Frequency dependence of the vertex function for the fRG and beyond

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Main collaborators

Stuttgart: Walter Metzner, Demetrio Vilardi (*next talk*)

Vienna: Nils Wentzell

ERG 2016, Trieste

Outline

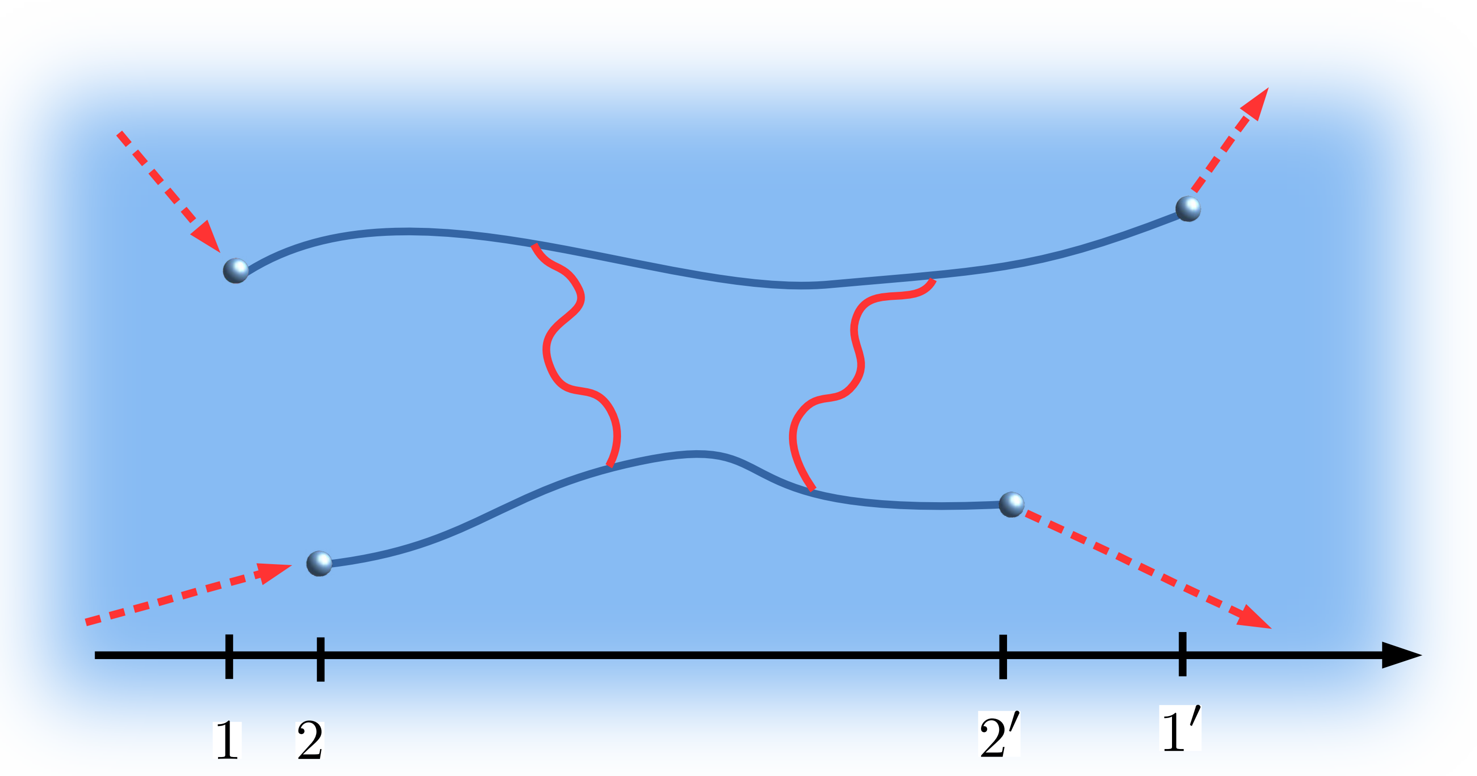
- Vertex frequency dependence (Part I)
  - Definitions, **1PI** and **2PI** vertexes
  - **Diagrammatic** understanding of the vertex structures
  - Vertex **decomposition** (and reconstruction)

- Towards strong coupling (Part II)
  - Apllication: combining **DMFT** and **fRG** (DMF^2RG)

    continued in next talk

1PI vertex in (fermionic) fRG

2-particle picture: $\langle c_{1'} c_{2'} c_2^\dagger c_1^\dagger \rangle$ $\neq \langle c_{1'} c_1^\dagger \rangle \langle c_{2'} c_2^\dagger \rangle - \langle c_{2'} c_1^\dagger \rangle \langle c_{1'} c_2^\dagger \rangle$



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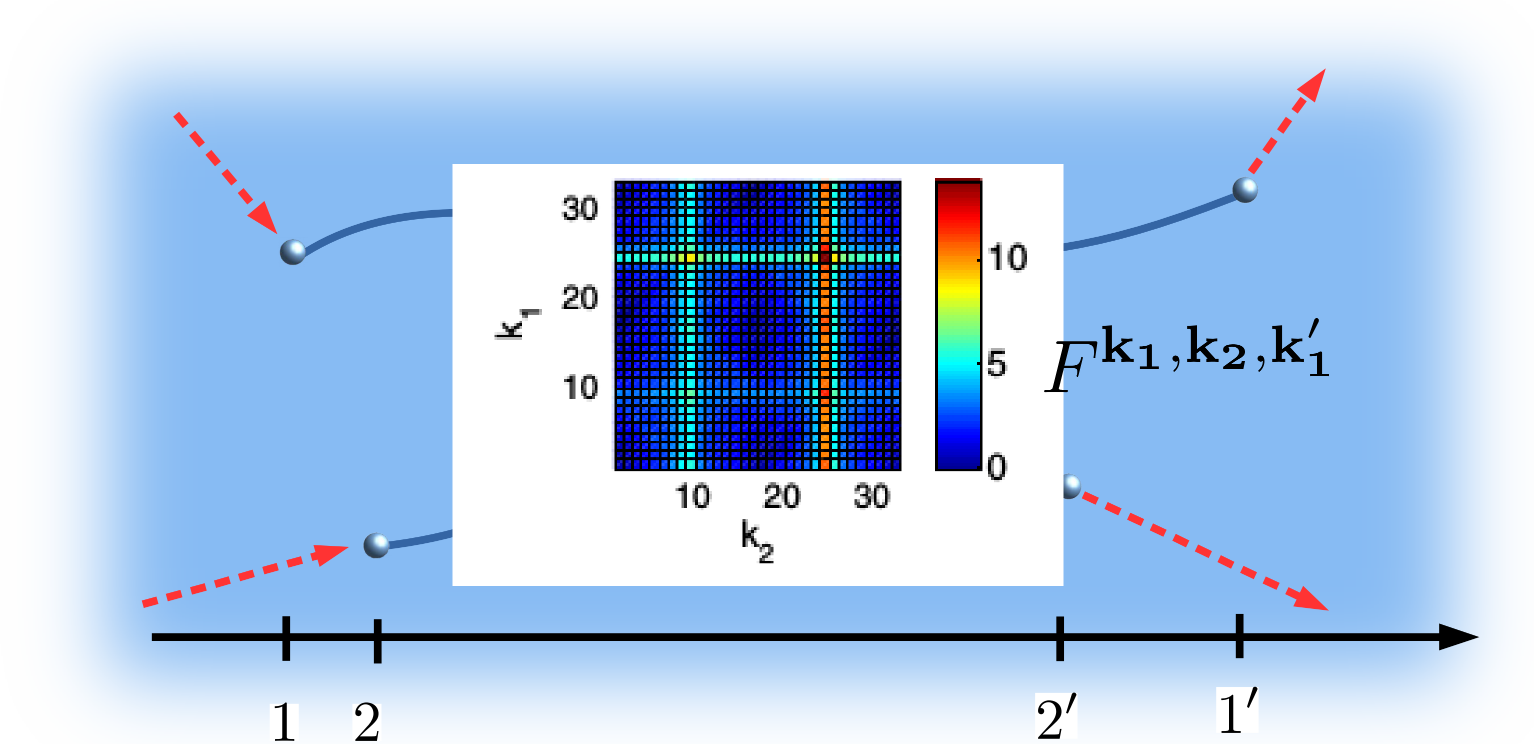

