Cooperative effects and long range interaction: Cooperative Shielding

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ICTP, July 28, Trieste, Italy

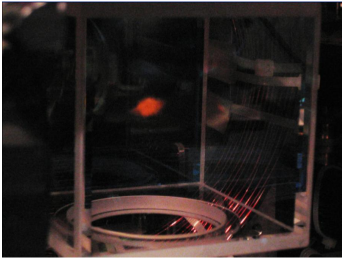
Motivations

- Long-range interacting systems: broken ergodicity (J. Stat. Phys. 116, 1435 (2004); PRL 95, 240604 (2005)); long-lasting out-of-equilibrium regimes (PRL 101, 260603 (2008).), ensemble inequivalence, QSS, etc...
- Surge of interest in cond-mat: cold atomic clouds, ion traps, light harvesting complexes, etc.. Even all to all interactions! (see Kurizki).
- *Cooperativity and Emergent Quantum properties*: Superconductivity, Superradiance, Macroscopic quantum tunnelling.
- Cooperativity, Functionality and Robustness. From quantum device to basic theoretical questions.

Cooperativity and long range interaction

Cold Atoms

Robin Kaiser Lab

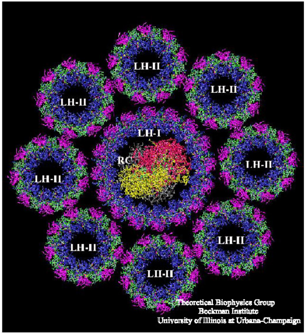


$$V_{ij} = -\frac{\cos(k_0 r_{ij})}{k_0 r_{ij}} - i\,\frac{\sin(k_0 r_{ij})}{k_0 r_{ij}},$$

arXiv:1604.07868, G.L.C., R. Kaiser, F. Borgonovi

Photosynthetic Complexes

LH1-RC complex of purple Bacteria



$$V_{ij} = \frac{3\gamma\left(\cos\phi_{i,j} - 3\cos\theta_i\cos\theta_j\right)}{4(k_0 r_{i,j})^3} - i\gamma\cos\phi_{i,j}$$

G.L.C., F. Borgonovi, V.I. Tsifrinovich, M. Merkli and G.P. Berman, The Journal of Physical Chemistry C, 116, 22105 (2012).

Spreading of Perturbations: Lieb-Robinson bounds

- Spreading of information: quantum computing, thermalization
- Short-Range, Lieb-Robinson bounds: In short range systems, spreading is linear, velocity is finite and independent of the systems size.
- Long-Range: Breaking of LR bounds.

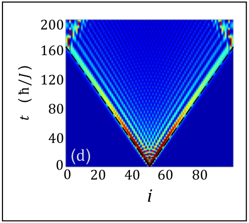
1d Many Body Hamiltonian:

$$H = B \sum_k \sigma_k^z + J \sum_{i<j} \frac{\sigma_i^x \sigma_j^x}{|i-j|^\alpha}$$

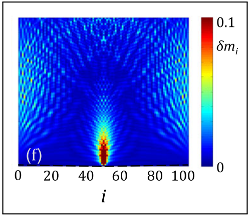
Long Range: $1/r^\alpha$, long range $\alpha < 1$, short range $\alpha > 1$.
Spreading of perturbations

P. Hauke and L. Tagliacozzo, PRL **11**, 207202 (2013).

$\alpha = 3$



$\alpha = 0.7$



Average polarization $\delta m_i = \langle S_i^z \rangle + 1/2$ *vs* time.

Contradictory features in Long-Range

Ion Traps experiment

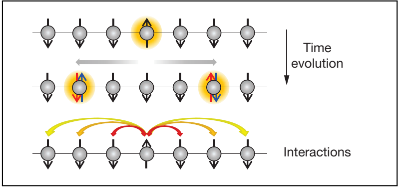
1d Many Body Hamiltonian:

$$H = B\sum_k \sigma_k^z + J\sum_{i<j} \frac{\sigma_i^x \sigma_j^x}{|i-j|^\alpha}$$

with $0 \leq \alpha \leq 3$.

Breaking of Lieb-Robinson bounds in Ion Trap

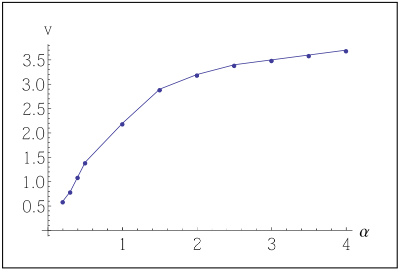
Richerme et al., Nature Letter **511**, 198 (2014); P. Jurcevic et. al., Nature, **511**, 202 (2014).



Theoretical work:

Suppression of the velocity of spreading with the increase of the interaction range α.

M. Kastner, New J. Phys. **17**, 063021 (2015)



Cooperative Shielding can help to explain such contradictory features

Cooperative Shielding

- Given a system $H = H_0 + V$, we can eliminate V from the dynamics.
- Existence of subspaces where the propagation of information is determined by an effective short range Hamiltonian, even in presence of strong long range.

Cooperative Shielding

Long Range: $H = H_0 + V$

V does not affect the evolution (*shielding*) up to a time scale that grows with N (*cooperativity*).

The Shielding effect

- Let us consider a system:

$$H = H_0 + V, \qquad \text{with} \qquad [H_0, V] = 0$$

with V highly degenerate $V|v_k\rangle = v|v_k\rangle$

- if $|\psi_0\rangle = \sum_{k=1}^{g} c_k|v_k\rangle$, V contributes only with global phase:

$$|\psi(t)\rangle = e^{iHt}|\psi_0\rangle = e^{ivt}e^{iH_0 t}|\psi_0\rangle$$

We have shielding from V!!. H_0: emerging Hamiltonian.

- If initial state contains different eigenvalues of V, dynamics is affected by V.
- What if $[H_0, V] \neq 0$?
- What if spectrum of V is not degenerate? What is the connection with long range? What is the emergent Hamiltonian?

