Functional renormalization group approach to the continuum limit of Group Field Theories

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Albert Einstein Institute

8th International Conference on the Exact Renormalization Group
ERG2016
ICTP, Trieste, Italy, EU
23/09/2016

Plan of the talk

- GFTs : what are they?
  - general formalism
  - relation with other QG approaches
- continuum limit in GFT (and QG)
- FRG analysis of GFT models
  - general set-up
  - overview of results
  - FRG analysis of an abelian rank-d TGFT
- effective continuum physics
  - cosmology from GFT (and QG)
  - GFT condensate cosmology
  - bouncing cosmologies from GFT

Part I:
the GFT formalism

Group field theories

(Boulatov, Ooguri, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, Baratin, ……)

QFT of spacetime, not defined on spacetime

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Quantum field theories over group manifold G (or corresponding Lie algebra) $\varphi : G^{\times d} \rightarrow \mathbb{C}$

relevant classical phase space for "GFT quanta": $(\mathcal{T}^* G)^{\times d} \simeq (\mathfrak{g} \times G)^{\times d}$

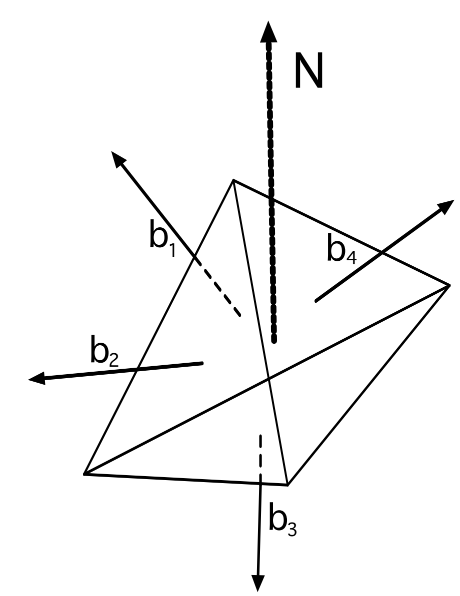
can reduce to subspaces in specific models depending on conditions on the field

d is dimension of "spacetime-to-be"; for gravity models, G = local gauge group of gravity (e.g. Lorentz group)

example: d=4 $\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \rightarrow \mathbb{C}$

arguments of GFT field: $b_i \in \mathfrak{su}(2)$

| b | ~ J = irrep of SU(2) ~ "area of triangles"



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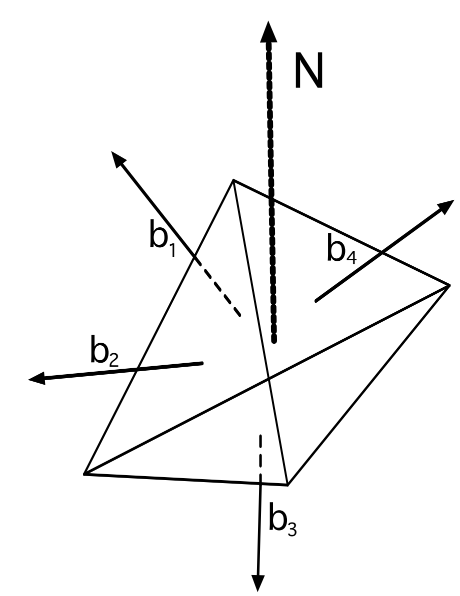

very general framework; interest rests on specific models/use
(most interesting QG models are for Lorentz group in 4d)

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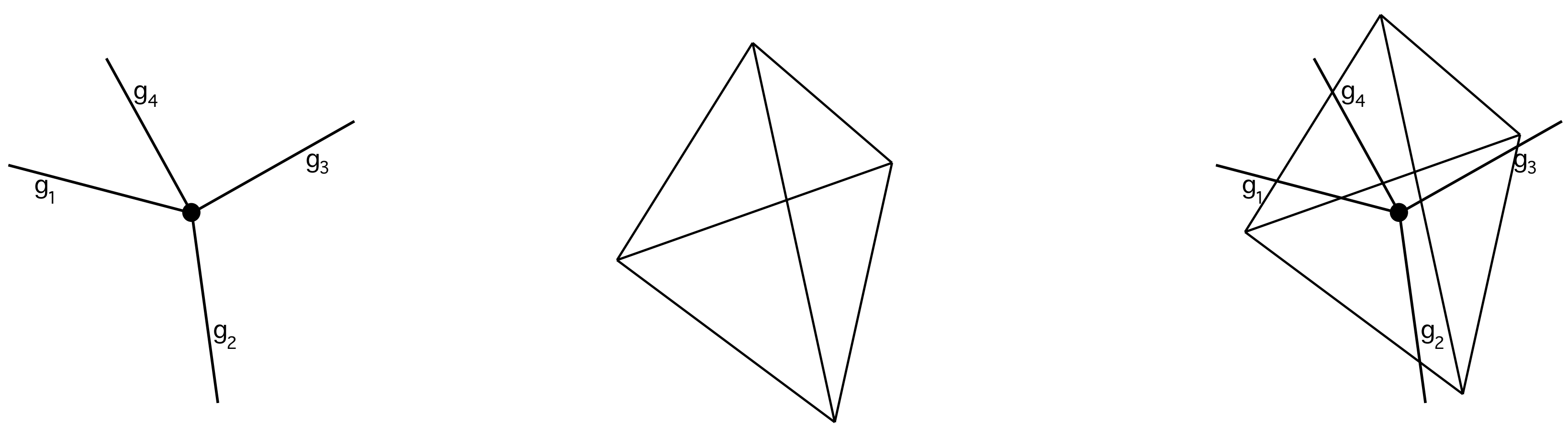

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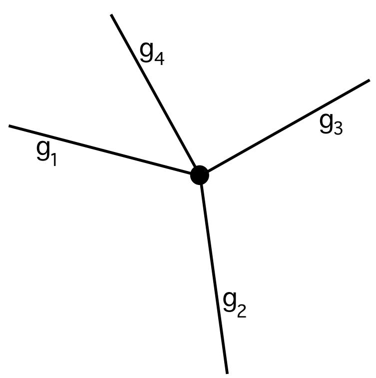
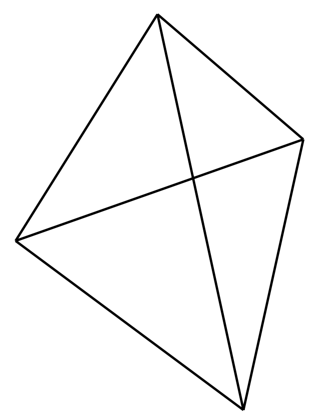
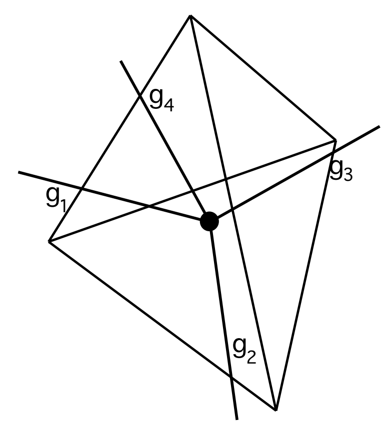
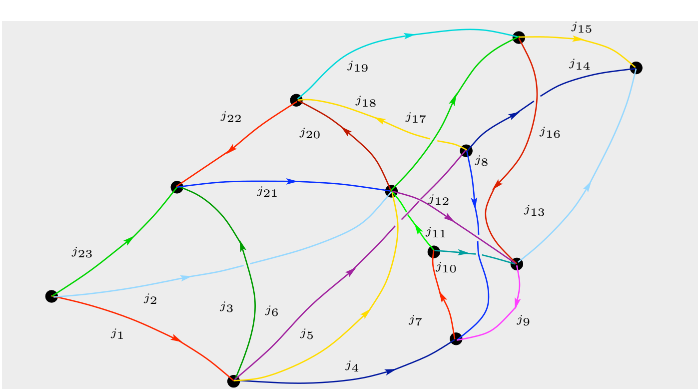
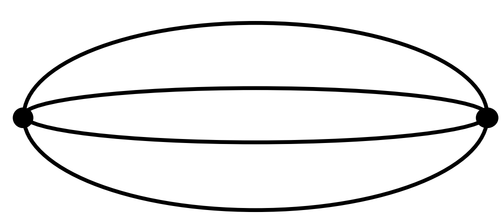


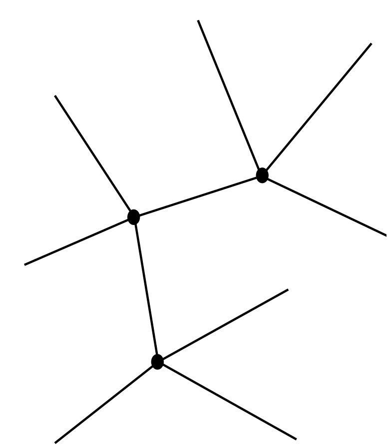


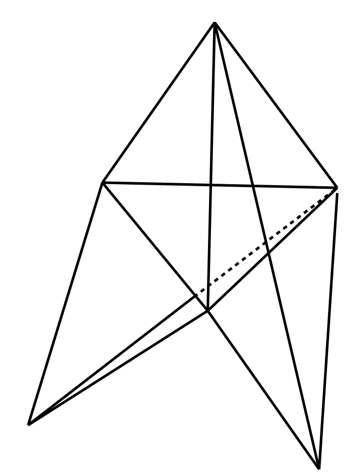


generic quantum state: arbitrary collection of spin network vertices (including glued ones) or tetrahedra (including glued ones)









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classical action: kinetic (quadratic) term + (higher order) interaction (convolution of GFT fields)

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

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combinatorics of field arguments in interaction: gluing of 5 tetrahedra across common triangles, to form 4-simplex ("building block of spacetime")

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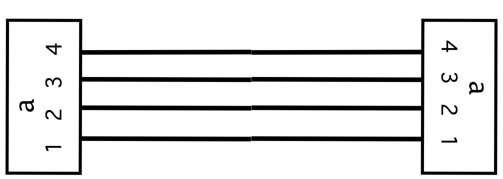

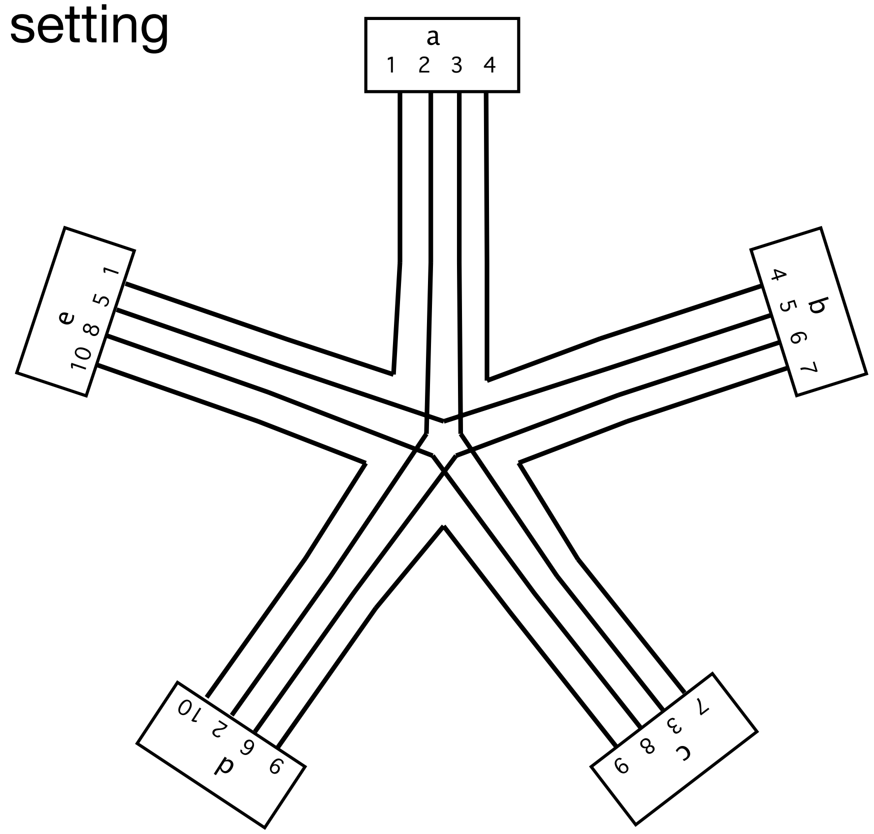

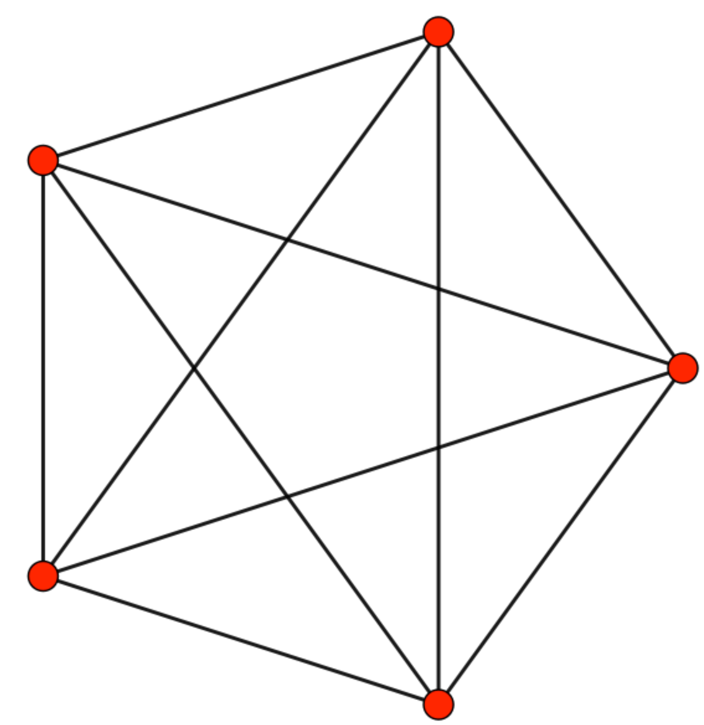

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Feynman perturbative expansion around trivial vacuum

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Feynman diagrams (obtained by convoluting propagators with interaction kernels) =

= stranded diagrams dual to cellular complexes of arbitrary topology

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equivalently:

- spin foam models (sum-over-histories of spin networks ~ covariant LQG)

  Reisenberger,Rovelli, '00

- lattice path integrals (with group+Lie algebra variables)

