UV Completion of Some UV Fixed Points

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Talk mostly based on

- L. Fei, S. Giombi, IK, arXiv:1404.1094
- S. Giombi, IK, arXiv:1409.1937
- L. Fei, S. Giombi, IK, G. Tarnopolsky, arXiv:1411.1099
- L. Fei, S. Giombi, IK, G. Tarnopolsky, arXiv:1507.01960
- L. Fei, S. Giombi, IK, G. Tarnopolsky, arXiv:1607.05316

The Gross-Neveu Model

$$\mathcal{L}_{\mathrm{GN}} = \bar{\psi}_j \not{\partial} \psi^j + \frac{g}{2}(\bar{\psi}_j \psi^j)^2 \qquad j = 1, \ldots N_f$$

- In 2 dimensions it has some similarities with the 4-dimensional QCD.
- It is asymptotically free and exhibits dynamical mass generation.
- Similar physics in the 2-d O(N) non-linear sigma model with N>2.
- In dimensions slightly above 2 both the O(N) and GN models have weakly coupled UV fixed points.

2+ ε expansion

- The beta function and fixed-point coupling are

$$\beta = \epsilon g - (N-2)\frac{g^2}{2\pi} + (N-2)\frac{g^3}{4\pi^2} + (N-2)(N-7)\frac{g^4}{32\pi^3} + \mathcal{O}(g^5)$$

$$g_* = \frac{2\pi}{N-2}\epsilon + \frac{2\pi}{(N-2)^2}\epsilon^2 + \frac{(N+1)\pi}{2(N-2)^3}\epsilon^3 + \mathcal{O}(\epsilon^4)\,,$$

- $N = N_f \mathrm{tr}\mathbf{1} = 4N_f$ is the number of 2-component Majorana fermions.
- Can develop 2+ε expansions for operator scaling dimensions, e.g. Gracey; Kivel, Stepanenko, Vasiliev

$$\Delta_\psi = \frac{1}{2} + \frac{1}{2}\epsilon + \frac{N-1}{4(N-2)^2}\epsilon^2 - \frac{(N-1)(N-6)}{8(N-2)^3}\epsilon^3 + \mathcal{O}(\epsilon^4)\,,$$

$$\Delta_\sigma = 1 - \frac{1}{N-2}\epsilon - \frac{N-1}{2(N-2)^2}\epsilon^2 + \frac{N(N-1)}{4(N-2)^3}\epsilon^3 + \mathcal{O}(\epsilon^4)\,, \qquad \sigma \sim \bar{\psi}\psi$$

- Similar expansions in the O(N) sigma model with N>2.
  Brezin, Zinn-Justin

4-ε expansion

- The O(N) sigma model is in the same universality class as the O(N) model:

$$S = \int d^d x \left(\frac{1}{2}(\partial \phi^i)^2 + \frac{\lambda}{4}(\phi^i \phi^i)^2 \right)$$

- It has a weakly coupled Wilson-Fisher IR fixed point in 4-ε dimensions.
- Using the two ε expansions, the scalar CFTs with various N may be studied in the range 2<d<4. This is an excellent practical tool for CFTs in d=3.

The Gross-Neveu-Yukawa Model

- The GNY model is the UV completion of the GN model in d<4 Zinn-Justin; Hasenfratz, Hasenfratz, Jansen, Kuti, Shen

$$\mathcal{L}_{\rm GNY} = \frac{1}{2}(\partial_\mu \sigma)^2 + \bar\psi_j \partial\!\!\!/ \psi^j + g_1 \sigma \bar\psi_j \psi^j + \frac{1}{24} g_2 \sigma^4$$

- IR stable fixed point in 4-ε dimensions

$$\beta_{g_1} = -\frac{\epsilon}{2} g_1 + \frac{N+6}{2(4\pi)^2} g_1^3 + \frac{1}{(4\pi)^4}\Big(-\frac{3}{4}(4N+3) g_1^5 - 2 g_1^3 g_2 + \frac{g_1 g_2^2}{12}\Big)$$

$$\beta_{g_2} = -\epsilon g_2 + \frac{1}{(4\pi)^2}\Big(3 g_2^2 + 2N g_1^2 g_2 - 12 N g_1^4\Big) + \frac{1}{(4\pi)^4}\Big(96 N g_1^6 + 7 N g_1^4 g_2 - 3 N g_1^2 g_2^2 - \frac{17 g_2^3}{3}\Big)$$

$$\frac{(g_1^*)^2}{(4\pi)^2} = \frac{1}{N+6}\epsilon + \frac{(N+66)\sqrt{N^2+132N+36} - N^2 + 516N + 882}{108(N+6)^3}\epsilon^2$$

$$\frac{g_2^*}{(4\pi)^2} = \frac{-N+6+\sqrt{N^2+132N+36}}{6(N+6)}\epsilon$$

- Operator scaling dimensions

$$\Delta_\sigma = 1 - \frac{3}{N+6}\epsilon + \frac{52N^2 - 57N + 36 + (11N+6)\sqrt{N^2+132N+36}}{36(N+6)^3}\epsilon^2$$

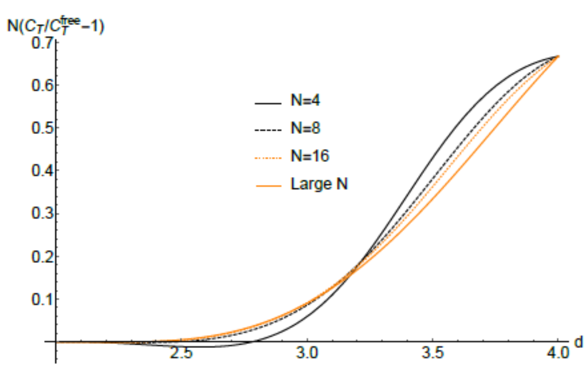
$$\Delta_\psi = \frac{3}{2} - \frac{N+5}{2(N+6)}\epsilon + \frac{-82N^2 + 3N + 720 + (N+66)\sqrt{N^2+132N+36}}{216(N+6)^3}\epsilon^2$$

$$\Delta_{\sigma^2} = d - 2 + \gamma_{\sigma^2} = 2 + \frac{\sqrt{N^2+132N+36} - N - 30}{6(N+6)}\epsilon$$