Cooperative Shielding in many-body.

Experimentally accessible 1d spin $1/2$ Hamiltonian:

$$
\begin{aligned}
H &= H_0 + V, \\
H_0 &= B \sum_{n=1}^{L} \sigma_n^z \\
V &= \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x .
\end{aligned}
\tag{1}
$$

- $\alpha < 1$: long range. $\alpha > 1$: short range.

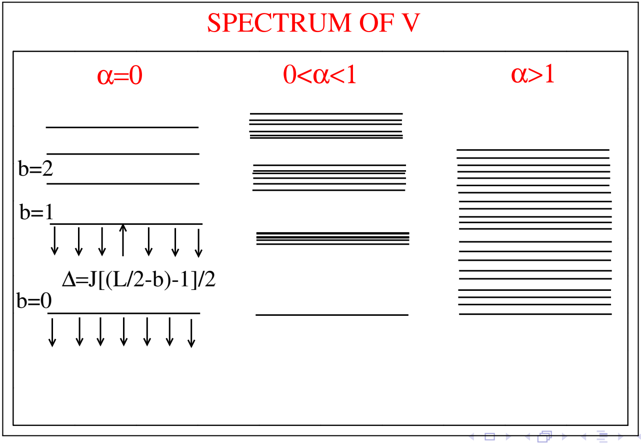
Cooperative Shielding in Many-Body Systems with Long-Range Interaction L. F. Santos, F. Borgonovi, and GLC PRL **116**, 250402 (2016).

Spectrum of V

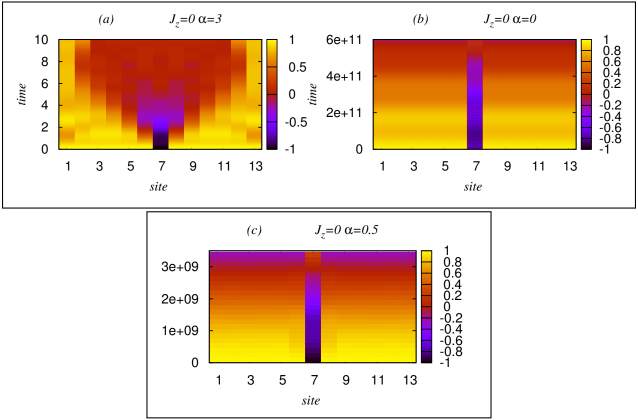
The case $\alpha = 0$:

$$V = J\sum_{n<m} \sigma_n^x \sigma_m^x = \frac{JM_x^2}{2} - \frac{JL}{2} \quad \text{where} \quad M_x = \sum_n \sigma_n^x$$

$$V_b = J(L/2 - b)^2/2 - JL/2, \quad \text{where} \quad b = 0, 1, \dots L/2$$



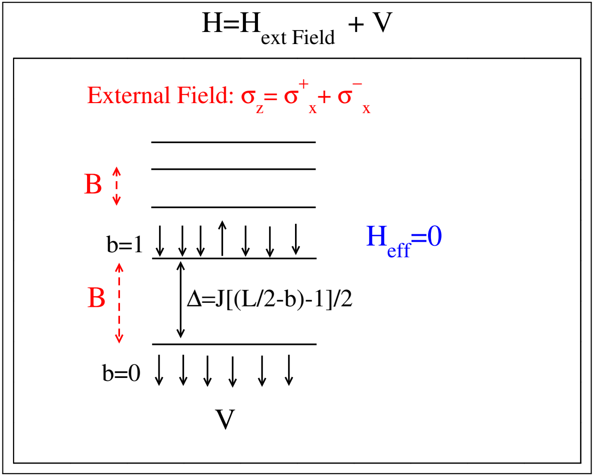
Light-cones



Initial State:

$$|\psi_0\rangle = |\uparrow, \uparrow, .., \downarrow, .., \uparrow, \uparrow\rangle_X$$

a) $B = 0.5, \alpha = 3$ light-cone;
b) $B = 0.5, \alpha = 0$ localization without disorder;
c) $B = 0.5, \alpha = 0.5$



$P_{leak} \propto (W/J)^2/L$ for random field and no NN interaction

Cooperative Shielding in many-body.

Experimentally accessible spin $1/2$ Hamiltonian:

$$H = H_0 + V, \tag{2}$$

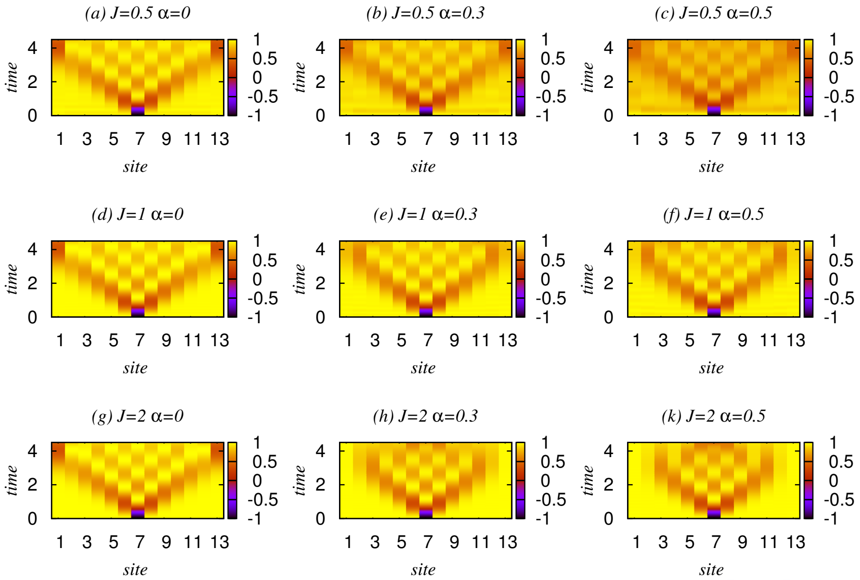
$$H_0 = \sum_{n=1}^{L-1} J_z \sigma_n^z \sigma_{n+1}^z,$$

$$V = \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x.$$

- $\alpha < 1$: long range. $\alpha > 1$: short range.