Review: Metzner *et al.*,RMP '12

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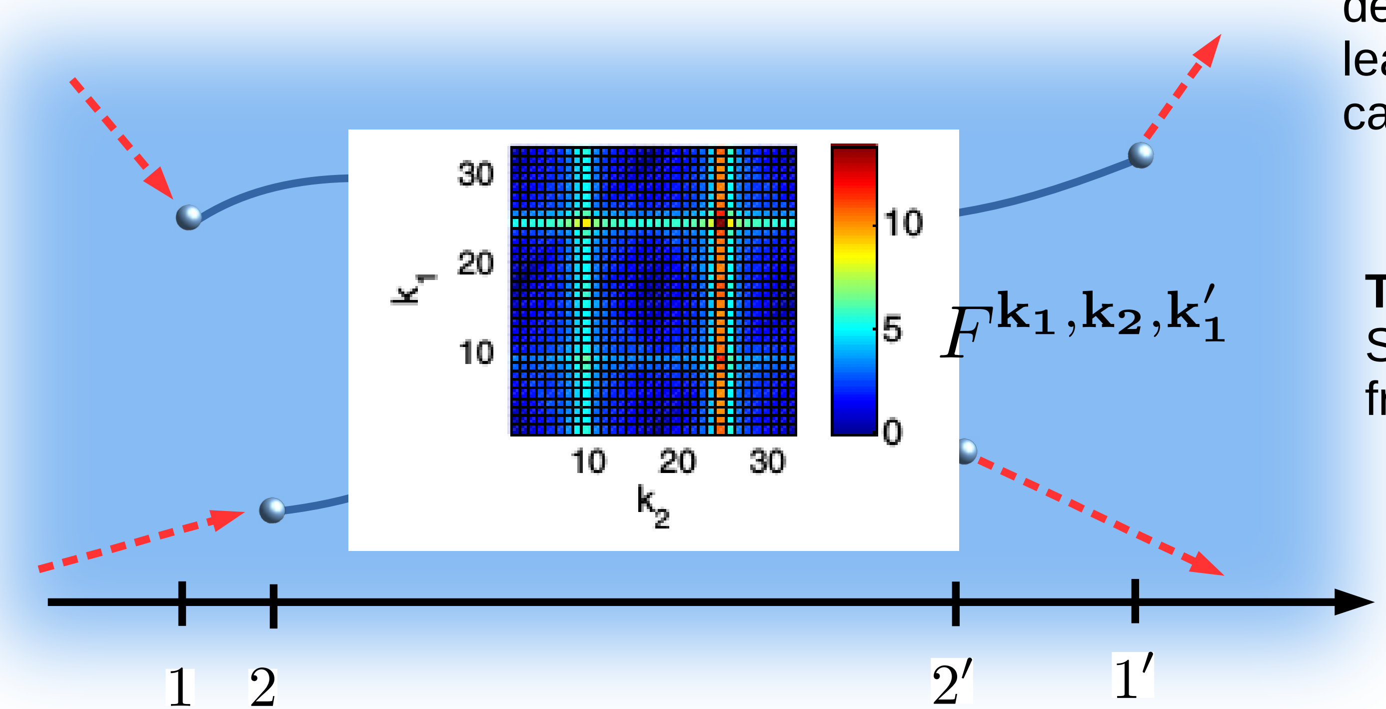


fRG:
Momentum dependence→ leading instabilities; calculation of susceptibilities

This talk:
Systematic analysis of frequency dependence

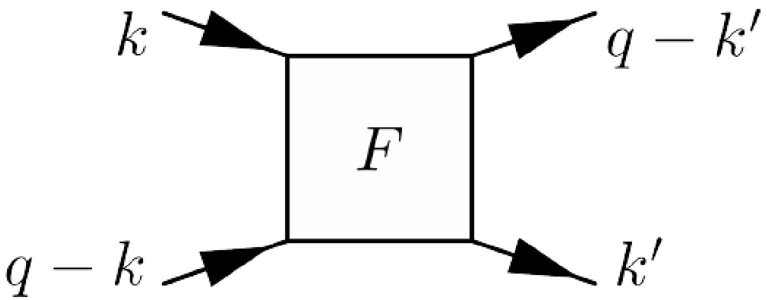
Review: Metzner *et al.*,RMP '12

Notation conventions

$k = (i\omega, \mathbf{k})$ "fermionic" 4-vector

$q = (i\Omega, \mathbf{q})$ "bosonic" 4-vector

$F_{pp}^{kk'q} =$



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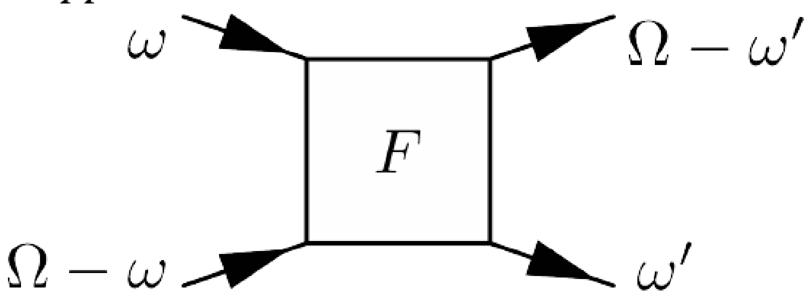
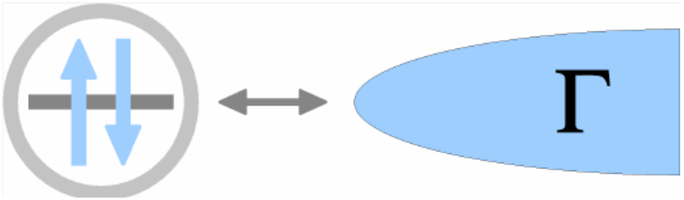


SIAM in wide-band limit



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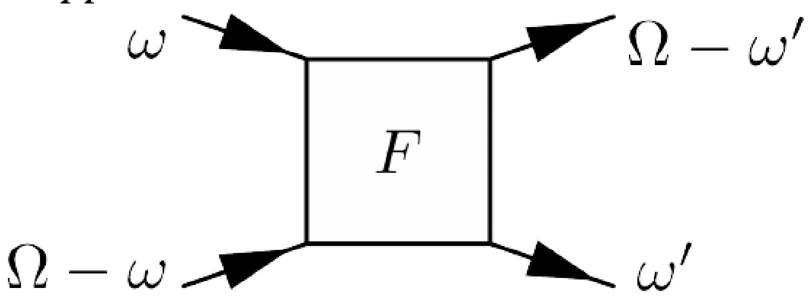
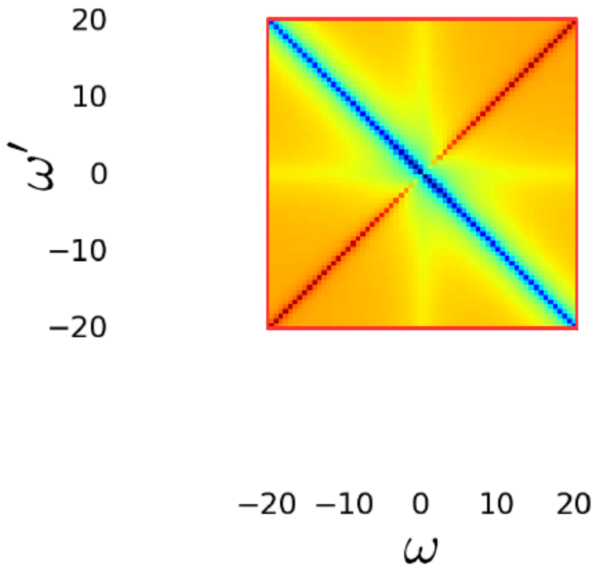
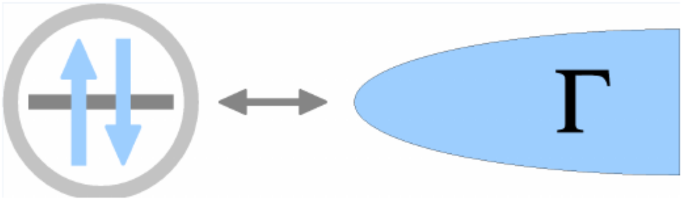




Computed in ***fRG***

SIAM in wide-band limit



Plot a fixed transfer frequency $i\Omega$

Diagonal & **horizontal**
frequency structure →
Large frequency behavior

Decomposing the vertex

Parquet equation

$$F = \Lambda_{2\mathrm{PI}} + \phi_{pp} + \phi_{ph} + \phi_{\overline{ph}}$$

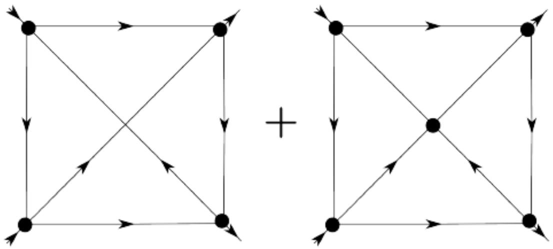
Decomposing the vertex

Parquet equation

$$F = \Lambda_{\mathrm{2PI}} + \phi_{pp} + \phi_{ph} + \phi_{\overline{ph}}$$

2-particle irreducible

$\Lambda_{\mathrm{2PI}} =$  $+$

 $+ \ldots$

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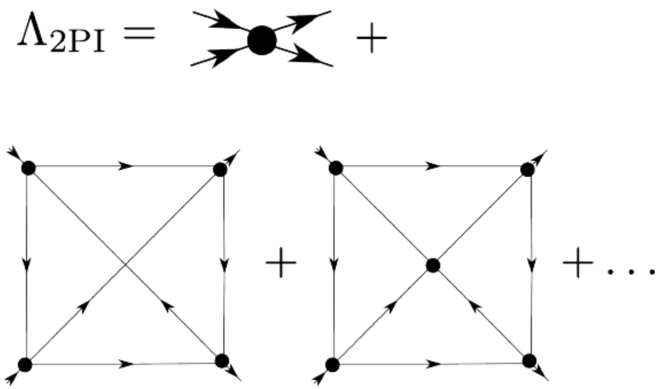

