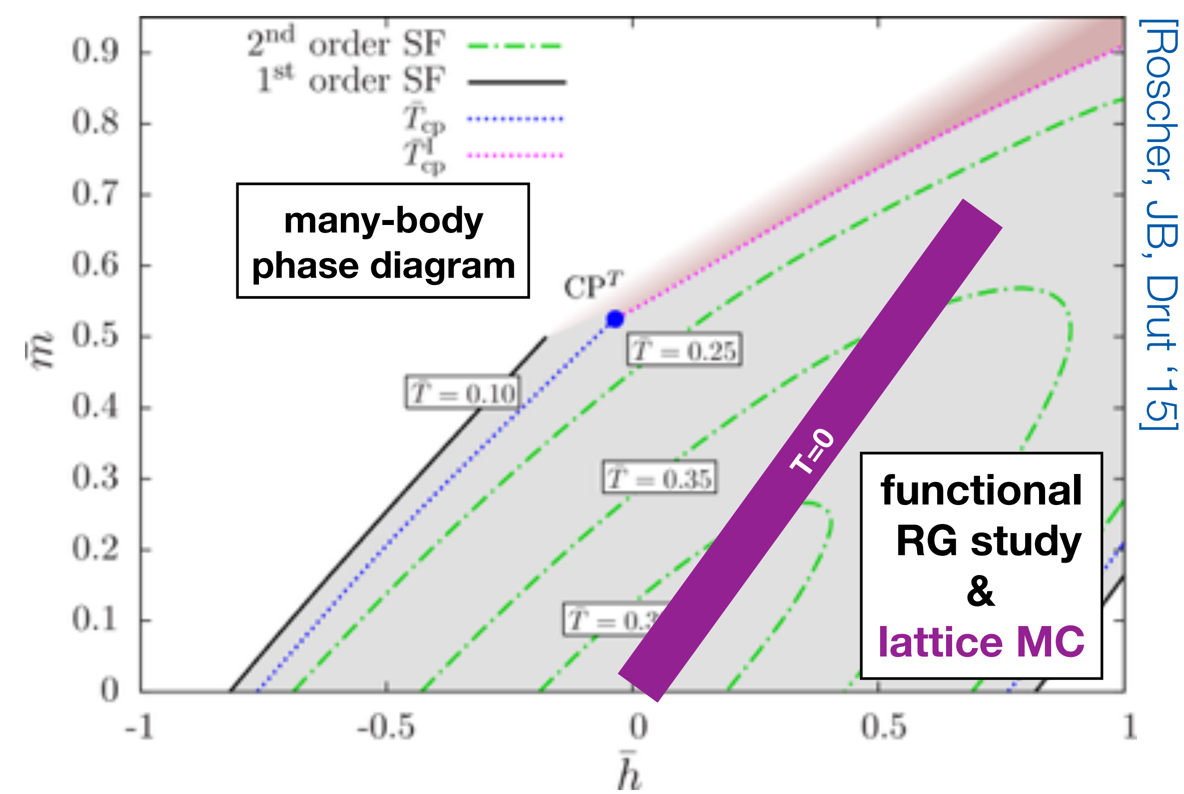
$\psi_{\mathrm{gs}}(\vec{x}_1, \ldots, \vec{x}_N) \Leftrightarrow n_{\mathrm{gs}}(\vec{x})$

[Hohenberg & Kohn '65]

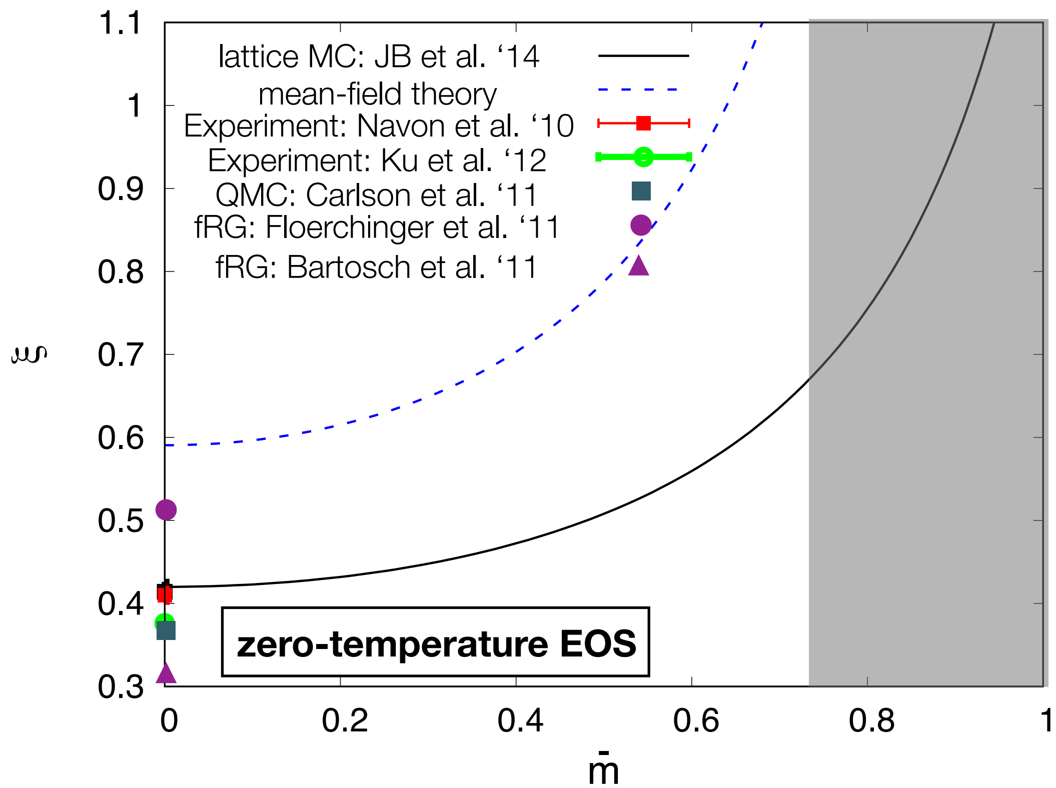
[picture taken from Hitchcock's 'Psycho' (1960)]

