Strongly interacting Fermi gas in two dimensions

Tilman Enss (Universität Heidelberg)

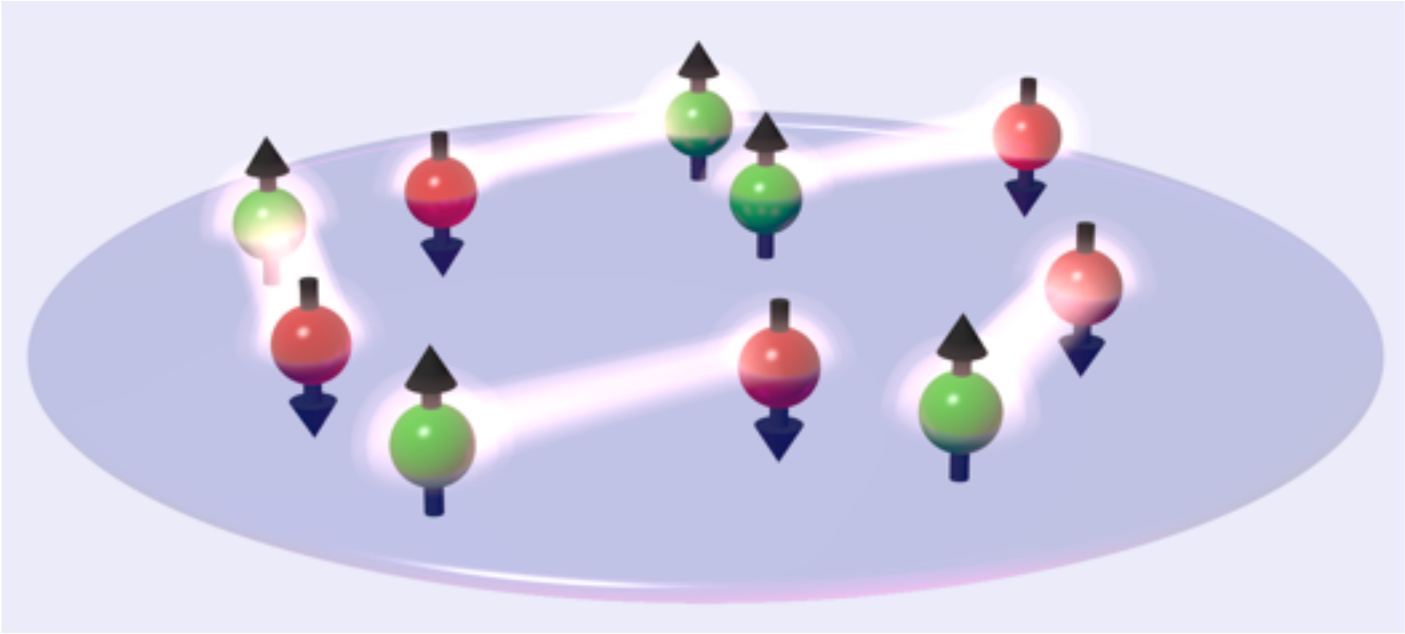
in collaboration with
M. Bauer, M.M. Parish (Cambridge)
I. Boettcher, S. Jochim group (Heidelberg)

Exact Renormalization Group Conference
ICTP Trieste, 22 Sep 2016

UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

Motivation

- understand properties of strongly interacting Fermi gas



- **thermodynamics:** equation of state (EoS), density n(μ,T,a), pressure P(μ,T,a)
- **transport & dynamical properties...**
- dilute gas of nonrelativistic ↑ and ↓ fermions with contact interaction

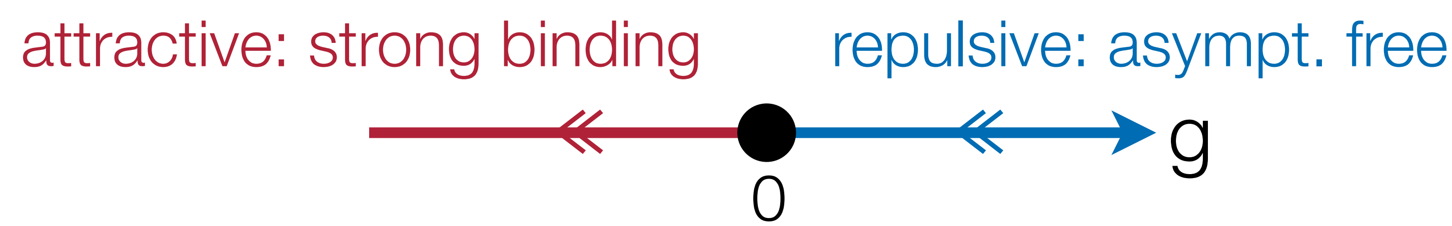
$$\mathcal{H} = \int d\mathbf{x} \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big(-\frac{\hbar^2 \nabla^2}{2m} - \mu_\sigma \Big) \psi_\sigma + g_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

2D Fermi gas

$$\mathcal{L} = \sum_\sigma \bar{\psi}_\sigma \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \psi_\sigma + g_0 \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow$$

- vacuum ($\mu=0$) classically scale invariant with $z=2$, $\dim[g_0]=0$
- exact beta function:

$$\frac{dg}{d\ell} = -\frac{g^2}{2}$$



- log. running coupling, energy scale dependence breaks scale invariance

Holstein 1993; Pitaevskii & Rosch 1997

exact 2D scattering amplitude:

always bound state!

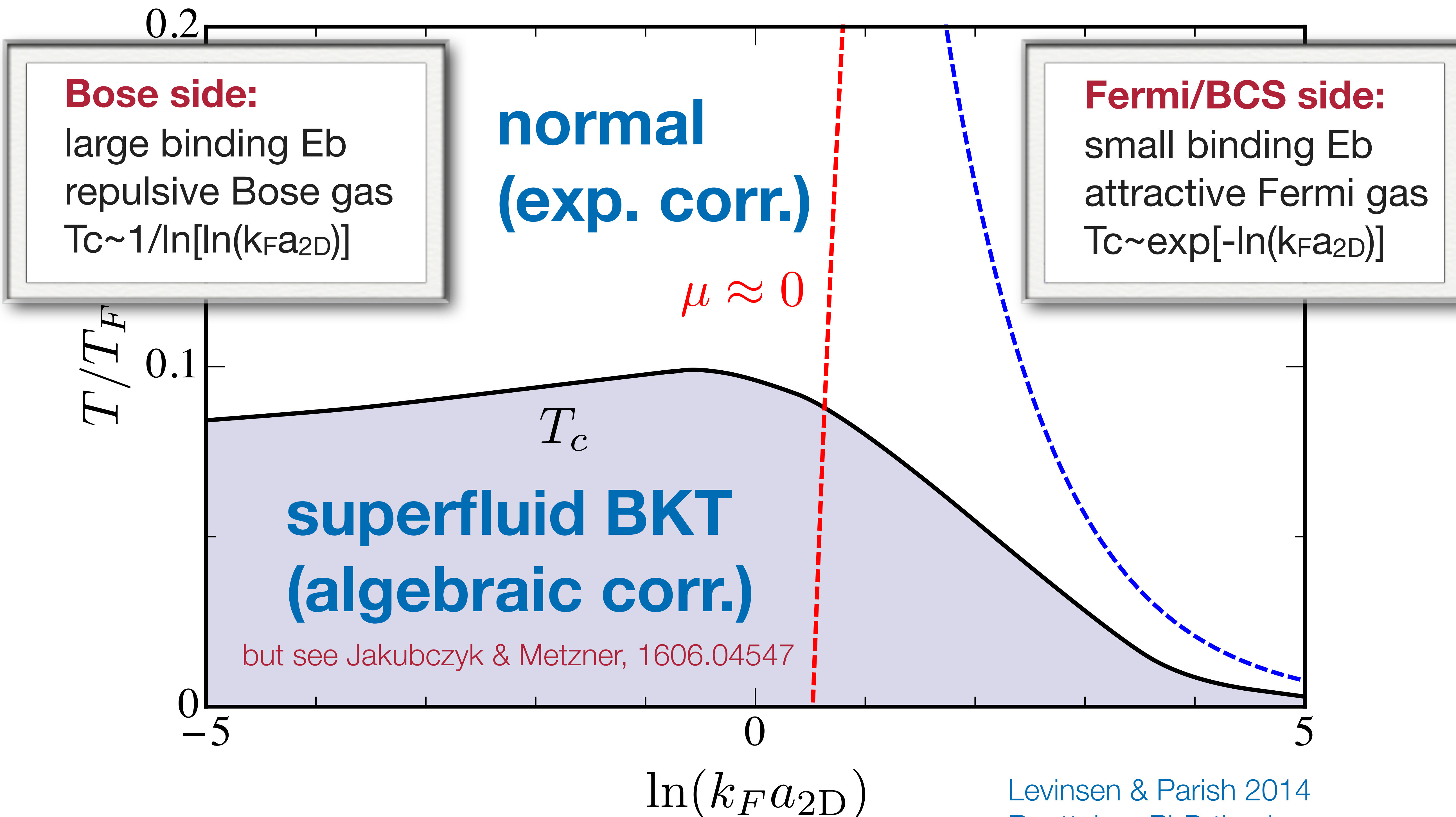
3D: $f(k) = \dfrac{1}{-1/a_{3D} - ik}$ → 2D: $f(k) = \dfrac{1}{\ln(1/k^2 a_{2D}^2) + i\pi}$

$$\varepsilon_B = \frac{\hbar^2}{m a_{2D}^2}$$

- typical scale $k=k_F$: expansion parameter $g=-1/\ln(k_F a_{2D})$

Adhikari 1986

Phase diagram of 2D Fermi gas



Levinsen & Parish 2014
Boettcher, PhD thesis

Thermodynamics

- dilute gas ($k_F r_0 \ll 1$): **universal properties** depend only on T/T_F and $k_F a_{2D}$