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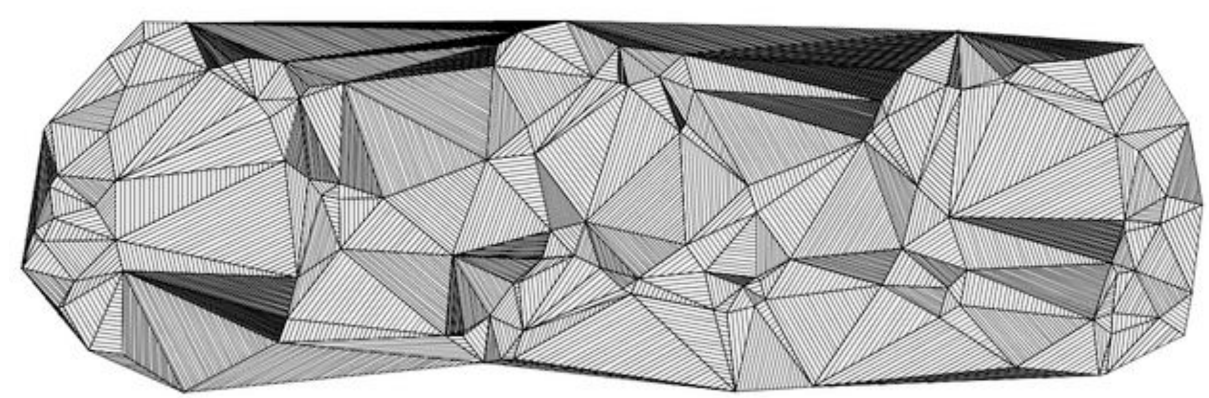

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GFT as lattice quantum gravity:

dynamical triangulations + quantum Regge calculus

GFTs and Loop Quantum Gravity

second quantized version of Loop Quantum Gravity

but dynamics not derived from canonical quantization of GR

(DO, 1310.7786 [gr-qc])

DO, J. Ryan, J. Thurigen, ‘14

(LQG spin network states ~ many-particles states, “particle” ~ spin network vertex)

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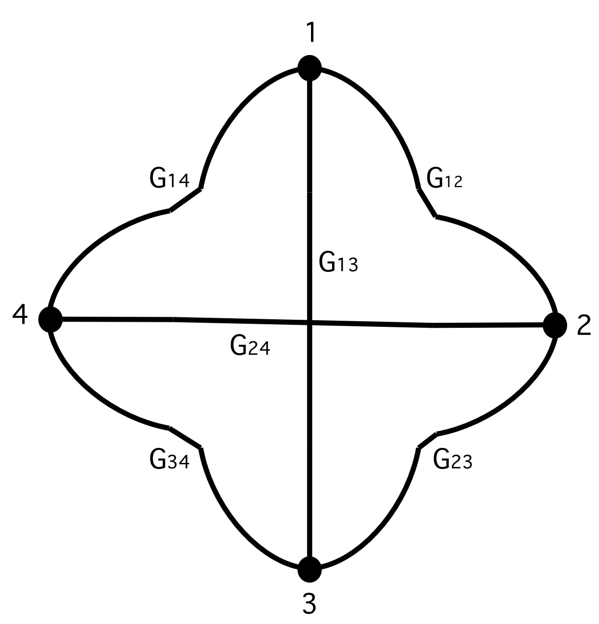

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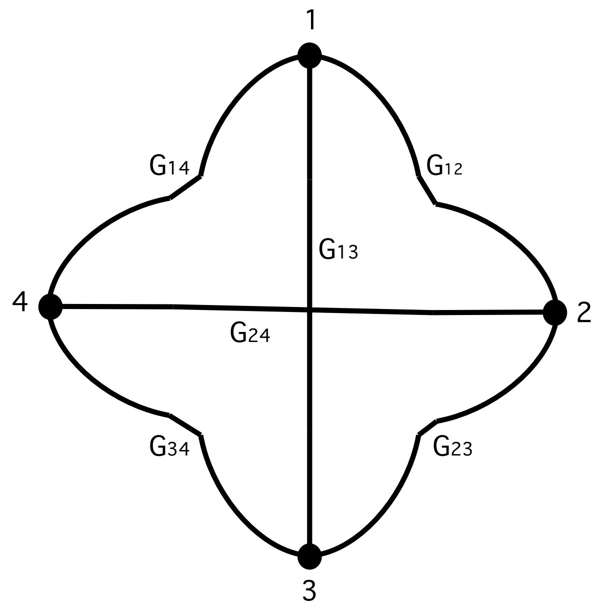

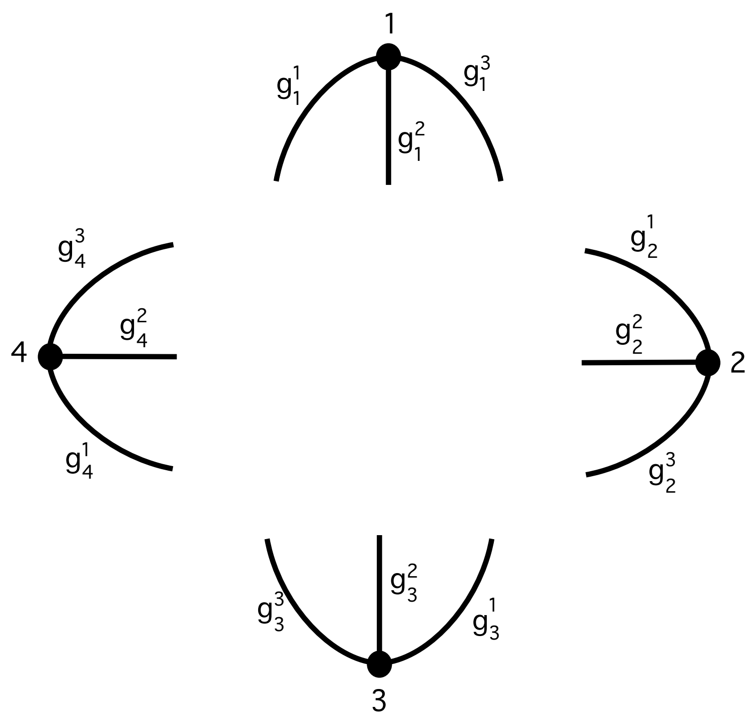

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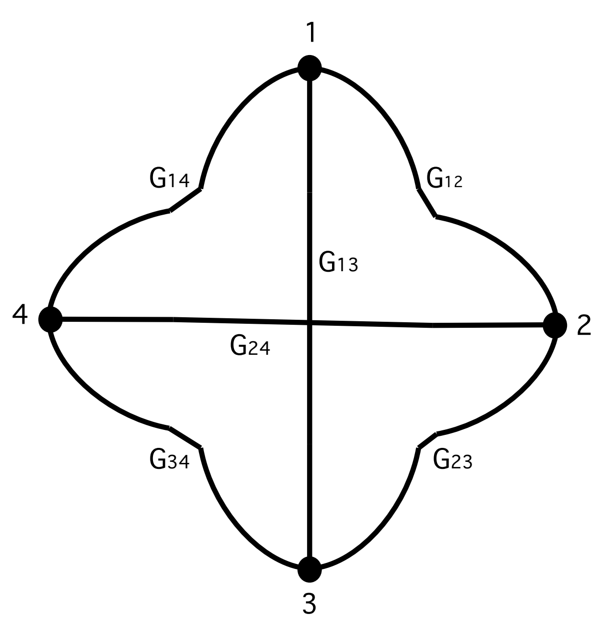

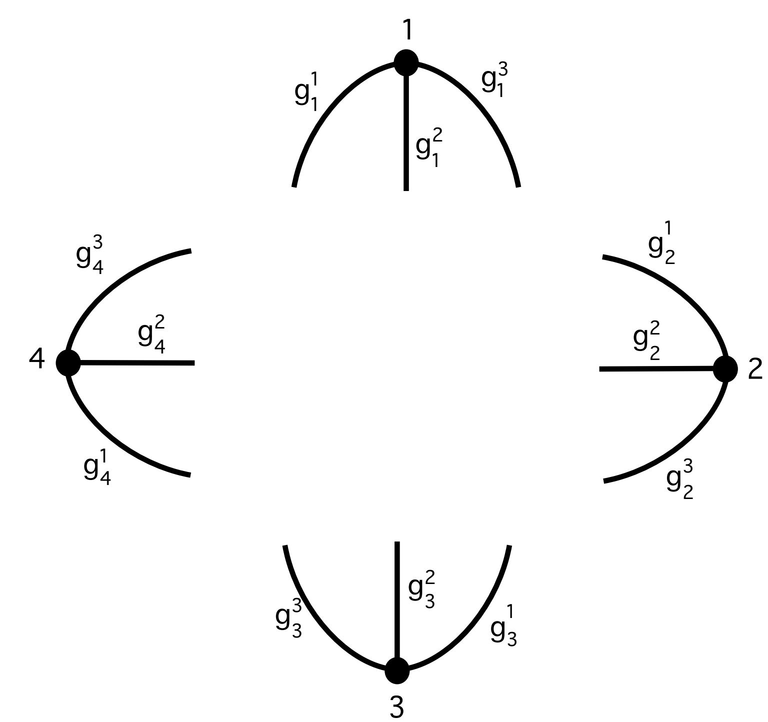

GFT Hilbert space = Fock space of open spin network vertices - contains any LQG state (all spin networks)

any LQG observable has a 2nd quantised, GFT counterpart

choice of LQG dynamics (Hamiltonian constraint operator) translates into choice of GFT action

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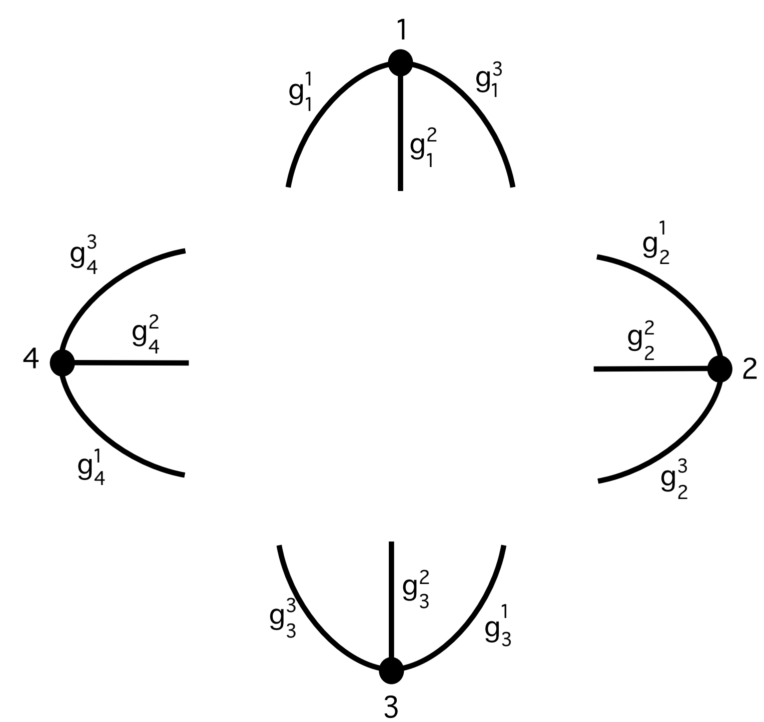

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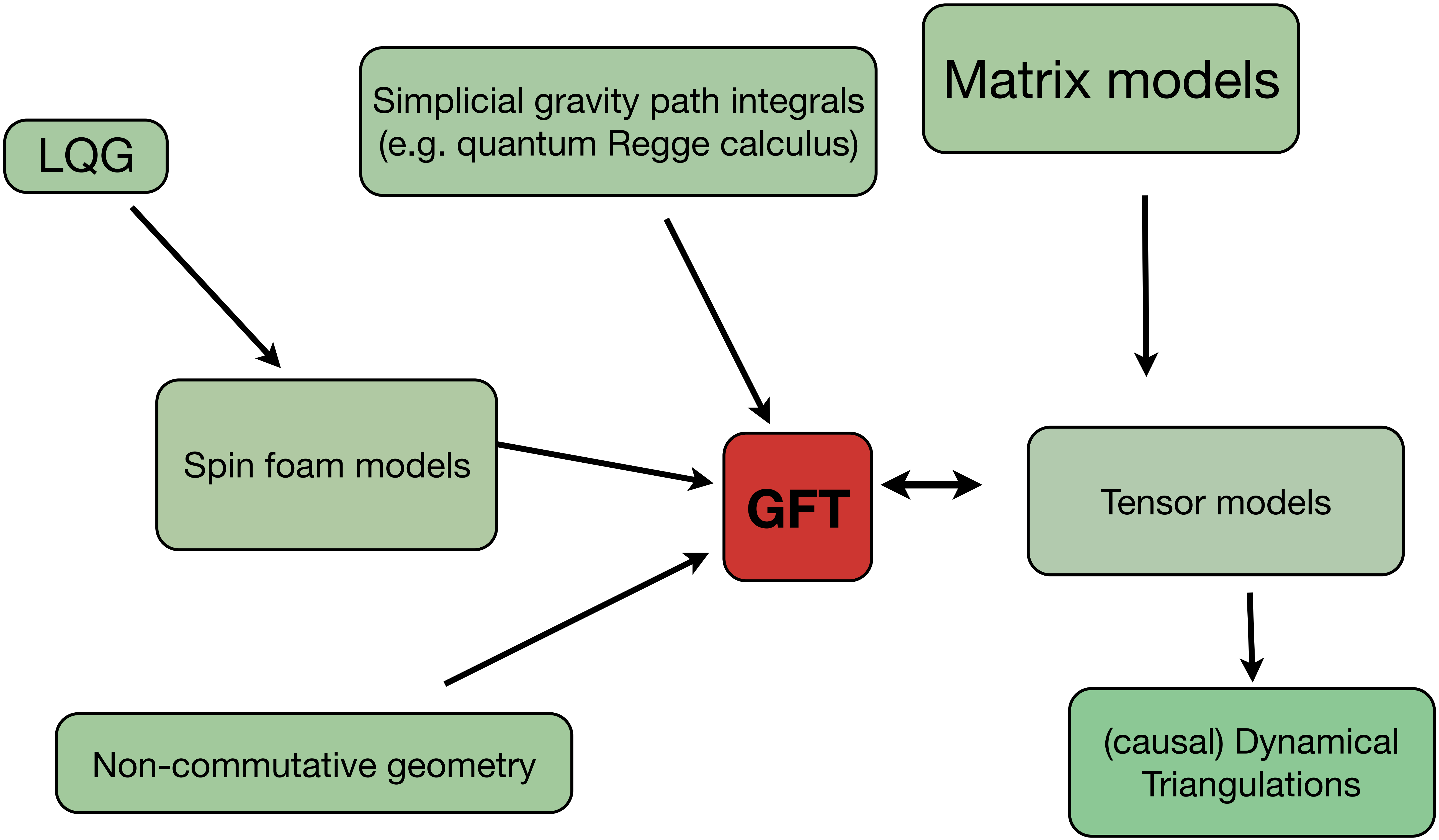
QFT methods (i.e. GFT reformulation of LQG and spin foam models) useful to address physics of large numbers of LQG d.o.f.s, i.e. many and refined graphs (continuum limit)

Group Field Theory: crossroad of approaches



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  - GFT perturbative renormalization

  —-> renormalizability of GFT for given discrete gravity path integral/spin foam amplitudes

  - GFT symmetries (at both classical and quantum level) Ben Geloun, '11; Girelli, Livine, '11; Baratin, Girelli, DO, '11

  —-> in particular, those with geometric interpretation (e.g. diffeomorphisms) Kegeles, DO, '15

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- how to define the continuum limit (of the LQG/SF dynamics or, equivalently, of discrete gravity path integral)?

    controlling quantum dynamics of more and more interacting degrees of freedom

    new analytic tools from QFT embedding

- Non-perturbative GFT renormalization and phase diagram - what are the QG phases? which one is geometric?
- Extraction of effective continuum dynamics in different phases

(as in QFT for condensed matter systems….)