- Using the two ε expansions, we can study the Gross-Neveu CFTs in the range 2<d<4.
- Another interesting observable

Diab, Fei, Giombi, IK, Tarnopolsky

$$\langle T_{\mu\nu}(x_1) T_{\lambda\rho}(x_2)\rangle = C_T \frac{I_{\mu\nu,\lambda\rho}(x_{12})}{(x_{12}^2)^d}$$



Sphere Free Energy in Continuous d

- A natural quantity to consider is Giombi, IK

$$\tilde{F} = \sin(\pi d/2) \log Z_{S^d} = -\sin(\pi d/2) F$$

- In odd d, this reduces to IK, Pufu, Safdi

$$\tilde{F} = (-1)^{\frac{d+1}{2}} F = (-1)^{\frac{d-1}{2}} \log Z_{S^d}$$

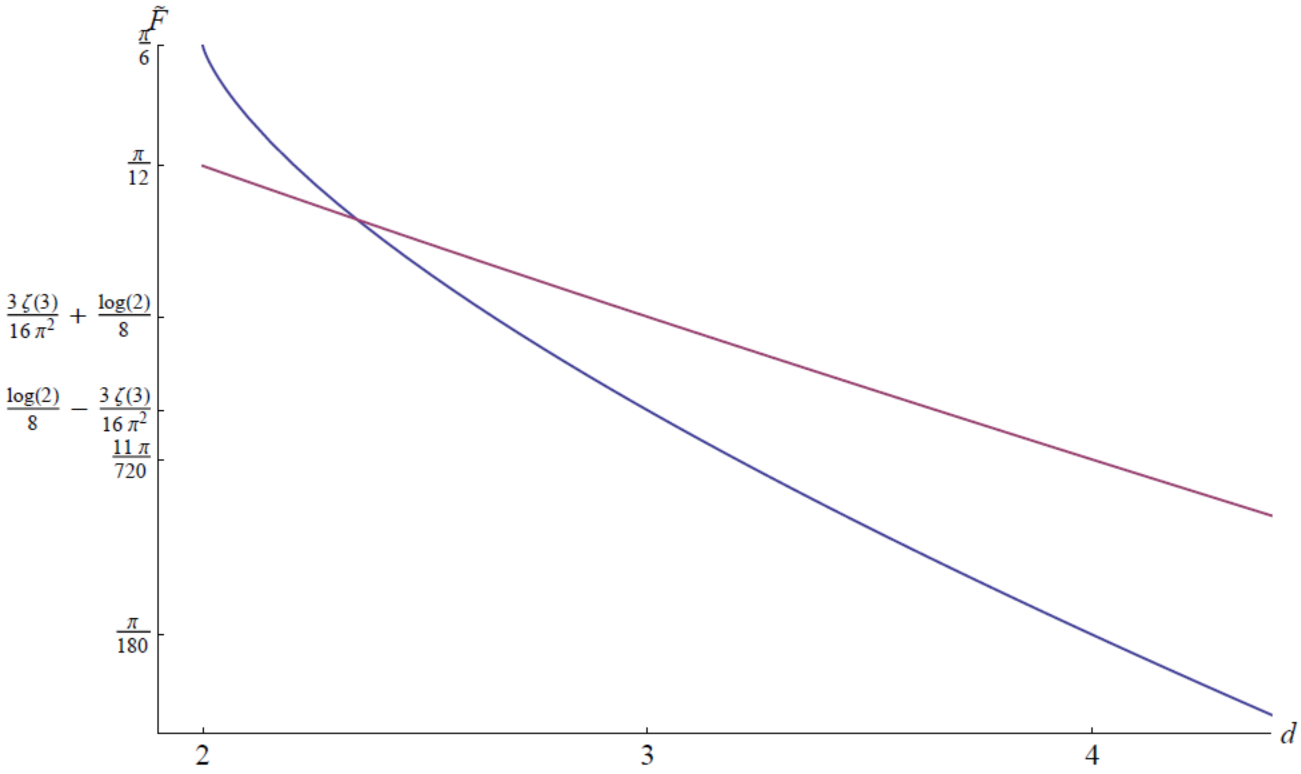
- In even d, -log Z has a pole in dimensional regularization whose coefficient is the Weyl *a*-anomaly. The multiplication by $\sin(\pi d/2)$ removes it.
- $\tilde{F}$ smoothly interpolates between *a*-anomaly coefficients in even and ``F-values" in odd d.
- Gives the universal entanglement entropy across d-2 dimensional sphere. Casini, Huerta, Myers

Free Conformal Scalar and Fermion

$$\tilde{F}_s = \frac{1}{\Gamma(1+d)} \int_0^1 du\, u \sin \pi u\, \Gamma\left(\frac{d}{2}+u\right) \Gamma\left(\frac{d}{2}-u\right) ,$$

$$\tilde{F}_f = \frac{1}{\Gamma(1+d)} \int_0^1 du\, \cos\left(\frac{\pi u}{2}\right) \Gamma\left(\frac{1+d+u}{2}\right) \Gamma\left(\frac{1+d-u}{2}\right)$$

- Smooth and positive for all d.



Sphere Free Energy for the O(N) Model

- At the Wilson-Fisher fixed point it is necessary to include the curvature terms in the Lagrangian *Fei, Giombi, IK, Tarnopolsky*

$$\frac{\eta_0}{2}\mathcal{R}\sigma^2 + a_0 W^2 + b_0 E + c_0 \mathcal{R}^2$$

$$E = \mathcal{R}_{\mu\nu\lambda\rho}\mathcal{R}^{\mu\nu\lambda\rho} - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}^2$$

- The 4-ε expansion then gives

$$\begin{aligned}\tilde{F}_{\rm IR} =& N\tilde{F}_s(\epsilon) - \frac{\pi N(N+2)\epsilon^3}{576(N+8)^2} - \frac{\pi N(N+2)(13N^2+370N+1588)\epsilon^4}{6912(N+8)^4} \\ &+ \frac{\pi N(N+2)}{414720(N+8)^6}\big(10368(N+8)(5N+22)\zeta(3) - 647N^4 - 32152N^3 \\ &-606576N^2 - 3939520N + 30\pi^2(N+8)^4 - 8451008\big)\,\epsilon^5 + \mathcal{O}(\epsilon^6)\end{aligned}$$

- The 2+ε expansion in the O(N) sigma model is plagued by IR divergences. It has not been developed yet, but we know the value in d=2 and can use it in the Pade extrapolations.

