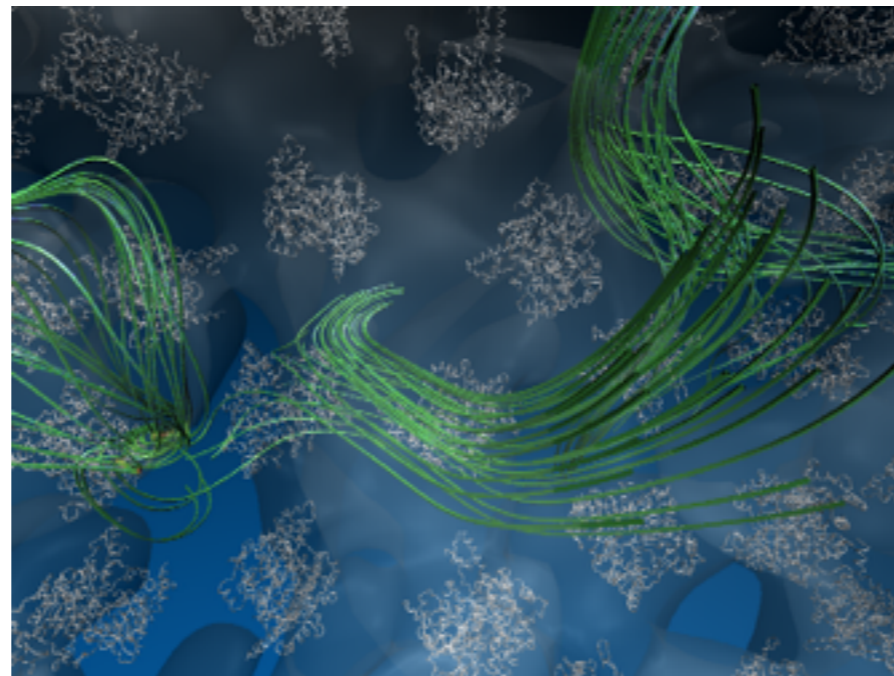


GRC.water.2016

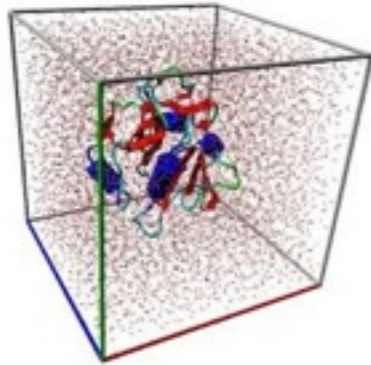
Proteins in cell

modelling cell-like environments...



F.Sterpone
LBT/UPR9080
IBPC, CNRS, Paris

Proteins in silico



years



70's

80's

90's

00's

Parallel
SuperComputing

DE Shaw
Anton Computer

GPU

PME / RESPA

McCammon, Gelin, Karplus
Nature (1977)

Levitt & Sharon PNAS (1988)

BPTI in vacuum

BPTI in water

μ s-ms

9 ps

210 ps

10...100 ns

MD mainly confined to a single protein in solution or in membrane!

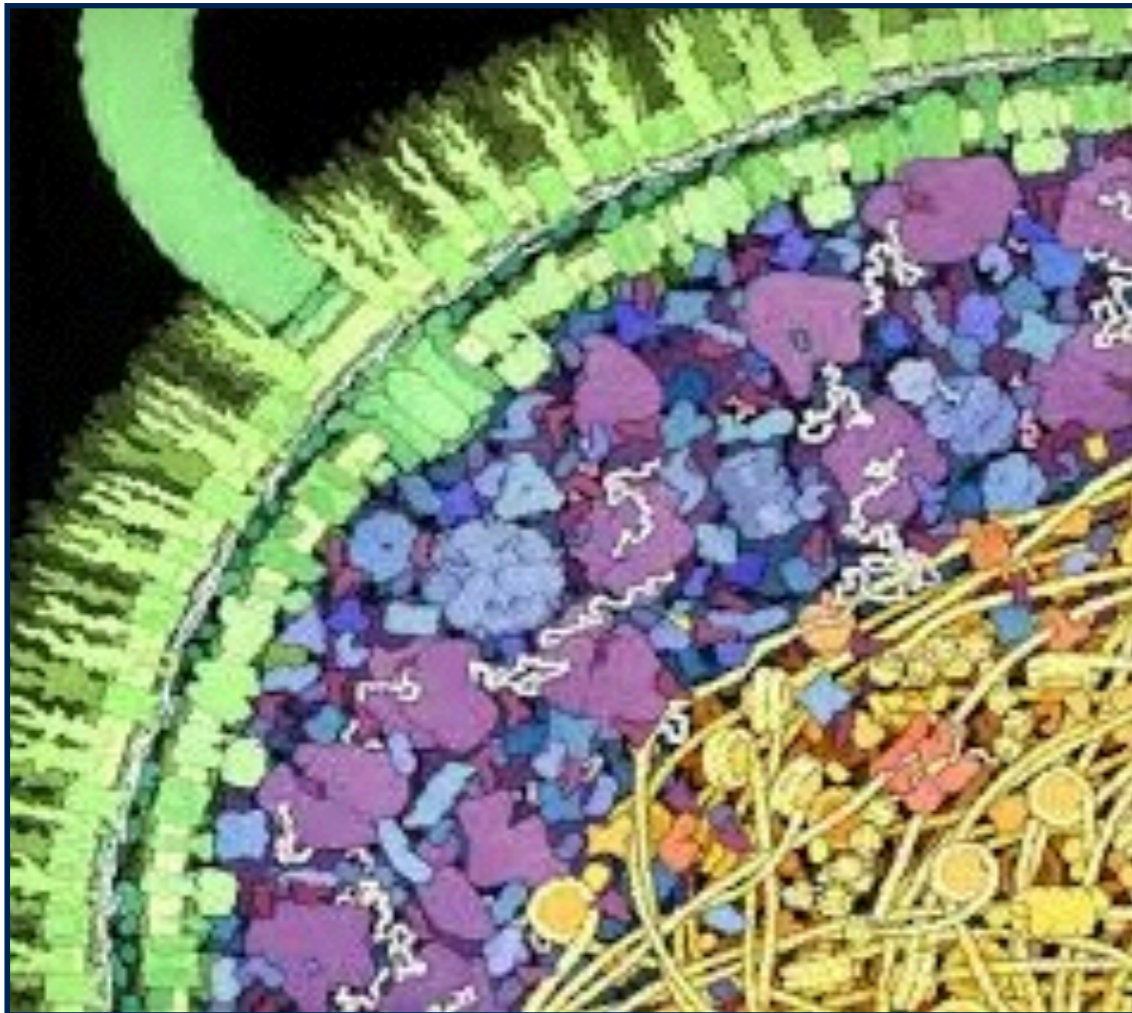
Proteins in the Cell

Ellis, Trends in Biochemical Sci (2001)

... some keywords

heterogenous distributions
trafficking
functional cascades
network of interactions

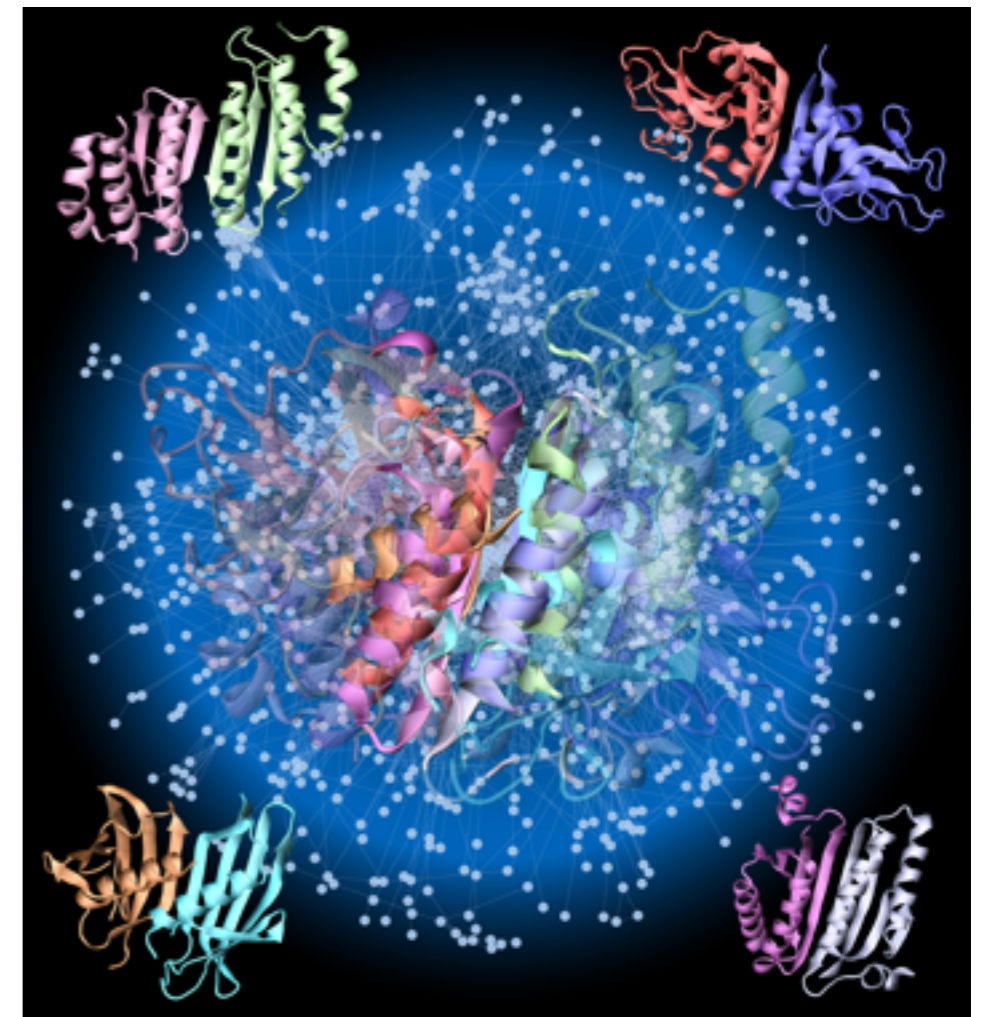
model of E. coli



Goodsell, Scripps, California, US

... composition

water 70 % in mass
macromolecules ~30 %
ions < 1%

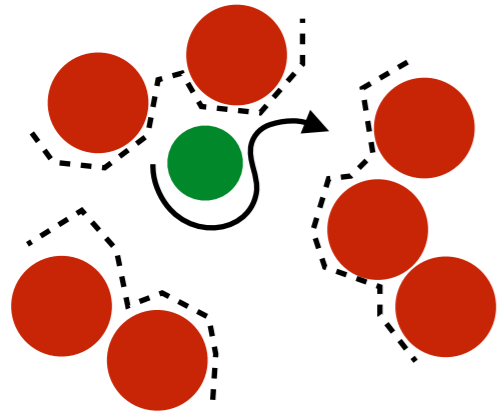


Protein diffusion in cells

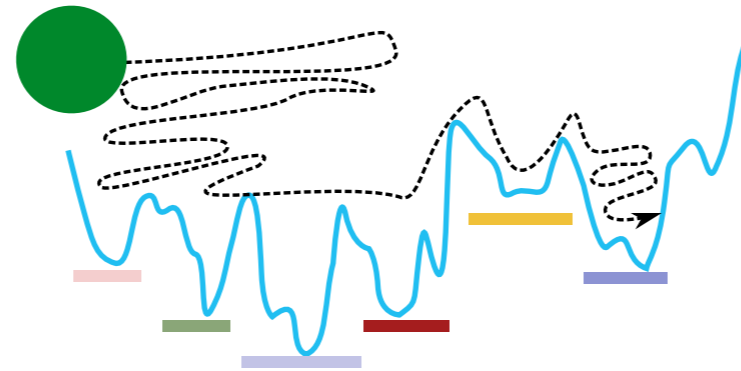
Dix, Verkman, Ann Rev Biophys (2008)

crowding effects on mobility

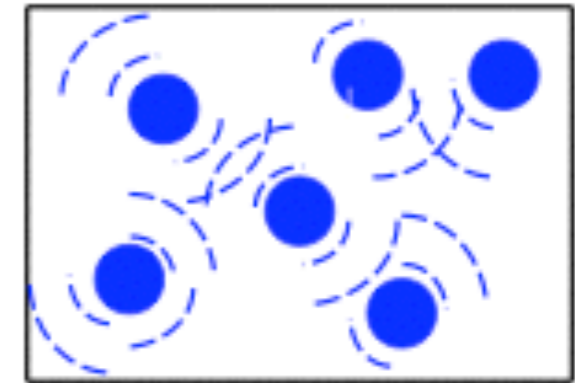
geometric disorder



distribution of times for jumping

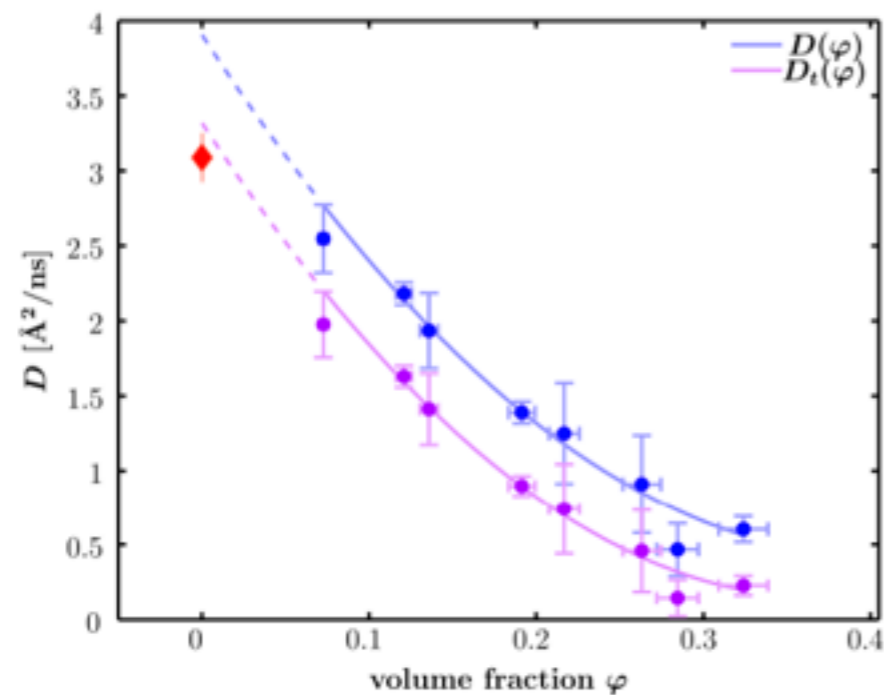


solvent mediated interactions

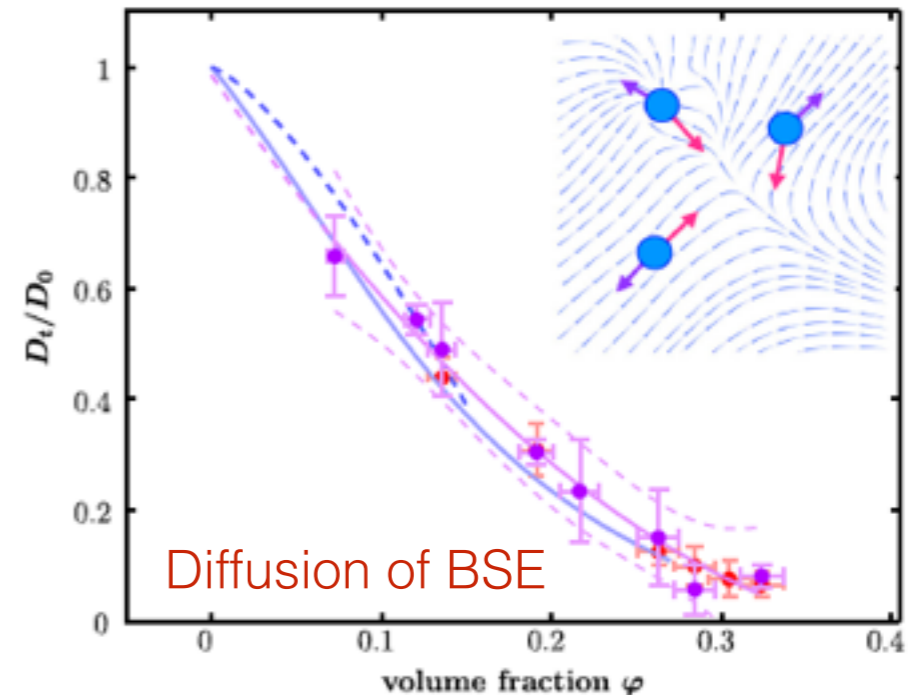


e.g. Neutron Scattering timescale nanoseconds

slowdown is due to solvent-mediated interactions



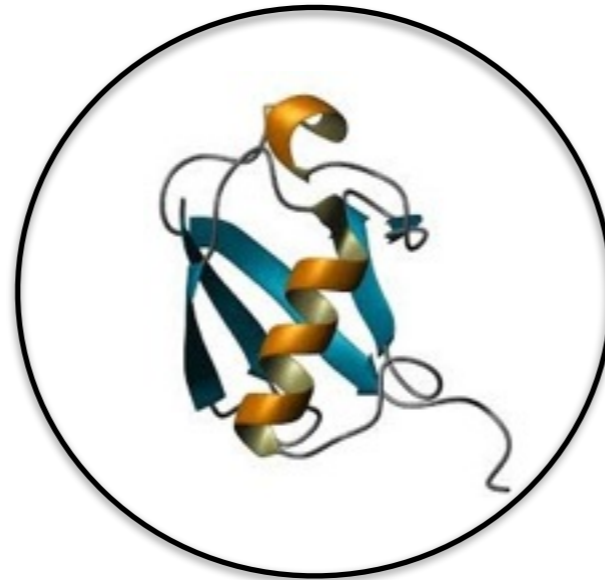
slowdown by crowding



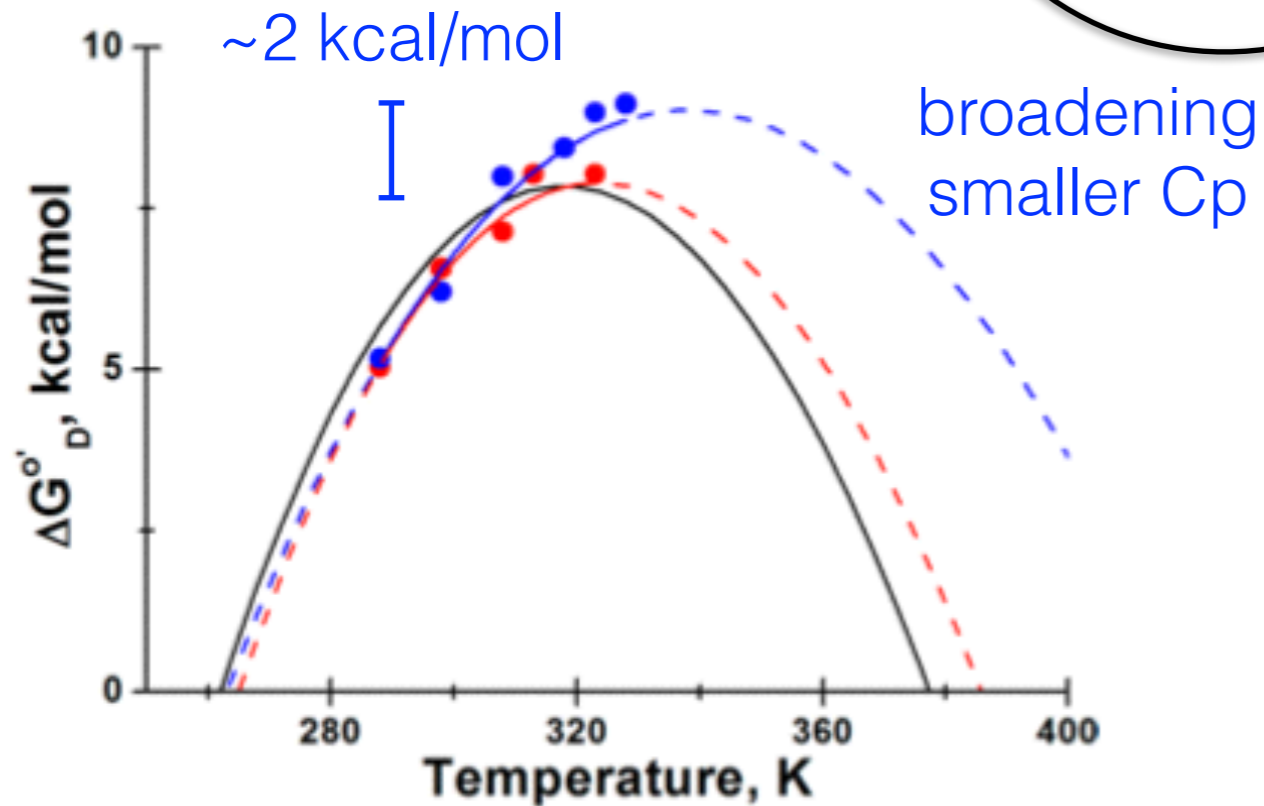
Roosen-Runge et al. PNAS (2011)



Stability of ubiquitin

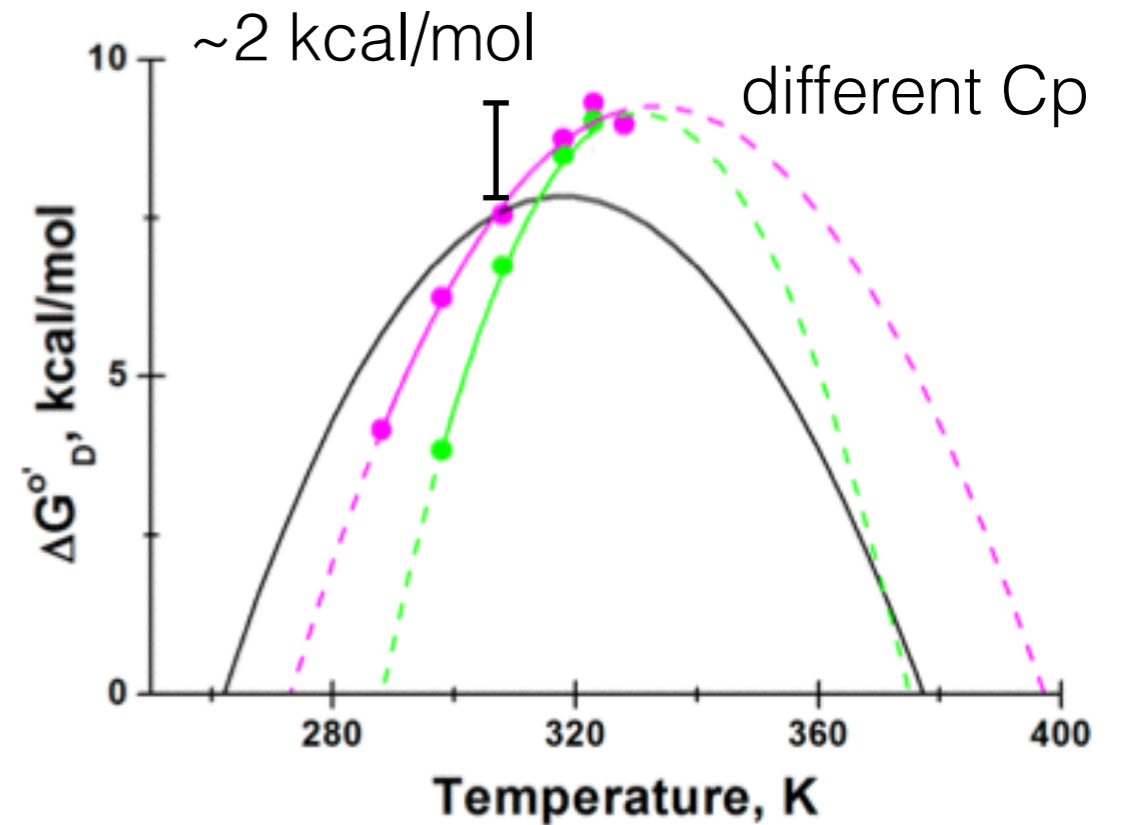


- Ficoll 100 g/L
- PVP 100 g/L



Crowding by polymers

- lysozyme 100 g/L
- BSA 100 g/L



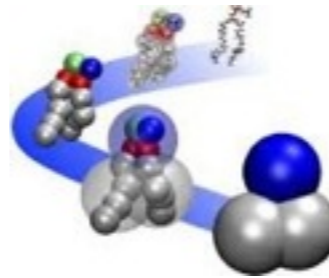
Crowding by proteins

Toward simulations of cell-like environments

All atom approach: too challenging

1000 proteins in solution at $\phi \sim 30\%$
 $\sim 90,000,000$ particles

Implicit solvent coarse-grained: good



but solvent mediated interactions ?

