Collective effects in light scattering: from Dicke Sub- and Superradiance to Anderson localisation

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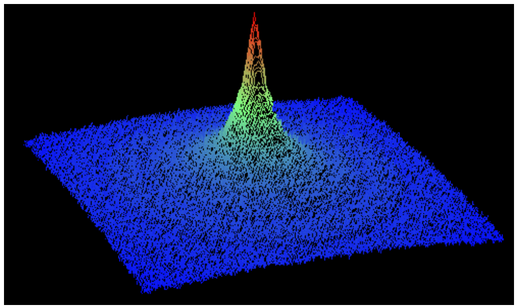
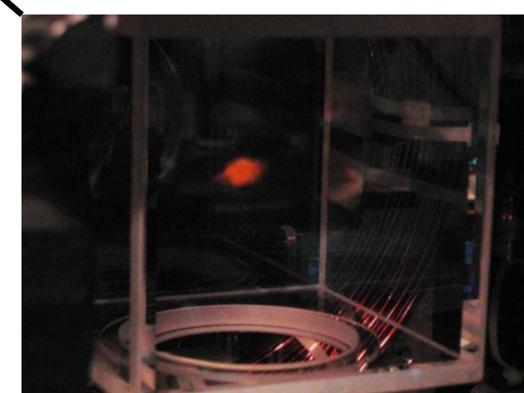
Conference on Long-Range-Interacting Many Body Systems: from Atomic to Astrophysical Scales

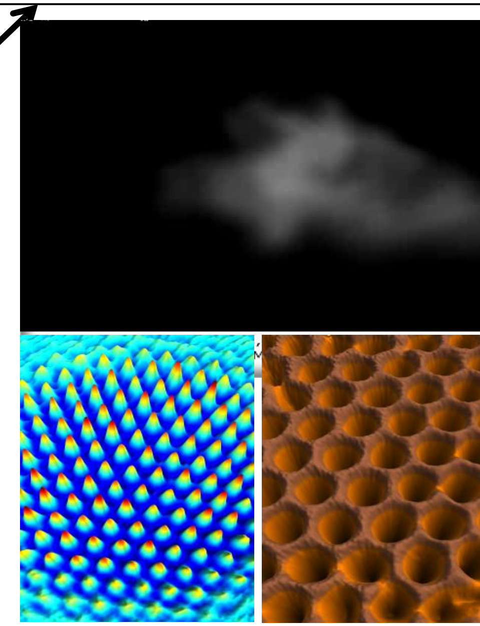
Trieste, Italy July 25th – 29th 2016

Dicke vs Anderson



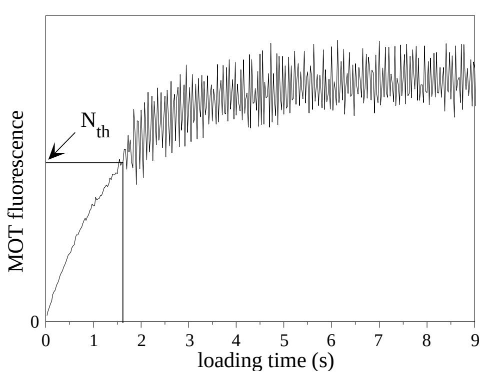
plasma physics / pattern formation

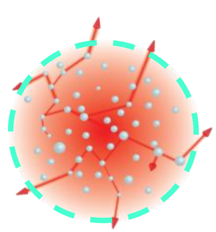


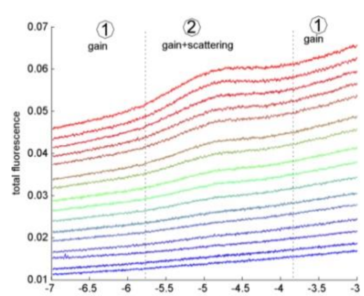


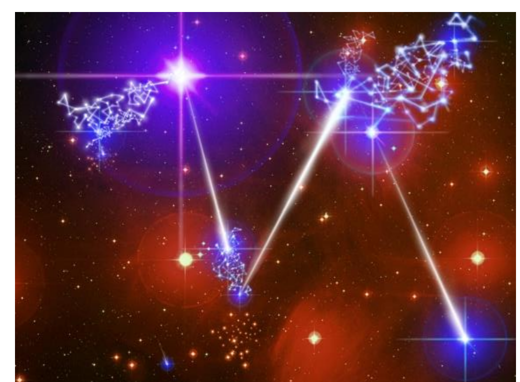
Nature Photonics 8, 321 (2014)

astrophysics
(self-oscillations, random lasing, Lévy flight of photons)





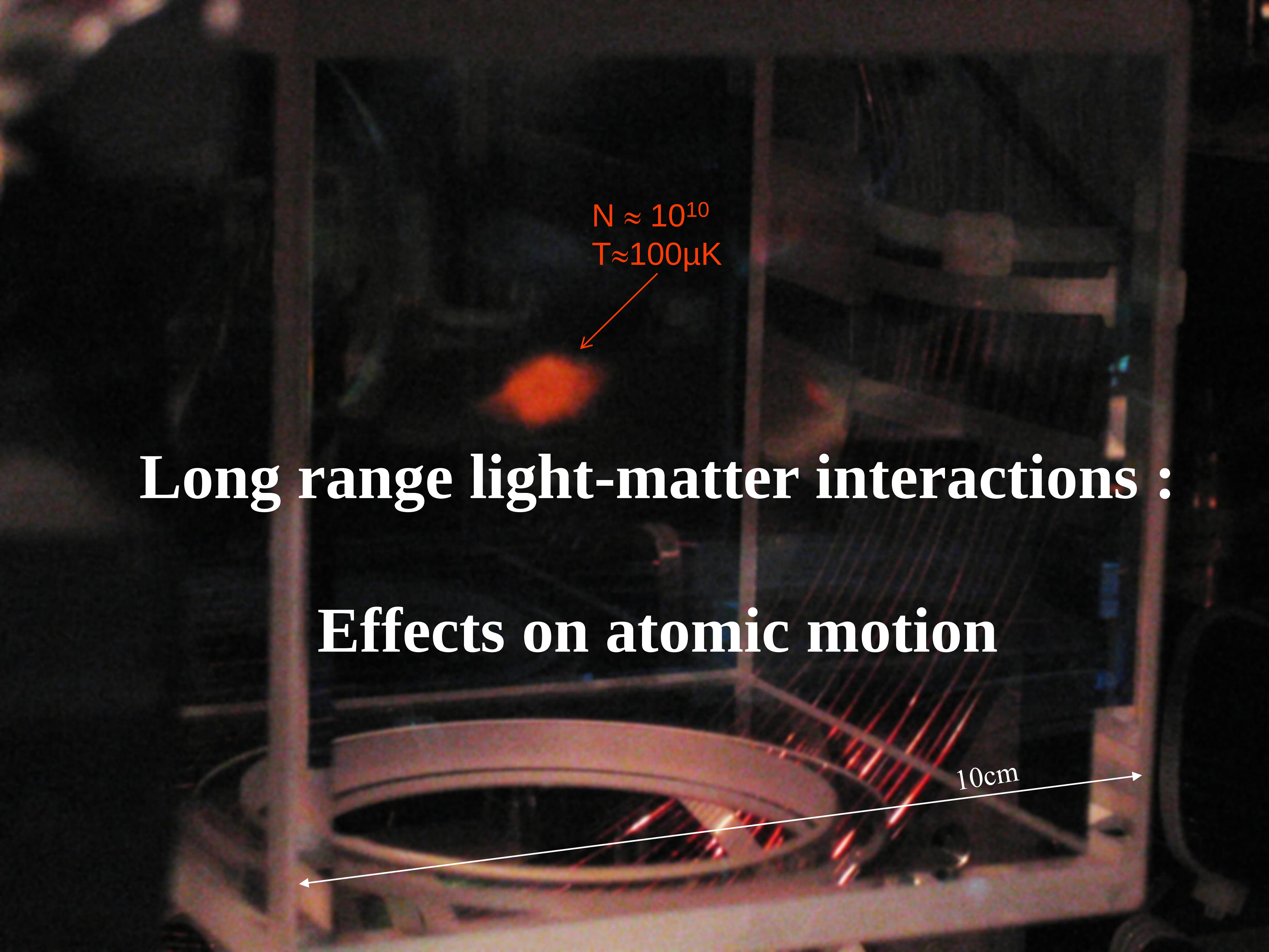




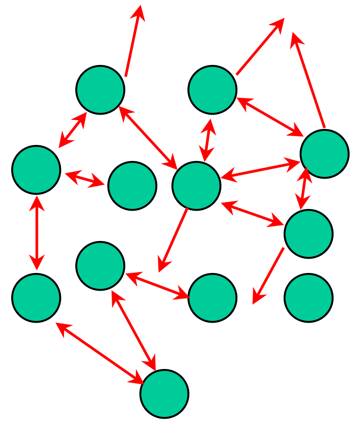
Nature Physics 9, 357 (2013)

Long range light-matter interactions :

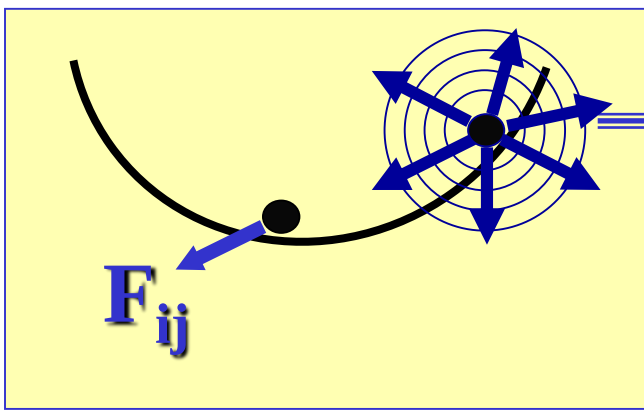
Effects on atomic motion



Mechanical Effects of Multiple Scattering of light



Coulomb type force



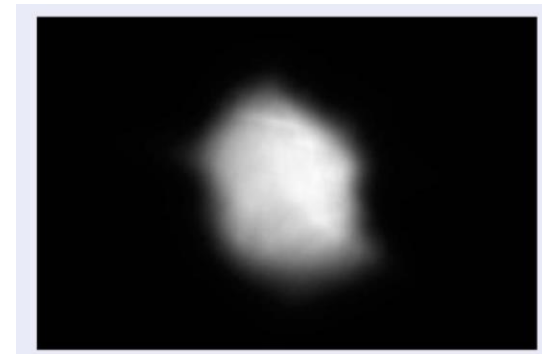
$$I_{ij} \propto P_{diff} / r_{ij}^2$$

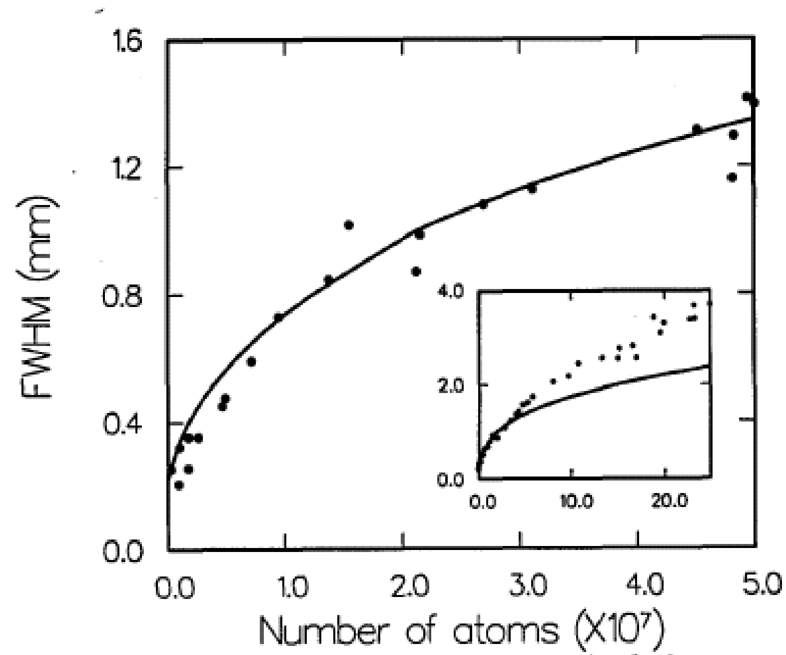
$$F_{ij} = \frac{q_{eff}^2}{4\pi\varepsilon_0 r^2}$$

long range component (C_3/r^3, $1/r^2$, $1/r$)
of **resonant dipole-dipole interaction**

MOT size :

‘One Component Plasma’



