the DNA of asymptotic safety

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based on work with Andrew Bond and Tugba Buyukbese

ICTP
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ERG2016

19 - 23 September 2016
Miramare - Trieste, Italy

standard model

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local QFT for fundamental interactions

strong nuclear force
weak force
electromagnetic force

degrees of freedom

spin 0 (the Higgs has finally arrived)
spin 1/2 (quite a few)
spin 1

perturbatively renormalisable & predictive

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local QFT for fundamental interactions

strong nuclear force
weak force
electromagnetic force

open challenges

many...

what comes beyond the SM?
how does gravity fit in?


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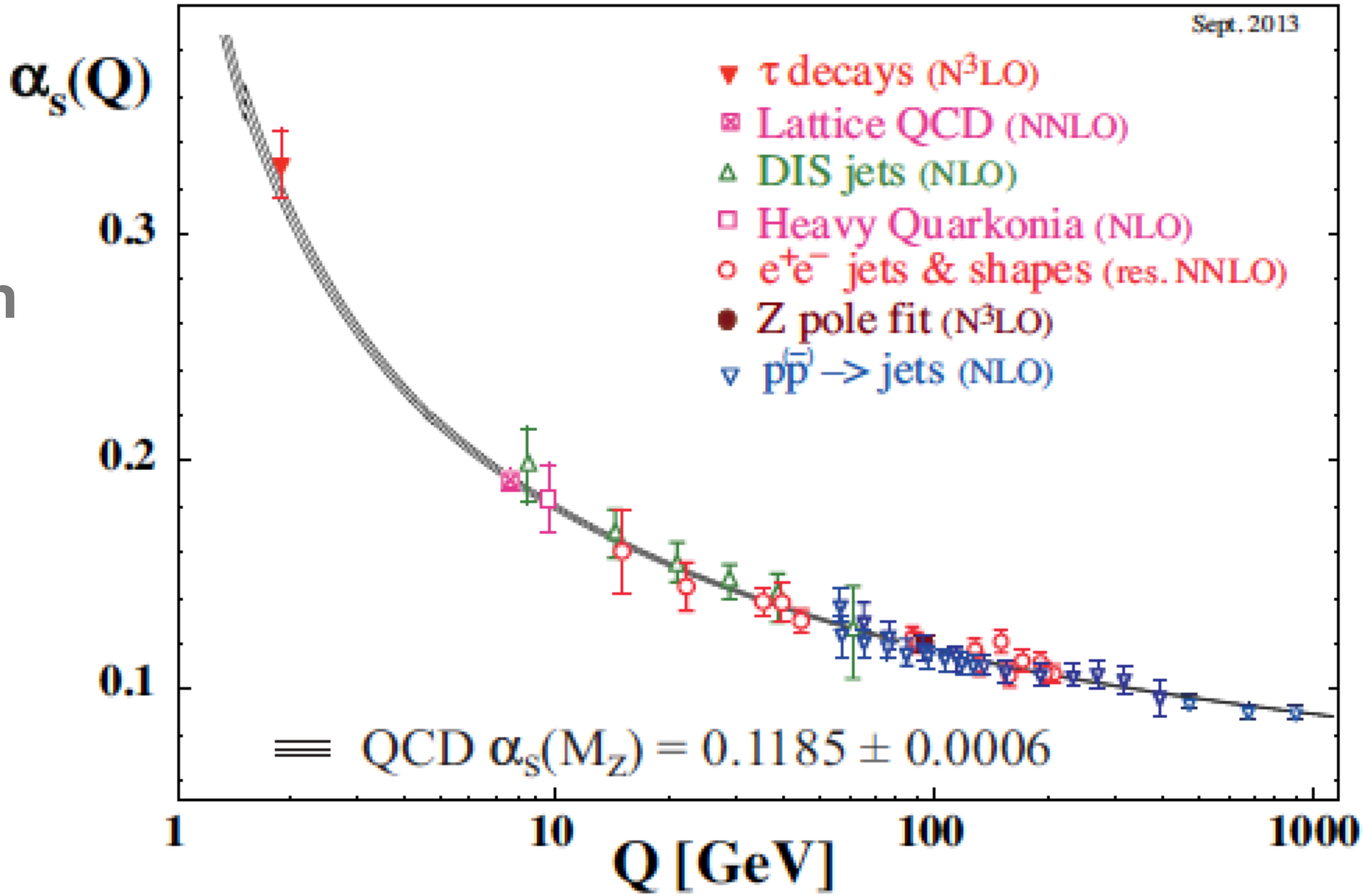
asymptotic freedom

triumph of QFT



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asymptotic freedom

complete asymptotic freedom in 4D

all couplings achieve non-interacting UV fixed point

fields	cAF
scalars	no
scalars with fermions	no
non-Abelian gauge fields	yes
non-Abelian fields with fermions	yes*
non-Abelian fields, fermions, scalars	yes*

*) provided certain conditions hold true


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idea:

some or all couplings achieve
interacting UV fixed point

Wilson '71
Weinberg '79

if so, new opportunities for
BSM physics & quantum gravity

proof of existence:

4D gauge-Yukawa theory with
exact asymptotic safety

Litim, Sannino, 1406.2337
(see talk by F Sannino)

asymptotic safety

today:

weakly interacting fixed points of general 4d gauge theories

conditions for asymptotic safety

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asymptotic safety

Bond, Litim 1608.00519

results

case	gauge group	matter	Yukawa	asymptotic safety
a)	simple	fermions in irreps	No	No
b)	simple or abelian	fermions, any rep	No	No
		scalars, any rep	No	No
		fermions and scalars, any rep	No	No
c)	semi-simple, with or without abelian factors	fermions, any rep	No	No
		scalars, any rep	No	No
		fermions and scalars, any rep	No	No
d)	simple or abelian	fermions and scalars, any rep	Yes	Yes *)
e)	semi-simple, with or without abelian factors	fermions and scalars, any rep	Yes	Yes *)

*) provided certain auxiliary conditions hold true

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fixed points

gauge theory

$$\alpha = \frac{g^2}{(4\pi)^2} \qquad \beta = -B\,\alpha^2 + C\,\alpha^3 + \mathcal{O}(\alpha^4)$$

weakly coupled fixed point

$$0 < \alpha^* = B/C \ll 1$$

competition between matter and gauge fields

$$B = \frac{2}{3}\left(11C_2^G - 2S_2^F - \frac{1}{2}S_2^S\right)$$

$$C = 2\left[\left(\frac{10}{3}C_2^G + 2C_2^F\right)S_2^F + \left(\frac{1}{3}C_2^G + 2C_2^S\right)S_2^S - \frac{34}{3}(C_2^G)^2\right]$$

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fixed points

gauge theory

$$\alpha = \frac{g^2}{(4\pi)^2}$$

$$\beta = -B\,\alpha^2 + C\,\alpha^3 + \mathcal{O}(\alpha^4)$$

weakly coupled fixed point $0 < \alpha^* = B/C \ll 1$

competition between matter and gauge fields

$B, C > 0:$

asymptotic freedom
Caswell-Banks-Zaks **IR FP**

$B, C < 0:$

asymptotic safety **UV FP**
no examples

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fixed points

gauge theory

$$\alpha = \frac{g^2}{(4\pi)^2} \qquad \beta = -B\,\alpha^2 + C\,\alpha^3 + \mathcal{O}(\alpha^4)$$

UV fixed point

$$C = \frac{2}{11}\left[2S_2^F\left(11C_2^F + 7C_2^G\right) + 2S_2^S\left(11C_2^S - C_2^G\right) - 17B\,C_2^G\right]$$

fermions — scalars — 1-loop

no go theorem, case a)

Caswell '74

fixed points

gauge theory

$$\alpha = \frac{g^2}{(4\pi)^2} \qquad \beta = -B\,\alpha^2 + C\,\alpha^3 + \mathcal{O}(\alpha^4)$$

UV fixed point

$$C = \frac{2}{11}\left[2S_2^F\left(11C_2^F + 7C_2^G\right) + 2S_2^S\left(11C_2^S - C_2^G\right) - 17B\,C_2^G\right]$$