Part II:
the continuum limit of GFTs

The problem of the continuum limit in QG

new (non-geometric, non-spatio-temporal) physical degrees of freedom (“building blocks”) for space-time

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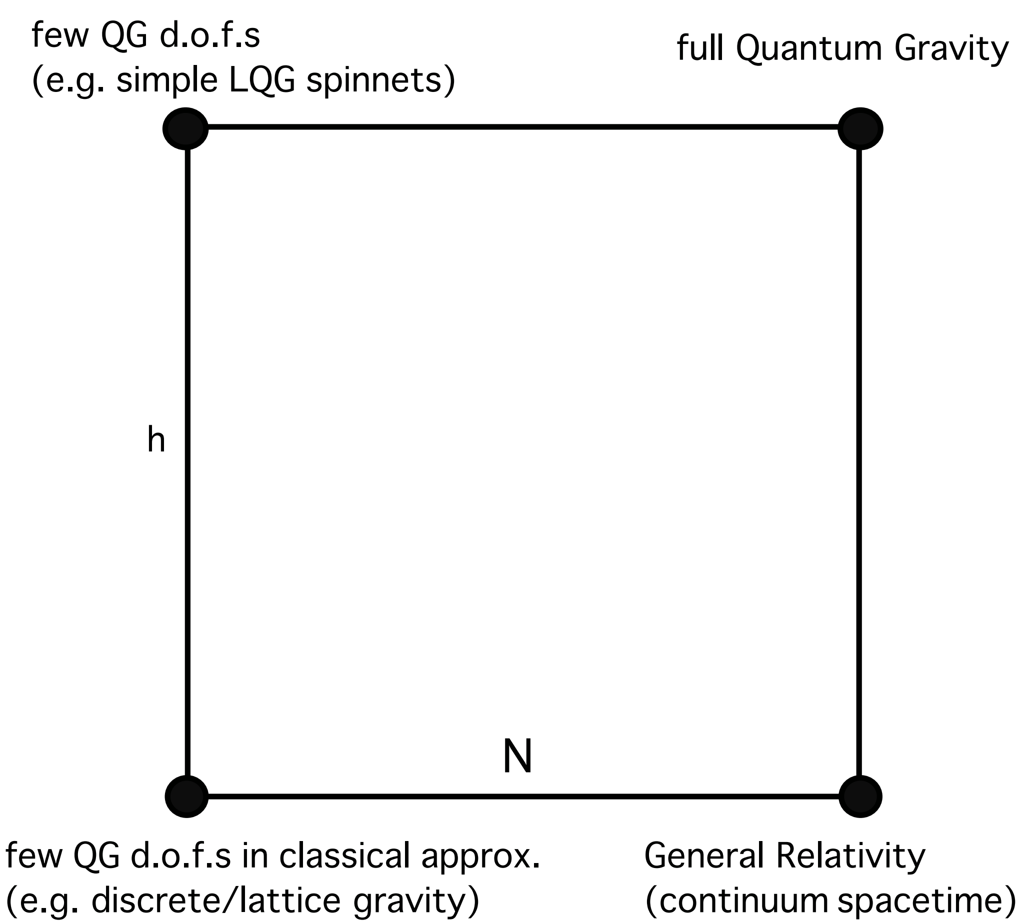

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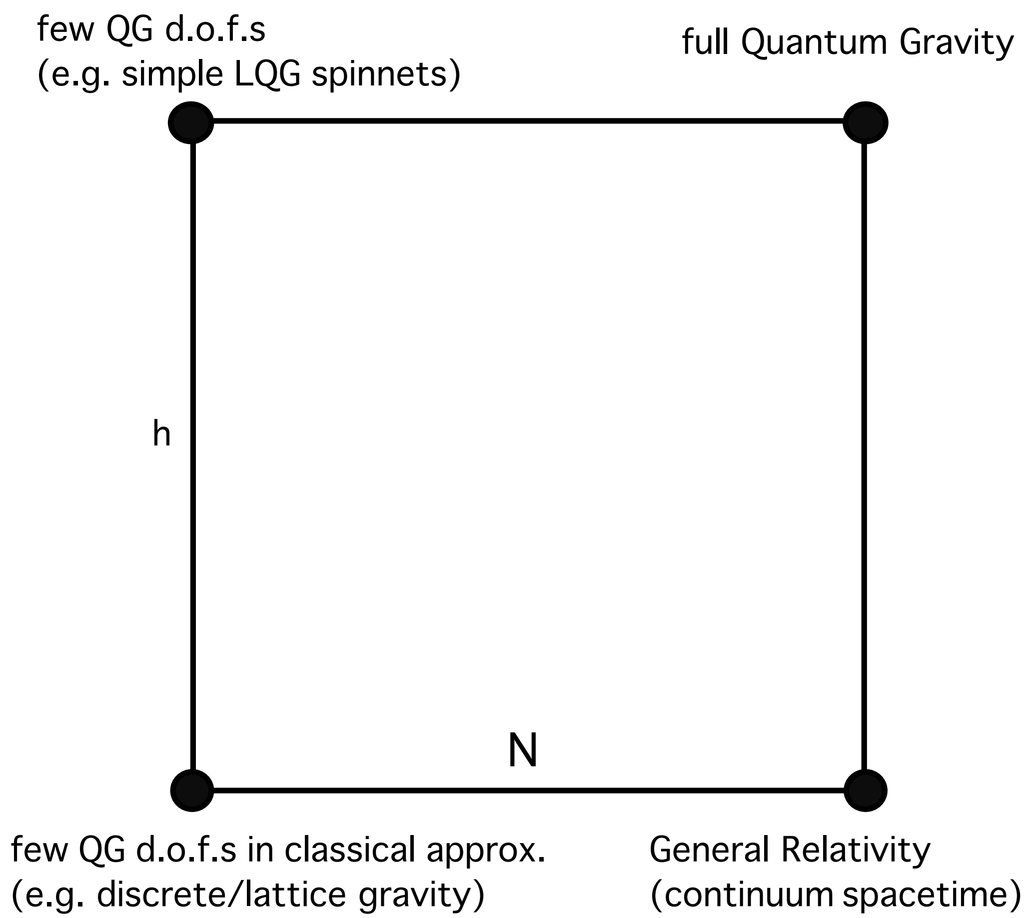
(collective behaviour of many interacting degrees of freedom):
continuum approximation

h-direction: classical approximation



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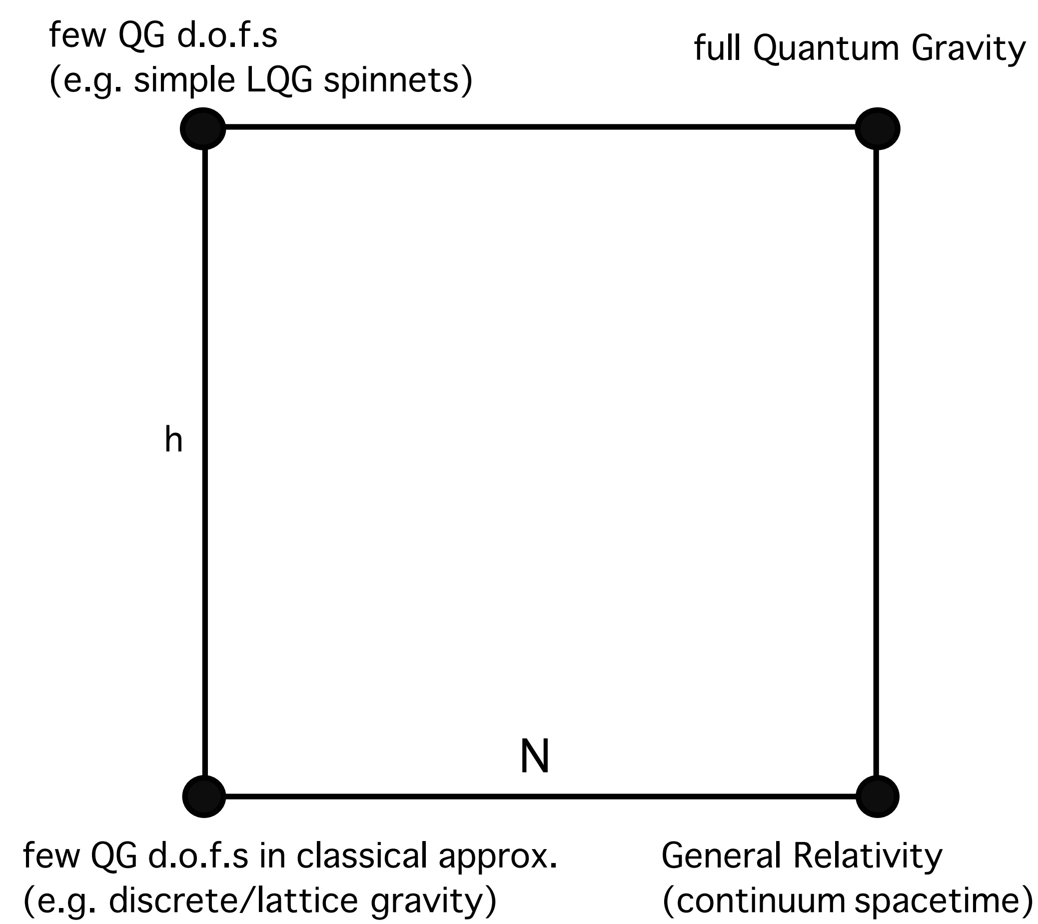
“well-understood” in spin foam models and discrete gravity



Problem of the continuum in QG: role of RG

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collective behaviour of (interacting) fundamental d.o.f.s should lead to different macroscopic phases, separated by phase transitions

- for a non-spatio-temporal QG system (e.g. LQG in GFT formulation), which of the macroscopic phases is described by a smooth geometry with matter fields?

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in specific GFT case:

- treat GFT models as analogous to atomic QFTs in condensed matter systems

- need to understand effective dynamics at different “GFT scales”: RG flow of effective actions & phase structure & phase transitions

Continuum limit of GFT (and LQG, discrete gravity etc)

the issue:

controlling quantum dynamics of more and more interacting degrees of freedom

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- control GFT quantum dynamics for boundary states involving (superpositions of) large graphs

-compute- spin foam amplitudes for finer complexes and corresponding sum over complexes

up to infinite refinement (infinite number of degrees of freedom), at least in simple approximations

Continuum limit of GFT (and LQG, discrete gravity etc)

the issue:

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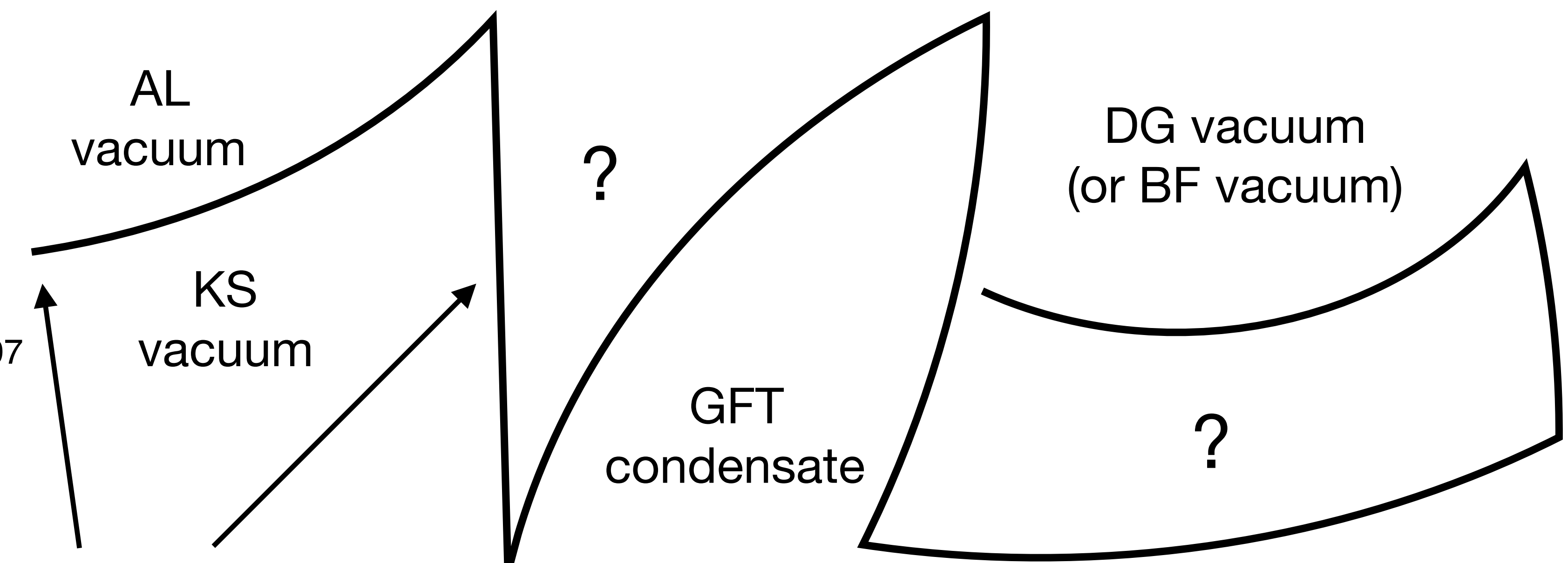

Ashtekar, Lewandowski, '94 Koslowski, Sahlmann, '10 Dittrich, Geiller, '14, '15

Gielen, DO, Sindoni, '13 Kegeles, DO, Tomlin, to appear

Part III:
the FRG analysis of GFTs

GFT renormalisation - general scheme

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

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use group structures (Killing form, topology, etc) to define notion of scale and to set up mode integration

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most results for Tensorial GFTs