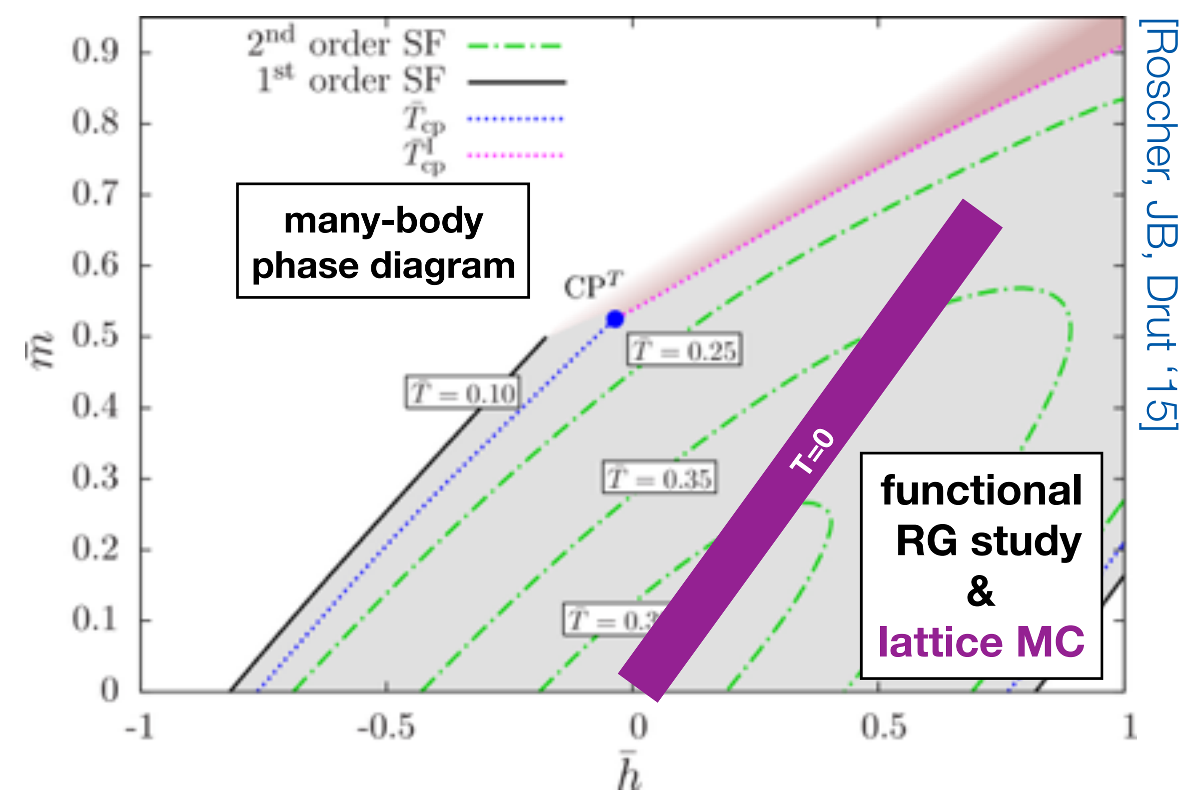
Theory “versus” Experiment





Theory “versus” Experiment





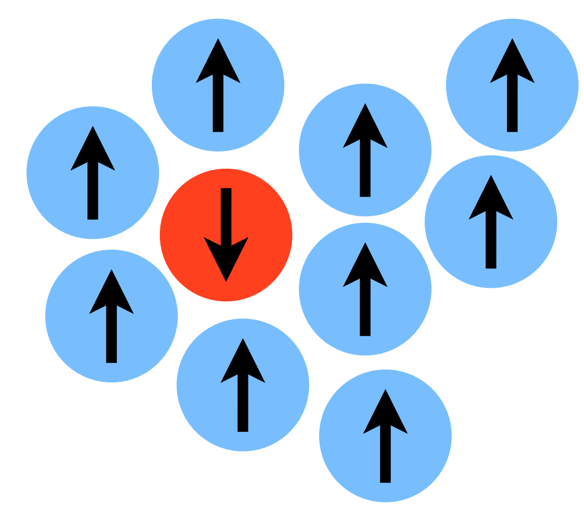
- Fermi polaron (“N+1-body problem”)

[**MC**: Carlson & Reddy ’05; Lobo et al. ’06; Prokof’ev & Svistunov ’08; Pilati & Giorgini ’08;
Variational: Chevy ’07; Bulgac & Forbes ’07; Ku, Braun & Schwenk ’09;
functional RG: Enss & Schmidt ’11]