- **Contact density:** probability to find up and down in same place [S. Tan 2008]

$$C = m^2 g_0^2 \left\langle \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow (r) \right\rangle$$

$$\frac{dE}{d \ln a_{2D}} = \frac{C}{2\pi m}$$

adiabatic theorem for internal energy E via Hellmann-Feynman

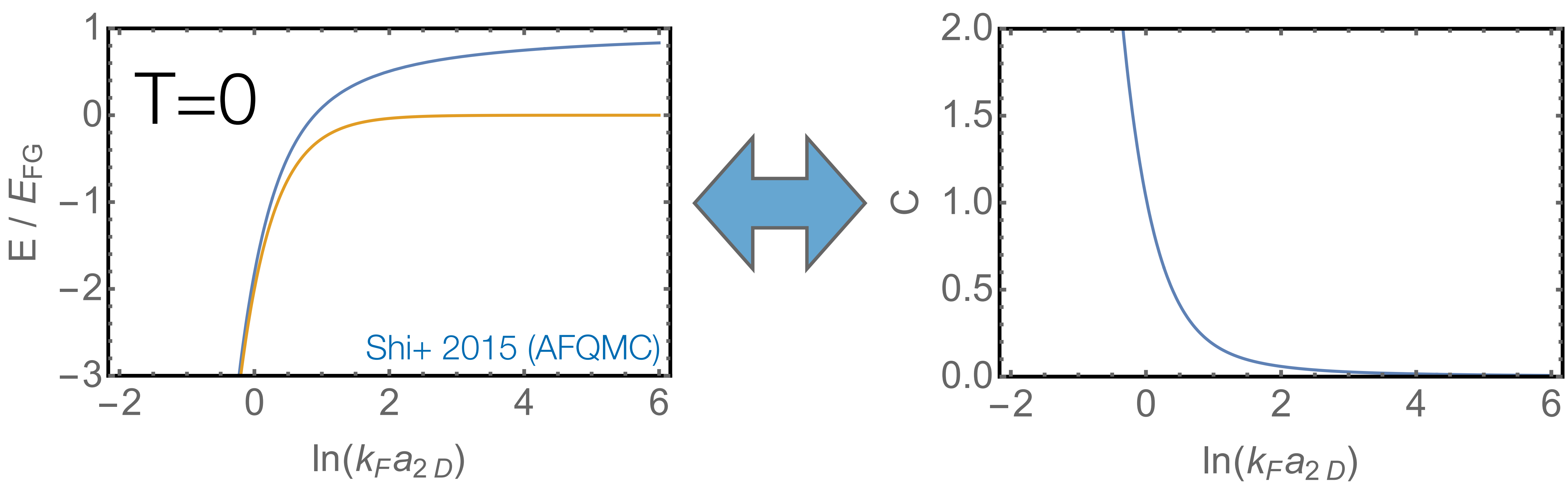
Thermodynamics

- dilute gas ($k_F r_0 \ll 1$): **universal properties** depend only on T/T_F and $k_F a_{2D}$

- **Contact density:** probability to find up and down in same place [S. Tan 2008]

$$C = m^2 g_0^2 \left\langle \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow (r) \right\rangle \qquad \frac{dE}{d \ln a_{2D}} = \frac{C}{2\pi m}$$

adiabatic theorem for internal energy E via Hellmann-Feynman



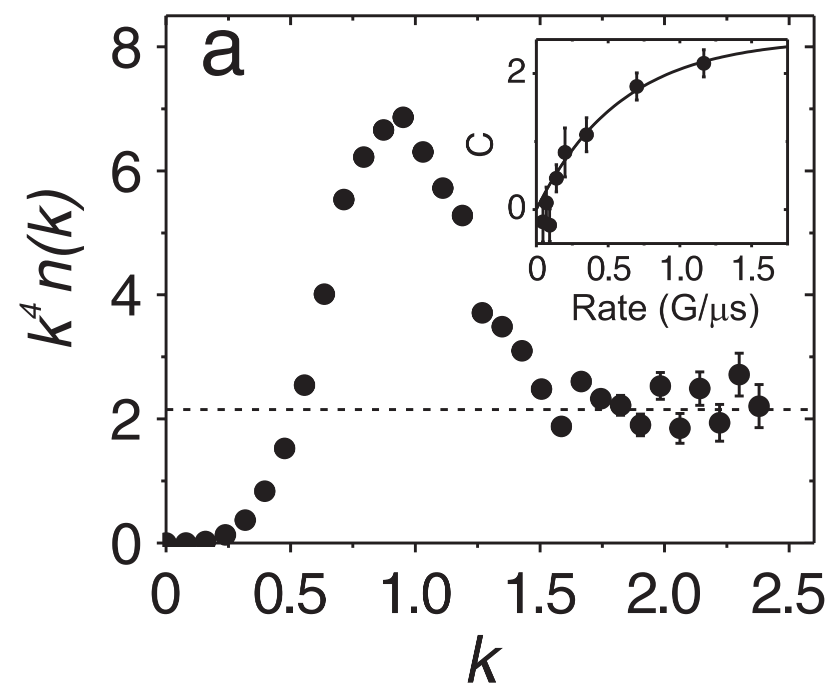
Universal relations

- dilute gas: Contact determines **UV limit of correlation functions**

- momentum distribution

$$n(k) \xrightarrow{k \gg k_F} \frac{C}{k^4}$$

Stewart+ PRL 2010



- internal energy

$$E = \sum_\sigma \int^\Lambda \frac{d^2k}{(2\pi)^2} \frac{k^2}{2m} n_\sigma(k) - \ln(\Lambda a_{2D}) \frac{C}{2\pi m}$$

kinetic **interaction energy**

contact in fRG for 3D Fermi gas: Boettcher, Diehl, Pawlowski & Wetterich 2013

Pressure

scale invariant:

$$P = E$$

interacting 2D Fermi gas:

$$P = E + \frac{C}{4\pi m}$$

breaks scale invariance!

Pressure

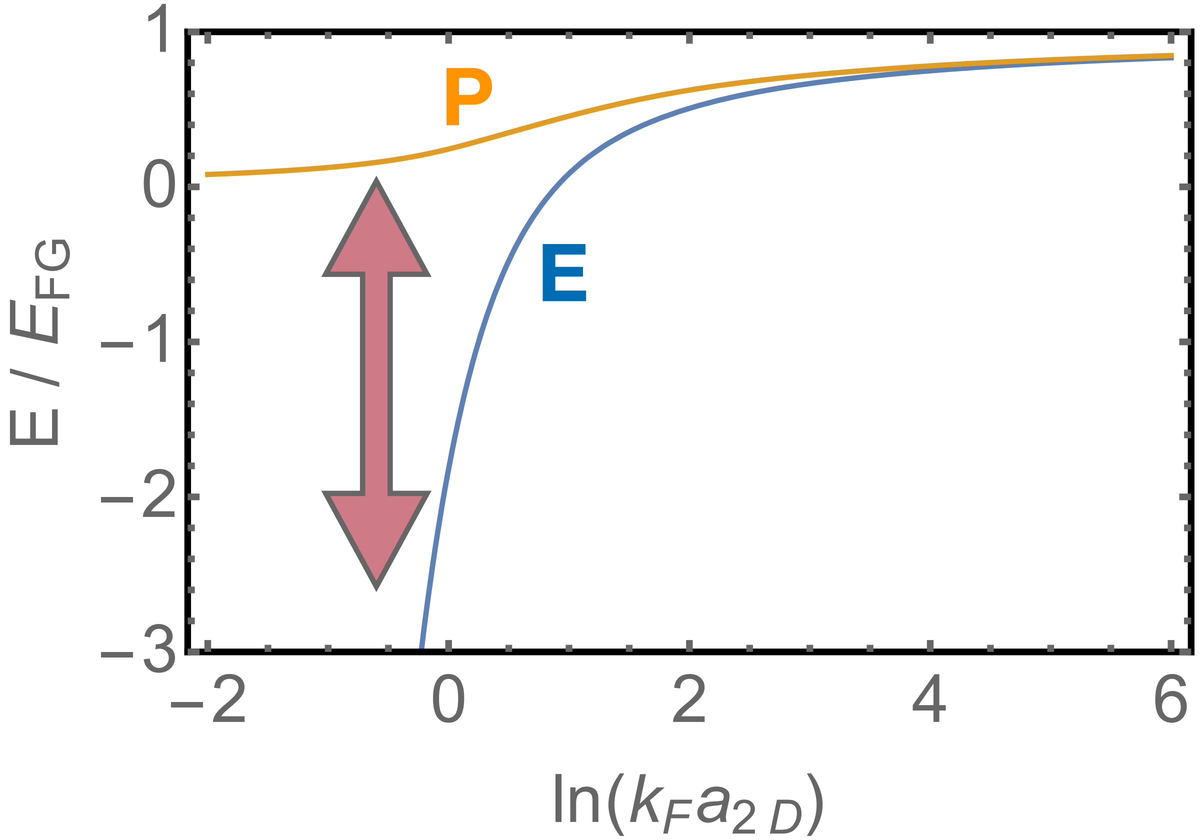
scale invariant:

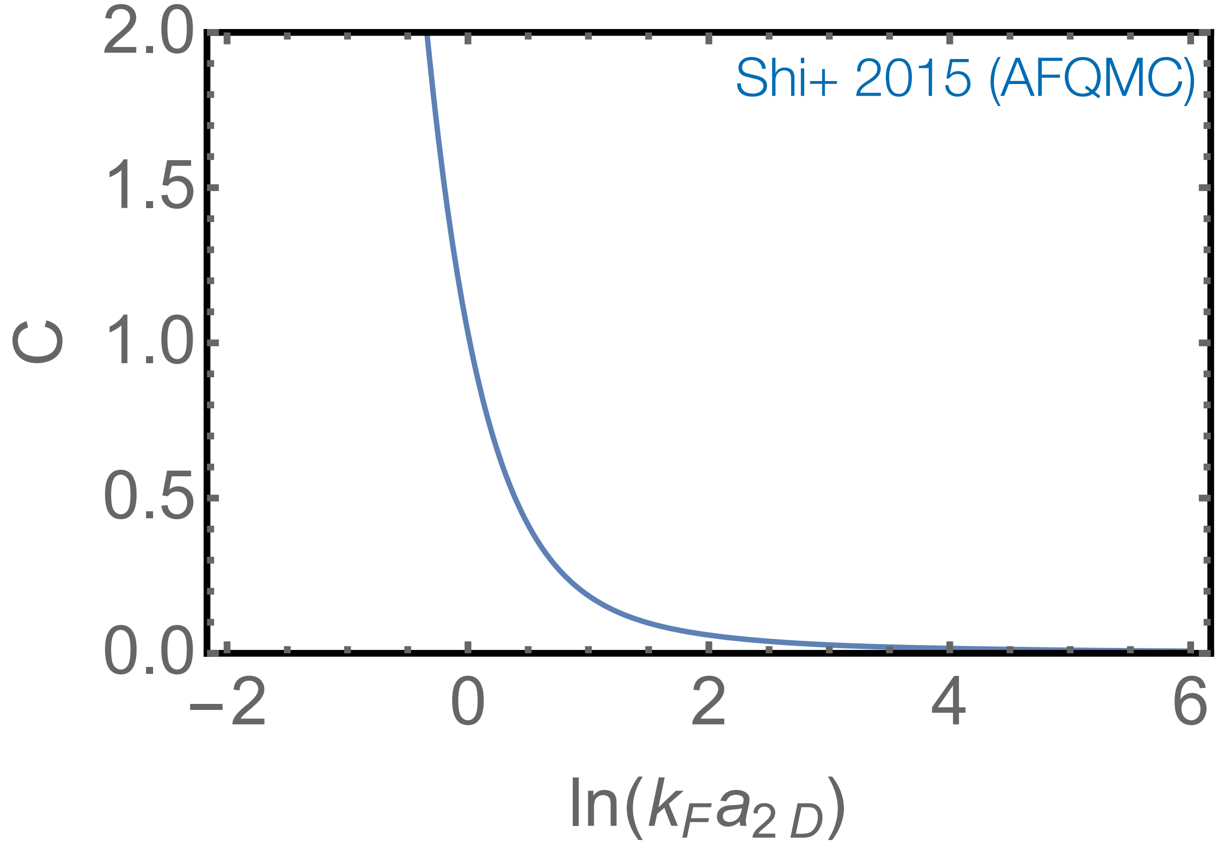
$$P = E$$

interacting 2D Fermi gas:

$$P = E + \frac{C}{4\pi m}$$

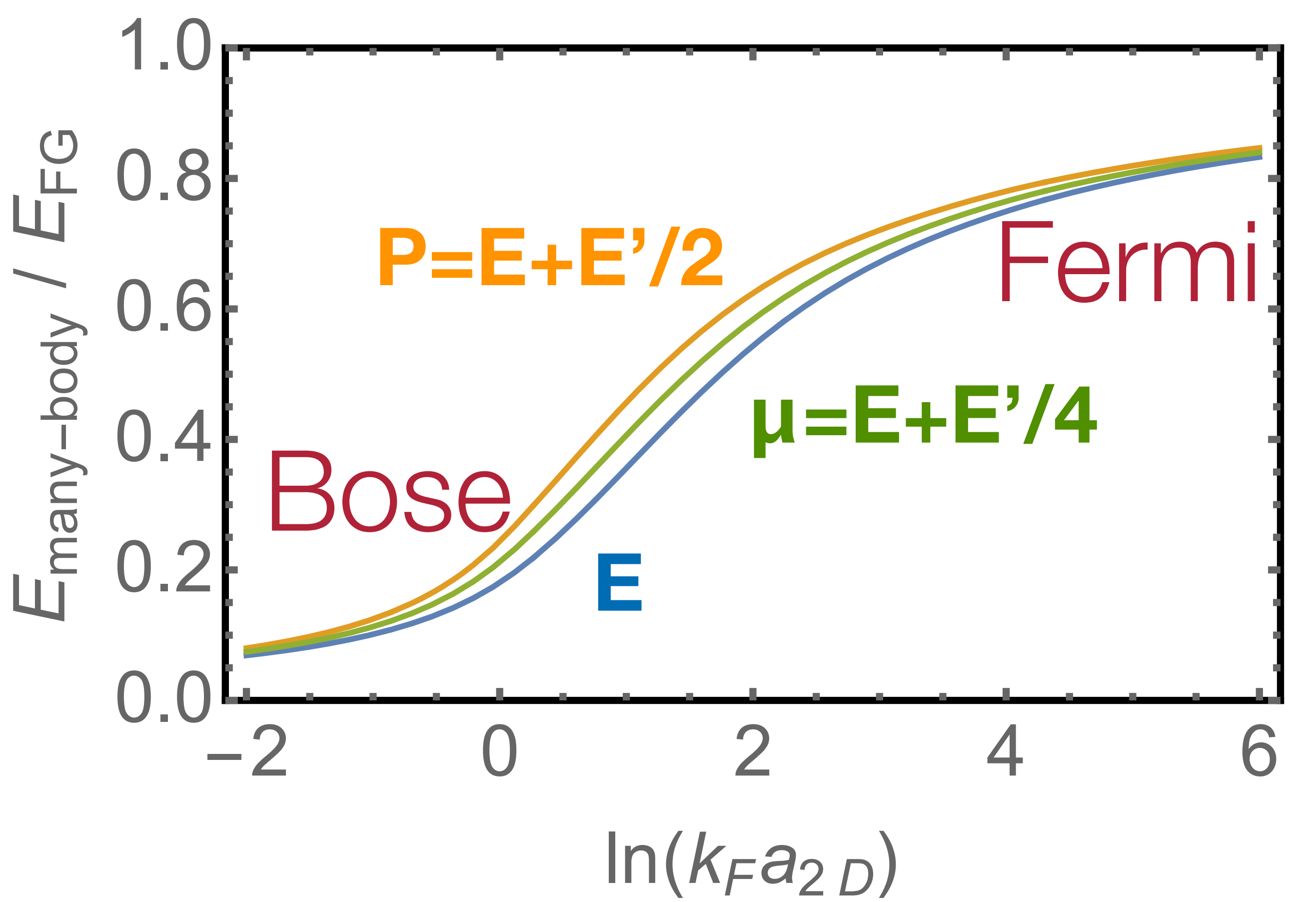
breaks scale invariance!

