Quantum gravity on foliated spacetime asymptotically safe and sound

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H. Gies, B. Knorr, S. Lippholdt and F. Saueressig, PRL 116 (2016) 211302

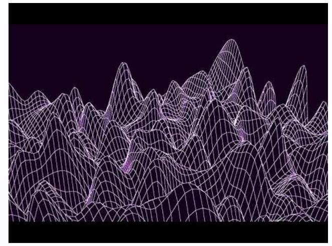
J. Biemans, A. Platania and F. Saueressig, arXiv:1609.04813

ERG2016

Trieste, Sept. 23nd, 2016

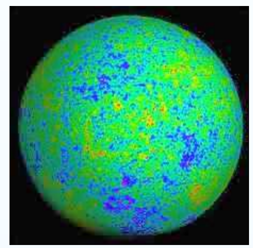
Goal: a consistent theory describing gravity on all scales

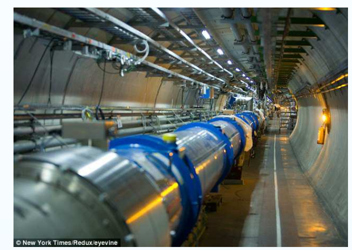
10^{-35} m



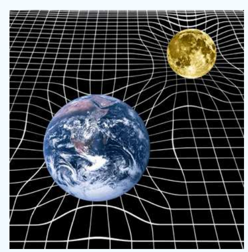
10^{-29} m

10^{-19} m

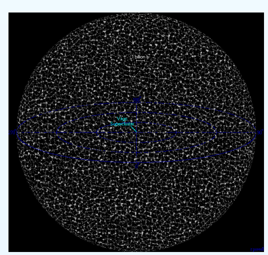




10^{12} m



10^{26} m



Quantum Gravity from the renormalization group

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- ensures the absence of UV-divergences

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  $\Longleftrightarrow$ experimental determination of relevant parameters
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c) classical limit

- RG-trajectories have part where GR is good approximation
- recover gravitational physics captured by general relativity:
  (perihelion shift, gravitational lensing, nucleo-synthesis, $\ldots$)

Perturbative quantization of General Relativity

Dynamics of General Relativity governed by Einstein-Hilbert action

$$S^{\mathrm{EH}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} \big[- R + 2\Lambda \big]$$

- Newton's constant G_N has negative mass-dimension

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Wilsonian picture of perturbative renormalization:

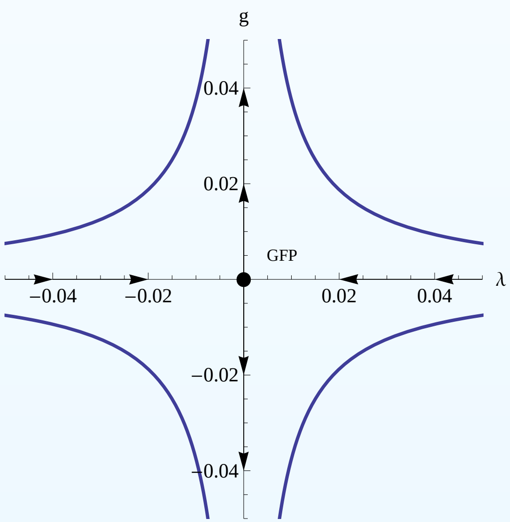
$\Rightarrow$ dimensionless coupling constant attracted to GFP (free theory) in UV

- introduce dimensionless coupling constants

$$g_k = k^2 G_N \,, \qquad \lambda_k \equiv \Lambda k^{-2}$$

- GFP: flow governed by mass-dimension:

$$k\partial_k g_k = 2g + \mathcal{O}(g^2)$$
$$k\partial_k \lambda_k = -2\lambda + \mathcal{O}(g)$$



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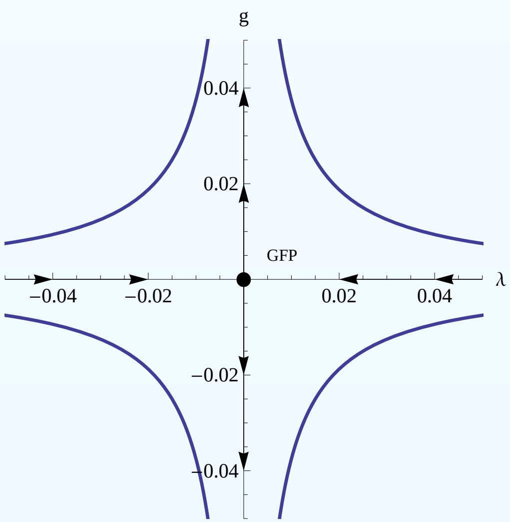

General Relativity is not asymptotically free

Quantizing general relativity

[M. H. Goroff and A. Sagnotti, Phys. Lett. B160 (1985) 81]
[M. H. Goroff and A. Sagnotti, Nucl. Phys. B266 (1986) 709]
[A. E. M. van de Ven, Nucl. Phys. B378 (1992) 309]

Einstein-Hilbert action

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{g}\, R$$

gives rise to a two-loop divergence:

$$S^{\mathrm{div}} = \frac{1}{\epsilon} \frac{209}{2880} \frac{1}{(16\pi^2)^2} \int d^4x \sqrt{g}\, C_{\mu\nu}{}^{\rho\sigma}\, C_{\rho\sigma}{}^{\alpha\beta}\, C_{\alpha\beta}{}^{\mu\nu}\,.$$

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- divergence not of Einstein-Hilbert form:
  $\Longrightarrow$ cannot be absorbed into bare Newton's constant
- need to add new C^3-interaction to bare action
  $\Longrightarrow$ introduce a new free parameter
  $\Longrightarrow$ hallmark of perturbative non-renormalizability

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General Relativity is perturbatively non-renormalizable

Proposals for UV completions of gravity

- Gaussian Fixed Point (GFP)
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Gravity

Outline

- Introduction
- Asymptotic Safety: recent developments
- Goroff-Sagnotti counterterm in Asymptotic Safety
- real-time computations in quantum gravity
- the unitarity question: a brief comment
- Summary and outlook