Sphere Free Energy for the GN CFT

- The 4-ε expansion

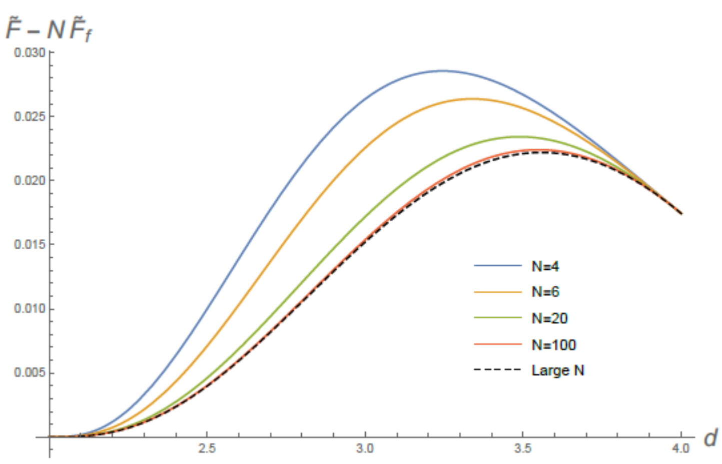
$$\tilde{F} = N\tilde{F}_f + \tilde{F}_s - \frac{N\pi\epsilon^2}{96(N+6)} - \frac{1}{31104(N+6)^3}\Big(161N^3 + 3690N^2 + 11880N + 216 + \left(N^2+132N+36\right)\sqrt{N^2+132N+36}\Big)\pi\epsilon^3 + \mathcal{O}(\epsilon^4)$$

- The 2+ε expansion is under good control; no IR divergences:

$$\tilde{F} = N\tilde{F}_f + \frac{N(N-1)\pi\epsilon^3}{48(N-2)^2} - \frac{N(N-1)(N-3)\pi\epsilon^4}{32(N-2)^3} + \mathcal{O}(\epsilon^5)$$

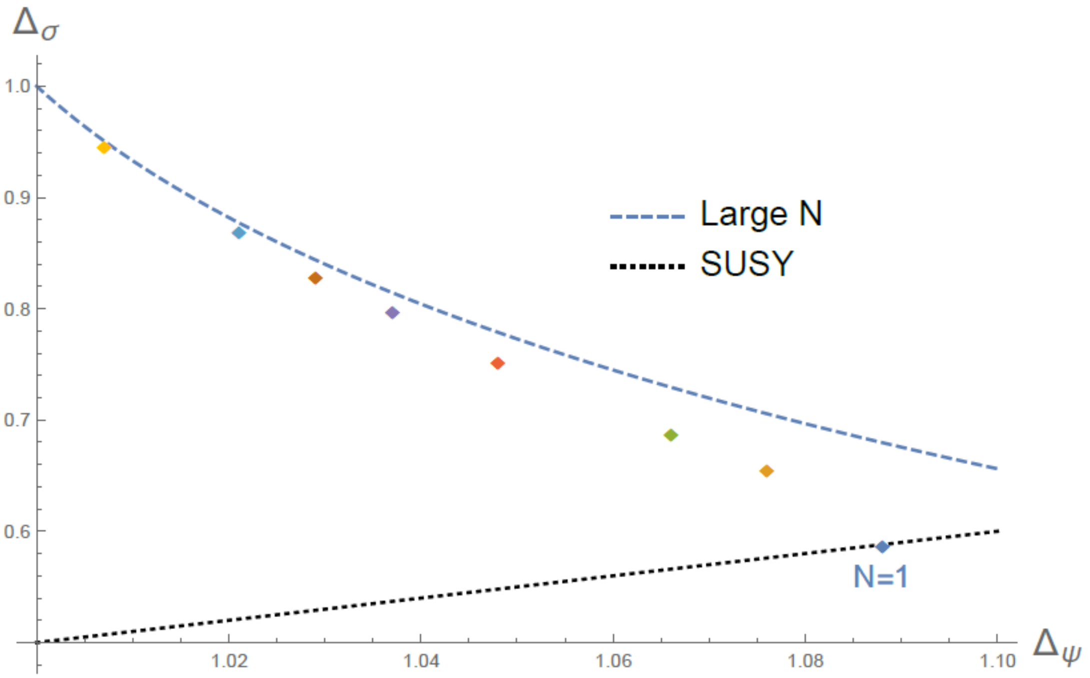
- It is a pleasure to Pade.
- Once again,

$$\tilde{F}_{UV} > \tilde{F}_{IR}$$



Summary for the 3-d GN CFTs

N	3	4	5	6	8	20	100
Δ_ψ (Pade$_{[4,2]}$)	1.066	1.048	1.037	1.029	1.021	1.007	1.0013
Δ_σ (Pade$_{[4,2]}$)	0.688	0.753	0.798	0.829	0.87	0.946	0.989
Δ_{σ^2} (Pade$_{[1,5]}$)	2.285	2.148	2.099	2.075	2.052	2.025	2.008
$F/(NF_f)$ (Pade$_{[4,4]}$)	1.091	1.060	1.044	1.034	1.024	1.008	1.0014



Emergent Global Symmetries

- Renormalization Group flow can lead to IR fixed points with enhanced symmetry.
- The minimal 3-d Yukawa theory for one Majorana fermion and one real pseudo-scalar was conjectured to have "emergent supersymmetry."
  Scott Thomas, unpublished seminar at KITP.
- The fermion mass is forbidden by the time reversal symmetry.
- After tuning the pseudo-scalar mass to zero, the theory is conjectured to flow to a $\mathcal{N}$=1 supersymmetric 3-d CFT.