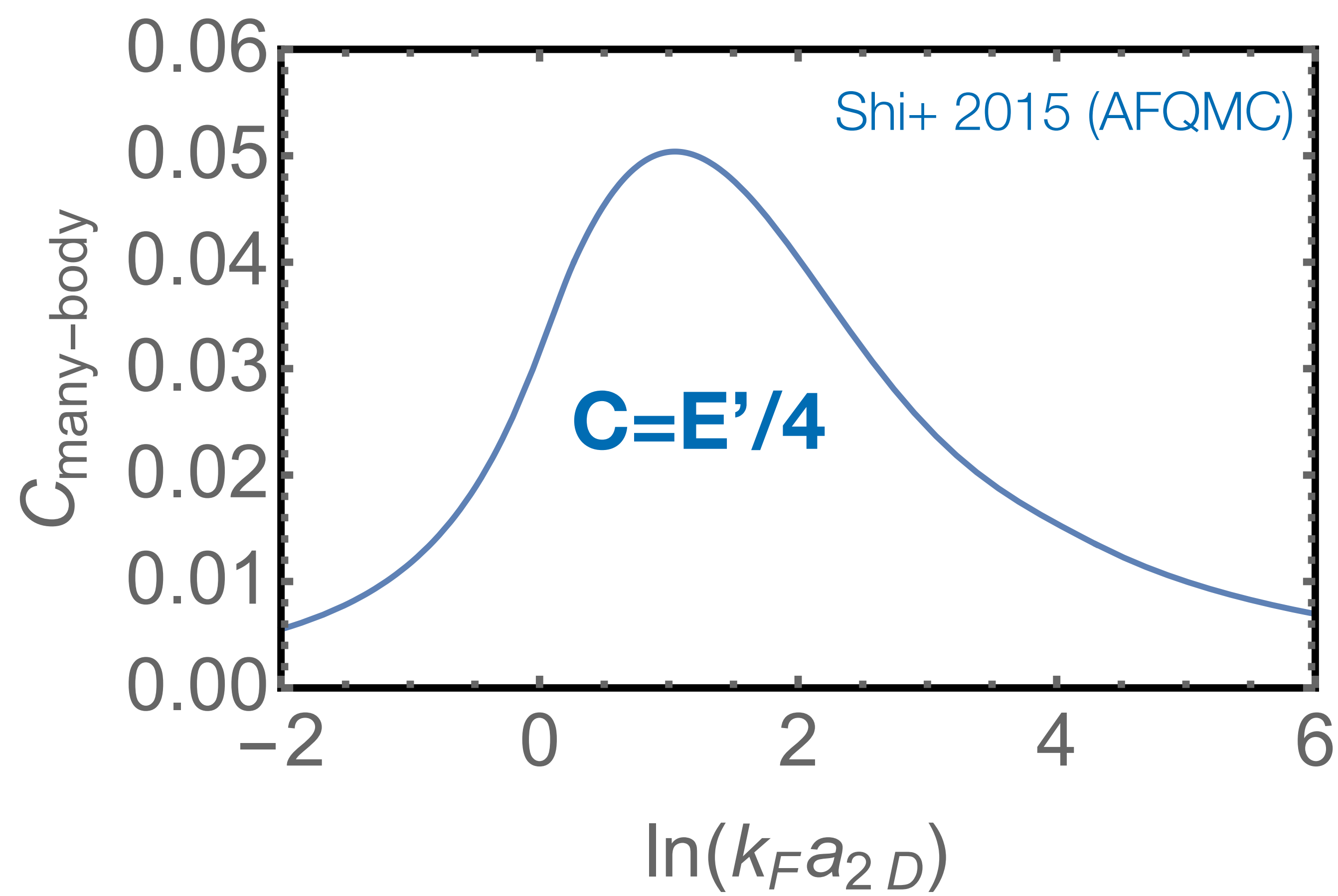




Ground-state energy

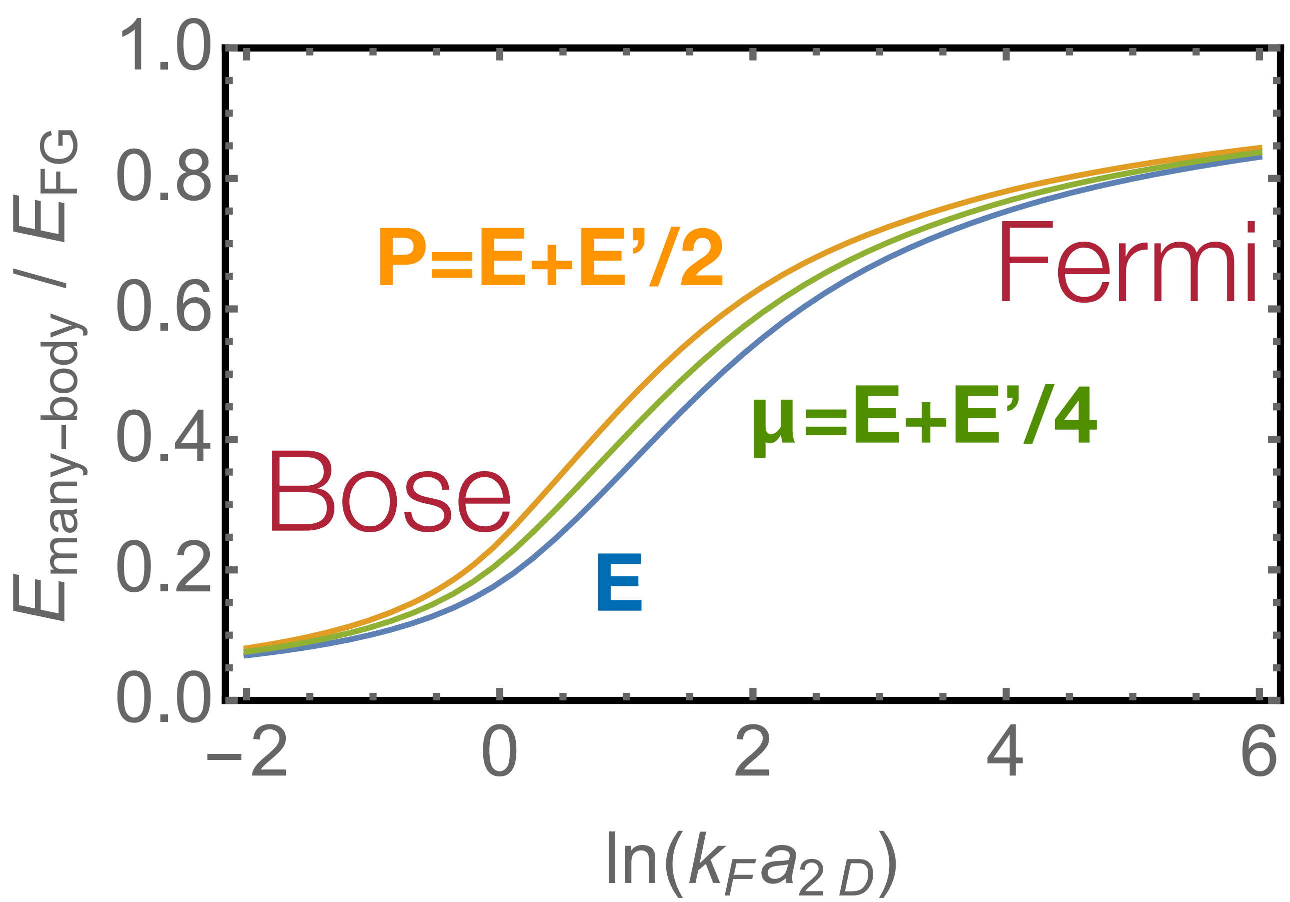
subtract two-body binding energy:

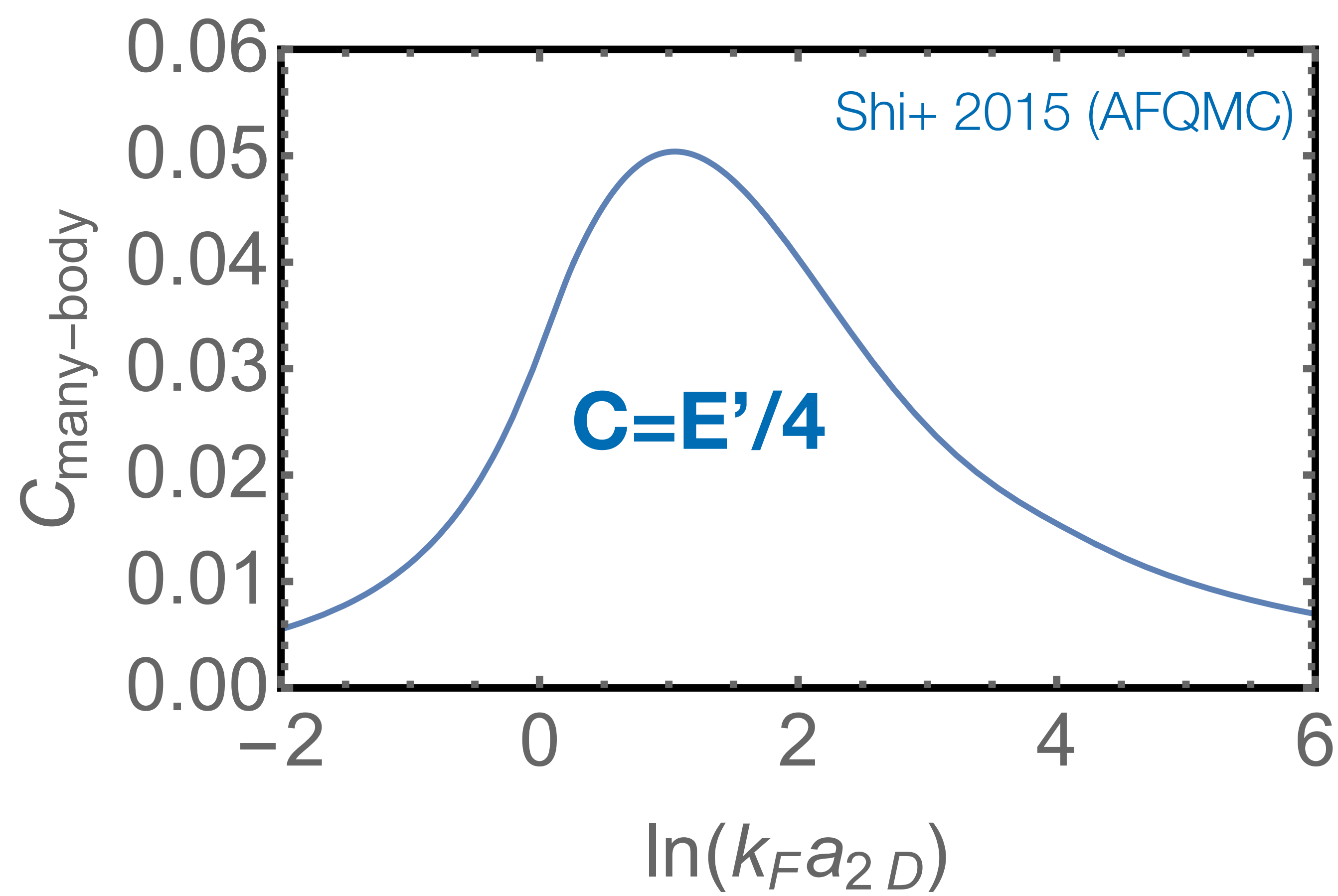




Ground-state energy

subtract two-body binding energy:

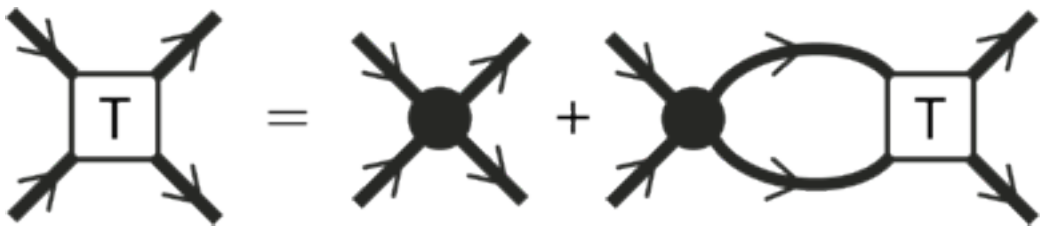


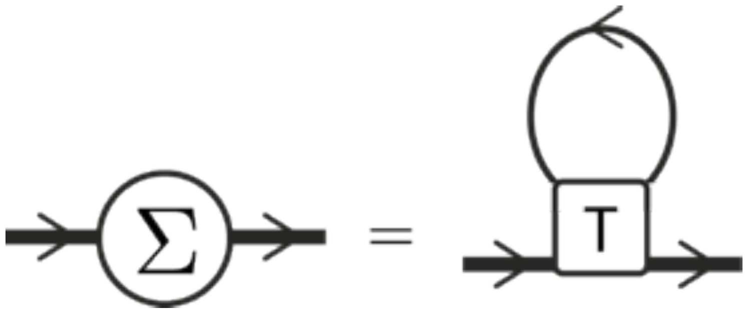


Fermi liquid theory:
Engelbrecht & Randeria 1992

$$\frac{E}{E_{FG}} = 1 + g + (\tfrac{3}{4} - \ln 2)g^2 + \ldots \qquad [g = -\frac{1}{\ln(k_F a_{2D})}]$$