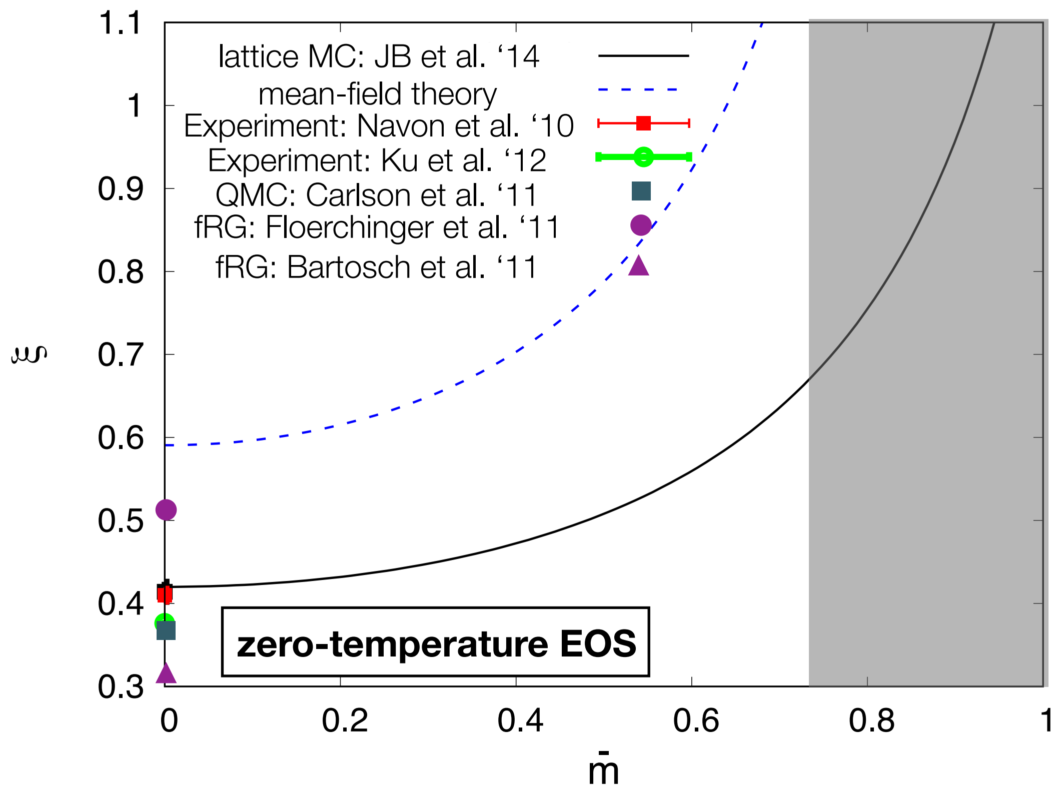
[not discussed in this talk]

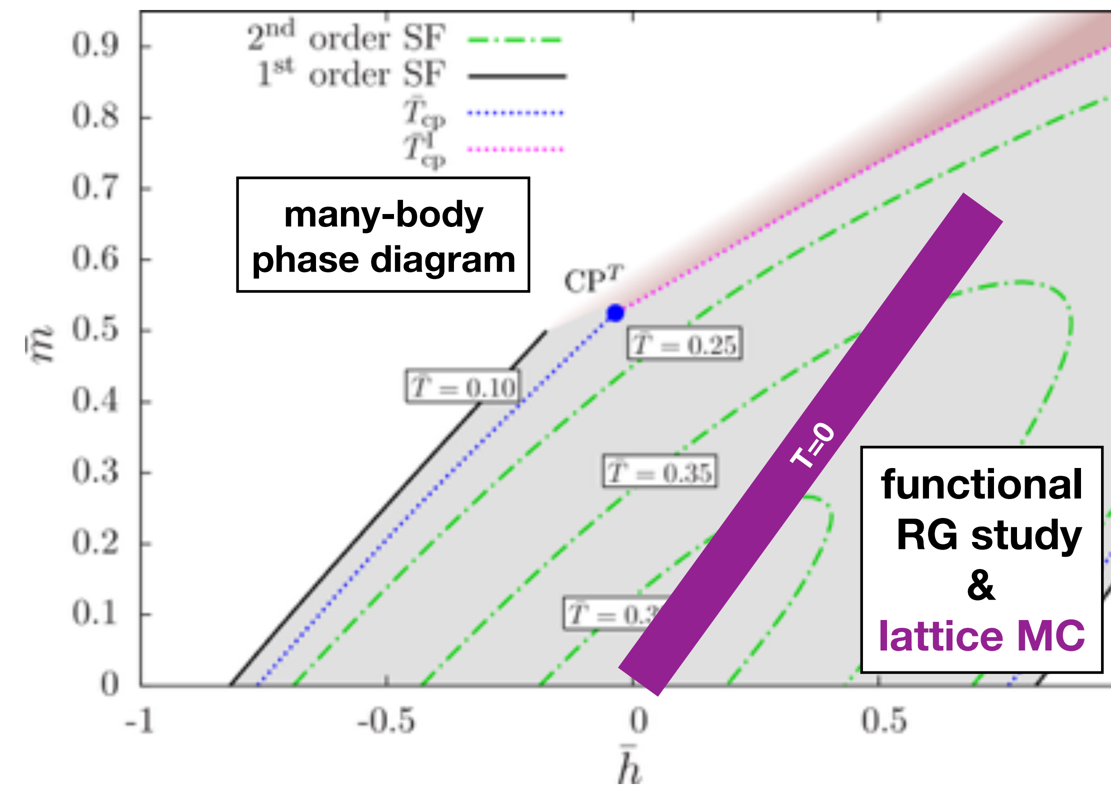
$$\eta = \mu_\downarrow/\epsilon_{\mathrm{F}} \approx -0.615 \ldots -0.57$$