NN+ LONG RANGE

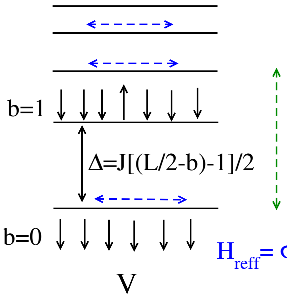
Shielding


(a) J=0.5 α=0
(b) J=0.5 α=0.3
(c) J=0.5 α=0.5
(d) J=1 α=0
(e) J=1 α=0.3
(f) J=1 α=0.5
(g) J=2 α=0
(h) J=2 α=0.3
(k) J=2 α=0.5
time
site
0
2
4
1 3 5 7 9 11 13
1
0.5
0
-0.5
-1

Invariant Subspaces II

$$H = H_{NN} + V$$

$$NN: \sigma^z_n \sigma^z_{n+1} = \sigma^+_n \sigma^-_{n+1} + \sigma^-_n \sigma^+_{n+1} + \sigma^+_n \sigma^+_{n+1} + \sigma^-_n \sigma^-_{n+1}$$



$$H_{reff} = \sigma^+_n \sigma^-_{n+1} + \sigma^-_n \sigma^+_{n+1}$$

$P_{leak} \propto (J_z/J)^2/L$ for NN interaction only.

(Cooperative) Zeno Dynamics

- QZE: Observation freeze dynamics in invariant subspaces.

$$H = H_0 + KH_{meas}$$

As K increases, eigensubspace of H_{meas} becomes invariant.

- Zeno Hamiltonian: in our case: $H = H_0 + V_{LR}$, $V_{LR} \leftarrow H_{meas}$.

$$H_Z = \sum_b [P_b H_0 P_b + V_b P_b] =$$

$$= diag(H_0) + \sum_b V_b P_p$$

where P_b are the projectors on the eigensubspace of V corresponding to the eigenvalues V_b.

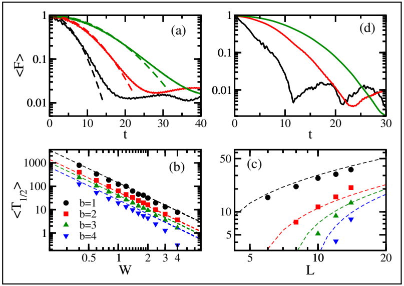
For $\alpha = 0$ $H_{eff} = H_Z$!

For ext field: $H_z = 0$**, for NN:**

$$H_z = \frac{J_z}{4}(\sigma_n^+ \sigma_{n+1}^- + hc)$$

Zeno Fidelity:

$$F(t) = |\langle \Psi(0)| e^{iH_z t} e^{-iHt} |\Psi(0)\rangle|^2$$



Fidelity decay slows down with N!

Partial Conclusions

- For $\alpha = 0$, Zeno Hamiltonian describes the dynamics up to time scale increasing with *N*! *Cooperative Shielding*
- Also for $0 < \alpha < 1$ we saw signatures of shielding. Explanation is more difficult since the bands of *V* are not degenerate and *NN* interaction connects states within each band.

Classical *vs* Quantum Shielding

Questions:

- Is it a classical or quantum effect?
- Is the energy gap essential?
- What if we rescale the long range term $(J/N^{1-\alpha})$?
- Classical case...continuum spin of modulus one.

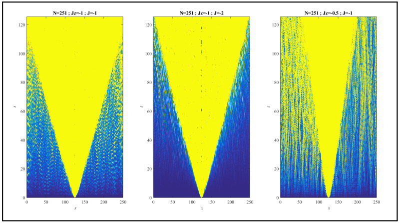
The classical model:

$$H = \sum_{j=1}^{N} h_j S_j^z + J_z \sum_{j=1}^{N-1} S_j^z S_{j+1}^z + \frac{J}{2N^{1-\alpha}} \sum_{j,m\neq j} \frac{S_j^x S_m^x}{|\mathbf{r}_j - \mathbf{r}_m|^\alpha},$$

Classical *vs* Quantum Shielding II

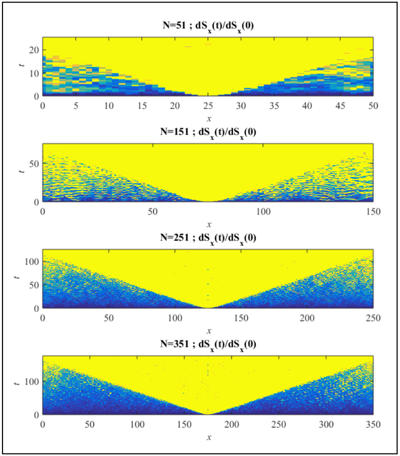
Simulations: $B = W = 0$, and $J_z = 1, J = 1/N^{1-\alpha}, \alpha = 0.5$, $E \approx 0.95 E_{min}$.

Shielding



Spreading of perturbations depends only on J_z and not on J! Short Ranged spreading.

Cooperativity



Short-ranged spreading last for longer time as we increase N!

Conclusions and Perspectives

1. *Cooperative Shielding* is able to explain contradictory behaviour of Long Range Systems.
2. Shielding allows to control quantum dynamics: spreading of information strongly depends on the initial state.
3. Classical *vs* Quantum Shielding. (R. Bachelard, USP, Sao Carlos, Brazil).
4. CS allows localization even in presence of LR: cold atoms.

Cooperative Shielding

THANK YOU!!!