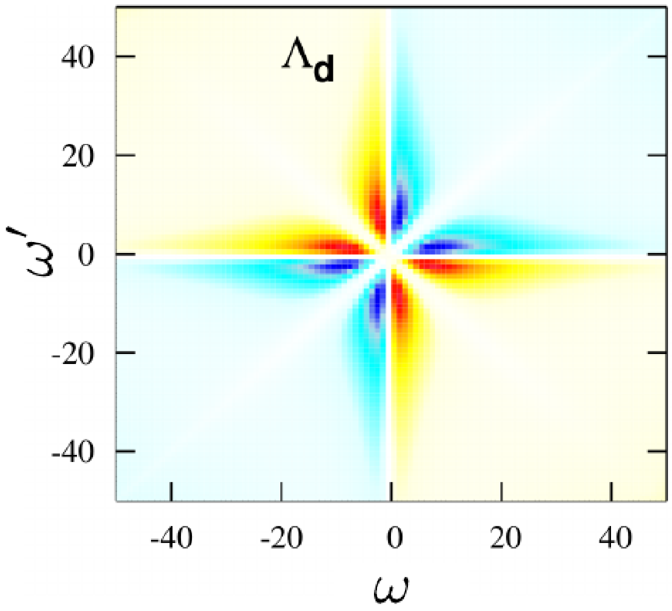

Rohringer,Valli and Toschi,PRB'12

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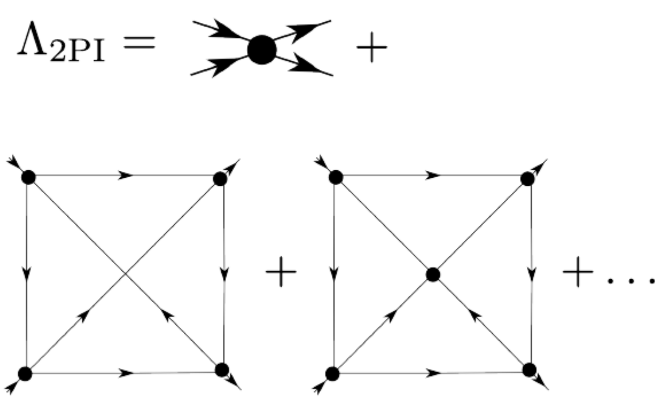
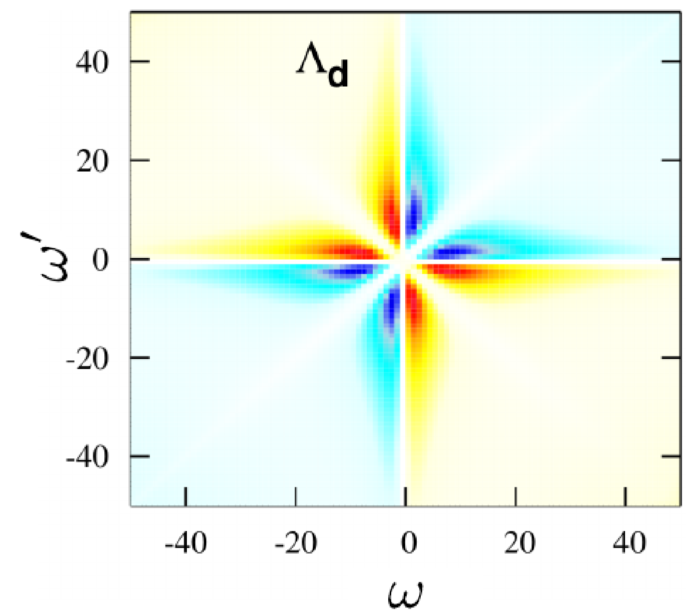


ED result



No two-particle irreducible terms in fRG at one-loop truncation level

Rohringer,Valli and Toschi,PRB'12

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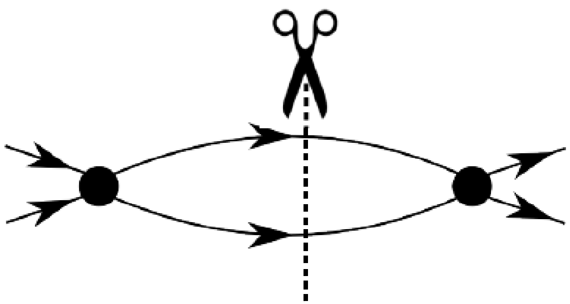
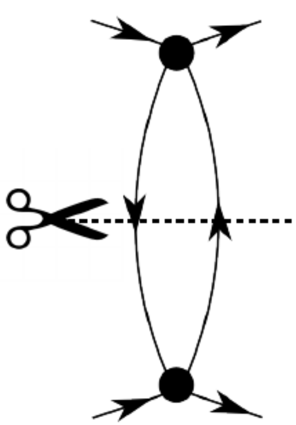
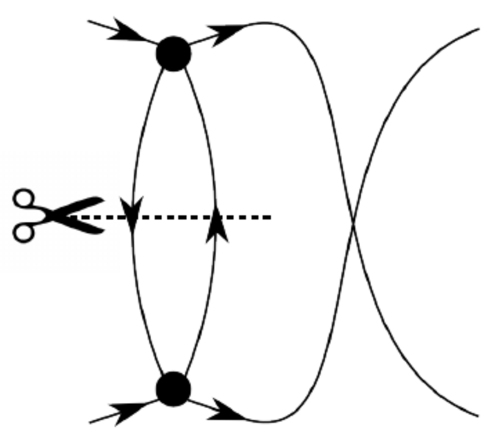
$$F = \Lambda_{\mathrm{2PI}} + \underbrace{\phi_{pp} + \phi_{ph} + \phi_{\overline{ph}}}_{\text{2-particle reducible}}$$

fRG: integrate each channel separately







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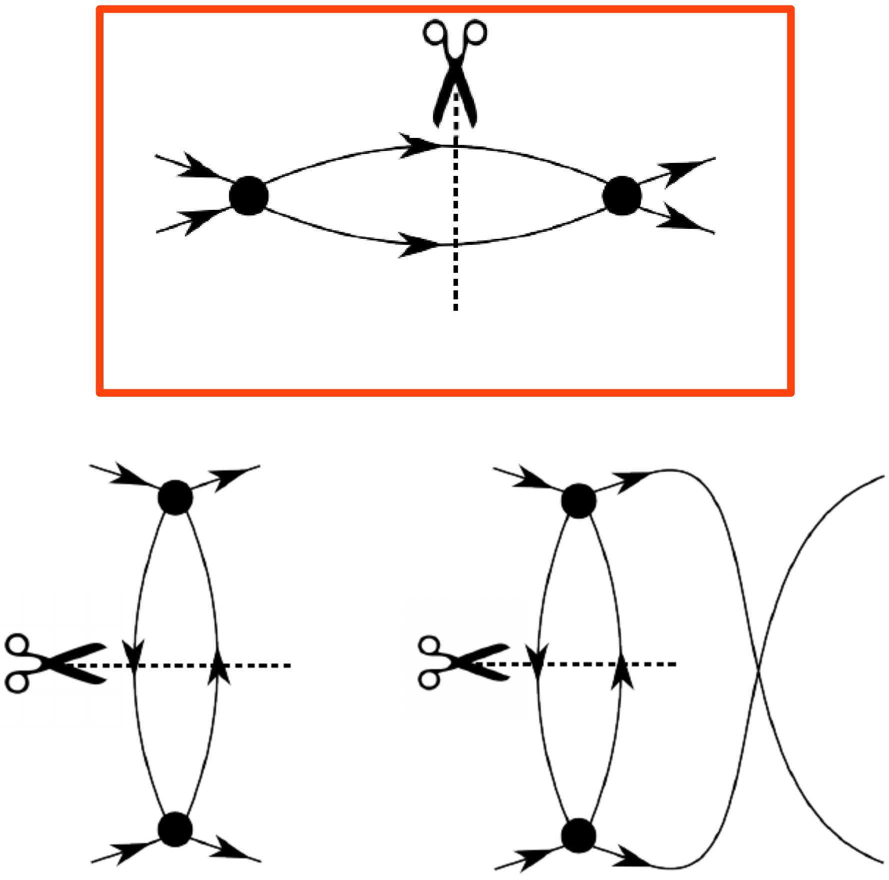

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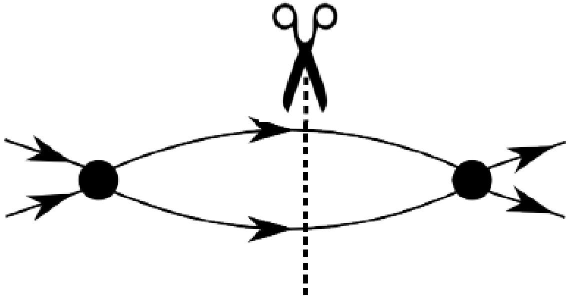

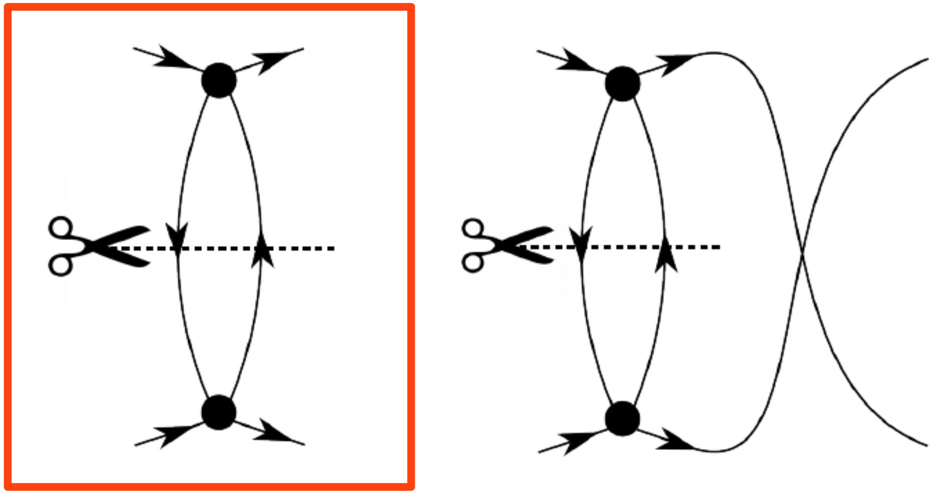

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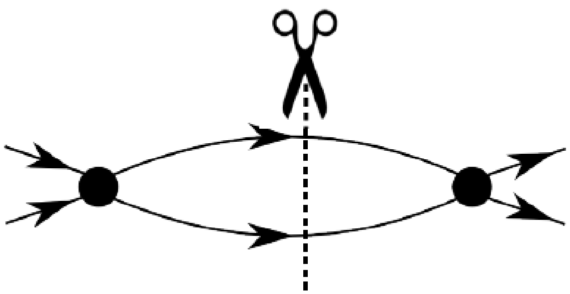

