Collective effects in light scattering: from Dicke Sub- and Superradiance to Anderson localisation

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Conference on Long-Range-Interacting Many Body Systems: from Atomic to Astrophysical Scales
Trieste, Italy July 25th – 29th 2016
Dicke vs Anderson

plasma physics / pattern formation

astrophysics
(self-oscillations, random lasing, Lévy flight of photons)

Nature Photonics 8, 321 (2014)

Nature Physics 9, 357 (2013)
Long range light-matter interactions:

Effects on atomic motion

$N \approx 10^{10}$
$T \approx 100 \mu K$
Mechanical Effects of Multiple Scattering of light

\[ F_{ij} = \frac{q_{eff}^2}{4\pi\varepsilon_0 r^2} \]

long range component \((C_3/r^3, 1/r^2, 1/r)\) of resonant dipole-dipole interaction
MOT size:

‘One Component Plasma’

bad for BEC $\Rightarrow$ Multiple scattering to be avoided...

from Phys.Rev. Lett. 64, 408 (1990)
Self Sustained Oscillation of MOT

« Cepheid » type instability:
Unstable Competition between compression and radiation pressure induced repulsion

complex spatio-temporel evolution!

G. Labeyrie, F. Michaud, R. K.

T. Pohl, G. Labeyrie, R. K.
Photon bubbles

\[
\frac{\partial I}{\partial t} - \nabla \cdot (D \nabla I) = -\gamma_a I
\]

\[
\frac{\partial n}{\partial t} + \nabla \cdot (nv) = 0
\]

\[
\frac{\partial v}{\partial t} + (v \cdot \nabla)v = \frac{F}{m} - \frac{\nabla P}{nm} - \nu v
\]

\[
\nabla \cdot F = q_n
\]


Photon bubble experiments (in Lisbon and Nice)
Looking at the internal degrees of freedom of the atoms
Multiple Scattering of Light in Atomic samples: Disorder vs cooperative effects

Anderson Localization

Multiple Scattering

“Local”

Interferences

“Global”

Dicke States
The case for Anderson:

‘Random walk of photons’
Wave propagation in disordered media:

< 1958: on average: interferences washed out: random walk / diffusion
  Light: radiation trapping in stars
  Electrons: metal (Drude model)

1958: P.W. Anderson: vanishing diffusion for strong disorder!

- Solid State Physics:
  Metal-Insulator Transitions for electrons

- Light Scattering:
  Semiconductor powder, White Paint, Atoms

- Matter Waves:
  BEC in Disordered Potential, Kicked Rotator

- Acoustics:
  Aluminium Beads

- NMR:
  Nuclear Spins
Anderson Localization of non interacting waves in 1,2 and 3D

Scaling theory of localization: Abrahams et al., PRL 42, 673 (1979)

$g$: dimensionless conductance

$\beta(g) = \frac{\partial \ln g}{\partial \ln L}$

In 3D: threshold for disorder

Ioffe-Regel criterion: $k\ell=1$

$g \approx e^{-L}$

$\frac{\partial \ln g}{\partial \ln L} \approx \ln g$

No microscopic theory
self consistent theory of localization,
numerical simulations of toy systems
Anderson Localization of Light in 3D: phase transition $\Rightarrow$ strong scattering required

**Semi-conductor powder**

D. Wiersma et al., Nature 1997
T. v. der Beek et al., PRB 85 115401 (2012)

$\Rightarrow$ Not observed so far

**White Paint**

C. Aegerter et al., EPL 2006
F. Scheffold et al., Nat. Photon. 7, 934 (2013)
Weak Localisation = precursor of strong Localisation?

Coherence after resonant scattering with atoms!

See also: M. Havey’s group

Theory:
- no “exact” solution
- diagrammatic approach

![Diagram showing R and L approximations]

Excellent agreement (no free parameter)

Towards strong localization of light: dense atomic clouds

Ioffe-Regel: \( k \ell \approx 1 \)

Dynamical Breakdown

Strong Localization of Light

Weak Localization of Light

Dynamical Breakdown

BEC

Dipole Trap (Havey, Browaeys)

\( k \ell < 1 \)

\( k \ell \approx 3 \)

\( k \ell \approx 1000 \)
Light scattering from point dipoles: $1/r$ outgoing wave
Building up a refractive index « ab inito » (from individual atoms)

\[
E_0 \rightarrow E_{sc} \rightarrow \beta_i \quad \beta_m
\]

\[
\dot{\beta}_j(t) = -\frac{i}{2} \Omega e^{i\kappa_0 \cdot r_j} + \left( i \Delta - \frac{\Gamma}{2} \right) \beta_j(t)
\]

\[
E_{sc}(r) = -\frac{\hbar \Gamma}{2d} \sum_{j=1}^{N} \beta_j \frac{e^{i\kappa_0 |r - r_j|}}{k_0 |r - r_j|}
\]

\[
\frac{\Gamma}{2} \sum_{m \neq j}^{N} \exp\left( ik_0 |r_j - r_m| \right)
\]

\(\beta_i\): amplitude of dipole i
Spherical gaussian cloud: emission diagram

Cloud of atoms

Far field emission diagram

Incoherent model (particles trajectories, scattering in ‘empty modes’)

Mesoscopic physics: Weak localization (waves beyond mean field)

S. Bromley et al., Nat. Comm. 7, 11039 (2016)
Theory: Effective Hamiltonian

\[ H_{\text{eff}} = \left( \hbar \omega_0 - i \frac{\hbar \Gamma_0}{2} \right) \sum_i S_i^z + \frac{\hbar \Gamma_0}{2} \sum_{i \neq j} V_{ij} S_i^+ S_j^- \]