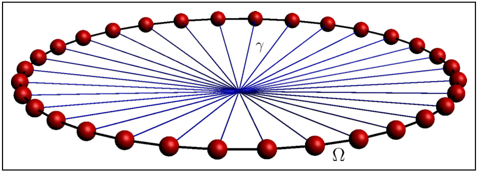
Cooperative Shielding in tight binding models

1d Anderson model with long range hopping:

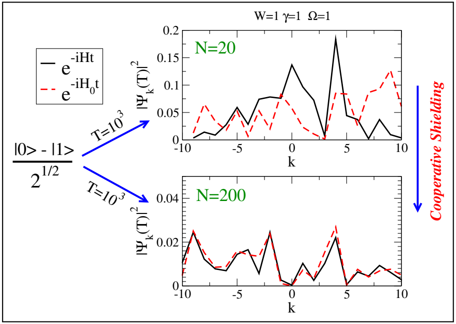
$$H = H_{\mathrm{NN}} + V_{\mathrm{LR}} + D =$$

$$H_{\mathrm{NN}} = -\Omega \sum_{\langle i,j \rangle} (|j\rangle\langle i| + |i\rangle\langle j|)$$

$$V_{\mathrm{LR}} = -\gamma \sum_{i \neq j} \frac{|i\rangle\langle j|}{r_{\mathrm{i,j}}^{\alpha}}; \qquad D = \sum_i \epsilon_i^0 |i\rangle\langle i|$$



Shielding with Disorder?



GLC, F. Borgonovi and R. kaiser, arXiv:1604.07868v1

Perspectives: many body open quantum systems

Fano, Rev. Mod. Phys. **64**, 313 (1992), Nambu

A common mechanism of collective phenomena

U. Fano

Department of Physics and The James Franck Institute, University of Chicago, Chicago, Illinois 60637

A common thread is followed through diverse phenomena: Weak interactions lock seemingly independent variables into a collective state of enhanced or depressed energy. A rather novel perspective is thus afforded on superconductivity and on nuclear and particle physics.

plasmon excitation, nuclear physics, superconductivity

superradiance-superconductivity

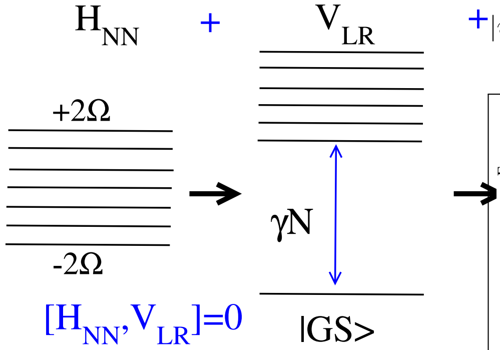
sion only. Application of this approach to superconductivity will allow us to bypass apparent complexities of the BCS theory, which are rooted in its historical development and persist in current texts. It will also stress the effective strength of interactions over the role of specific mechanisms. Superfluidity will not be dealt with *per se,* but is presumed to differ from superconductivity mainly in that it concerns mass displacements instead of charge displacements.

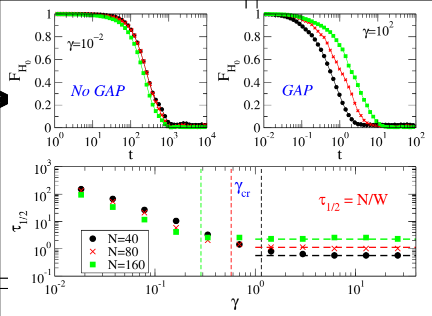
Cooperative Shielding

Fidelity: (Loschmidt echo)

$$F(t) = |\langle\psi_0|e^{iH_0t/\hbar}e^{-iHt/\hbar}|\psi_0\rangle|^2$$

+ $|\psi_0\rangle$: random superposition of excited states.

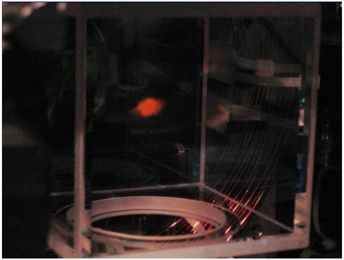




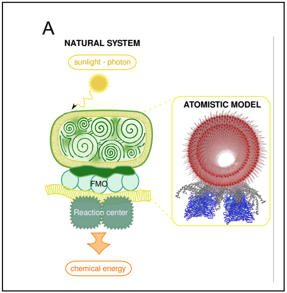
Experimental Relevance

Cold Atomic Clouds

Robin Kaiser (CNRS, France)



Transport Photosynthetic Complexes



SESC in Exciton Wires

J. Feist and F. J. Garcia-Vidal

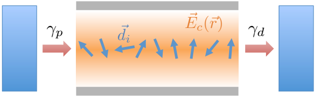
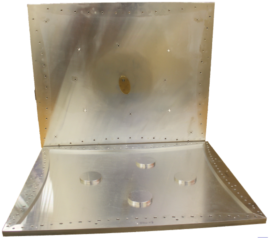


FIG. 1. Sketch of the model system. A 1D chain of (possibly disordered) quantum emitters with dipole moments $\vec{d}_i$ inside a cavity with cavity mode $\vec{E}_c(\vec{r})$. Excitons are pumped into the system from the left reservoir with rate γ_p. The exciton current is measured by the excitons reaching the sink reservoir on the right, coupled through incoherent decay of the last emitter with rate γ_d.

Microwave Cavity, U. Kuhl (LPMC, France).



Is Localization possible with long range?

- LONG RANGE HOPPING

  Levitov, PRL **64**, 547 1990: "IT IS KNOWN THAT IN SYSTEMS WITH DIMENSION d WITH $r^{-\alpha}$ INTERACTION, LOCALIZATION CAN EXIST ONLY IF $\alpha > d$. FOR $\alpha \leq d$ A DIVERGING NUMBER OF RESONANCES DESTROYS LOCALIZED STATES".

  ANDERSON (1958): More distant sites are not important because the probability of finding one with the right energy increases much more slowly with distance than the interaction decreases

Localization and Long Range

Levitov Argument: EPL (1989):

Resonance Condition: $|V_{i,j}| > |\Delta E_{i,j}|$

Cosider two spheres: $2^k R < r_{i,j} < 2^{k+1} R$, the volume is

$$Vol_k = \frac{28\pi}{3}(2^k R)^3 \qquad N_k = \rho Vol_k$$

Probability of Resonance: V_k/W, with

$$V_k = A/(2^k R)^\alpha$$

.