Brownian Dynamics + Hydrodynamics Interactions (Oseen/Rotne-Prager-Yamakawa)

poor scalability with size!

diffusion/folding

Ermak&McCammon JCP(1978)

Frembgen-Kesner&Elcock JCTC (2009)

Mikhailov&Kapral PNAS (2015)

Lipska et al JCP (2016)

molecular motor

Goldtvik et al JPC (2016)

aggregation

Ando&Skolnick BJ (2013)

MD + Multiple Particle Collision

mobility/catalysis

Malevanets&Kapral, JCP(1999)

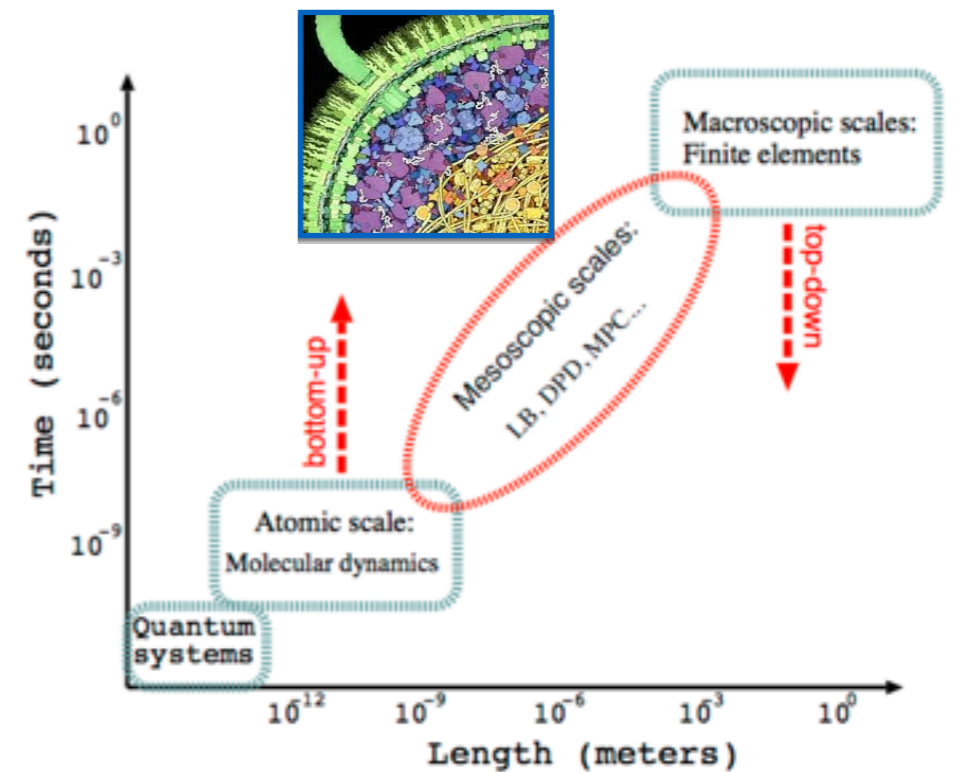
Schfield et al, JCP(2012)

MD + Lattice Boltzmann accounting kinetics of solvent

colloid/polymers

Ahlich&Dunweg, JCP(1999)

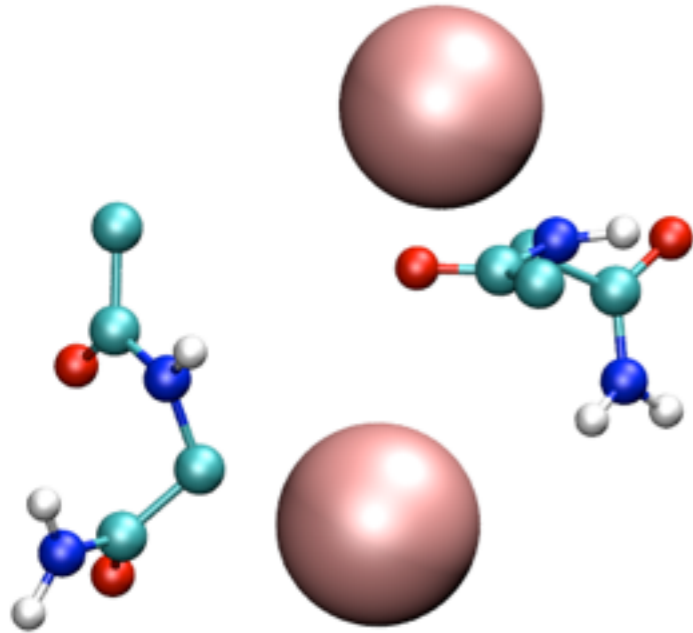
Limbach, CPC(2006)



← excellent scalability with size!

The coarse-grained model OPEP

Chebaro et al, JPCB(2102), FS et al. JNCS(2014), FS et al, ChemSocRev(2014)



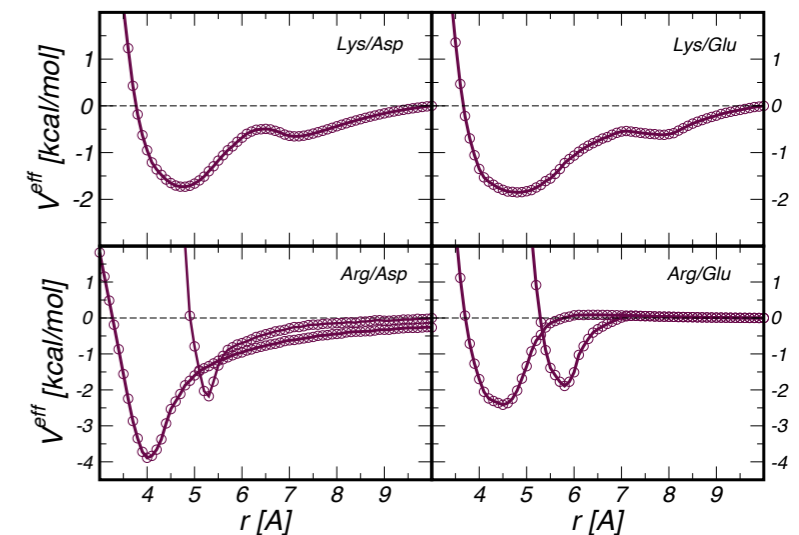
Optimized Potential for Efficient protein structure Prediction

back-bone atomistic resolution
Side-chain I bead

water and electrostatic free

cooperative term for HBs
ad hoc potential for ion-pairs

Ion-Pair Potentials



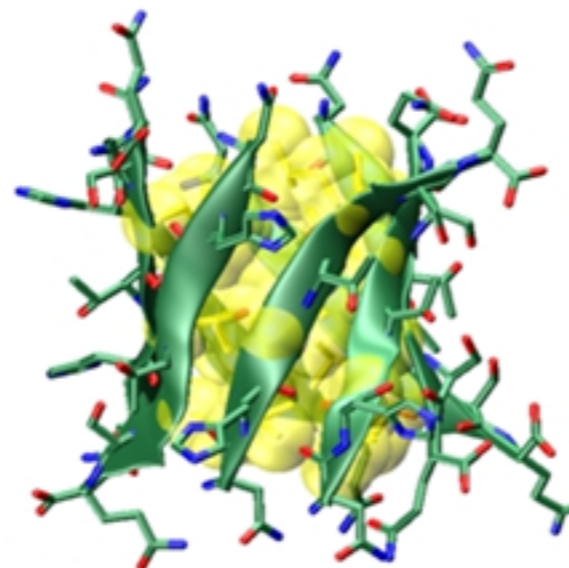
Techniques / Applications

Molecular Dynamics, Monte Carlo

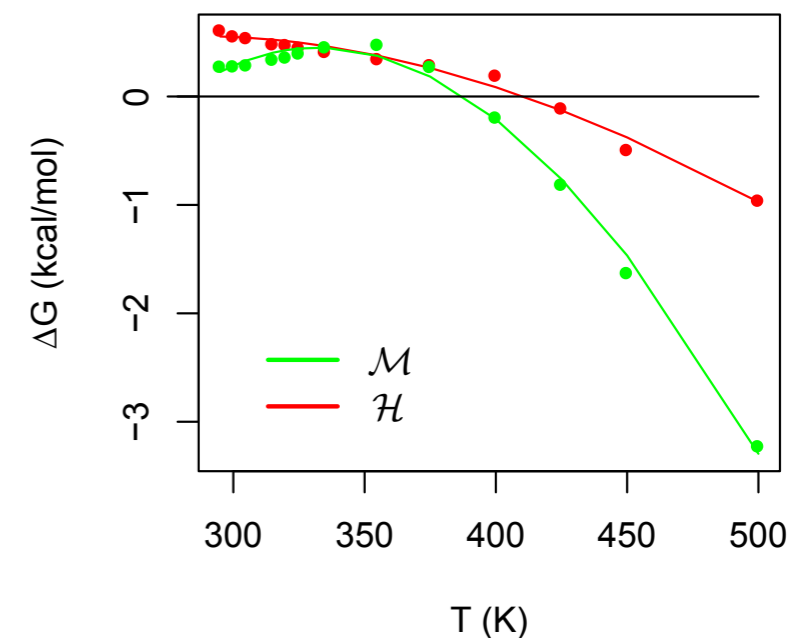
REMD, Simulated Tempering,
Metadynamics

Folding/Unfolding,
Amyloid aggregation

Amyloid oligomer structures



Different stability for mesophilic/thermophilic proteins



A solute in fluid

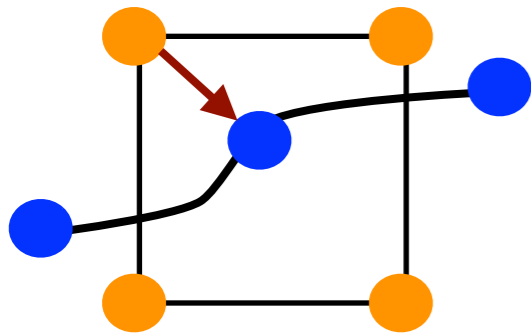
coupling MD and LB

FS et al Chem Soc Rev (2014), FS et al. JCTC (2013), FS et al. JCTC (2015), MK,PD,FS JNCS (2015)

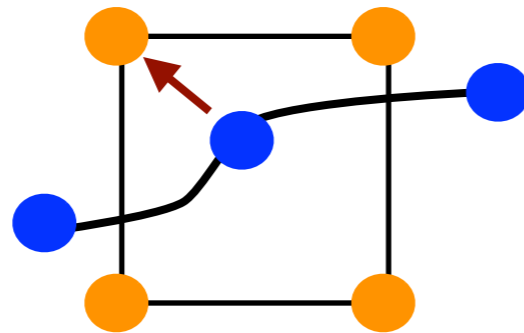
fluid → *molecule*

$$\mathbf{F}_{drag,i} = -m\gamma(\mathbf{v}_i - \mathbf{u}_i)$$

interpolation



extrapolation on grid

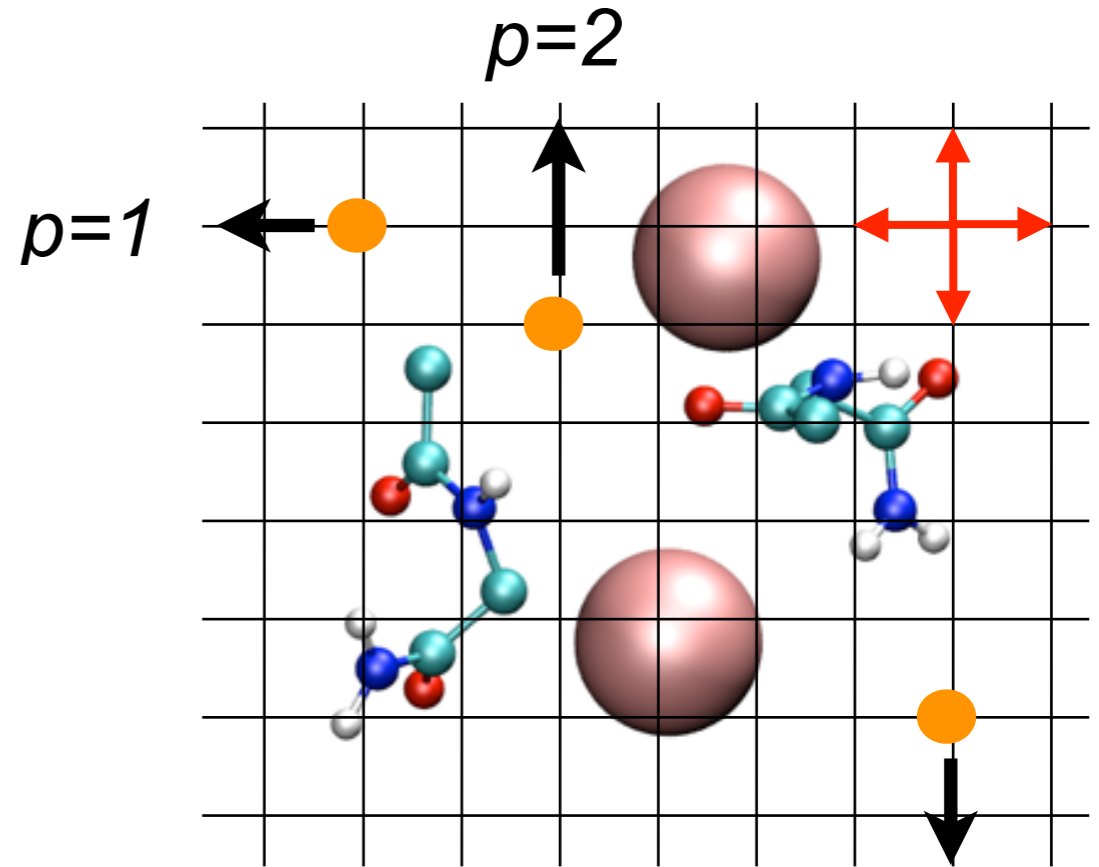


● Molecular Particle *i*

molecule → *fluid*

$$S_p(\mathbf{x}, t) = \omega_p \sum_{\text{mesh } i} [\mathbf{F}_{drag,i} + \mathbf{F}_{r,i}] \cdot \mathbf{c}_p / c_s^2$$

mesh *random force*



● Lattice Boltzmann Particle

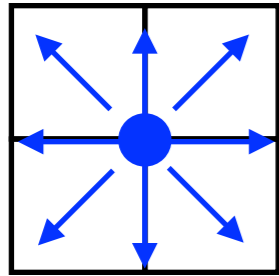
← Discrete velocity (c_p)

$f_p(\mathbf{x}, t)$ Velocity distribution along p th direction

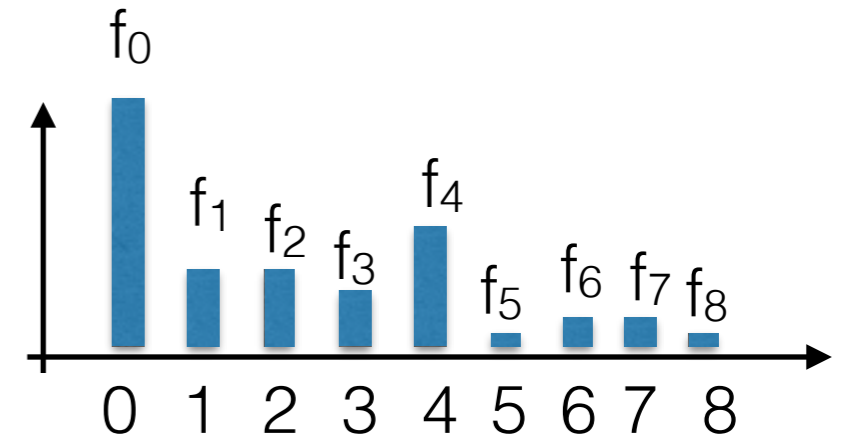
$$\mathbf{u}(\mathbf{x}, t) = 1/\rho \sum_p f_p \mathbf{c}_p$$

A solute in fluid

coupling MD and LB



ie 2 dimensional lattice D2Q9



streaming

BGK collision

$$f_p(x + c_p \Delta t, t + \Delta t) = f_p(x, t) - \omega \Delta t (f_p - f_p^{eq})(x, t) + S_p \Delta t$$

Solute feedback
+
noise

➔ Relaxation Time

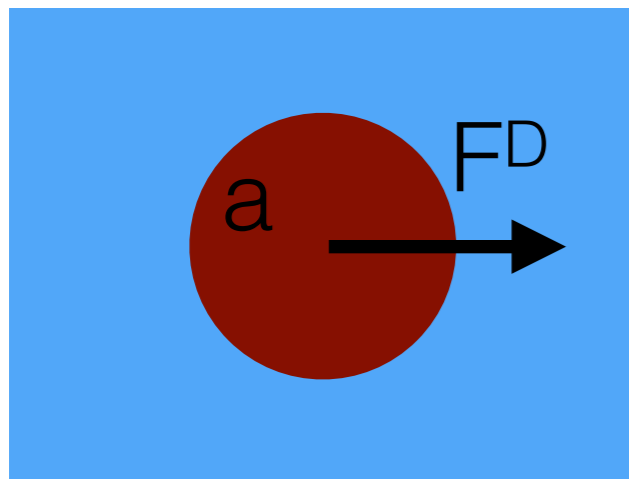
$$\omega \sim 1/\nu \quad \nu = \text{Fluid Kinematic Viscosity}$$

Water 0.1 - 0.166 [$\text{\AA}^2 / \text{fs}$]

➔ Solute/Fluid coupling

microscopic friction γ used as independent parameter

Hynes, Kapral, Weinber, JCP(1978)



$$F^D = -\gamma(v - u)$$

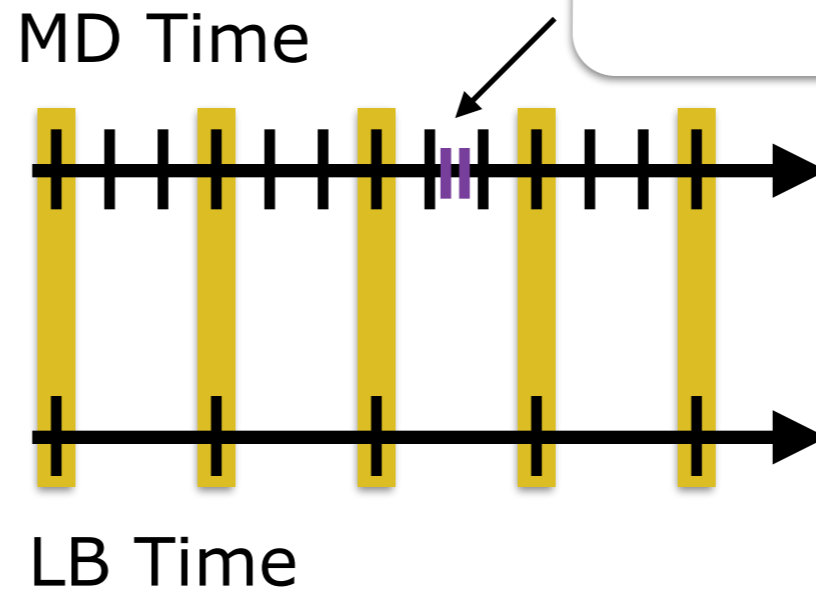
$$\gamma = 6\pi\eta a$$

Macroscopic Stokes Law relates γ and η

A solute in fluid

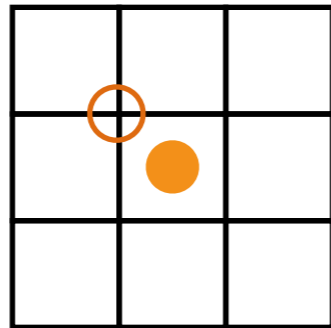
Multi-scaling

LB Time and MD Time



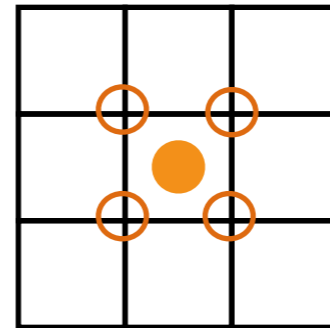
Grid Interpolation

Nearest Point



cheap

Multiple support



expensive



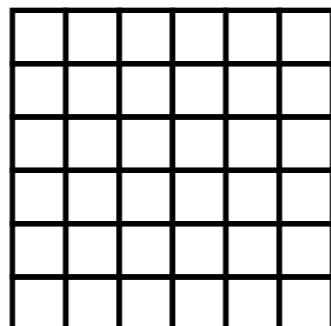
lattice



particle

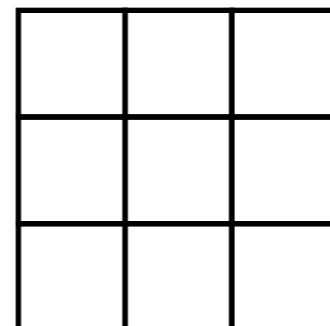
Grid Resolution

High Res



expensive

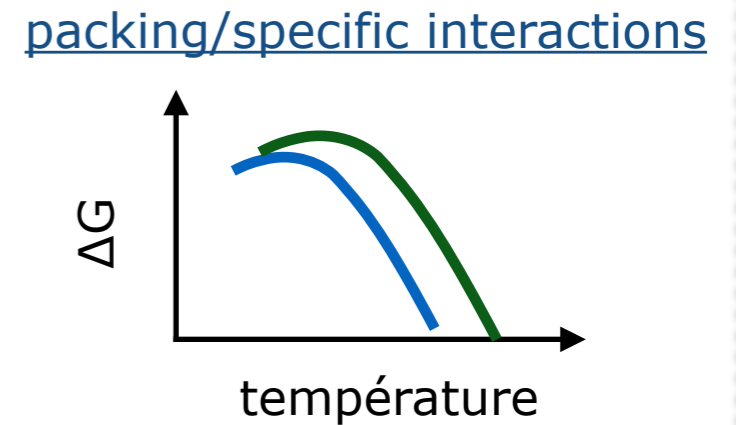
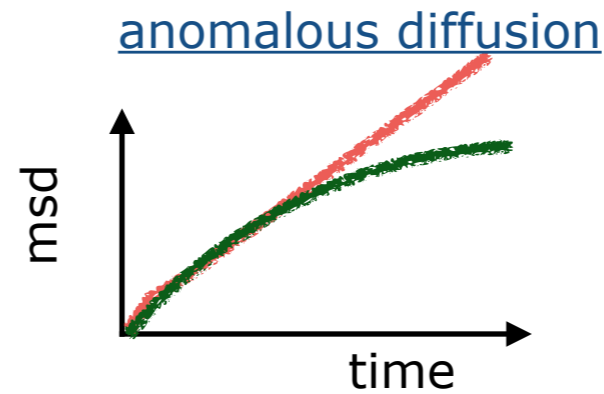
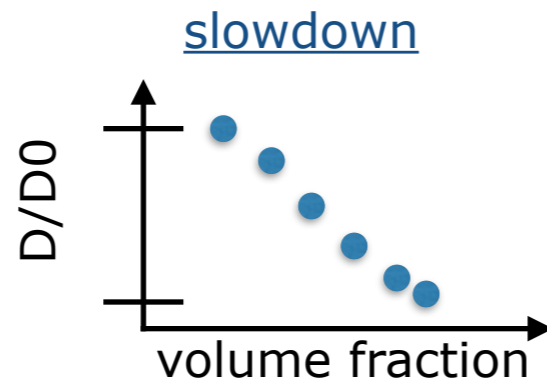
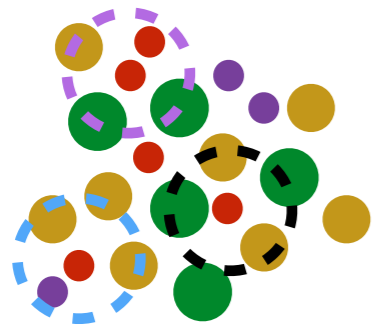
Low Res



cheap

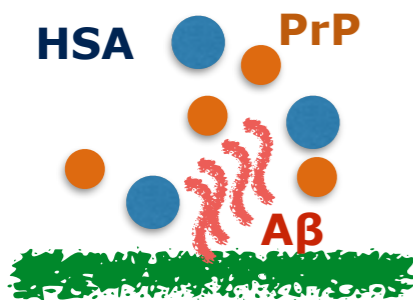
CPU Time $\sim N_g^3$

Diffusion/Stability



P. Derreumaux (LBT), experiments G. Pielak (Univ North Caroline)

Aggregation Amyloid



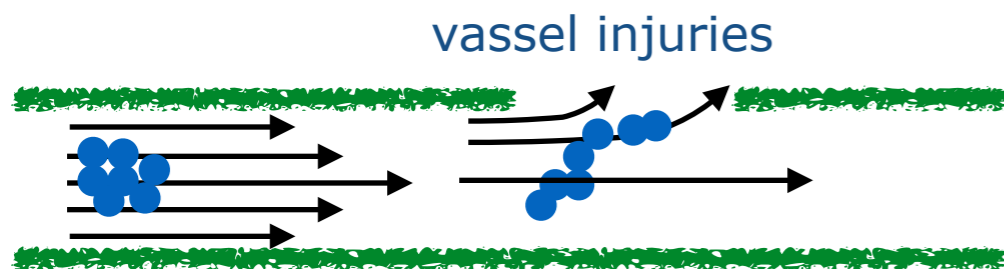
Problems
crowding and aggregation, co-receptor, membrane

CG model for lipids

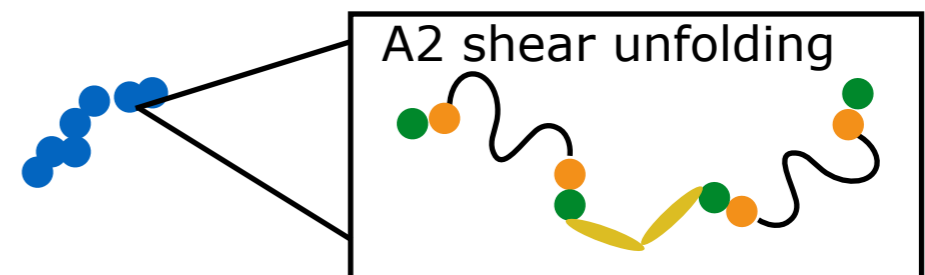


P. Derreumaux (LBT), experiments U. Rezai (INSERM)

Shear-Flow effect



VWF (multimeric protein) senses shear-flow (vessel injuries)



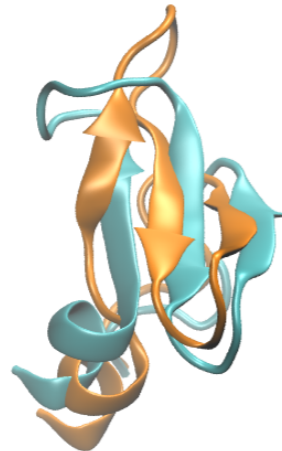
Muphy/OPEP: Protein Stability

FS et al. JCTC (2015)