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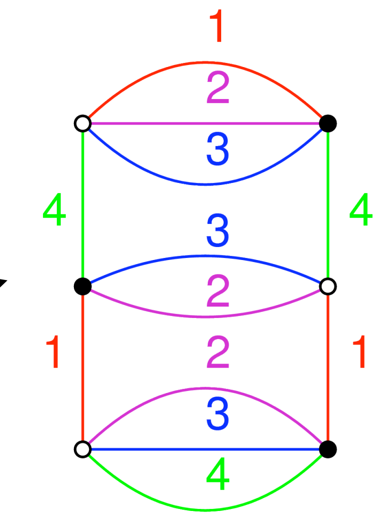

indexed by bipartite d-colored graphs ("bubbles")
dual to d-cells with triangulated boundary

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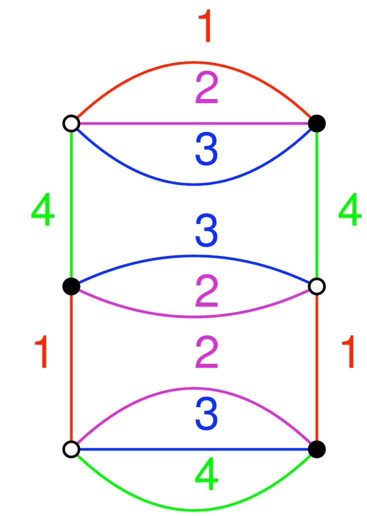

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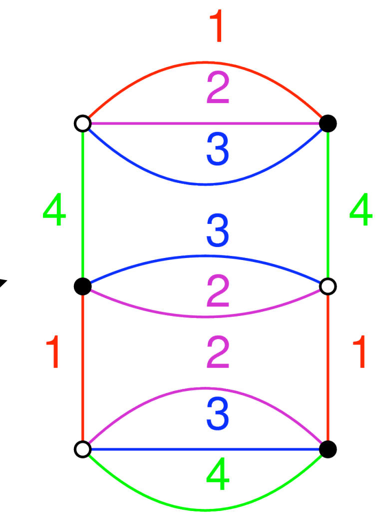

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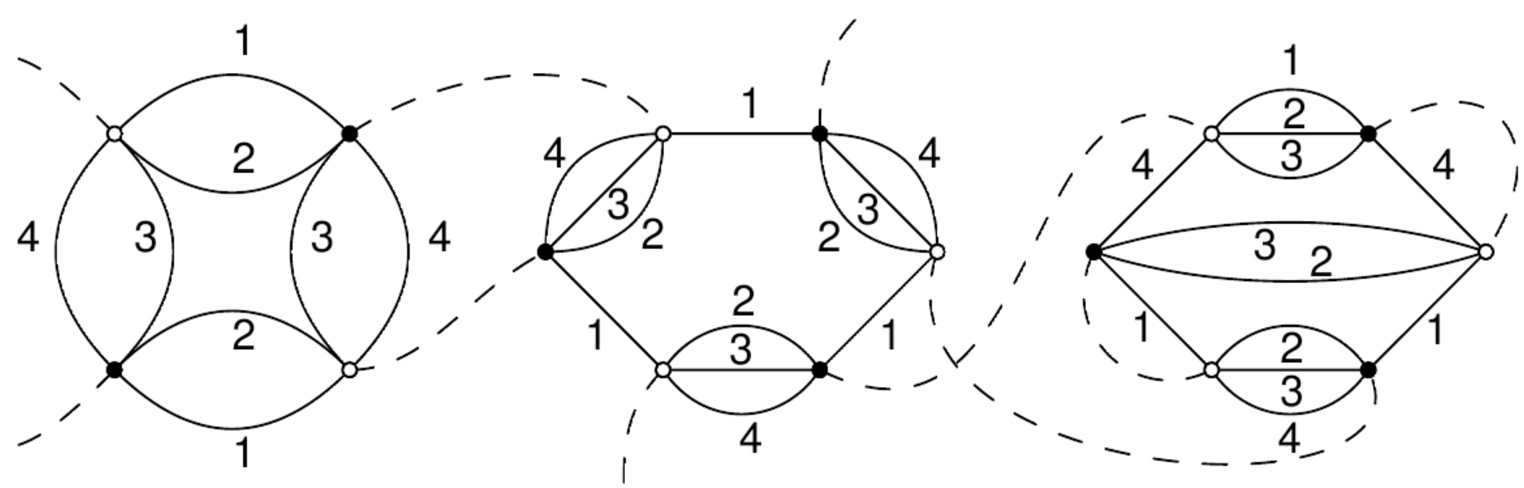

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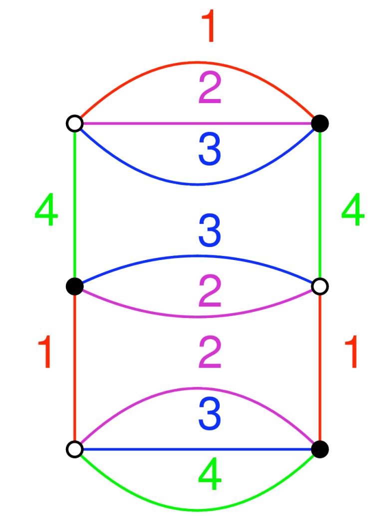

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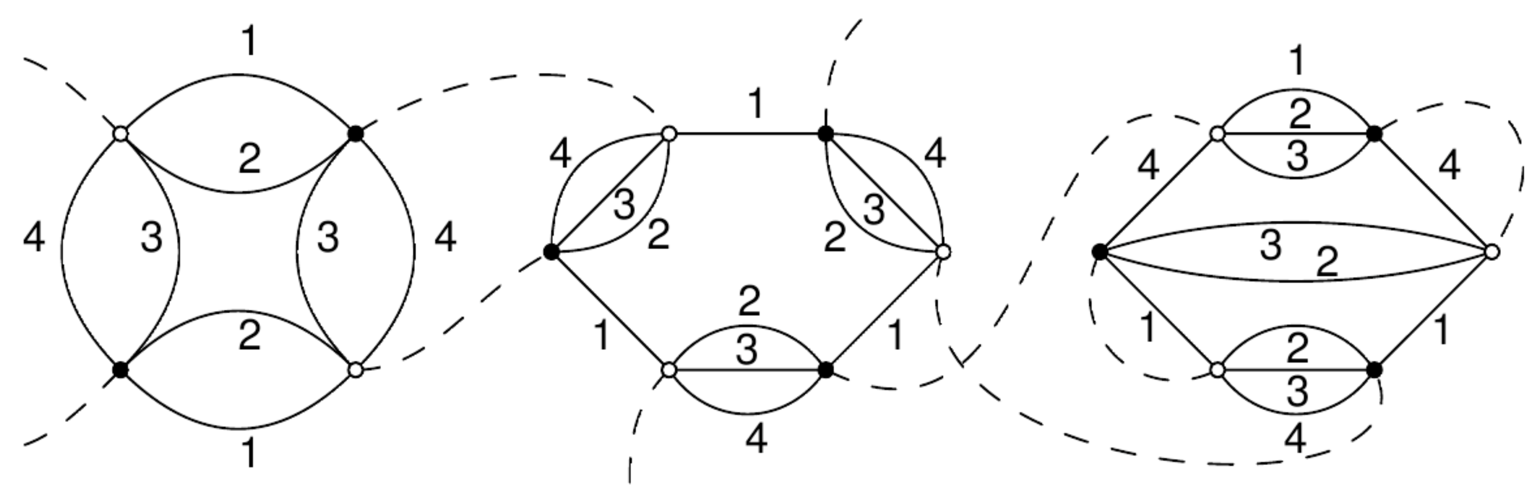
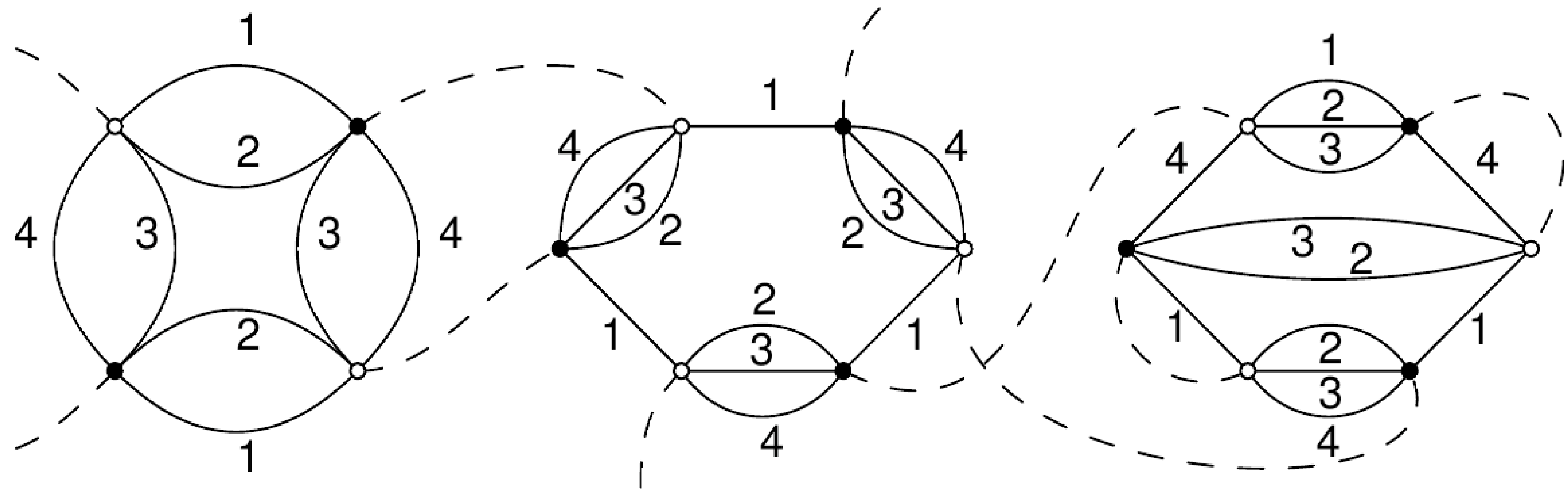


require generalization of notions of "connectedness", "contraction of high subgraphs", "locality", Wick ordering, ...

taking into account internal structure of Feynman graphs, full combinatorics of dual cellular complex, results from crystallization theory (dipole moves)

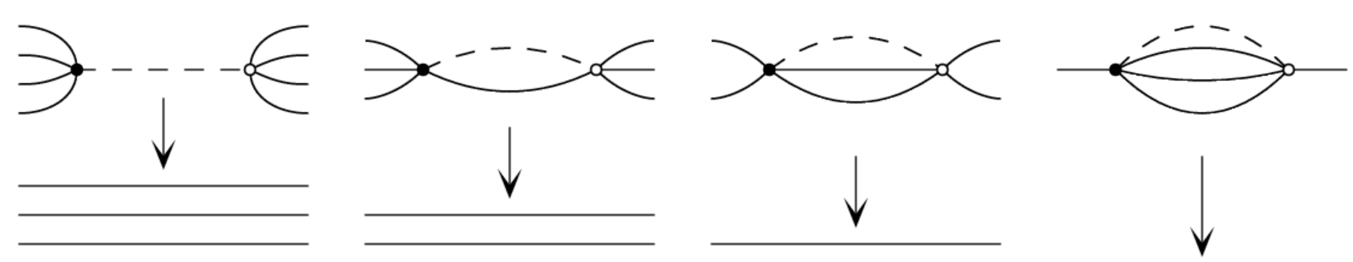
TGFT renormalization

example of Feynman diagram



- building blocks: coloured bubbles, dual to d-cells with triangulated boundary
- glued along their boundary (d-1)-simplices
- parallel transports (discrete connection) associated to dashed (color 0, propagator) lines
- faces of color i = connected set of (alternating) lines of color 0 and i

“contraction of internal line” ~ dipole contraction



GFT Renormalization: "geometric" interpretation?

consistent with cosmological interpretation of classical GFT fields
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- GFT “UV” cut-off $N \sim J_{max}$
- RG flow: J_{max} ——> small J
- (perturbative) GFT renormalizability: UV fixed point as J_{max} ——> oo

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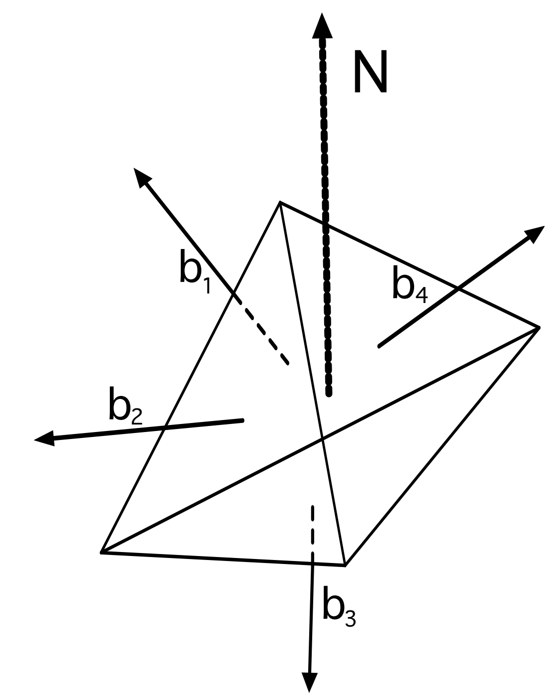

from LQG
from Regge calculus

arguments of GFT field: $b_i \in \mathfrak{su}(2)$ gravity case: d=4

| b | ~ J = irrep of SU(2) ~ “area of triangles”

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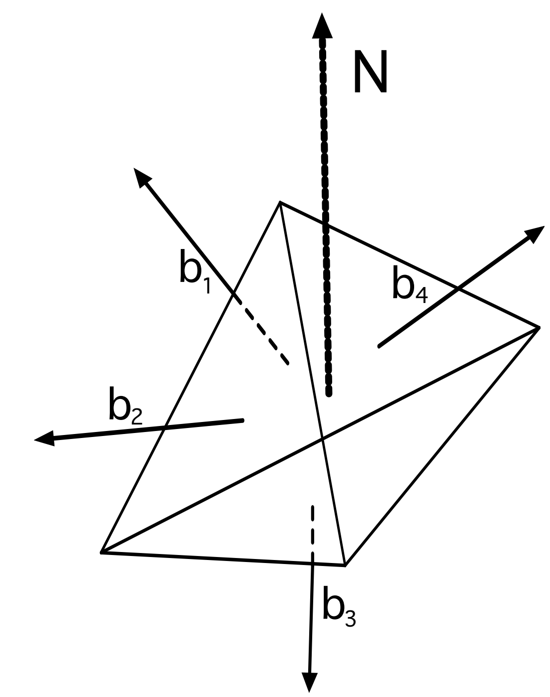