fermions — scalars — 1-loop

must have

$$C_2^S < \frac{1}{11}C_2^G$$

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quadratic Casimirs

highest weights, Racah formula

$$C_2(\Lambda) = \frac{1}{2}(\Lambda, \Lambda + 2\delta)$$

$$\delta = (1, 1, \ldots, 1)$$

weight metrics G

$$(u, v) \equiv \sum_{ij} G_{ij}\, u^i\, v^j$$

fundamental weights

$$(\Lambda_k)^i = \delta^i_k$$

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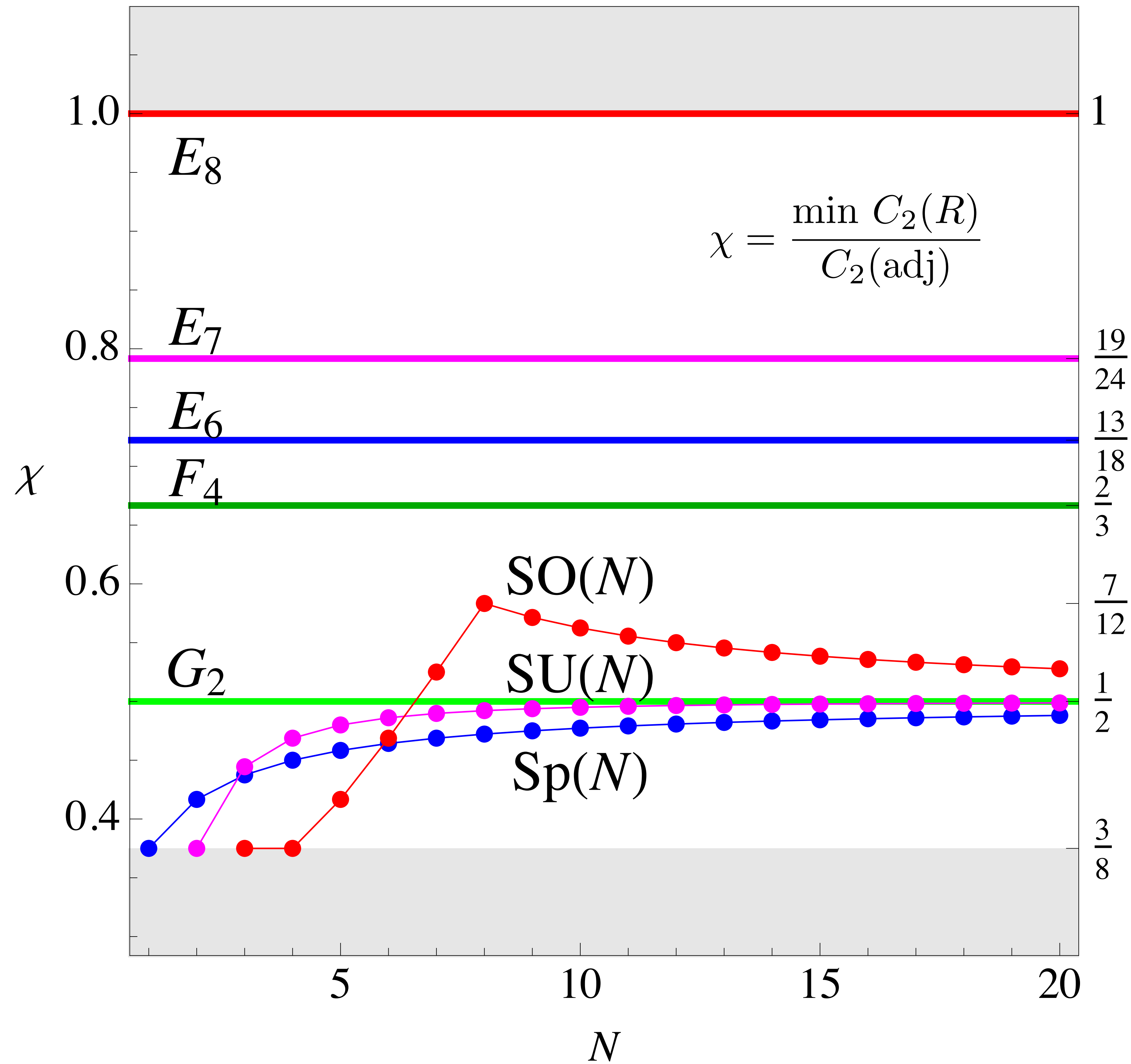
quadratic Casimirs

result

symmetry	range	min C_2	C_2(adj)	χ	irrep with smallest C_2
$SU(N)$	$N \geq 2$	$\frac{N^2-1}{2N}$	N	$\frac{1}{2}\left(1-\frac{1}{N^2}\right)$	fundamental $\mathbf{N}$ and $\overline{\mathbf{N}}$
$SO(N)$	$3 \leq N \leq 7$	$\frac{1}{16}N(N-1)$	$N-2$	$\frac{N}{16}\frac{N-1}{N-2}$	fundamental spinors $\mathbf{2}^{\lceil N/2 \rceil - 1}$
	$N = 8$	$\frac{7}{2}$	6	$\frac{7}{12}$	fundamental vector $\mathbf{8_v}$ and fundamental spinors $\mathbf{8_s}$, $\mathbf{8_c}$
	$N \geq 9$	$\frac{1}{2}(N-1)$	$N-2$	$\frac{N-1}{2(N-2)}$	fundamental $\mathbf{N}$
$Sp(N)$	$N \geq 1$	$\frac{1}{4}(2N+1)$	$N+1$	$\frac{2N+1}{4(N+1)}$	fundamental $\mathbf{2N}$
E_8		30	30	1	adjoint $\mathbf{248}$
E_7		$\frac{57}{4}$	18	$\frac{19}{24}$	fundamental $\mathbf{56}$
E_6		$\frac{26}{3}$	12	$\frac{13}{18}$	fundamental $\mathbf{27}$ and $\overline{\mathbf{27}}$
F_4		6	9	$\frac{2}{3}$	fundamental $\mathbf{26}$
G_2		2	4	$\frac{1}{2}$	fundamental $\mathbf{7}$

quadratic Casimirs



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no go theorems I

must have $$C_2^S < \frac{1}{11} C_2^G$$

instead, we find $$C_2^S \geq \frac{3}{8} C_2^G$$

implication: $$B \leq 0 \quad \Rightarrow \quad C > 0$$

no go theorem, case b)

no go theorems II

more gauge factors $$\beta_a = \alpha_a^2\,(-B_a + C_{ab}\,\alpha_b) + \mathcal{O}(\alpha^4)$$

interacting fixed points $$B_a = C_{ab}\,\alpha_b^*$$

non-trivial mixing $$C_{ab} = 4\left(C_2^{F_b}\,S_2^{F_a} + C_2^{S_b}\,S_2^{S_a}\right) \qquad (a \neq b)$$

positive definite $$C_{ab} \geq 0$$

implication: $$B_a \leq 0 \quad \Rightarrow \quad C_{ab} \geq 0 \quad \text{for all } b$$

no go theorem, case c)

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asymptotic safety

result

case	gauge group	matter	Yukawa	asymptotic safety
a)	simple	fermions in irreps	No	No
b)	simple or abelian	fermions, any rep	No	No
		scalars, any rep	No	No
		fermions and scalars, any rep	No	No
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***) provided certain auxiliary conditions hold true**

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matter couplings

scalar self-couplings **No** (start at 3- or 4-loop)

Yukawa couplings

scalar self-couplings	No (start at 3- or 4-loop)
Yukawa couplings	Yes (start at 2-loop)

Yukawa couplings

scalar self-couplings **No** (start at 3- or 4-loop)