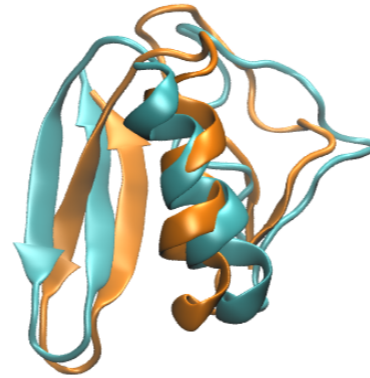
1CLB



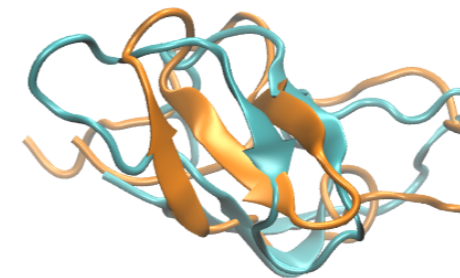
1E0L



1FCL



1SHG

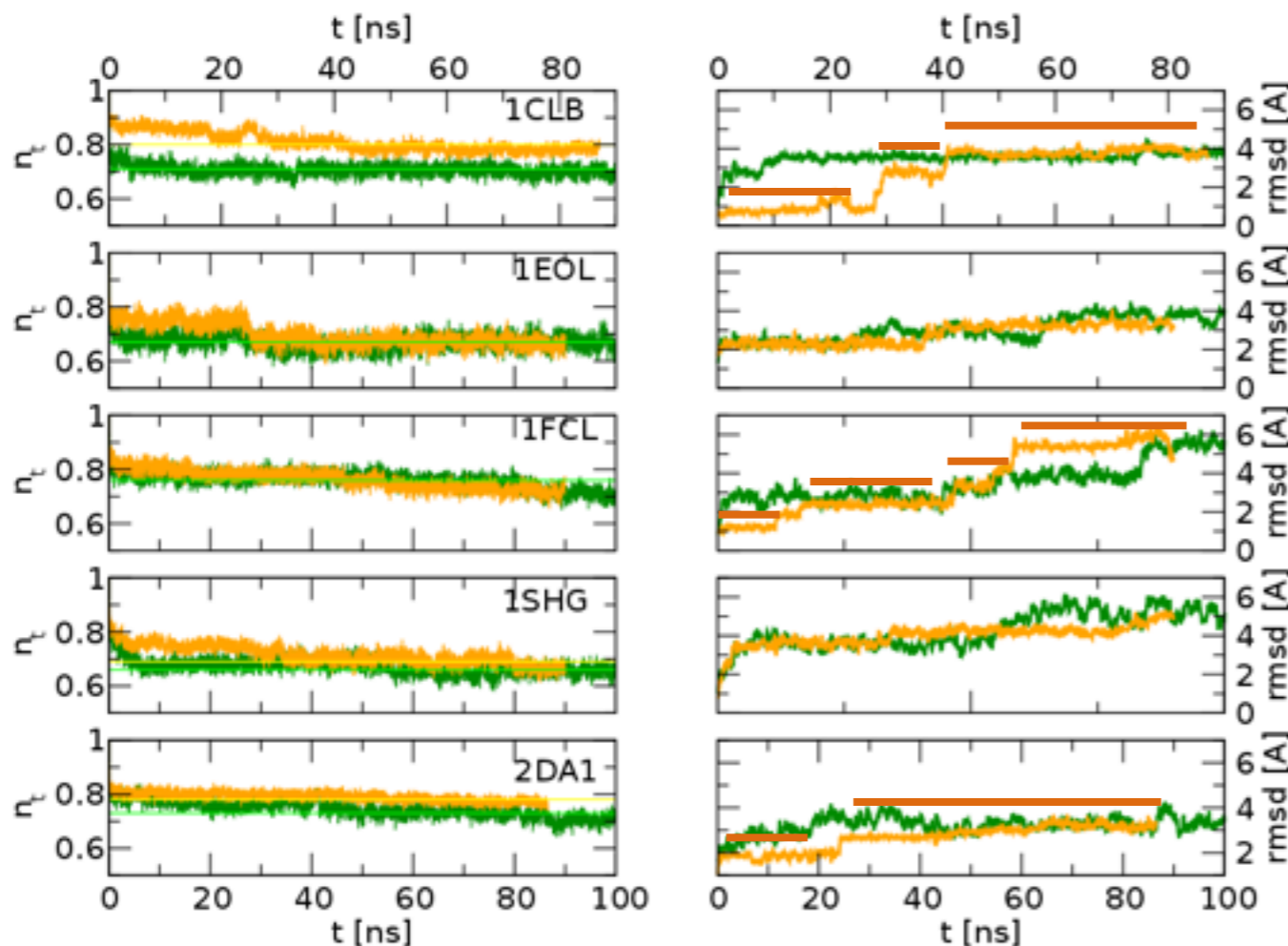


2DA1



Hydrodynamics

Langevin



RMSD [A]

1CLB 3.7

1E0L 3.4

1FCL 5.5

1SHG 5.3

2DA1 3.3

Same Stability in Langevin Dynamics and with Hydrodynamics

when RMSD is high
arrangement of secondary structures,
only minor instabilities

Hydrodynamics:: longer living states



Muphy/OPEP: Protein diffusion

Protein mobility in "cell" Massive simulation

FS et al ChemSocRev (2014)

Simulations

17576 Rat I yeast proteins (*S. Pombe*)
Rat I = 4013 CG particles
17756 GPU Titan Supercomputer (Oak Ridge)
for **Gordon Bell Prize 2013 (SuperComputing)**

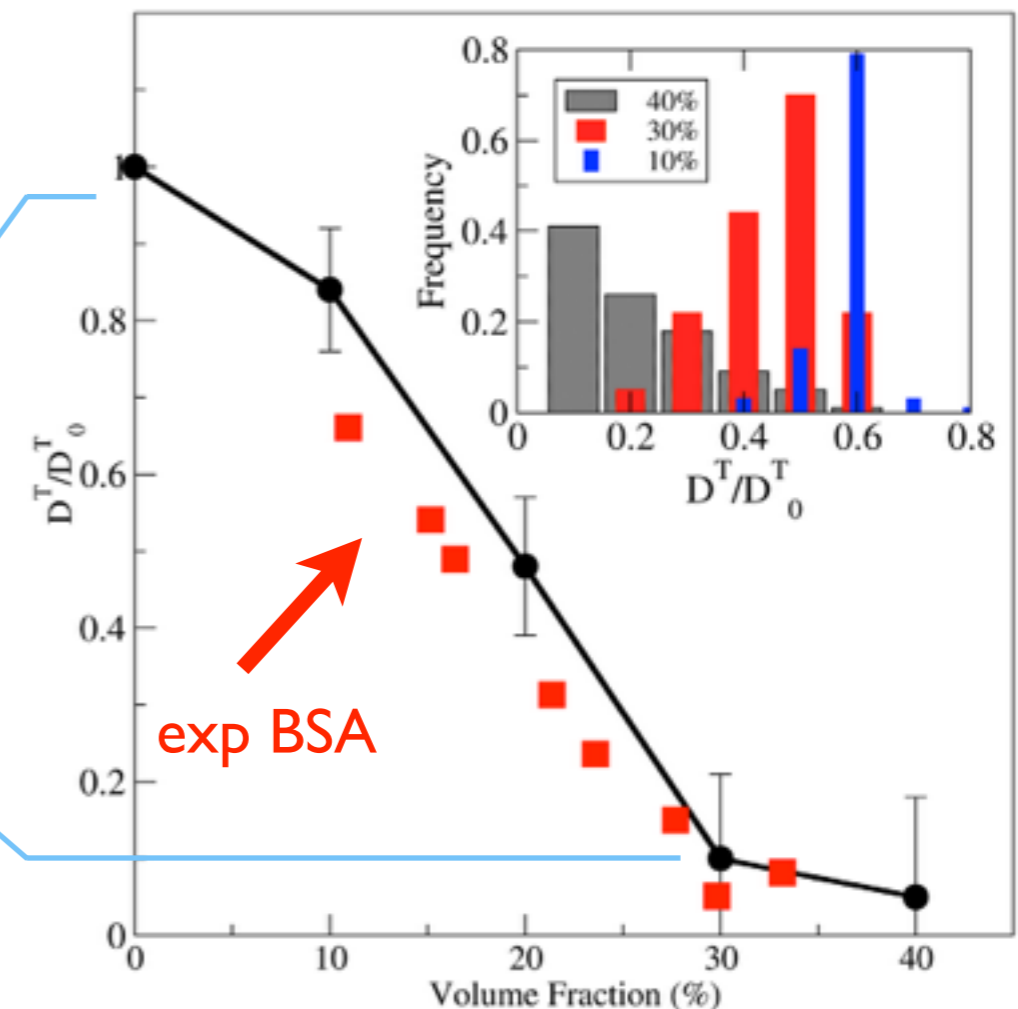


Simulation time ~30 ns

Authors

Bernaschi / Bisson / Fatica / Melchionna

Crowding $\Phi=30\%$
Diffusion slowdown $D/D_0 \sim 1/10$



*Roosen-Runge et al. PNAS(2011)

Muphy/OPEP: Protein diffusion

Protein mobility in "cell" Elastic Network for Proteins

Chiricotto, Derreumaux, **FS**, Melchionna, Philos. Tran. (2016)

Simulations

70 Cl2 protein

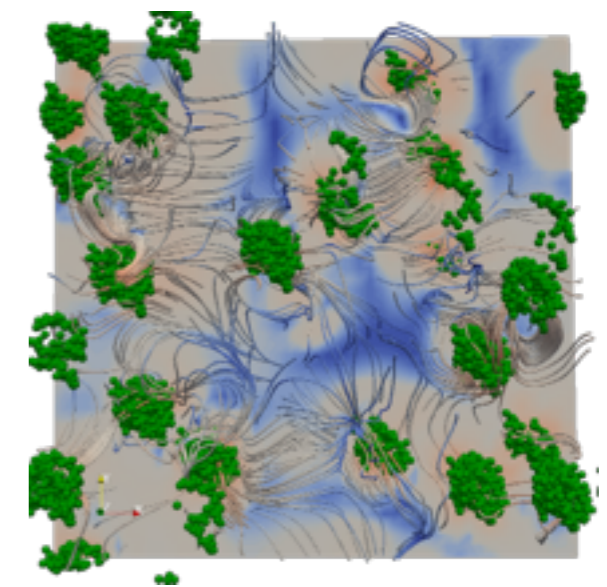
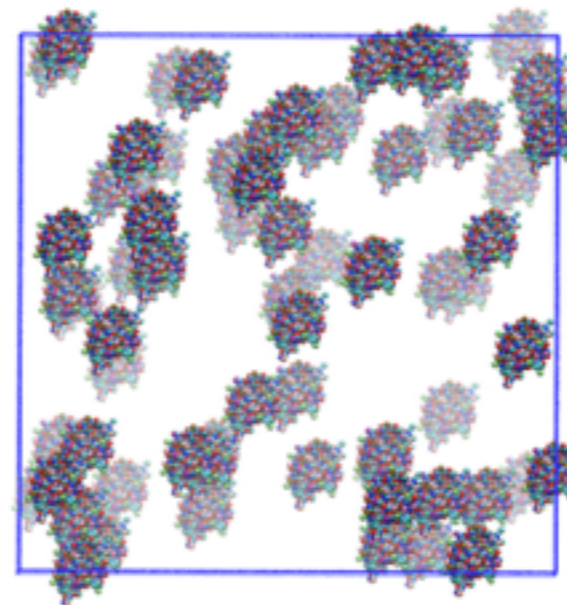
Cl2= 331 CG particles

Elastic Network (EN) or protein

Box L=135, 145, 160, 180, 250 Å.

Simulation time 0.5 μ s

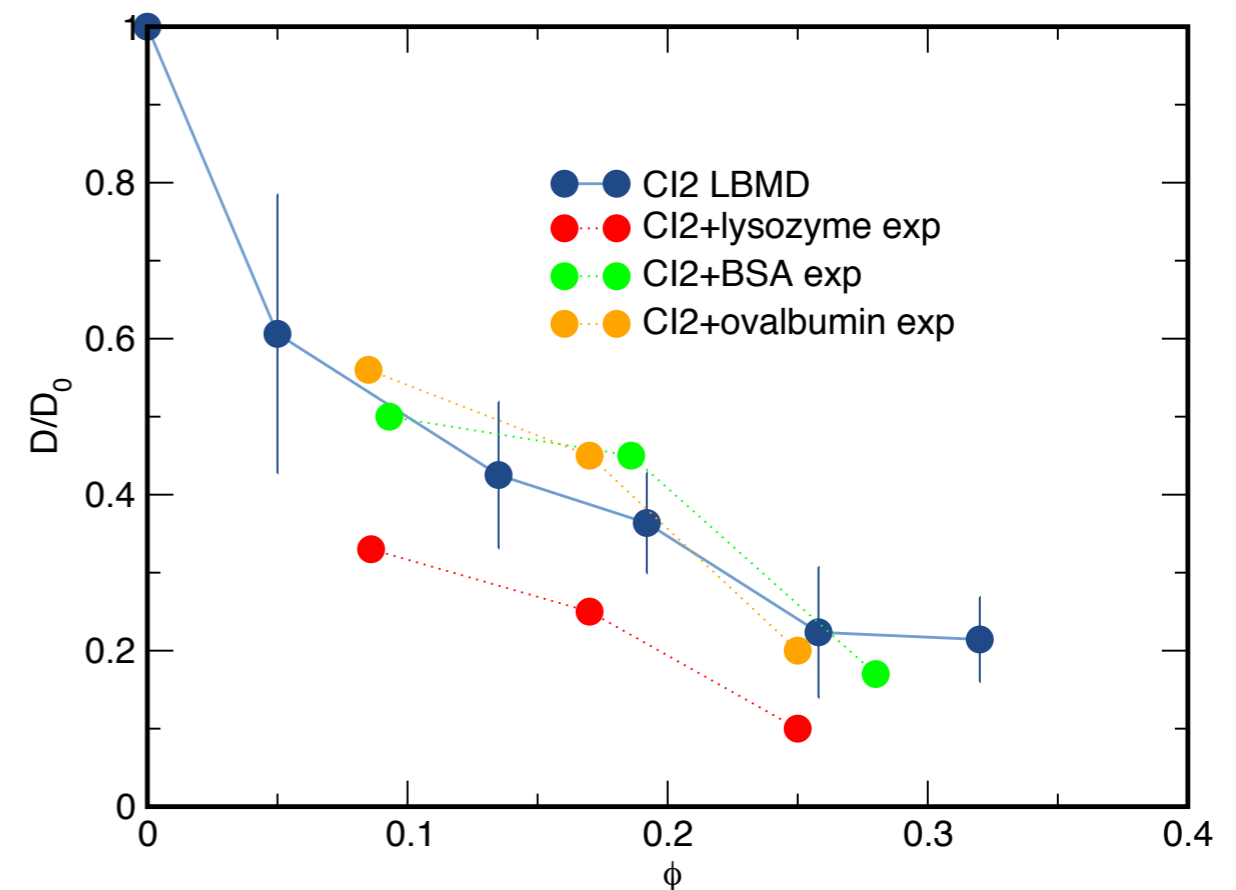
$D_0 \sim 14 \text{ \AA}^2/\text{ns}$ (*exp 15 $\text{\AA}^2/\text{ns}$)



Fluid Streamlines

EN cheap&good
possibility to modulate flexibility
time scale up to μ s

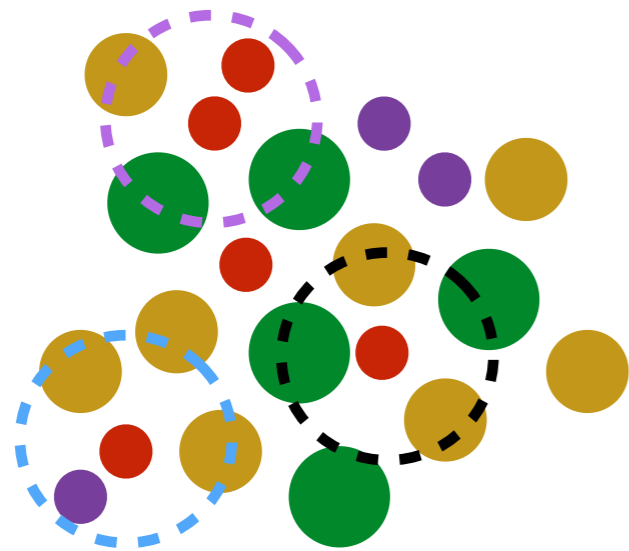
€ affordable \$
64 < cores < 512



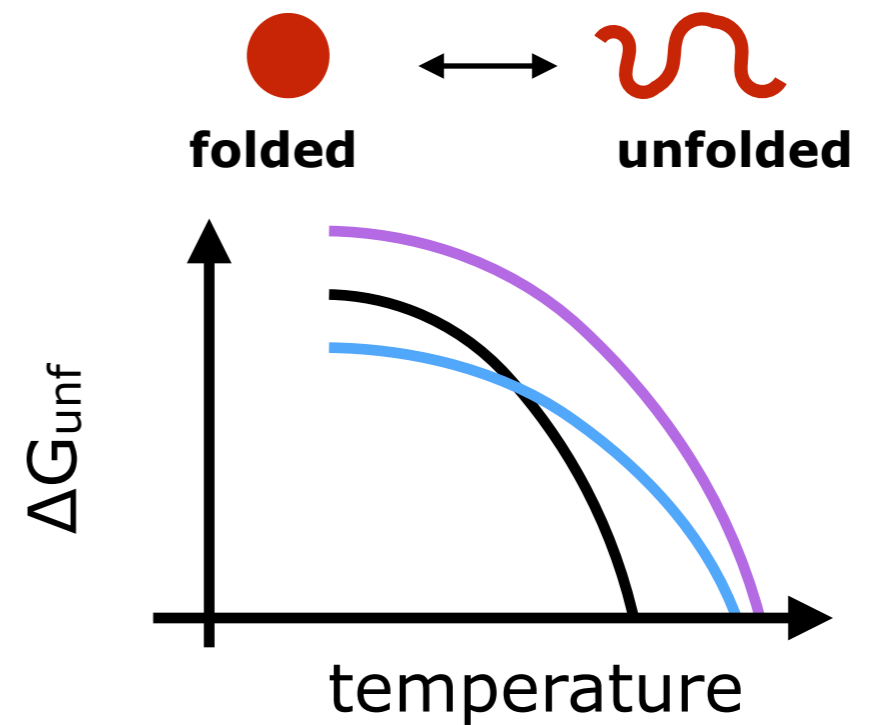
*Wang, Li, Pielak JACS (2010)

Muphy/OPEP: Back-map...

Protein mobility in "cell"
Shake, sample, and back-map



LBMD :: sampling local packing



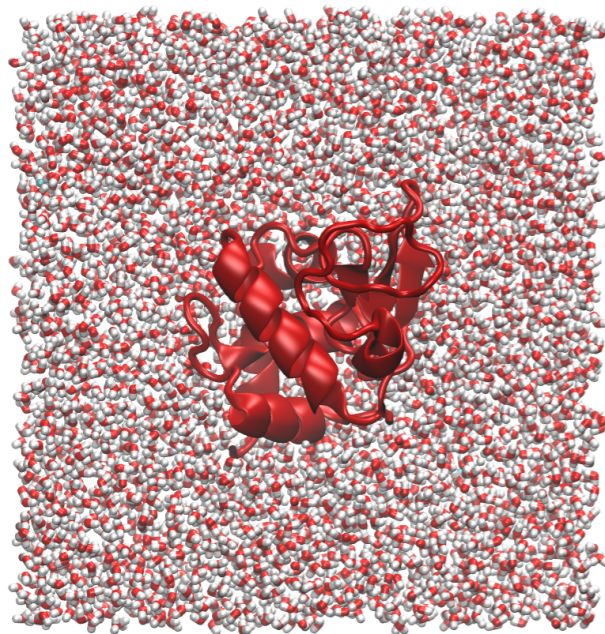
Enhanced Sampling :: stability curves
@CG level or all-atoms (back-map)

Muphy/OPEP: Back-map...

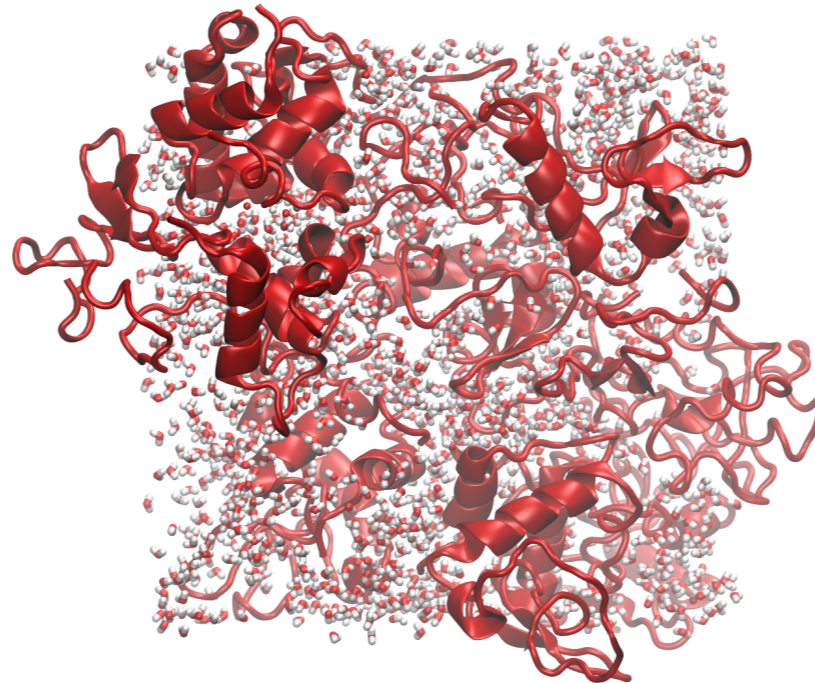
Protein mobility in "cell"

Katava et al in preparation

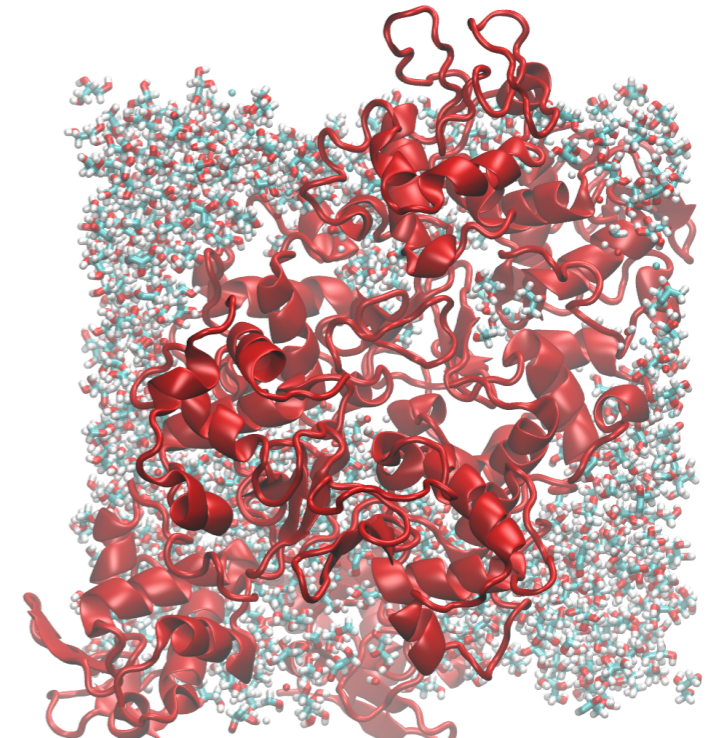
lysozyme in powder + REST2



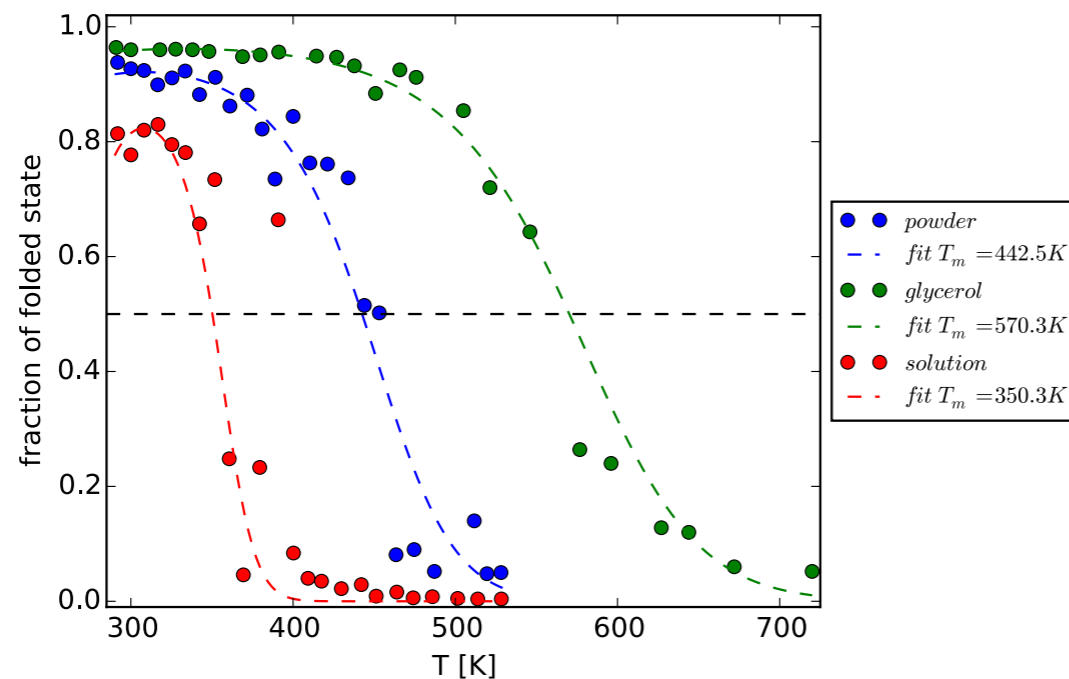
lysozyme in H₂O



lysozyme powder + H₂O
(h=0.3)



lysozyme powder + glycerol
(h=0.3)



REST2 to sample f/u

thermal response via
corresponding state principle
experimental trend reproduced

Stirnemann&FS, JCTC (2015)



Muphy/OPEP: amyloid aggregation

FS et al. JCTC (2015)

Hydrodynamics speeds up peptides aggregation

Simulations

18 monomers $A\beta_{16-22}$

KLVFFAE

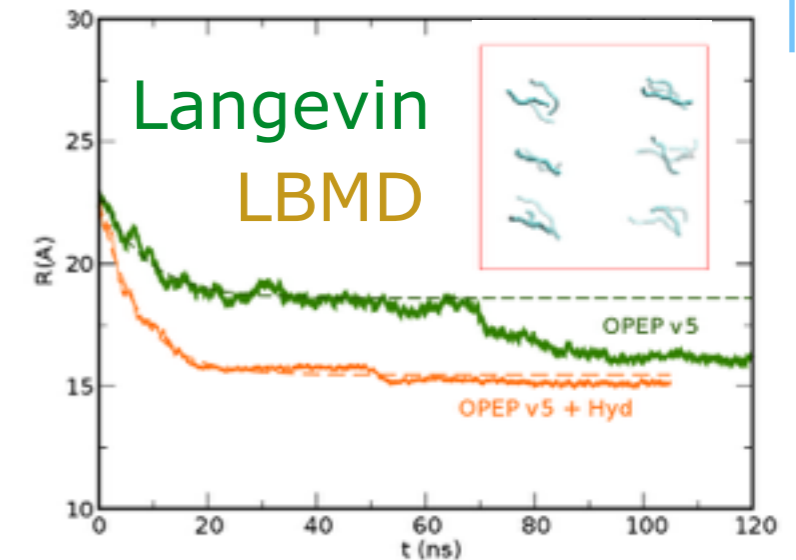
CH3-CO and NH2 terminal

Cubic box 65 Å

$c = 100$ mM

Two-step mechanism

Speed up of both collapses (45%)



Hydrodynamics

enhance diffusivity of proteins

Frembgen-Kesner&Elcock JCTC (2009)