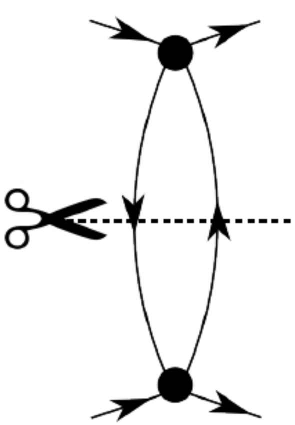

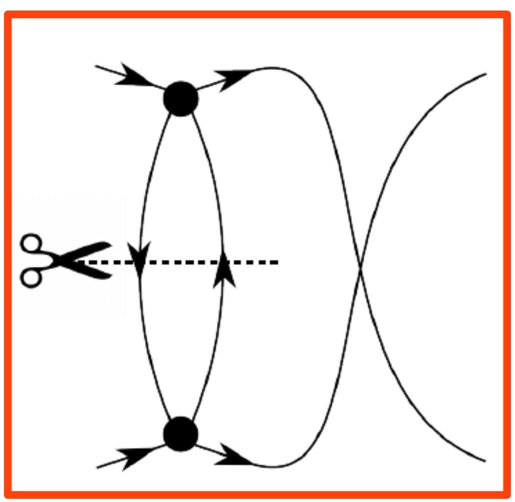

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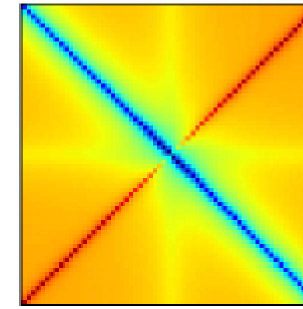

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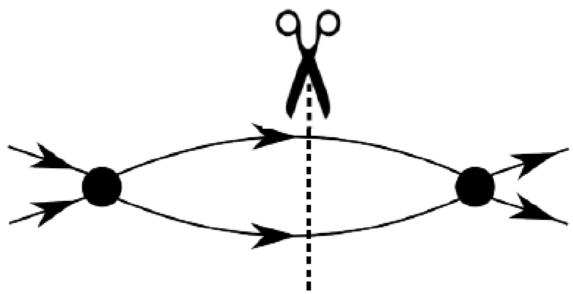

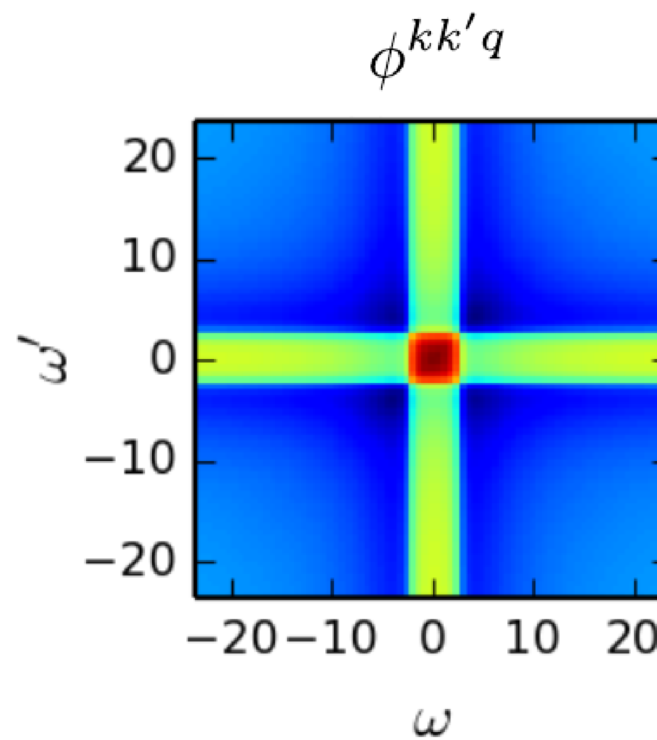

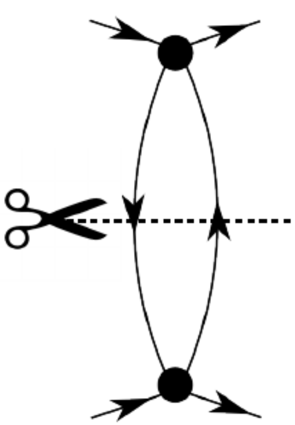

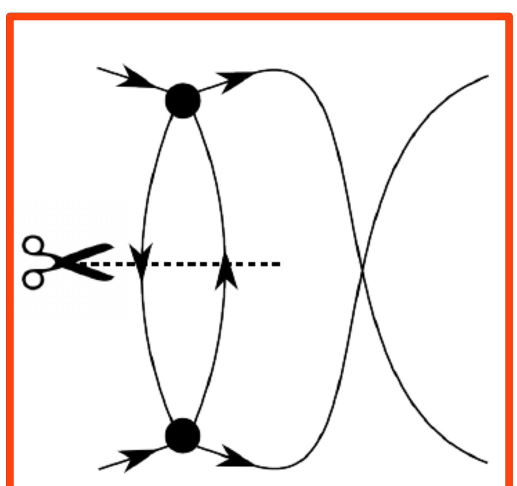

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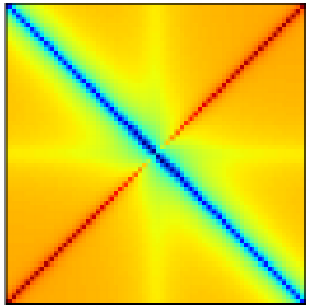

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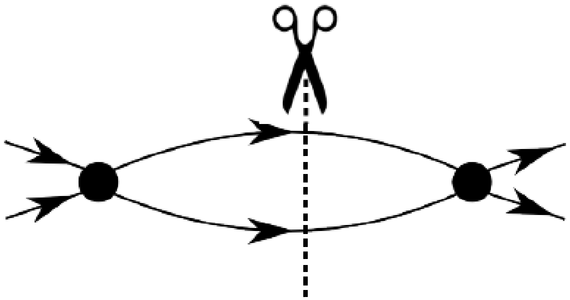
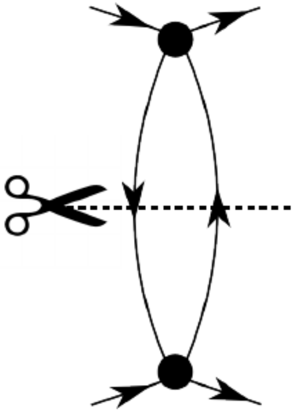
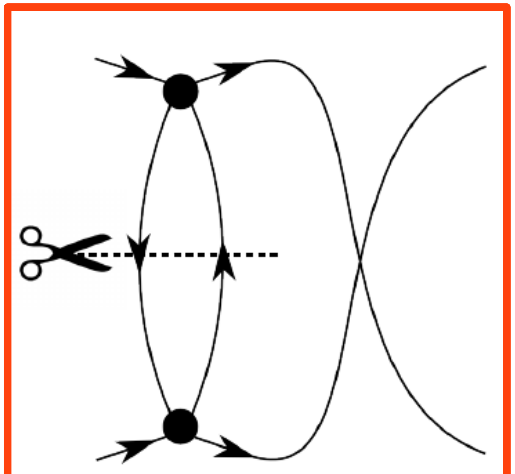
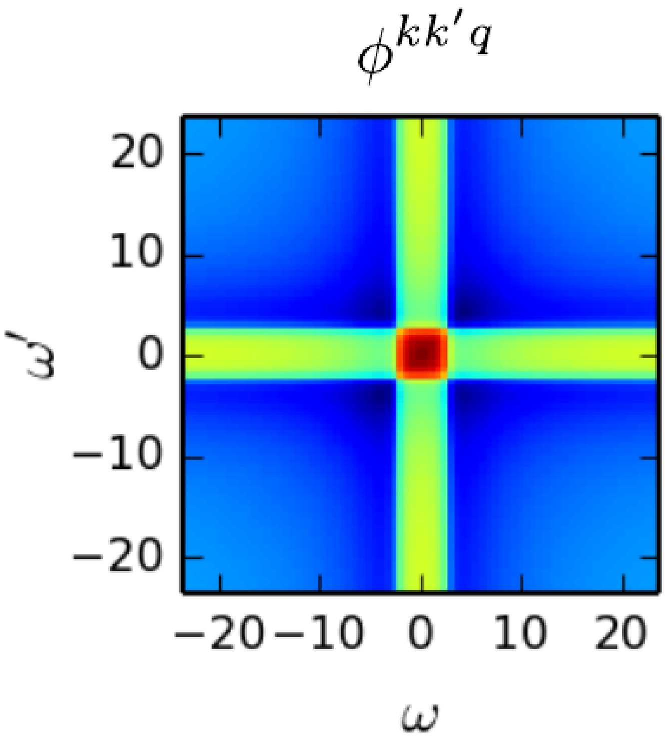






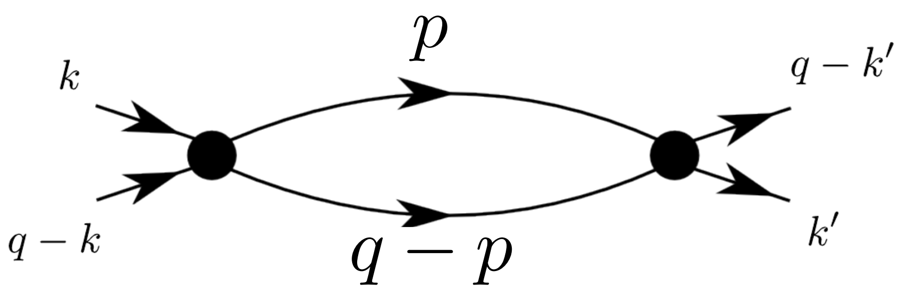


Can one further understand the structures?