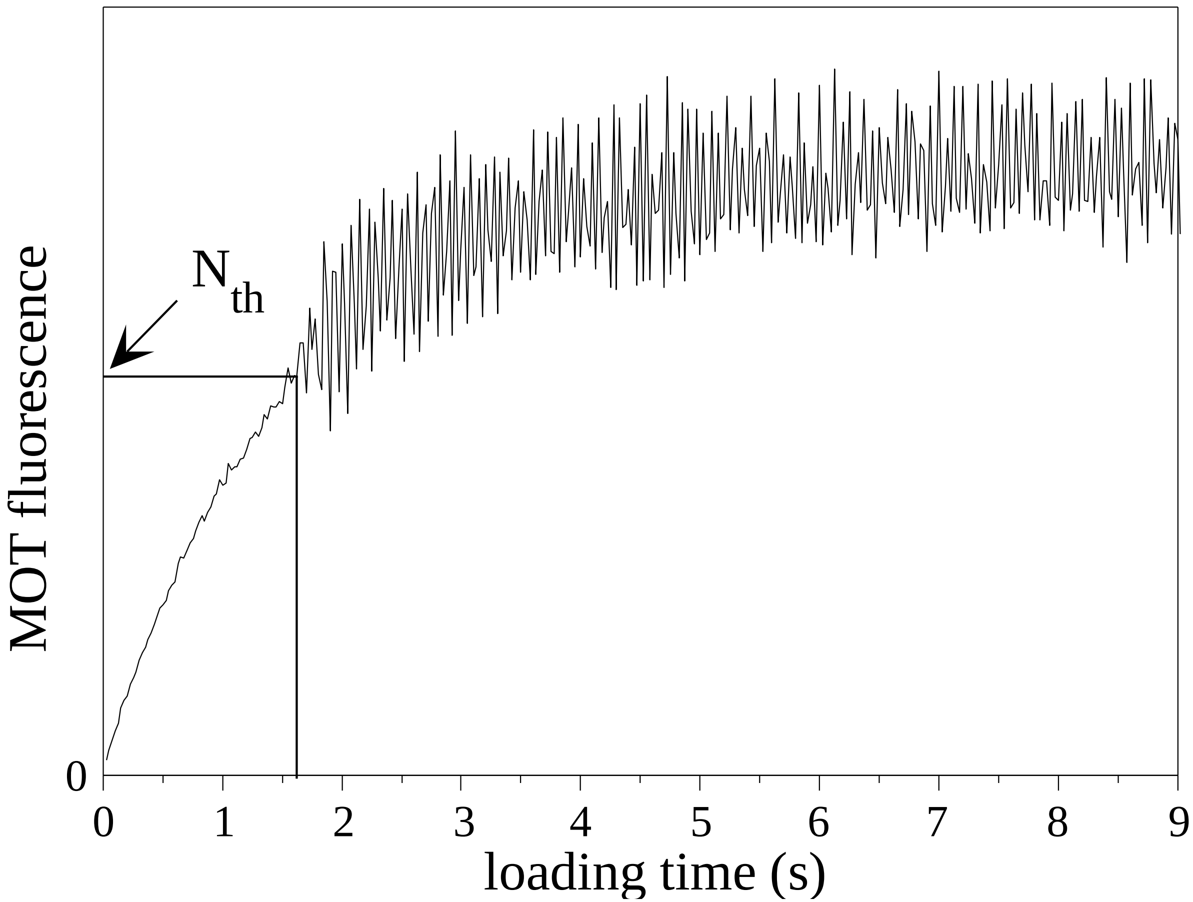


from Phys.Rev. Lett. **64**, 408 (1990)

bad for BEC ⇒ Multiple scattering to be avoided…

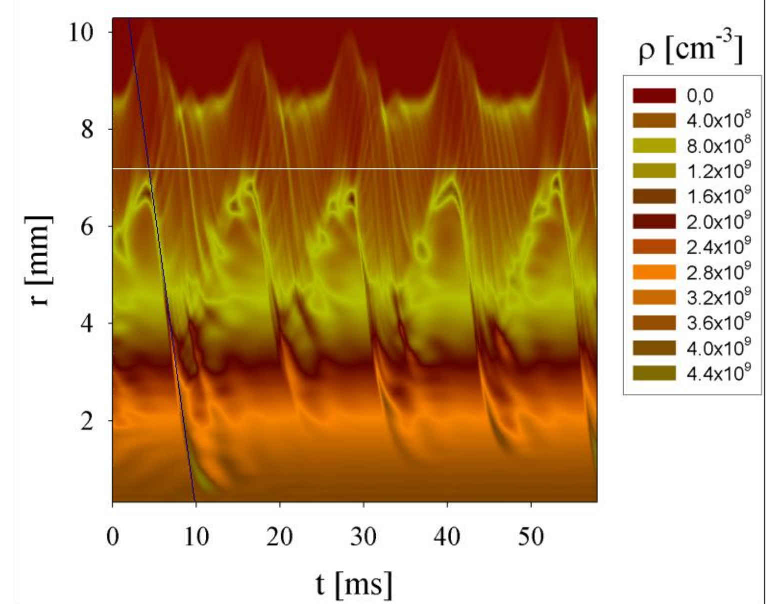
Self Sustained Oscillation of MOT

« Cepheid » type instability :
Unstable Competition between compression and radiation pressure induced repulsion



G. Labeyrie, F. Michaud, R. K.
Phys. Rev. Lett. 96, 023003 (2006)

complex spatio-temporel evolution !



T. Pohl, G.Labeyrie, R.K.
Phys. Rev. A 74, 023409 (2006)

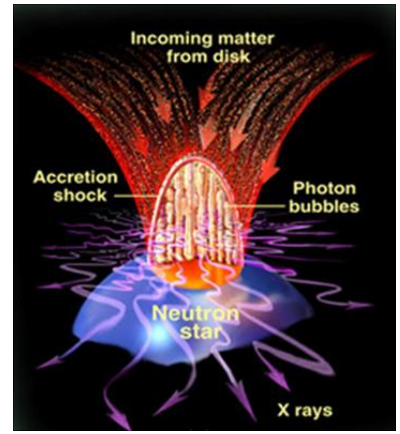
Photon bubbles

$$\frac{\partial}{\partial t} I - \nabla \cdot (D \nabla I) = -\gamma_a I$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0$$

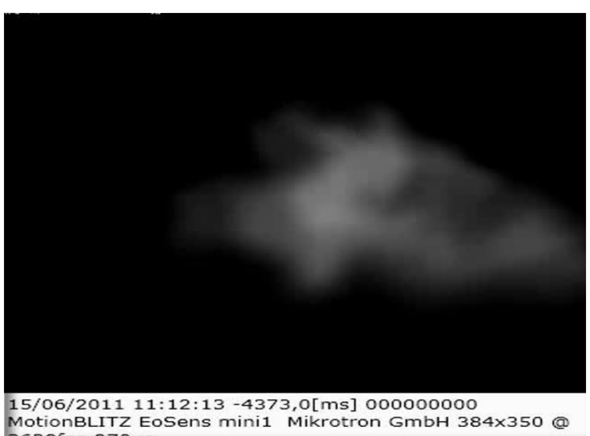
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \frac{\mathbf{F}}{m} - \frac{\nabla P}{nm} - \nu \mathbf{v}$$

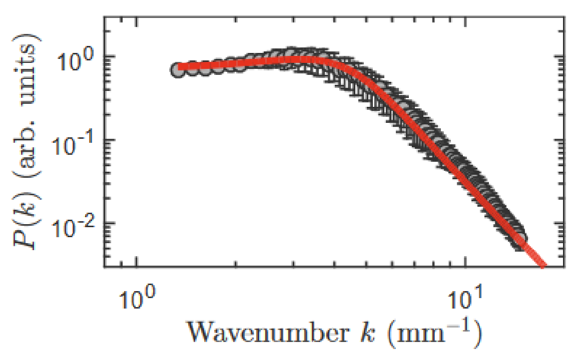
$$\nabla \cdot \mathbf{F} = Qn$$

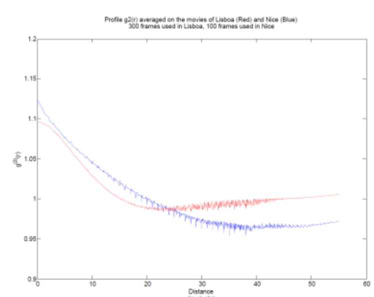


T. Mendonca, R. K., Phys. Rev. Lett,108,033001 (2012)

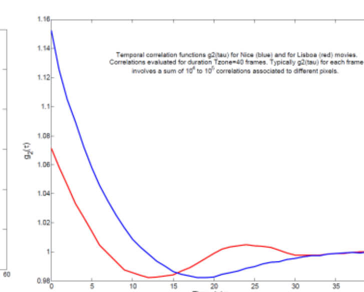
Photon bubble experiments (in Lisbon and Nice)







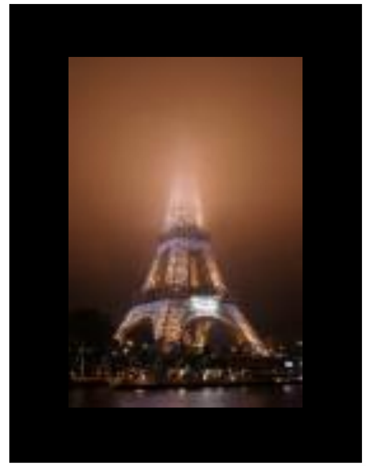
$g^2(r)$



$g^2(t)$

Looking at the internal degrees of freedom of the atoms

Multiple Scattering of Light in Atomic samples : Disorder vs cooperative effects



Anderson Localization

Multiple Scattering

Dicke States

"Local"

"Global"

Interferences

The case for Anderson :

'Random walk of photons'

Wave propagation in disordered media :

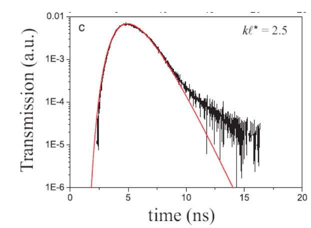
< 1958 : on average : interferences washed out : random walk / diffusion

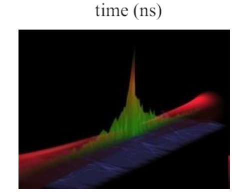
Light : radiation trapping in stars

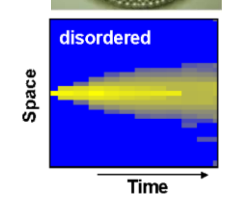
Electrons : metal (Drude model)

1958 : P.W. Anderson : vanishing diffusion for strong disorder !

- **Solid State Physics :**
  Metal-Insulator Transitions for electrons
- **Light Scattering :**
  Semiconductor powder, White Paint, **Atoms**
- **Matter Waves :**
  BEC in Disordered Potential, Kicked Rotator
- **Accoustics :**
  Aluminium Beads
- **NMR :**
  Nuclear Spins



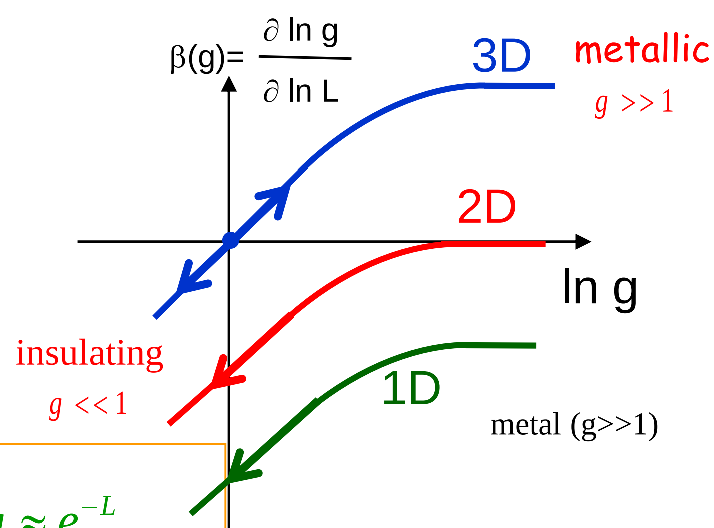




Anderson Localization of non interacting waves in 1,2 and 3D

- **Scaling theory of localization :** Abrahams et al., PRL **42**, 673 (1979)

g : dimensionless conductance



$$g \approx e^{-L}$$

$$\frac{\partial \ln g}{\partial \ln L} \approx \ln g$$

In 3D : threshold for disorder

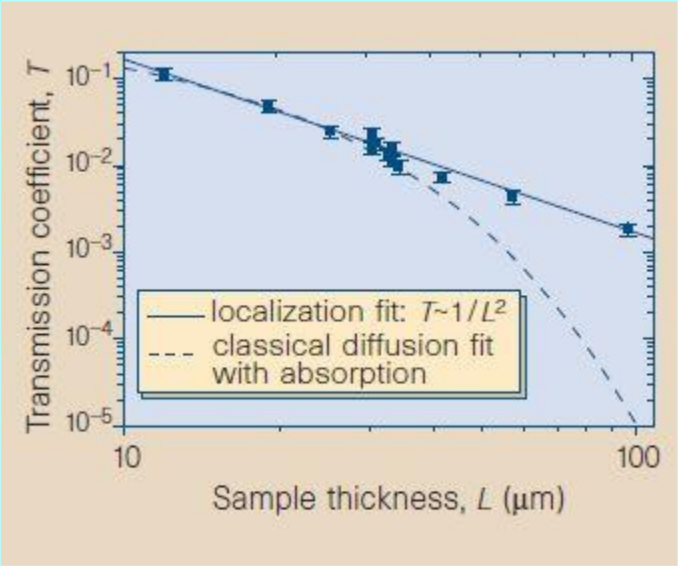
Ioffe-Regel criterion : $k\ell=1$

- **No microscopic theory**
  self consistent theory of localization,
  numerical simulations of toy systems

Anderson Localization of Light in 3D :

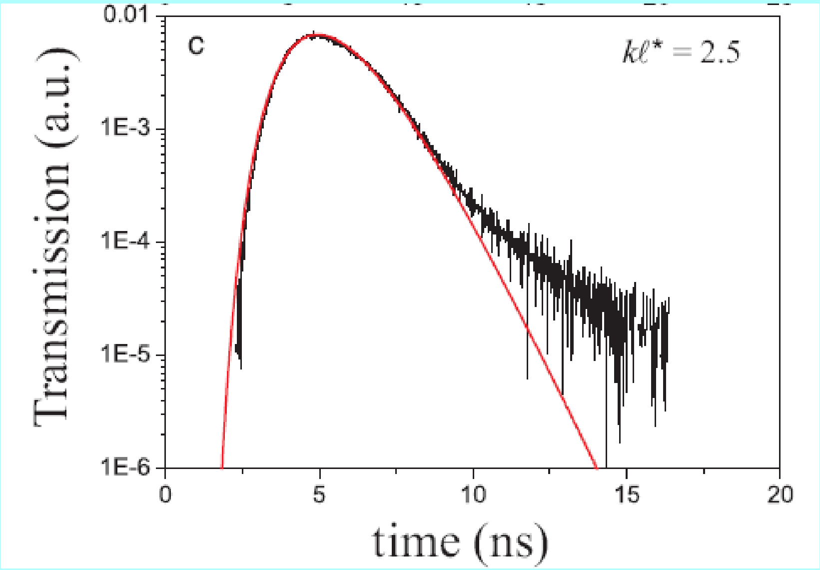
phase transition $\Rightarrow$ strong scattering required