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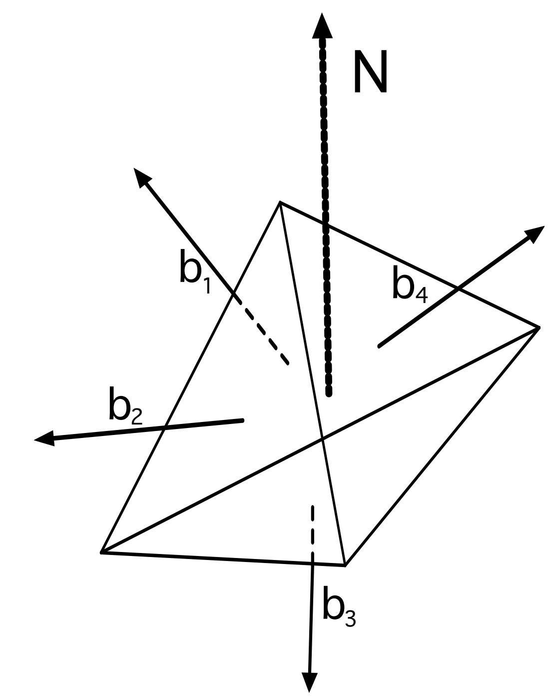

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- CAUTION in interpreting things geometrically outside continuum geometric approx

- e.g. expect “physical” continuum areas $A \sim < J > < n >$
- expect proper continuum geometric interpretation (and effective metric field) for $< J >$ small, $< n >$ large, A finite (not too small), and small curvature

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GFT perturbative renormalisation

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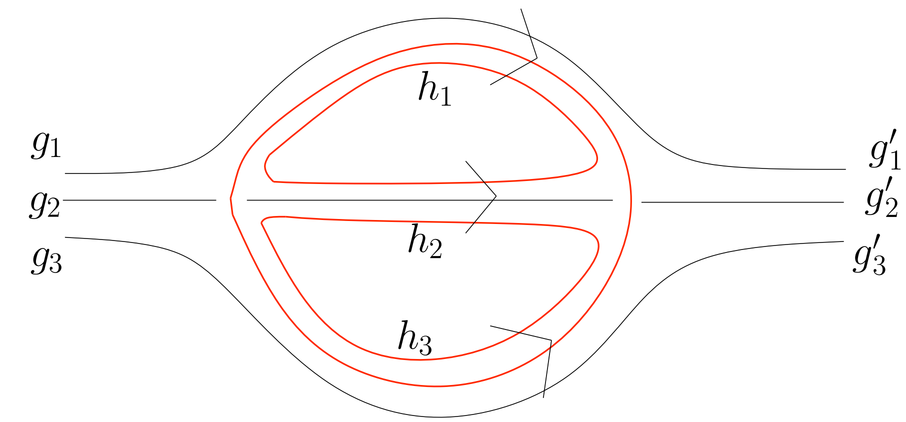

step by step, towards renormalizable 4d gravity models:

- scale indexed by group representations
- interplay between algebraic data and combinatorics of diagrams

- calculation of some radiative corrections T. Krajewski, J. Magnen, V. Rivasseau, A. Tanasa, P. Vitale, '10; A. Riello, '13; Bonzom, Dittrich, '15
- finiteness results in 3d simplicial models (Boulatov with Laplacian kinetic term) Ben Geloun, Bonzom, '11; Ben Geloun, '13

- renormalizable TGFT models (3d, 4d, and higher) - Laplacian + tensorial interactions

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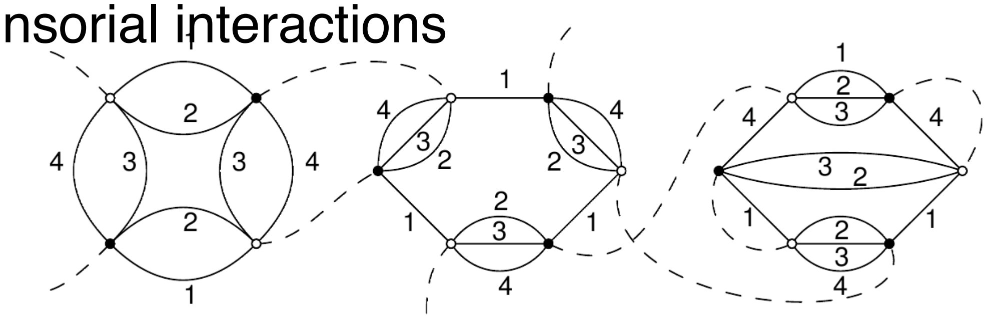

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—> non-abelian (SU(2))

— —> SO(4) or SO(3,1) with simplicity constraints: first results on BC-like 4d models

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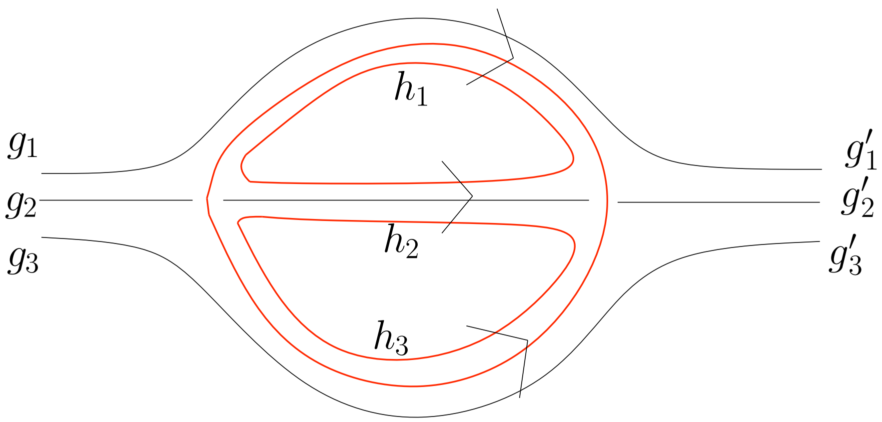

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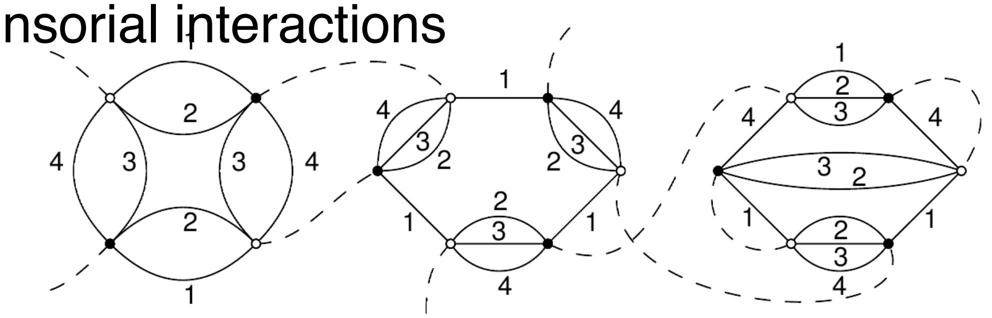

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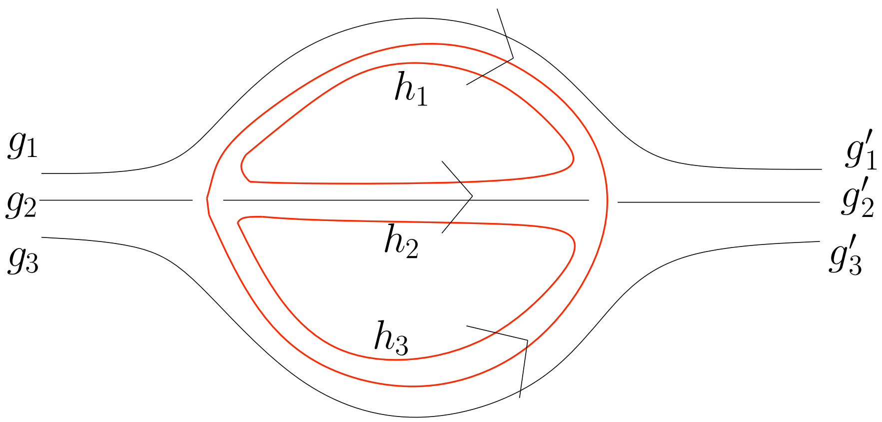

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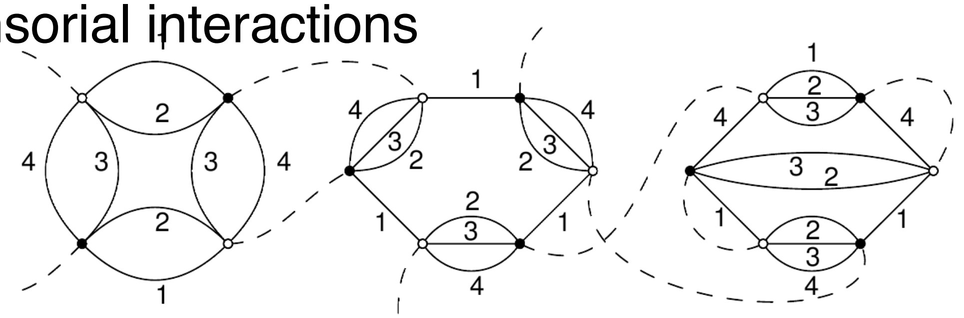

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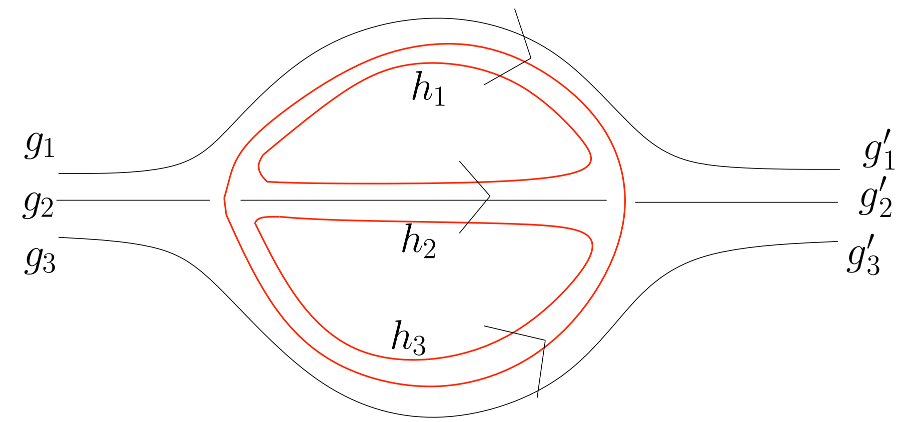

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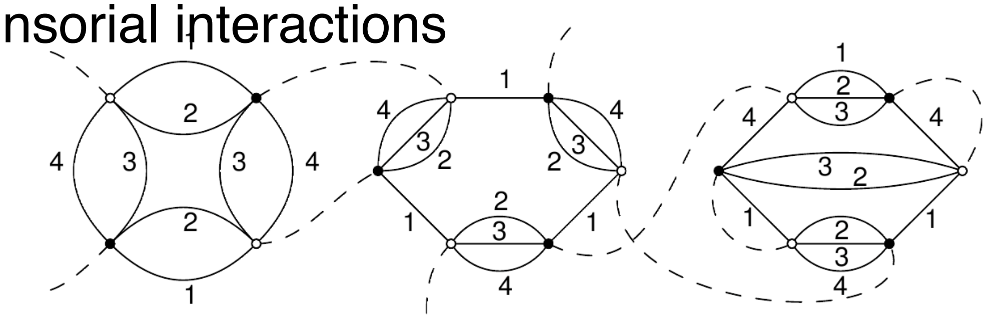

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Benedetti, Ben Geloun, DO, Martini, Lahoche, Carrozza, Douarte, ….

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two directions:

- GFT non-perturbative renormalization and "IR" fixed points (e.g. FRG analysis - e.g. a la Wetterich

  Benedetti, Ben Geloun, DO, Martini, Lahoche, Carrozza, Douarte, ….

- GFT constructive analysis

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  non-perturbative resummation of perturbative (SF) series

  variety of techniques:
  - intermediate field method
  - loop-vertex expansion
  - Borel summability

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FRG for (tensorial) GFT models

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(similar to matrix model but distinctively field-theoretic)

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- Polchinski formulation based on SD equations — Krajewski, Toriumi, ‘14
- general set-up for Wetterich formulation based on effective action
  - analysis of TGFT on compact U(1)^d — Benedetti, Ben Geloun, DO, ’14 ; Ben Geloun, Martini, DO, ’15, ’16, Benedetti, Lahoche, ’15; Douarte, DO, ‘16
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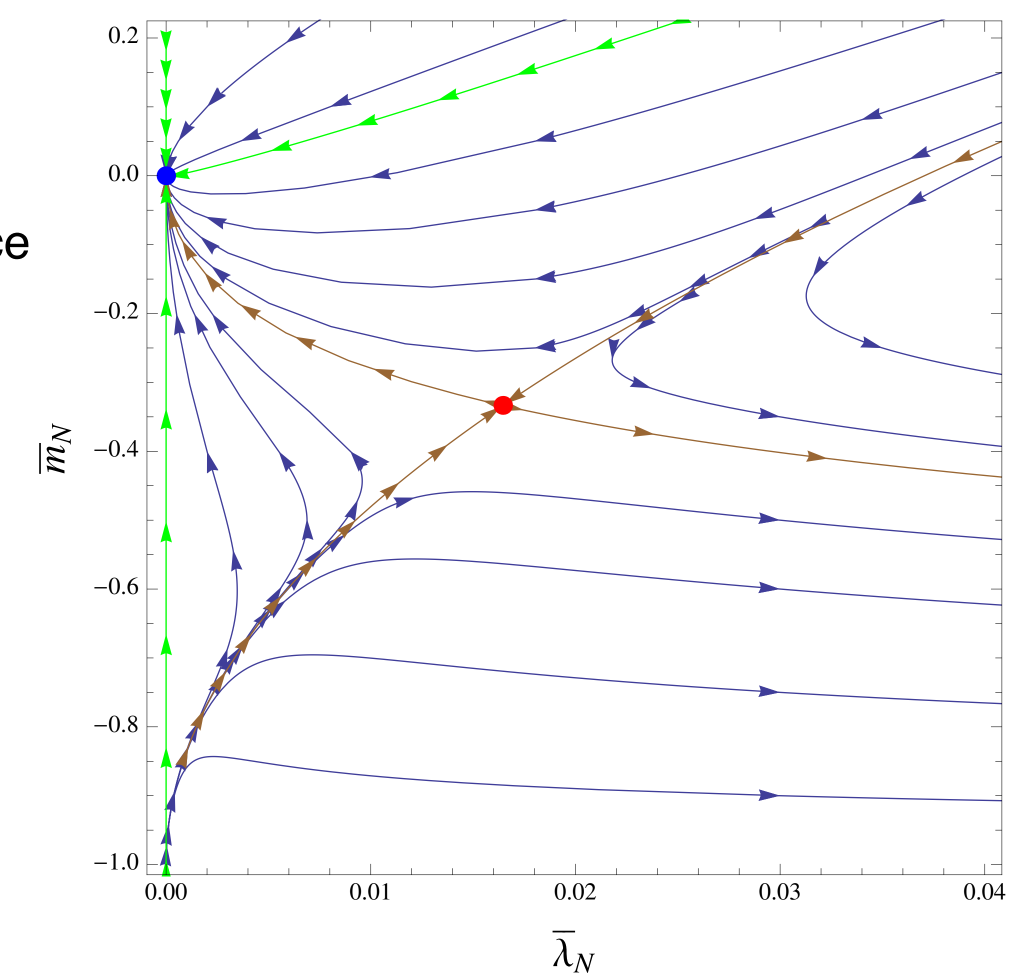
  - analysis of TGFT on non-compact R^d with gauge invariance
    - RG flow and phase diagram established
  - analysis of TGFT on SU(2)^3 — Carrozza, Lahoche, ‘16



generically (so far):

two FPs (Gaussian-UV, Wilson-Fisher-IR)

asymptotic freedom

one symmetric phase

one broken or condensate phase

FRG analysis of GFT models

D. Benedetti, J. Ben Geloun, DO, ‘14

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regularised path integral: $$\mathcal{Z}_k[J,\overline{J}] = e^{W_k[J,\overline{J}]} = \int d\phi d\overline{\phi}\; e^{-S[\phi,\overline{\phi}]-\Delta S_k[\phi,\overline{\phi}]+\mathrm{Tr}(J\cdot\overline{\phi})+\mathrm{Tr}(\overline{J}\cdot\phi)}$$