[Note: consistent with experimental constraints, Chevy ’07]



Theory “versus” Experiment





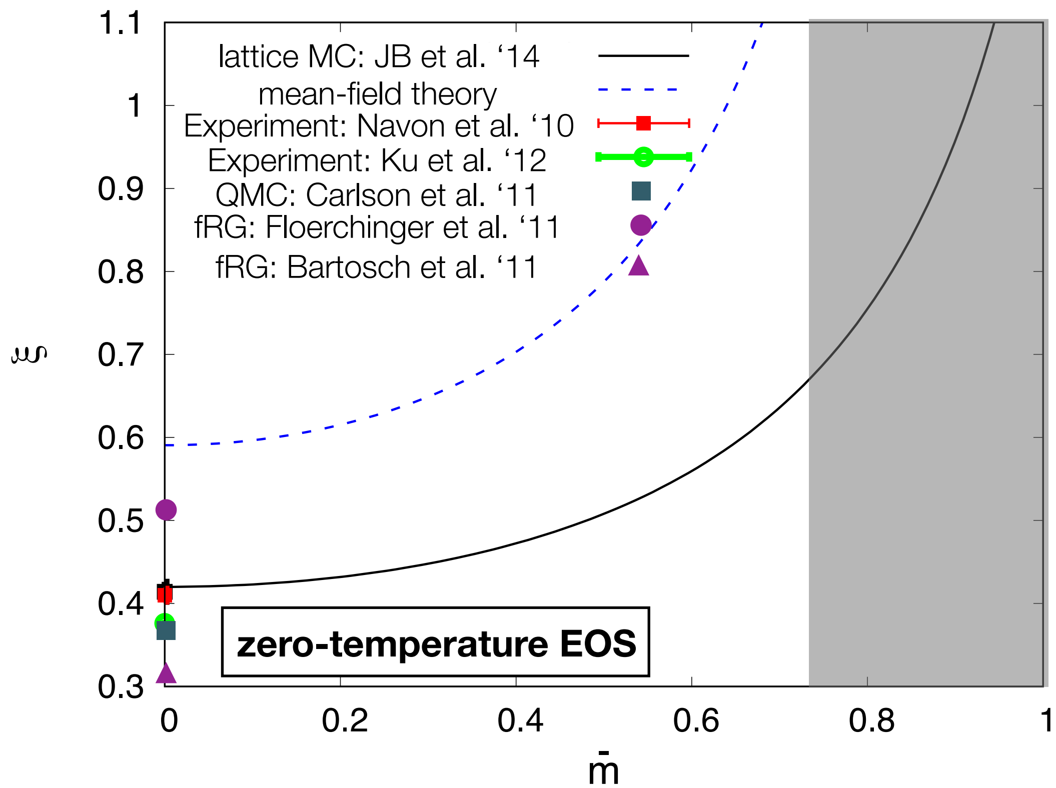
[Roscher, JB, Drut '15]

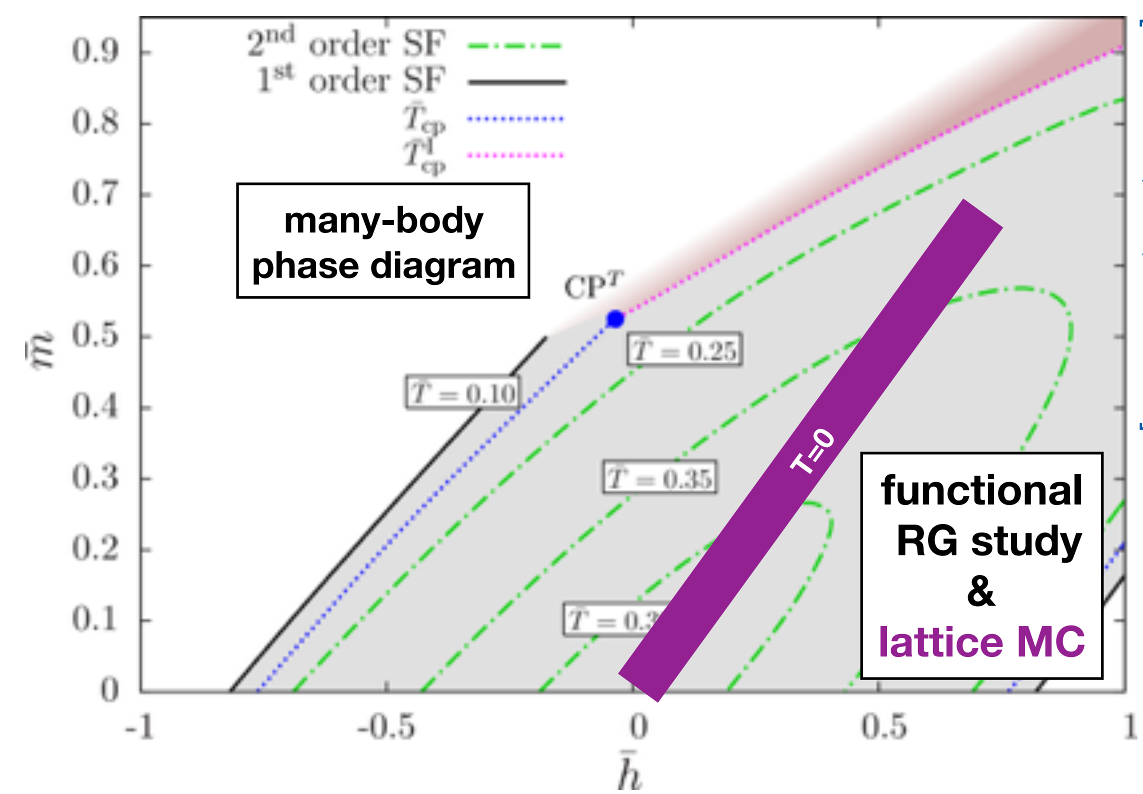
- phase transition temperature T_c/μ at zero imbalances:

functional RG	0.40

[Boettcher, JB, Herbst, Roscher, Pawlowski, Wetterich ’14]
[Roscher, JB, Drut ’15]

Theory "versus" Experiment





[Roscher, JB, Drut '15]

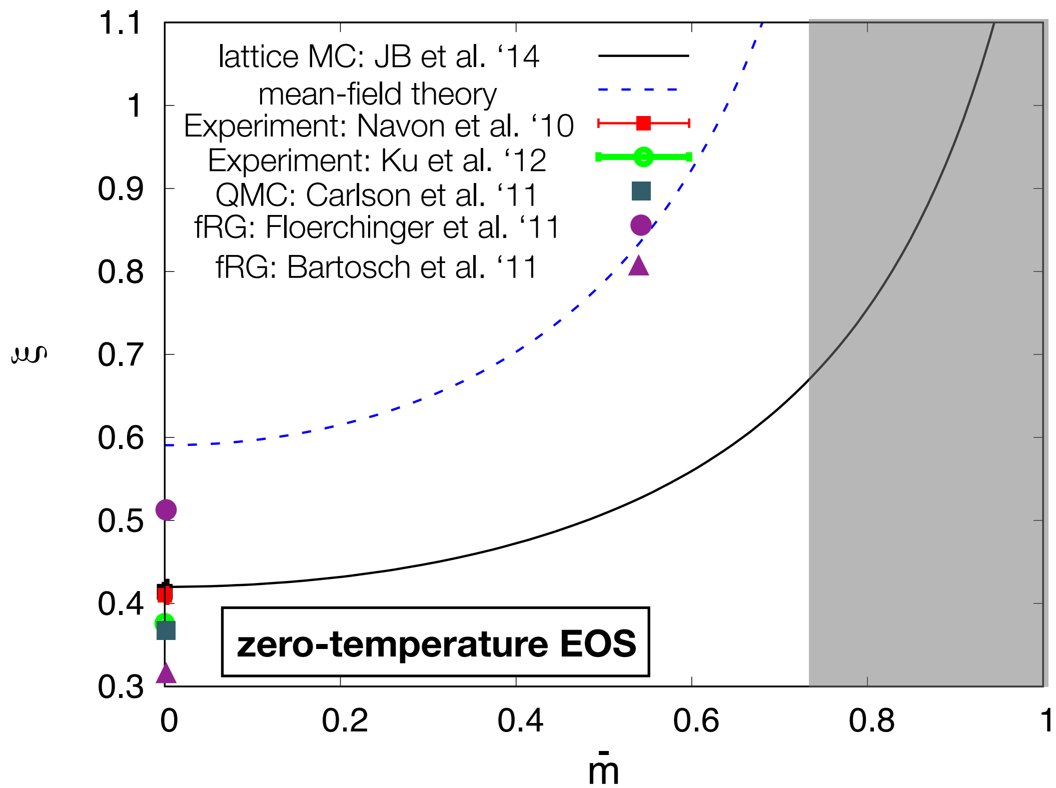
- phase transition temperature T_c/μ at zero imbalances:

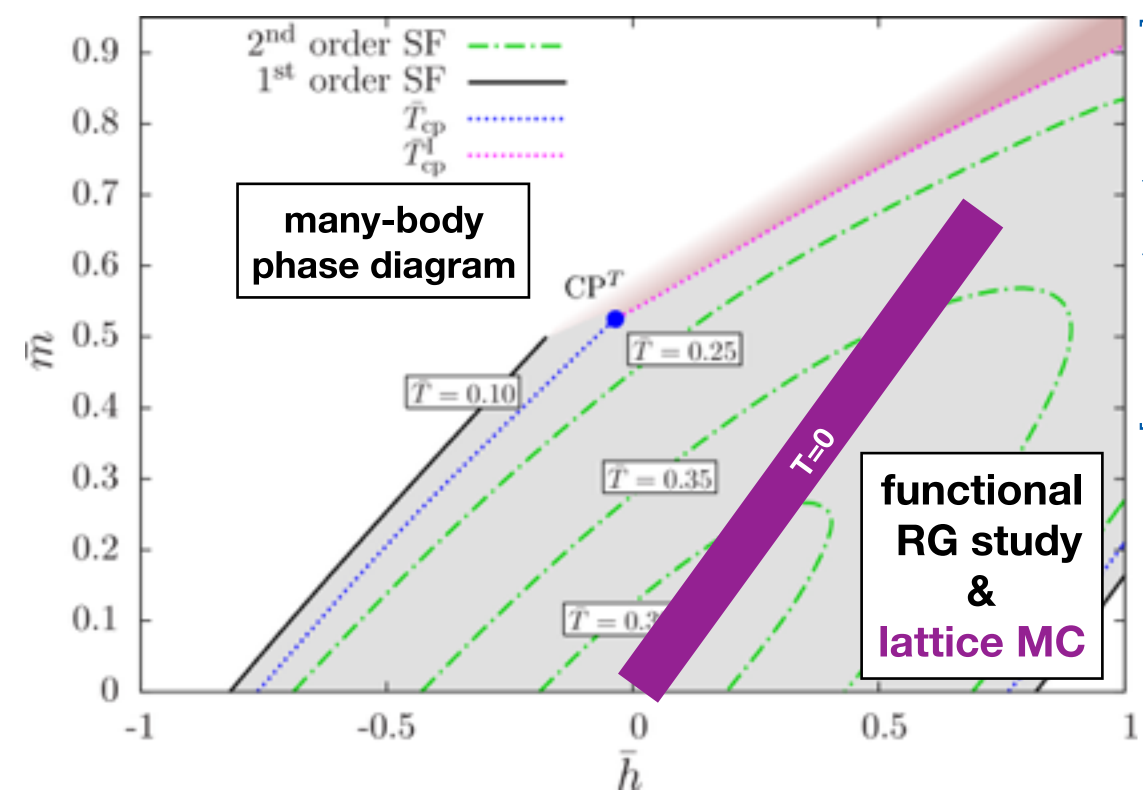
functional RG	0.40
QMC/lattice MC	0.35

[Boettcher, JB, Herbst, Roscher, Pawlowski, Wetterich '14]
[Roscher, JB, Drut '15]

[Bulgac, Drut, Magierski '08]