Superconformal Theory

- The UV lagrangian may be taken as

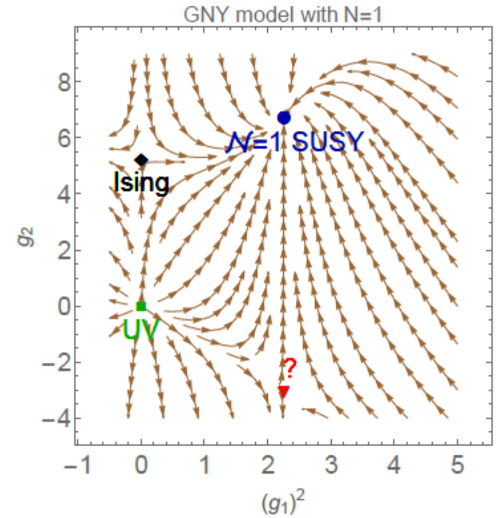
$$\mathcal{L}_{\mathcal{N}=1} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}\bar{\psi}\not{\partial}\psi + \frac{\lambda}{2}\sigma\bar{\psi}\psi + \frac{\lambda^2}{8}\sigma^4$$

- Has cubic superpotential $W \sim \lambda\Sigma^3$ in terms of the superfield $\Sigma = \sigma + \bar{\theta}\psi + \frac{1}{2}\bar{\theta}\theta f$
- Some evidence for its existence from the conformal bootstrap (but requires tuning of some operator dimensions). Iliesiu, Kos, Poland, Pufu, Simmons-Duffin, Yacoby; Bashkirov
- Condensed matter realization has been proposed: emergent SUSY may arise at the boundary of a topological superconductor. Grover, Sheng, Vishwanath

The Minimal Case: N=1

- For a single Majorana doublet the GN quartic interaction vanishes. Cannot use the 2+ε expansion to describe an interacting CFT.
- We have developed the 4-ε expansion by continuing the GNY model to N=1.
- $\sqrt{N^2+132N+36}$ equals 13.

$$\frac{(g_1^*)^2}{(4\pi)^2} = \frac{1}{7}\epsilon + \frac{3}{49}\epsilon^2 + \mathcal{O}(\epsilon^3)$$
$$\frac{g_2^*}{(4\pi)^2} = \frac{3}{7}\epsilon + \frac{9}{49}\epsilon^2 + \mathcal{O}(\epsilon^3)$$



- Consistent with the emergent SUSY relation!

$$3g_1^2 = g_2 = 3\lambda^2$$

More Evidence of SUSY for N=1

$$\Delta_\sigma = 1 - \frac{3}{7}\epsilon + \frac{1}{49}\epsilon^2 + \mathcal{O}(\epsilon^3)$$
$$\Delta_\psi = \frac{3}{2} - \frac{3}{7}\epsilon + \frac{1}{49}\epsilon^2 + \mathcal{O}(\epsilon^3)$$
$$\Delta_{\sigma^2} = 2 - \frac{3}{7}\epsilon + \frac{1}{49}\epsilon^2 + \mathcal{O}(\epsilon^3)$$

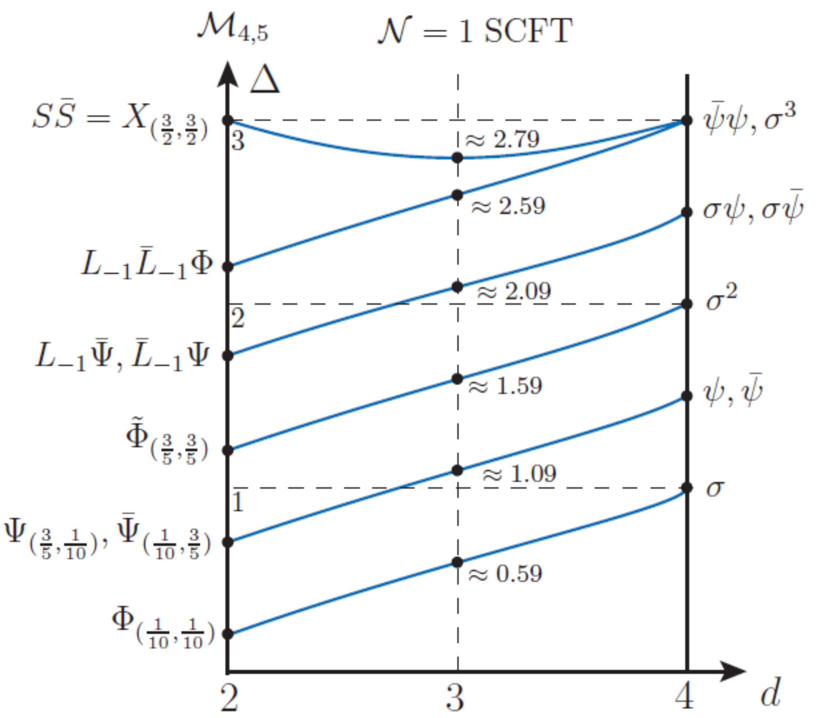
- Consistent with the SUSY relation

$$\Delta_{\sigma^2} = \Delta_\psi + \frac{1}{2} = \Delta_\sigma + 1$$

- We conjecture that it holds exactly for d< 4.
- Would be nice to test at higher orders in ε. This requires doing Yukawa theory at 3 loops and beyond.
- Pade to d=3 gives $\Delta_\sigma \approx 0.588$ which seems close to the bootstrap result. Iliesiu, Kos, Poland, Pufu, Simmons-Duffin, Yacoby

Continuation to d=2

- Gives an interacting superconformal theory.
- Likely the tri-critical Ising model with c=7/10.
- Pade extrapolation gives $\Delta_\sigma \approx 0.217$, close to dimension 1/5 of the energy operator in the (4,5) minimal model.
- Pade also gives $\tilde{F}/\tilde{F}_s \approx 0.68$, close to c=0.7.