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Wetterich equation expanded in field powers, with all possible contractions; truncation matching classical action

system of flow equations is generically non-homogeneous, because of combinatorial patterns of contractions

for compact groups, it is also non-autonomous, due to hidden scale (size of group)

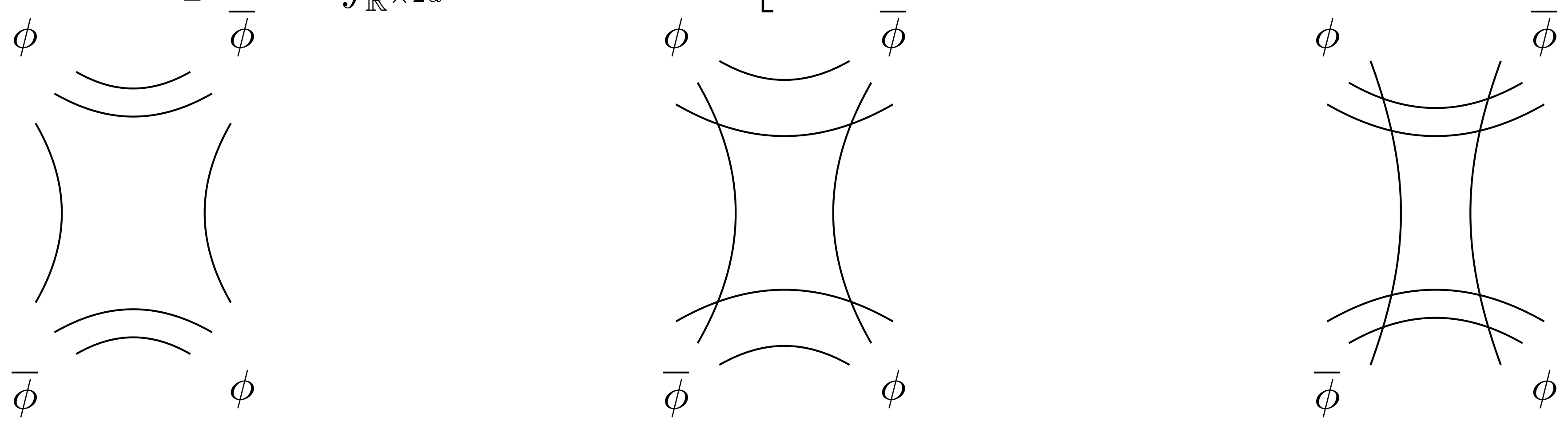
FRG analysis of a quartic abelian rank-d TGFT model

the model:

$G = \mathbb{R}$

$$S[\phi,\overline{\phi}] = (2\pi)^d \int_{\mathbb{R}^{\times d}} [dx_i]_{i=1}^d \, \overline{\phi}(x_1,\ldots,x_d)\left(-\sum_{s=1}^{d} \triangle_s + \mu\right)\phi(x_1,\ldots,x_d)$$

$$+\frac{\lambda}{2}(2\pi)^{2d}\int_{\mathbb{R}^{\times 2d}} [dx_i]_{i=1}^d [dx'_j]_{j=1}^d \Big[\phi(x_1,x_2,\ldots,x_d)\overline{\phi}(x'_1,x_2,\ldots,x_d)\phi(x'_1,x'_2,\ldots,x'_d)\overline{\phi}(x_1,x'_2,\ldots,x'_d)$$ + sym



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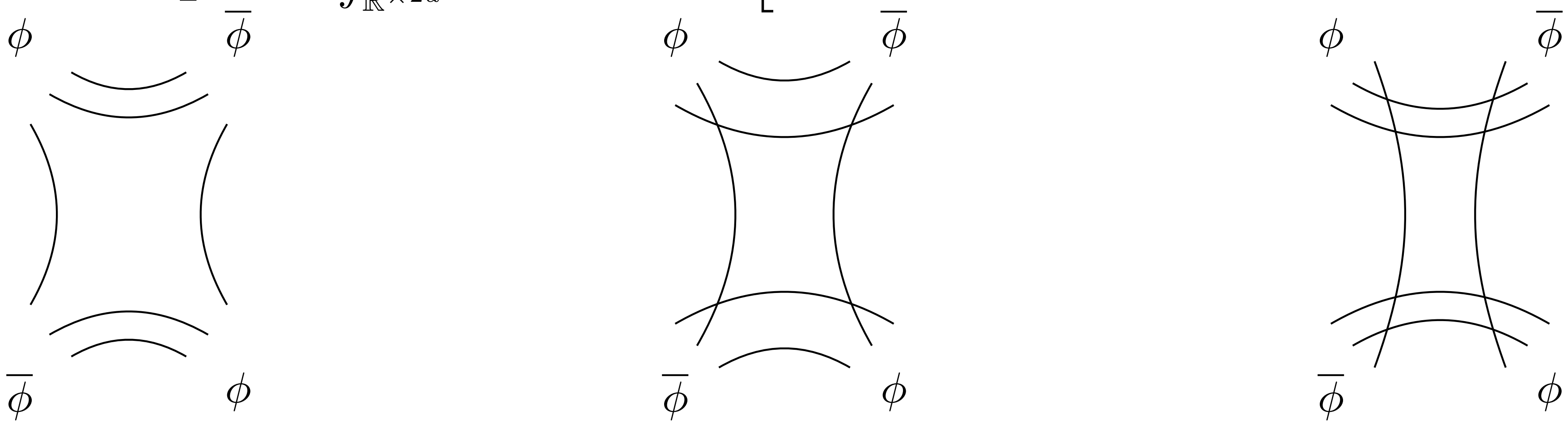

$$\Gamma_k[\varphi,\overline{\varphi}] = \int_{\mathbb{R}^{\times d}}[dp_i]_{i=1}^d \; \overline{\varphi}_{12\dots d}(Z_k\sum_s p_s^2 + \mu_k)\varphi_{12\dots d}$$

$$+\frac{\lambda_k}{2}\int_{\mathbb{R}^{\times 2d}}[dp_i]_{i=1}^d[dp'_j]_{j=1}^d\left[\varphi_{12\dots d}\overline{\varphi}_{1'2\dots d}\varphi_{1'2'\dots d'}\overline{\varphi}_{12'\dots d'} + \mathrm{sym}\Big\{1,2,\dots,d\Big\}\right]$$

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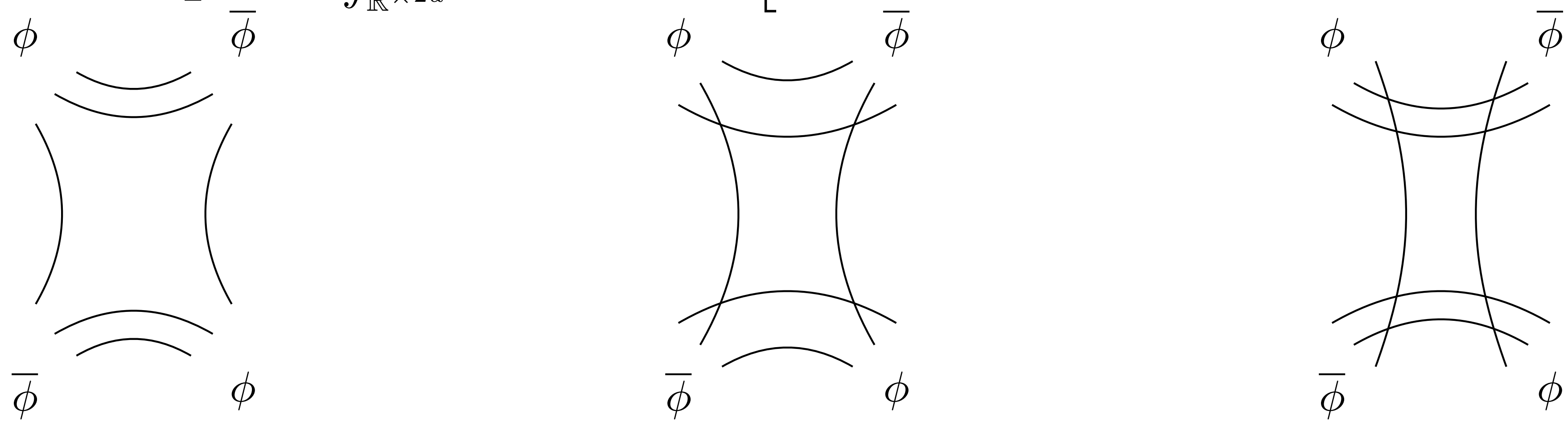

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- divergences in Wetterich equation due to non-compactness of group manifold
- non-locality of interactions prevents from using standard methods, e.g. local potential approx.
- thermodynamic limit must be taken carefully

step 1: compactly configuration space to U(1)^d, with $V = \left(\frac{2\pi}{l}\right)^d$

step 2: determine (non-standard) scaling of coupling constants

step 3: take non-compact limit so to regularise the most divergent contributions to the RG flow

FRG analysis of a quartic abelian rank-d TGFT model

scaling of couplings: $Z_k = \overline{Z}_k l^{\chi} k^{-\chi}, \quad \mu_k = \overline{\mu}_k \overline{Z}_k l^{\chi} k^{2-\chi}, \quad \lambda_k = \overline{\lambda}_k \overline{Z}_k^2 l^{\xi} k^{4-\xi}$

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(regularized) flow equations:

$$\eta_k = \frac{\overline{\lambda}_k l^{\xi} k^{\sigma}}{l^{2\chi} k^{2(2-\chi)} (1+\overline{\mu}_k)^2} \Big\{ (\eta_k - \chi) \Big[\frac{\pi^{\frac{d-1}{2}}}{\Gamma_E\left(\frac{d+1}{2}\right)} \frac{k^{d-1}}{l^{d-1}} + 2(d-1)\frac{k}{l} \Big] + 2\Big[(d-1)\frac{k}{l} + \frac{\pi^{\frac{d-1}{2}}}{\Gamma_E\left(\frac{d-1}{2}\right)} \frac{k^{d-1}}{l^{d-1}} \Big] \Big\} + \chi$$

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non-autonomous, non-homogeneous; matches TGFT on U(1)^d

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most divergent contributions finite for: $\xi = 2\chi + (d-1)$

and redefined anomalous dimension: $\eta_k' = \eta_k - \chi \qquad \eta_k = \frac{1}{\overline{Z}_k}\beta(\overline{Z}_k) = \frac{1}{Z_k}\beta(Z_k) + \chi$

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now can take thermodynamic limit

FRG analysis of a quartic abelian rank-d TGFT model

flow equations for couplings:

autonomous,

still non-homogeneous

$$\eta_k = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma_E\left(\frac{d-1}{2}\right)} \frac{\overline{\lambda}_k}{(1+\overline{\mu}_k)^2}\left[\frac{\eta_k}{d-1}+1\right]$$

$$\beta(\overline{\mu}_k) = \frac{-2d\,\pi^{\frac{d-1}{2}}}{\Gamma_E\left(\frac{d+1}{2}\right)} \frac{\overline{\lambda}_k}{(1+\overline{\mu}_k)^2}\left[\frac{\eta_k}{d+1}+1\right] - \eta_k\overline{\mu}_k - 2\overline{\mu}_k$$

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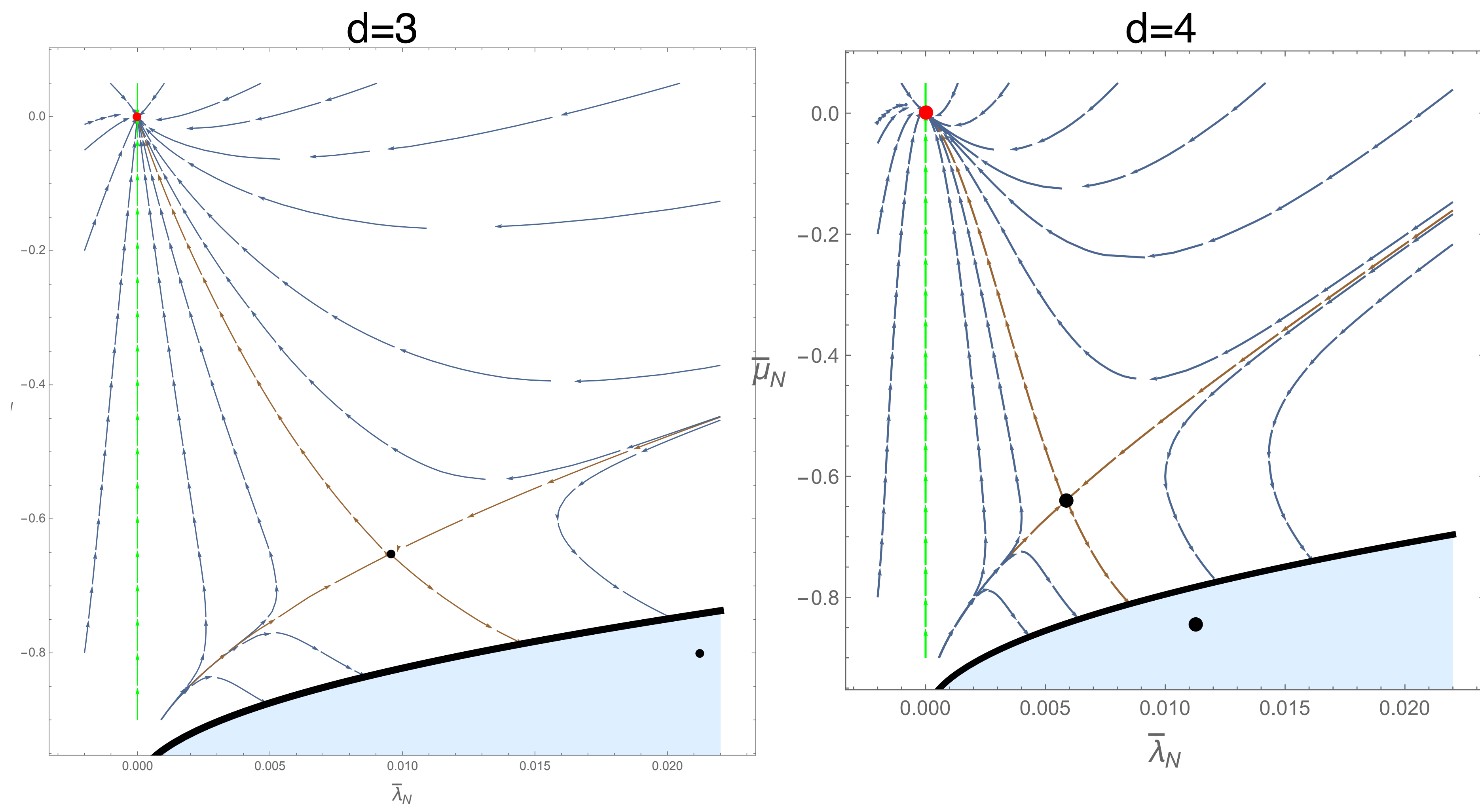