Semi-conductor powder



D.Wiersma et al., Nature 1997

White Paint



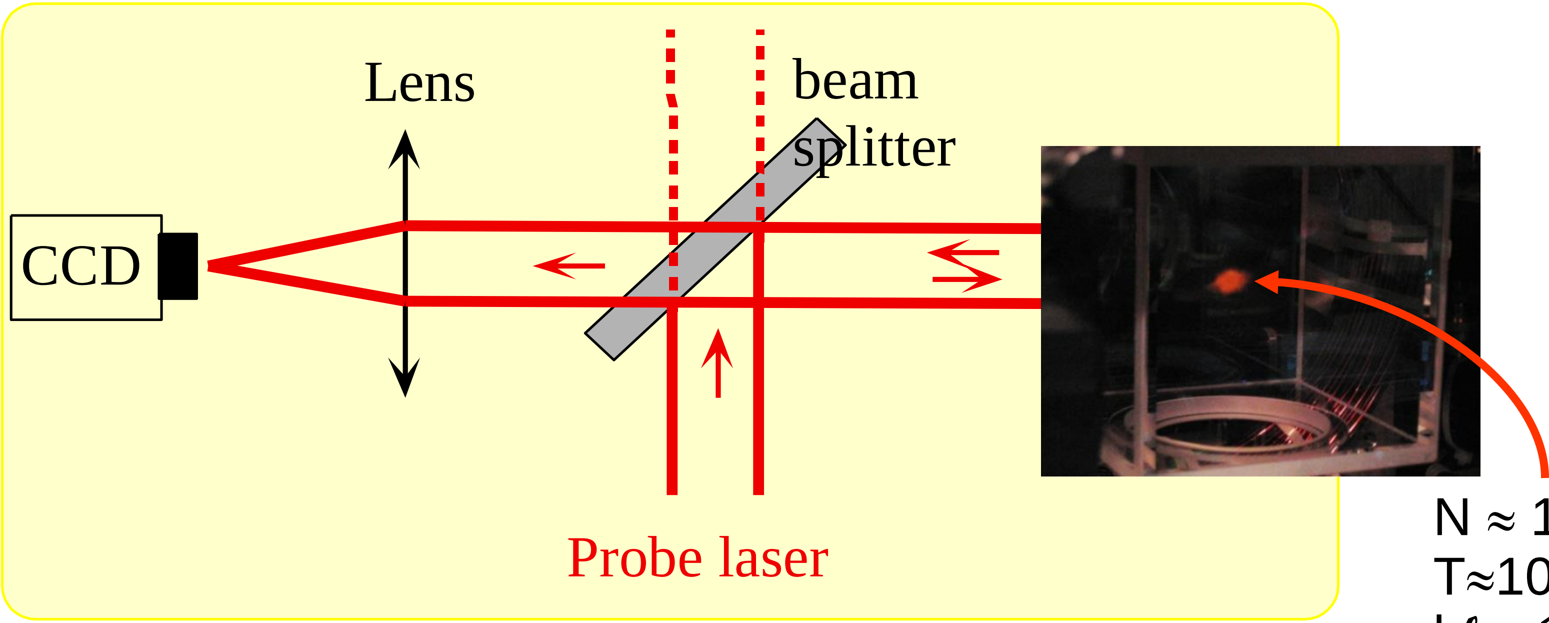
C.Aegerter et al., EPL 2006

F. Scheffold et al., Nature 398, 206(1999)
T. v. der Beek et al., PRB 85 115401 (2012)

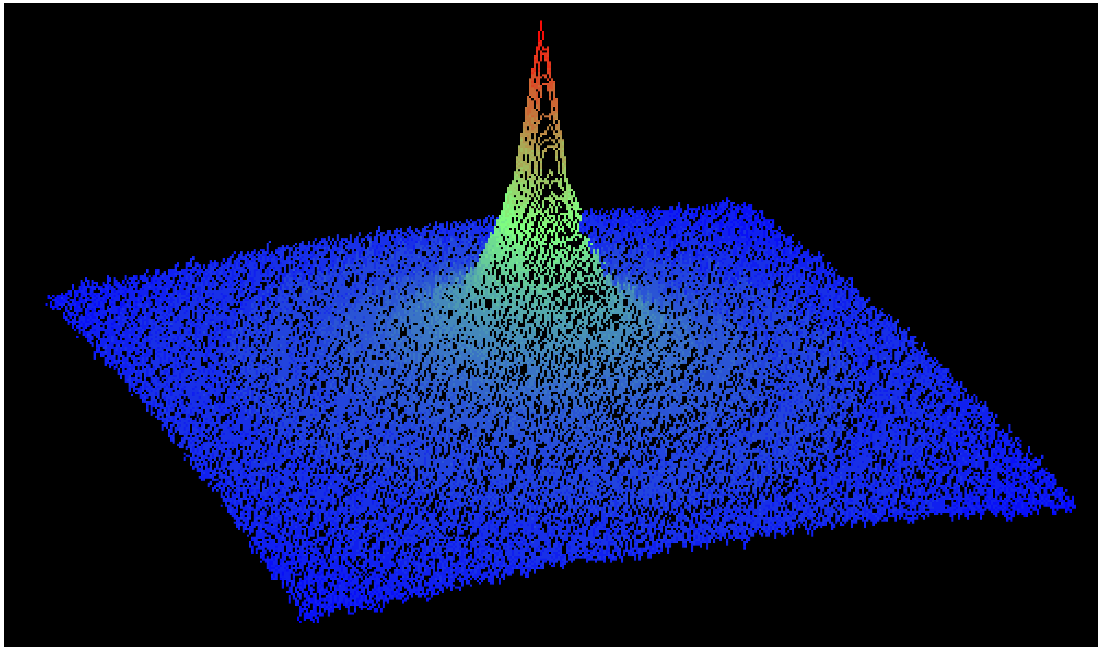
F. Scheffold et al., Nat. Photon. 7, 934 (2013)
T Sperling et al., New J. Phys. 18, 013039 (2016)

=> Not observed so far

Weak Localisation = precursor of strong Localisation?



$N \approx 10^{10}$
$T \approx 100\mu K$
$k\ell \approx 1000$



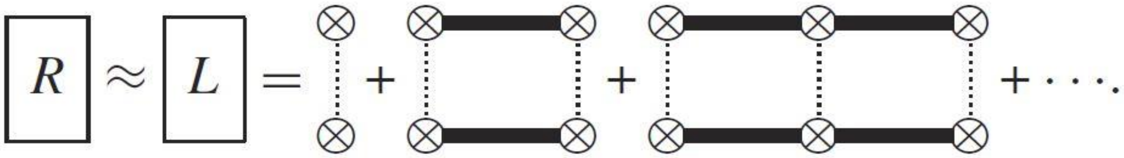
Coherence after resonant scattering with atoms !

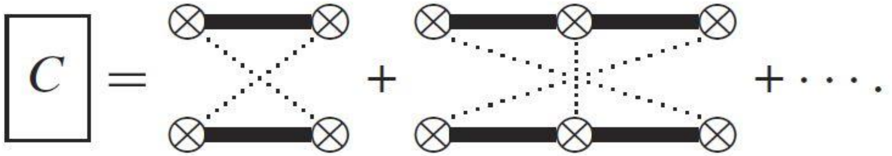
See also :
M. Havey's group

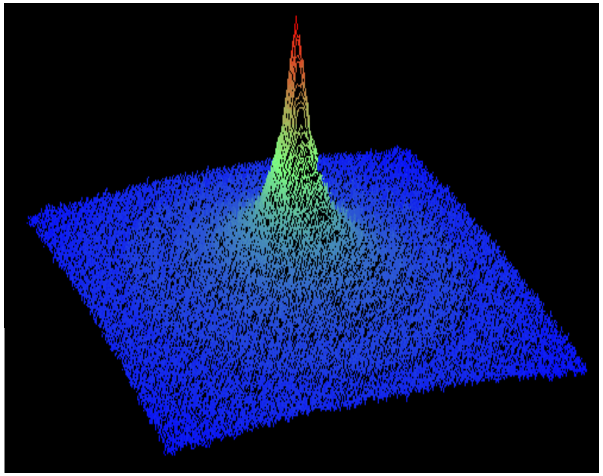
G. Labeyrie et al., Phys. Rev. Lett., **83**, 5266 (1999)

Theory :

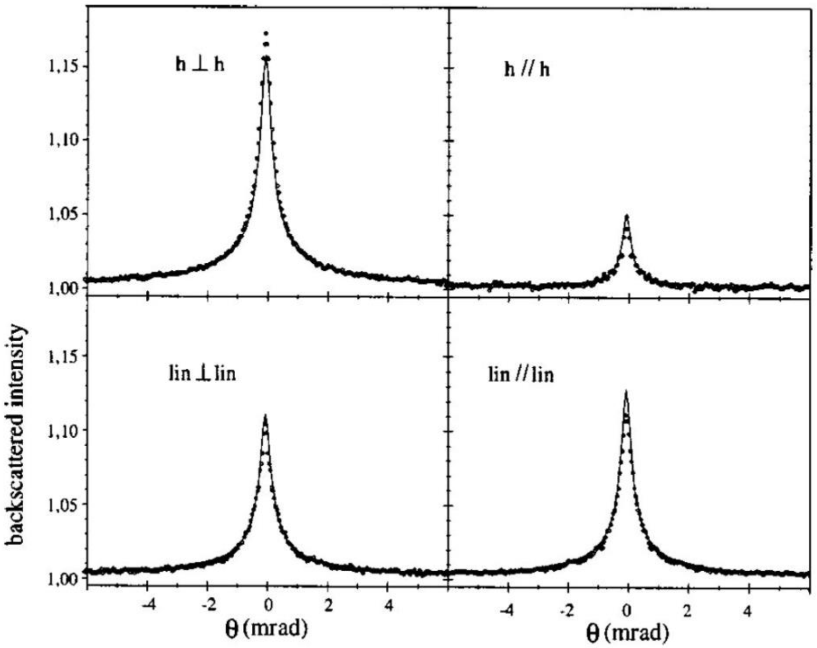
- no "exact" solution
- diagrammatic approach





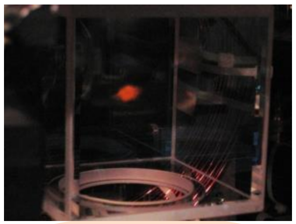


Excellent agreement
(no free parameter)

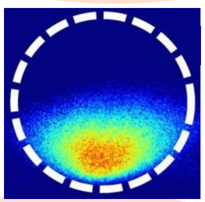


T. Jonckheere et al., Phys. Rev. Lett., **85**, 4269 (2000)

Towards strong localization of light : dense atomic clouds

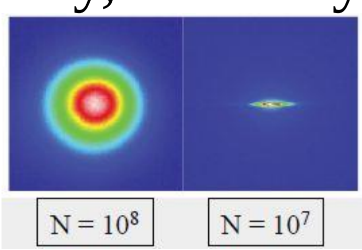


$k\ell \approx 1000$



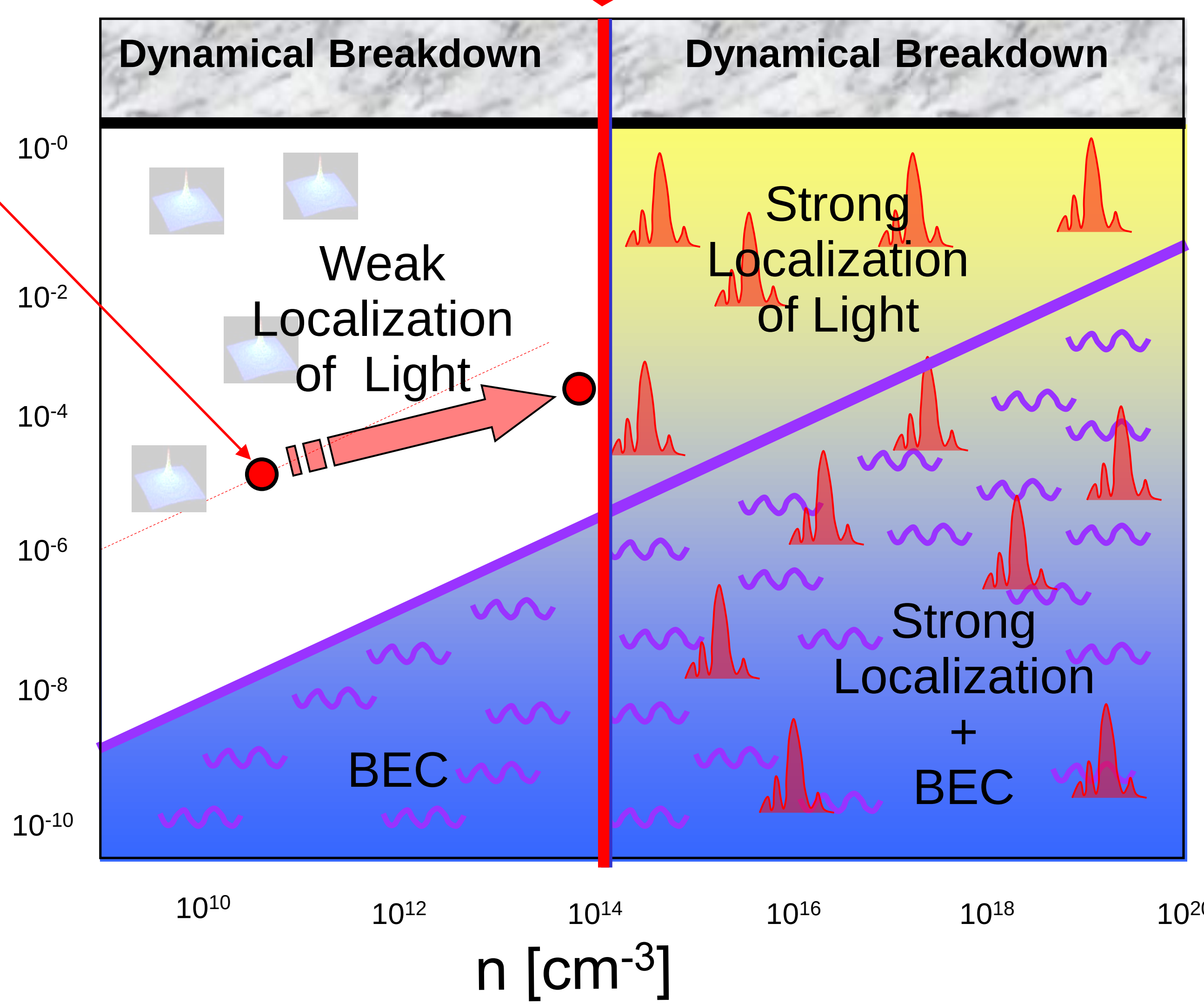
$k\ell \approx 3$

Dipole Trap
(Havey, Browaeys)