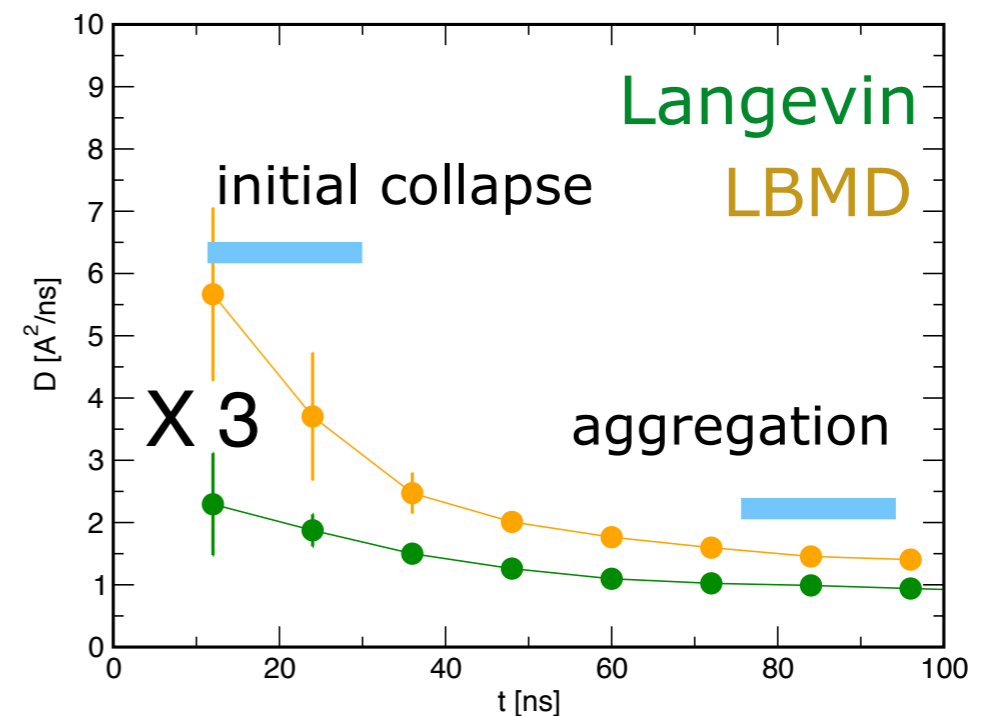
enhance aggregation of lipids

Ando&Skolnick BJ (2013)

enhance folding kinetics

Cieplak&Niewieczerza JCP (2009)

$A\beta_{16-22} \sim 23 \text{ Å}^2/\text{ns}$ (dilute solution)



HI enhance diffusivity

Muphy/OPEP: amyloid aggregation

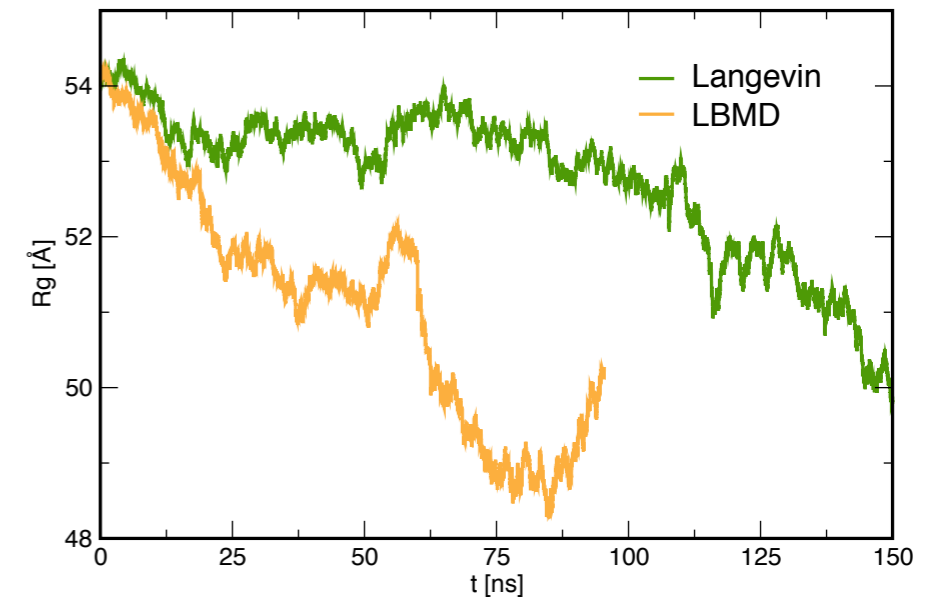
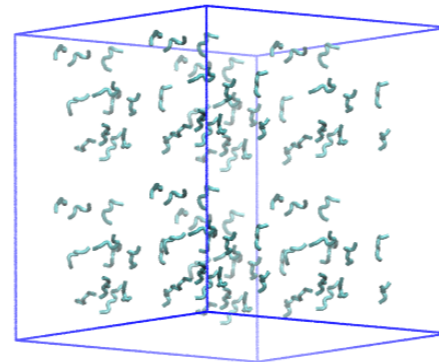
A β aggregation
toward big systems

100 monomer A β (16-22)

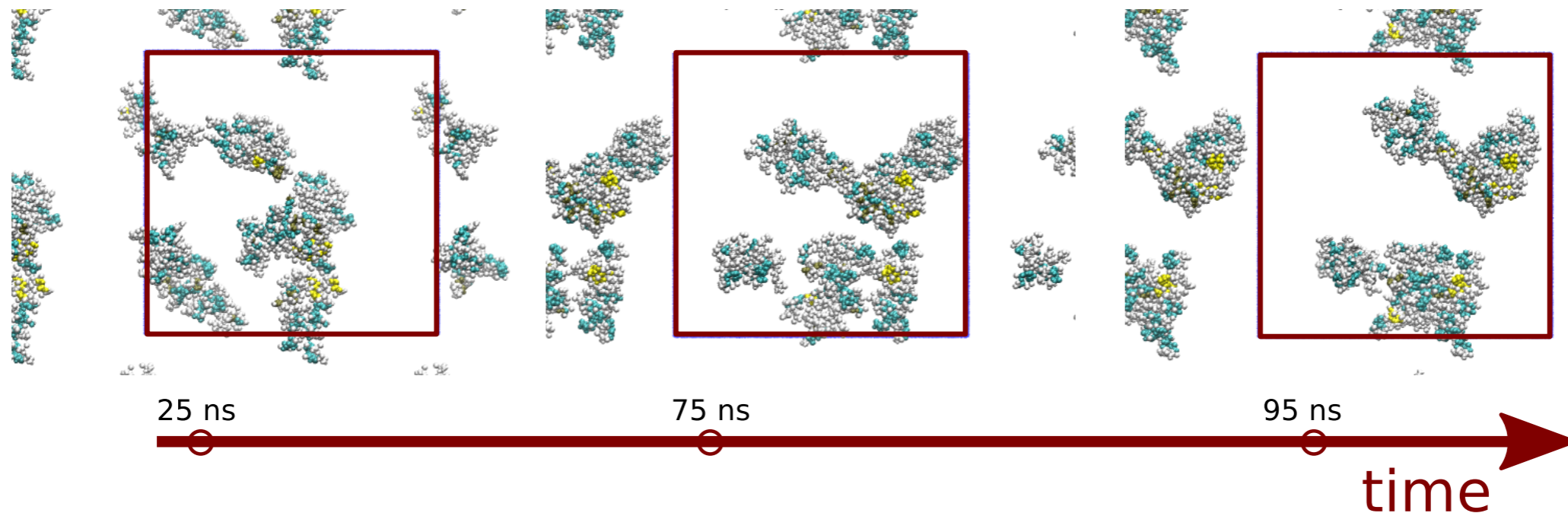
Chiricotto, et al JCP(2016)

Simulation

100 monomer A β (16-22)
all atoms equivalent 300k
L=150 Å
c~50 mM



HI speed up aggregation



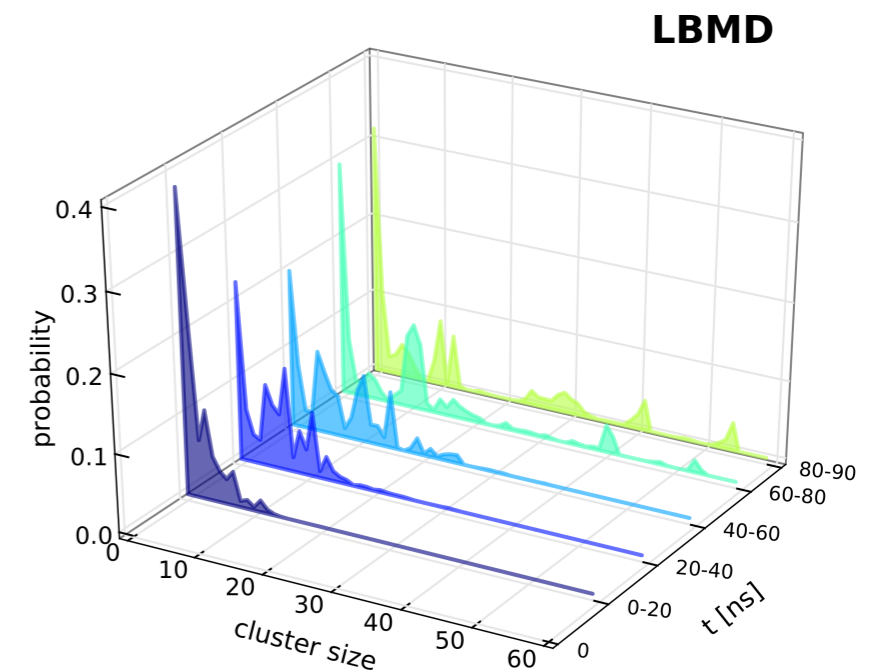
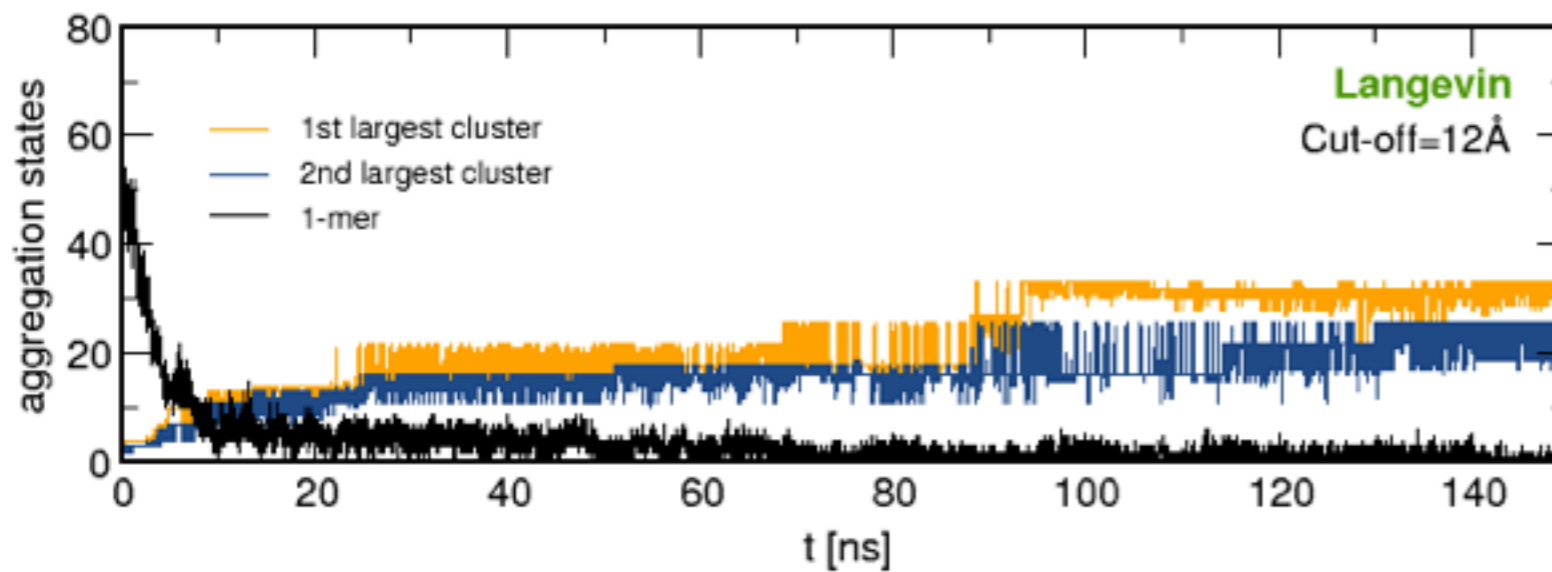
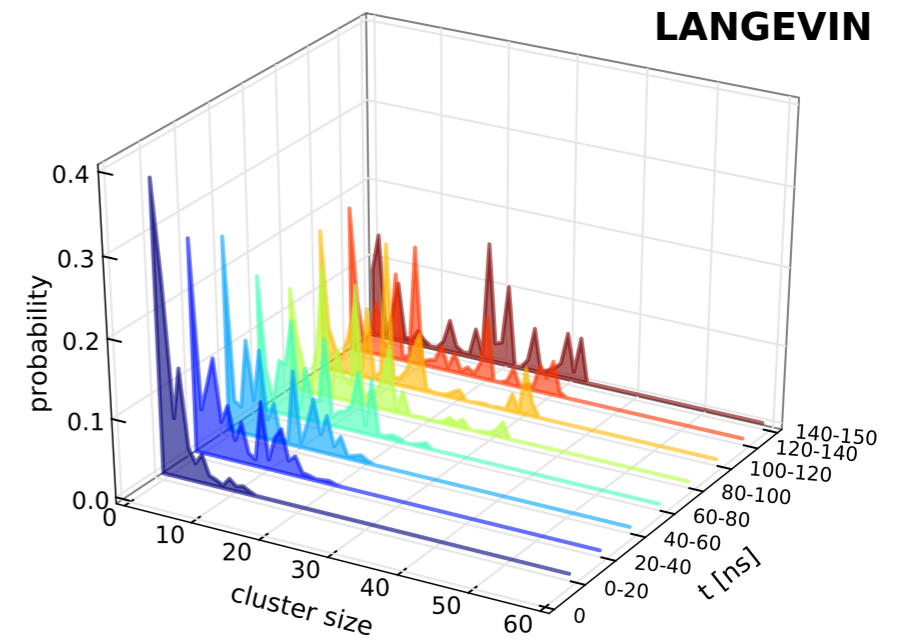
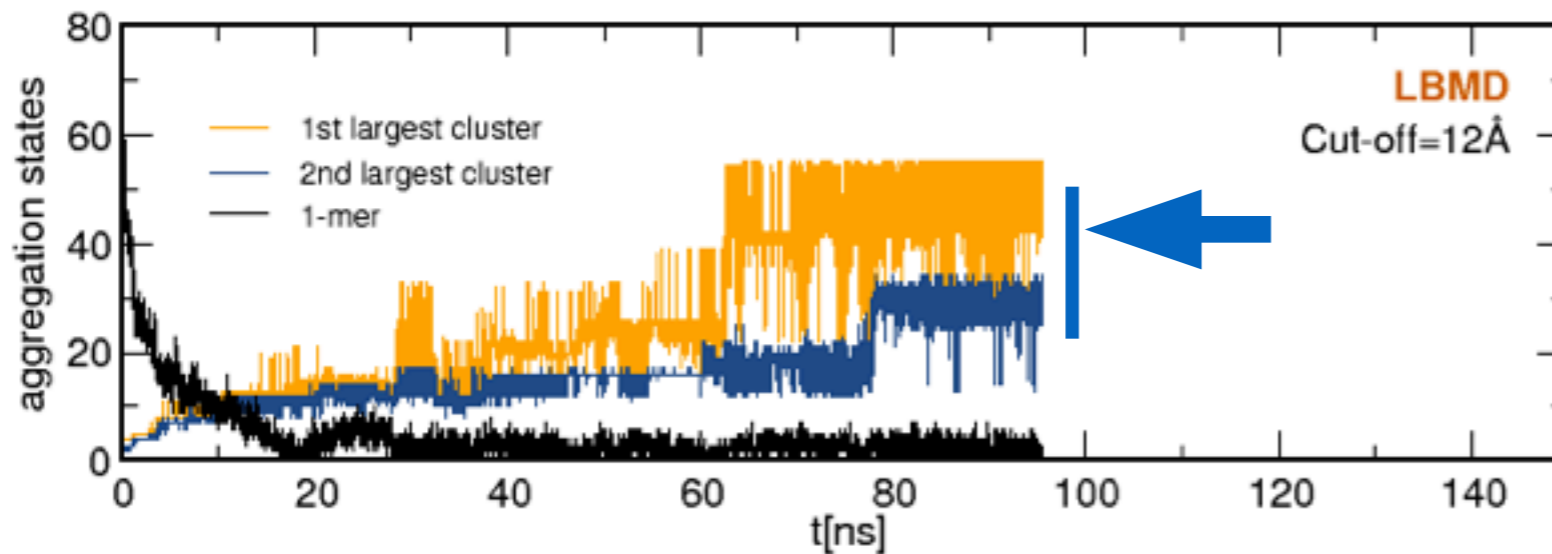
Muphy/OPEP: amyloid aggregation

A β aggregation
toward big systems

100 monomer A β (16-22)

Chiricotto, et al JCP(2016)

HI enhance cluster formation and exchange



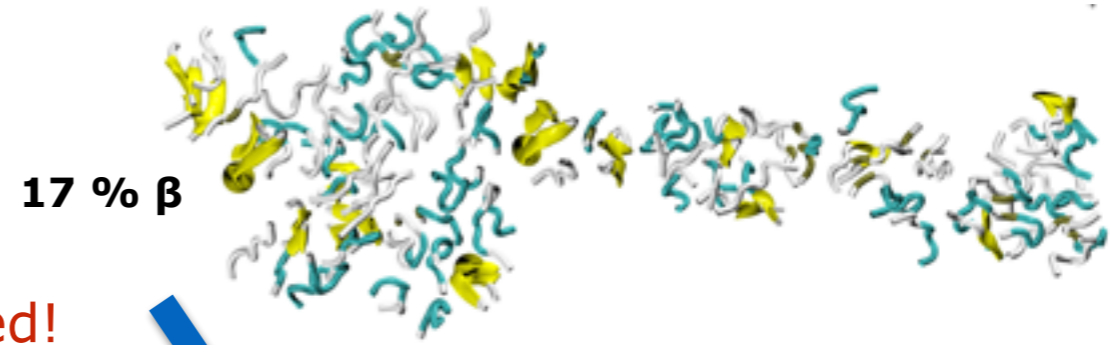
Muphy/OPEP: amyloid aggregation

A β aggregation
toward big systems

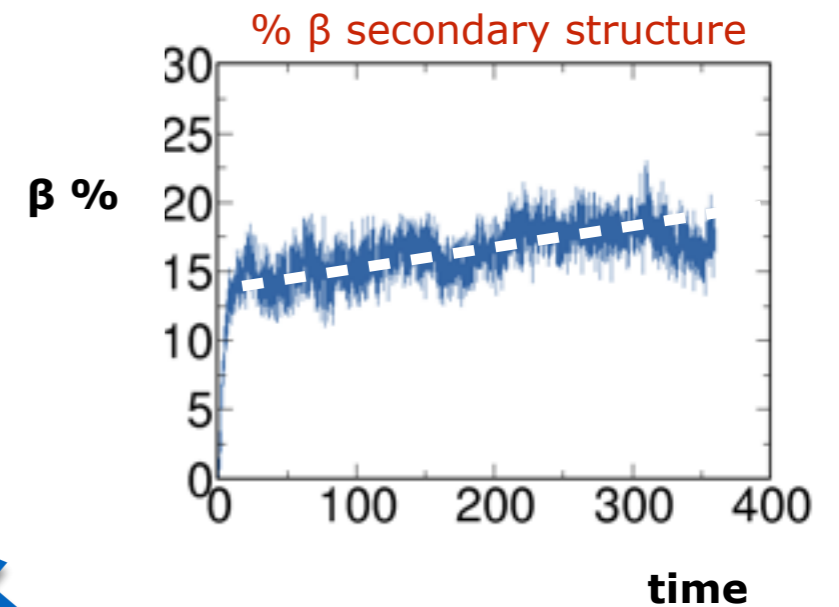
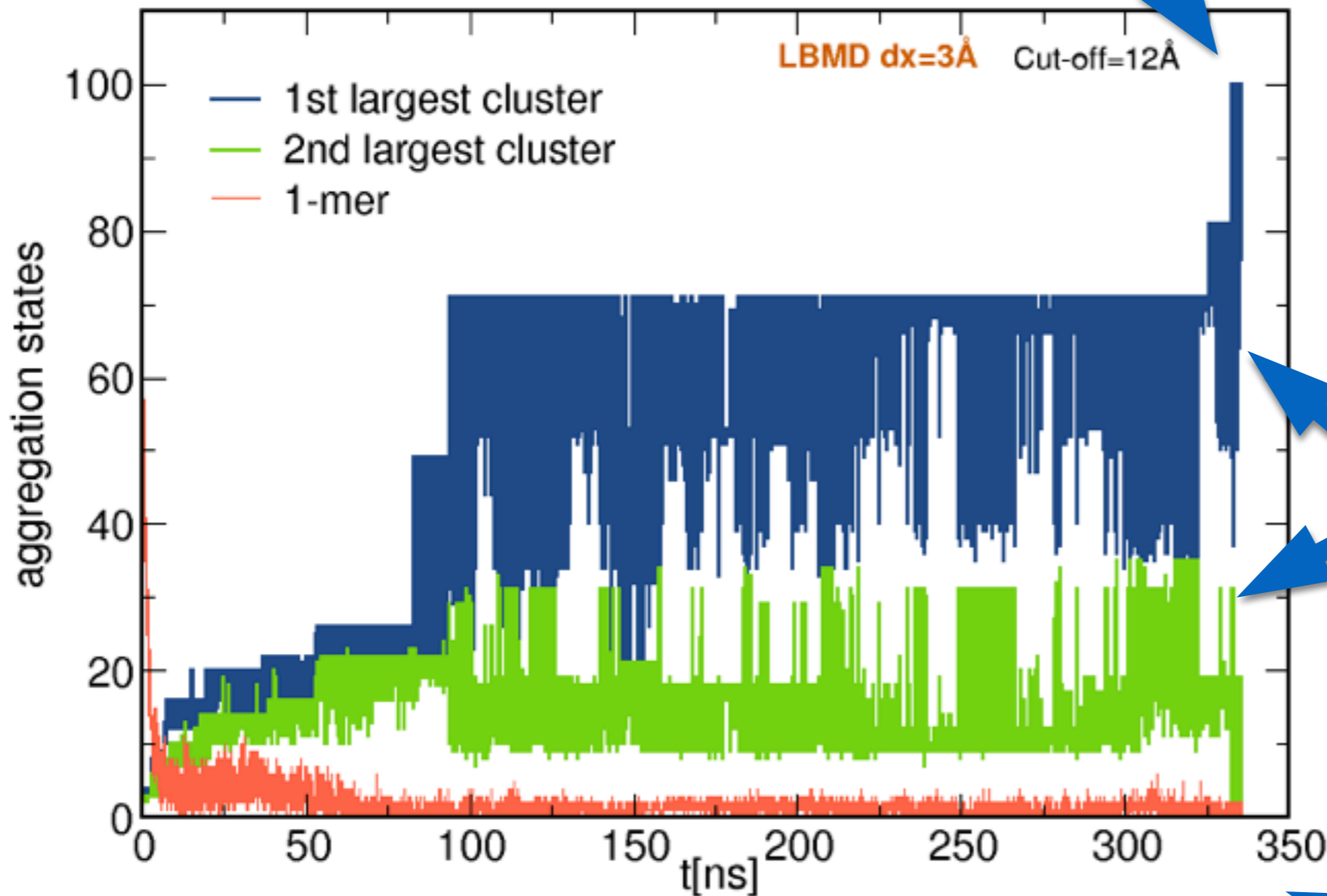
extending simulation time
lower resolution LB

100 monomer A β (16-22)

Chiricotto, et al JCP(2016)



Aggregation Completed!



fusion of largest and
second largest
clusters

timescale \sim 300/400 ns

Muphy/OPEP: amyloid aggregation

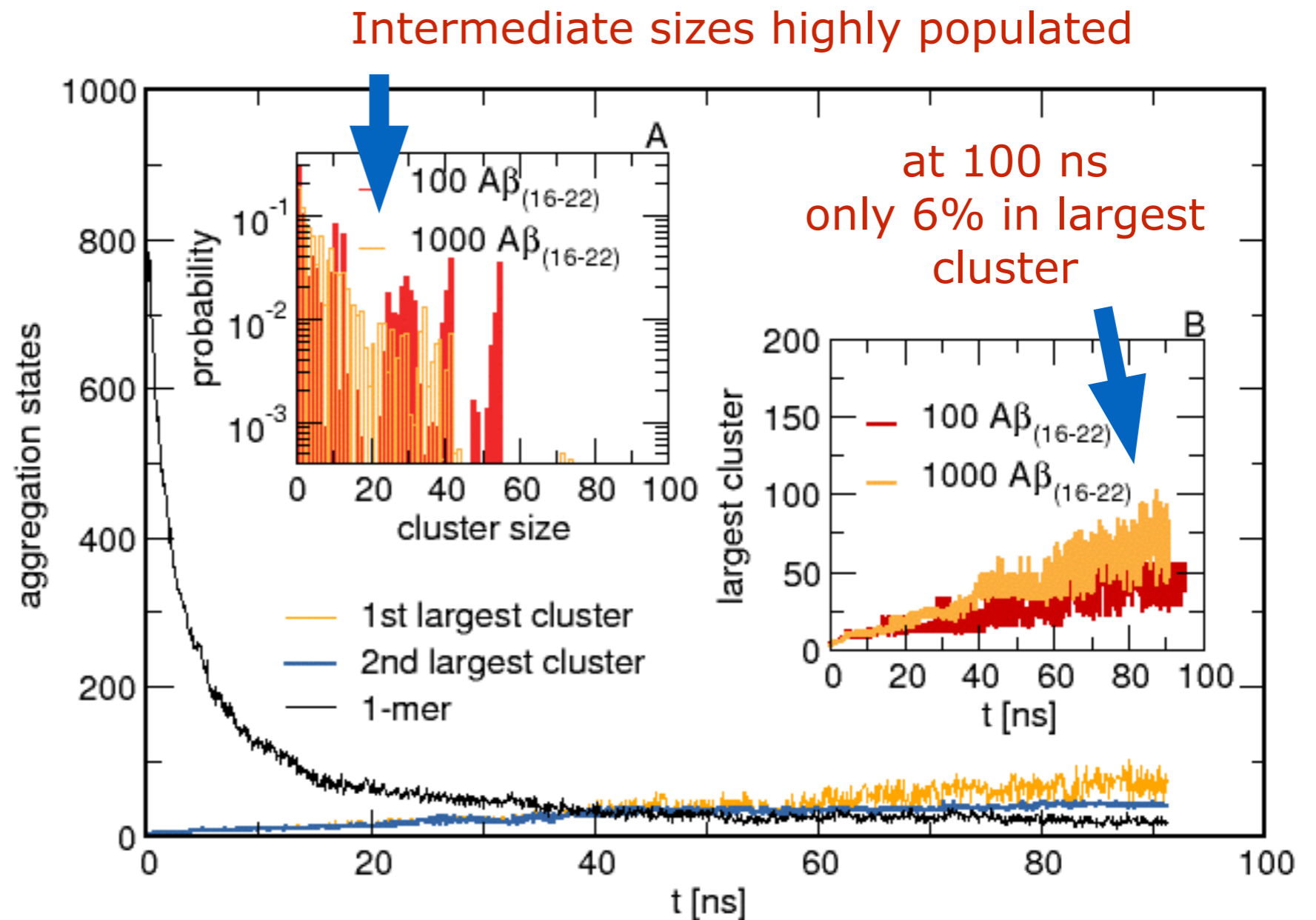
A β aggregation
toward big systems

1000 monomer A β (16-22)

Chiricotto, et al JCP(2016)