Number of Resonances:

$$N_{res} = \frac{V_k}{W} N_k \propto R^{3-\alpha} \to \infty \;\; \text{for} \;\; \alpha < d$$

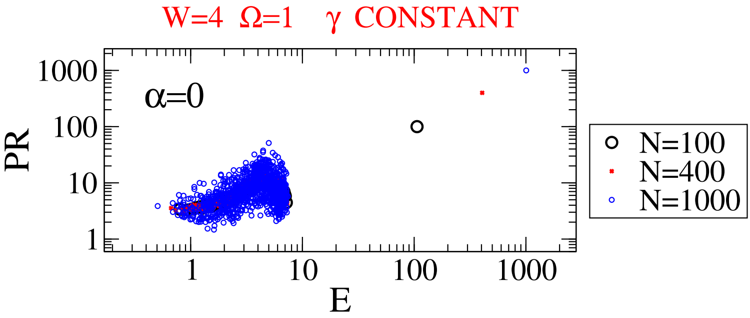
Random *vs* non-random long range

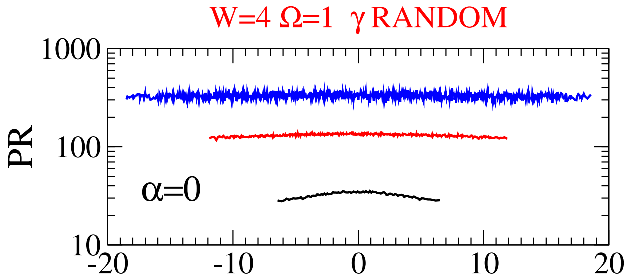
1. We consider a random coupling $\gamma_{i,j}$ is randomly distributed between -1 and $+1$.
2. Compute *PR vs E* for random and non random.

$$H = \sum_i E_i |i\rangle\langle i| - \Omega \sum_i |i\rangle\langle i+1| - \sum_{i,j} \frac{\gamma_{i,j}}{r_{i,j}^{\alpha}} |i\rangle\langle j|$$

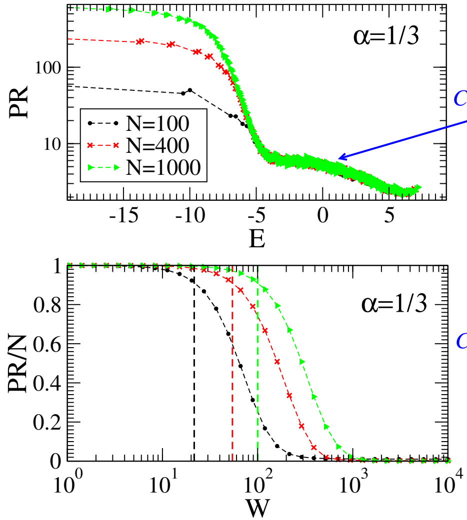
$\gamma_{i,j} = \gamma$ or $\gamma_{i,j}$ random.

Random *vs* non random: localization





Hermitian long range: Results I

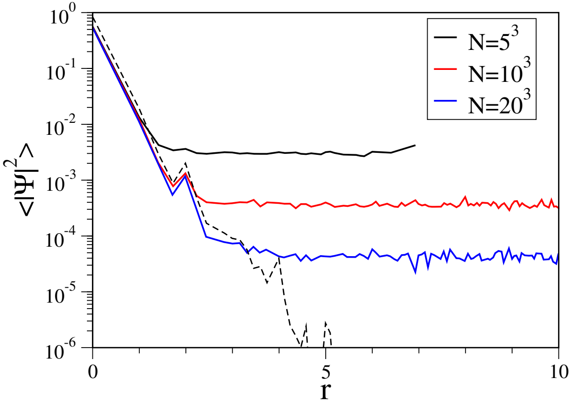


Cooperative Shielding

$$PR=(\Sigma_k|\Psi_k|^4)^{-1}$$

Cooperative Robustness

Structure of Eigenstates



hybrid states: *Anderson Peak + plateau*

SUPPRESSION OF EXPONENTIAL DECAY OF TRANSMISSION

COOPERATIVE SHIELDING

Shielding in cold atoms?

Our starting point is the effective Hamiltionian (E. Akkermans, A. Gero and R. Kaiser, PRL **101** 103602, 2008) for N two levels atoms system when only one photon is present

$$H_{eff} = \left(\hbar\omega_0 - \mathrm{i}\frac{\hbar\Gamma_0}{2}\right) S_z + \frac{\hbar\Gamma_0}{2}\sum_{i\neq j} V_{ij} S_i^+ \, S_j^- \,,$$

The potential is a random and complex-valued quantity and in a scalar approximation can be written as:

$$V_{ij} = -\frac{\cos(k_0 r_{ij})}{k_0 r_{ij}} - \mathrm{i}\,\frac{\sin(\mathrm{k_0 r_{ij}})}{\mathrm{k_0 r_{ij}}},$$

where $k_0 = 2\pi\lambda_0$ and $r_{ij} = |r_i - r_j|$. For $k_0 r_{i,j} \to 0$ Dicke limit, the same of Anderson Model!!

Consequences on Transport: Localization and long range.

- Localization: Absence of diffusion, Anderson 1958.
- LONG RANGE HOPPING

  Levitov, PRL **64**, 547 1990: "IT IS KNOWN THAT IN SYSTEMS WITH DIMENSION d WITH $r^{-\alpha}$ INTERACTION, LOCALIZATION CAN EXIST ONLY IF $\alpha > d$. FOR $\alpha \leq d$ A DIVERGING NUMBER OF RESONANCES DESTROYS LOCALIZED STATES".