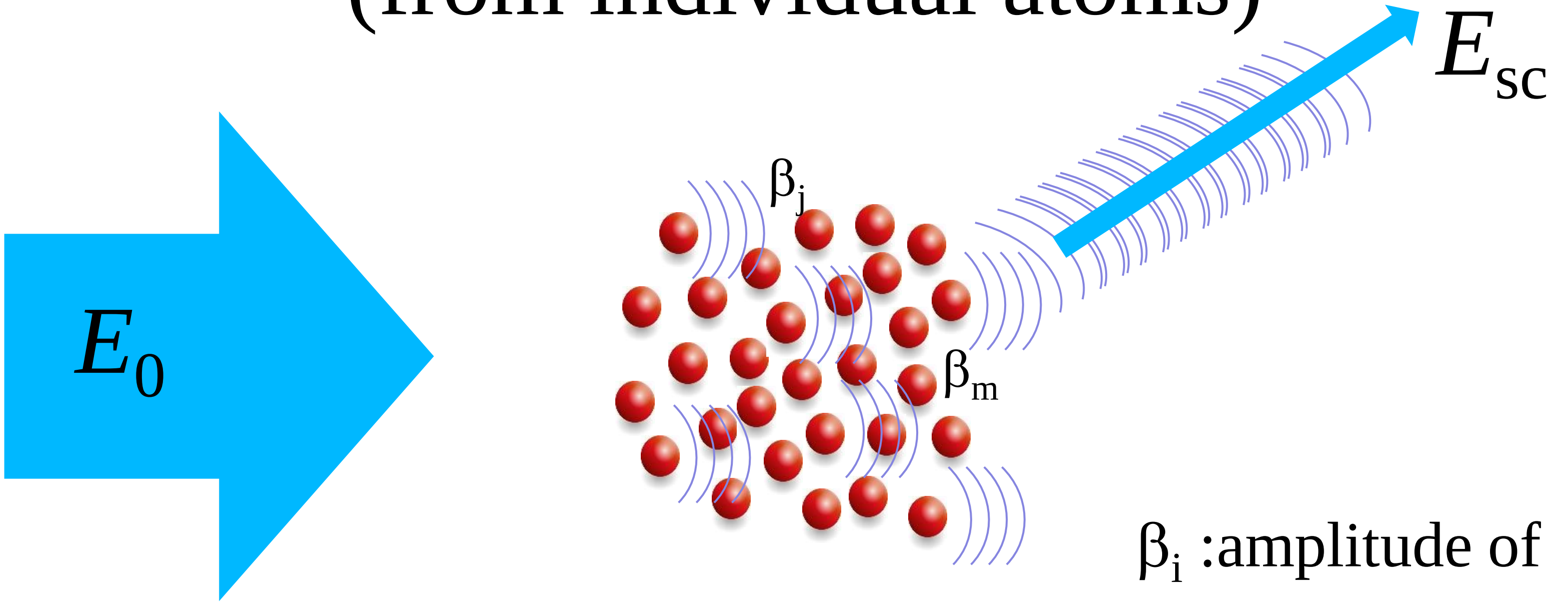
$k\ell < 1$

Ioffe-Regel : $k\ell \approx 1$



Ligth scattering from point dipoles : 1/r outgoing wave

Building up a refractive index « ab inito » (from individual atoms)



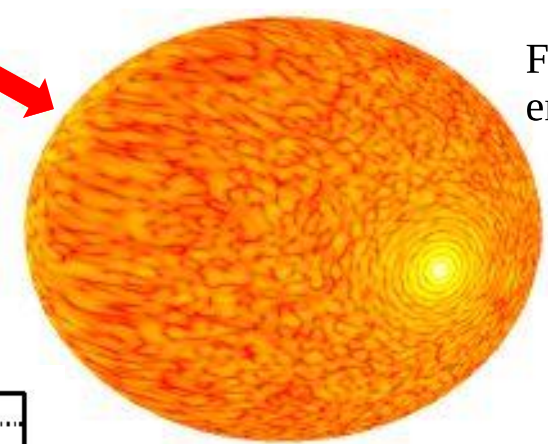
β_i :amplitude of dipole i

$$\dot{\beta}_j(t) = -\frac{i}{2}\Omega e^{i\mathbf{k}_0\cdot\mathbf{r}_j} + \left(i\Delta - \frac{\Gamma}{2}\right)\beta_j(t) - \frac{\Gamma}{2}\sum_{m\neq j}^{N}\beta_m \frac{\exp(ik_0 \mid \mathbf{r}_j - \mathbf{r}_m \mid)}{ik_0 \mid \mathbf{r}_j - \mathbf{r}_m \mid}$$

$$E_{\mathrm{sc}}(\mathbf{r}) = -\frac{\hbar\Gamma}{2d}\sum_{j=1}^{N}\beta_j \frac{e^{ik_0|\mathbf{r}-\mathbf{r}_j|}}{k_0|\mathbf{r}-\mathbf{r}_j|}$$

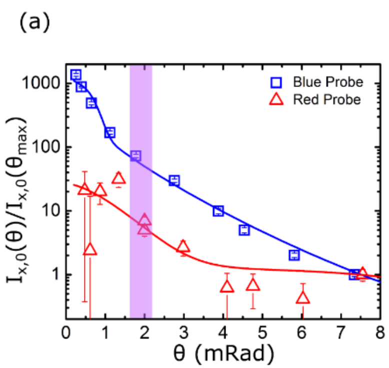
Spherical gaussian cloud : emission diagram

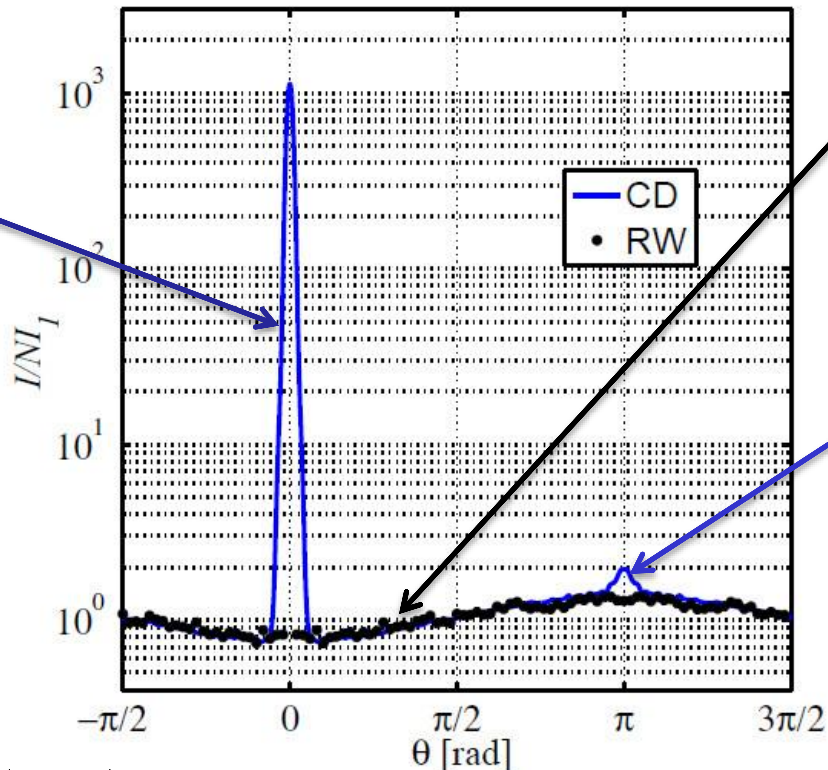
Cloud of atoms



Far field emission diagram

refractive index (mean field)





Incoherent model (particles trajectories, scattering in ‘empty modes’)

Mesoscopic physics: Weak localization (waves beyond mean field)

S. Bromley et al., Nat. Comm. 7, 11039 (2016)

Theory : Effective Hamiltonian

$$H_{eff} = (\hbar\omega_0 - i\frac{\hbar\Gamma_0}{2}) \sum_i S_i^z + \frac{\hbar\Gamma_0}{2} \sum_{i\neq j} V_{ij} S_i^{+} S_j^{-}$$

Diagonal : On site energy

Off diagonal : transport

$$V_{ij} = \beta_{ij} - i\gamma_{ij}$$

$$\beta_{ij} = \frac{3}{2}\left[-p\frac{\cos k_0 r_{ij}}{k_0 r_{ij}} + q\left(\frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^3} + \frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^2}\right)\right]$$

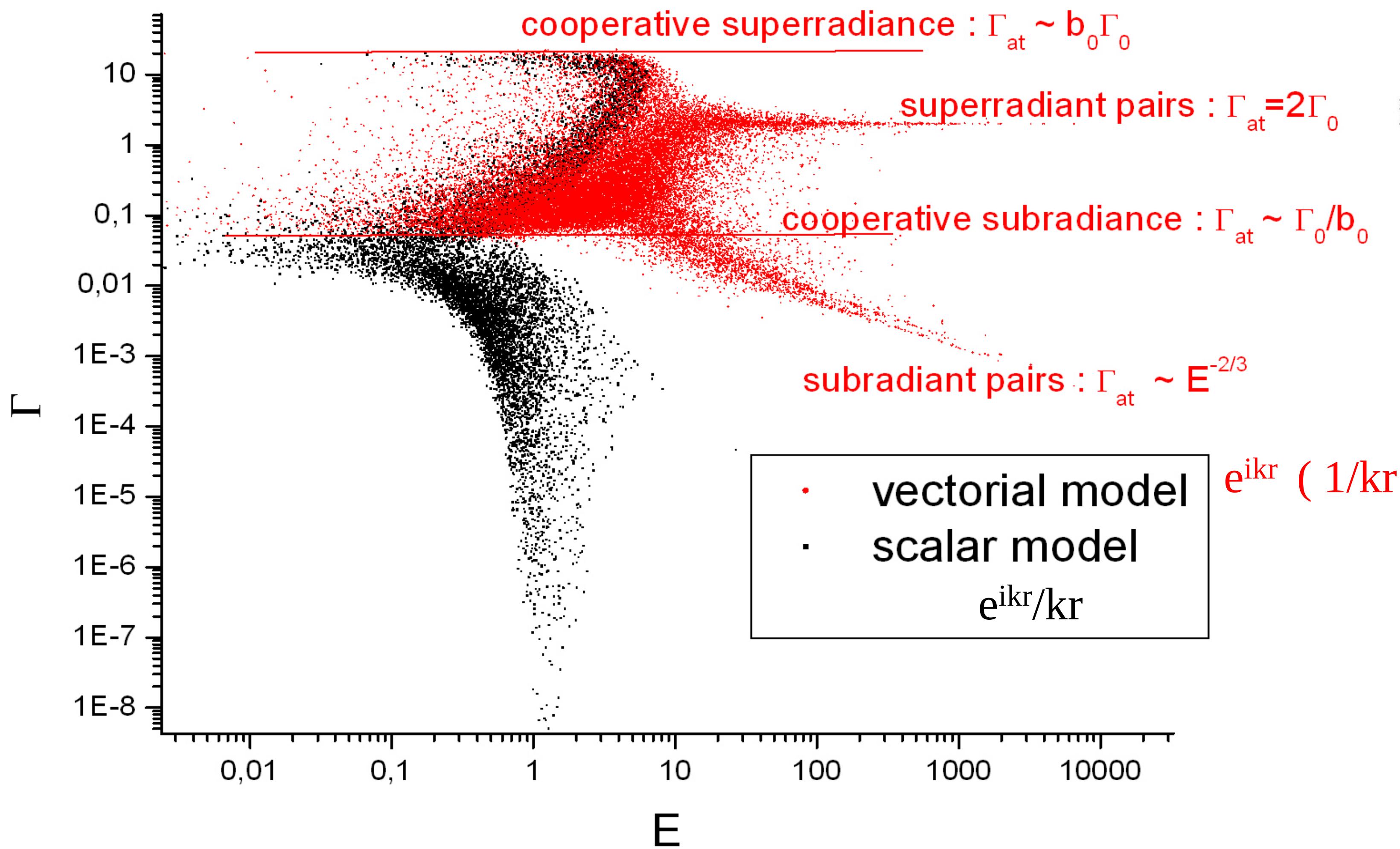
$$\gamma_{ij} = \frac{3}{2}\left[p\frac{\sin k_0 r_{ij}}{k_0 r_{ij}} - q\left(\frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^3} - \frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^2}\right)\right]$$

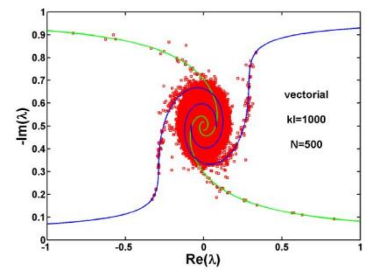
- Open System
- Reminiscent of Anderson Hamiltonian
- Heisenberg model with global coupling
- Long range hopping
- No decoherence (coupling to phonons, …)

