Critical exponents can be different on the two sides of a transition

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A bit of history

Before the 70's, quasi-systematic distinction between critical exponents in the ordered and disordered phases: α_+ and α_-, ν_+ and ν_-, ...

Then the RG arrived and, progressively, the distinction disappeared...

Arguments:

- if a theory is renormalizable in the symmetric phase, it is also in the spontaneously broken phase,
- the divergences are of the same nature,
- the scaling functions are identical,
- it is possible to avoid the singularity at the transition by adding a small source (magnetic field),
- there is a continuous path going from one phase to the other;
- Existence of a "proof" for the O(N) models.

But David Nelson arrived...

... in 1976! D. Nelson, Phys.Rev. 13, (1976) 2222.

He claimed that in the presence of discrete symmetries, the magnetic susceptibility in XY systems:

$$\chi = \frac{\partial M}{\partial B} \; , \qquad M = \langle \phi \rangle \;\; , \quad B = \text{magn. field}$$

can behave as:

$$\chi \propto \begin{cases} (T - T_c)^{-\gamma_+} & \text{for T} \to \text{T}_\text{c}^+ \\ (T_c - T)^{-\gamma_-} & \text{for T} \to \text{T}_\text{c}^- \end{cases}$$

with

$$\gamma_+ \neq \gamma_-$$

Since then...

... because of their relationship with

- pyrochlore, M.E. Zhitomirsky, P.C.W. Holdsworth R. and Moessner Phys. Rev. B 89 (2014) 140403,
- deconfined quantum critical points, J. Lou, A. W. Sandvik, et L. Balents, Phys. Rev. Lett. 99 (2007) 207203,
- the possibility of two distinct phase transitions in $d = 3$, M. Oshikawa, Phys. Rev. B 61 (2000) 3430; T. Okubo, K. Oshikawa, H. Watanabe, et N. Kawashima, Phys. Rev. B 91 (2015) 174417

XY systems with hexagonal anisotropy have been restudied in detail in $d = 3$.

Main focus: the existence for $\forall T < T_c$ of two correlation lengths, ξ and ξ' that scale around T_c^- with two different exponents ν and ν'.

Study of cubic anisotropy in $d = 3$ and by J. M. Carmona et al. who proposed that the exponent γ_T of the transverse susceptibility for $T < T_c$ is different from γ (but very small difference...).

J.M. Carmona, A. Pelissetto and E. Vicari, Phys. Rev. B 61,15136-15151 (2000).

The general idea under the form of a paradox

Consider a N-component system described by

$$\mathcal{H} = \mathcal{H}_{O(N)} + \lambda \int_x \tau(x)$$

where $\tau(x) = \tau(\phi_1(x), \phi_2(x), \cdots, \phi_N(x))$

- is invariant under under a discrete subgroup of $O(N)$,
- is irrelevant at the fixed point describing the phase transition.

τ is irrelevant $\Rightarrow$ we can neglect it for the long-distance physics $\Rightarrow$ the attractive fixed point is $O(N)$-invariant $\Rightarrow$ the critical physics is identical to the usual $O(N)$ one.

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WRONG

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- is invariant under under a discrete subgroup of $O(N)$
- is irrelevant at the fixed point describing the phase transition of the model.

Discrete symmetry $\Rightarrow$ no Goldstone bosons $\Rightarrow$ the susceptibilities (transverse and longitudinal) are finite for $\forall T < T_c \Rightarrow$ they diverge only when $T \to T_c^- \Rightarrow$ although irrelevant, $\tau(x)$ matters for their behavior at $T_c^- \Rightarrow$ difference with $T > T_c$ where $\tau(x)$ indeed plays no role at long distance.

$\tau(x)$ is a dangerously irrelevant operator for the susceptibilities.

NPRG is convenient (as usual!)

For concreteness, we consider the $N = 2$ model with $\mathbb{Z}_6$ anisotropy:
$\tau = (\phi_1 - \phi_2)^2(\phi_1^2 + 4\phi_1\phi_2 + \phi_2^2)^2$.

We use the LPA':

$$\Gamma_k[\phi] = \int_x \frac{Z_k}{2}[\nabla\phi(x)]^2 + U_k[\rho(x), \tau(x)],$$

$(\rho = 1/2(\phi_1^2 + \phi_2^2))$ and we perform a field-expansion around the minimum of U_k: $\rho = \kappa_k$ and $\tau = 0$ (no need to be functional but important to be nonperturbative).

$$U_k(\rho, \tau) = \frac{u_k}{2}(\rho - \kappa)^2 + \frac{u_{3;k}}{3!}(\rho - \kappa)^3 + \ldots + \lambda_{6;k}\tau + u_{1,1;k}(\rho - \kappa_k)\tau + \ldots$$

We turn the crank...

$$\partial_t \tilde{\kappa} = (2 - d - \eta_t)\tilde{\kappa} + \left(\frac{1}{2} + \frac{18\tilde{\kappa}\tilde{\lambda}_6}{\tilde{u}}\right) l_2(\tilde{m}_T^2) + \frac{3}{2} l_2(\tilde{m}_L^2)$$