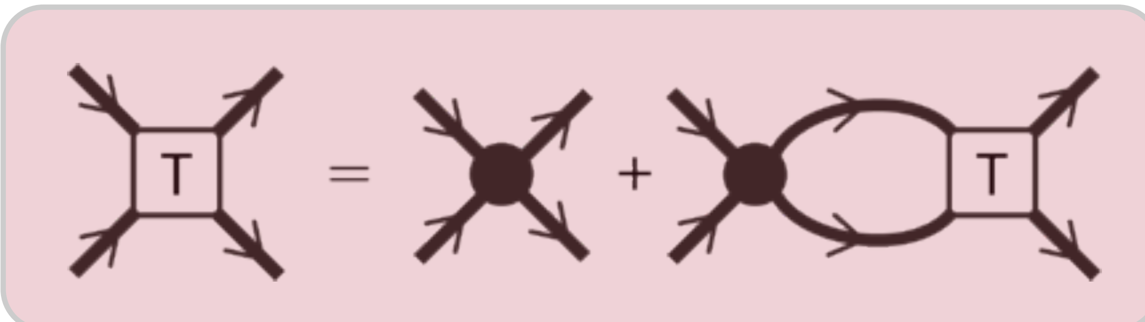
Many-body T-matrix

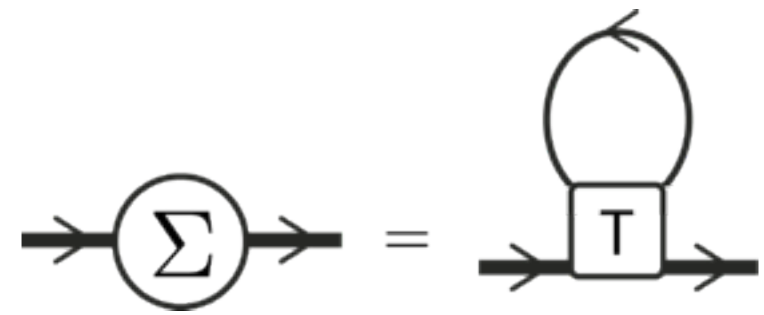




Nozières & Schmitt-Rink 1985;
2D: Engelbrecht & Randeria 1990

Many-body T-matrix





Nozières & Schmitt-Rink 1985;
2D: Engelbrecht & Randeria 1990

step 1: compute many-body T-matrix

two-body T-matrix: $$T_0(E) = \frac{4\pi/m}{\ln(\varepsilon_B/E) + i\pi}$$

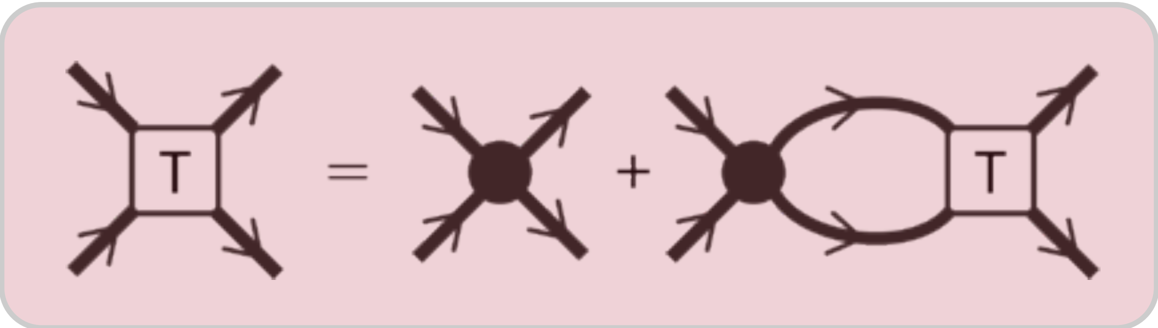
many-body: finite density medium scattering   Schmidt, Enss, Pietilä & Demler 2012

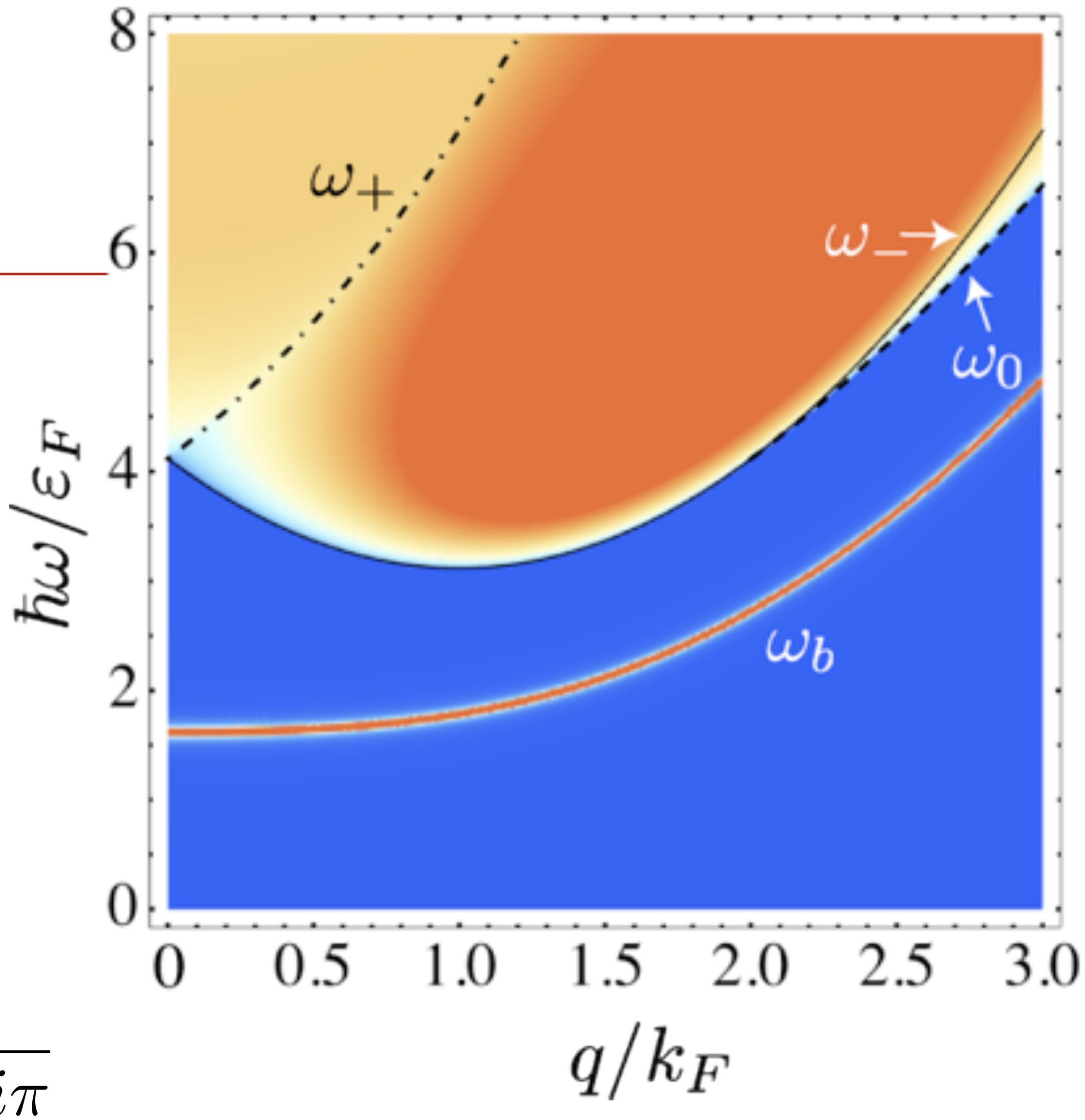
$$T^{-1}(\mathbf{q},\omega) = T_0^{-1}(\omega + i0 + \mu_\uparrow + \mu_\downarrow - \varepsilon_\mathbf{q}/2) + \int \frac{d^2k}{(2\pi)^2} \frac{n_F(\varepsilon_\mathbf{k} - \mu_\uparrow) + n_F(\varepsilon_{\mathbf{k}+\mathbf{q}} - \mu_\downarrow)}{\omega + i0 + \mu_\uparrow + \mu_\downarrow - \varepsilon_\mathbf{k} - \varepsilon_{\mathbf{k}+\mathbf{q}}}$$

we find compact solution

$$T(\mathbf{q},\omega) = T_0\Big(\frac{1}{2}z \pm \frac{1}{2}\sqrt{(z-\varepsilon_\mathbf{q})^2 - 4\varepsilon_F\varepsilon_\mathbf{q}}\Big) \qquad z = \omega + i0 - \varepsilon_F + \mu_\downarrow$$