Eigenvalues for N coupled dipoles

Important near field terms for high densities

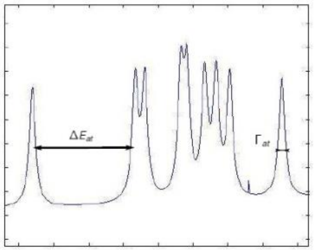
kl=0.1



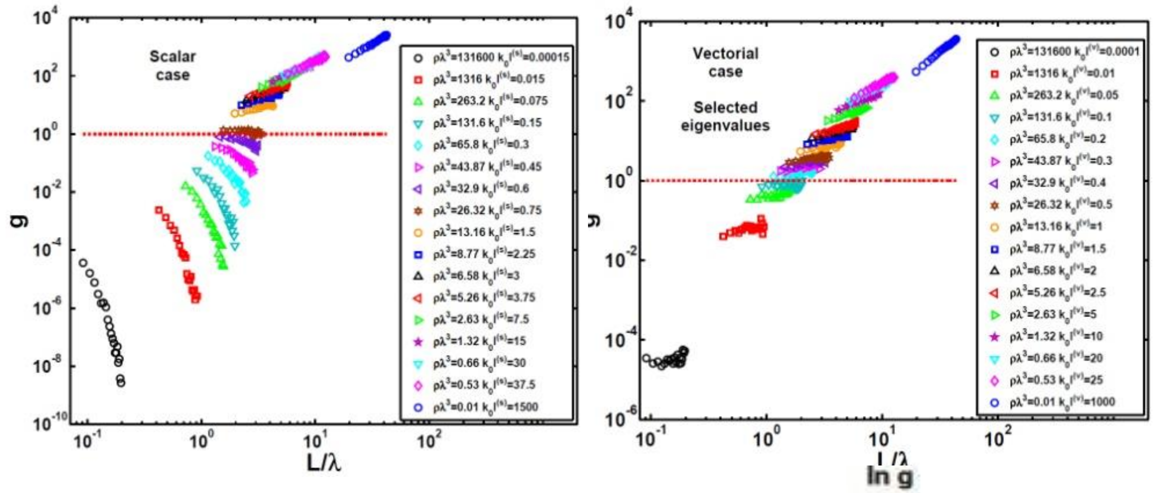


$e^{ikr}\ (1/kr + 1/kr^2 + 1/kr^3)$

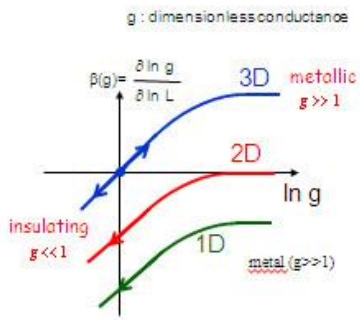
Resonance Overlap (« Thouless »)

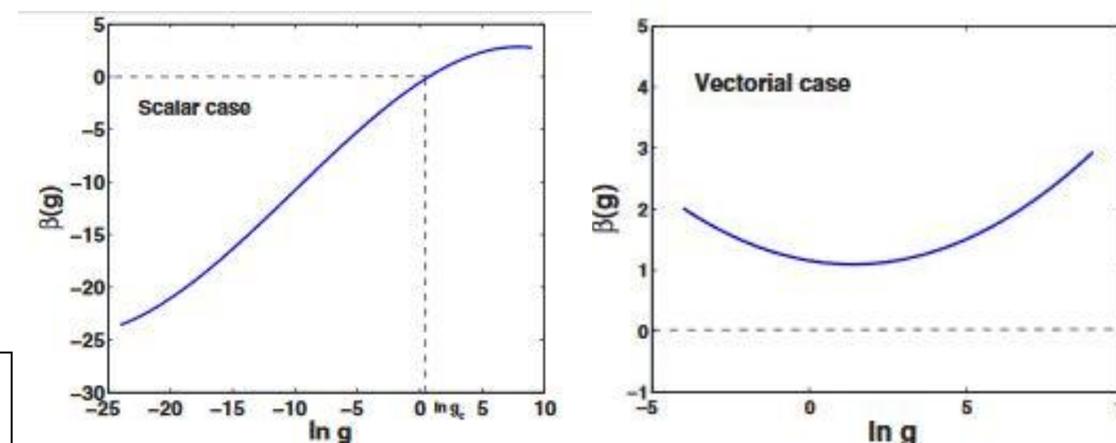


$$g = \left\langle \frac{1}{\langle 1/\Gamma \rangle \langle \Delta E \rangle} \right\rangle$$



Scaling function β(g)

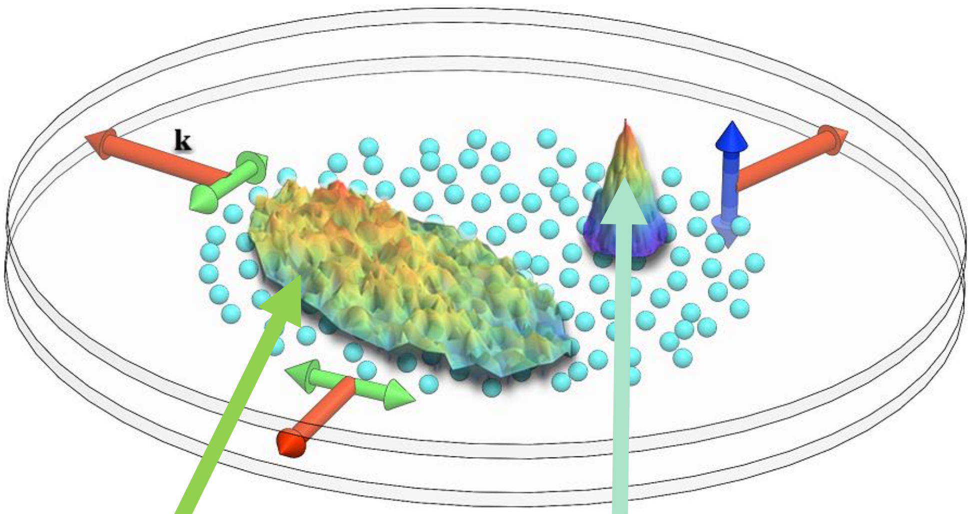




NO ANDERSON LOCALISATION FOR VECTORIAL LIGHT IN 3D ?

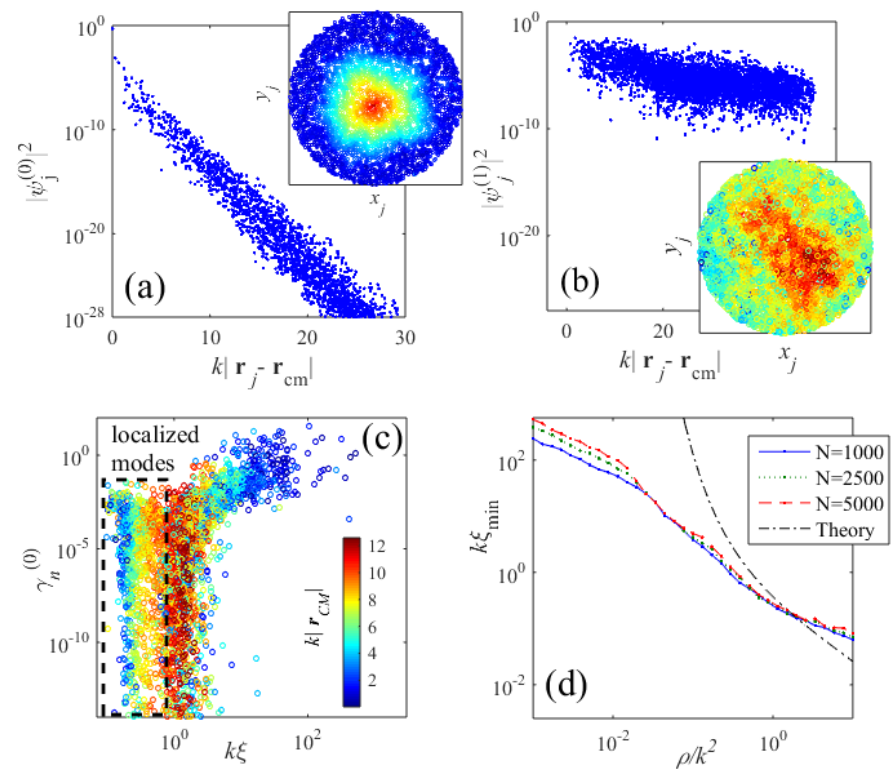
S. Skipetrov, I. Sokolov, PRL 112, 023905 (2014)
Bellando et al,. Phys. Rev. A 90, 063822 (2014)

TIME vs SPACE LOCALISATION (2D)



Spatially extended mode
(vectorial case)

Spatially localized mode
(scalar case)



Mode width NOT correlated to localisation length :
temporal vs spatial localisation

C. E. Maximo, N. Piovella, Ph. W. Courteille, R. K., R. Bachelard, PRA 92, 062702 (2015)

The quest for Dicke subradiance

1954 : Dicke super- and subradiant states

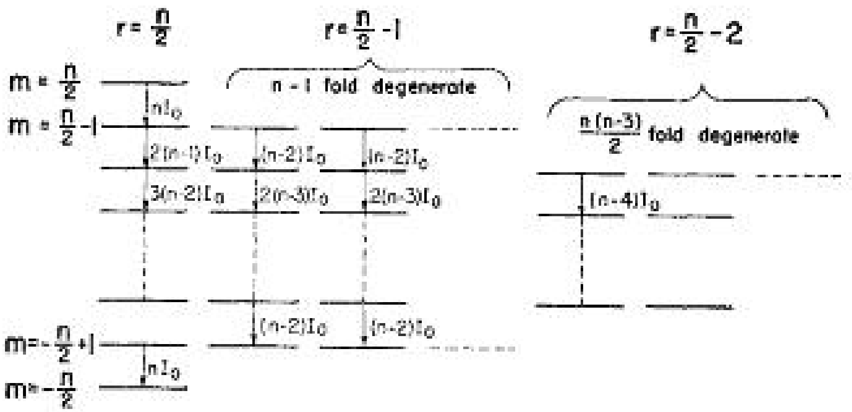
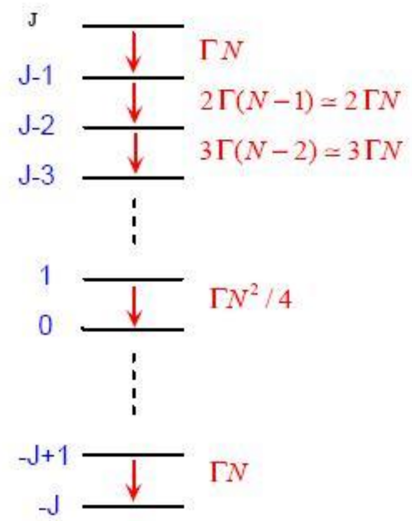
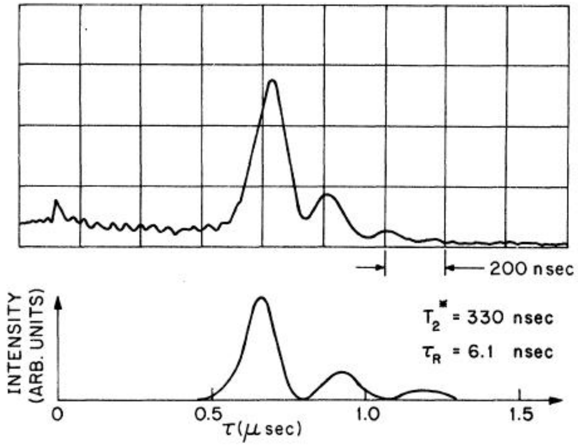


Fig. 1. Energy level diagram of an n-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$.