Yukawa couplings **Yes** (start at 2-loop)

$$\sim \frac{1}{2}(\mathbf{Y}^A)_{JL}\phi^A\,\psi_J\,\zeta\,\psi_L$$

$$\beta = \alpha^2\left(-B + C\,\alpha - 2\,Y_4\right)$$

$$Y_4 = \mathrm{Tr}[\mathbf{C}_2^F\,\mathbf{Y}^A\,(\mathbf{Y}^A)^\dagger]/d(G) \geq 0$$

Yukawa's slow down the running of the gauge

induced shift $B \to B' = B + 2Y_4^* > B$

fixed point $\alpha^* = \dfrac{B'}{C}$

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Yukawa couplings

Yukawa couplings $$\beta^A = \mathbf{E}^A(Y) - \alpha\,\mathbf{F}^A(Y)$$

Yukawa nullclines $\beta^A = 0$

$$\mathbf{Y}^A_* = 0$$ **Gauss**

$$\mathbf{Y}^A_* = \frac{g}{4\pi}\,\mathbf{C}^A$$ **int. FP**

Yukawa couplings

Yukawa couplings $\beta^A = \mathbf{E}^A(Y) - \alpha\,\mathbf{F}^A(Y)$

Yukawa nullclines $\beta^A = 0$

$$\mathbf{Y}_*^A = 0$$ Gauss

$$\mathbf{Y}_*^A = \frac{g}{4\pi}\,\mathbf{C}^A$$ int. FP

$$\Rightarrow Y_4^* = D \cdot \alpha$$

$$D = \mathrm{Tr}[\mathbf{C}_2^F\,\mathbf{C}^A\,(\mathbf{C}^A)^\dagger]/d(G) \geq 0$$

Yukawa contributions modify two-loop gauge term

Yukawa couplings

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Yukawa couplings

$$\beta^A = \mathbf{E}^A(Y) - \alpha\,\mathbf{F}^A(Y)$$

$$\beta = \alpha^2\,(-B + C\,\alpha - 2\,Y_4)$$

$$Y_4^* = D \cdot \alpha$$

$$\overset{!}{=} \alpha^2(-B + C'\alpha)$$

induced shift $C \to C' = C - 2D < C$

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Yukawa couplings

Yukawa couplings $\beta^A = \mathbf{E}^A(Y) - \alpha\,\mathbf{F}^A(Y)$

$$\beta = \alpha^2\,(-B + C\,\alpha - 2\,Y_4)$$

$$\overset{!}{=} \alpha^2(-B + C'\alpha)$$

$$Y_4^* = D \cdot \alpha$$

induced shift $C \to C' = C - 2D < C$

reliable fixed point $\alpha_* = B/C'$ even if B<0 and C>0

impossible without Yukawa's

necessary condition for asymptotic safety $C' < 0$

case d)

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more gauge couplings

$$\beta_a = \alpha_a^2\,(-B_a\ + C_{ab}\,\alpha_b - 2\,Y_{4,a})$$

Yukawa-induced shift $\quad B_a \to B'_a = B_a + 2\,Y^*_{4,a}$

fixed points $\quad B'_a = C_{ab}\alpha^*_b$

novel solutions including UV FPs if B<0 and B’>0

Yukawas may compensate gauge contributions

necessary condition for asymptotic safety $\quad B'_a > 0$

case e)

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asymptotic safety

result

case	gauge group	matter	**Yukawa**	**asymptotic safety**
a)	simple	fermions in irreps	**No**	**No**
b)	simple or abelian	fermions, any rep	**No**	**No**
		scalars, any rep	**No**	**No**
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*) **provided certain auxiliary conditions hold true**

interacting FPs

case	gauge group	Yukawa	parameter	interacting FPs	type
a)	simple	**No**	$B > 0$ and $C > 0$	Banks-Zaks	**IR**
b)	semi-simple, no $U(1)$ factors	**No**	all $B_a > 0$	Banks-Zaks and products thereof	**IR**
c)	simple	**Yes**	$B > 0$ and $C > 0 > C'$	Banks-Zaks	**IR**
	simple	**Yes**	$B > 0$ and $C > C' > 0$	BZ and GYs	**IR**
	simple or abelian	**Yes**	$B < 0$ and $C' < 0$	gauge-Yukawas	**UV/IR**
d)	semi-simple, with or without $U(1)$ factors	**Yes**	all $B'_a > 0$	BZs and GYs and products thereof	**UV/IR**