Asymptotic Safety

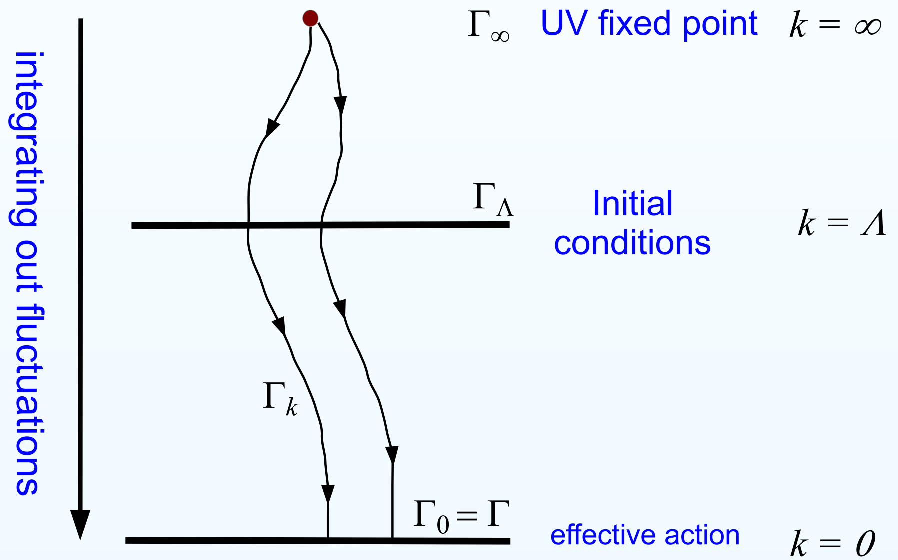
status report

RG flows: effective average action for gravity

C. Wetterich, Phys. Lett. **B301** (1993) 90
M. Reuter, Phys. Rev. D **57** (1998) 971

central idea: integrate out quantum fluctuations shell-by-shell in momentum-space

$$k\partial_k \Gamma_k[h_{\mu\nu}, \bar{g}_{\mu\nu}] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$



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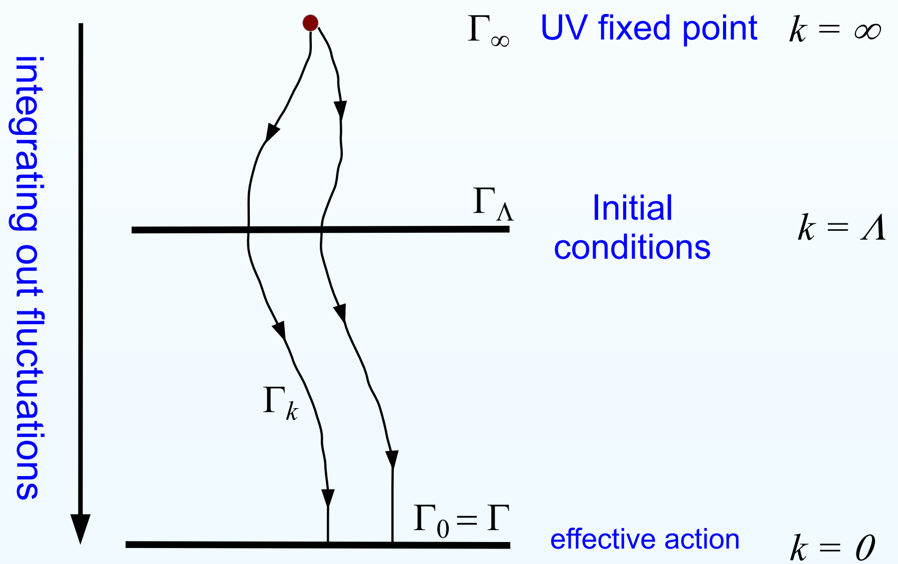

- ⊖ background formalism: required for defining RG scale k
- ⊕ independent of a "preset" fundamental action
- ⊕ flexible approximation schemes

Projecting the RG flow of $\Gamma_k^{\mathrm{grav}}[g]$

Einstein-Hilbert truncation

functional $f(R)$-truncation

$R^2 + C^2$-truncation

$\vdots$			
R^8	$\cdots$		
R^7	$\cdots$		
R^6	$\cdots$		
R^5	$\cdots$		
R^4	$\cdots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$	$R \Box R$	+ 7 more
R^2	$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$	
R			
$\mathbb{1}$			

key results: pure gravity

- **non-Gaussian fixed point established in a wide range of approximations**
  - derivative expansion at the background level
  - vertex expansions

[N. Christiansen, B. Knorr, J. Meibohm, J. M. Pawlowski and M. Reichert, arXiv:1506.07016]

- **low number of relevant parameters ($\simeq$ 3)**

[R. Percacci and A. Codello, arXiv:0705.1769]
[P.F. Machado and F. Saueressig, arXiv:0712.0445]
[D. Benedetti, P.F. Machado and F. Saueressig, arXiv:0901.2984]
[K. Falls, D. F. Litim, K. Nikolakopoulos and C. Rahmede, arXiv:1301.4191]

- **non-Gaussian fixed point in $d = 2$ is a unitary CFT**

[A. Nink and M. Reuter, arXiv:1512.06805]

- **structural analysis (gauge/background-dependence, c-theorems, . . .)**

[D. Becker and M. Reuter, arXiv:1404.4537]
[D. Becker and M. Reuter, arXiv:1412.0468]
[A. Codello, G. D'Odorico and C. Pagani, arXiv:1502.02439]
[H. Gies, B. Knorr and S. Lippoldt, arXiv:1507.08859]
[P. Labus, T. R. Morris and Z. H. Slade, arXiv:1603.04772]
[J. A. Dietz, T. R. Morris and Z. H. Slade, arXiv:1605.07636]

key results: gravity coupled to matter

- **gravity + scalars: asymptotic safety survives 1-loop counterterm**

  [D. Benedetti, P.F. Machado and F. Saueressig, arXiv:0902.4630]

- **non-Gaussian fixed point compatible with standard-model matter**

  [R. Percacci and D. Perini, hep-th/0207033]
  [P. Dona, A. Eichhorn and R. Percacci, arXiv:1311.2898]
  [J. Meibohm, J. M. Pawlowski and M. Reichert, arXiv:1510.07018]