Higher Spin AdS/CFT

- When N is large, the O(N) and GN models have an infinite number of higher spin currents whose anomalous dimensions are of order 1/N.
- Their singlet sectors have been conjectured to be dual to the Vasiliev interacting higher-spin theories in d+1 dimensional AdS space.
- One passes from the dual of the free to that of the interacting large N theory by changing boundary conditions at AdS infinity. IK, Polyakov; Leigh, Petkou; Sezgin, Sundel; for a recent review, see Giombi's TASI lectures

Interacting CFT's

- A scalar operator $\mathcal{O}(x^\mu)$ in d-dimensional CFT is dual to a field $\Phi(z, x^\mu)$ in AdS_{d+1} which behaves near the boundary as z^Δ
- There are two choices $$\Delta_\pm = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2}$$
- If we insist on unitarity, then Δ_- is allowed only in the Breitenlohner-Freedman range IK, Witten

$$-(d/2)^2 < m^2 < -(d/2)^2 + 1$$

- Flow from a large N CFT where $\mathcal{O}(x^\mu)$ has dimension Δ_- to another CFT with dimension Δ_+ by adding a double-trace operator. Witten; Gubser, IK
- Can flow from the free d=3 scalar model in the UV to the Wilson-Fisher interacting one in the IR. The dimension of scalar bilinear changes from 1 to 2 +O(1/N). The dual of the interacting theory is the Vasiliev theory with Δ=2 boundary conditions on the bulk scalar.
- The 1/N expansion is generated using the Hubbard-Stratonovich auxiliary field.

$$S = \int d^d x \left(\frac{1}{2}(\partial\phi^i)^2 + \frac{1}{2}\sigma\phi^i\phi^i - \frac{\sigma^2}{4\lambda} \right)$$

- In 2<d<4 the quadratic term may be ignored in the IR:

$$Z = \int D\phi D\sigma \, e^{-\int d^dx \left(\frac{1}{2}(\partial\phi^i)^2 + \frac{1}{2\sqrt{N}}\sigma\phi^i\phi^i\right)}$$

$$= \int D\sigma \, e^{\frac{1}{8N}\int d^dx d^dy \, \sigma(x)\sigma(y) \, \langle\phi^i\phi^i(x)\phi^j\phi^j(y)\rangle_0 + \mathcal{O}(\sigma^3)}$$

- Induced dynamics for the auxiliary field endows it with the propagator

$$\langle\sigma(p)\sigma(-p)\rangle = 2^{d+1}(4\pi)^{\frac{d-3}{2}}\Gamma\left(\frac{d-1}{2}\right)\sin(\frac{\pi d}{2})\,(p^2)^{2-\frac{d}{2}} \equiv \tilde{C}_\sigma(p^2)^{2-\frac{d}{2}}$$

$$\langle\sigma(x)\sigma(y)\rangle = \frac{2^{d+2}\Gamma\left(\frac{d-1}{2}\right)\sin(\frac{\pi d}{2})}{\pi^{\frac{3}{2}}\Gamma\left(\frac{d}{2}-2\right)}\frac{1}{|x-y|^4} \equiv \frac{C_\sigma}{|x-y|^4}$$

- The 1/N corrections to operator dimensions are calculated using this induced propagator. For example,

$$\Delta_\phi = \frac{d}{2} - 1 + \frac{1}{N}\eta_1 + \frac{1}{N^2}\eta_2 + \dots$$

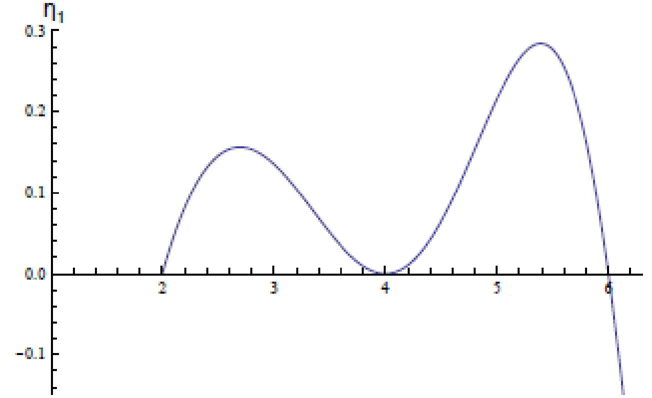
- For the leading correction need

$$\frac{1}{N}\int \frac{d^d q}{(2\pi)^d} \frac{1}{(p-q)^2} \frac{\tilde{C}_\sigma}{(q^2)^{\frac{d}{2}-2+\delta}}$$

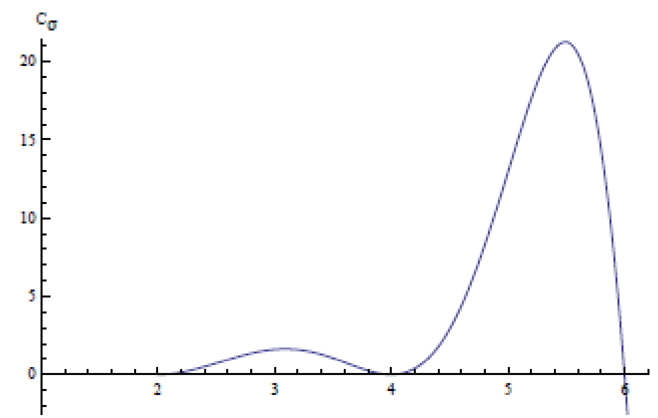
- δ is the regulator later sent to 0.

$$\eta_1 = \frac{\tilde{C}_\sigma (d-4)}{(4\pi)^{\frac{d}{2}} d\Gamma(\frac{d}{2})} = \frac{2^{d-3}(d-4)\Gamma\left(\frac{d-1}{2}\right)\sin\left(\frac{\pi d}{2}\right)}{\pi^{\frac{3}{2}}\Gamma\left(\frac{d}{2}+1\right)}$$

- When the leading correction is negative, the large N theory is non-unitary.
- It is positive not only for 2<d< 4, but also for 4<d<6.