phase diagrams

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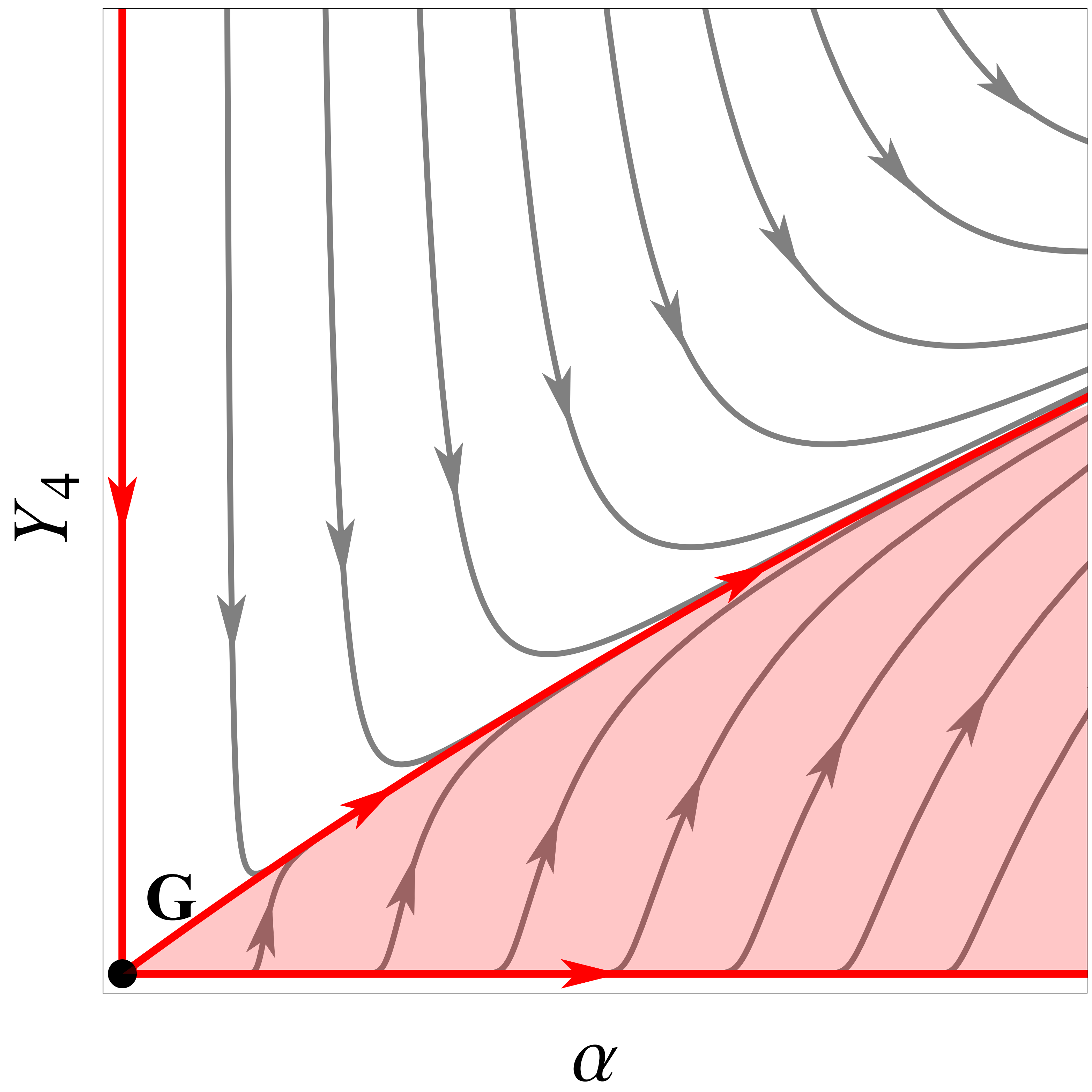
phase diagrams of **simple gauge theories**

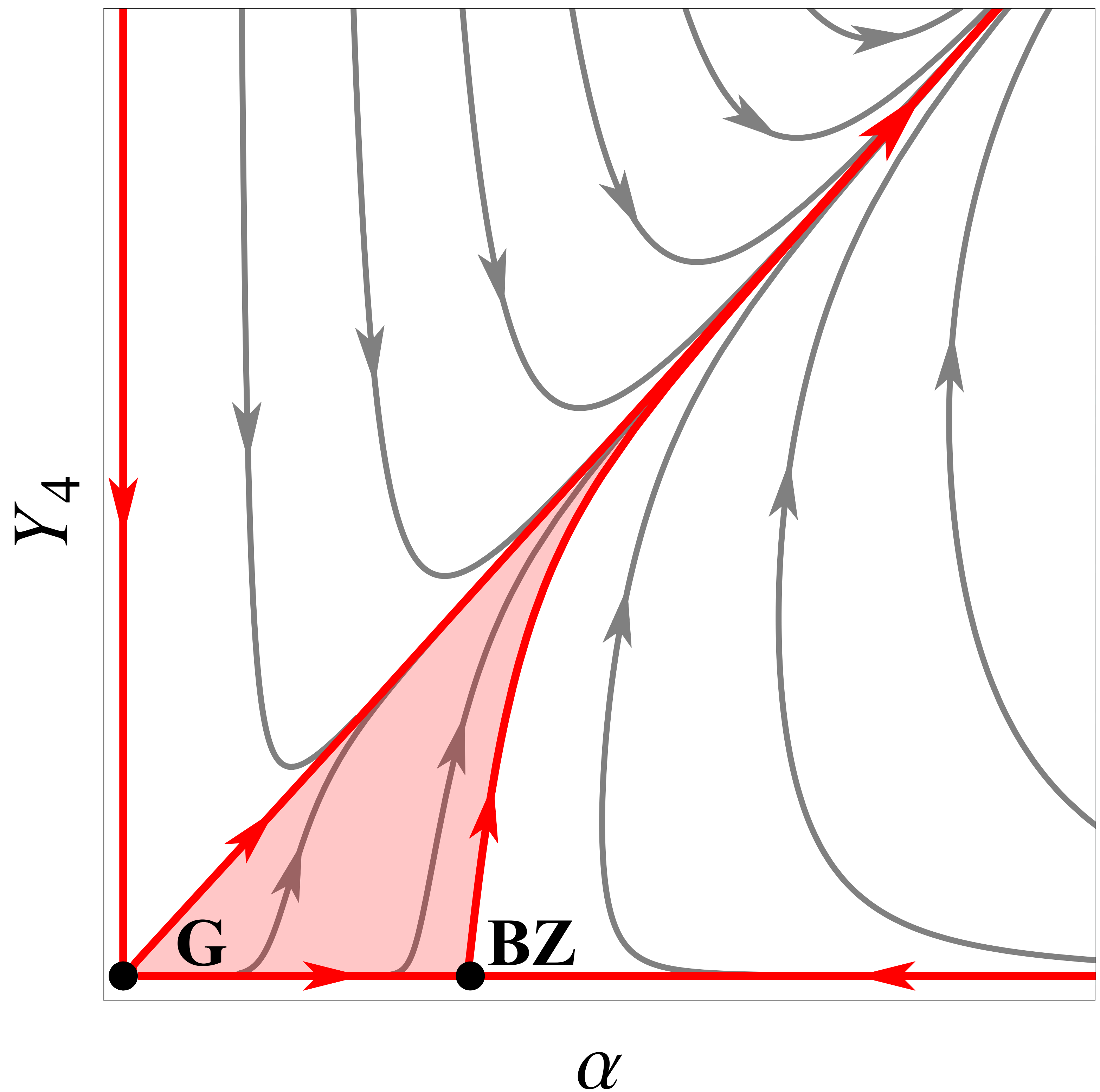
parameters	B, C	matter content
	C'	Yukawa structure