- **prediction of the Higgs mass $m_H \simeq 126$ GeV**

  [M. Shaposhnikov and C. Wetterich, arXiv:0912.0208]

- **investigations of gravity-Higgs-Yukawa systems on the way**

  [K. Y. Oda and M. Yamada, arXiv:1510.03734]
  [A. Eichhorn, A. Held and J. M. Pawlowski, arXiv:1604.02041]

- **cosmology-inspired gravity-matter systems**

  [R. Percacci and G. P. Vacca, arXiv:1501.00888]
  [A. Bonanno and A. Platania, arXiv:1507.03375]
  [N. Ohta, R. Percacci and G. P. Vacca, arXiv:1511.09393]
  [I. D. Saltas, arXiv:1512.06134]
  [T. Henz, J. M. Pawlowski and C. Wetterich, arXiv:1605.01858]
  [K. Falls, D. F. Litim, K. Nikolakopoulos and C. Rahmede, arXiv:1607.04962]

. . . and many open questions

- does the NGFP indeed come with complex critical exponents?
- what is the role of the background field?
- how to treat truncations with an infinite number of couplings?
- is there an intrinsic definition of the NGFP?
  (correlation functions, conformal field theory data, . . .)
- does Asymptotic Safety manifests in phenomenological signatures?

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!!! WORK AHEAD !!!

Does the

Goroff-Sagnotti counterterm

destroy asymptotic safety?

[H. Nicolai]

including the two-loop counterterm in Γ_k

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Projecting the gravitational RG flow on the Einstein-Hilbert action

Einstein-Hilbert truncation: Γ_k retains two running couplings: G_k, Λ_k

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β-functions for dimensionless couplings $g_k \equiv k^2 G_k\,,\ \lambda_k \equiv \Lambda_k k^{-2}$

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Gaussian fixed point at $g^* = 0, \lambda^* = 0$:

- saddle point in the g-λ-plane

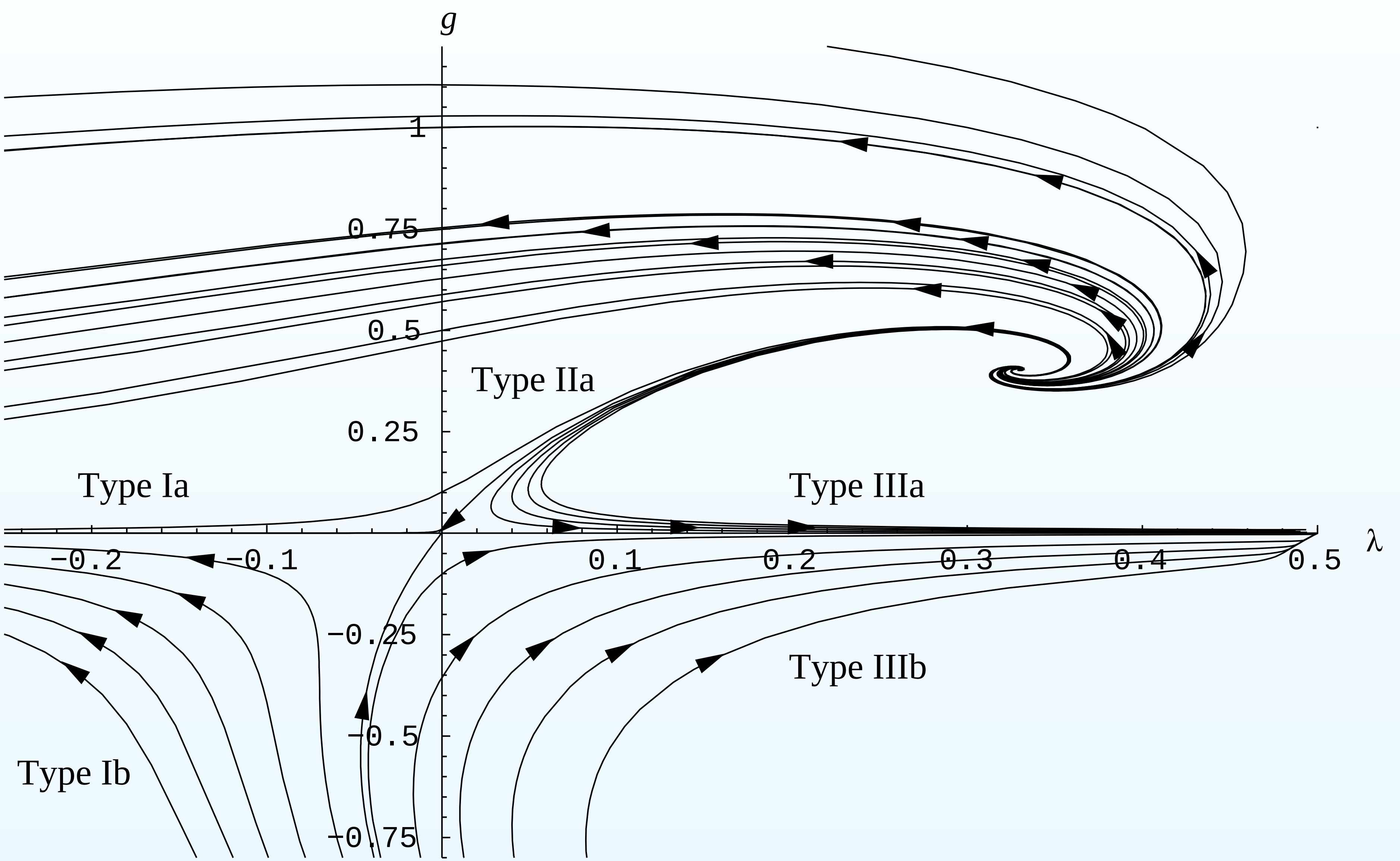
non-Gaussian fixed point at $g^* > 0, \lambda^* > 0$:

- UV attractive in g_k, λ_k

asymptotic safety: non-Gaussian fixed point is UV completion for gravity

Einstein-Hilbert-truncation: the phase diagram

M. Reuter and F. Saueressig, Phys. Rev. D **65** (2002) 065016 [hep-th/0110054]



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H. Gies, B. Knorr, S. Lippoldt and F. Saueressig, arXiv:1601.01800

Supplement Einstein-Hilbert action by Goroff-Sagnotti term: $\Gamma_k^{\rm grav} = \Gamma_k^{\rm EH} + \Gamma_k^{\rm GS}$

$$\Gamma_k^{\rm EH} = \frac{1}{16\pi G_k} \int d^4x \sqrt{g} \, [-R + 2\Lambda_k]$$

$$\Gamma_k^{\rm GS} = \bar{\sigma}_k \int d^4x \sqrt{g} \, C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$$

dimensionless coupling constants

$$\lambda = \Lambda_k \, k^{-2} \, , \qquad g = G_k \, k^2 \, , \qquad \sigma = \bar{\sigma}_k \, k^2$$

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flow of g, λ is not changed $\Longleftrightarrow$ EH-sector has NGFP

Goroff-Sagnotti does not feed into Einstein-Hilbert sector

FRGE uses the background field method:

$\Longrightarrow$ obtain beta functions as expansion in background curvature

Contribution of $\Gamma_k^{\rm GS}$ to the flow equation

$$\Gamma_k^{\rm GS} = \bar{\sigma}_k \int d^4x \sqrt{g}\, C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$$

Goroff-Sagnotti term enters trace as contribution to $\delta^2 \Gamma_k^{(2)}$:

$$\delta^2 \Gamma_k^{\rm GS}\Big|_{g=\bar{g}} \sim \sigma\, \bar{C}_{\alpha\beta}{}^{\mu\nu} + \mathcal{O}(\bar{R}^2)$$

- $\delta^2 \Gamma_k^{\rm GS}$ starts at linear order in the background curvature, but:

$$\mathrm{tr}\, \bar{C}_{\alpha\beta}{}^{\mu\nu} = 0$$

Goroff-Sagnotti does not give rise to a structure $\int d^4x \sqrt{g} R$

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2. surprise: the beta-function for σ_k is cubic

$$\beta_\sigma = c_0 + (2 + c_1) \, \sigma + c_2 \, \sigma^2 + c_3 \, \sigma^3$$

β_σ is cubic in σ

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$$\delta^2 \Gamma_k^{\rm GS}\big|_{g=\bar{g}} \sim \sigma\, \bar{C}_{\alpha\beta}{}^{\mu\nu} + \mathcal{O}(\bar{R}^2)$$

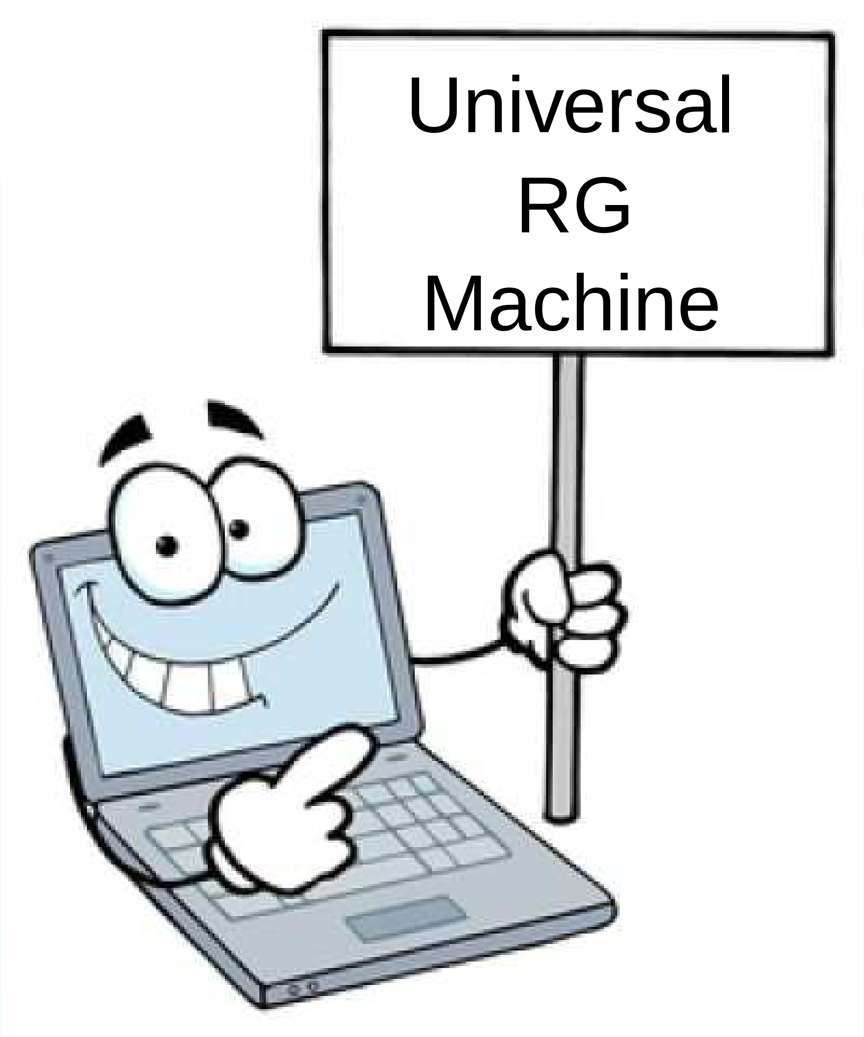
- beta function of σ is read off from the $\bar{C}^3$-term:

$$\left(\delta^2 \Gamma_k^{\rm GS}\right)^n \propto \sigma^n\, \bar{C}^n + \mathcal{O}(\sigma^{n-1})$$

β_σ can at most be cubic in σ

$$\beta_\sigma = c_0 + (2 + c_1)\,\sigma + c_2\,\sigma^2 + c_3\,\sigma^3$$

Compute the coefficients c_i



After one month of CPU time crunching 900 vertex-insertions

Beta functions of the Goroff-Sagnotti projection

flow in the Einstein-Hilbert sector is unchanged:

$$\beta_g = (2 + \eta_N)\, g\,,$$

$$\beta_\lambda = (\eta_N - 2)\, \lambda + \frac{g}{2\pi} \left(\frac{5}{1-2\lambda} - 4 - \frac{5}{6}\eta_N \frac{1}{1-2\lambda} \right).$$