Diagrammatic classification

$$\phi = \mathcal{K}_1 + \mathcal{K}_2 + \overline{\mathcal{K}_2} + \mathcal{R}$$

Assumption: *Bare* interaction
Local and frequency independent



$$\propto U^2 \sum_{\omega_p} \int d\mathbf{p}\ \ G(p)G(q-p)$$

Lowest order: Dependence on transfer arguments only,
often used in fRG:

Karrasch*, et al.*,JPCM 2008 (frequencies);
Husemann and Salmhofer, PRB 2009 (momenta)
Bauer, Heyder and von Delft, PRB 2014 (inhomogeneous systems)

Generalization this argument for higher order diagrams?

Diagrammatic classification

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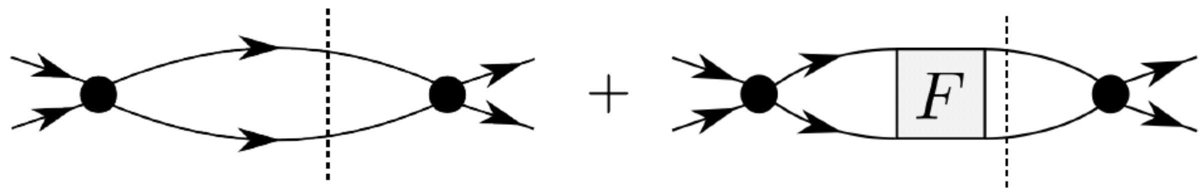
$\mathcal{K}_1^q =$



$$\mathcal{K}_1^q = U^2 \chi^q \rightarrow$$ Direct connection with susceptibilities

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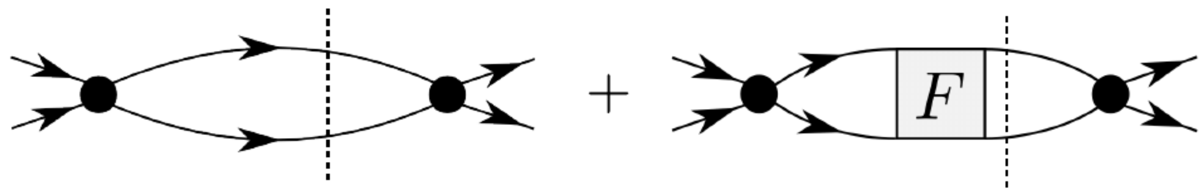

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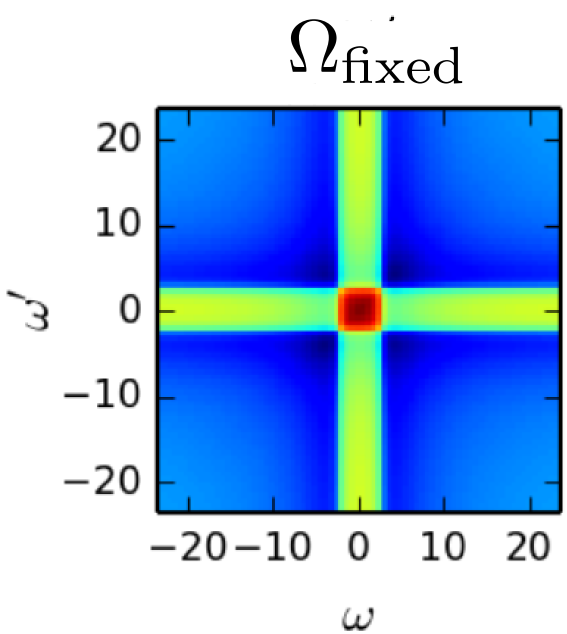

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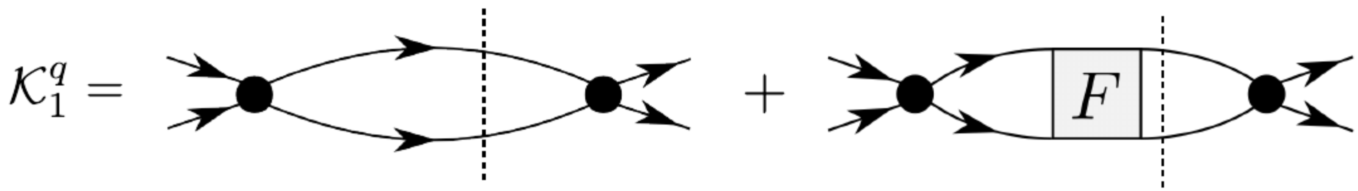

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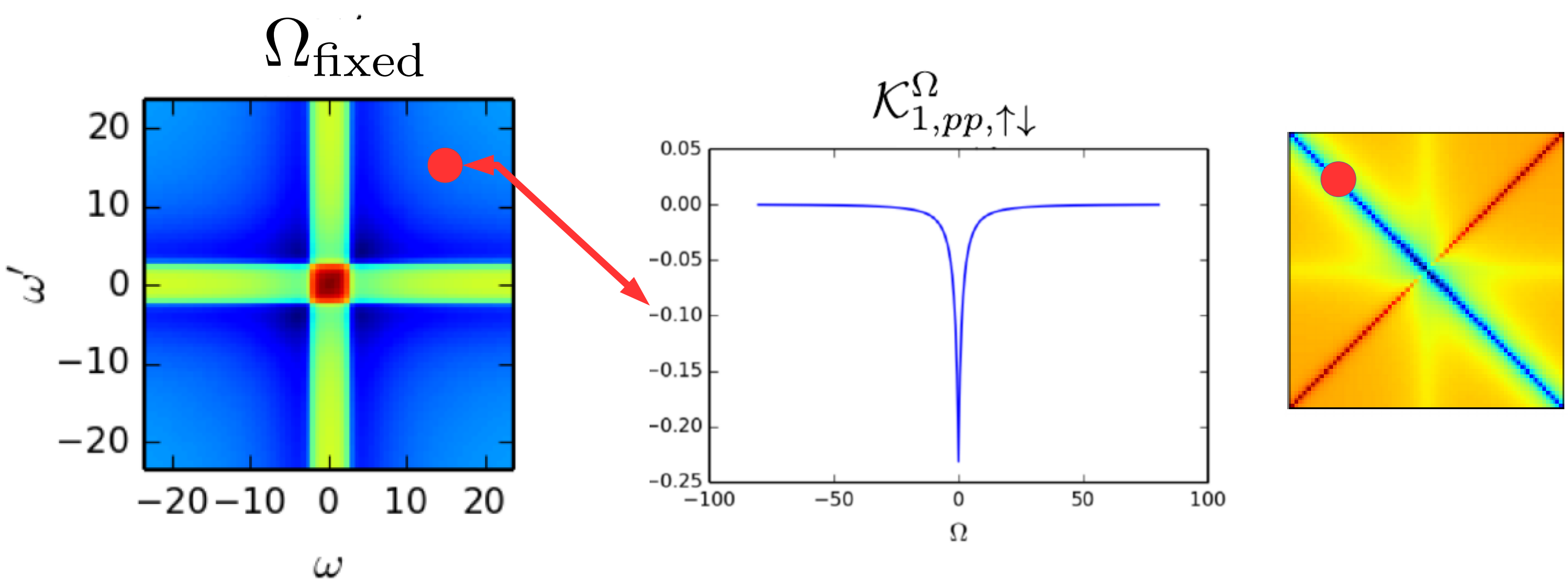

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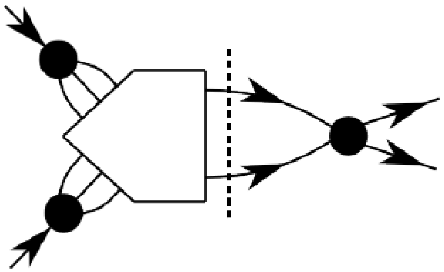
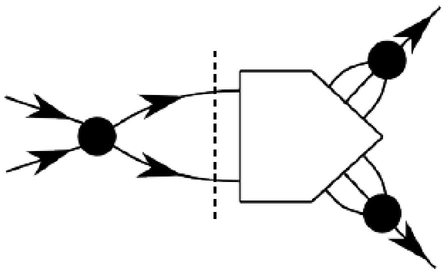
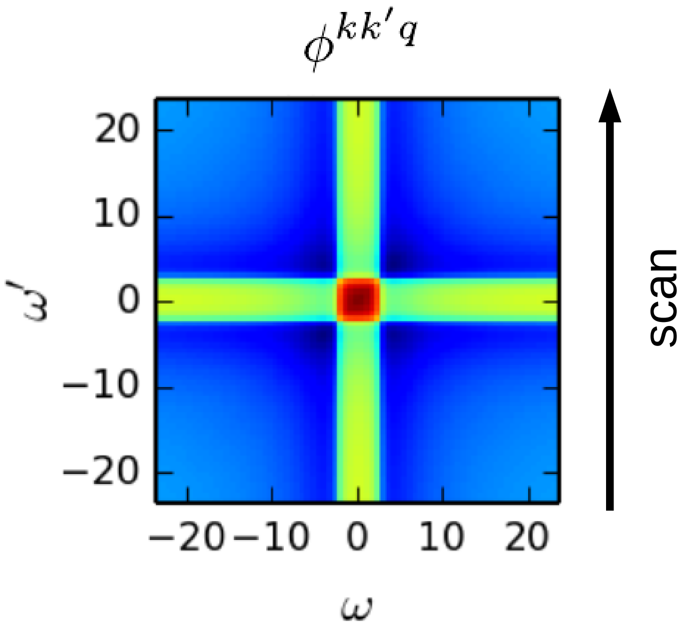
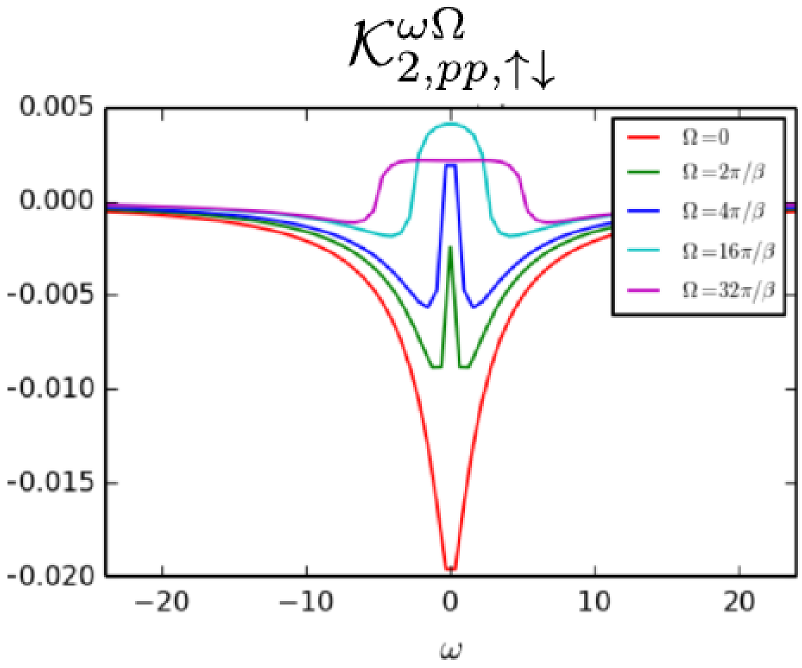