Many-body T-matrix





step 1: compute many-body T-matrix

two-body T-matrix: $$T_0(E) = \frac{4\pi/m}{\ln(\varepsilon_B/E) + i\pi}$$

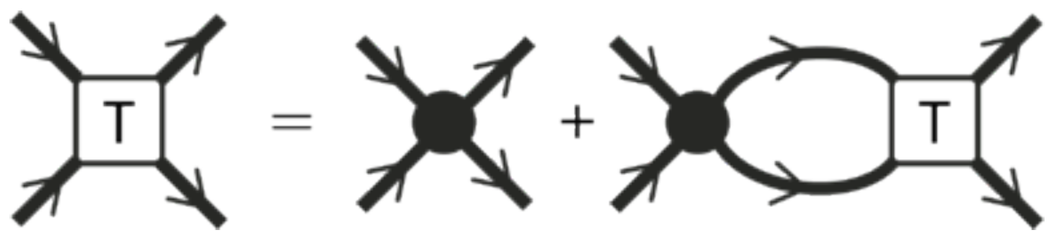
many-body: finite density medium scattering Schmidt, Enss, Pietilä & Demler 2012

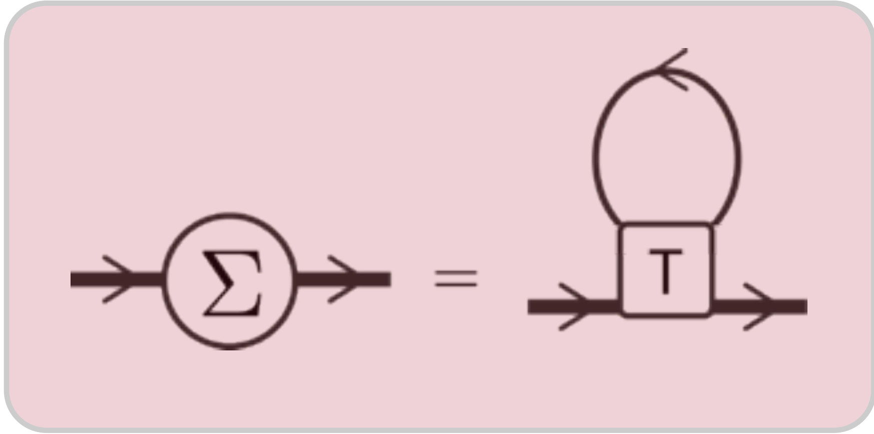
$$T^{-1}(\mathbf{q},\omega) = T_0^{-1}(\omega + i0 + \mu_\uparrow + \mu_\downarrow - \varepsilon_\mathbf{q}/2) + \int \frac{d^2k}{(2\pi)^2} \frac{n_F(\varepsilon_\mathbf{k} - \mu_\uparrow) + n_F(\varepsilon_{\mathbf{k}+\mathbf{q}} - \mu_\downarrow)}{\omega + i0 + \mu_\uparrow + \mu_\downarrow - \varepsilon_\mathbf{k} - \varepsilon_{\mathbf{k}+\mathbf{q}}}$$

we find compact solution

$$T(\mathbf{q},\omega) = T_0\left(\frac{1}{2}z \pm \frac{1}{2}\sqrt{(z-\varepsilon_\mathbf{q})^2 - 4\varepsilon_F\varepsilon_\mathbf{q}}\right) \qquad z = \omega + i0 - \varepsilon_F + \mu_\downarrow$$

Fermion spectral function





step 2: fermion self-energy

$$\Sigma_\downarrow(\mathbf{p},\omega) = \int_{k<k_F} \frac{d^2k}{(2\pi)^2}\, T(\mathbf{k}+\mathbf{p}, \varepsilon_\mathbf{k} - \mu_\uparrow + \omega)$$

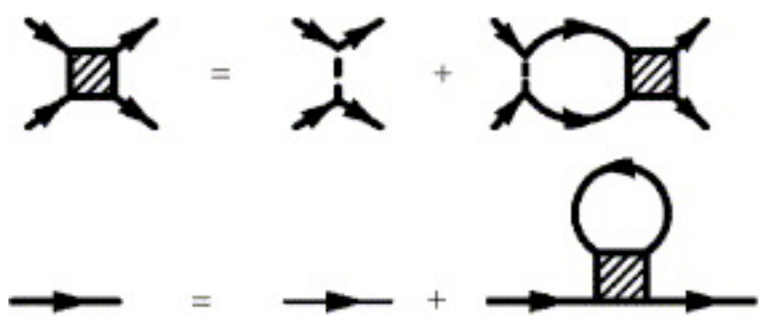
step 3: spectral function

$$\mathcal{A}_\downarrow(\mathbf{p},\omega) = -2\,\mathrm{Im}\,\frac{1}{\omega + i0 + \mu_\downarrow - \varepsilon_\mathbf{p} - \Sigma_\downarrow(\mathbf{p},\omega)}$$

contains full information about
energy spectrum, quasiparticle weights, decay rates...

Luttinger-Ward approach

- repeated particle-particle scattering dominant in dilute gas:

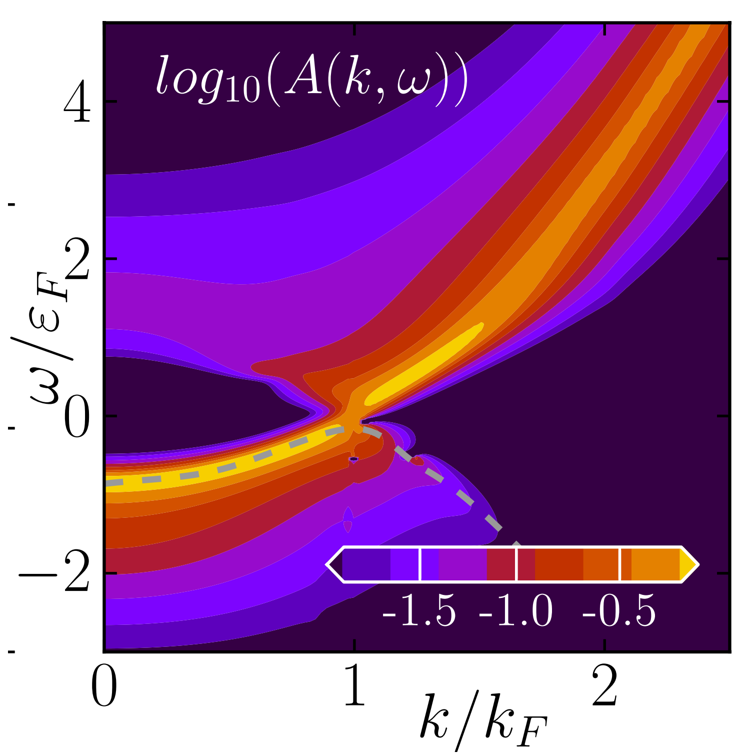


self-consistent T-matrix Haussmann 1993, 1994; Haussmann et al. 2007

self-consistent fermion propagator
(400 momenta / 400 Matsubara frequencies)

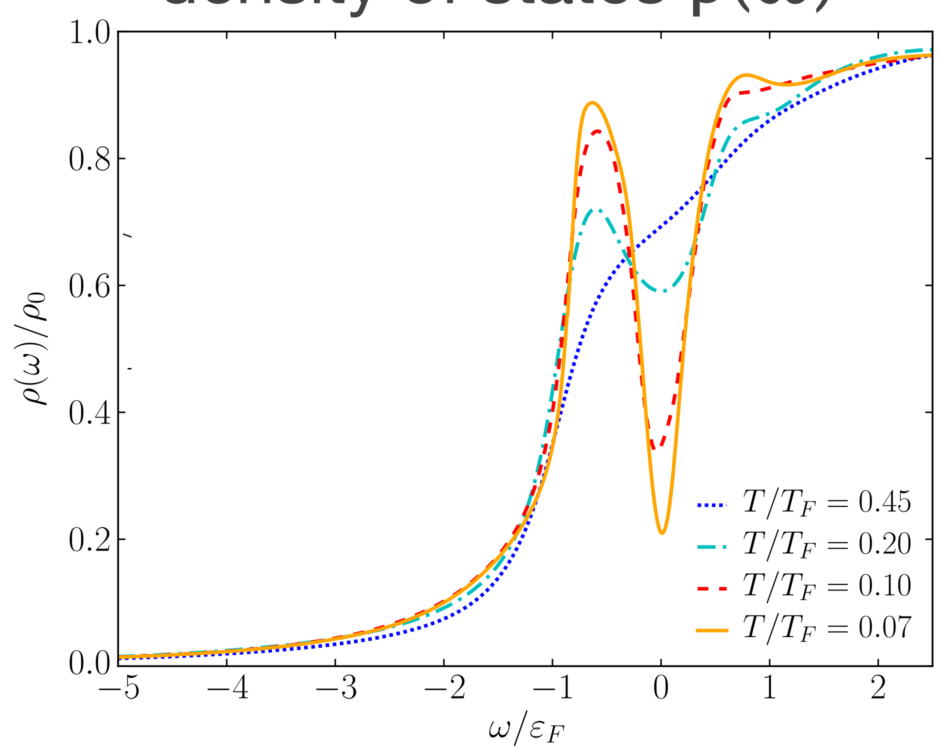
Bauer, Parish, Enss PRL 2014

- spectral function A(k,ω)



$$\int \frac{d^2k}{(2\pi)^2} A(k,\omega)$$

density of states ρ(ω)