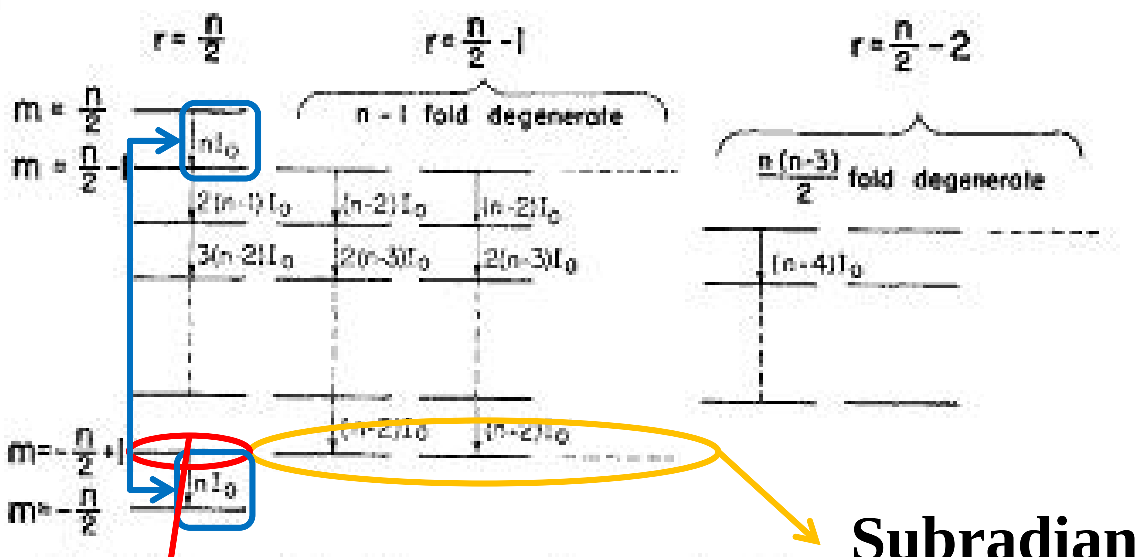
R. Dicke 1954

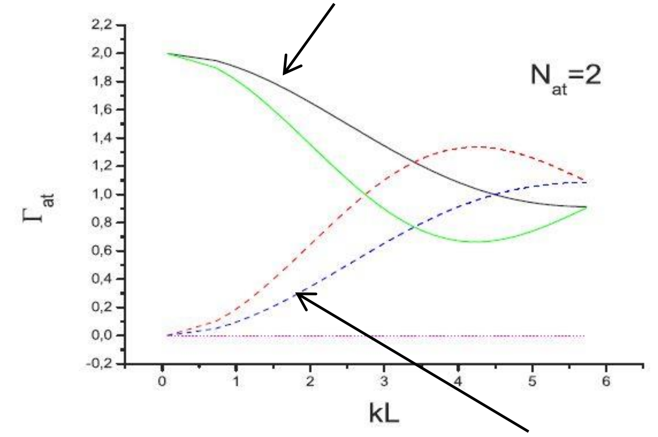


First experimental observation of superradiance

Feld et al. 1973

Single photon excitation / low intensity limit

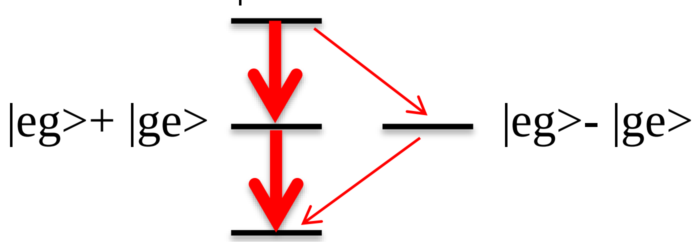




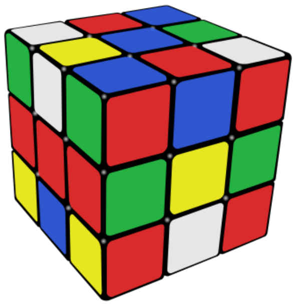
Subradiant

N-1 metastable states

$\Gamma_{max} \sim N\,\Gamma_0$

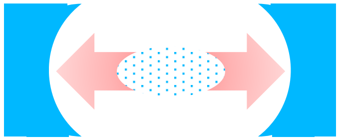


Extended Volume : $\mathbf{b_0=N_{at}/N_{modes}}$

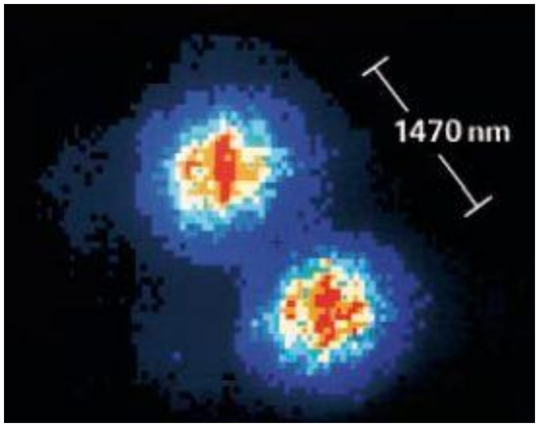


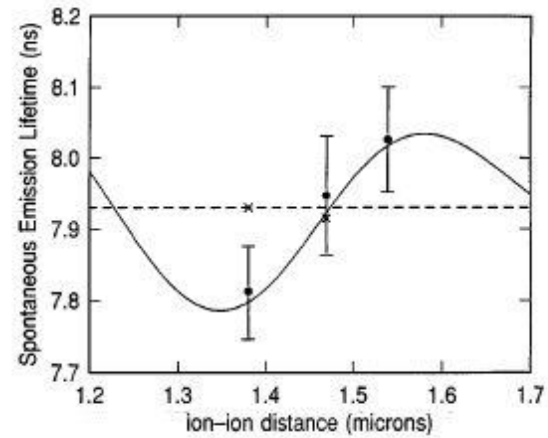
Cooperativity without cavity

(also Random lasing)



Subradiant pairs : N=2

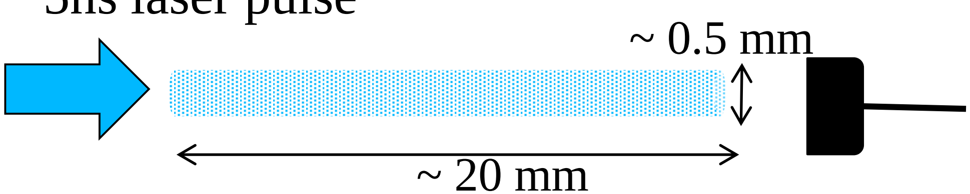




R. G. DeVoe, R. G. Brewer, PRL 76, 2049 (1996).

Forward ‘subradiance echo’ from inverted system

5ns laser pulse



Pencil shape excitation

τ_{nat}=7ns

λ_3

λ_4

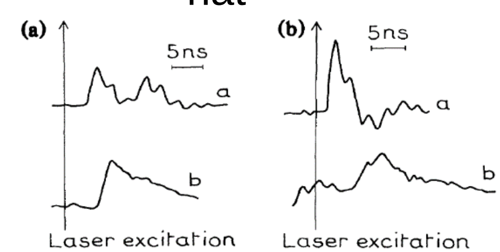


FIG. 2. Typical superradiant signals. (a) corresponds to the excitation with saturated linearly polarized light from the $4p_{3/2}$ level; (b), the $4p_{1/2}$ level. In both cases, traces *a* and *b* correspond respectively to $4d \rightarrow 5p$ and to $5p \rightarrow 5s$ transitions. The visible oscillations of the signals are most likely due to the hyperfine structure of the $5p_{1/2}$ level.

D. Pavolini et al. , Phys. Rev. Lett. 54, 1917 (1985)

Fragile subradiance

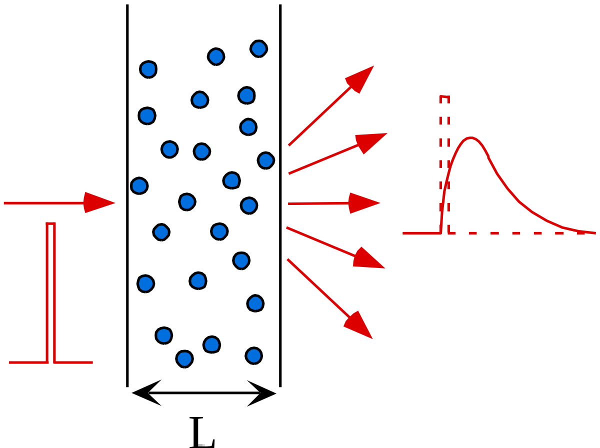
Single Photon Dicke subradiance for N two level systems (in free space, N>>2) has **not been observed**

- Does **not** require large spatial densities
  (near field effect maybe even bad : Gross&Haroche 1982)

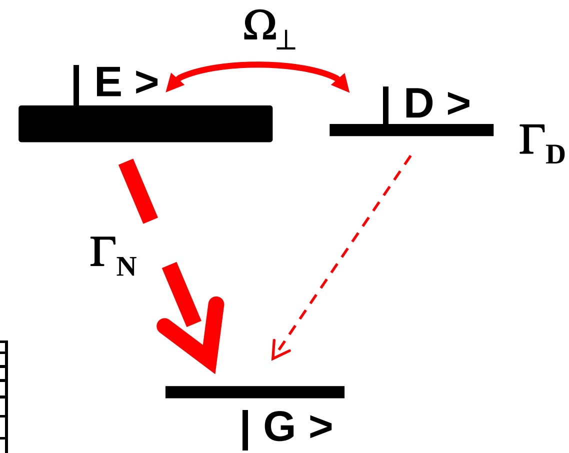
- Requires large optical densities in all directions (b_0>>1)

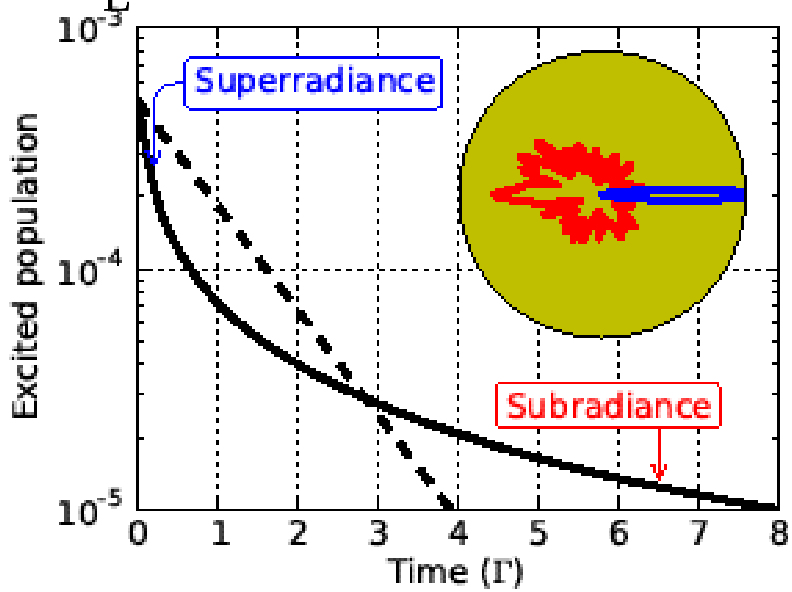
- Exploits the 1/r **long range** dipole-dipole interaction

Time dependent experiments : coherent scattering



Superradiance = bright state
Subradiance = metastable 'dark' states





Numerical Simulation
of N driven coupled dipoles

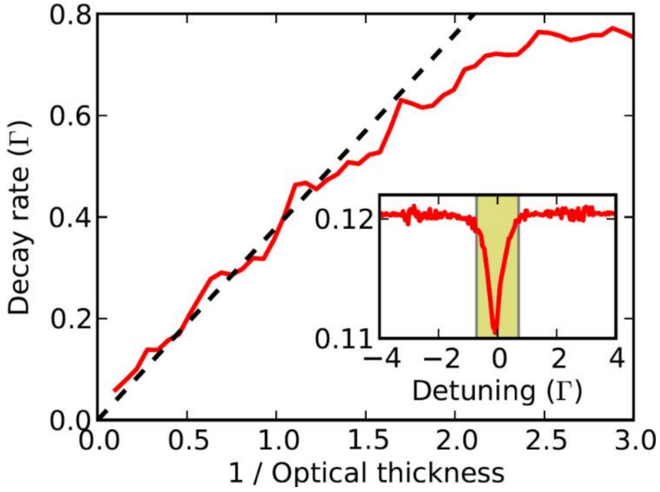
PRL **108,** 123602 (2012) PHYSICAL REVIEW LETTERS week ending 23 MARCH 2012

Controlled Dicke Subradiance from a Large Cloud of Two-Level Systems

Tom Bienaimé,[1] Nicola Piovella,[2] and Robin Kaiser[1]

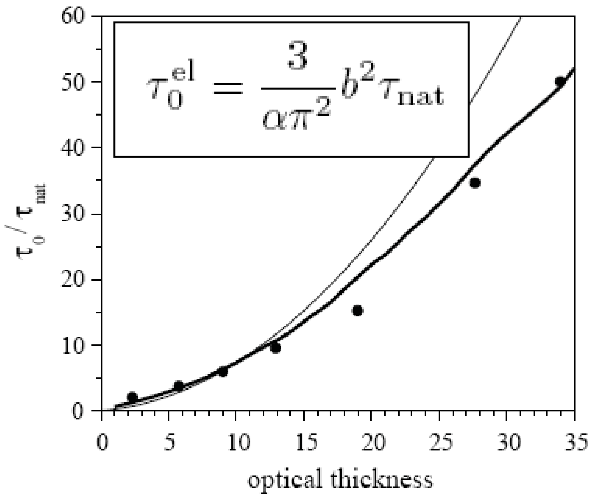
Subradiance vs incoherent scattering

$t_{sub} \propto b_0$



- Does not require large spatial densities
- Requires large optical densities

$t_{Rad.Trap.} \propto b(\delta)^2$

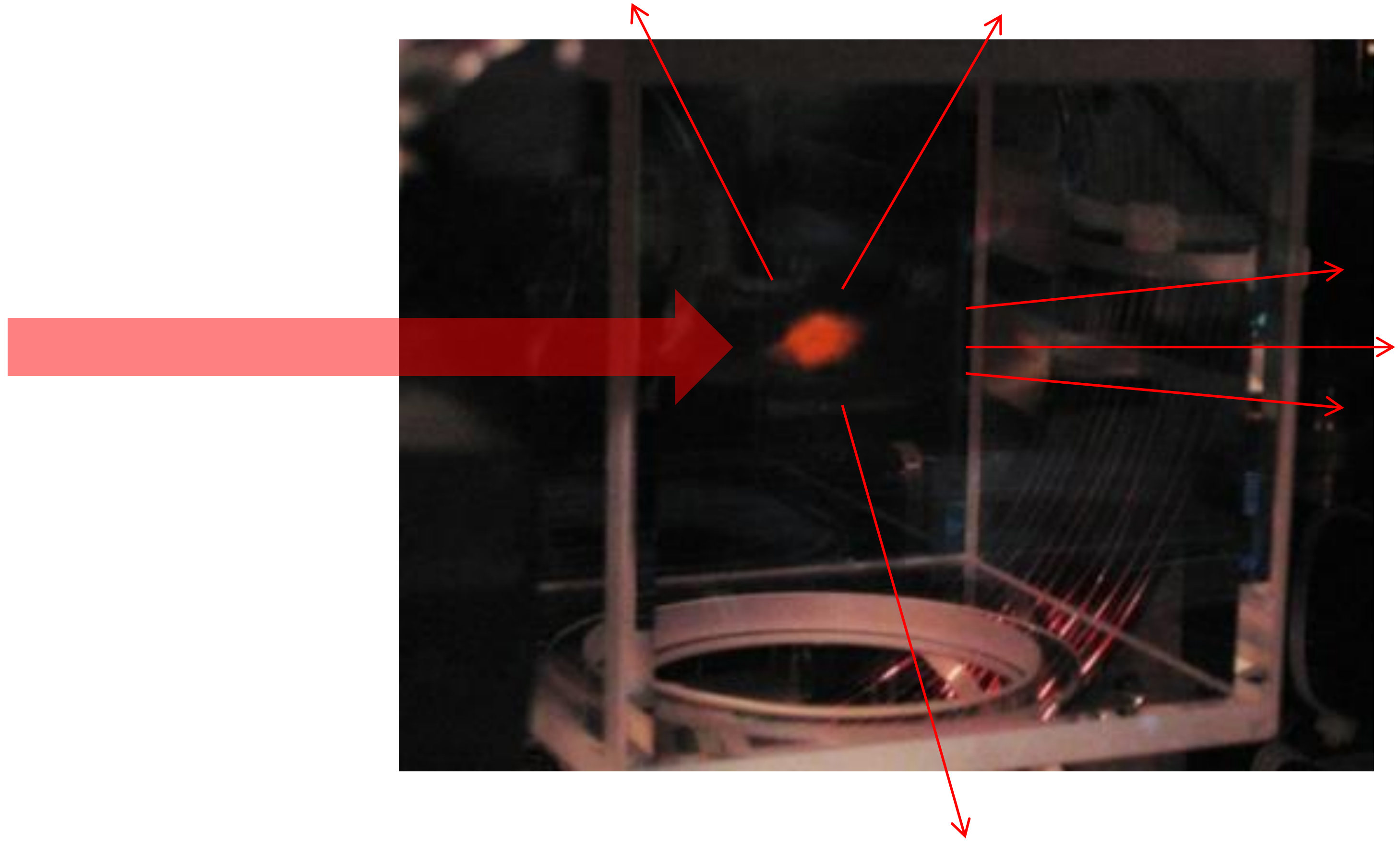


- Random walk of photons (without interference)
- Diffusion equation

$t_{Anderson} \propto \exp\{b(\delta)\}$

- Density Threshold ?