Beta function for the Goroff-Sagnotti coupling

$$\beta_\sigma = c_0 + (2 + c_1)\, \sigma + c_2\, \sigma^2 + c_3\, \sigma^3$$

Coefficients c_i are functions of g, λ:

$$c_0 = \frac{1}{64\pi^2(1-2\lambda)} \left(\frac{2-\eta_N}{2(1-2\lambda)} + \frac{6-\eta_N}{(1-2\lambda)^3} - \frac{5\eta_N}{378} \right),$$

$$c_1 = \frac{3g}{16\pi(1-2\lambda)^2} \left(5(6 - \eta_N) + \frac{23(8-\eta_N)}{8(1-2\lambda)} - \frac{7(10-\eta_N)}{10(1-2\lambda)^2} \right),$$

$$c_2 = \frac{g^2}{2(1-2\lambda)^3} \left(\frac{233(12-\eta_N)}{10} - \frac{9(14-\eta_N)}{7(1-2\lambda)} \right),$$

$$c_3 = \frac{6\pi g^3 (18-\eta_N)}{(1-2\lambda)^4} \neq 0\,.$$

Beta functions of the Goroff-Sagnotti projection

flow in the Einstein-Hilbert sector is unchanged:

$$\beta_g = (2 + \eta_N)\, g\,,$$

$$\beta_\lambda = (\eta_N - 2)\, \lambda + \tfrac{g}{2\pi} \left(\tfrac{5}{1-2\lambda} - 4 - \tfrac{5}{6} \eta_N \tfrac{1}{1-2\lambda} \right).$$

Beta function for the Goroff-Sagnotti coupling

$$\beta_\sigma = c_0 + (2 + c_1)\,\sigma + c_2\,\sigma^2 + c_3\,\sigma^3$$

Coefficients c_i are functions of g, λ:

$$c_0 = \tfrac{1}{64\pi^2(1-2\lambda)} \left(\tfrac{2-\eta_N}{2(1-2\lambda)} + \tfrac{6-\eta_N}{(1-2\lambda)^3} - \tfrac{5\eta_N}{378} \right)\,,$$

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$g_* > 0$ implies $c_3 > 0 \Longleftrightarrow$ NGFP extents to GS-projection

Fixed point structure of the Goroff-Sagnotti projection

Gaussian fixed point is shifted:

$$\text{GFP}^{\text{GS}}: \qquad \lambda_* = 0, \quad g_* = 0, \quad \sigma_* = -\frac{7}{128\pi^2}.$$

- stability coefficients indicate: GFP is a saddle point

$$\theta_g = -2\,, \qquad \theta_\lambda = 2\,, \qquad \theta_\sigma = -2\,.$$

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$$\text{NGFP}^{\text{GS}}: \qquad \lambda_* = 0.193, \quad g_* = 0.707, \quad \sigma_* = -0.305.$$

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$$\theta_{1,2}^{\text{NGFP}} = 1.475 \pm 3.043i\,, \qquad \theta_3^{\text{NGFP}} = -79.39\,.$$

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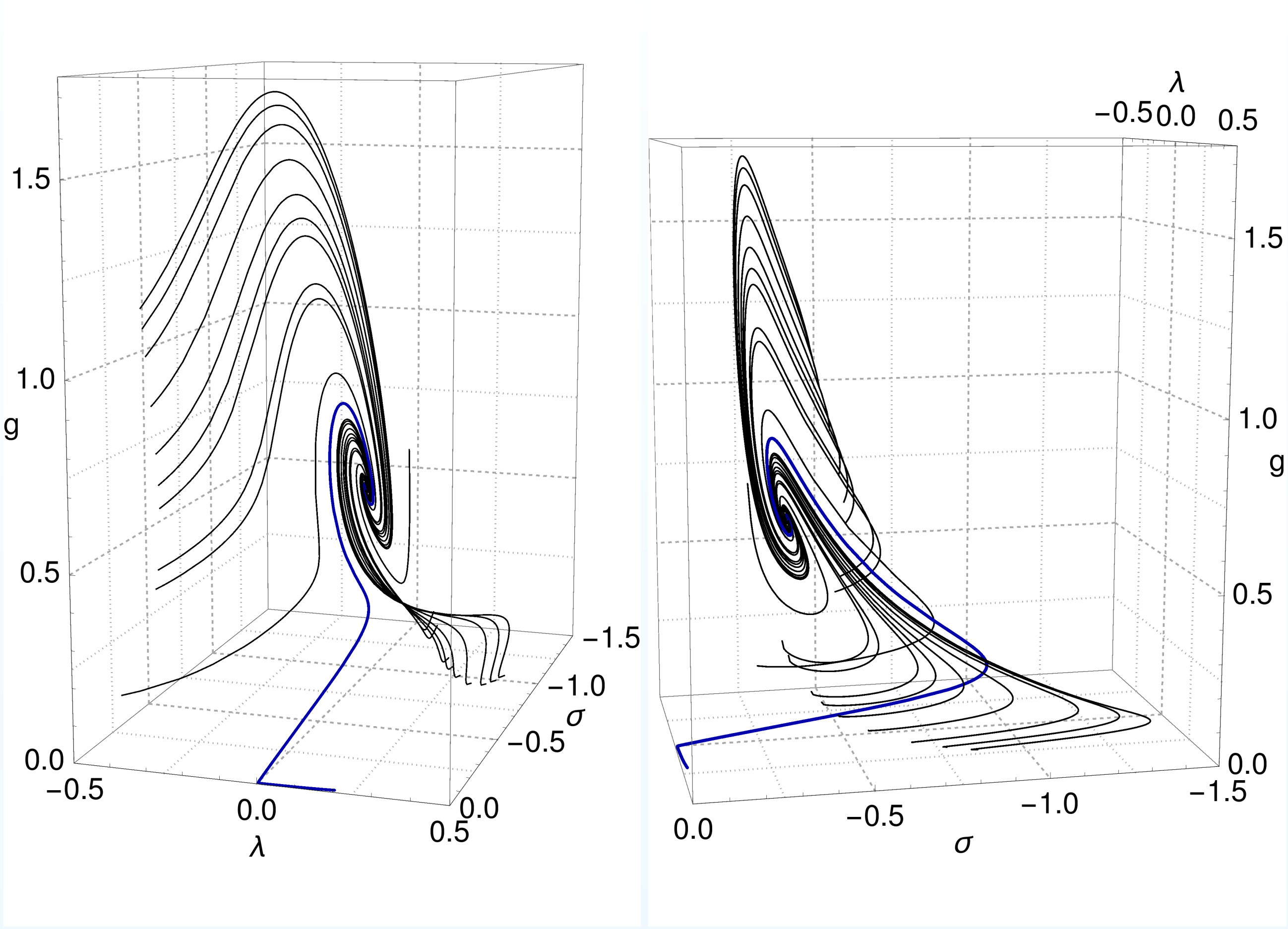
NGFP: counterterm does not increase number of free couplings!