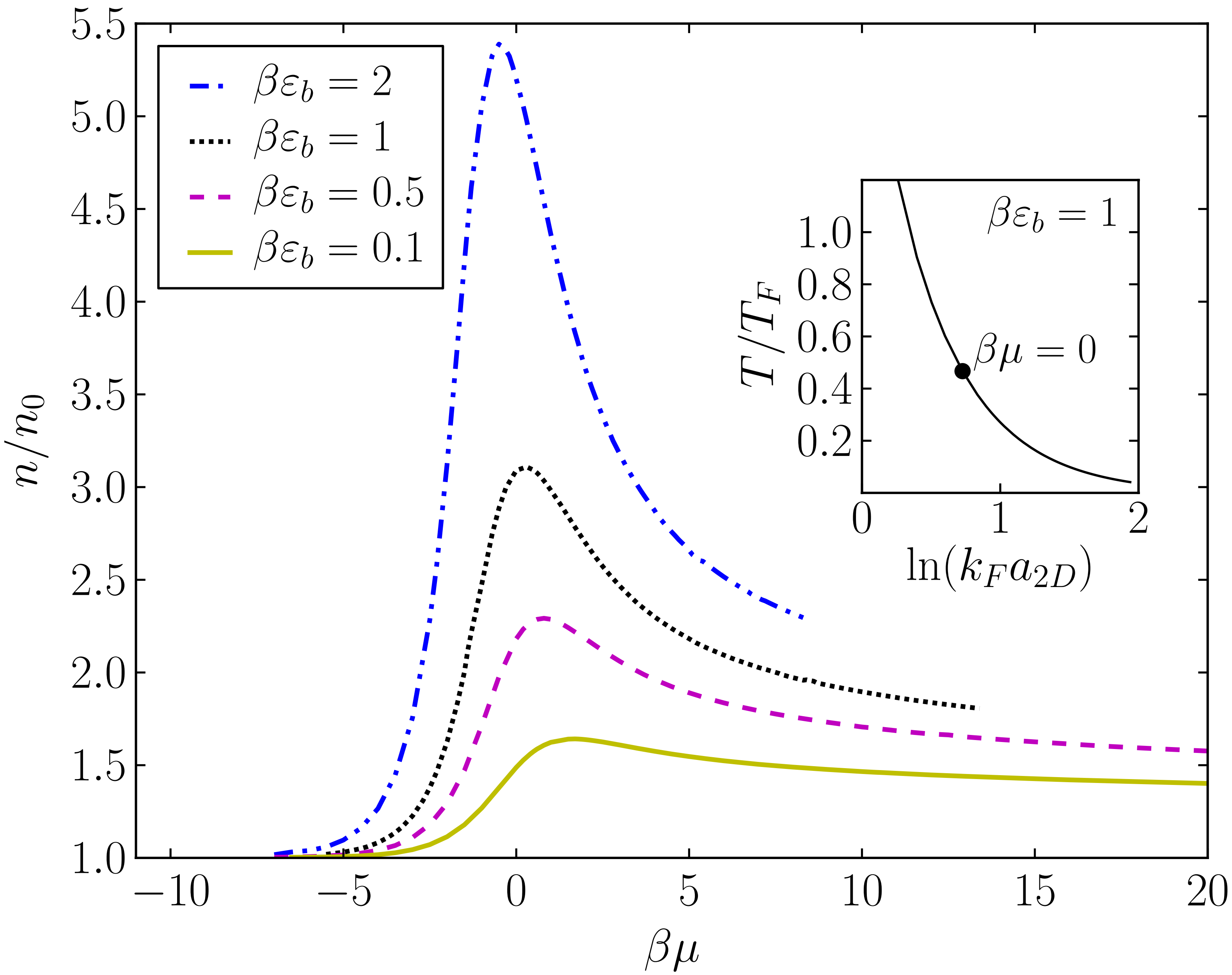


$$n = 2\int d\omega\, f(\omega)\rho(\omega)$$

density

Density equation of state: theory

- **maximum** & **density driven crossover**



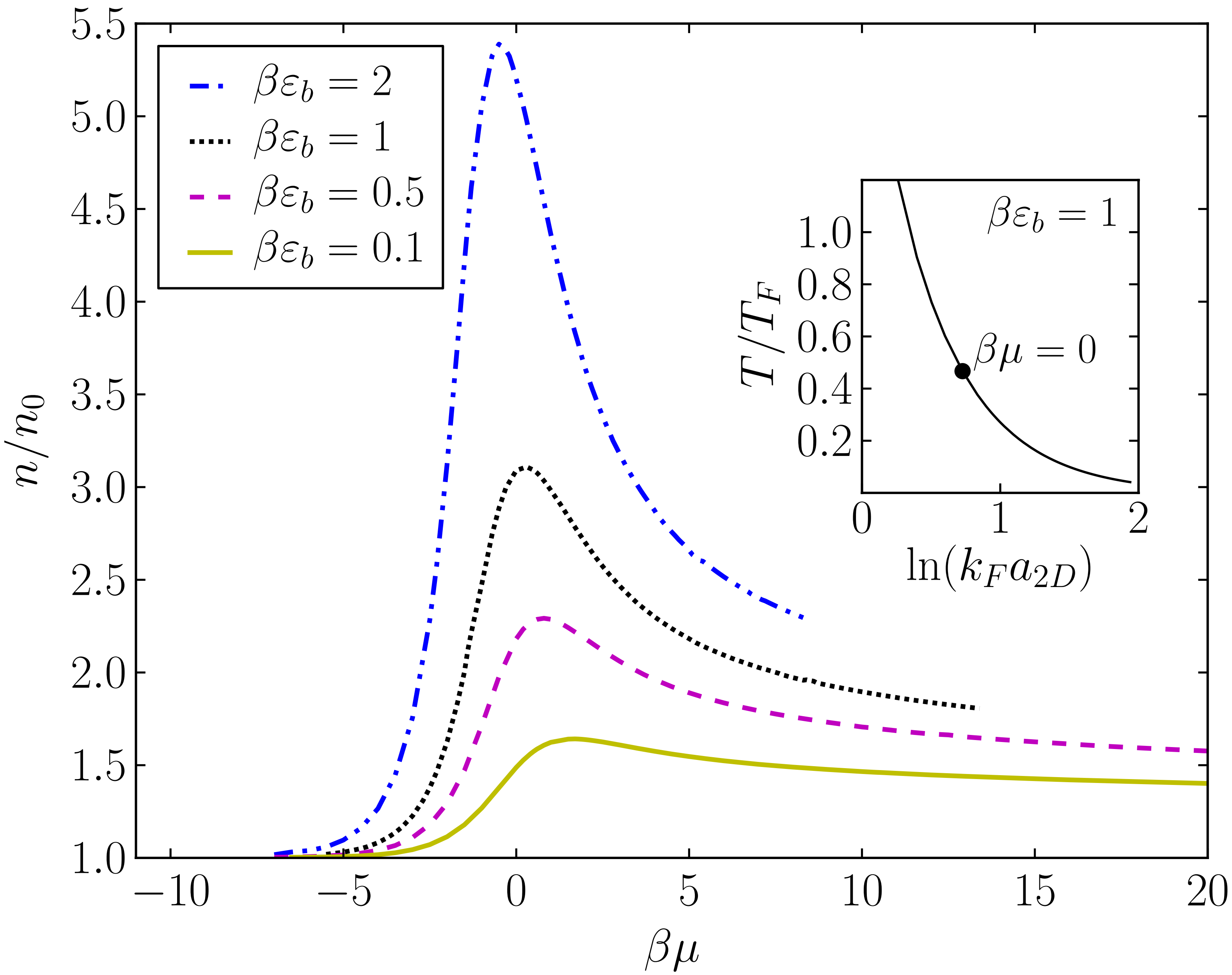
Bauer, Parish & Enss PRL 2014

$$n = 2\int d\omega\, f(\omega)\rho(\omega)$$

$$n_0 = 2\ln(1 + e^{\beta\mu})/\lambda_T^2$$

Density equation of state: theory

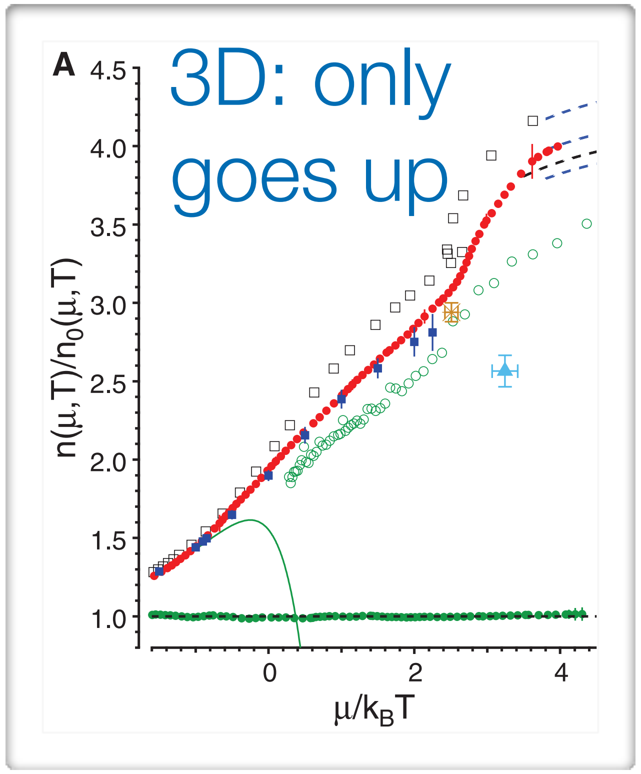
- **maximum** & **density driven crossover**