Experiment



detector

$N=10^9$ ^{87}Rb

$T=50\ \mu K$

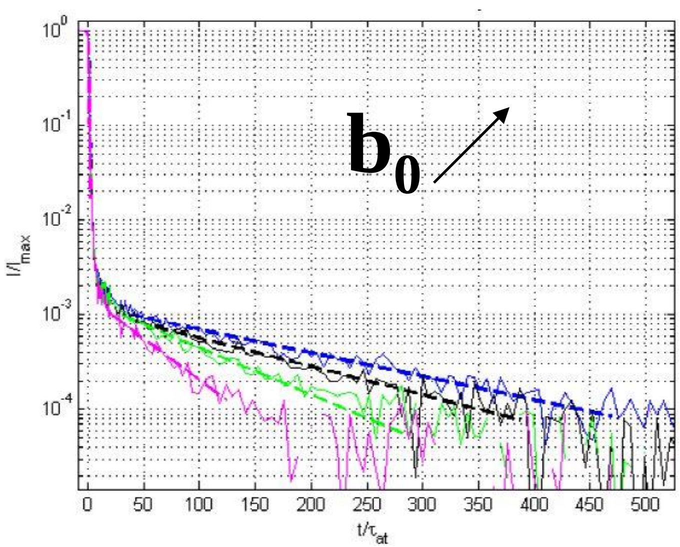
$R=1$ mm

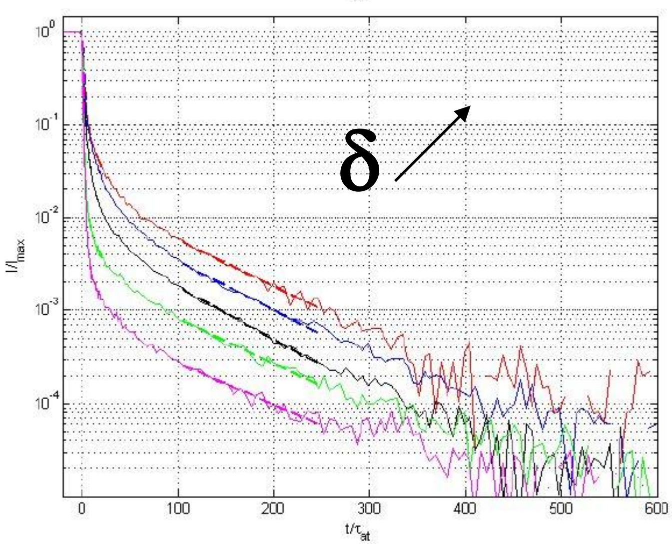
$\rho=10^{11}/cc$

$b_0 = 20\ldots100$

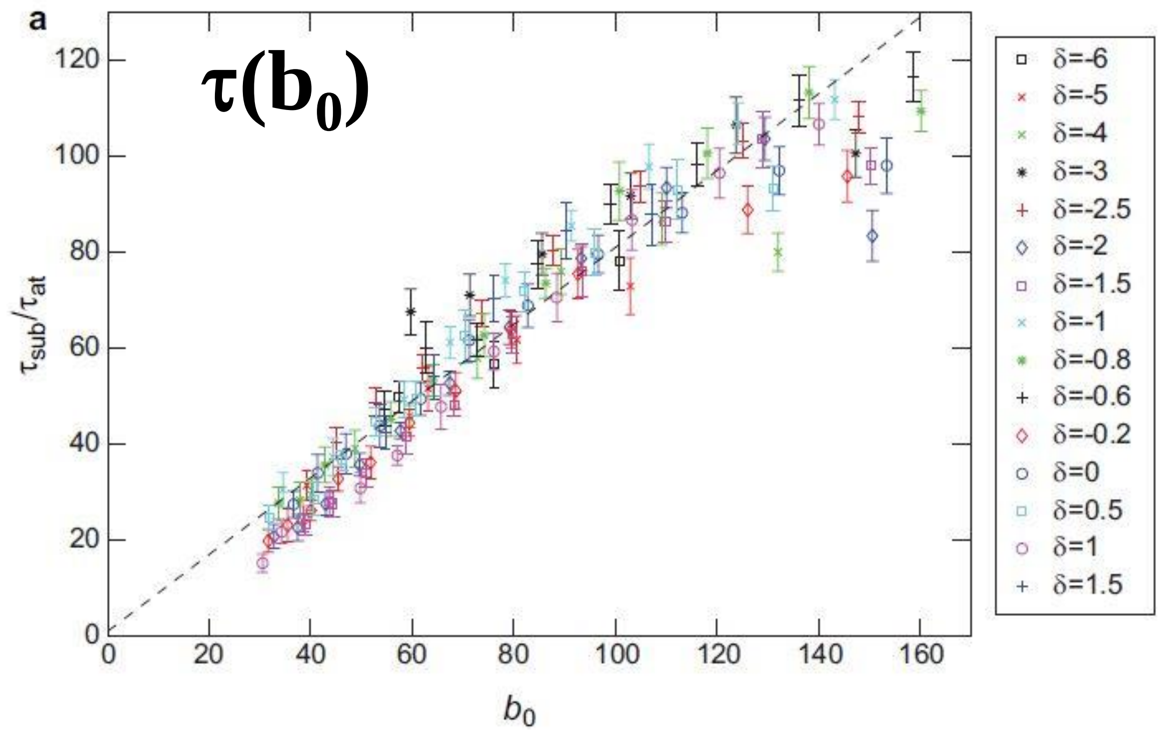
Experimental results

Long decay at b(δ)<1 ☺





Scaling with SYSTEM SIZE !



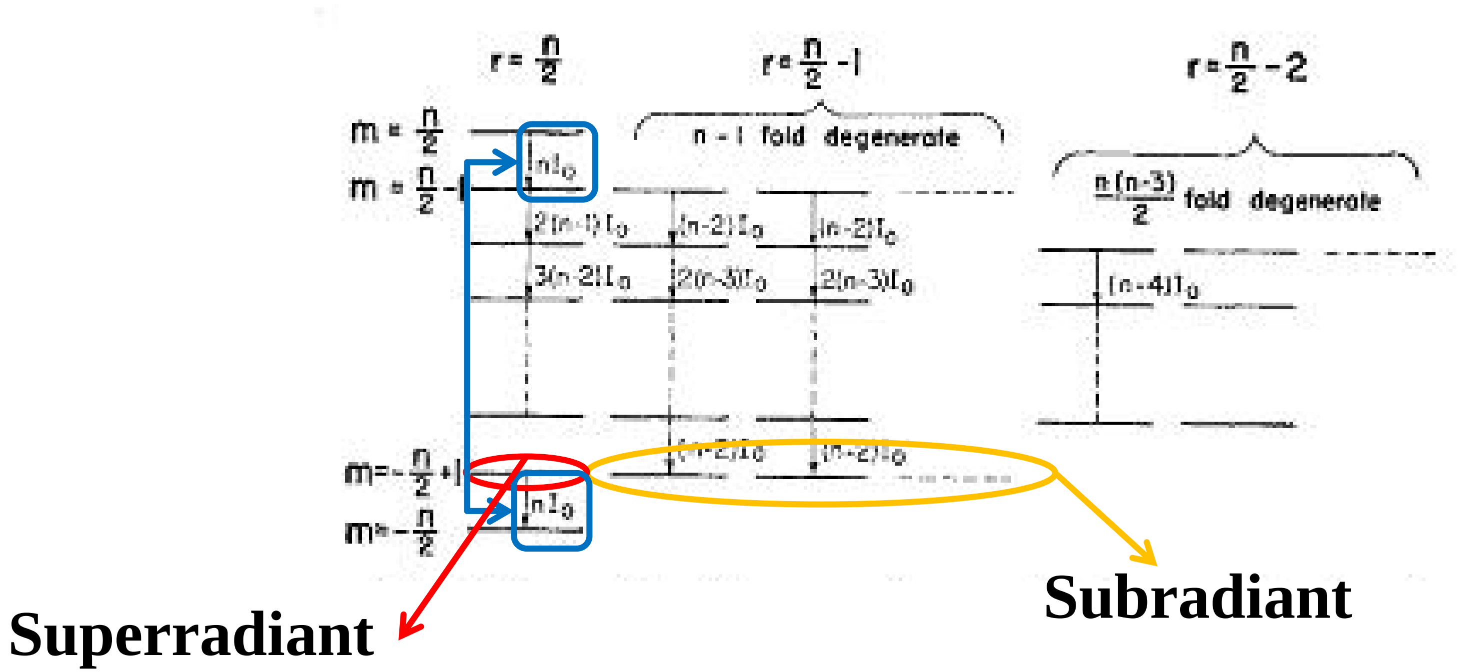
Increases as $b_0 = \rho\, \sigma\, L$ ☺

Selected for a Viewpoint in *Physics*
PRL 116, 083601 (2016) PHYSICAL REVIEW LETTERS week ending 26 FEBRUARY 2016

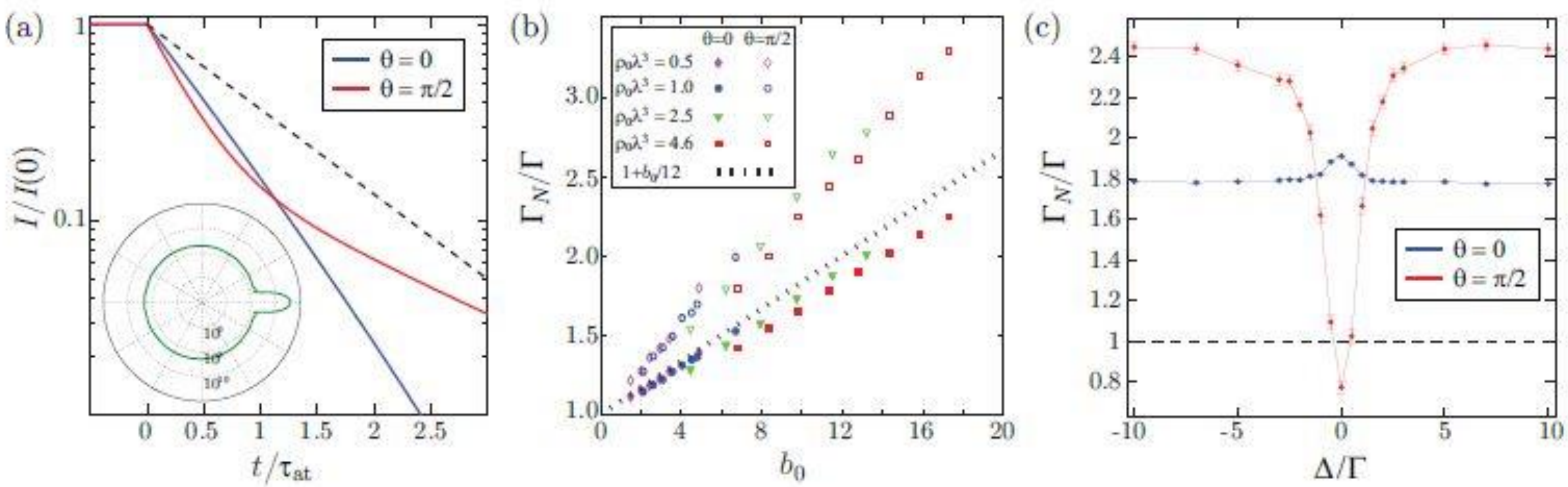
Subradiance in a Large Cloud of Cold Atoms

William Guerin,[1,*] Michelle O. Araújo,[1,2] and Robin Kaiser[1]

Single Photon Super- vs Subradiance
The ‘super’ of ‘single photon Dicke states’