Simulation

1000 monomer A β (16-22)
all atoms equivalent 2,4Mio
L=300 Å
c=55 mM



Comparing 100 A β and 1000 A β

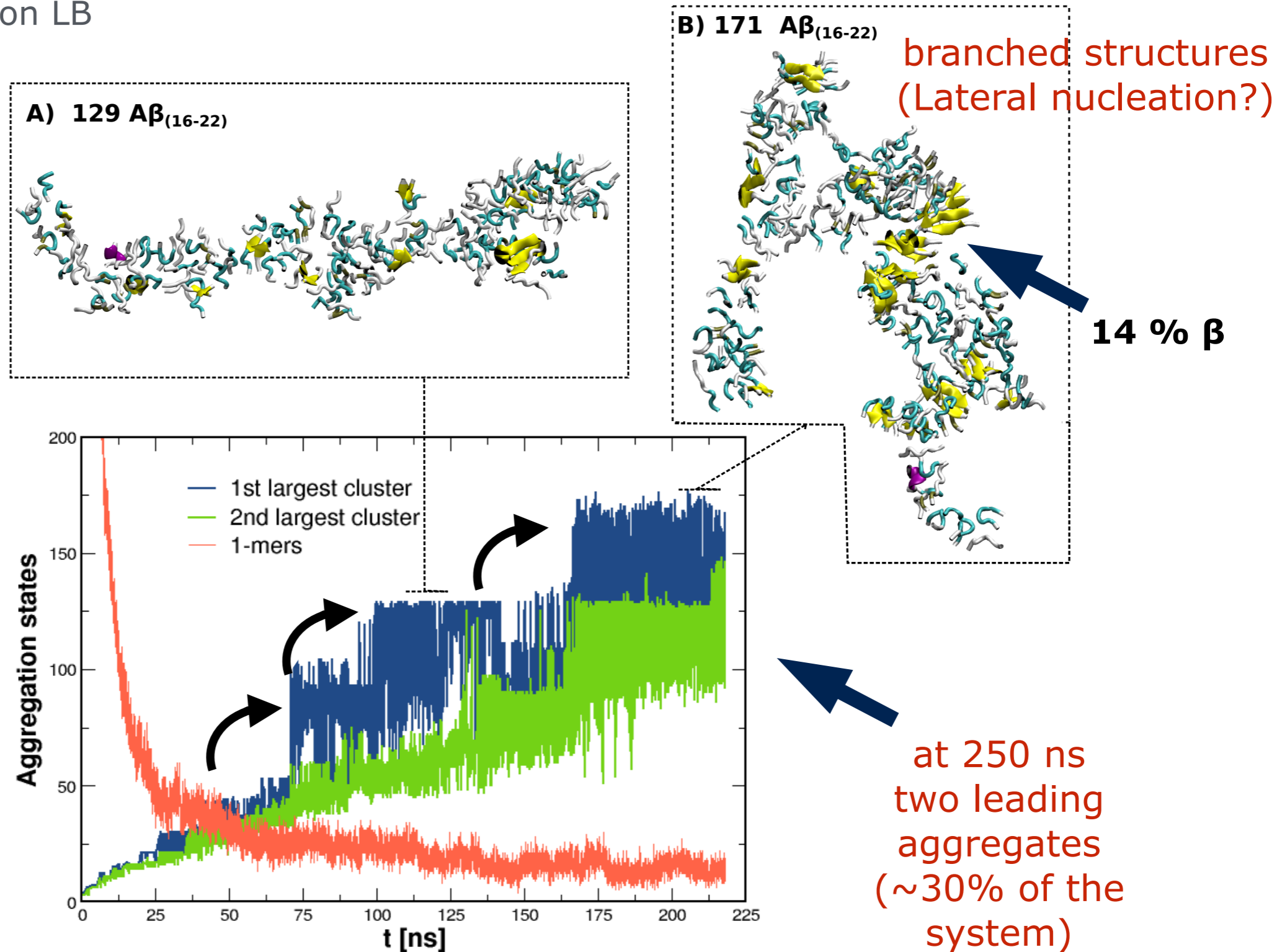
Muphy/OPEP: amyloid aggregation

A β aggregation
toward big systems

extending simulation time
lower resolution LB

1000 monomer A β (16-22)

Chiricotto, et al JCP(2016)

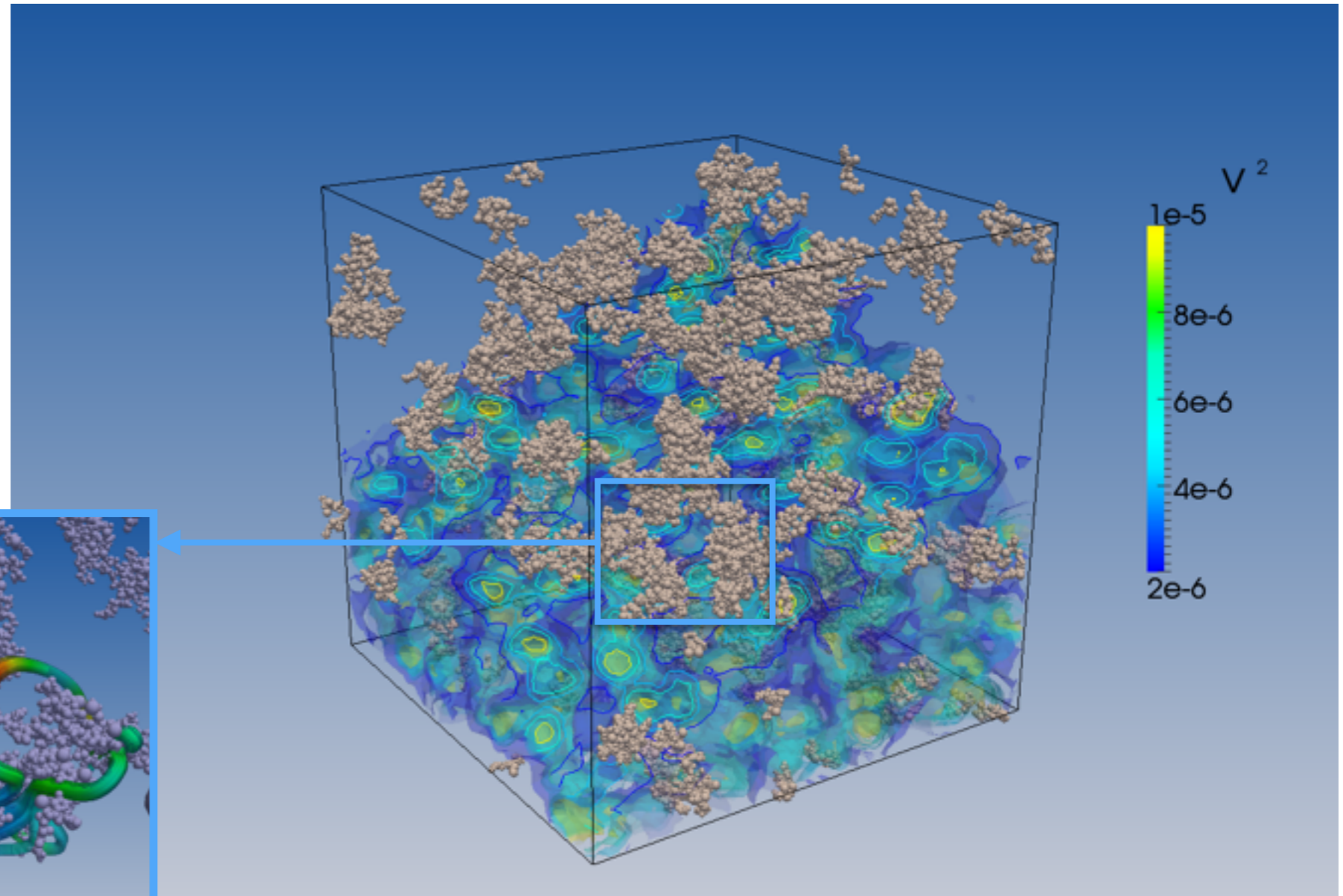




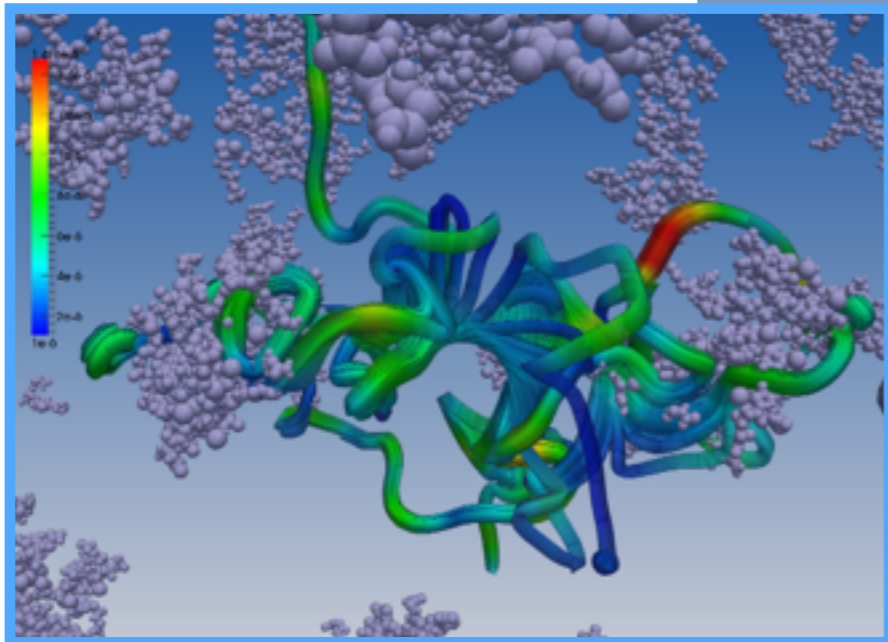
Muphy/OPEP: amyloid aggregation

1000 monomer $A\beta(16-22)$

Fluid iso-kinetic surfaces



Local fluid streamlines



Protein unfolding in shear flow

Simulations

β -hairpin (GB1, fragment 41-56)
 Box 50x50x50 Å
 Shear Rate 10^{10} s^{-1}

SHEAR

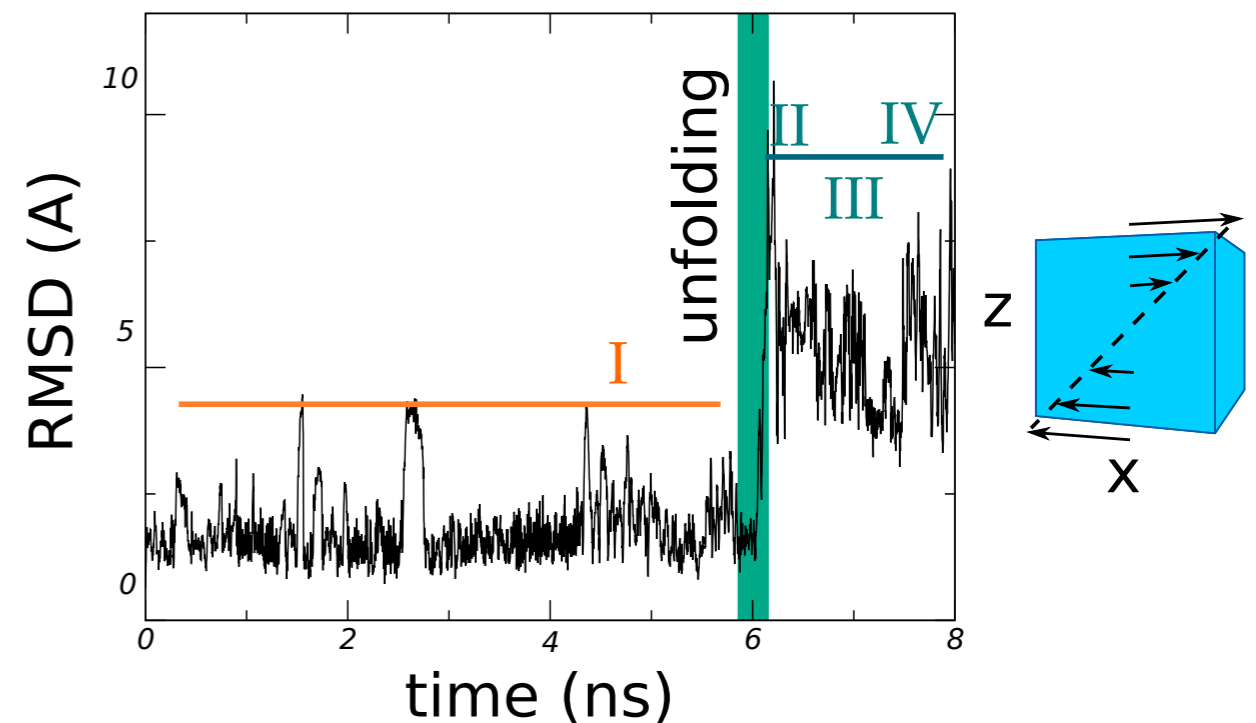
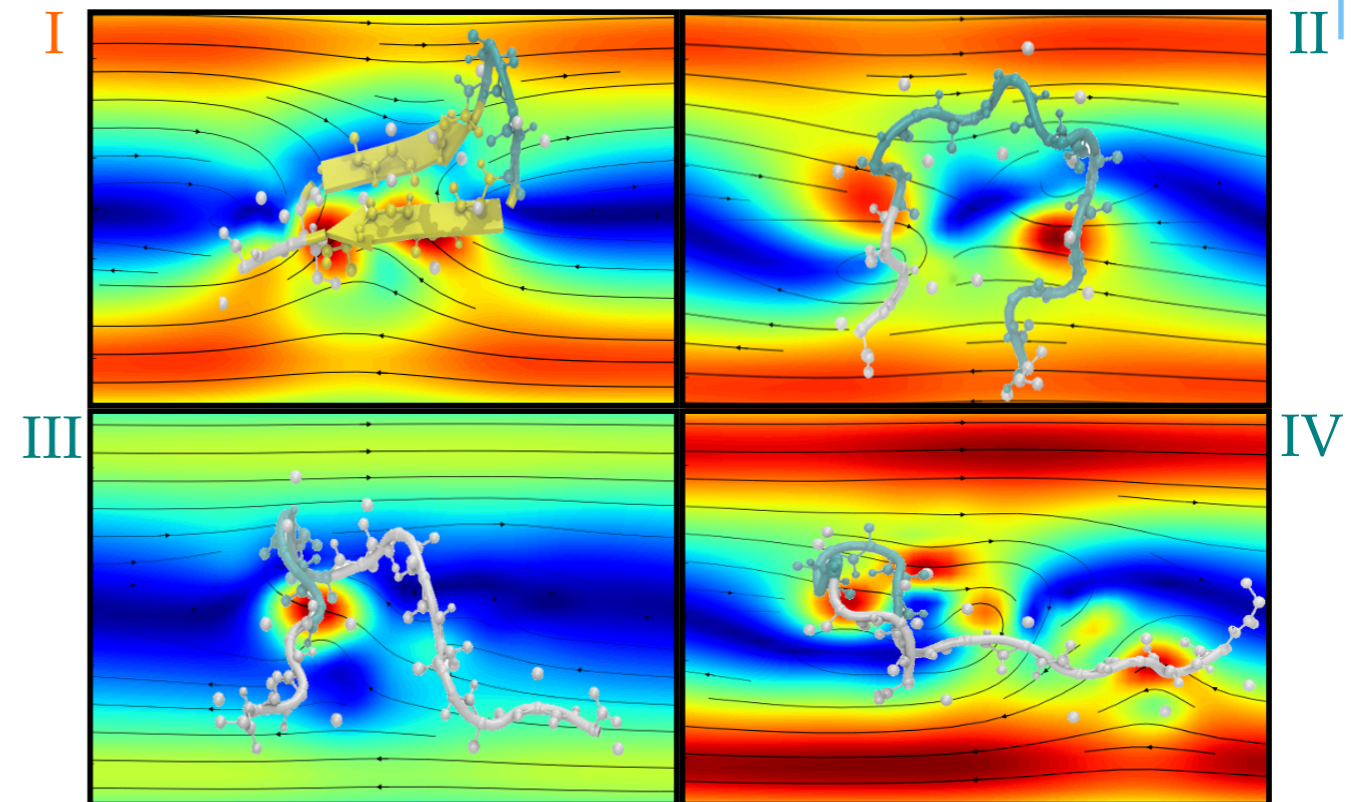
a route to probe mechanical stability

OPEN QUESTION

do proteins unfold under shear?

Protein unfolds!

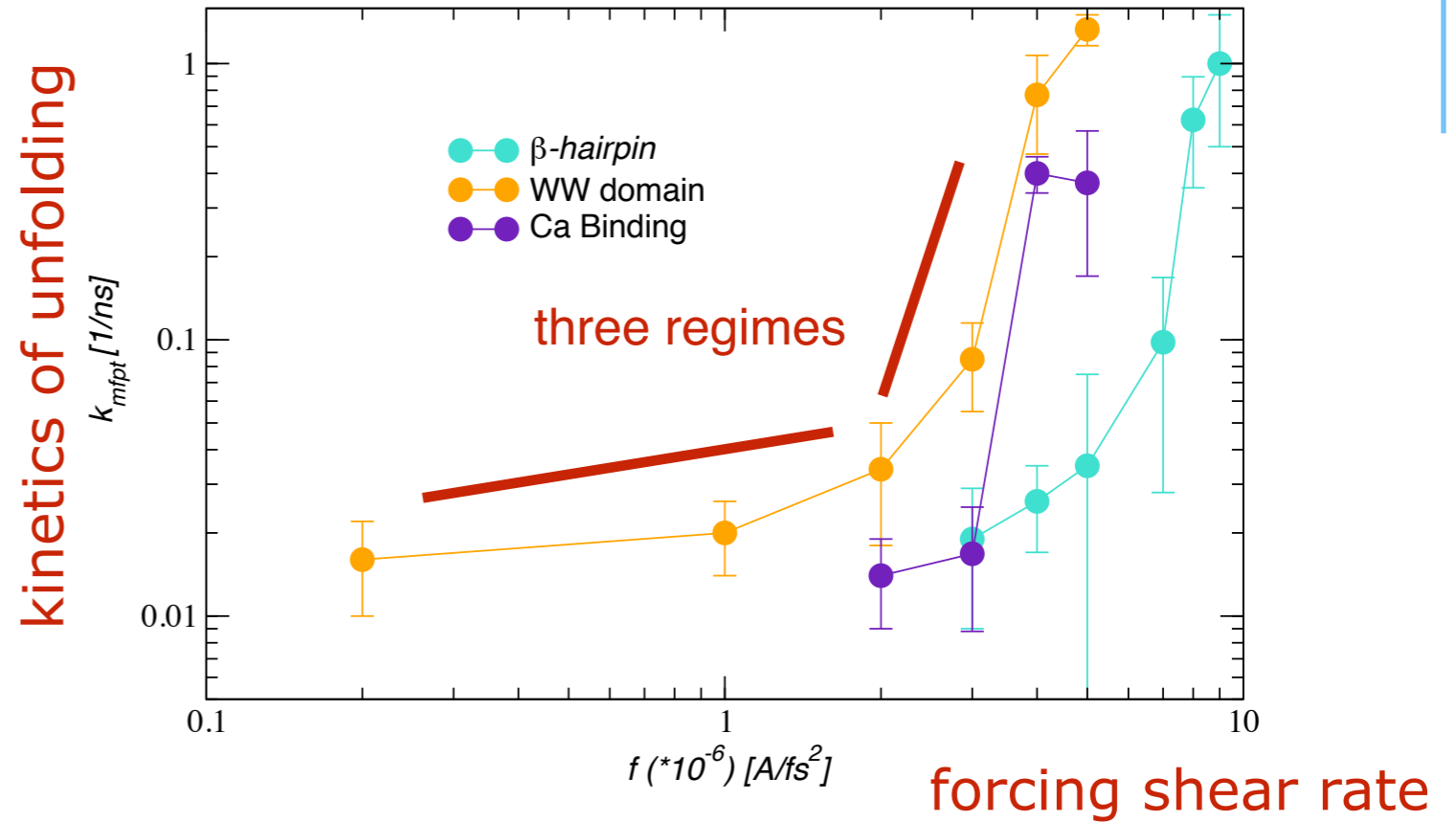
but at very high shear rate (*)