phase diagrams



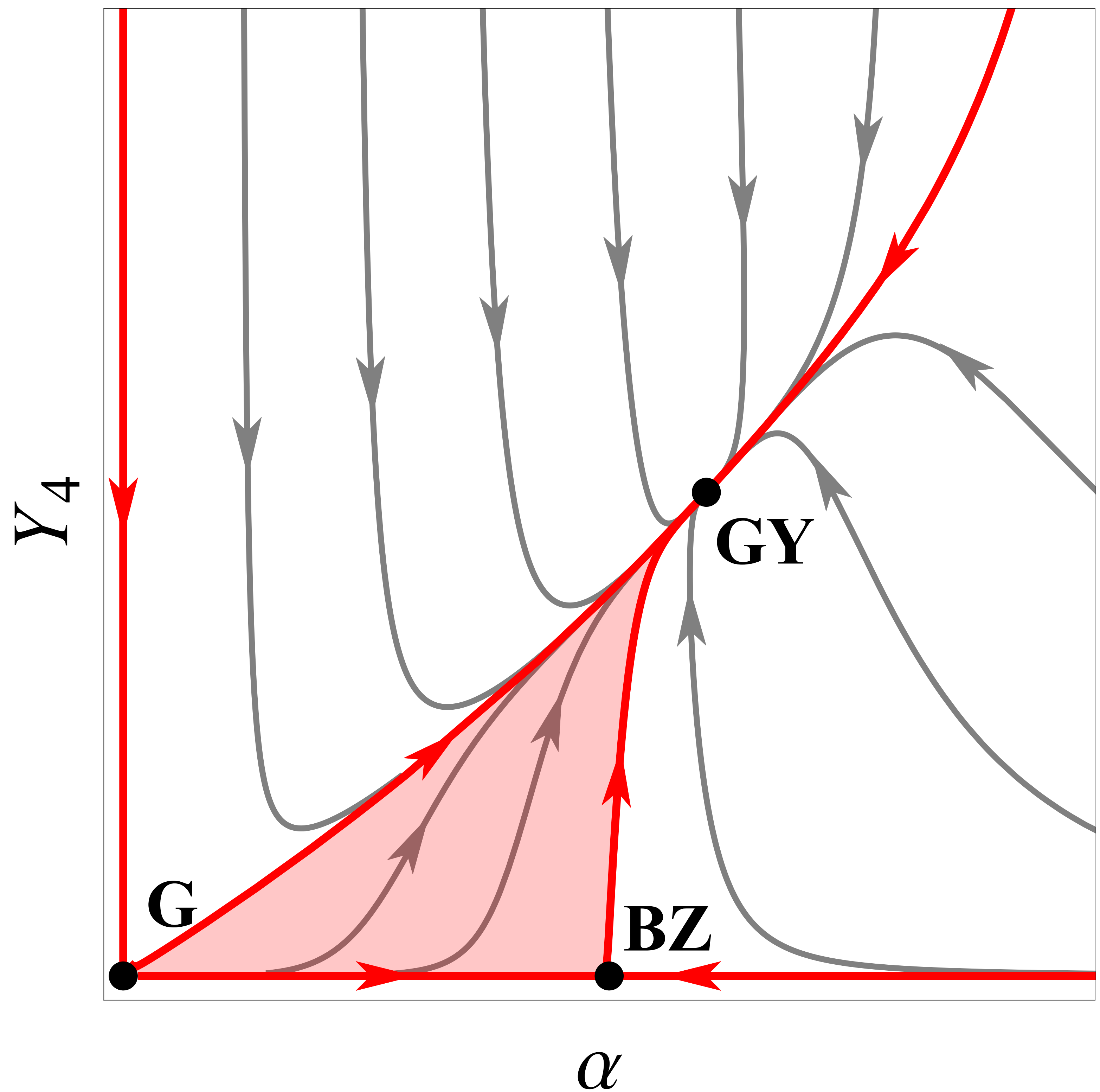
$(B > 0)$

phase diagrams



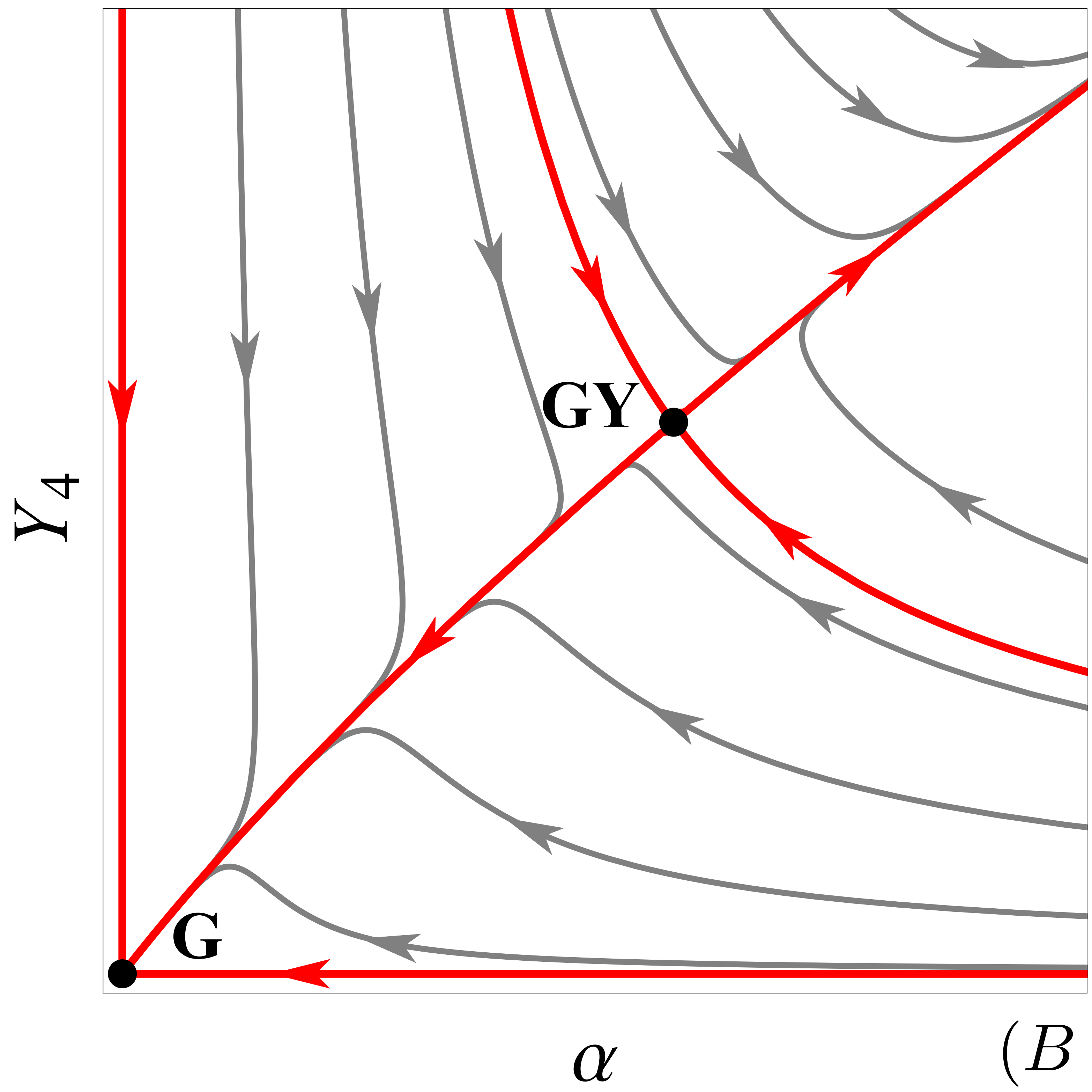
$(B, C > 0)$

phase diagrams



$(B, C, C' > 0)$

phase diagrams



$(B < 0, C > 0 > C')$

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extensions I

interacting UV FPs with exact asymptotic safety exist for simple gauge theories

Litim, Sannino, 1406.2337

but: do interacting UV FPs with exact asymptotic safety exist for semi-simple gauge theories?