  ANDERSON (1958): More distant sites are not important because the probability of finding one with the right energy increases much more slowly with distance than the interaction decreases
- Resonance Condition: $|V_{i,j}| > |\Delta E_{i,j}|$

$$N_{res} = \frac{V_k}{W} N_k \propto R^{d-\alpha} \to \infty \ \ for \ \ \alpha < d$$

Experimental Relavance: $\alpha = 0$ case

- Ion Traps, valence electron can be in two states: spin $1/2$.

$$H = \sum_k h_k \sigma_k^z + J_{NN} \sum [\sigma_x^k \sigma_x^{k+1} + \sigma_y^k \sigma_y^{k+1}] + J_{LR} \sum \frac{[\sigma_x^l \sigma_x^m + \sigma_y^l \sigma_y^m]}{r_{lm}^\alpha}$$

  $\alpha = 0, 0.5, 3$. Equivalent to our model in the single excitation manifold.

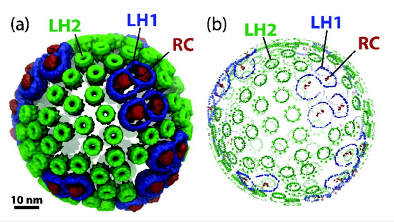
- Light Harvesting models: $H_{\text{eff}} = H_0 + H_{em} = H_0 + \Delta - i\gamma Q/2$,

$$\begin{aligned} Q_{i,j} &= \gamma \cos \phi_{i,j} \\ \Delta_{i,j} &= \tfrac{3\gamma}{4(k_0 r_{i,j})^3} (\cos \phi_{i,j} - 3 \cos \theta_i \cos \theta_j) \end{aligned} \qquad (3)$$

- Superconducting grains, Excitonic transport, Microwave cavities..

Purple Bacteria photosynthetic complexes

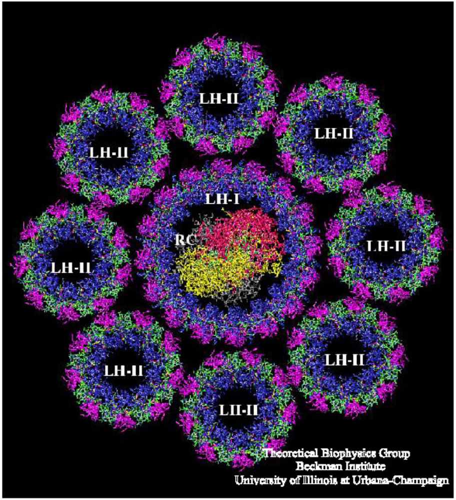
Purple Bacteria photosynthetic complex



Spherical chromatophore from Rhodobacter sphaeroides. J. Strumpfer et al., Phys. Chem. Lett. **3**, 536 (2012)

Photosynthetic Complexes

LH1-RC complex: 32 chromophores on a circle, and 4 central chromophores.



Superradiance Transition (ST)

Simplified H_{eff}:

$$H_{\mathrm{eff}} = H_0 - \frac{i}{2}\gamma Q \quad \mathcal{E}_k = E_k^0 - i\Gamma_k/2$$

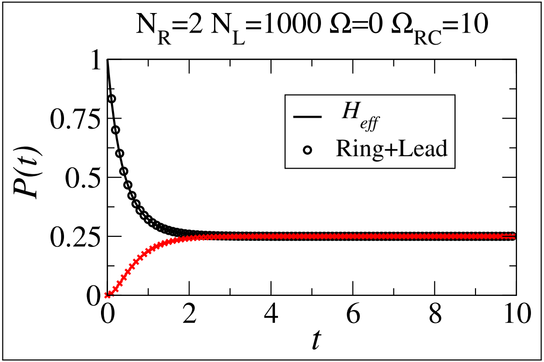
- *Non Superradiant regime*: for $\gamma \ll 1$: $\mathcal{E}_i = E_0^i - \frac{i}{2}\gamma Q_{ii}$.
- *Superradiant regime*: for $\gamma \gg 1$: $Q_{i,j} = \sum_{c=1}^{M} A_i^c A_j^c$. Only M with $\Gamma \neq 0$

Roughly, the transition occurs when

$$\langle\Gamma\rangle/D \approx 1,$$

where D is the mean level spacing of H_0.
Signature of Superradiance: non monotonic behavior of Γ_k.

Superradiance is a general phenomenon!



One state and continuum: $c_0(t) = e^{-i(E_0+\delta)t/\hbar - \gamma t/2\hbar}$,

$\gamma = \hbar w_{FGR} = 2\pi |A_i(E)|^2 \rho$

Many states and continuum: $H_{eff} = H_0 + \Delta - i\gamma/2Q$

$$Q_{i,j} = 2\pi A_i(E)(A_j(E))^* \rho(E)$$

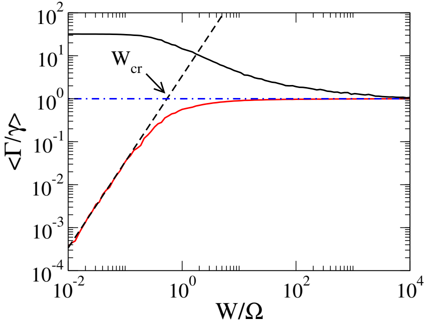


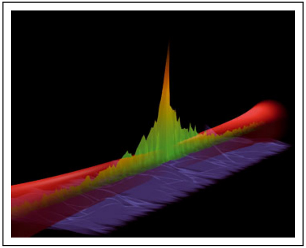
Figure: Average decay widths for a ring with $N = 32$, $\gamma = 10^{-3}$, $\Omega = 1$ are plotted *vs* the disorder strength, W.