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general features independent of rank-d:

Gaussian-UV FP, Wilson-Fisher-IR FP

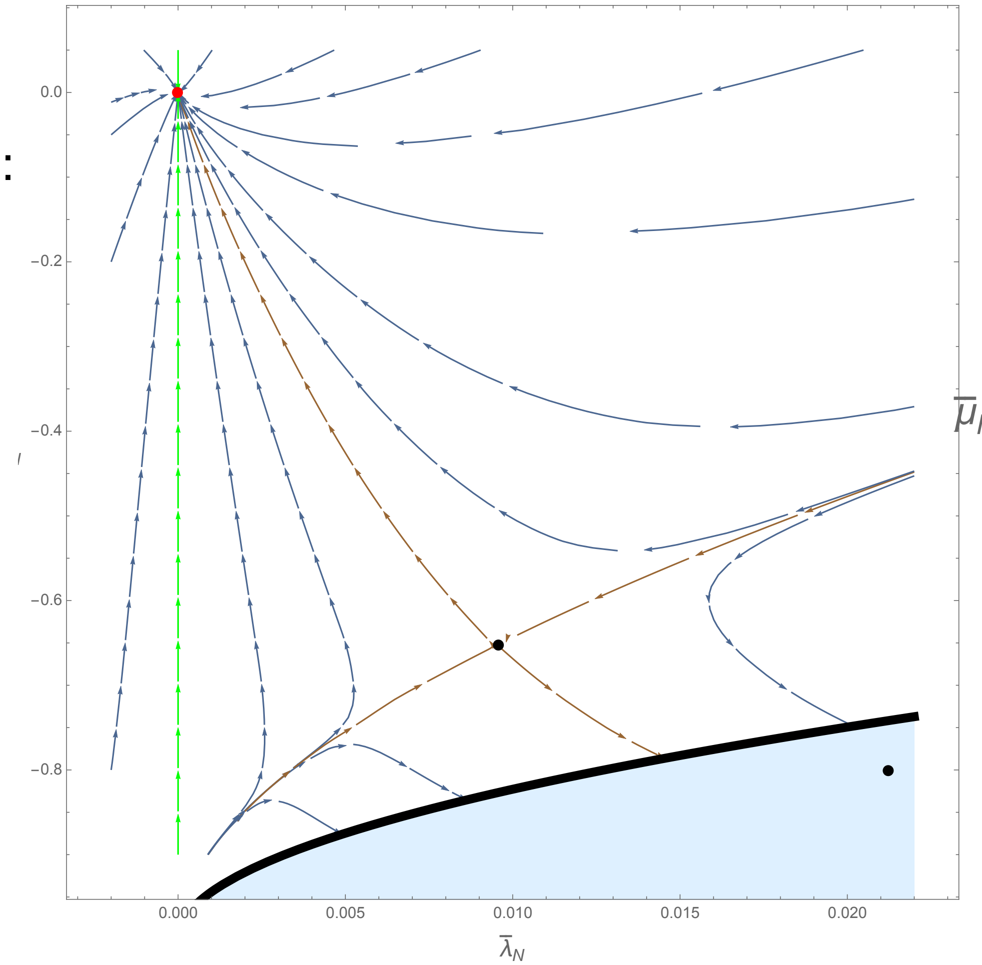
asymptotic freedom

one symmetric phase

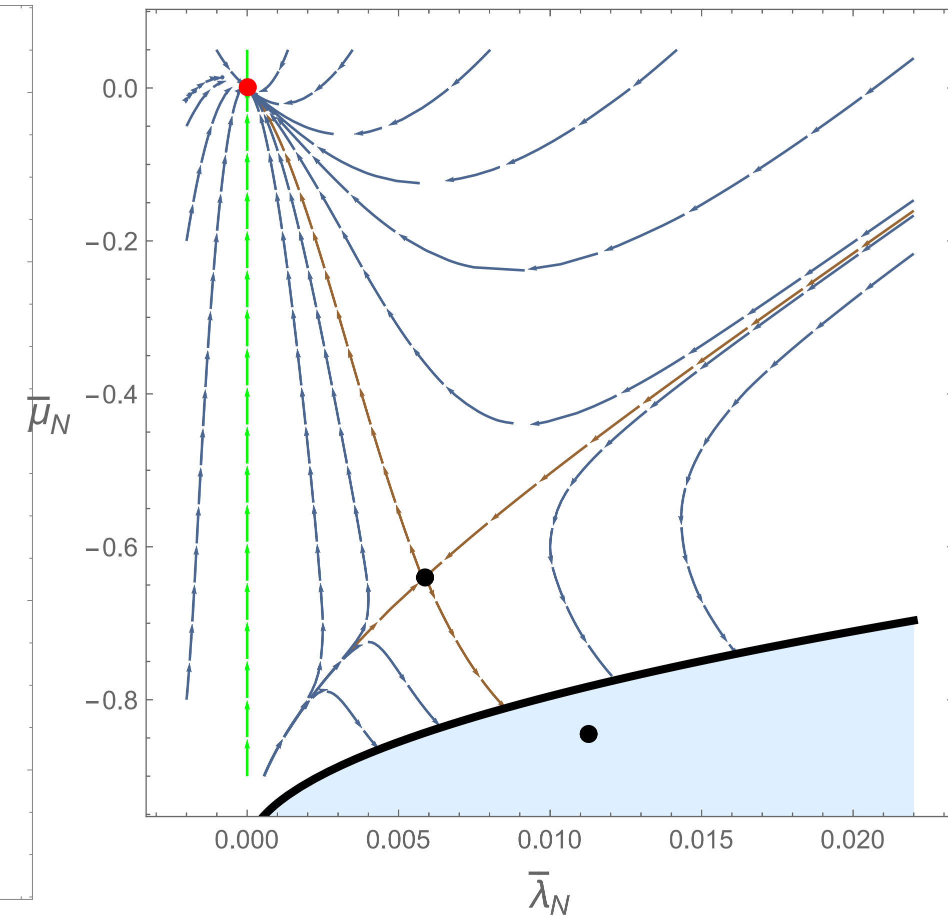
one broken or condensate phase

2nd non-G IR FP at negative coupling

d=3



d=4



FRG analysis of a quartic abelian rank-d TGFT model

similar model with gauge invariance (imposed in both kinetic and interaction terms):

$$\Gamma_k[\varphi,\overline{\varphi}] = \int d\mathbf{p}\,\overline{\varphi}(\mathbf{p})\Big[Z_k\Sigma_s p_s^2 + \mu_k\Big]\varphi(\mathbf{p})\delta(\Sigma p)$$

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similar RG flow equations, different scaling dimensions of couplings:

$$\eta_k' = \frac{d\lambda_k}{(1+\overline{\mu}_k)^2} \frac{\pi^{\frac{}{2}}}{(d-1)^{\frac{3}{2}}} \left\{ \eta_k' \frac{1}{\Gamma_E\left(\frac{d}{2}\right)} + \frac{2}{\Gamma_E\left(\frac{d-2}{2}\right)} \right\}$$

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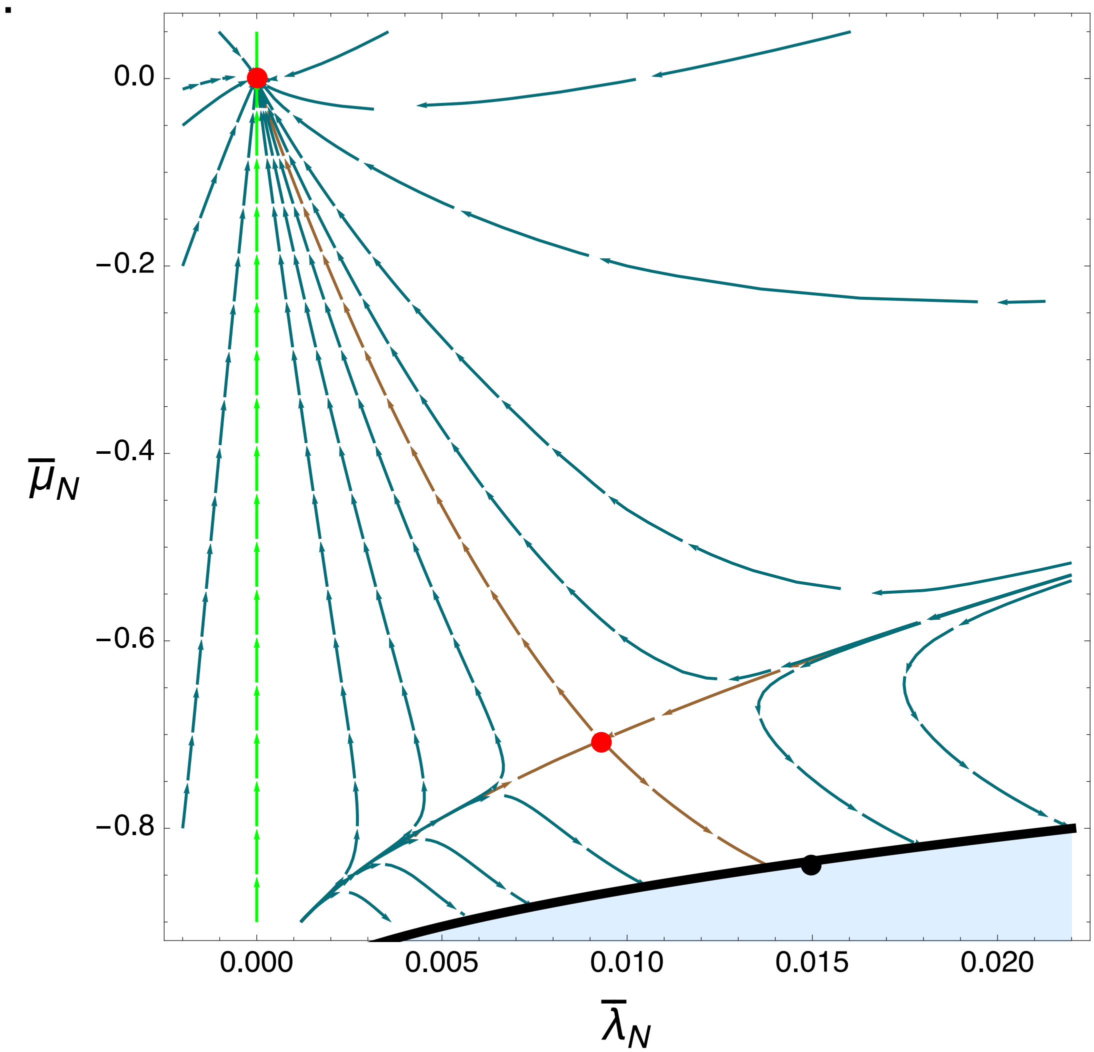

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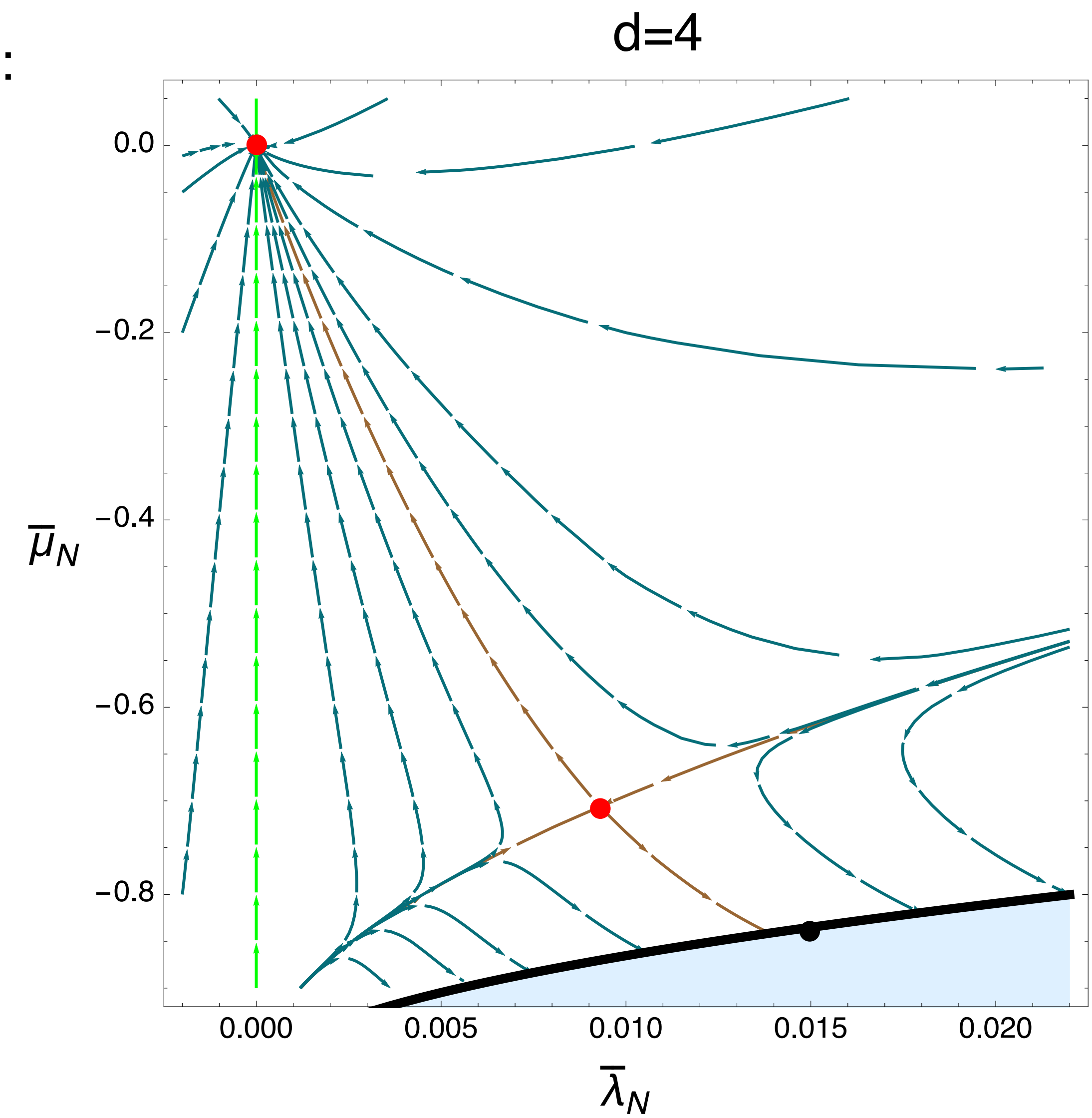

again, general features independent of rank-d:

Gaussian-UV FP (asymptotic freedom), Wilson-Fisher-IR FP

symmetric phase + broken or condensate phase

2nd non-G IR FP at negative coupling

Part IV:
effective continuum physics from GFTs

Quantum spacetime:
the difficult path from microstructure to cosmology

the issue:

identify relevant phase for effective continuum geometry

extract effective continuum dynamics and relate it to GR

is GR a good effective description of LQG/SF/GFT in some approximation (in one continuum phase)?