Yes! (talk by Andrew Bond)

→ space of UV FP solutions is non-empty

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extensions II

what is the impact of higher-dimensional invariants?

tool: functional RG

(see **poster** by Tugba Buyukbese)

results:

fixed point persists
effective potential remains stable

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extensions II

Lagrangean

$$L_{\rm YM} = -\frac{1}{2}\,{\rm Tr}\,F^{\mu\nu}F_{\mu\nu}$$

$$L_F = {\rm Tr}\left(\overline{Q}\,i\not{D}\,Q\right)$$

$$L_Y = y\,{\rm Tr}\left(\overline{Q}\,H\,Q\right)$$

$$L_H = {\rm Tr}\,(\partial_\mu H^\dagger\,\partial^\mu H)$$

$$L_U = -u\,{\rm Tr}\,(H^\dagger H)^2$$

$$L_V = -v\,({\rm Tr}\,H^\dagger H)^2\,.$$

Litim, Sannino, 1406.2337

further scalar invariants

$$v_k(i_1, i_2) = u_k(i_1) + i_2 c_k(i_1)$$

$$u_k(i_1) = \sum_{j=2}^{N_i} \frac{(4\pi)^{2j-2} i_1^j \lambda_{2j-2}}{N_f^{2j-2}}$$

$$c_k(i_1) = \sum_{i=0}^{N_i} \frac{(4\pi)^{2j+2} i_1^j \lambda_{2j+1}}{N_f^{2j+1}}$$

$$i_1 = {\rm Tr}(h^\dagger h)$$

$$i_2 = {\rm Tr}\left((h^\dagger h)^2 - \frac{1}{N_f}({\rm Tr} h^\dagger h)^2\right)$$

Buyukbese, Litim (in prep.)

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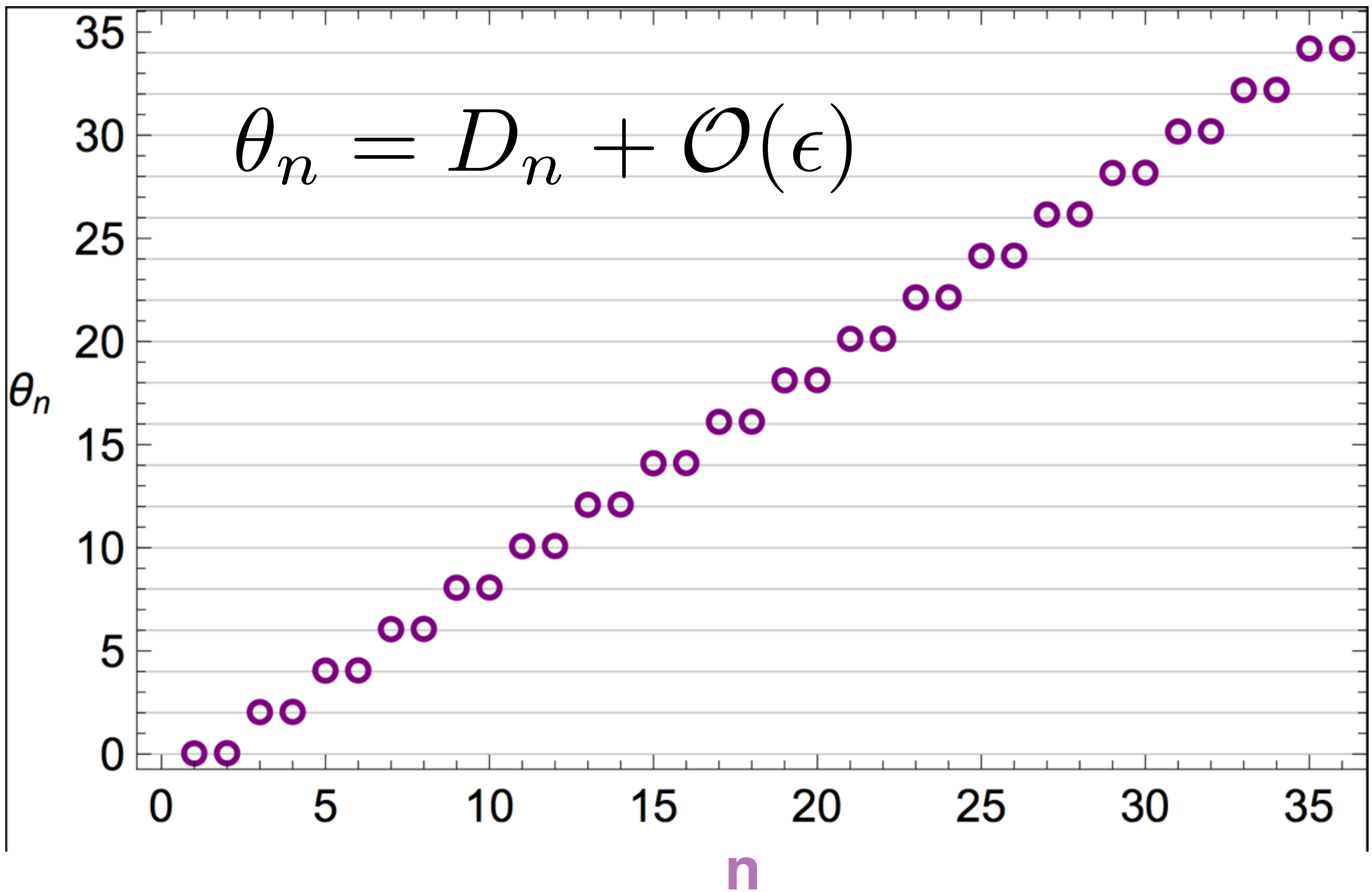
extensions II

results: **exact eigenvalue spectrum**



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conclusions

identified all weakly interacting fixed points of general 4D gauge theories - rich spectrum

- strict **no go theorems** together with necessary and sufficient **conditions for asymptotic safety** for general 4D gauge theories

- Yukawa interactions pivotal for asymptotic safety

asymptotic safety persist beyond canonically marginal invariants

window of opportunities for BSM