COOPERATIVE ROBUSTNESS TO DISORDER for $N\gamma/4\Omega \gg 1$:

$$W_{cr} \equiv ST \quad W_{cr} \propto \gamma N; \quad (W_{cr} \propto \Gamma_{SR})$$

INTRO: Anderson Localization

- Absence of diffusion
- Anderson Model:
  $H = \sum E_i |i><i| + \sum \Omega_{i,k} |i><k| + h.c.$
- localization only if Ω decays faster then D
- $|\psi| \sim e^{-|x-x_0|/\xi}$
- $< lnG > \propto -\xi L$: non ohmic!
- 1D: $\xi \propto l$,
  where l: elastic mean free path
- 2D: $\xi \propto le^{al}$
- 3D: Anderson transition



Exponential density profile (green) shows the atomic matter waves localized by an optical disorder (in blue) within a laser wave guide (red). Nature 453, 891-894 (12 June 2008), Juliette Billy et all. Laboratoire Charles Fabry de l'Institut d'Optique, Palaiseau (CNRS / UniversitÃÂ Paris Sud-XI / Institut Optique graduate school). www.atomoptic.fr

Interplay of superradiance and disorder

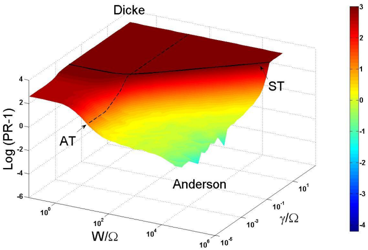


Figure: $Log(PR - 1)$ is plotted in the γ/Ω and W/Ω plane. (a) refers to the state with the largest width, which become superradiant above the ST. In both panels the system is a cubic lattice with $N = 10 \times 10 \times 10$ sites and $\Omega = 1$.

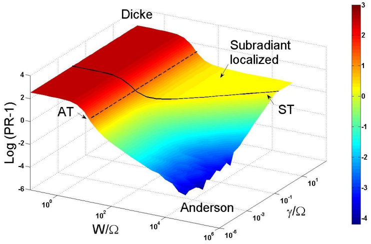


Figure: $Log(PR - 1)$ is plotted in the γ/Ω and W/Ω plane. $N - 1$ states which become subradiant above the ST.

Region I (at the left of the AT line)

Region II (right of the AT line and below the ST line)

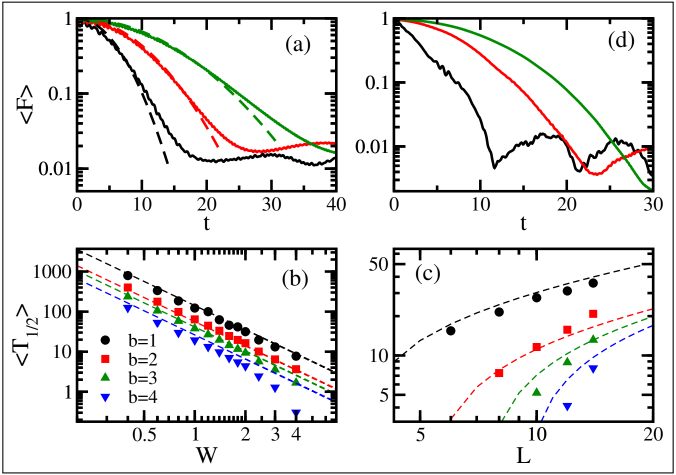
Region III (right of the AT line and above the ST line):

superradiant state is fully delocalized,

subradiant states are hybrid and localized.

subradiant localized regime.

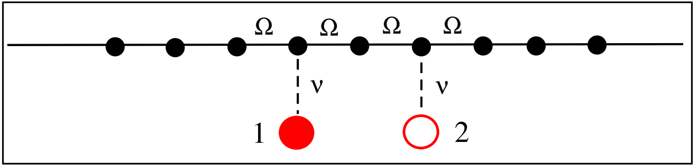
Zeno Fidelity

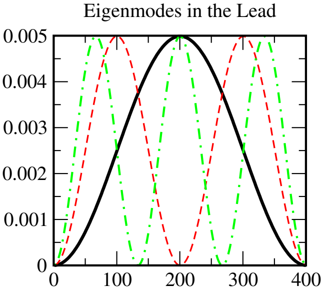


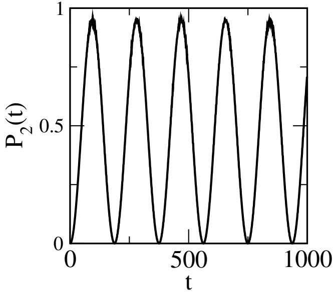
$$T_{1/2} \propto J\sqrt{L}/W^2$$

Mediated Long Range

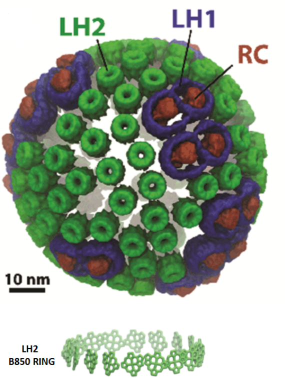
Effective Long Range interaction mediated by an external mode