- **Diagonal:**
  - On site energy
- **Off diagonal:**
  - Transport

\[ V_{ij} = \beta_{ij} - i \gamma_{ij} \quad \beta_{ij} = \frac{3}{2} \left[ -p \frac{\cos k_0 r_{ij}}{k_0 r_{ij}} + q \left( \frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^3} + \frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^2} \right) \right] \]
\[ \gamma_{ij} = \frac{3}{2} \left[ p \frac{\sin k_0 r_{ij}}{k_0 r_{ij}} - q \left( \frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^3} - \frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^2} \right) \right] \]

- Open System
- Reminiscent of Anderson Hamiltonian
- Heisenberg model with global coupling
- Long range hopping
- No decoherence (coupling to phonons, …)
Eigenvalues for N coupled dipoles

Important near field terms for high densities

$kl=0.1$

cooperative superradiance: $\Gamma_{at} \sim b_0 \Gamma_0$

superradiant pairs: $\Gamma_{at} = 2\Gamma_0$

cooperative subradiance: $\Gamma_{at} \sim \Gamma_0/b_0$

subradiant pairs: $\Gamma_{at} \sim E^{-2/3}$

$e^{ikr} \left( \frac{1}{kr} + \frac{1}{kr^2} + \frac{1}{kr^3} \right)$

- vectorial model
- scalar model

$e^{ikr}/kr$
Resonance Overlap (« Thouless »)

Scaling function $\beta(g)$

NO ANDERSON LOCALISATION FOR VECTORIAL LIGHT IN 3D?

S. Skipetrov, I. Sokolov, PRL 112, 023905 (2014)
TIME vs SPACE LOCALISATION (2D)

Spatially extended mode (vectorial case)

Spatially localized mode (scalar case)

Mode width NOT correlated to localisation length: temporal vs spatial localisation

The quest for Dicke subradiance
1954: Dicke super- and subradiant states

Fig. 1. Energy level diagram of an $n$-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$.

First experimental observation of superradiance

Feld et al. 1973
Single photon excitation / low intensity limit

\[ \Gamma_{\text{max}} \sim N \Gamma_0 \]

Subradiant pair

Superradiant pair

Extended Volume: \[ b_0 = \frac{N_{\text{at}}}{N_{\text{modes}}} \]

Cooperativity without cavity (also Random lasing)
Subradiant pairs : N=2


Forward ‘subradiance echo’ from inverted system

5ns laser pulse

\[ \tau_{\text{nat}} = 7 \text{ns} \]

Pencil shape excitation

Fragile subradiance

Single Photon Dicke subradiance for $N$ two level systems (in free space, $N \gg 2$) has **not been observed**

- **Does not** require large spatial densities (near field effect maybe even bad: Gross&Haroche 1982)
- Requires large optical densities in all directions ($b_0 \gg 1$)
- Exploits the $1/r$ **long range** dipole-dipole interaction
Time dependent experiments: coherent scattering

Superradiance = bright state
Subradiance = metastable ‘dark’ states

Numerical Simulation of N driven coupled dipoles

PRL 108, 123602 (2012)
Subradiance vs incoherent scattering

\[ t_{\text{sub}} \propto b_0 \]

\[ t_{\text{Rad.Trap.}} \propto b(\delta)^2 \]

- Does not require large spatial densities
- Requires large optical densities
- Random walk of photons (without interference)
- Diffusion equation

\[ t_{\text{Anderson}} \propto \exp\{b(\delta)\} \]
- Density Threshold ?
Experiment

N = $10^9 \ ^{87}\text{Rb}$
T = 50 $\mu$K
R = 1 mm
$\rho = 10^{11}/cc$

$b_0 = 20 \ldots 100$

detector
Experimental results

Long decay at $b(\delta) < 1$ 😊 Scaling with SYSTEM SIZE!

Increases as $b_0 = \rho \sigma L$ 😊
Single Photon Super- vs Subradiance
The ‘super’ of ‘single photon Dicke states’

Superradiant

Subradiant
Superradiance in dilute and large cloud of cold atoms

Off-axis Superradiance ≠ forward superradiance

Combining Anderson and Dicke Toy Model: Open Disordered System:
A. Biella, F. Borgonovi, R. K., G.L. Celardo, EPL, 103, 57009 (2013)

3D Anderson model on 10x10x10 lattice hoping ($\Omega$) + disorder ($W$) + opening ($\gamma$)

\[ H_0 = \sum_{j=1}^{N} E_j |j\rangle \langle j| + \Omega \sum_{\langle i,j \rangle} (|j\rangle \langle i| + |i\rangle \langle j|) \]

\[ (H_{\text{eff}})_{ij} = (H_0)_{ij} - \frac{i}{2} \sum_c A^c_i (A^c_j)^* = (H_0)_{ij} - i\frac{\gamma}{2} Q_{i,j} \]

All sites coupled to one single decay channel: $Q_{ij}=1$
Hybrid Subradiant States « decoupled » from outside world
Outlook:

- **Subradiance vs Radiation trapping**

  Radiation trapping for small beam and intermediate regimes: subradiance dominant at long times

- **Towards Anderson of subradiant Dicke states**
Perspectives:

• Anderson Localisation of Light in Cold Atoms
  With diagonal disorder / magnetic field?
  Identify experimental signatures
  Technical issues: Rb 😞 … Yb 😊

• Long range interactions and mechanical effects
  Photon bubbles
  Debye Screening
  Long range attractive forces in 3D
  (mimic ‘gravity’ in the lab?)

• Super- and subradiant states:
  fast and slow relaxation: connection to quasi-stationary states?
Precious Collaborators

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Thank you for your attention