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also work by:

C. Rovelli, F. Vidotto (spin foam context); E. Alesci, F. Cianfrani (canonical LQG context); …..

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end result of (any) proper construction:

basic variable is “single-patch density” with arguments the geometric data of minisuperspace

cosmology is (non-linear) dynamics for such density and for geometric (global) observables computed from it

From Quantum Gravity to Cosmological hydrodynamics

key strategy:

coarse graining of QG configurations

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one special case:

quantum condensates (BEC)

effective hydrodynamics directly read out of microscopic quantum dynamics (in simplest approximation)

(Quantum) Cosmology from GFT condensates

S. Gielen, DO, L. Sindoni, PRL, arXiv:1303.3576 [gr-qc]; JHEP, arXiv:1311.1238 [gr-qc]

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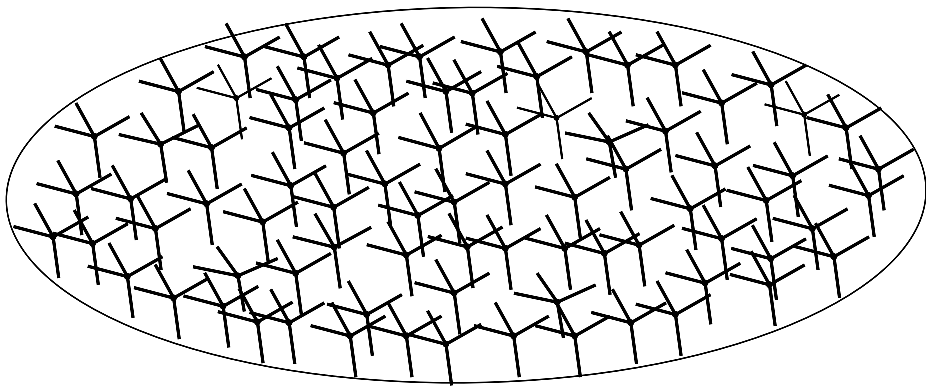
e.g. (simplest): $|\sigma\rangle := \exp(\hat{\sigma})\,|0\rangle$

GFT field coherent state $\hat{\sigma} := \int d^4g\ \sigma(g_I)\hat{\varphi}^\dagger(g_I) \qquad \sigma(g_I k) = \sigma(g_I)$



superposition of
infinitely many SN dofs

described by single collective wave function
(depending on homogeneous anisotropic geometric data)

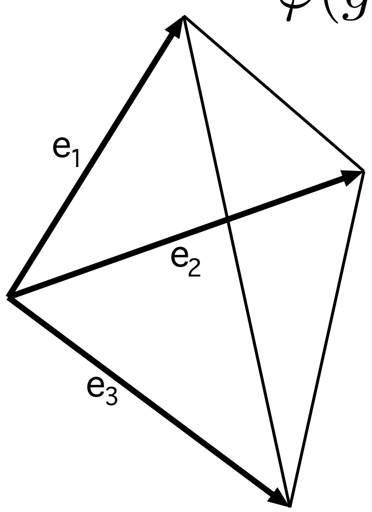
$$\sigma(\mathcal{D}) \qquad \mathcal{D} \simeq \{\text{geometries of tetrahedron}\} \simeq$$
$$\simeq \{\text{continuum spatial geometries at a point}\} \simeq$$
$$\simeq \text{minisuperspace of homogeneous geometries}$$

GFT states and approximate continuum geometries

- work with GFT with simplicial geometric interpretation (A,B=0,1,2,3; i,j,k = 1,2,3)

$$\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \rightarrow \mathbb{C}$$

describes geometric tetrahedron



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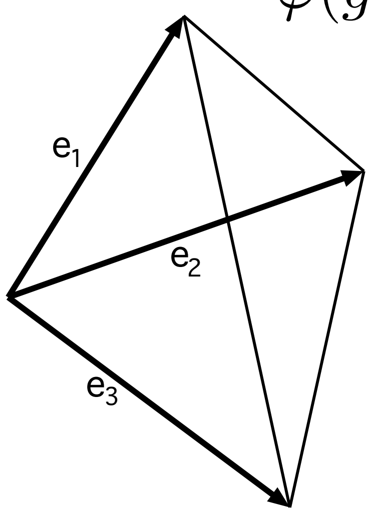

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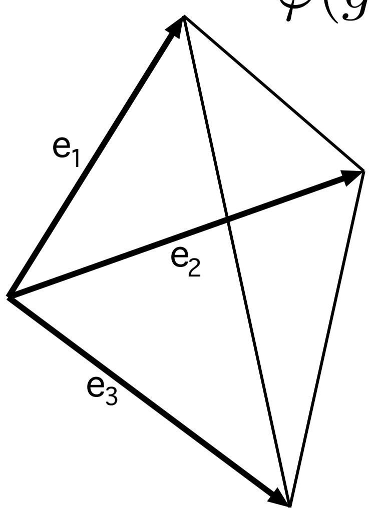

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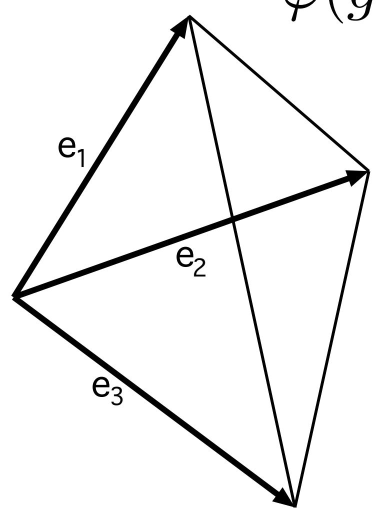

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interpretation: spatial metric coefficients (and conjugate variables) "sampled" at N points

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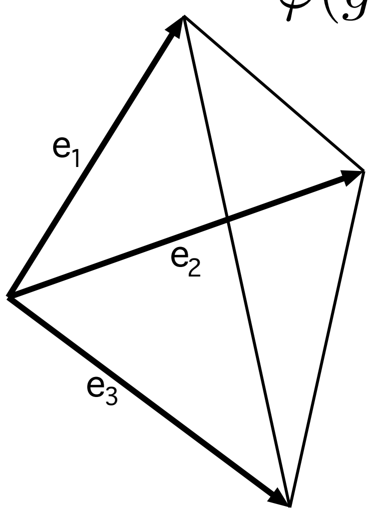

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- classical criterion for homogeneity (for GFT data): $g_{ij(m)} = g_{ij(k)} \quad \forall k, m = 1, \ldots, N$

i.e. all GFT quanta are labelled by the same (gauge invariant) data

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similar constructions in LQG (Alesci, Cianfrani) and LQC (Bojowald, Wilson-Ewing, ...)

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QG (GFT) analogue of Gross-Pitaevskii hydrodynamic equation in BECs
is
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(Quantum) Cosmology from GFT condensates

S. Gielen, DO, L. Sindoni, PRL, arXiv:1303.3576 [gr-qc]; JHEP, arXiv:1311.1238 [gr-qc]

S. Gielen, ’14; G. Calcagni, ’14; L. Sindoni, ’14; S. Gielen, DO, ’14; S. Gielen, ’14; S. Gielen, ’15; DO, L. Sindoni, E. Wilson-Ewing, ‘16

problem 1:
identify quantum states in fundamental theory with continuum spacetime interpretation

Quantum GFT condensates are continuum homogeneous (quantum) spaces

described by single collective wave function
(depending on homogeneous anisotropic geometric data)

problem 2:
extract from fundamental theory an effective macroscopic dynamics for such states

following procedures of standard BEC

QG (GFT) analogue of Gross-Pitaevskii hydrodynamic equation in BECs
is
non-linear and non-local extension of (loop) quantum cosmology equation for collective wave function

cosmology as QG hydrodynamics!!!

Effective cosmological dynamics from GFT

follow closely procedure used in real BECs

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single-particle GFT condensate:

$$|\sigma\rangle := \exp(\hat{\sigma})\,|0\rangle \qquad \hat{\sigma} := \int d^4g\; \sigma(g_I)\hat{\varphi}^\dagger(g_I) \qquad \sigma(g_I k) = \sigma(g_I)$$

superposition of infinitely many SN dofs

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from truncation of SD equations for GFT model

applied to (coherent) GFT condensate state, gives equation for "wave function":

$$\int [dg_i']\, \tilde{\mathcal{K}}(g_i, g_i')\sigma(g_i') + \lambda \frac{\delta \tilde{\mathcal{V}}}{\delta\varphi(g_i)}|_{\varphi\equiv\sigma} = 0$$

basically (up to some approximations), the "classical GFT eqns"

similar equations to M. Bojowald et al., arXiv:1210.8138 [gr-qc]

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no perturbative (spin foam) expansion -

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non-linear and non-local extension of quantum cosmology-like equation for "collective wave function"

QG (GFT) analogue of Gross-Pitaevskii hydrodynamic equation in BECs

Emergent bouncing cosmology from full QG

DO, Sindoni, Wilson-Ewing, '16

- starting from (generalised) EPRL model for 4d Lorentzian QG (simplicial interactions, G=SU(2), dynamics encodes embedding into SL(2,C) ~ simplicity constraints)

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- coupling of free massless scalar field (+ truncation at lowest order ~ slowly varying field)

$\hat{\varphi}(g_v) \to \hat{\varphi}(g_v, \phi)$ $\qquad$ $K_2(g_{v_1}, g_{v_2}, \phi_1, \phi_2) = K_2(g_{v_1}, g_{v_2}, (\phi_1 - \phi_2)^2)$

$$\mathcal{V}_5(g_{v_a}, \phi_a) = \mathcal{V}_5(g_{v_a}) \prod \delta(\phi_a - \phi_1)$$

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- reduction to isotropic condensate configurations (depending on single spin variable j):

$$|\sigma\rangle \sim \exp\left(\int \mathrm{d}g_v \mathrm{d}\phi\, \sigma(g_v, \phi)\hat{\phi}^\dagger(g_v, \phi)\right)|\mathbf{0}\rangle \qquad \sigma(g_v, \phi) \rightarrow \sigma_j(\phi)$$

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- effective condensate hydrodynamics (non-linear quantum cosmology):

$$A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) + w_j \sigma_j(\phi)^4 = 0$$

functions A, B, w define the details of the EPRL model

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$$A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) + w_j \sigma_j(\phi)^4 = 0$$

$$\rho_j'' - \frac{Q_j^2}{\rho_j^3} - m_j^2 \rho_j \approx 0$$

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- two (approximately) conserved quantities (per mode):

$$E_j = A_j |\partial_\phi \sigma_j(\phi)|^2 - B_j |\sigma_j(\phi)|^2 + \frac{2}{5} \mathrm{Re}\left(w_j \sigma_j(\phi)^5\right)$$

$$Q_j = -\frac{i}{2}\left[\bar{\sigma}_j(\phi)\partial_\phi \sigma_j(\phi) - \sigma_j(\phi)\partial_\phi \bar{\sigma}_j(\phi)\right]$$

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universe volume (at fixed "time")

$$V(\phi) = \sum_j V_j \bar{\sigma}_j(\phi)\sigma_j(\phi) = \sum_j V_j \rho_j(\phi)^2 \qquad V_j \sim j^{3/2} \ell_{\mathrm{Pl}}^3$$

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momentum of scalar field (at fixed "time")

$$\pi_\phi = \langle \sigma | \hat{\pi}_\phi(\phi) | \sigma \rangle = \hbar \sum_j Q_j$$

constant of motion ~ continuity equation

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momentum of scalar field (at fixed "time")

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energy density of scalar field (at fixed "time")

$$\rho = \frac{\pi_\phi^2}{2V^2} = \frac{\hbar^2 (\sum_j Q_j)^2}{2(\sum_j V_j \rho_j^2)^2}$$

Emergent bouncing cosmology from full QG

DO, Sindoni, Wilson-Ewing, '16

effective dynamics for volume - generalised Friedmann equations:

$$\left(\frac{V'}{3V}\right)^2 = \left(\frac{2\sum_j V_j\,\rho_j\sqrt{E_j - \frac{Q_j^2}{\rho_j^2} + m_j^2\rho_j^2}}{3\sum_j V_j\rho_j^2}\right)^2$$

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$$\exists j \; / \; \rho_j(\phi) \neq 0 \; \forall \phi$$

$$V = \sum_j V_j \rho_j^2$$

remains positive at all times

generic quantum bounce!

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approx. classical Friedmann eqns if $m_j^2 \approx 3G_N$

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$$\left(\frac{V'}{3V}\right)^2 = \frac{4\pi G}{3}\left(1 - \frac{\rho}{\rho_c}\right) + \frac{V_{j_o} E_{j_o}}{9V}$$

$$\rho_c = 6\pi G \hbar^2 / V_{j_o}^2 \sim (6\pi / j_o^3)\rho_{\text{Pl}}$$

LQC-like modified dynamics!

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LQC-like modified dynamics!

can show that

1) generic solutions approximate such simple condensates at late times — Gielen, '16 — De Cesare, Pithis, Sakellariadou, '16

2) GFT interactions can make primordial acceleration last enough e-folds to avoid need for inflation

Thank you for your attention!