Theory “versus” Experiment



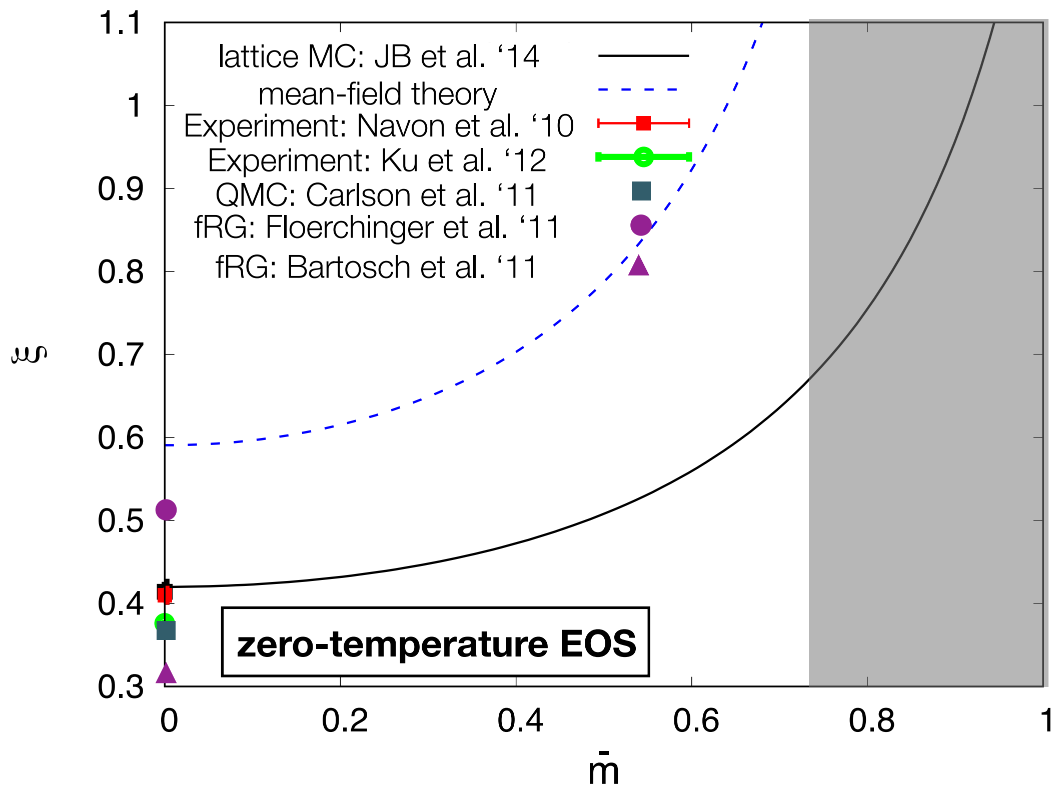


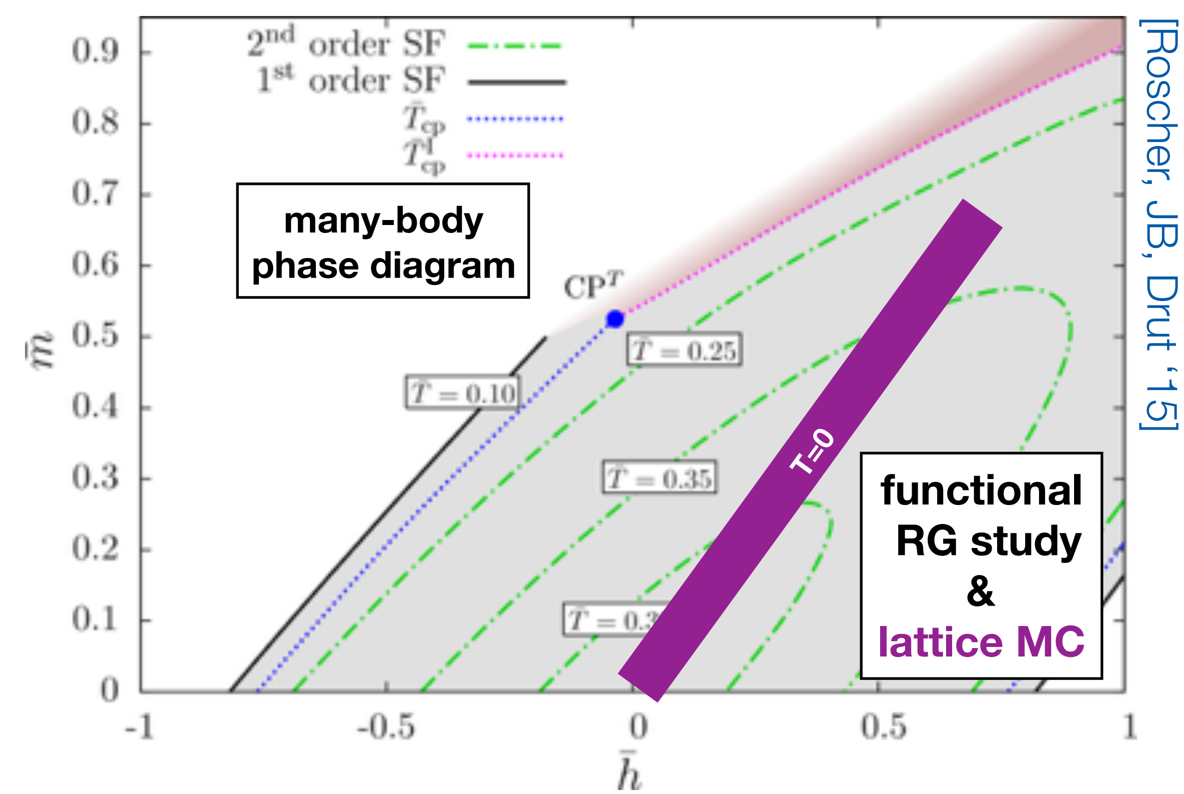
[Roscher, JB, Drut ‘15]

- phase transition temperature T_c/μ at zero imbalances:

functional RG	0.40	[Boettcher, JB, Herbst, Roscher, Pawlowski, Wetterich ’14] [Roscher, JB, Drut ’15]
QMC/lattice MC	0.35	[Bulgac, Drut, Magierski ’08]
DDMC	0.399	[Goulko, Wingate ’10]