Phase portrait of the Goroff-Sagnotti projection

H. Gies, B. Knorr, S. Lippoldt and F. Saueressig, arXiv:1601.01800

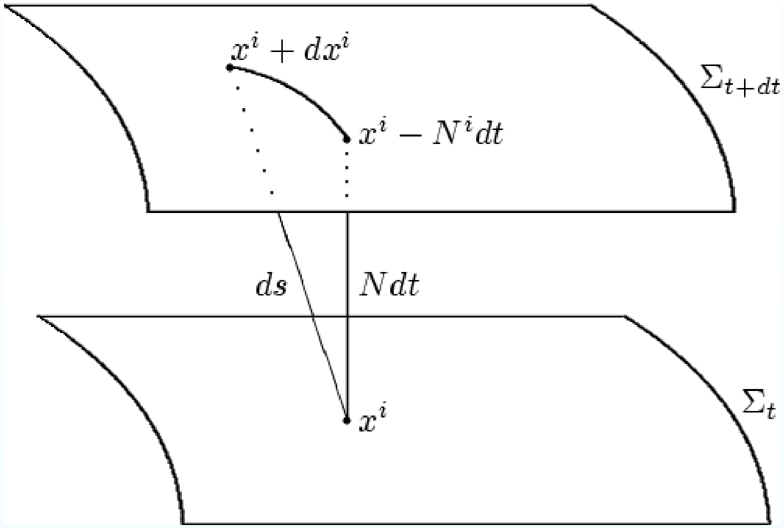


blue trajectory: crossover to classical regime intact renormalization group flows

in the presence of a causal structure

Introducing time through the ADM formalism

Preferred "time"-direction via foliation of space-time



- foliation structure $\mathcal{M}^{d+1} = \mathbb{R} \times \mathcal{M}^d$ with $y^\mu \mapsto (t, x^a)$:

$$ds^2 = N^2 dt^2 + \sigma_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right)$$

- fundamental fields: $g_{\mu\nu} \mapsto (N, N_i, \sigma_{ij})$

$$g_{\mu\nu} = \begin{pmatrix} N^2 + N_i N^i & N_j \\ N_i & \sigma_{ij} \end{pmatrix}$$

Off-shell flows in the ADM formalism

Einstein-Hilbert action in ADM variables

$$\Gamma_k^{\rm grav} = \frac{1}{16\pi G_k} \int dt d^d x N \sqrt{\sigma} \left[K_{ij} K^{ij} - K^2 - R + 2\Lambda_k \right]$$

background: flat Friedmann-Robertson-Walker spacetime

$$\bar{N} = 1 \, , \qquad \bar{N}_i = 0 \, , \qquad \bar{\sigma}_{ij} = a^2(t) \delta_{ij} \, .$$

fluctuations: adapted to cosmological perturbation theory

$$\hat{N}_i = u_i + \partial_i \, \frac{1}{\sqrt{\Delta}} \, B \, , \qquad \partial^i \, u_i = 0$$

$$\hat{\sigma}_{ij} = h_{ij} - \left(\bar{\sigma}_{ij} + \partial_i \partial_j \, \frac{1}{\Delta} \right) \psi + \partial_i \partial_j \, \frac{1}{\Delta} \, E + \partial_i \frac{1}{\sqrt{\Delta}} v_j + \partial_j \, \frac{1}{\sqrt{\Delta}} \, v_i \, .$$

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unique gauge-fixing of diffeomorphisms

$$F = \partial_t \, \hat{N} + \partial^i \hat{N}_i - \tfrac{1}{2} \partial_t \hat{\sigma} + \tfrac{2(d-1)}{d} \bar{K} \hat{N} \, ,$$

$$F_i = \partial_t \hat{N}_i - \partial_i \hat{N} - \tfrac{1}{2} \partial_i \hat{\sigma} + \partial^j \hat{\sigma}_{ji} + (d-2) \bar{K}_{ij} \hat{N}^j \, ,$$

Gauge-fixing in the ADM formalism (flat space)

$$\delta^2\Gamma_k^{\rm grav}$$

Index	matrix element of $32\pi G_k\,\delta^2\Gamma_k^{\rm grav}$
$h\,h$	$-\partial_t^2+\Delta-2\Lambda_k$
$v\,v$	$2\big[-\partial_t^2-2\Lambda_k\big]$
$E\,E$	$-\Lambda_k$
$\psi\,\psi$	$-(d-1)\big[(d-2)\,(-\partial_t^2+\Delta)-(d-3)\,\Lambda_k\big]$
$\psi\,E$	$-\frac{1}{2}(d-1)\big[-\partial_t^2-2\Lambda_k\big]$
$\hat{N}\,\hat{N}$	0
$B\,B$	0
$u\,u$	$2\,\Delta$
$u\,v$	$-2\,\partial_t\sqrt{\Delta}$
$B\,\psi$	$2\,(d-1)\sqrt{\Delta}\,\partial_t$
$\hat{N}\,\psi$	$2\,(d-1)\big[\Delta-\Lambda_k\big]$
$\hat{N}\,E$	$-2\,\Lambda_k$