Protein unfolding in shear flow

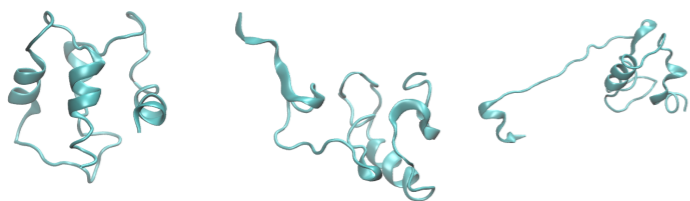
Mechanical weakness

...where unfolding starts
 to be compare to
T and Force unfolding

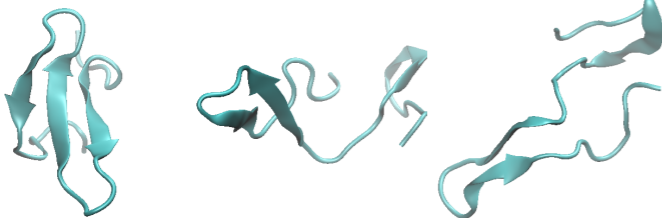


time \rightarrow

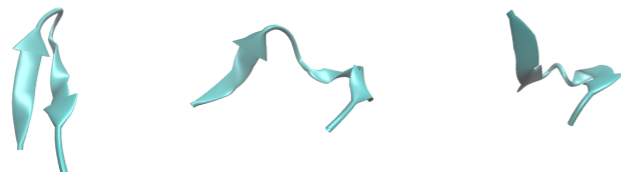
Ca binding



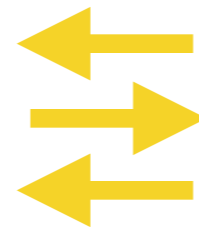
ww domain



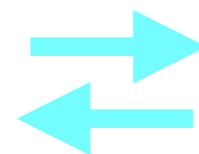
β -hairpin



3 α helices



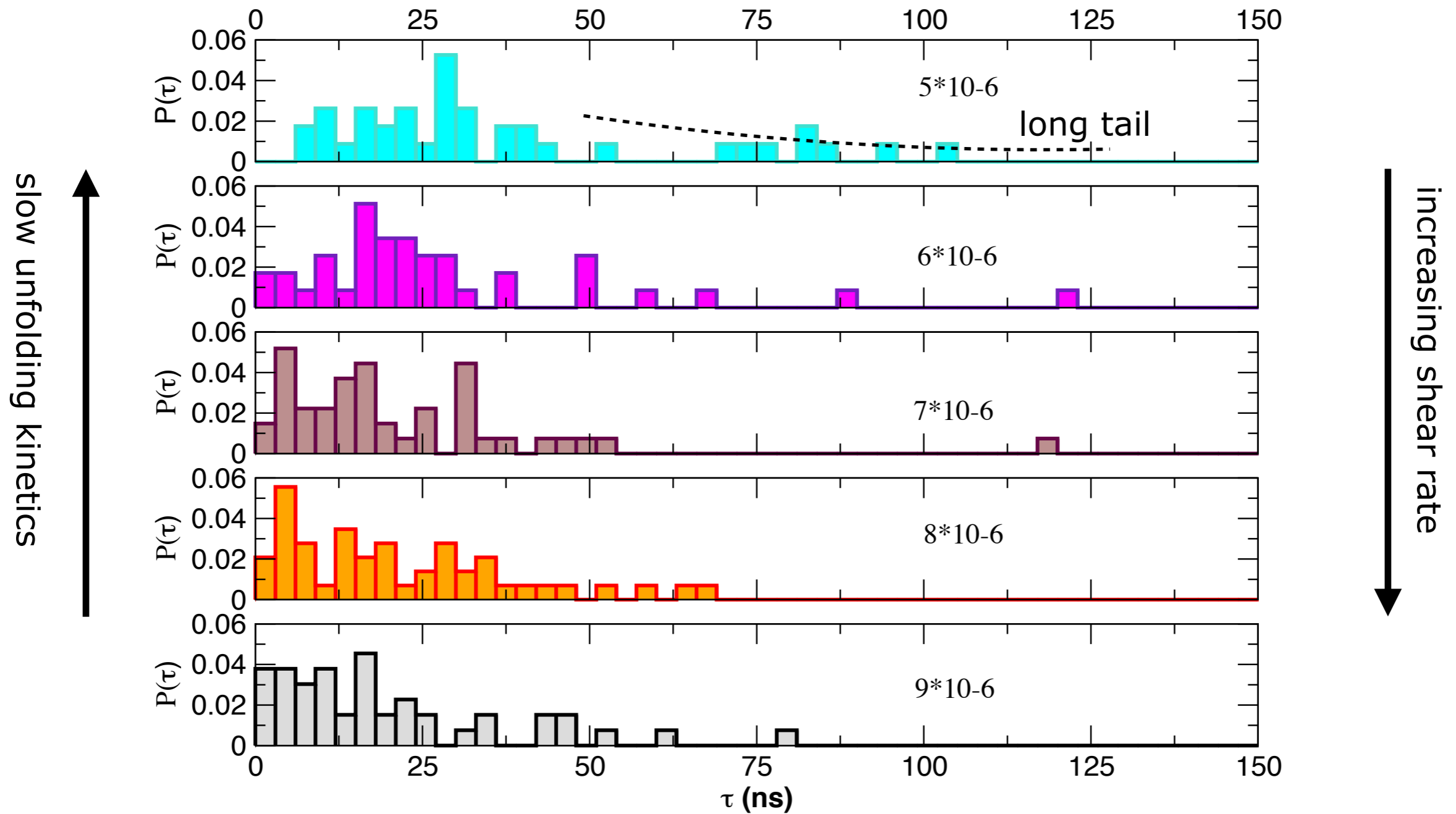
β sheets



β hairpin

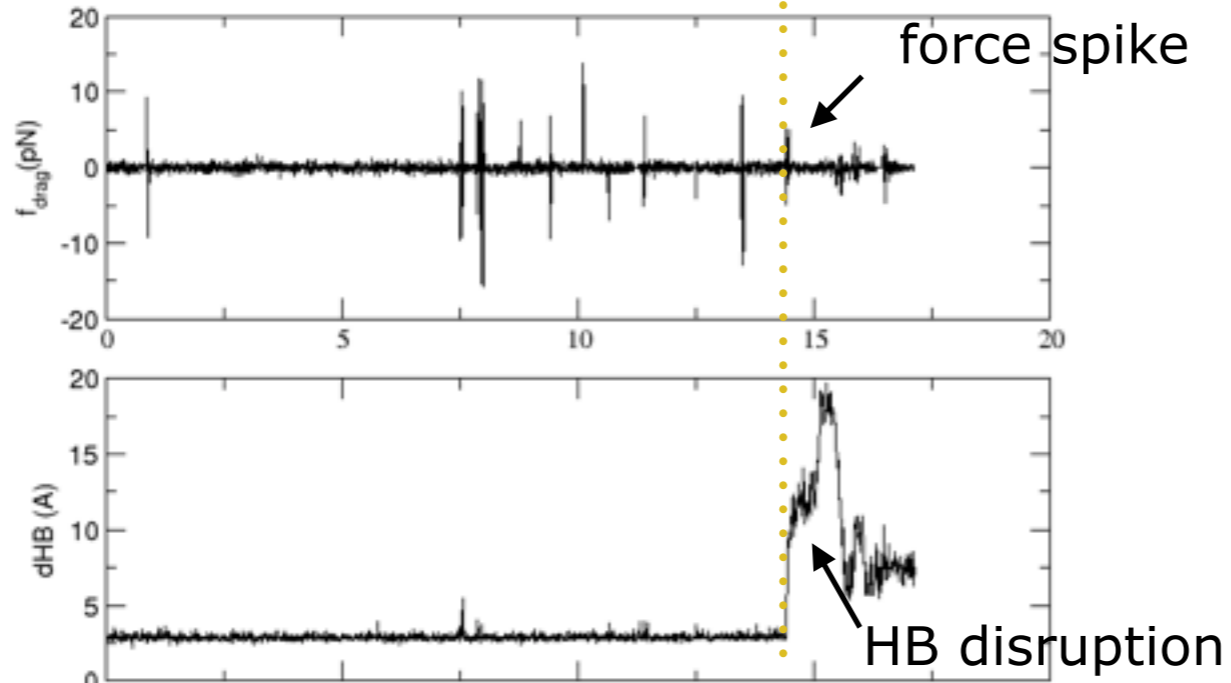
Protein unfolding in shear flow

distribution of unfolding time for **β -hairpin**

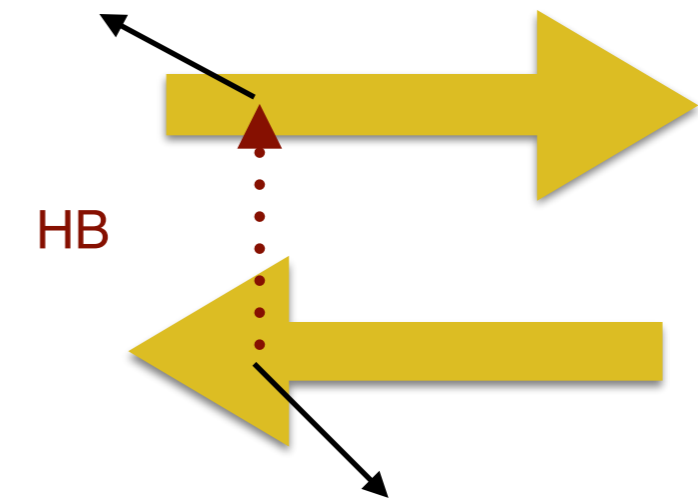


Protein unfolding in shear flow

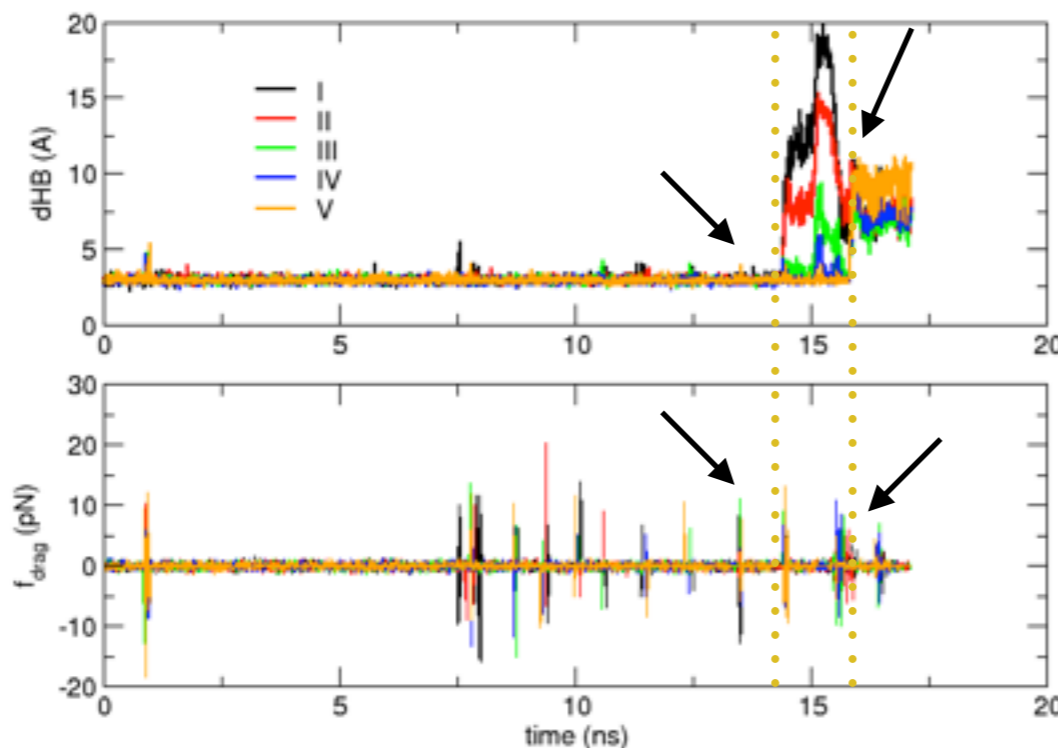
reaction coordinate only sporadically aligned with shear gradient



solvent drag force

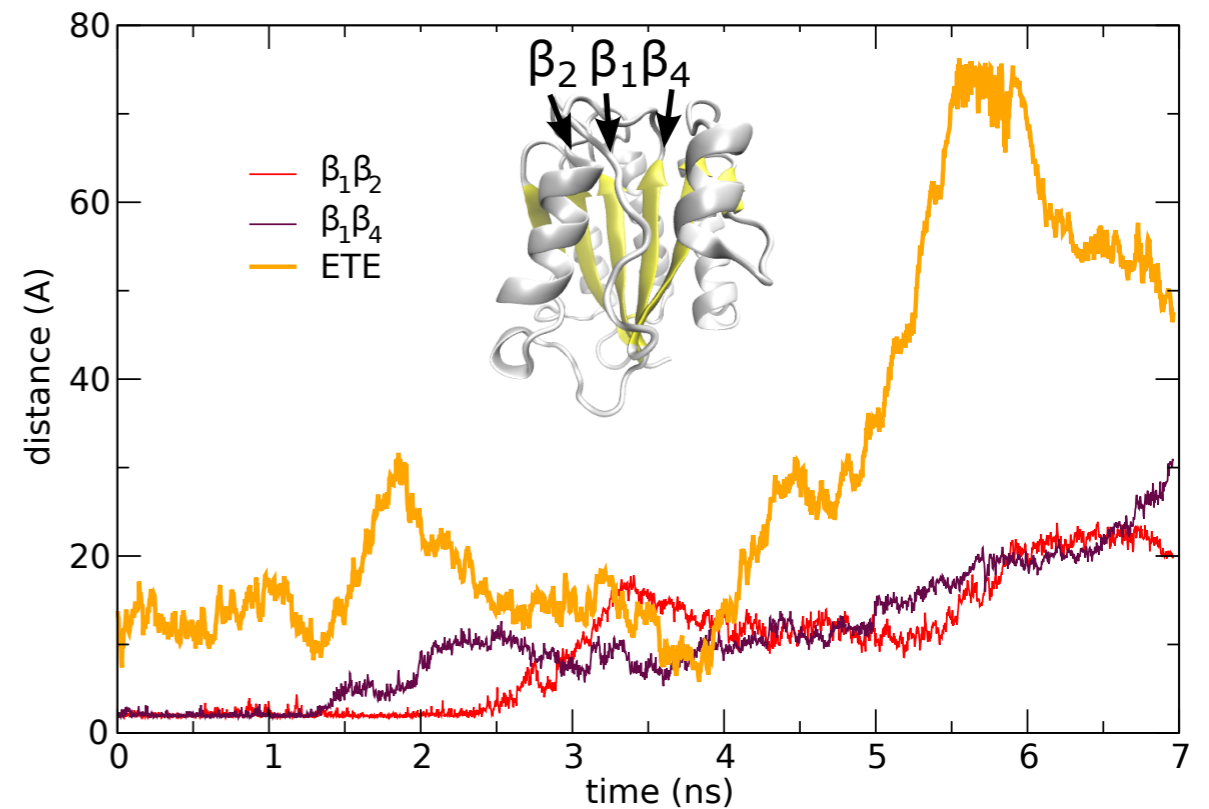
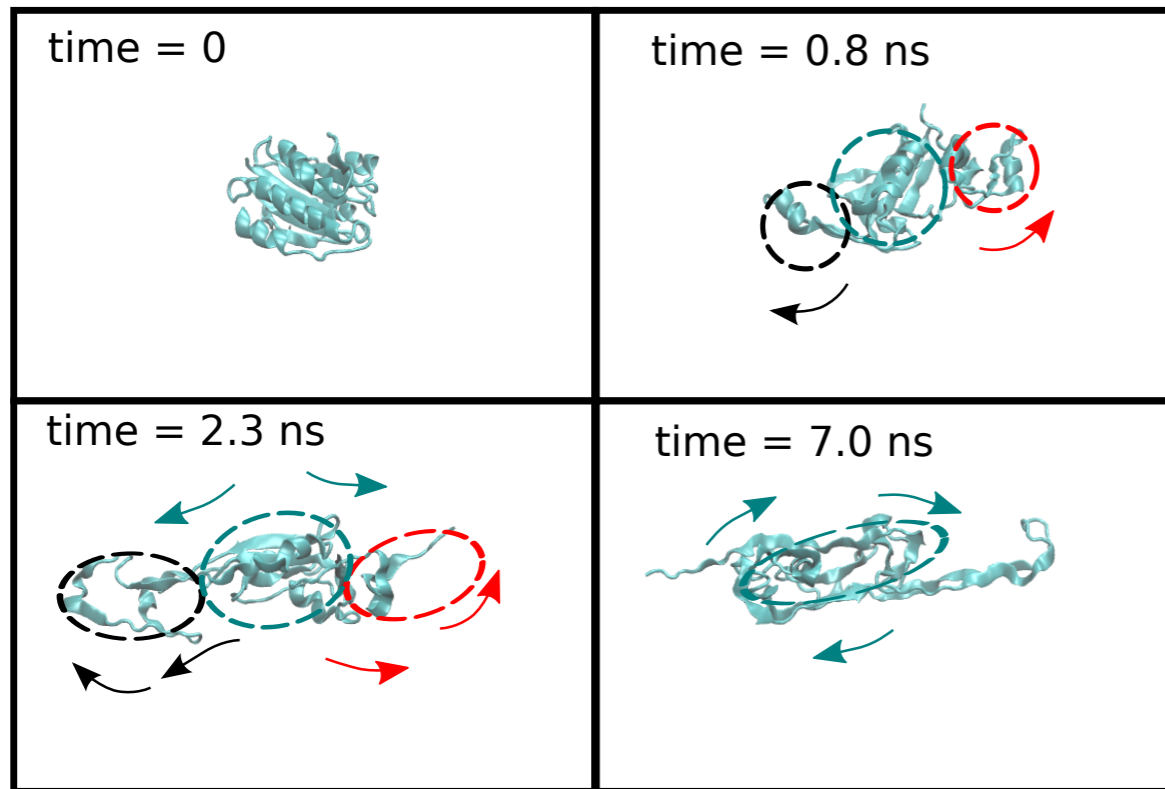


$$f_{\text{drag}} = \text{force on HB}_{\text{donor}} - \text{HB}_{\text{acceptor}} \text{ projected on HB direction}$$



HBs break at different times as effect of different spikes

test on large domain **A2 VWF**



unfolding steps:

1. separation peripheral domains
2. crack of hydrophobic cores (3 β -sheets)

Crowding in silico

Kesner&Elcock Biophys Rev (2013)

years

1996

Bicout&Field

ribosomes, tRNAs, proteins

spheres {r}
Langevin Dynamics

$N_p \sim 340$



2008

Ellison et al.

composition as in E.coli

spheres {r}
specialised Random Dynamics

$N_p \sim 1.7$ million

2010

McGuffee&Elcock

composition as in E.coli

all atoms
rigid body BD

$N_p \sim 1000$

Ando&Skolnick
composition as in E.coli
spheres
Stokesian Dynamics



Here we are

Feig&Sugita
all atoms
+ water

Hydrodynamics is included

Diffusion only mildly slowed down by excluded volume, only including Hydrodynamics there is agreement with experiments

Proteins in cell: a new frontier for Computational Biology

Effect of crowding on diffusion, stability and aggregation is a major challenge for both experiments and simulations

Simplified models are necessary but is key to keep solvent effects

Brownian Dynamics with Hydrodynamics is a standard way

Lattice Boltzmann coupled to MD is a valuable alternative

Friends & “Money”

M. Chiricotto (IBPC, Paris)

P. Derreumaux (IBPC, Paris)

S. Melchionna (CNR, Rome)



IDRIS::big-challenge

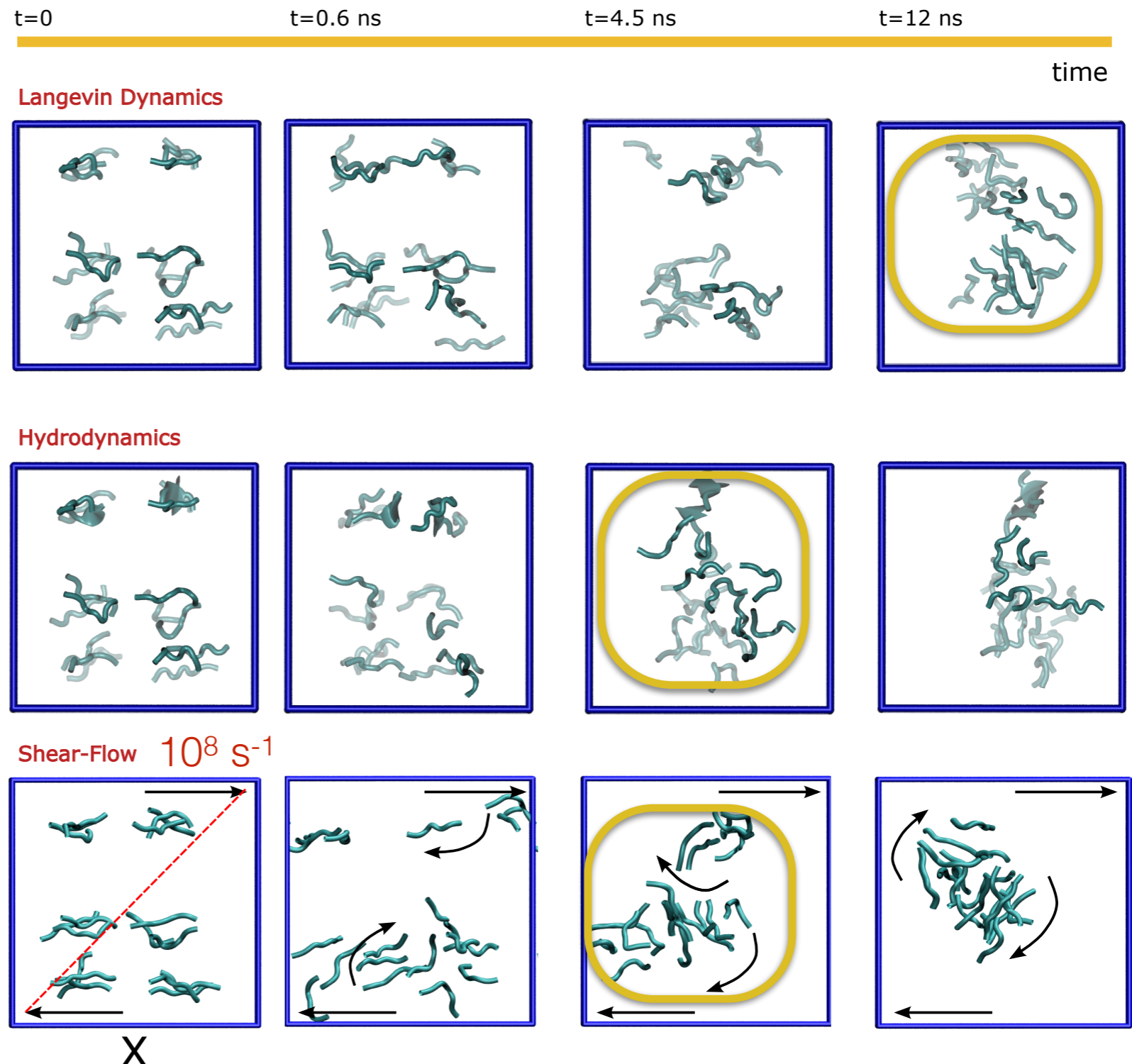


*LBX::Dynamo
Gral*



Thermos

<http://x-proteins.blogspot.fr>



SHEAR

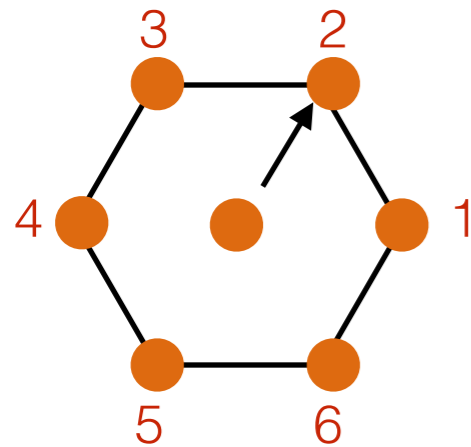
A model for understanding agitation processes and steps of fibril growth.

OPEN QUESTION

do amyloids aggregate faster under shear?

Same overall kinetics
but different
peptide organisation

Lattice Gas Cellular Automate

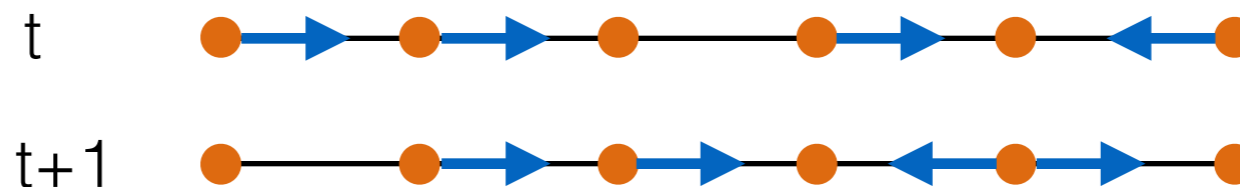


- Rules
- each site hosts up to 6 particles
 - particles move along 6 discrete velocities
 - 2 particles sitting on same site NOT move along same directions

● occupation site $n_i(x,t)=1,0$ Lattice State for N site = 2^{6N}
 for site i where particles go $|n_1n_2n_3n_4n_5n_6\rangle$ i.e. $|001010\rangle$

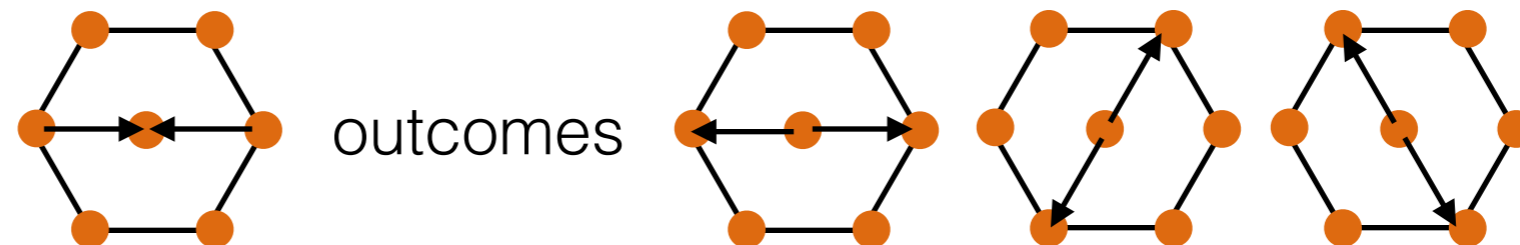
Streaming

$$n_i(\vec{x} + \vec{e}_i, t + 1) = n_i(\vec{x}, t)$$



Collision

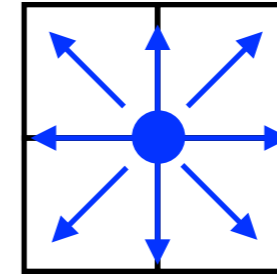
Operator altering the vector state n (+1,-1,0) $n'_i - n_i = \Omega(\vec{n})$



LGCA crude BUT essential :
 conserve particle number, total momentum, if lattice is good invariant rotation!!!!

Lattice Boltzmann Method

Distribution function of displacement
discrete states $\mathbf{f}(\mathbf{x},\mathbf{t})$



$$f_i = \langle n_i \rangle$$

ie 2 dimensional lattice D2Q9

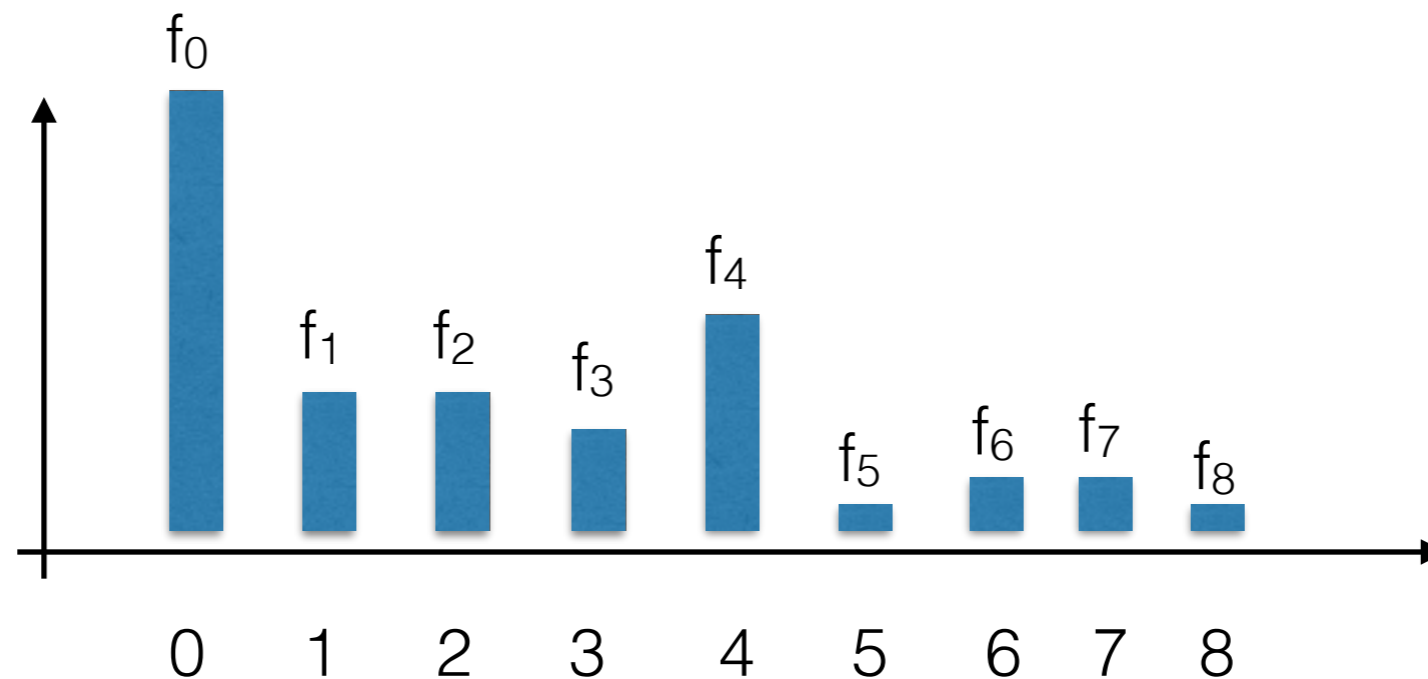
9 velocity directions

Macroscopic velocity

Fluid Density

$$\rho = \sum_{a=0}^8 f_a$$

$$\mathbf{u} = \frac{1}{\rho} \sum_{a=0}^8 f_a \mathbf{e}_a$$



$i=0,\dots,M$ directions

Kinetic equation $f_i(\mathbf{x} + \mathbf{e}_i \Delta x, t + 1) = f_i(\mathbf{x}, t) + \Omega_i(f(\mathbf{x}, t))$

Total Mass Conservation $\sum_i \Omega_i = 0$ Total momentum conservation $\sum_i \Omega_i \mathbf{e}_i = 0$

Problems

Construct a functional form for collisions

Derive macroscopic hydrodynamic equation, Navier-Stokes

Strategy

Focus on long-wave-length and low-frequency

$$\Delta x \sim \epsilon \quad \Delta t \sim \epsilon$$

incremental length and time are small and of the same order ϵ

The collision term

Multi-scale separation

$$f_i = f_i^{eq} + \epsilon f_i^{neq}$$

$$f_i^{neq} = f_i^1 + \epsilon f_i^2 + O(\epsilon^2)$$

equilibrium fluid

$$\sum_i f_i^{eq} = \rho$$

$$\sum_i f_i^{eq} \mathbf{e}_i = \rho \mathbf{u}$$

$$\sum_i f_i^k \mathbf{e}_i = 0$$

$$\sum_i f_i^k = 0$$

perturbation

Taylor expansion of collision term

$$\Omega_i(f) = \Omega_i(f^{eq} + \epsilon f^{neq})$$

Equilibrium condition

$$\Omega_i(f^{eq}) = 0$$



Linearised collision

$$\Omega_i(f) = \frac{\partial \Omega_i(f^{eq})}{\partial f_j} (f_j - f_j^{eq})$$

BGK

if one single rate for relaxation to equilibrium

$$\Omega_i(f) = -(f_i - f_i^{eq})/\tau$$

BGK==Bhatnagar-Gross-Krook

Starting from Boltzmann Equation

Probability in phase-space $d\mathbf{x}d\mathbf{p}$ at time t $f(\vec{x}, \vec{p}, t)$ Core of Kinetic Theory

Evolution of one-body distribution $D_t f = \underbrace{[\partial_t + \frac{\vec{p}}{m} \cdot \partial_{\vec{x}} + \vec{F} \cdot \partial_{\vec{p}}]}_{\text{Evolution of one particle distribution}} f(\vec{x}, \vec{p}, t) = \underbrace{C_{12}}_{\text{Collision}}$

C_{12} depends hierarchically on 2,3,4... n body

BGK for C_{12}

$$-\frac{1}{\tau}(f - f^{eq})$$

Assume Maxwell-Boltzmann
Expansion of MB

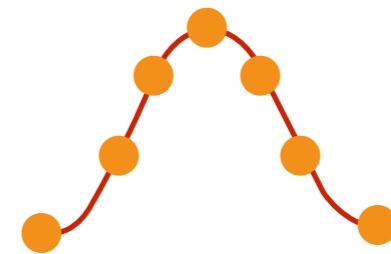
$$f^{eq} \simeq A \exp(-3/2\chi^2) [1 + 3(\chi \cdot \mathbf{u}) + \frac{9}{2}(\chi \cdot \mathbf{u})^2 - \frac{3}{2}u^2]$$