Theory “versus” Experiment

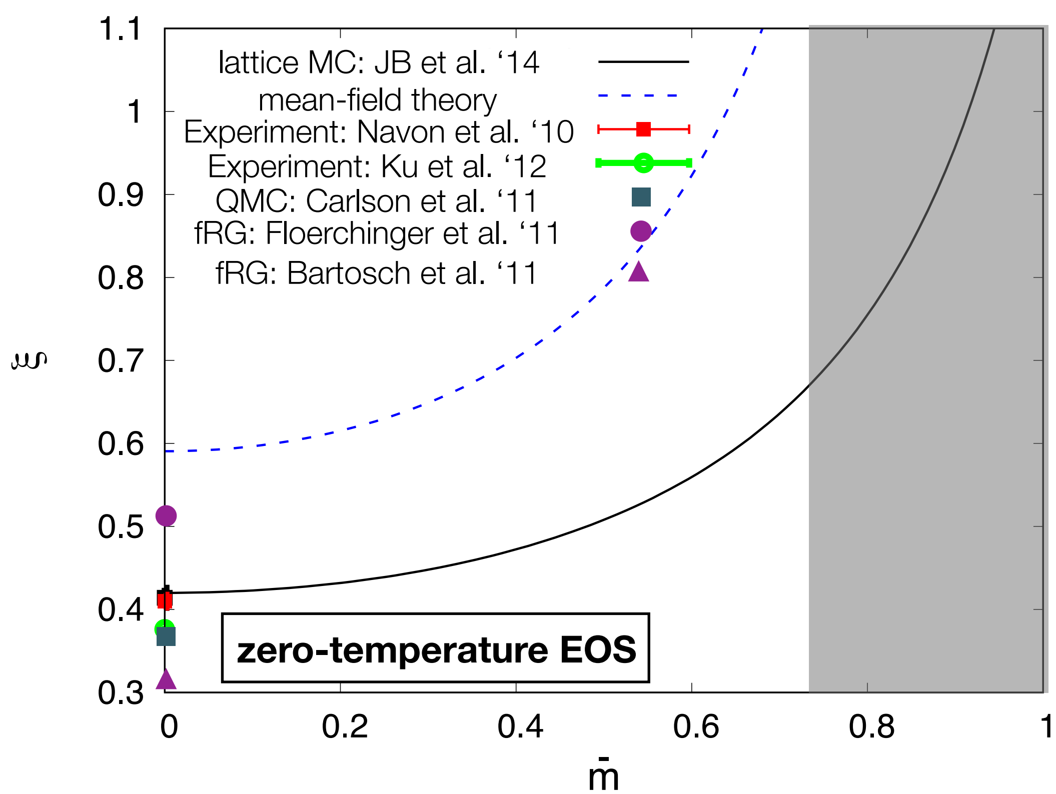


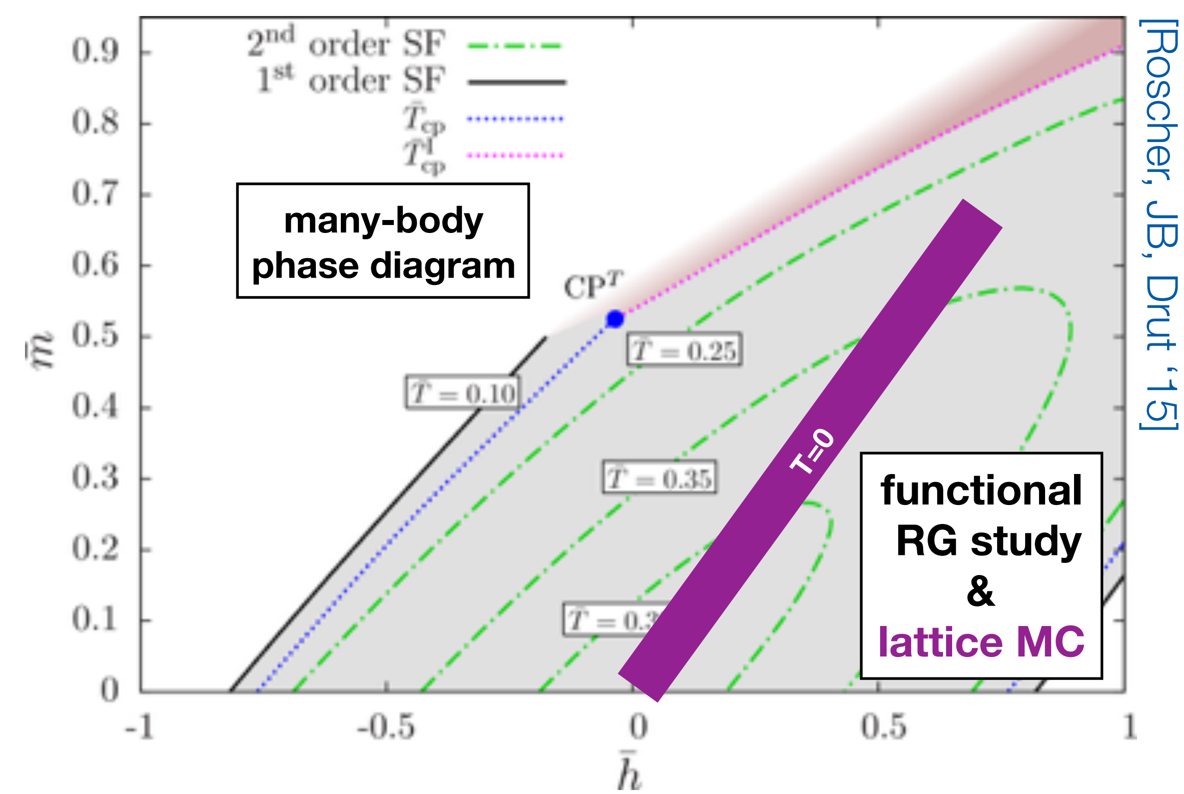


- phase transition temperature T_c/μ at zero imbalances:

functional RG	0.40	[Boettcher, JB, Herbst, Roscher, Pawlowski, Wetterich ’14] [Roscher, JB, Drut ’15]
QMC/lattice MC	0.35	[Bulgac, Drut, Magierski ’08]
DDMC	0.399	[Goulko, Wingate ’10]
Experiment	0.40	[Ku et al. ‘12]

Theory "versus" Experiment





- very good agreement between theory and experiment with respect to the phase transition in the balanced limit
- **experiments** with different **mass imbalances** are now in reach (6Li, 40K, 161Dy, 163Dy and 167Er)

Summary

- overall: very good agreement between theoretical predictions and experiments for balanced unitary Fermi gases
- EOS of mass-imbalanced Fermi gases
  - testable prediction for future experiments
- strong indications for the existence of a crystalline phase at large mass imbalances
  - (very) exact coordinates?
  - what is the “structure” of the “crystal”?

Summary

- overall: very good agreement between theoretical predictions and experiments for balanced unitary Fermi gases
- EOS of mass-imbalanced Fermi gases
  - testable prediction for future experiments
- strong indications for the existence of a crystalline phase at large mass imbalances
  - (very) exact coordinates?
  - what is the “structure” of the “crystal”?

STAY TUNED