$\overline{\mathcal{K}_2}^{k'q} =$







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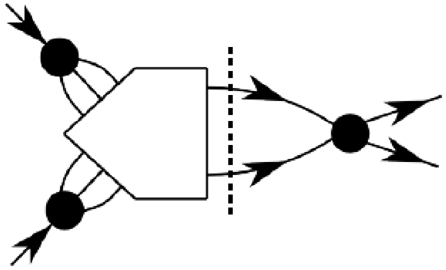
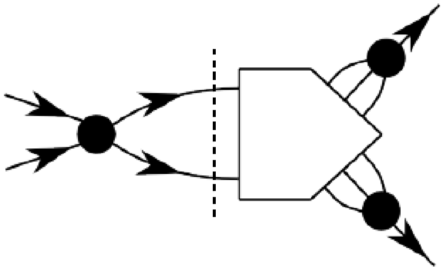
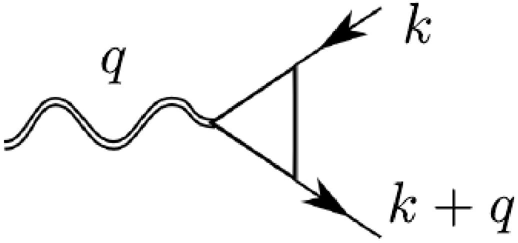
$\mathcal{K}_2^{kq} =$ 

$\overline{\mathcal{K}}_2^{k'q} =$ 

$$n_q = \sum\!\!\!\!\!\!\int dk \sum_\sigma c_\sigma^\dagger(k) c_\sigma(k+q)$$

$$\langle T n_q c_\sigma(k+q) c_\sigma^\dagger(k) \rangle_c \sim \sum_{\sigma'} \left(\mathcal{K}^q_{1,ph,\sigma\sigma'} + \mathcal{K}^{kq}_{2,ph,\sigma\sigma'} \right)$$



Direct connection to fermion-boson vertices!

Diagrammatic classification

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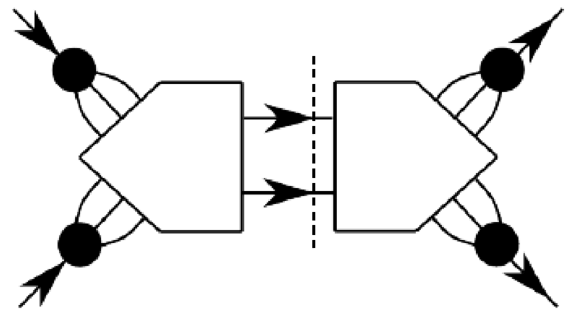
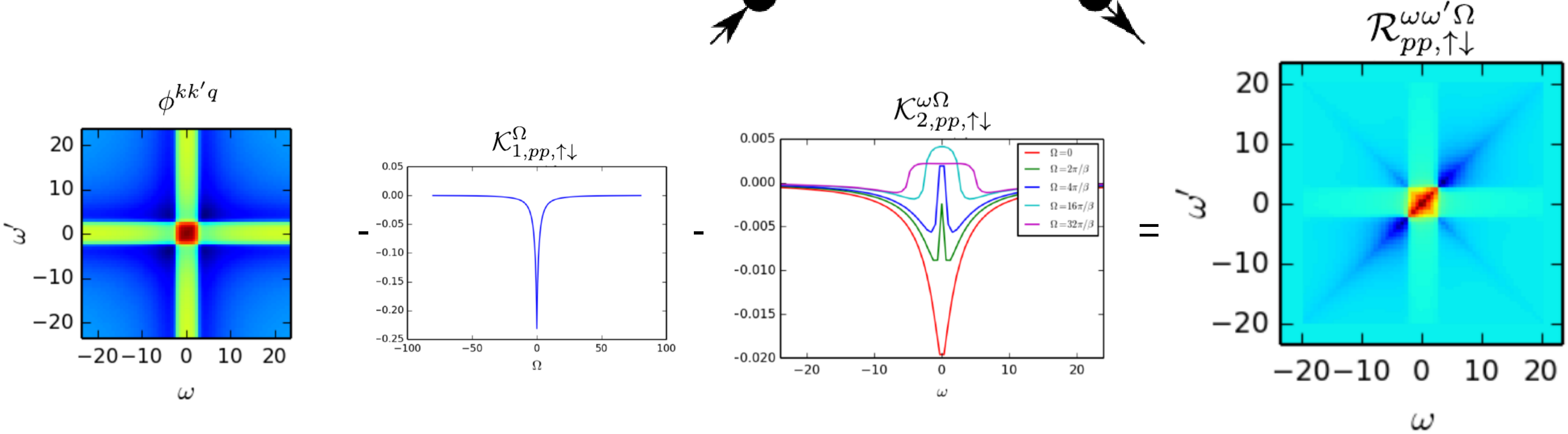
$$\mathcal{R}^{kk'q} =$$

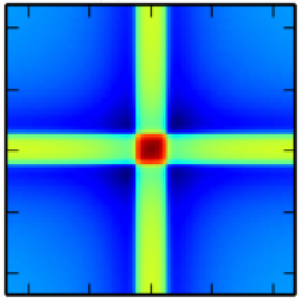




- Subleading *at weak coupling*
- Full argument dependence: numerically expensive
- Possibly relevant for ***d*-wave scattering**

Vertex: decomposition and reconstruction

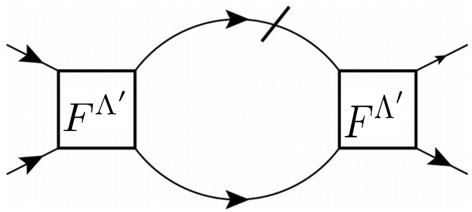
Computed in
a finite box



Vertex: decomposition and reconstruction

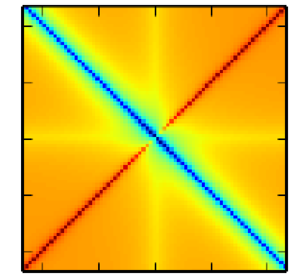
fRG: separate channel integration

$$\phi_{pp}^{\Lambda} = \int_{\Lambda_0}^{\Lambda} d\Lambda'$$



$$+\phi_{pp}^{\Lambda_0}$$

From the full vertex: Bethe-Salpeter equations



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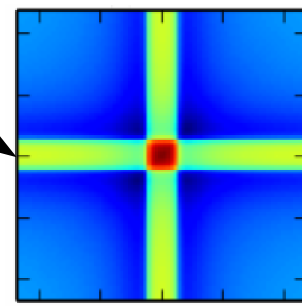

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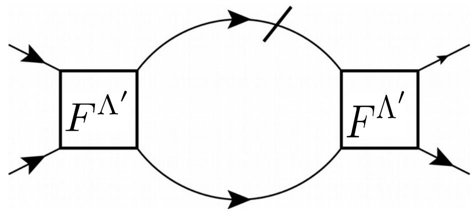

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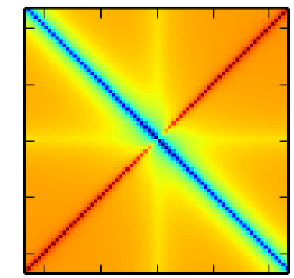

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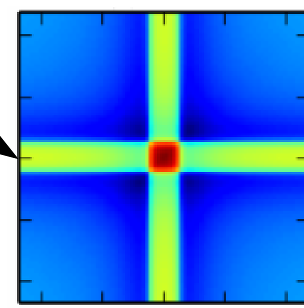
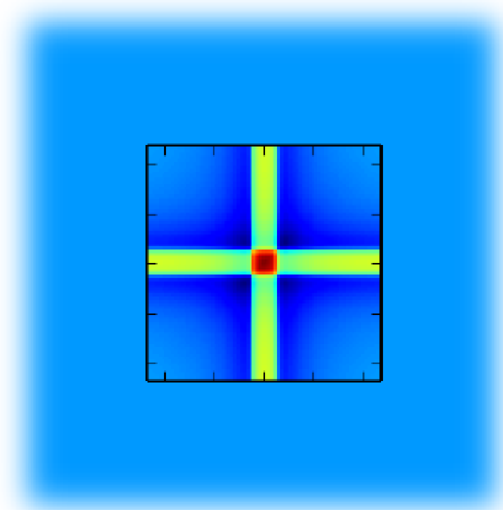