- The 2-point function coefficient C_σ is similar



Towards Interacting 5-d O(N) Model

- Scalar large N model with $\frac{\lambda}{4}(\phi^i\phi^i)^2$ interaction has a good UV fixed point for 4<d<6. Parisi
- In $4+\epsilon$ dimensions $\beta_\lambda = \epsilon\lambda + \frac{N+8}{8\pi^2}\lambda^2 + \dots$
- So, the UV fixed point is at a negative coupling

$$\lambda_* = -\frac{8\pi^2}{N+8}\epsilon + O(\epsilon^2)$$

- At large N, conjectured to be dual to Vasiliev theory in AdS_6 with Δ_- boundary condition on the bulk scalar. Giombi, IK, Safdi
- Check of 5-dimensional F-theorem $-F = \log Z_{S^5}$

$$F_{\rm UV}^{(1)} - F_{\rm IR}^{(1)} = -\frac{3\zeta(5)+\pi^2\zeta(3)}{96\pi^4} \approx -0.0016$$

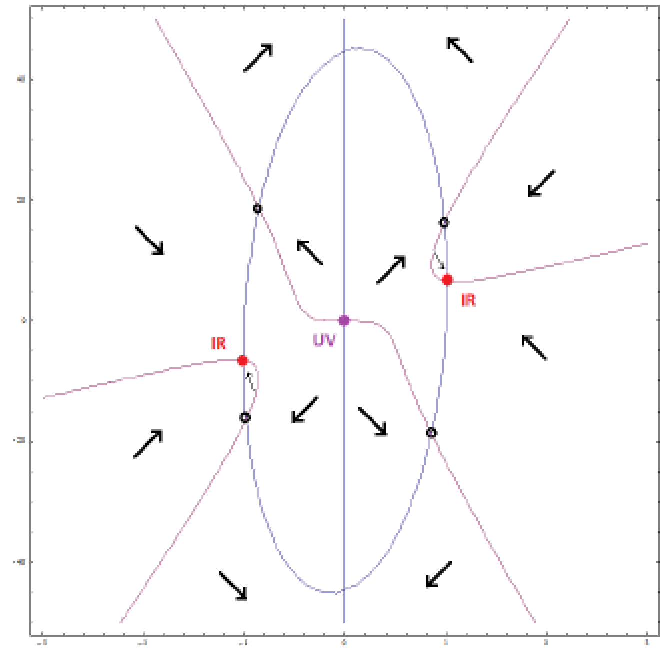
Perturbative IR Fixed Points

- Work in $d = 6 - \epsilon$ with O(N) symmetric cubic scalar theory $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{g_1}{2}\sigma(\phi^i\phi^i) + \frac{g_2}{6}\sigma^3$
- The beta functions Fei, Giombi, IK

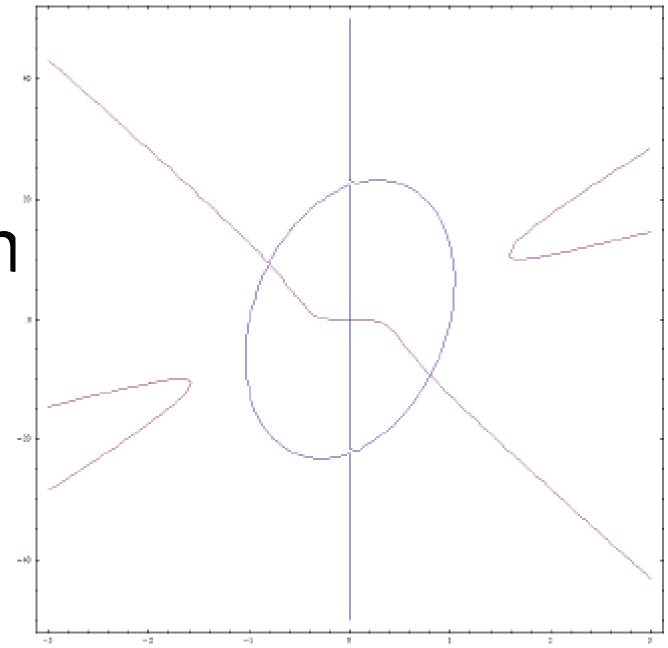
$$\beta_1 = -\frac{\epsilon g_1}{2} + \frac{(N-8)g_1^3 - 12g_1^2 g_2 + g_1 g_2^2}{12(4\pi)^3}$$
$$\beta_2 = -\frac{\epsilon g_2}{2} + \frac{-4N g_1^3 + N g_1^2 g_2 - 3g_2^3}{4(4\pi)^3}$$

- For large N, the IR stable fixed point is at real couplings $g_{1*} = \sqrt{\frac{6\epsilon(4\pi)^3}{N}}$ $g_{2*} = 6g_{1*}$

RG Flows



- Here is the flow pattern for N=2000



- The IR stable fixed points go off to complex couplings for N < 1039. Large N expansion breaks down very early!

- The dimension of sigma is $\Delta_\sigma = 2 - \frac{\epsilon}{2} + \frac{N g_1^2 + g_2^2}{12(4\pi)^3}$
- At the IR fixed point this is $2 + 40\frac{\epsilon}{N}$
- Agrees with the large N result for the O(N) model in d dimensions:

Petkou (1995)

$$2 + \frac{4}{N}\frac{\Gamma(d)}{\Gamma(d/2-1)\Gamma(1-d/2)\Gamma(d/2)\Gamma(d/2+1)}$$

- For N=0, the fixed point at imaginary coupling may lead to a description of the Lee-Yang edge singularity in the Ising model. Michael Fisher (1978)
- For N=0, Δ_σ is below the unitarity bound $2 - \frac{\epsilon}{2}$
- For N>1039, the fixed point at real couplings is consistent with unitarity in $d = 6 - \epsilon$