\mathbf{u} fluid velocity

χ particle velocity

Discrete $f \rightarrow f^D$
Obtain weight for lattice

$$f_i^D = W_i f_i$$



● discrete velocity \mathbf{e}_i



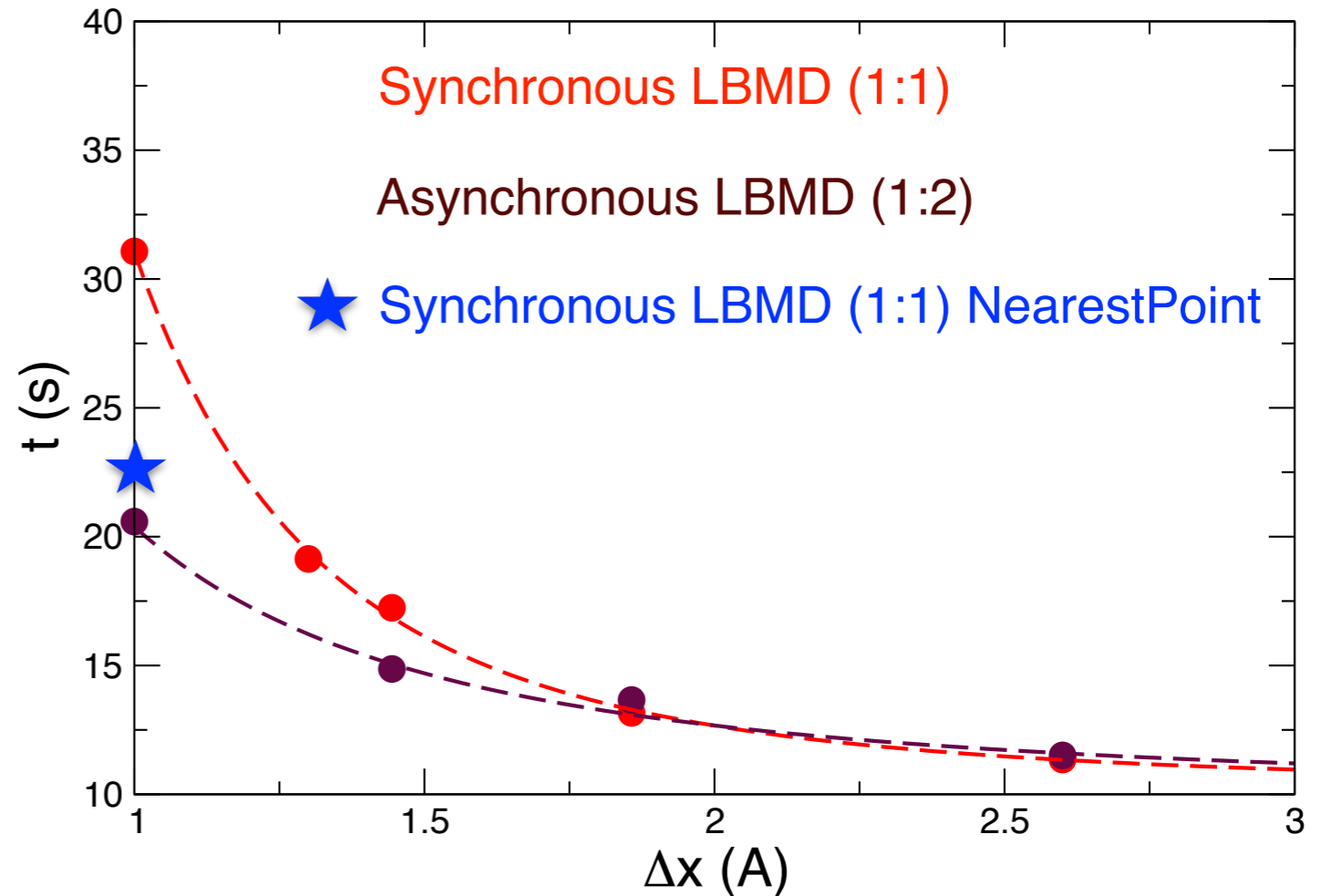
$$f_i(\mathbf{x} + \mathbf{e}_i \Delta x, t + 1) = f_i(\mathbf{x}, t) + \Omega_i(f(\mathbf{x}, t))$$

LBM

A solute in fluid

Multi-scaling

time for 1000 MD step
solute 864 particle
 $L_x=L_y=L_z=65 \text{ \AA}$
Intel Xeon 5660



MD time + mapping

$$t = A * (\Delta x^{-\alpha} + t_0)$$

LBM

Synchronous $\alpha \sim 3$

Asynchronous $\alpha \sim 2$

$t_0 \sim 12 \text{ s}$

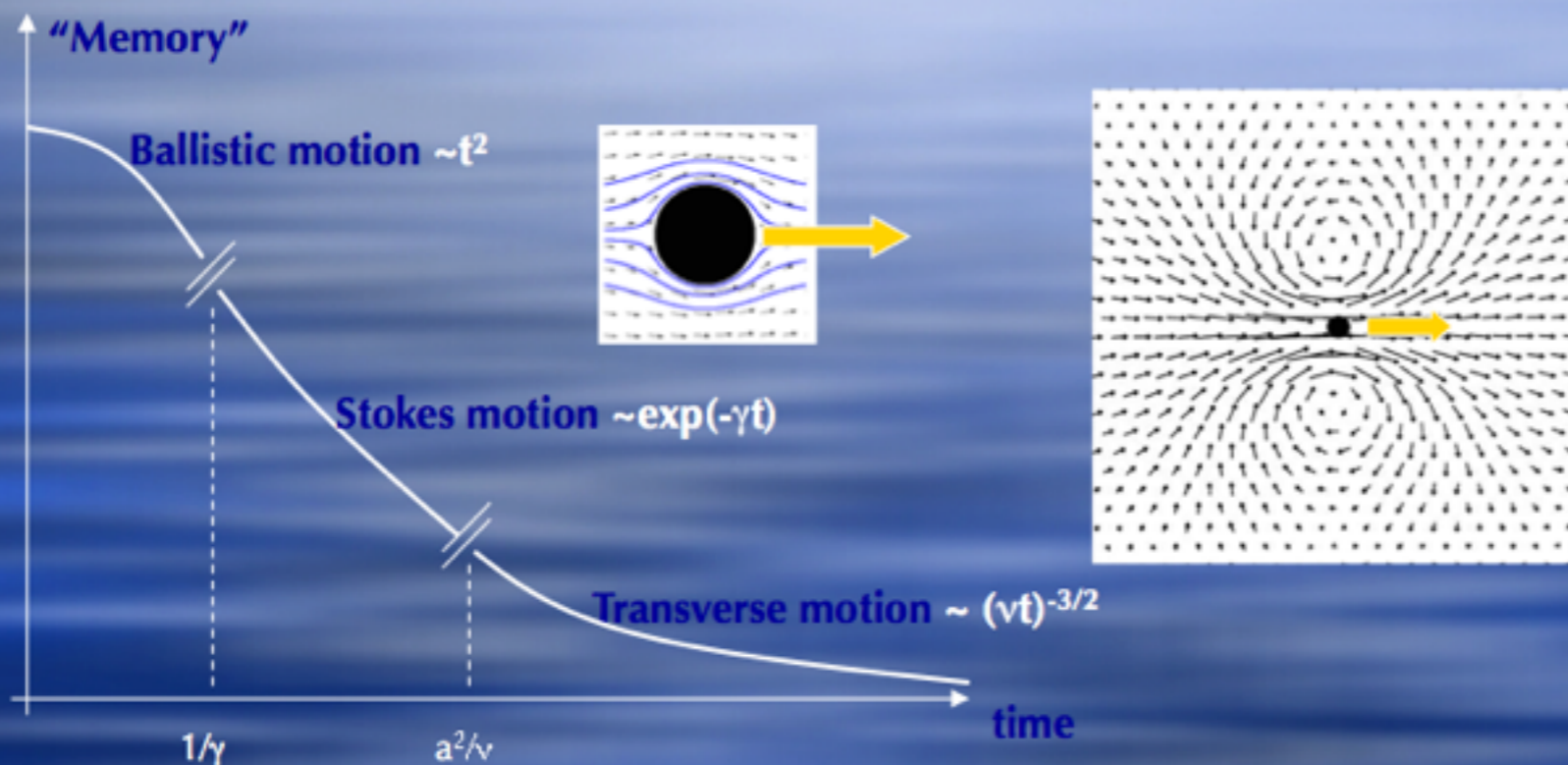
$\text{time}_{\text{NearestPoint}} / \text{time}_{\text{MultipleSupport}} \sim 0.7$

A solute in fluid

Multi-scaling

Coarse-graining?

A well-known example: the kicked particle



- Coarse-graining would not consider the time evolution of coherent motion
- Hydrodynamics pervasive down to the micro scale
- Many-body motion would be increasingly hard to treat
- Handling systematic flows problematic

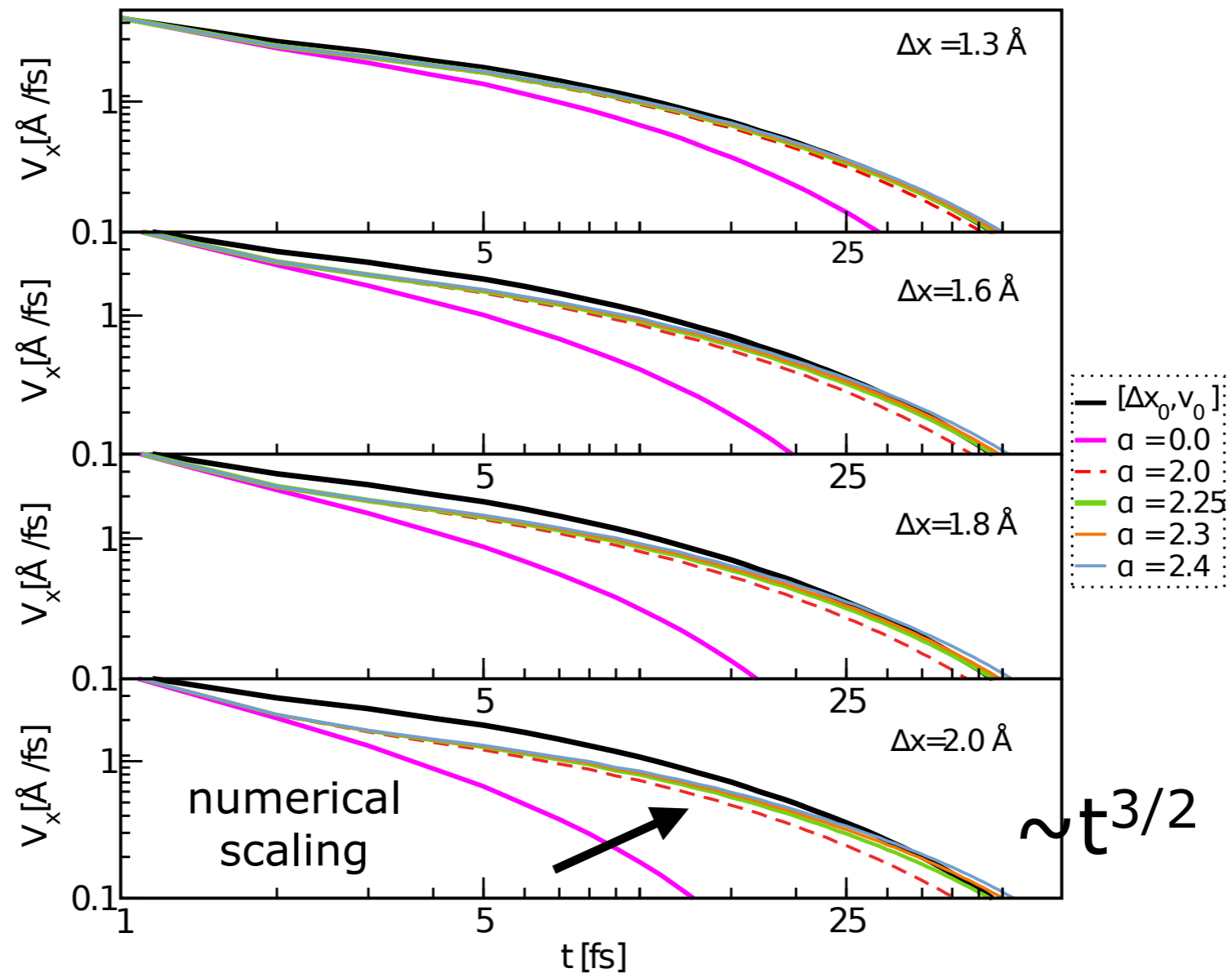
A solute in fluid

Multi-scaling

$$\nu = \nu_0 (\Delta x_0 / \Delta x)^\alpha$$

$$[\nu] = [L^2] / [t]$$

expected $\alpha=2$



Questions:

How molecularity affects dimensional scaling?

How molecule length scale affects scaling at lower and lower resolution?

A solute in fluid

Multi-scaling

Chapman-Enskog

$$\nu = c_s^2 \left(\frac{1}{\omega} - \frac{\Delta t}{2} \right)$$

LB :: viscosity
MD :: timestep

1 monomers $A\beta_{16-22}$
 $T=0$ K
 $v(t=0) = 10^{-4}$ A/fs
 $\Delta x = 1$ A

Reference Relaxation

$\tau_0 = 1$ fs

$\nu_0 = 0.1666 \rightarrow 0.1666 A^2/fs$

Relaxation 1

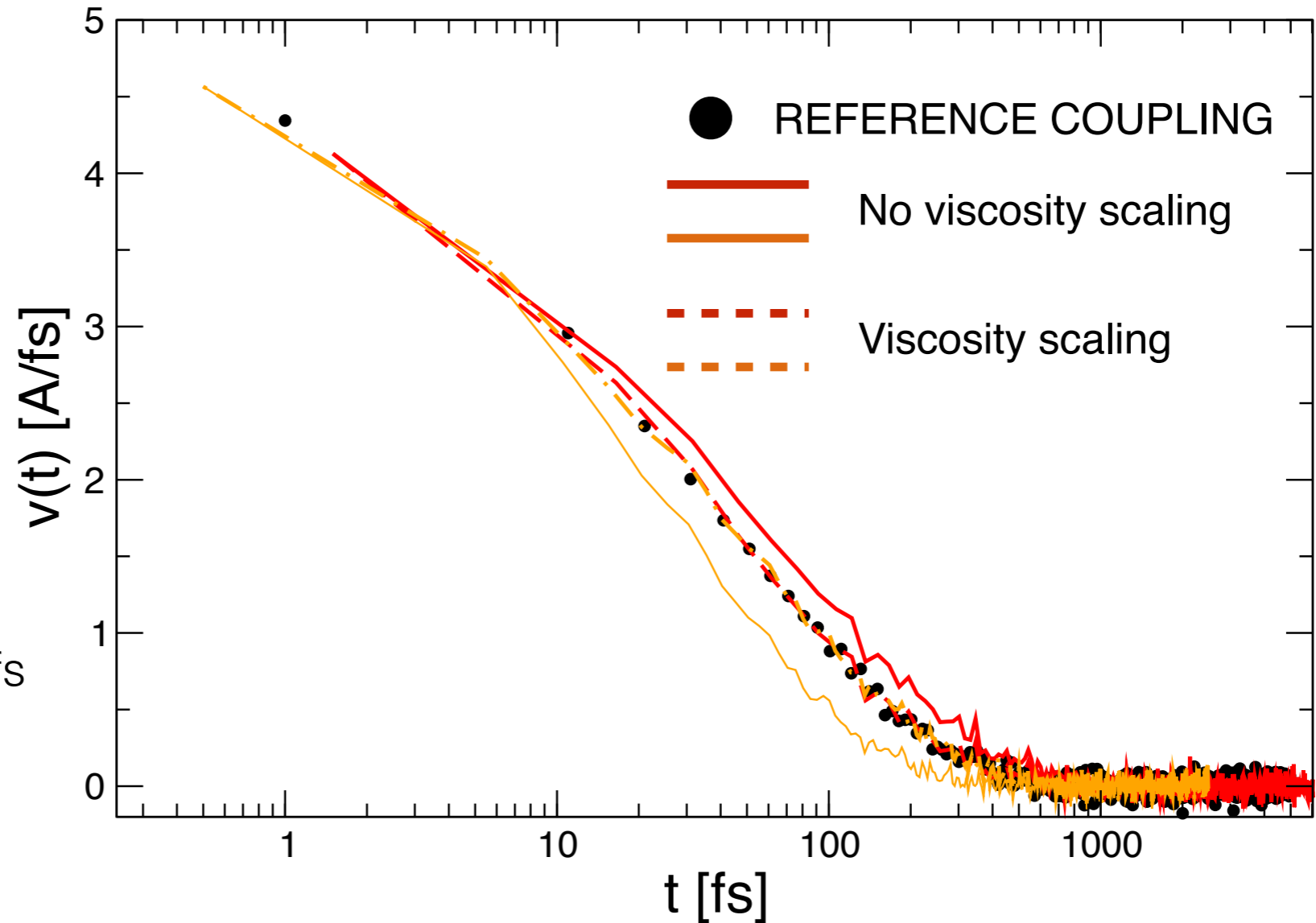
$\tau = 1.5$ fs

$\nu = 0.1666 [\Delta X^2/\Delta t]$

Relaxation 2

$\tau = 0.5$ fs

$\nu = 0.1666 [\Delta X^2/\Delta t]$



Scaling

$$\nu = \nu_0 (\tau / \tau_0)$$

Brownian Dynamics

The phase-space distribution function $W(\{r\},\{p\},t)$

The Fokker-Plank eq

$$\rightarrow D_t W = \sum_{i,j} \frac{\partial}{\partial p_i} \gamma_{ij} (m_j^{-1} p_j W + kT \frac{\partial W}{\partial p_j})$$

friction tensor incorporating hydrodynamic interactions
(2 body)

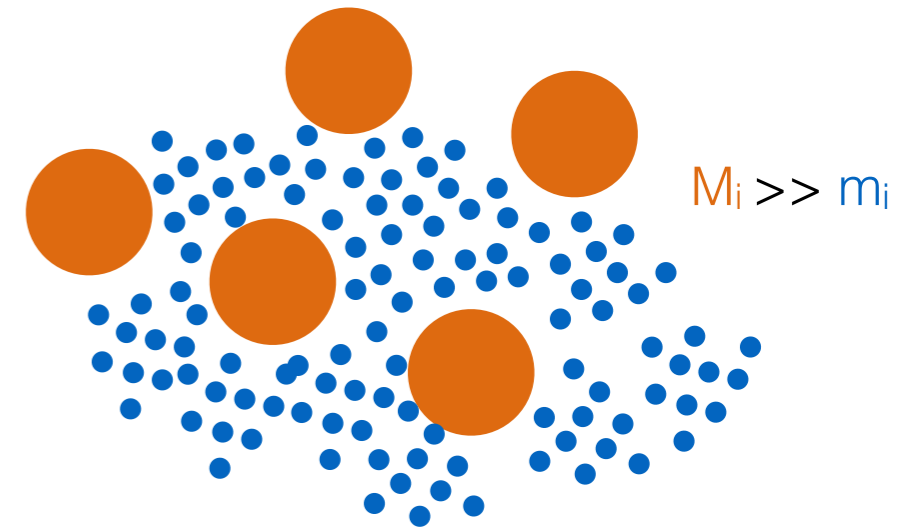
From the Fokker-Plank eq to the Diffusion eq, $W(\{r\},t)$

$$\rightarrow \frac{\partial W}{\partial t} = \sum_{i,j} \frac{\partial}{\partial r_i} D_{i,j} \left(\frac{\partial W}{\partial r_j} - \frac{1}{kT} F_j W \right)$$

Diffusion tensor
related to friction

inter-particles + external

N Brownian Particles in Fluid



$M_i \gg m_i$

main assumptions

p_i relaxes more rapidly than r_i

F_i smooth for Δr_i

$v(t+\Delta t)$ and $v(t)$ uncorrelated

time

$t=0$

solution at first order of $W(r,t+\Delta t)$

extract $\{r\}$ from W

.....

$\{r\}$

$$\langle \Delta r_i(\Delta t) \rangle = \sum_j \left(\frac{\partial D_{ij}}{\partial r_j} + \frac{D_{ij}}{kT} F_j \right) \Delta t$$

$\{r\}$

$W(r,0)$

$W(r,t)$

$$\langle \Delta r_i(\Delta t) \Delta r_j(\Delta t) \rangle = 2D_{ij} \Delta t$$

Brownian Dynamics

Ermak-McCammon

Time evolution of particles

Langevin eq + Hydrodynamics

$$\rightarrow M_i \dot{v}_i = - \sum_j \gamma_{ij} v_j + F_i + \sum_j \alpha_{ij} f_j$$

white noise

friction tensor
for hydrodynamic interactions
(2 body)

N Brownian Particles in Fluid



$$r_i = r_i^0 + \sum_j \frac{\partial D_{ij}^0}{\partial r_j} \Delta t + \sum_j \frac{D_{ij}^0 F_{ij}^0}{kT} \Delta t + \underline{R_i(\Delta t)}$$

Evaluated before displacement

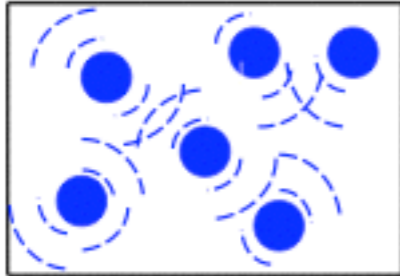
Random Displacement
(Depends on $(D_{ij})^{1/2}$)

Problem : Functional form for D_{ij}



Brownian Dynamics

idea : sphere perturbs a fluid, this in turn acts on a probe sphere at distance r



expansion for $r \gg a$ (far field contribution)
NB a term for near field ($r < 2a$) can be added!
(Lubrication Forces)

Oseen Tensor

$$D_{ij} = \frac{kT}{6\pi\eta a} \delta_{ij}$$

$$D_{ij} = \frac{kT}{6\pi\eta r_{ij}} \left(I + \frac{\mathbf{r}_{ij}\mathbf{r}_{ij}}{r_{ij}^2} \right)$$

$$\sum_j \frac{\partial D_{ij}}{\partial r_j} = 0$$

Rotne-Prager Tensor

$$D_{ij} = \frac{kT}{6\pi\eta a} \delta_{ij}$$

$$D_{ij} = \frac{kT}{8\pi\eta r_{ij}} \left[\left(I + \frac{\mathbf{r}_{ij}\mathbf{r}_{ij}}{r_{ij}^2} \right) + \frac{2a^2}{r_{ij}^2} \left(\frac{1}{3} I - \frac{\mathbf{r}_{ij}\mathbf{r}_{ij}}{r_{ij}^2} \right) \right]$$

Brownian Dynamics

Random Displacement

Hydrodynamically Correlated Motion

$$\langle R_i(\Delta t) \rangle = 0$$

$$\langle R_i(\Delta t) R_j(\Delta t) \rangle = 2D_{ij} \Delta t$$

Long Range Nature HI $\sim 1/r$

$$\mathbf{D} = \mathbf{B}\mathbf{B}^T$$

$$\vec{R} = \mathbf{B}\vec{X}$$

Cost N^3

SQRT of D

X vector of $3N$ random numbers

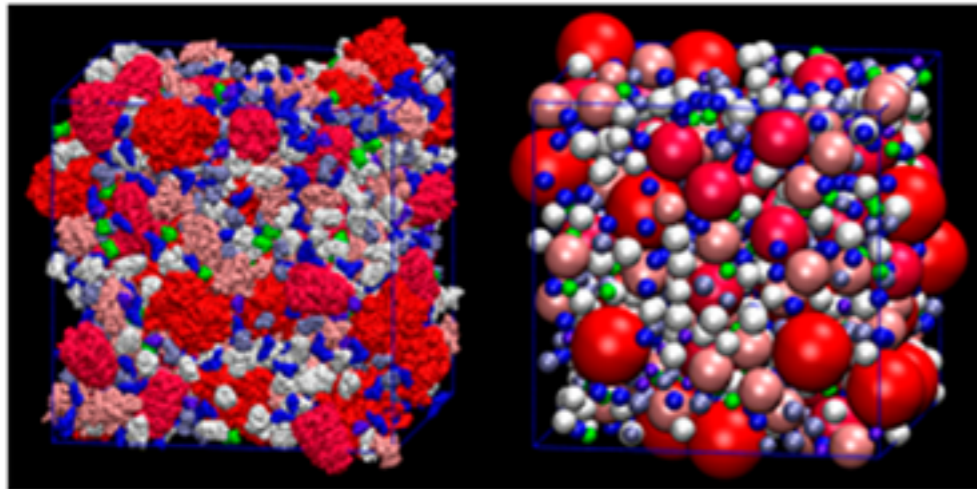
Strategies

Mean Field (from $D_{ij} \rightarrow D_i$)
Screening (remove long range r_{ij})
Long range part as PME

Crowding in silico

Skolnick group

Cell Crowding



● macromolecules as spheres
short range LJ interactions

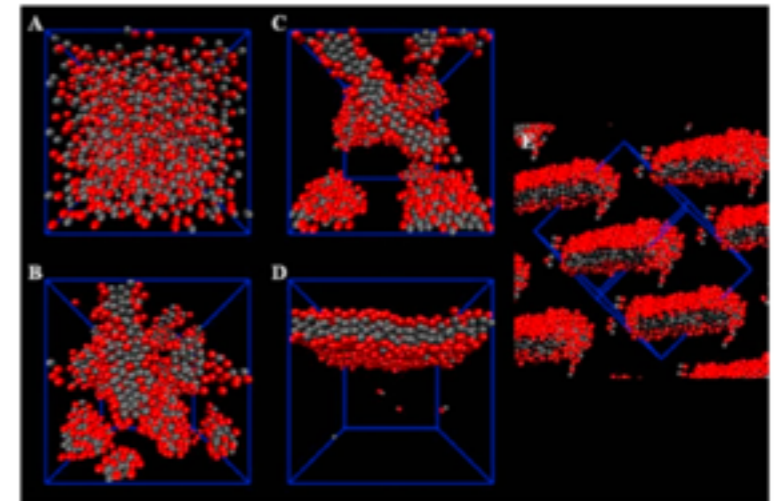
concentration 250/300/350 mg/mL
np 400 - 1200