Gauge-fixing in the ADM formalism (flat space)

$$\delta^2\Gamma_k^{\text{grav}} + S^{\text{gf}}$$

Index	matrix element of $32\pi G_k \left(\delta^2\Gamma_k^{\text{grav}} + S^{\text{gf}}\right)$
$h\,h$	$-\partial_t^2 + \Delta - 2\Lambda_k$
$v\,v$	$2\big[-\partial_t^2 + \Delta - 2\Lambda_k\big]$
$E\,E$	$\frac{1}{2}\left[-\partial_t^2 + \Delta - \Lambda_k\right]$
$\psi\,\psi$	$\frac{(d-1)(d-3)}{4}\big[-\partial_t^2 + \Delta - 2\Lambda_k\big]$
$\psi\,E$	$-\frac{1}{2}(d-1)\big[-\partial_t^2 + \Delta - 2\Lambda_k\big]$
$\hat{N}\,\hat{N}$	$-\partial_t^2 + \Delta$
$B\,B$	$-\partial_t^2 + \Delta$
$u\,u$	$2\left[-\partial_t^2 + \Delta\right]$
$u\,v$	~~$-2\,\partial_t\sqrt{\Delta}$~~
$B\,\psi$	~~$2\,(d-1)\sqrt{\Delta}\,\partial_t$~~
$\hat{N}\,\psi$	$(d-1)\left[-\partial_t^2 + \Delta - \Lambda_k\right]$
$\hat{N}\,E$	$-\partial_t^2 + \Delta - 2\,\Lambda_k$

RG flows in the ADM formalism

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fluctuations: adapted to cosmological perturbation theory

$$\begin{aligned} \hat{N}_i =& u_i + \partial_i \tfrac{1}{\sqrt{\Delta}} B \,, \qquad \partial^i u_i = 0 \\ \hat{\sigma}_{ij} =& h_{ij} - \left(\bar{\sigma}_{ij} + \partial_i \partial_j \tfrac{1}{\Delta} \right) \psi + \partial_i \partial_j \tfrac{1}{\Delta} E + \partial_i \tfrac{1}{\sqrt{\Delta}} v_j + \partial_j \tfrac{1}{\sqrt{\Delta}} v_i \,. \end{aligned}$$

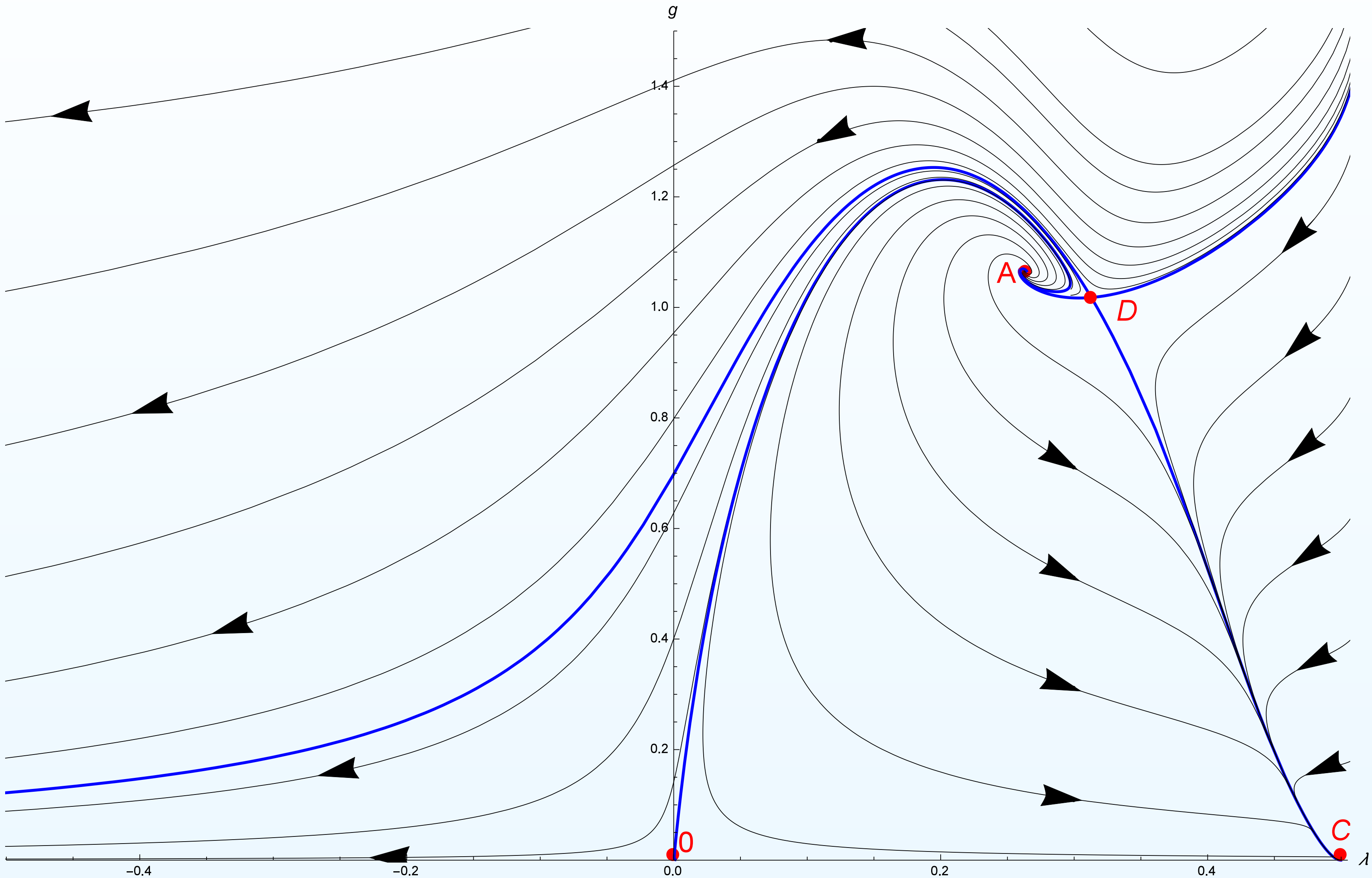
unique gauge-fixing of diffeomorphisms

$$\begin{aligned} F =& \partial_t \hat{N} + \partial^i \hat{N}_i - \tfrac{1}{2} \partial_t \hat{\sigma} + \tfrac{2(d-1)}{d} \bar{K} \hat{N} \,, \\ F_i =& \partial_t \hat{N}_i - \partial_i \hat{N} - \tfrac{1}{2} \partial_i \hat{\sigma} + \partial^j \hat{\sigma}_{ji} + (d-2) \bar{K}_{ij} \hat{N}^j \,, \end{aligned}$$