Three Loop Analysis

- The beta functions are found to be

$$
\begin{aligned}
\beta_1 =& -\frac{\epsilon}{2}g_1 + \frac{1}{12(4\pi)^3}g_1\left((N-8)g_1^2 - 12g_1g_2 + g_2^2\right)\\
&- \frac{1}{432(4\pi)^6}g_1\left((536+86N)g_1^4 + 12(30-11N)g_1^3g_2 + (628+11N)g_1^2g_2^2 + 24g_1g_2^3 - 13g_2^4\right)\\
&+ \frac{1}{62208(4\pi)^9}g_1\Big(g_2^6(5195-2592\zeta(3)) + 12g_1g_2^5(-2801+2592\zeta(3))\\
&- 8g_1^2g_2^4(1245+119N+7776\zeta(3)) + g_1^4g_2^2(-358480+53990N-3N^2-2592(-16+5N)\zeta(3))\\
&+ 36g_1^5g_2(-500-3464N+N^2+864(5N-6)\zeta(3))\\
&- 2g_1^6(125680-20344N+1831N^2+2592(25N+4)\zeta(3)) + 48g_1^3g_2^3(95N-3(679+864\zeta(3)))\Big)\\
\beta_2 =& -\frac{\epsilon}{2}g_2 + \frac{1}{4(4\pi)^3}\left(-4Ng_1^3 + Ng_1^2g_2 - 3g_2^3\right)\\
&+ \frac{1}{144(4\pi)^6}\left(-24Ng_1^5 - 322Ng_1^4g_2 - 60Ng_1^3g_2^2 + 31Ng_1^2g_2^3 - 125g_2^5\right)\\
&+ \frac{1}{20736(4\pi)^9}\Big(-48N(713+577N)g_1^7 + 6272Ng_1^2g_2^5 + 48Ng_1^3g_2^4(181+432\zeta(3))\\
&- 5g_2^7(6617+2592\zeta(3)) - 24Ng_1^5g_2^2(1054+471N+2592\zeta(3))\\
&+ 2Ng_1^6g_2(19237N-8(3713+324\zeta(3))) + 3Ng_1^4g_2^3(263N-6(7105+2448\zeta(3))\Big)
\end{aligned}
$$

- The epsilon expansions of scaling dimensions agree in detail with the large N expansion at the UV fixed point of the quartic O(N) model:

$$\begin{aligned}
\Delta_\phi &= \frac{d}{2} - 1 + \gamma_\phi \\
&= 2 - \frac{\epsilon}{2} + \left(\frac{1}{N} + \frac{44}{N^2} + \frac{1936}{N^3} + \dots\right)\epsilon + \left(-\frac{11}{12N} - \frac{835}{6N^2} - \frac{16352}{N^3} + \dots\right)\epsilon^2 \\
&\quad + \left(-\frac{13}{144N} + \frac{6865}{72N^2} + \frac{54367/2 - 3672\zeta(3)}{N^3} + \dots\right)\epsilon^3, \\
\Delta_\sigma &= \frac{d}{2} - 1 + \gamma_\sigma \\
&= 2 + \left(\frac{40}{N} + \frac{6800}{N^2} + \dots\right)\epsilon + \left(-\frac{104}{3N} - \frac{34190}{3N^2} + \dots\right)\epsilon^2 \\
&\quad + \left(-\frac{22}{9N} + \frac{47695/18 - 2808\zeta(3)}{N^2} + \dots\right)\epsilon^3.
\end{aligned}$$

- Continues to work at four loop order. Gracey

Critical N

- What is the critical value of N below which the perturbatively unitary fixed point disappears?
- Need to find the solution of

$$\beta_1 = 0\,, \qquad \beta_2 = 0,$$
$$\frac{\partial\beta_1/\partial g_1}{\partial\beta_1/\partial g_2} = \frac{\partial\beta_2/\partial g_1}{\partial\beta_2/\partial g_2}$$

- This gives

$$N_{\text{crit}} = 1038.266 - 609.840\epsilon - 364.173\epsilon^2 + \mathcal{O}(\epsilon^3)$$

(Meta) Stability

- Since the UV lagrangian is cubic, does the theory make sense non-perturbatively?
- When the CFT is studied on S^d or $R \times S^{d-1}$ the conformal coupling of scalar fields to curvature renders the perturbative vacuum meta-stable. In 6-ε dimensions, scaling dimensions may have imaginary parts of order exp (- A N/ε)
- Metastability of the 5-d O(N) model also suggested by applications of Exact RG.

Mati; Eichhorn, Janssen, Scherer

Conformal Bootstrap in 5-d

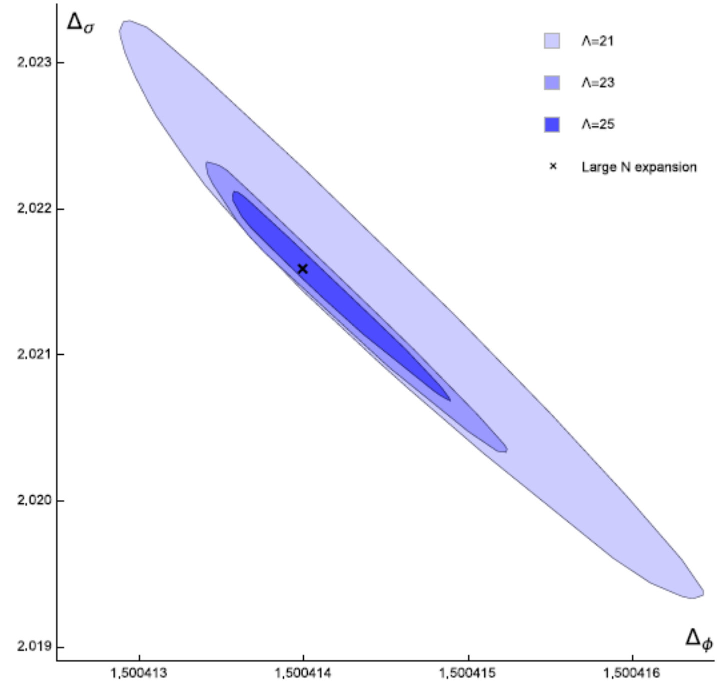
- Recent results using mixed correlators in the O(500) model show good agreement with the 1/N expansion. Z. Li, N. Su; see also S. Chester, S. Pufu, R. Yacoby

$$\Delta_\phi = \frac{3}{2} + \frac{0.216152}{N} - \frac{4.342}{N^2} - \frac{121.673}{N^3} + \ldots$$

$$\Delta_\sigma = 2 + \frac{10.3753}{N} + \frac{206.542}{N^2} + \ldots$$

- The shrinking island similar that seen for O(N) in d=3.