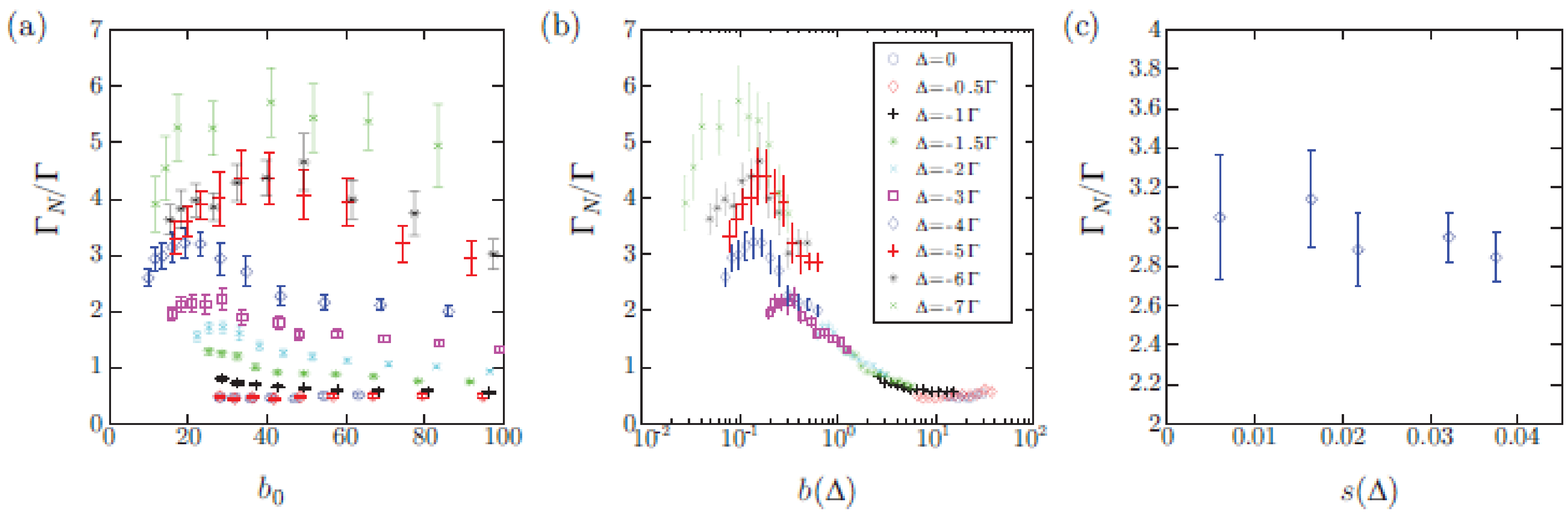


Superradiance in dilute and large cloud of cold atoms



Coupled Dipoles (Numerics)

Off-axis Superradiance ≠ forward superradiance



Experiments

M. O. Araujo, I. Kresic, R. K., W. Guerin, to appear in PRL (2016)

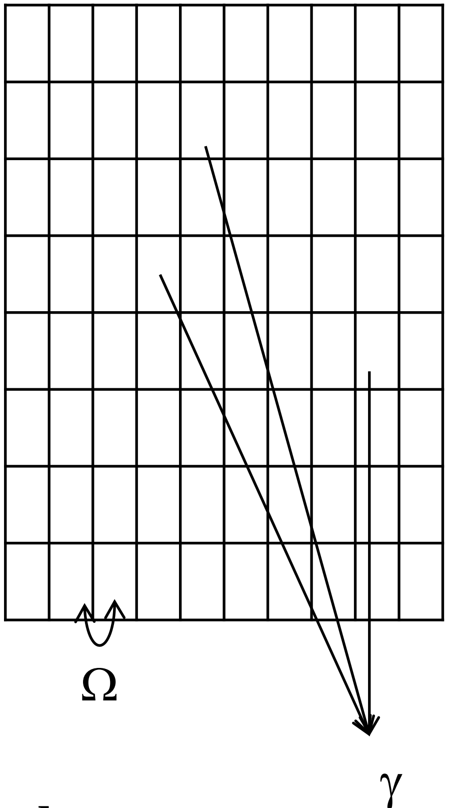
Combining Anderson and Dicke Toy Model : Open Disordered System:

A. Biella, F. Borgonovi, R. K., G.L. Celardo, EPL, 103, 57009 (2013)

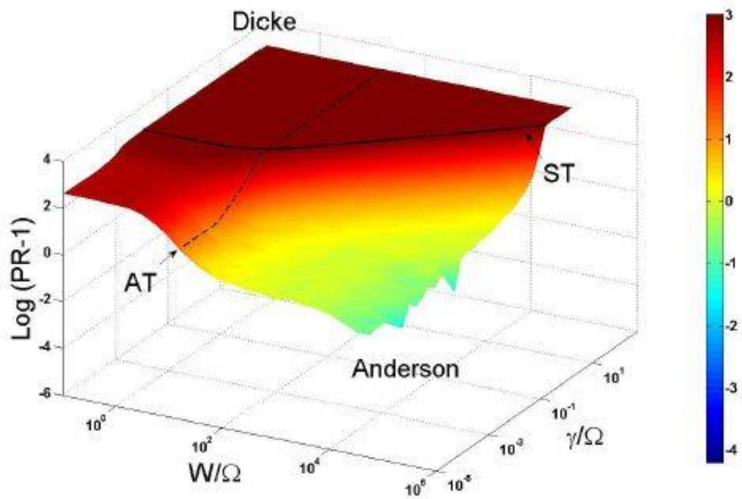
3D Anderson model on 10x10x10 lattice
hoping (Ω) + disorder (W) + opening (γ)

$$H_0 = \sum_{j=1}^{N} E_j |j\rangle\langle j| + \Omega \sum_{\langle i,j\rangle} \left(|j\rangle\langle i| + |i\rangle\langle j|\right)$$

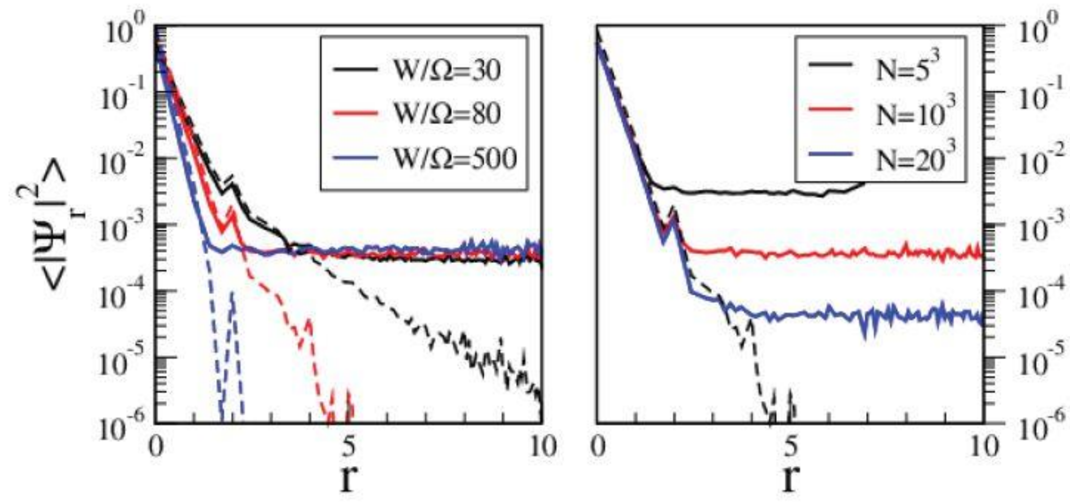
$$(H_{\rm eff})_{ij} = (H_0)_{ij} - \frac{i}{2}\sum_c A_i^c (A_j^c)^* = (H_0)_{ij} - i\frac{\gamma}{2} Q_{i,j}$$



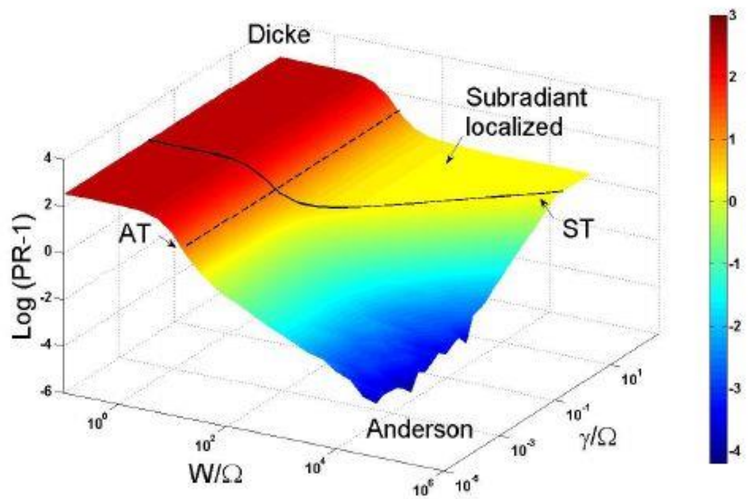
All sites coupled to one single decay channel : $Q_{ij}=1$



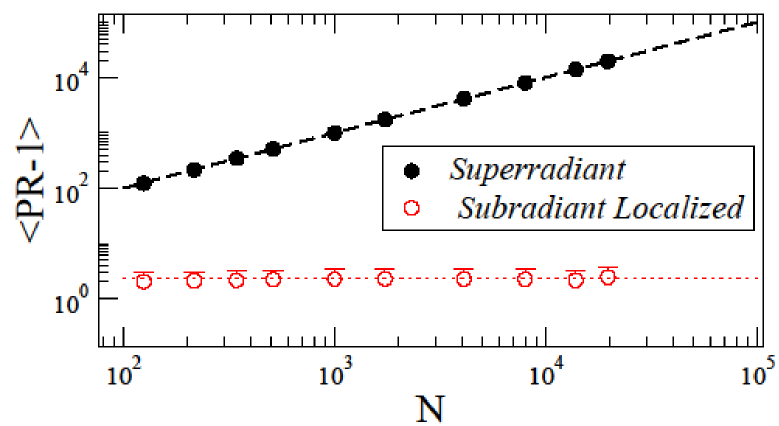
(a)



Hybrid Subradiant States
« decoupled » from outside world



(b)

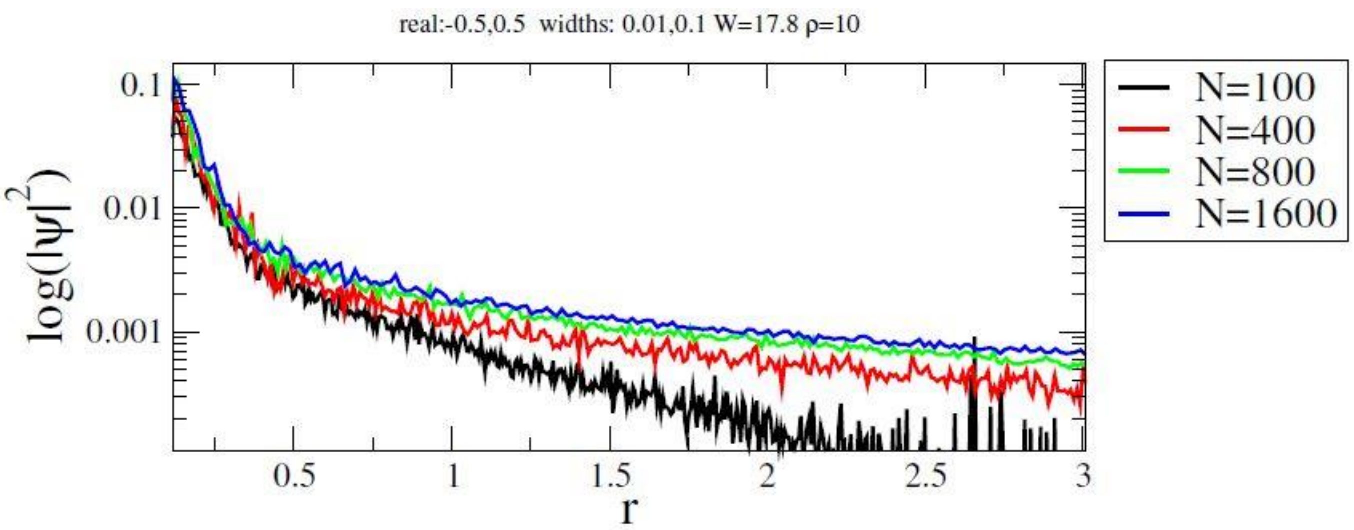


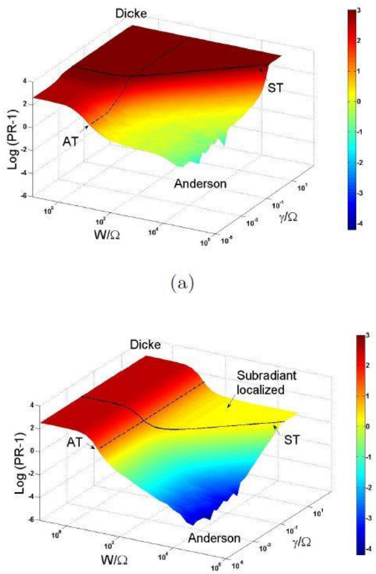
Outlook :

- **Subradiance vs Radiation trapping**

Radiation trapping for small beam and intermediate regimes: subradiance dominant at long times

- **Towards Anderson of subradiant Dicke states**





(a)

(b)

Perspectives :

- **Anderson Localisation of Light in Cold Atoms**

With diagonal disorder / magnetic field ?
Identify experimental signatures
Technical issuses : Rb ☹ …Yb ☺

- **Long range interactions and mechanical effects**

Photon bubbles
Debye Screening
Long range attractive forces in 3D
(mimic ‘gravity’ in the lab?)

- **Super- and subradiant states :**

fast and slow relaxation : connection to quasi-stationnary states ?

Precious Collaborators

INLN:

G. Labeyrie, W. Guerin, M. Fouché, M. Araujo, I. Kresic

Collaborations :

E. Akkermans, A. Gero

L. Celardo, F. Borgonovi, A. Biella, M. Angeli

R. Bachelard, P. Courteille, N. Piovella, C. Maximo

J. Ye, A. M. Rey, M. Lukin, J. Schachenmayer, et al.

J. Barré, B. Marcos, A.Olivettti, D.Metivier

Thank you for your attention