Stokesian Dynamics
(BD + far + near + mb interactions)

$$D^L/D^0 \sim 0.1 \text{ for } c=300 \text{ mg/mL}$$

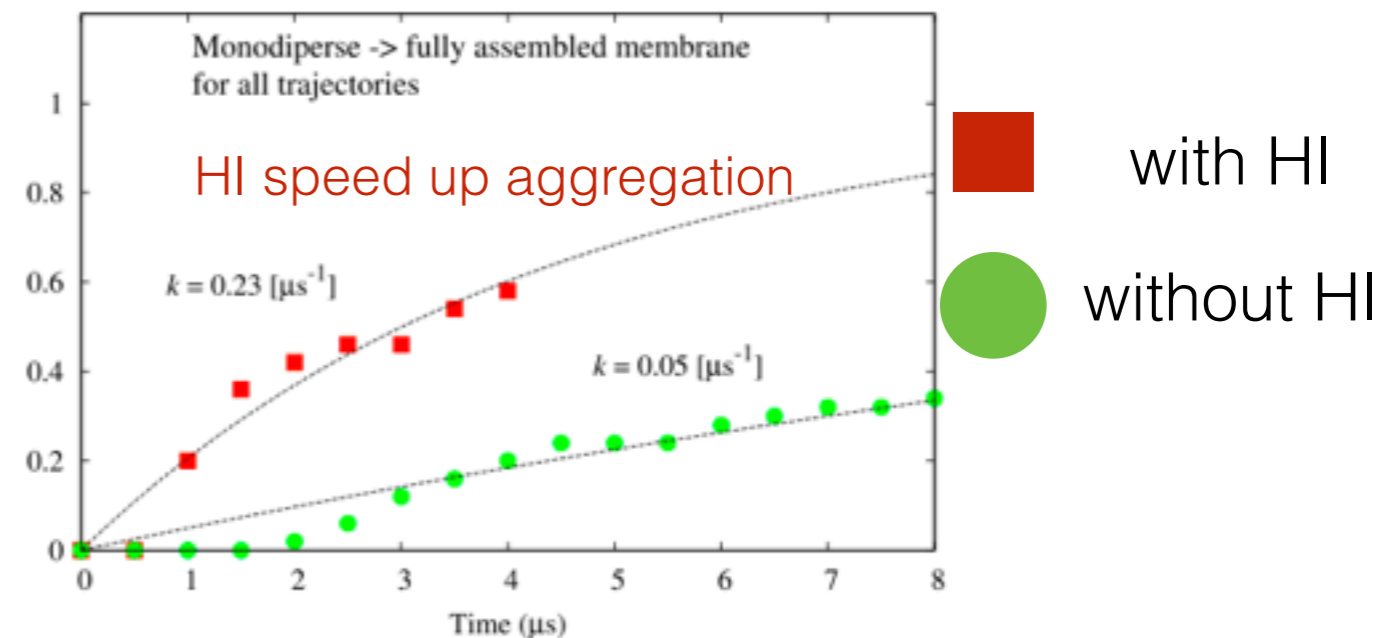
Ando&Skolnick, PNAS (2010), BJ (2013)

Lipid Aggregation



● ● lipid as dumbbell model

BD + Rotne-Prager-Yamakawa



Muphy/OPEP: Protein Relaxation

Tuning the coupling

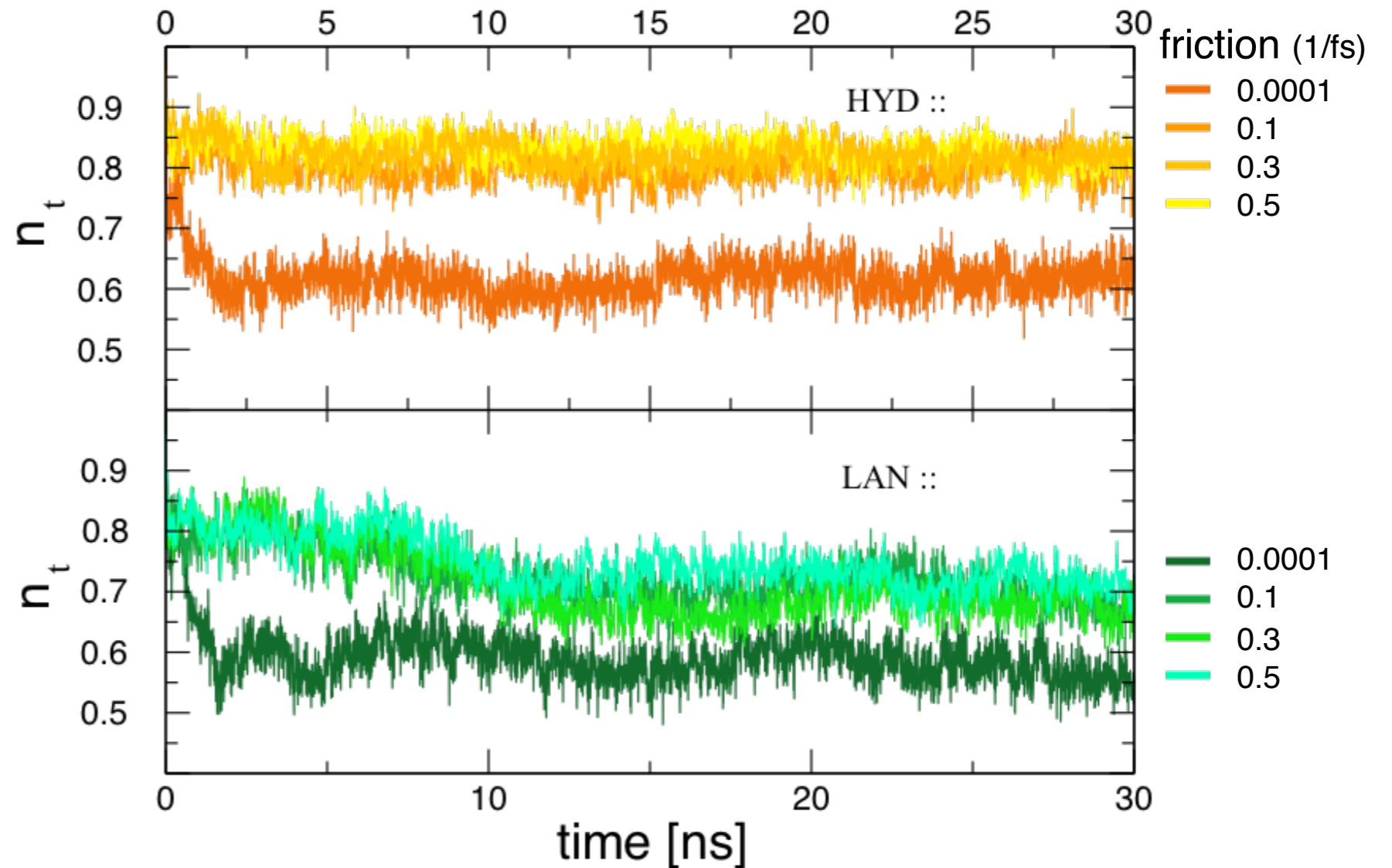
$$\mathbf{F}_{drag,i} = -m\gamma(\mathbf{v}_i - \mathbf{u}_i)$$

Hydrodynamics

Fluctuations around initial state
Drift at weak coupling

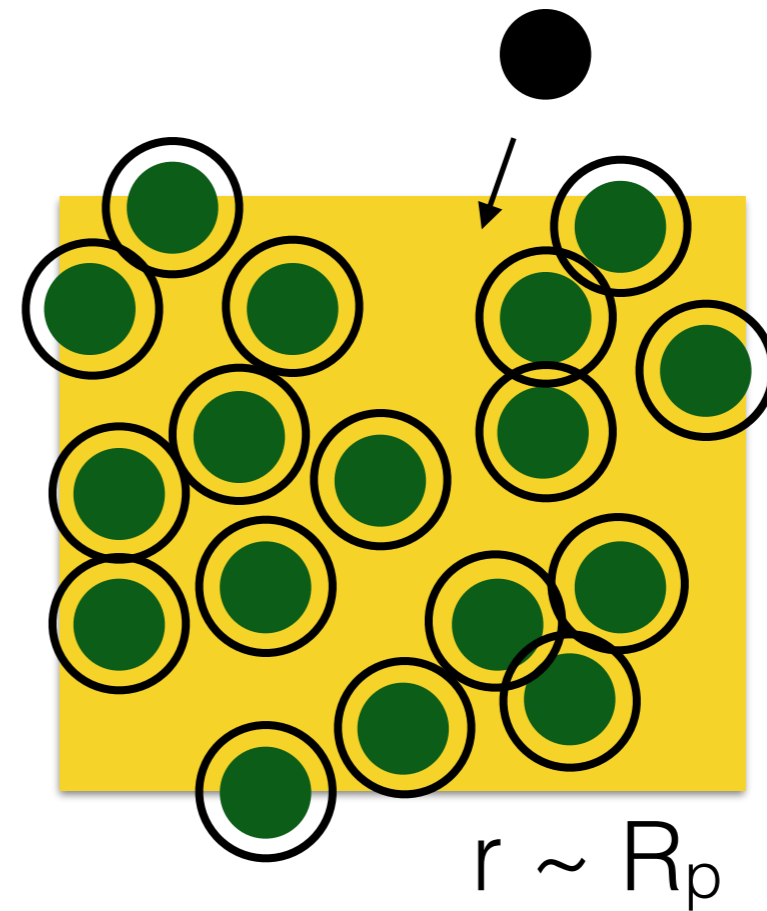
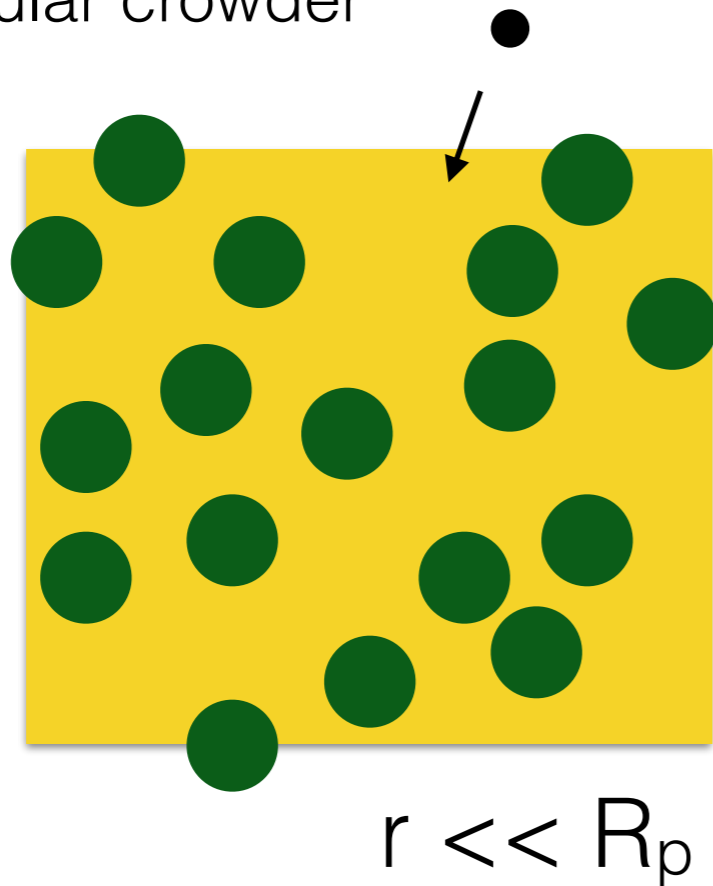
Langevin Dynamics

Drift for all coupling



Fluid pumps/dissipates energy and controls solute relaxation

● Molecular crowder



in Cell 20-40 % of volume is occupied by macromolecules

the effect of excluded volume is non-linear with size of “probe”

chemical potential

$$\mu_i = \mu_i^{ideal} + \mu_i^{nonideal}$$

effect of concentration

$$\mu_i^{ideal} = \mu_i^0 + kT \ln c_i$$

effect of intermolecular interactions

$$\mu_i^{nonideal} = kT \ln \gamma_i$$

macromolecules get closer under crowding



free energy of interactions between
species i and the other
macromolecules

Crowding enhances the effect of concentrations

$$kT \ln \gamma_i c_i$$

thermodynamic activity

free energy of confinement

$$\Delta F^c = -kT \frac{Z^0}{Z^c}$$

partition function

$$\int_{\Omega} dr^{3N} e^{-\beta U}$$

 volume accessible

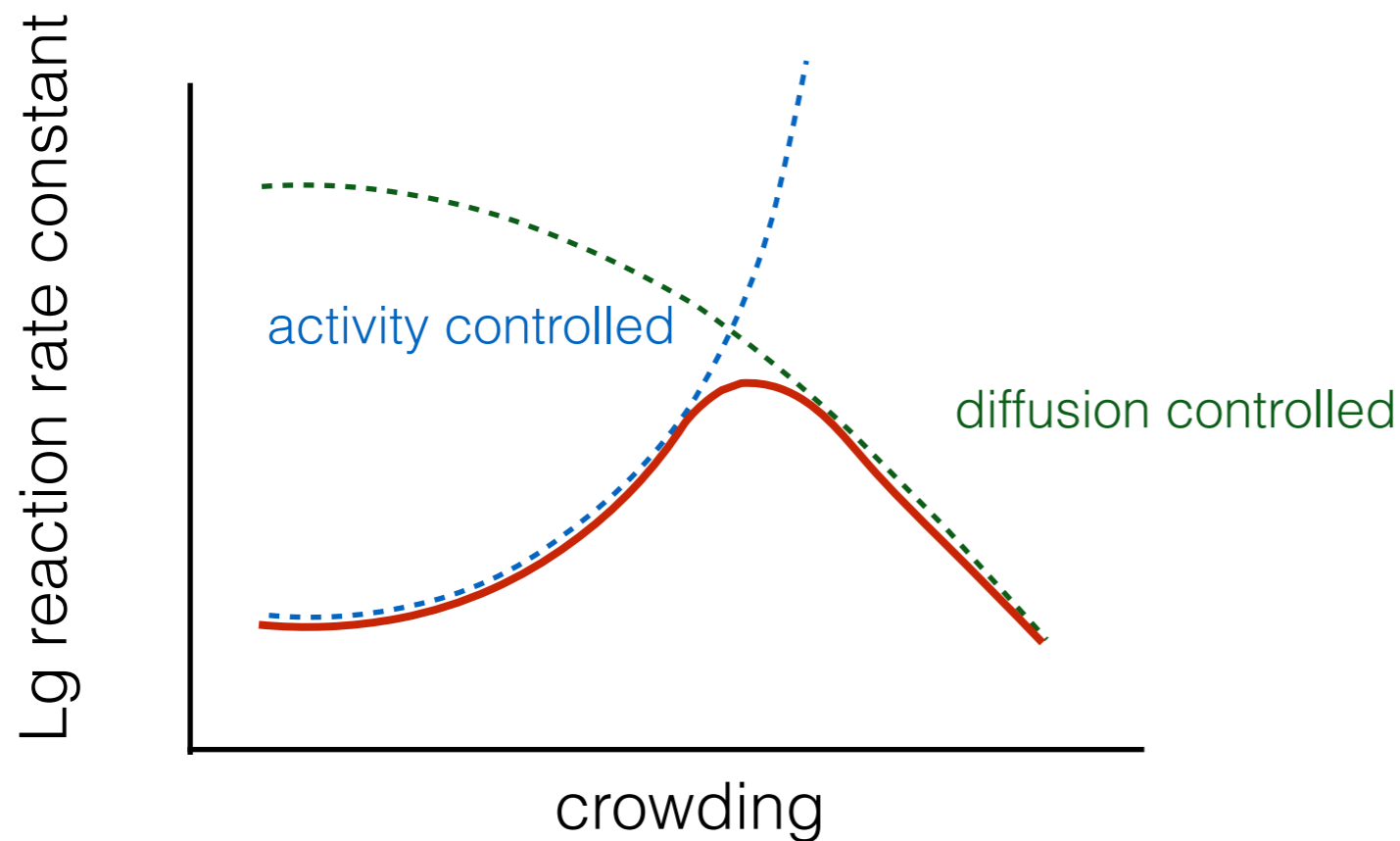
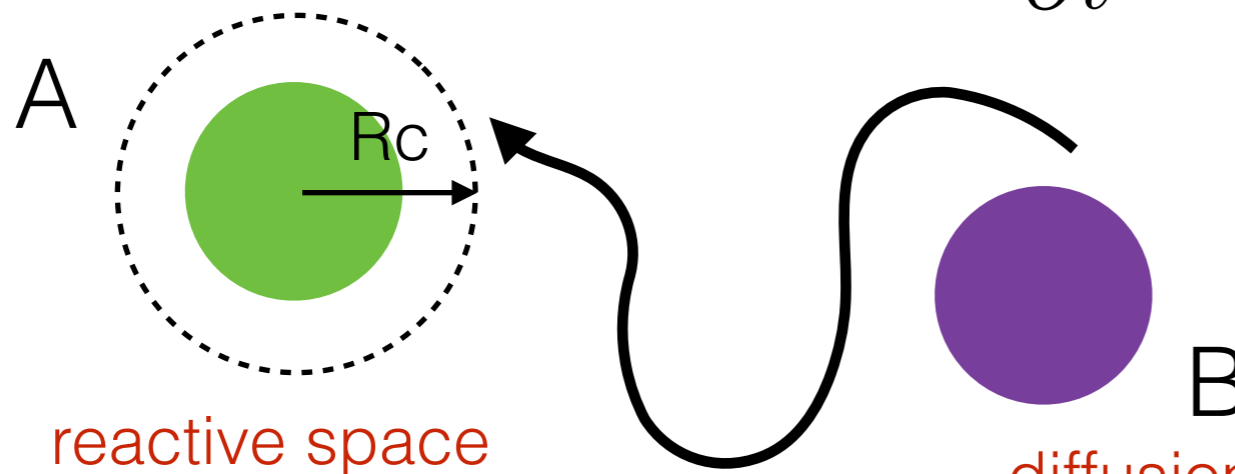
Crowding affects both thermodynamics and kinetics

overall exclude volume

exclude volume of TS configurations

species A and B react (k_s)

$$\frac{\partial C_B}{\partial t} = \underbrace{D \nabla^2 C_B}_{\text{diffusive term}} - \underbrace{k_s C_B C_A}_{\text{reactive term}}$$

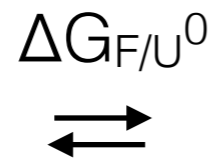
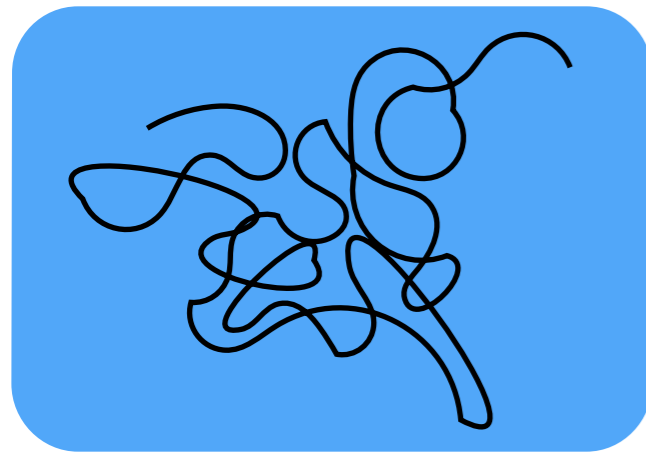


crowding enhances activity
crowding reduces diffusivity

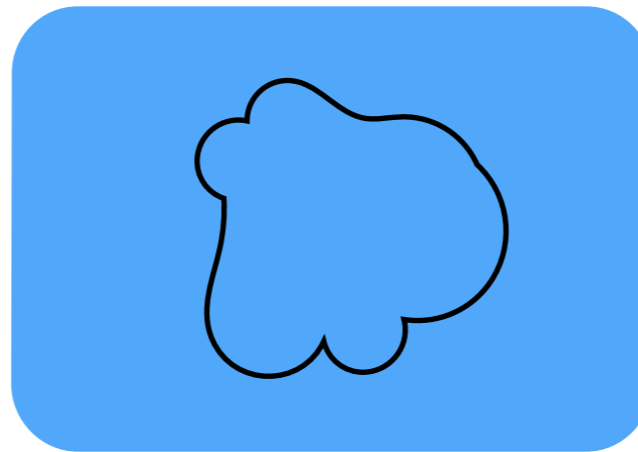
Protein stability

crowding

unfolded state



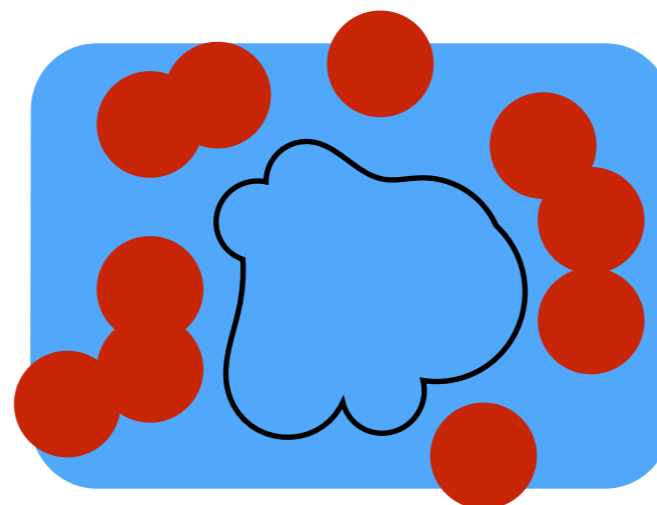
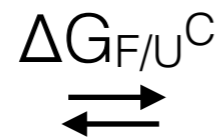
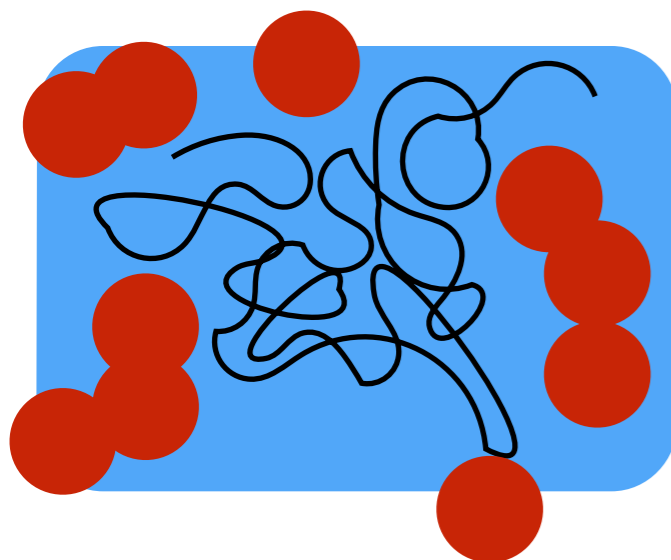
folded state



 dilute solution



stability under crowding
reduces to estimate
free energy of confinement
for Folded and Unfolded States

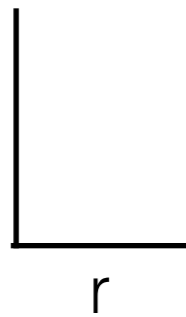


 crowders

Confining the folded state

Scaled Particle Theory

hard sphere



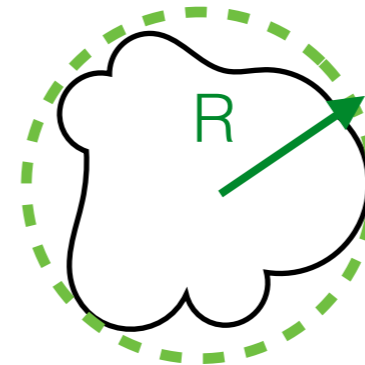
$$V(r) = \infty \quad r < \sigma$$

$$V(r) = 0 \quad r > \sigma$$

$$\Delta G_F^{Conf} = -kT \ln(1 - \phi) + \sum_{i=1}^3 A_i Q^i$$

$$Q = \phi / (1 - \phi)$$

$$A_i = f(R)$$

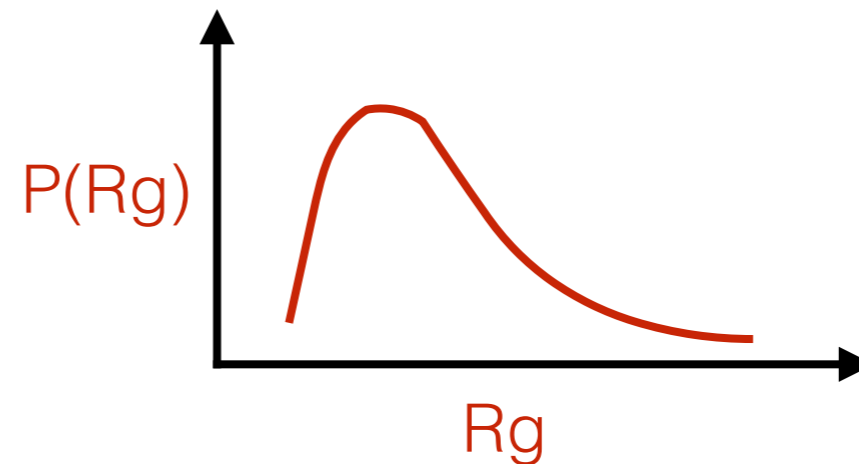
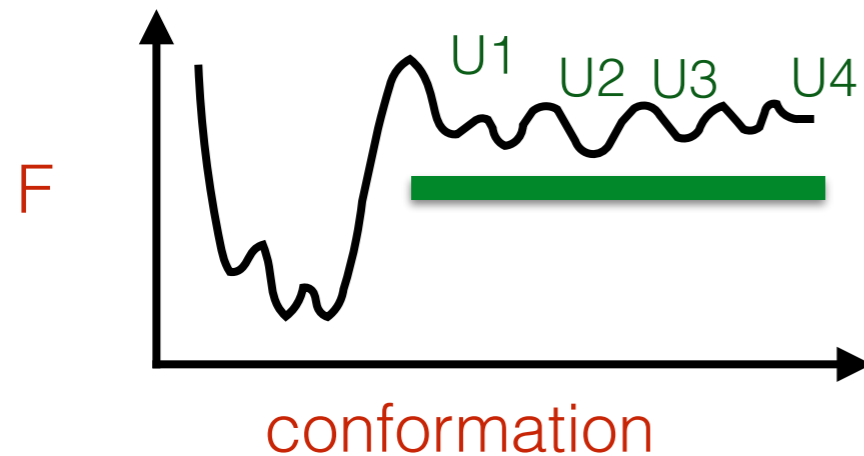


folded state == globular state

Free energy to transfer an ideal spherical particle of radius R in environment with occupied volume ϕ

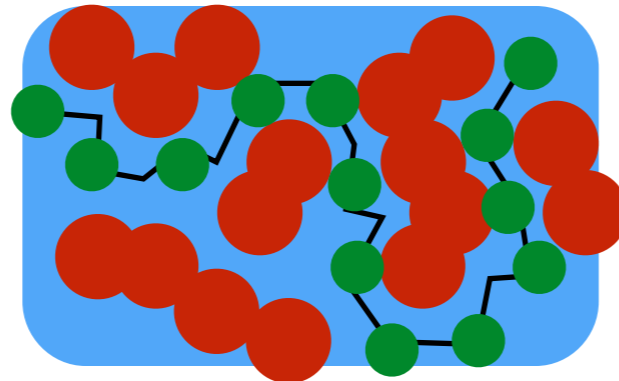
Confining the unfolded state

PROBLEM : unfolded state NOT globular



STRATEGIES

Random Walk
in crowded space



low ϕ Folded is stabilised
high ϕ Unfolded is stabilised

Generate representative Unfolded states from $P(R_g)$
and try to insert in the crowded space or compute Excluded Volume