F. Kos, D. Simmons-Duffin, D. Poland, A. Vichi



Conclusions

- The ε-expansions in the O(N), Gross-Neveu, Nambu-Jona-Lasinio, and other vectorial CFTs, are useful for applications to condensed matter and statistical physics.
- They provide "checks and balances" for the new numerical results using the conformal bootstrap.
- They serve as nice playgrounds for the RG inequalities (C-theorem, a-theorem, F-theorem) and for the higher spin AdS/CFT and dS/CFT correspondence.

- Some small values of N are special cases where there are enhanced IR symmetries.
- Yukawa CFTs in d<4 can exhibit emergent supersymmetry.
- Found a new description of the meta-stable fixed points of the scalar O(N) model in 4<d<6 valid for sufficiently large N.
- Interesting results about the 5-d O(N) model using the conformal bootstrap, Exact RG.
- Could the phase transition in 5-d be very weakly first order for large N?

Extra Slides: Higher-Spin dS/CFT

- To construct non-unitary CFTs dual to higher spin theory in de Sitter space, replace the commuting scalar fields by anti-commuting ones. Anninos, Hartman, Strominger
- The conjectured dual to minimal Vasiliev theory in dS_4 is the interacting Sp(N) model introduced earlier LeClair, Neubert

$$S = \int d^3x \left(\frac{1}{2} \Omega_{ij} \partial_\mu \chi^i \partial^\mu \chi^j + \frac{\lambda}{4} (\Omega_{ij} \chi^i \chi^j)^2 \right)$$

- In d>4 this quartic theory has a UV fixed point at large N.
- Consider instead the cubic Sp(N) invariant theory, which is weakly coupled in 6-ε dimensions.

$$S = \int d^dx \left(\frac{1}{2}\Omega_{ij}\partial_\mu\chi^i\partial^\mu\chi^j + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}g_1\Omega_{ij}\chi^i\chi^j\sigma + \frac{1}{6}g_2\sigma^3 \right)$$

- The beta functions are related to those of the O(N) theory via N-> -N
- For Sp(N) there are IR stable fixed points at imaginary couplings for all positive even N.

Symmetry Enhancement for N=2

- The N=2 model may be written as

$$S = \int d^d x \left(\partial_\mu \theta \partial^\mu \bar{\theta} + \frac{1}{2} (\partial_\mu \sigma)^2 + g_1 \sigma \theta \bar{\theta} + \frac{1}{6} g_2 \sigma^3 \right)$$

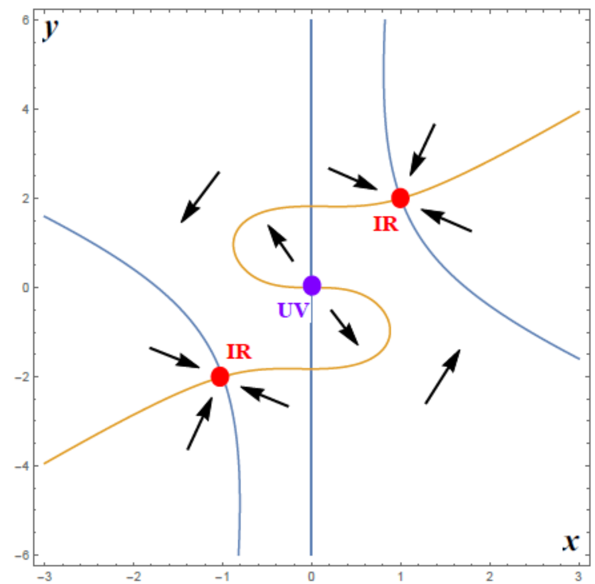
- At the fixed point

$$g_2^* = 2 g_1^* \,, \qquad g_1^* = i \sqrt{\frac{(4\pi)^3 \epsilon}{5}} \left(1 + \frac{67}{180} \epsilon + O(\epsilon^2) \right)$$

- There is symmetry enhancement from Sp(2) to the supergroup Osp(1|2)

$$\delta\theta = \sigma\alpha \ , \quad \delta\bar{\theta} = \sigma\bar{\alpha} \ , \quad \delta\sigma = -\alpha\bar{\theta} + \bar{\alpha}\theta$$

- Defining $g_1 = i\sqrt{\frac{(4\pi)^3\epsilon}{5}}x,\ g_2 = i\sqrt{\frac{(4\pi)^3\epsilon}{5}}y$



- The scaling dimensions of commuting and anti-commuting scalars are equal

$$\Delta_\sigma = \Delta_\theta = 2 - \frac{8}{15}\epsilon - \frac{7}{450}\epsilon^2 - \frac{269 - 702\zeta(3)}{33750}\epsilon^3$$

Connection with the Potts Model

- (n+1) state Potts model can be described in 6-ε dimensions by a cubic field theory of n scalar fields Zia, Wallace

$$S = \int d^d x \left(\partial_\mu \phi^i \partial^\mu \phi^i + \frac{1}{6} g d_{ijk} \phi^i \phi^j \phi^k \right)$$

$$d^{ijk} = \sum_{\alpha=1}^{n+1} e_\alpha^i e_\alpha^j e_\alpha^k$$

- The vectors e_α^i describe the vertices of the n-dimensional generalization of tetrahedron.

- The 6-ε expansions have been developed for any q-state Potts model.
- We find that, in the formal limit q-> 0, they are the same as at the fixed point with the emergent Osp(1|2) symmetry.
- The zero-state Potts model can be defined on a lattice using the spanning forest model, and Monte Carlo results for scaling exponents are available in d=3,4,5 where the model has second order phase transitions. Deng, Garoni, Sokal