→ Extract $\mathcal{K}_1$

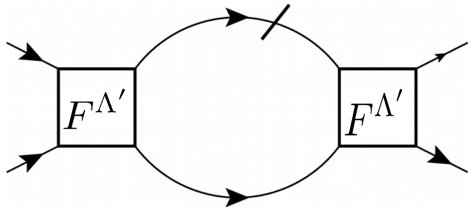
Extend ϕ →



Vertex: decomposition and reconstruction

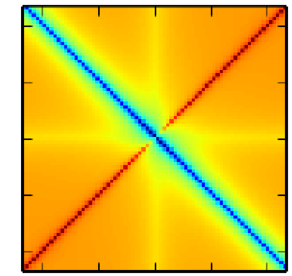
fRG: separate channel integration

$$\phi_{pp}^{\Lambda} = \int_{\Lambda_0}^{\Lambda} d\Lambda'$$

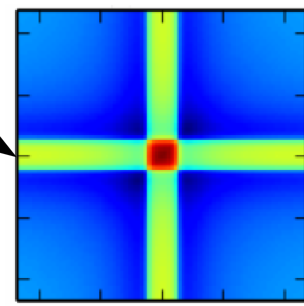


$$+\phi_{pp}^{\Lambda_0}$$

From the full vertex: Bethe-Salpeter equations



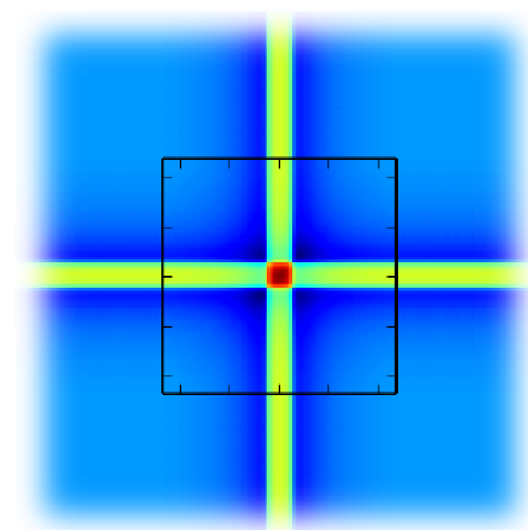
Computed in a finite box



→ Extract $\mathcal{K}_1$

→ Extract $\mathcal{K}_2, \ \overline{\mathcal{K}_2}$

Extend ϕ



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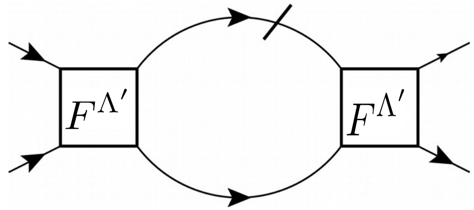

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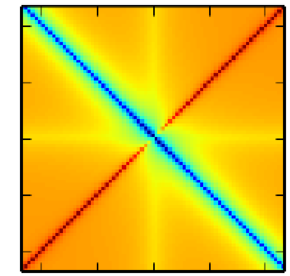

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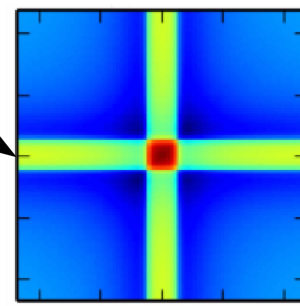
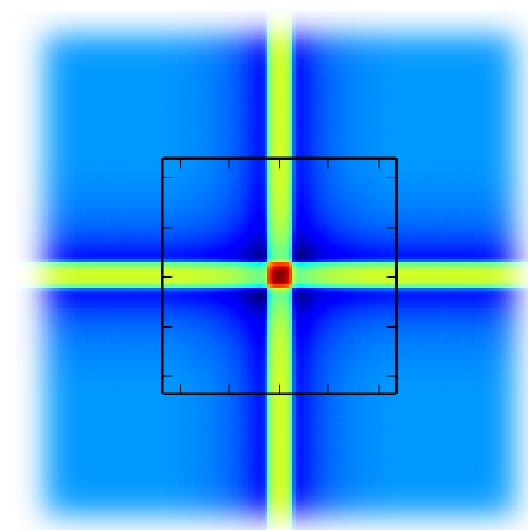


→ Extract $\mathcal{K}_1$

→ Extract $\mathcal{K}_2, \ \overline{\mathcal{K}_2}$ → Extend ϕ →

→ Extract $\mathcal{R}$ → Check that it decays inside the box



Part I: conclusions

1. The interaction vertex shows a nontrivial frequency structure
2. The vertex structure can be understood diagrammatically
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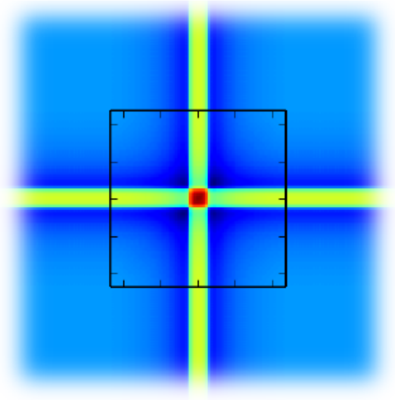


Part II: DMF^{2}RG and strong coupling

- Dynamical mean field theory in a nutshell
- Starting fRG from a correlated starting point

Flowing from infinite to *d*-dimensions

- Goal: combine **non-perturbative local** physics from **DMFT** with **nonlocal** fluctuations from **fRG**

Georges et al., *RMP* 1996

↓

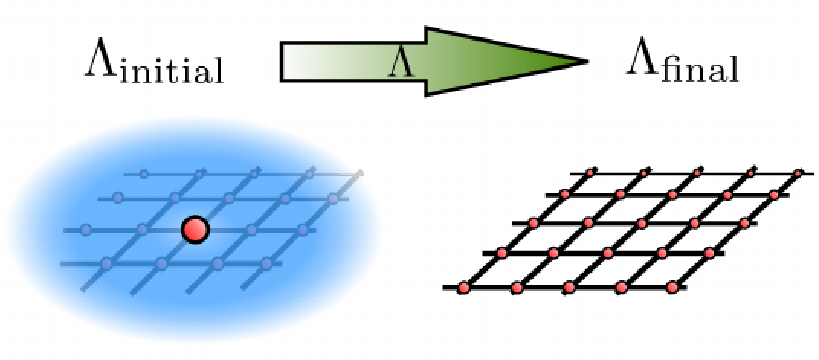
- In the **∞-dimensional** limit local approximation for the self-energy becomes exact

Metzner and Vollhardt, *PRB* 1989;

- Mapping on an Anderson Impurity model embedded in a self-consistent **frequency-dependent** bath (**MF in space**)

Georges and Kotliar, *PRB* 1992

- The Anderson Impurity model can be **exactly solved** (QMC, ED, …) good **starting point** for the flow equations

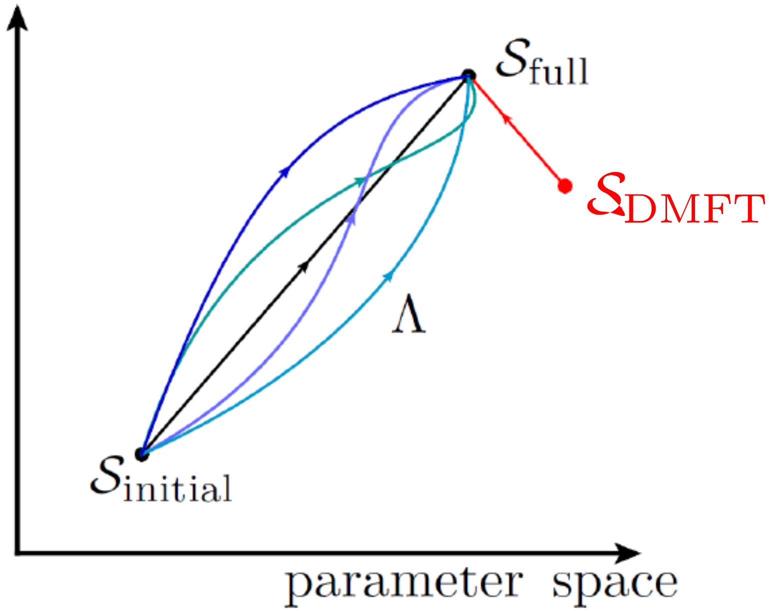


Conceptual steps:

(1) Approximate a lattice model with an **∞-dimensional** lattice (with the same DOS)
(2) **Exactly** solve the problem in infinite dimensions
(3) **Flow** from the infinite dimensional lattice to the original one using **fRG**

Taranto, *et al.*,*PRL* 2014;

Flowing from infinite to *d*-dimensions



Taranto, *et al.*,*PRL* 2014;