Bauer, Parish & Enss PRL 2014

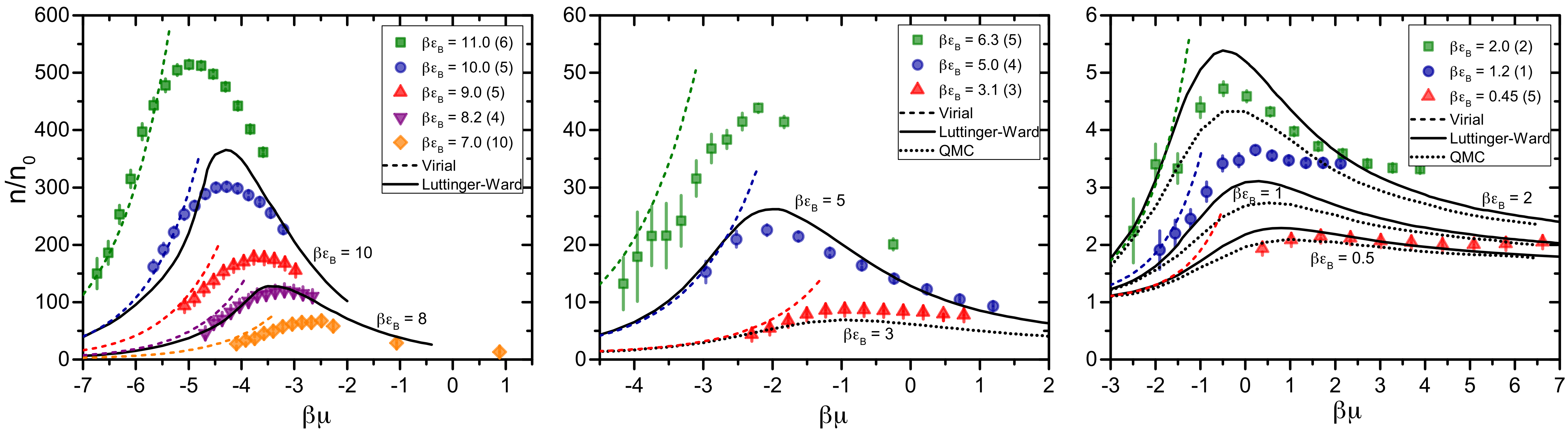
$$n = 2\int d\omega\, f(\omega)\rho(\omega)$$

$$n_0 = 2\ln(1+e^{\beta\mu})/\lambda_T^2$$



Ku+ Science 2012

Equation of state: cold atom experiment



Boettcher, ..., Jochim, Enss PRL 2016

High temperature: virial expansion

- virial expansion

$$n_\sigma \lambda_T^2 = \ln(1 + e^{\beta\mu}) + 2\Delta b_2 e^{2\beta\mu} + 3\Delta b_3 e^{3\beta\mu} + \cdots$$

$$\Delta b_2 = e^{\beta\varepsilon_B} - \int_{-\infty}^{\infty} \frac{\exp[-e^s/(2\pi)]\, ds}{\pi^2 + (s - \ln(2\pi\beta\varepsilon_B))^2}$$

Barth & Hofmann PRA 2014

- Bose limit ($\Delta b_2 \approx e^{\beta\varepsilon_B}$):

$$n_{\rm bos} \approx 2e^{2\beta\mu + \beta\varepsilon_B} \lambda_T^{-2} = e^{\beta\mu_{\rm bos}} \lambda_{\rm bos}^{-2}$$

- good variable between Fermi and Bose limits:

$$2\tilde{\mu} = 2\mu + \varepsilon_B = \mu_{\rm bos}$$

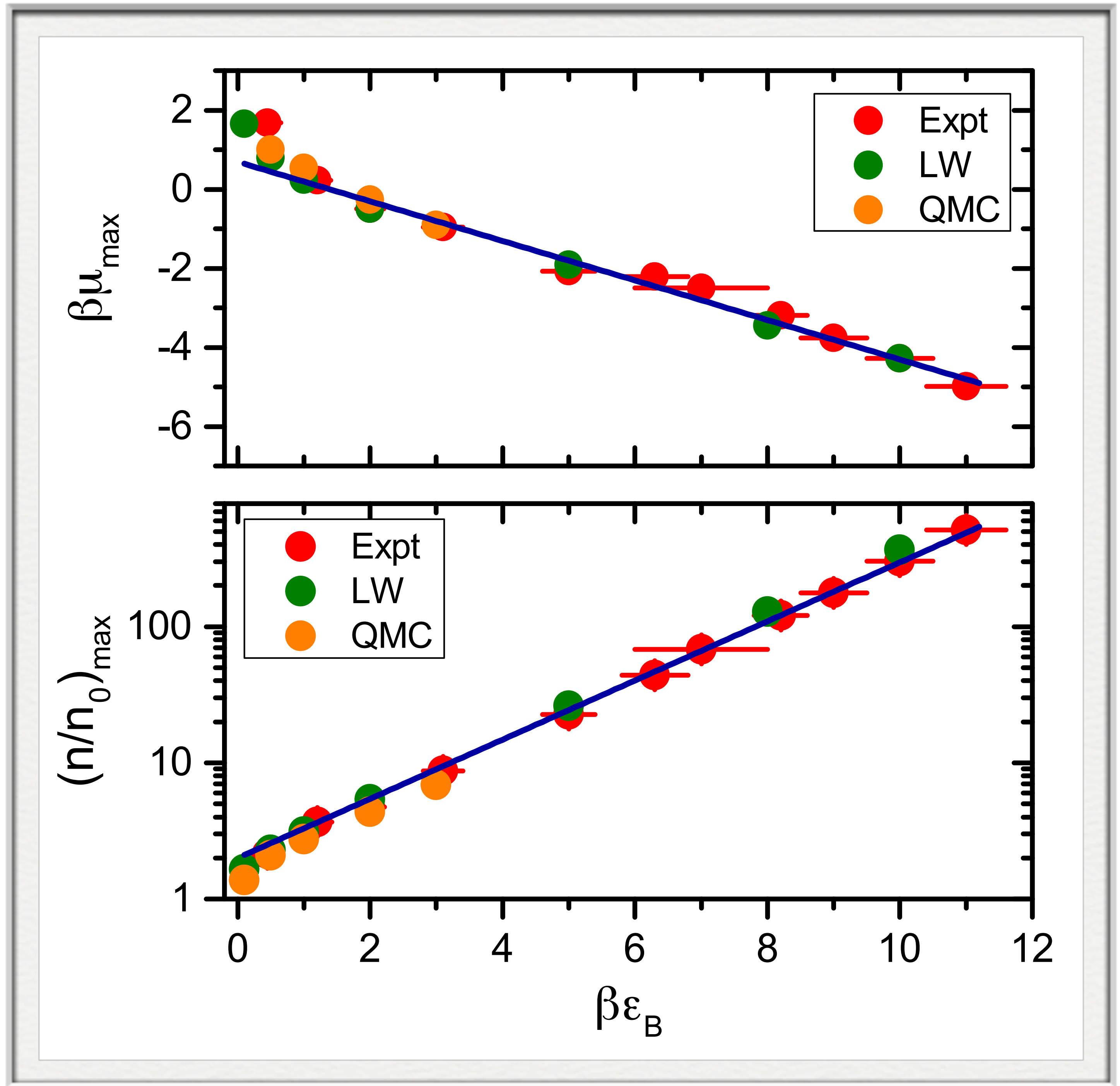
Scaling of density maximum

- **maximum** where $\tilde{\mu} \simeq 0$:

$$(\beta\mu)_{\max} \simeq -\frac{\beta\varepsilon_B}{2} + \ln 2$$

at density

$$(n/n_0)_{\max} \simeq 2e^{\beta\varepsilon_B/2}$$

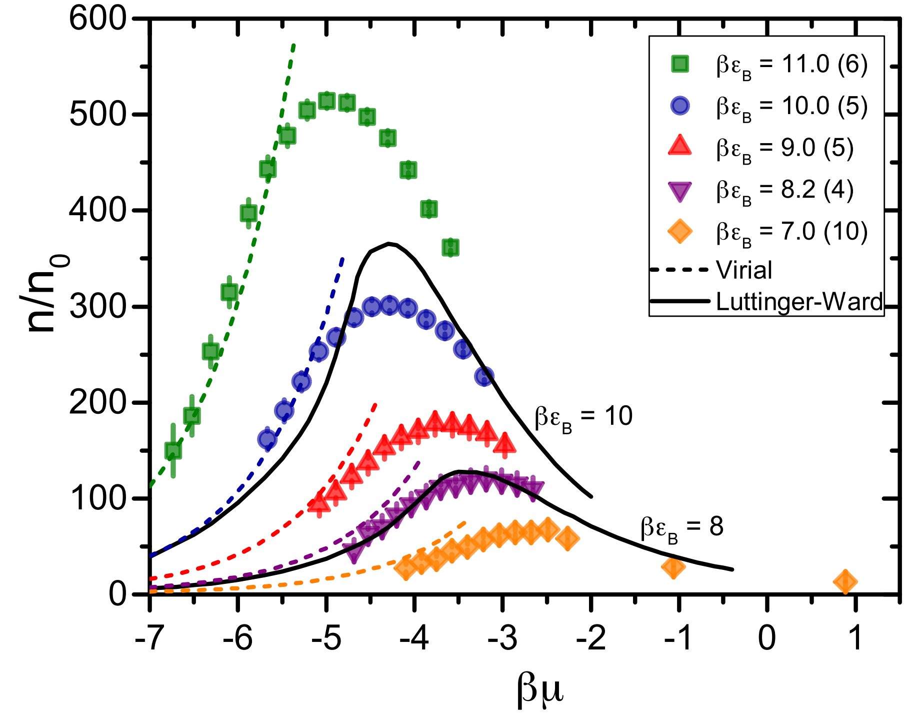


Boettcher, ..., Jochim, Enss PRL 2016

Summary & Outlook

- **2D Fermi gas:**
  scale invariance broken
  exact universal relations for dilute gas
  large density renormalization
  density driven crossover

  Bauer, Parish & Enss 2014
  Boettcher, ..., Jochim, Enss 2016



- **outlook:**
  pseudogap (from fRG...)
  (transverse) spin diffusion D0~hbar/m
  extension to low-temperature phase