well-defined Hessian $\Gamma_k^{(2)}$ with relativistic propagators

Einstein-Hilbert-truncation on cosmological background

J. Biemans, A. Platania and F. Saueressig, arXiv:1609.04813



towards testing unitarity

Ansatz for testing unitarity

D. Becker, C. Ripken and F. Saueressig, in preparation

Goal: include higher-derivative terms in Γ_k:

$$\Gamma_k = \Gamma_k^{\rm EH} + \frac{1}{2} \int d^dx \sqrt{g} \, [\phi f_k(\Delta) \phi] + \Gamma_k^{\rm gf} + \Gamma_k^{\rm ghost} \, .$$

- ideally: determine structure function $f_*(\Delta)$
- currently: only polynomials

$$f_k(\Delta) \simeq Z_{1,k} \left(\Delta + \sum_{n=2}^{N_\Delta} \bar{Y}_{n,k} \, \Delta^n \right)$$

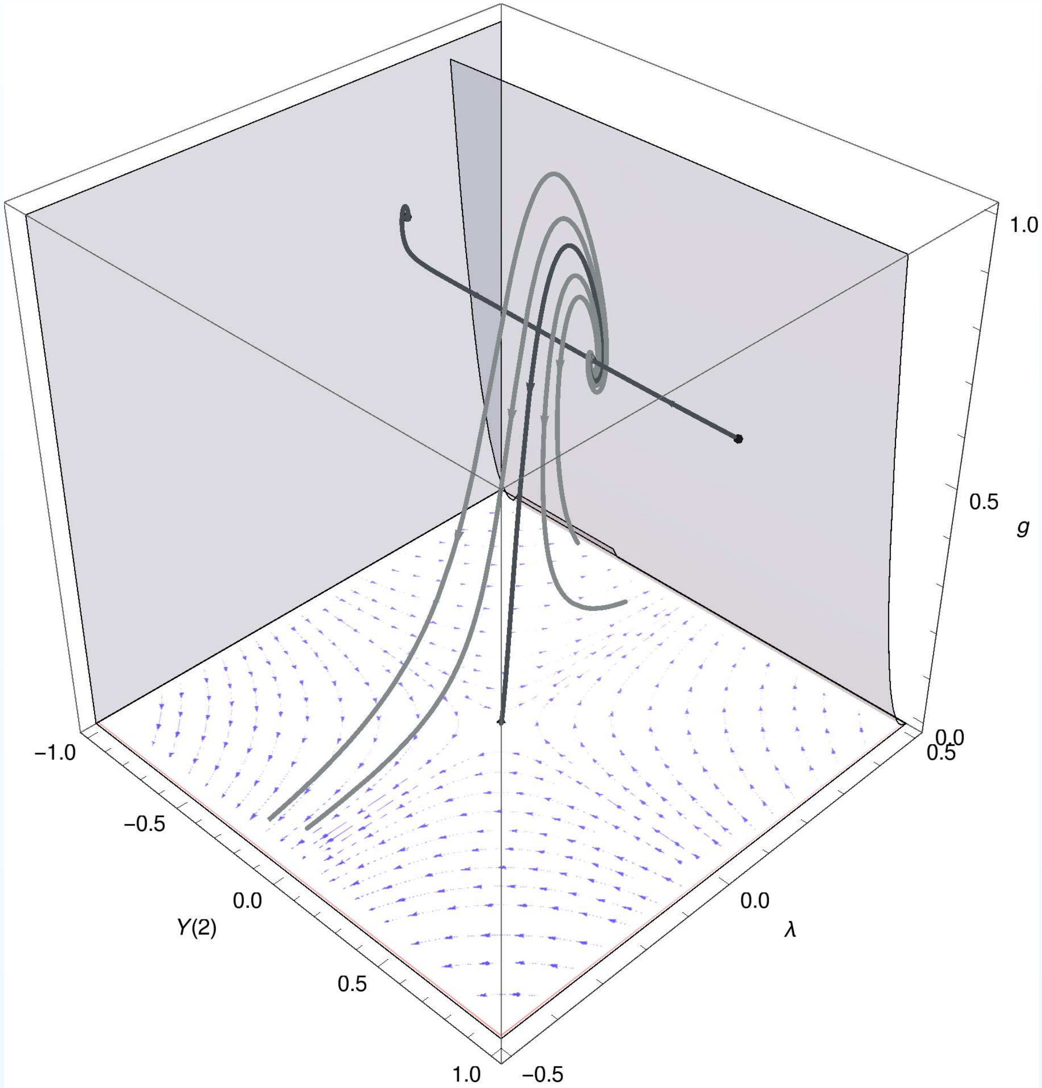
  - Ostrogradski-instability if $f_0(\Delta)$ has more than one real root

- ideally:

$$f_*(\Delta) = \Delta \, \tilde{f}_0(\Delta) \qquad \text{where} \qquad \tilde{f}_0(\Delta) \text{ positive function}$$

Phase diagram for $N_\Delta = 2$

D. Becker, C. Ripken and F. Saueressig, in preparation



summary and outlook

summary

gravity possesses a non-trivial renormalization group fixed point

- candidate for the completion of gravity at trans-Planckian scales
  - resolves non-renormalizability issue encountered in perturbation theory
  - stable against including perturbative counterterms
- transparent connection to classical low-energy physics
- extends to gravity-matter systems
- ADM decomposition:
  - bridge towards computing real-time correlation functions
  - tailored to cosmological investigations

3nd roadmap workshop

Quantum spacetime and the Renormalization Group

Lorentz Center Leiden (NL)

13 - 17 February 2017