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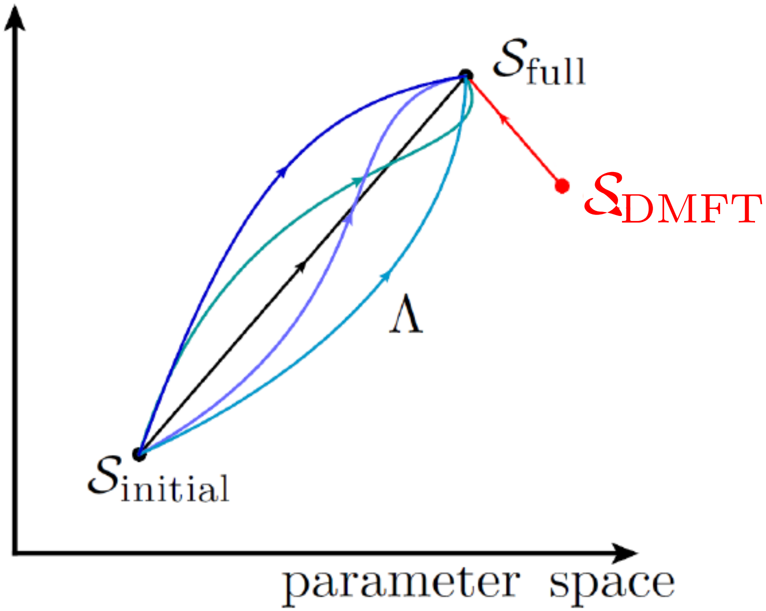
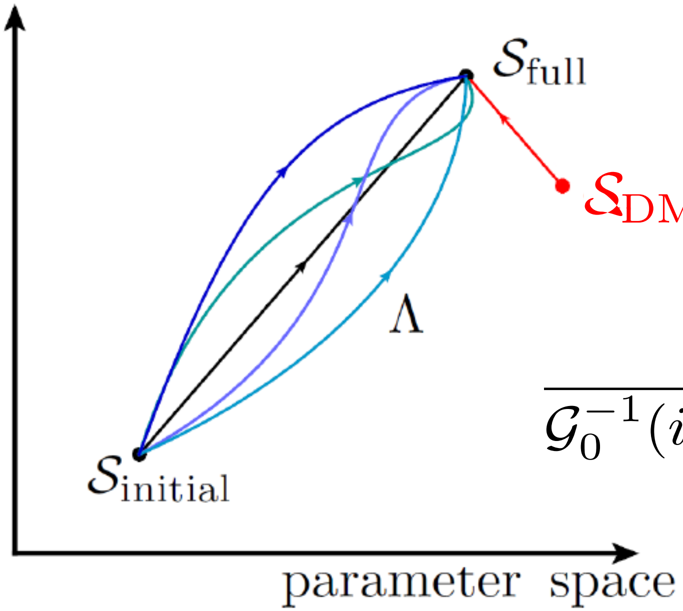


$$\mathcal{S}_{\text{DMFT}} = -\sum_{k,\sigma} \overline{\psi}_{k,\sigma} \mathcal{G}_0^{-1}(i\omega)\psi_{k,\sigma} + U\int_0^\beta d\tau \;\overline{\psi}_\uparrow(\tau)\psi_\downarrow(\tau)\overline{\psi}_\downarrow(0)\psi_\downarrow(0)$$

Taranto, *et al.*,*PRL* 2014;

Flowing from infinite to *d*-dimensions



DMFT self-consistency condition for the **Weiss field**

Georges,cond-mat/0403123 (2004)

$$\frac{1}{\mathcal{G}_0^{-1}(i\omega) - \Sigma_{\text{DMFT}(i\omega)}} = \sum_{\mathbf{k}} \frac{1}{G_0^{-1}(i\omega, \mathbf{k}) - \Sigma_{\text{DMFT}(i\omega)}}$$

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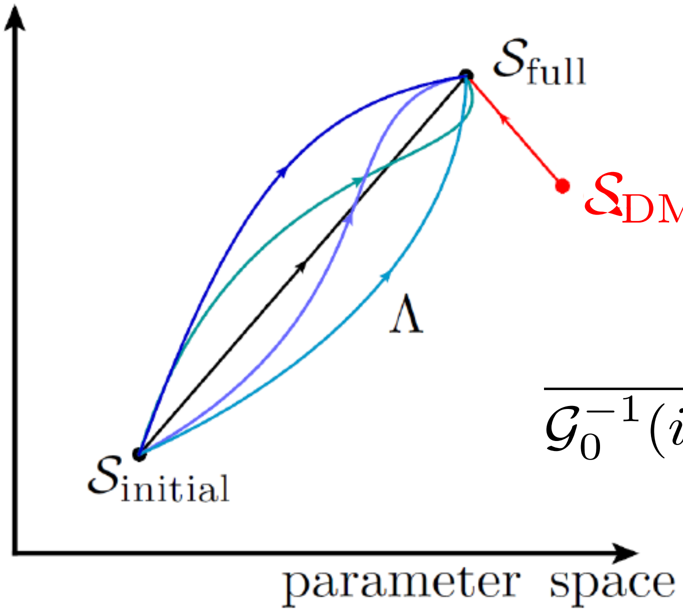

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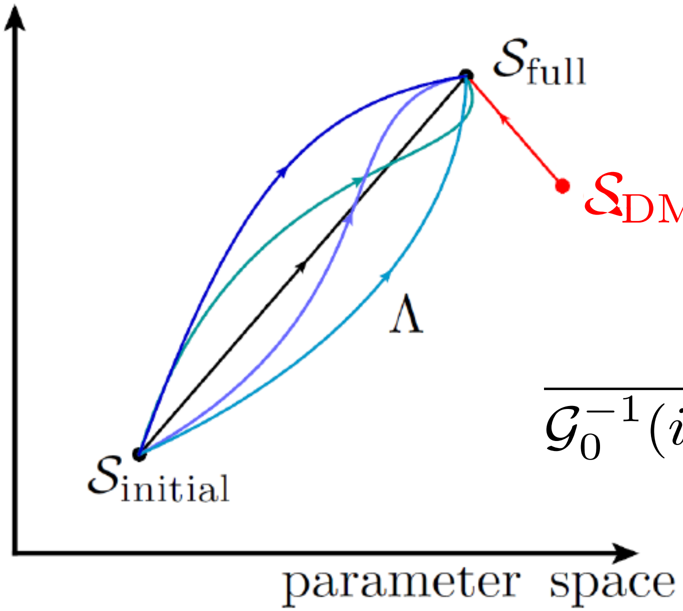
$$\mathcal{S}_{\text{full}} = -\sum_{k,\sigma} \overline{\psi}_{k,\sigma} G_0^{-1}(i\omega, \mathbf{k})\psi_{k,\sigma} + U \int_0^\beta d\tau \;\; \overline{\psi}_\uparrow(\tau)\psi_\downarrow(\tau)\overline{\psi}_\downarrow(0)\psi_\downarrow(0)$$

$$(G_0^\Lambda)^{-1} = \Lambda(G_0)^{-1} + (1-\Lambda)(\mathcal{G}_0)^{-1}$$

Taranto, *et al.,PRL* 2014;

Flowing from infinite to *d*-dimensions



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$$(G_0^\Lambda)^{-1} = R^\Lambda(i\omega, \mathbf{k})(G_0)^{-1} + \tilde{R}^\Lambda(i\omega, \mathbf{k})(\mathcal{G}_0)^{-1}$$

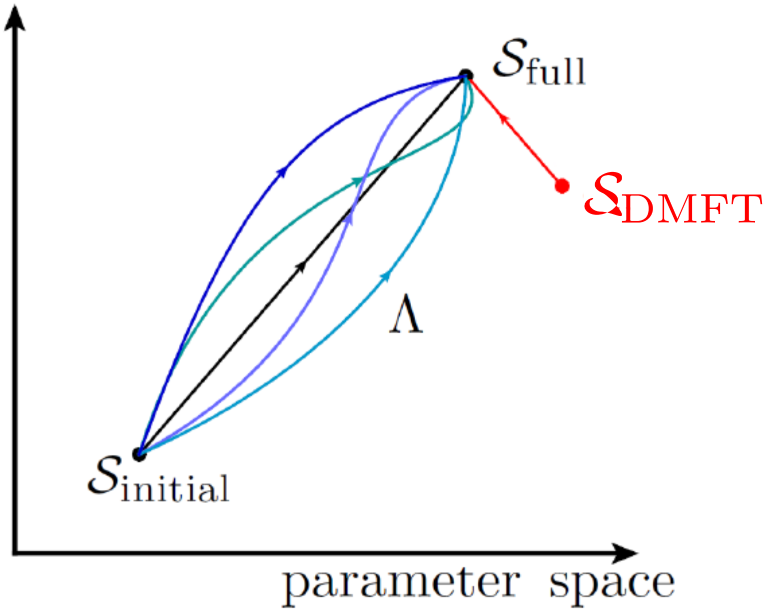
More freedom in the regulator

$$R^{\Lambda_{\text{ini}}}(i\omega, \mathbf{k}) = 0 \qquad R^{\Lambda_{\text{fin}}}(i\omega, \mathbf{k}) = 1$$

$$\tilde{R}^{\Lambda_{\text{ini}}}(i\omega, \mathbf{k}) = 1 \qquad \tilde{R}^{\Lambda_{\text{fin}}}(i\omega, \mathbf{k}) = 0$$

Taranto, *et al.,PRL* 2014;

Flowing from infinite to *d*-dimensions

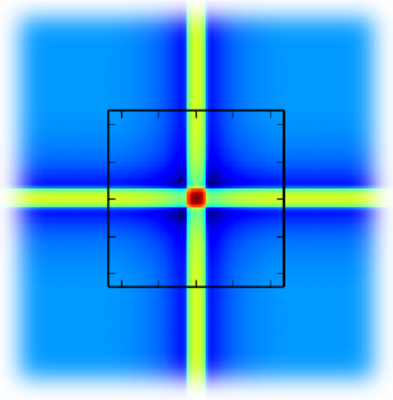


Initial conditions	Conventional fRG	**DMF²RG**
$\Sigma^{\Lambda_{ini}}$	0	$\Sigma^{\text{DMFT}}(i\omega)$
$F^{\Lambda_{ini}}$	U	$F_{\text{DMFT}}^{\omega_1\omega_2\omega_1'}$

Effect of the frequency dependence in the next talk

Taranto*, et al.,PRL* 2014;

Part I: conclusions



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