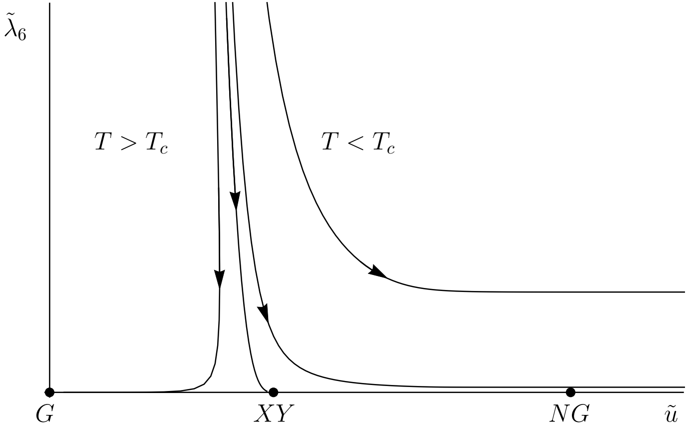
$$\partial_t \tilde{u} = (d - 4 + 2\eta_t)\tilde{u} - 18\tilde{\lambda}_6 l_2(\tilde{m}_T^2) + 9\tilde{u}^2 l_3(\tilde{m}_L^2) + (\tilde{u} + 36\tilde{\kappa}\tilde{\lambda}_6)^2 l_3(\tilde{m}_T^2)$$

$$\partial_t \tilde{\lambda}_6 = (2d - 6 + 3\eta_t)\tilde{\lambda}_6 + 15\tilde{\lambda}_6(\tilde{u} + 6\tilde{\kappa}\tilde{\lambda}_6)\frac{l_2(\tilde{m}_T^2) - l_2(\tilde{m}_L^2)}{\tilde{m}_L^2 - \tilde{m}_T^2}$$

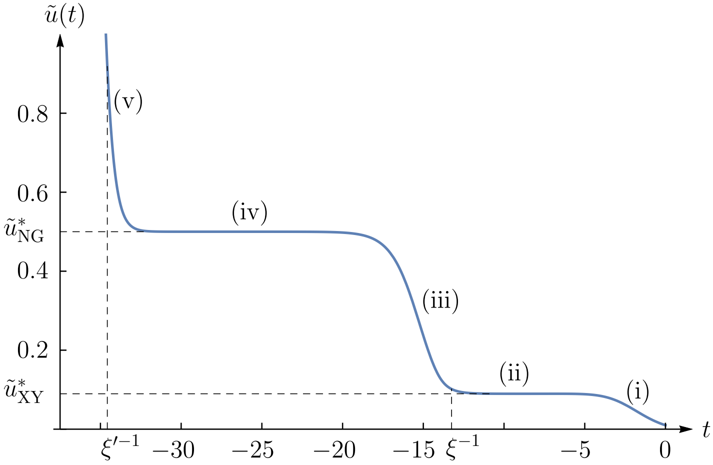
$$\tilde{m}_L^2 = 2\tilde{\kappa}\tilde{u} \text{ and } \tilde{m}_T^2 = 18\tilde{\kappa}^2\tilde{\lambda}_6$$

$$l_n(x) = 2\left(1 - \frac{\eta_k}{d+2}\right)\frac{1}{(1+x)^n}$$

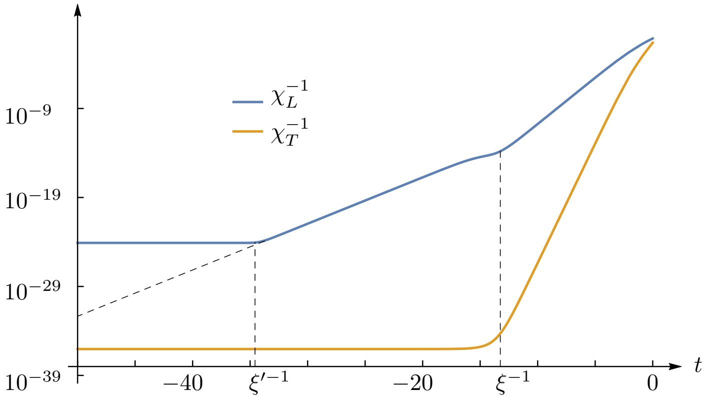
dimensionless flow in the $(\tilde{\lambda}_6, \tilde{u})$-plane



dimensionless flow of $\tilde{u}$



Flows of the inverse transverse and longitudinal susceptibilities



Scaling relations among exponents

ξ'^{-1} is defined by the scale where $\tilde{u}_k$ departs from its NG value (and u reaches a finite value). Equivalently, when $\chi_L(k)$ stops running.

For $k < \xi^{-1}$, the dimensionful minimum κ_k as well as $\lambda_{6,k}$ have (almost) reached their final values $\Rightarrow$ $\tilde{m}_T^2 = 18\tilde{\lambda}_{6,k}\tilde{\kappa}_k^2 \sim k^{-2}$ and $\tilde{m}_L^2 = 2\tilde{u}_k\tilde{\kappa}_k \sim k^{d-4}$.

$$\tilde{\kappa}_k \sim \tilde{\kappa}_{k=\xi^{-1}}\,(k\xi)^{2-d} \sim \tilde{\kappa}^*_{\mathrm{XY}}\,(k\xi)^{2-d}$$

$$\tilde{\lambda}_{6,k} \sim \tilde{\lambda}_{6,k=\xi^{-1}}\,(k\xi)^{2d-6} \sim \tilde{\lambda}_6^{\mathrm{in}}\,(\xi\Lambda)^{-|y_6|}(k\xi)^{2d-6}$$

where y_6 is the eigenvalue of the linearized flow around the O(2) fixed point in the λ_6-direction.

$\Rightarrow$ m_T^2 reaches a plateau $\Rightarrow$ $\tilde{m}_T^2 \sim k^{-2}$ while, for $\xi' < k < \xi$, $\tilde{m}_L^2 \sim k^{d-4}$.

$$\partial_t \tilde{u} = (d-4)\tilde{u} - 18\tilde{\lambda}_6 l_2(\tilde{m}_T^2) + 9\tilde{u}^2 l_3(\tilde{m}_L^2) + (\tilde{u} + 36\tilde{\kappa}\tilde{\lambda}_6)^2 l_3(\tilde{m}_T^2)$$

with $l_n(x) = 2/(1+x)^n$.
For $k < \xi^{-1}$, the term $l_3(\tilde{m}_T^2)$ starts to decrease when $\tilde{m}_T^2 \simeq 1 \Rightarrow$ the definition of ξ' is

$$\tilde{m}_T^2(k = \xi'^{-1}) = 1$$

Three new exponents

$$\xi' \sim (T_c - T)^{-\nu'}$$
$$\chi_{L,T} \sim (T_c - T)^{-\gamma_{L,T}}$$

with three new scaling relations

$$\nu' = \nu(1 + |y_6|/2)$$
$$\gamma_L = \gamma_+ + (4-d)\nu|y_6|/2$$
$$\gamma_T = \gamma_+ + \nu|y_6|$$

We have expanded the potential up to order 12:

$$\nu = 0.696$$
$$\eta = 0.044$$
$$\gamma_+ = \nu(2-\eta) = 1.36$$

Very large values of y_q: $y_{10} \simeq 9$ and $y_{12} \simeq 25$.

Symmetry	$\mathbb{Z}_4$	$\mathbb{Z}_5$	$\mathbb{Z}_6$	$\mathbb{Z}_8$	$\mathbb{Z}_{10}$	$\mathbb{Z}_{12}$
ν'	0.71	1.06	1.44	2.35	3.84	5.4
	0.72	1.05	1.6		2.8	
			1.45			
$\gamma_T - \gamma_+$	0.029	0.74	1.49	3.31	6.29	12.19
	0.06		1.58			

Table: Critical exponents in $d = 3$ for the $\mathbb{Z}_q$ invariant models.

The hierarchy/fine-tuning problem of the Standard Model

Together with N. Wschebor (Montevideo, Uruguay)

Pb: The ratio $\frac{\Lambda_{\rm UV}}{M_H}$ is very large... even if we do not know what $\Lambda_{\rm UV}$ is!
For $\Lambda_{\rm UV} = \Lambda_{\rm Planck}$, this ratio is 10^{17}.

The problem is similar to having ξ/a very large in Stat. Mech.: Requires a fine-tuning of the temperature (bare mass) to make it very close to the critical temperature $\Rightarrow$ very unnatural!

Solutions (?): Supersymmetry, technicolor, extra-space dimensions...

Is it possible to avoid these complicated solutions
and to produce naturally light scalars?

A toy model in $d = 4$: $O(4) \times \mathbb{Z}_q \to O(3)$

We need three Goldstone bosons that will be eaten by the gauge particles, $W^{\pm}$ and Z^0, (Higgs mechanism) and a naturally light Higgs particle: O(4)$\to$ O(3) and $\mathbb{Z}_q \to 1$.

Field content: a doublet of 4-component vectors $\phi_{\alpha,i}$ with $\alpha = 1, 2$ and $i = 1, \cdots, 4$. In the broken phase: three Goldstone bosons, one light and four heavy bosons.

Action: similar to above with the subtlety that we now have 4-component vectors.

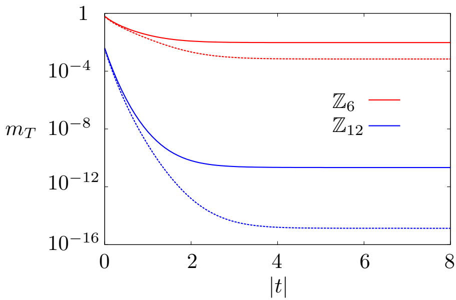
Problem with this toy model: the custodial symmetry imposing

$$\frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1$$

is not satisfied and the couplings with the fermions will not give them the correct (small) masses.

Anyway...

flow of the mass of the light scalar with a tuning of 1% and 10%



Work in progress: build a realistic model with a naturally light scalars and all couplings realistic.