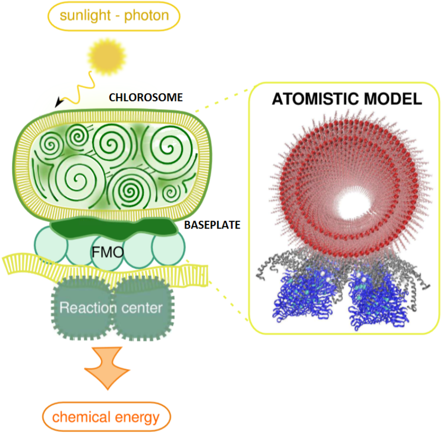




Purple Bacteria

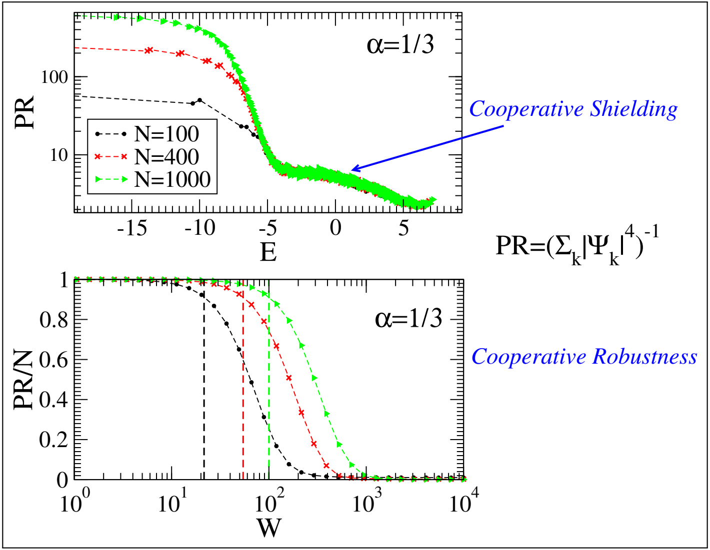


Green Sulfur Bacteria



Relation Structure-Functionality?

Long Range and Localization

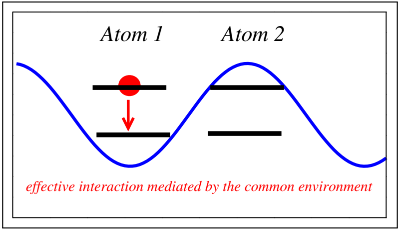


Effective interaction

One atom: $P(t) \propto e^{-\gamma t/\hbar}$, with $\gamma/\hbar$ from Fermi Golden Rule:

$$\gamma/\hbar = \frac{2\pi}{\hbar}|V|^2\rho$$

If I start with one atom $P_{1,2} \to 1/4$!



$$H_{eff} = \begin{pmatrix} E_0 - i\gamma/2 & -i\gamma/2 \\ -i\gamma/2 & E_0 - i\gamma/2 \end{pmatrix}$$

Complex Eigenvalues:
$\mathcal{E}_k = E_k^0 - i\Gamma_k/2$

Triplet: $|+\rangle = |1\rangle + |2\rangle$, with $\Gamma_+ = -\gamma$,

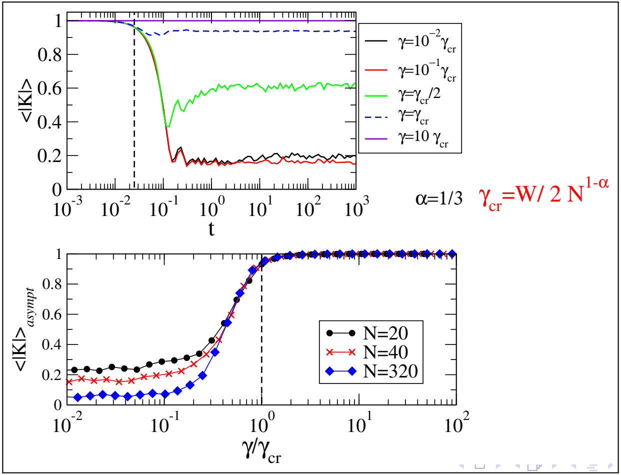
Singlet: $|-\rangle = |1\rangle - |2\rangle$, with $\Gamma_- = 0$,

$$|\psi(t)\rangle = \frac{e^{-i\mathcal{E}_+ t/\hbar}}{\sqrt{2}}|+\rangle + \frac{e^{-i\mathcal{E}_- t/\hbar}}{\sqrt{2}}|-\rangle$$

Generation of Entanglement coupled via common environment 2 qubits, zero temperature. F. Francica, S. Maniscalco, J. Piilo, F. Plastina and K.-A. Suominen, PRA **79**, 032310; S. Maniscalco, F. Francica, R. L. Zaffino, N. Lo Gullo and F. Plastina, PRL **100**, 090503 (2008).

Kuramoto parameter:

$$K = \frac{1}{N}\sum_{i=1}^{N}\frac{c_i}{|c_i|} = \frac{1}{N}\sum_i e^{i\theta_i} \qquad K = 1, 0$$



Localization and Long Range

Levitov Argument: EPL (1989):

Resonance Condition: $|V_{i,j}| > |\Delta E_{i,j}|$

Cosider two spheres: $2^k R < r_{i,j} < 2^{k+1} R$, the volume is

$$Vol_k = \frac{28\pi}{3}(2^k R)^3 \qquad N_k = \rho Vol_k$$

Probability of Resonance: V_k/W, with

$$V_k = A/(2^k R)^\alpha$$

.

Number of Resonances:

$$N_{res} = \frac{V_k}{W} N_k \propto R^{3-\alpha} \rightarrow \infty \;\; \text{for} \;\; \alpha < d$$

Fidelity Decay

$$H_0|E_0\rangle = E_0|E_0\rangle,\ H|E\rangle = E|E\rangle$$

$$F = |\langle E_0|e^{iH_0 t}e^{-iHt}|E_0\rangle|^2 = |\sum_E |\langle E|E_0\rangle|^2 e^{-i(E-E_0)t}|^2$$

In general for $H = H_0 + KV$, $E \propto K$ so that $\tau \propto 1/K$. When the Gap opens the eigenvalues of H are not perturbed in the same manner: Eigenvalues in the $Z = 0$ are not affected much.

Following Ref.s: G.L. Celardo, A. Biella, L. Kaplan, F. Borgonovi Fortschr. Phys. **61**, No. 2-3, 250-260 (2013).

V. V. Sokolov, I. Rotter, D. V. Savin and M. Müller, Phys. Rev. C **56**, 1031 (1997).

Analytical expression:

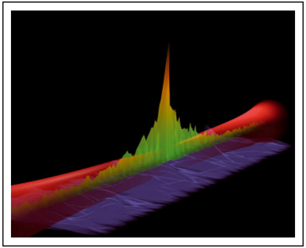
$$|E_\mu\rangle = \frac{1}{\sqrt{C_\mu}} \sum_{j^0=1}^{N} \frac{1}{E_\mu - E_j^0}|E_j^0\rangle$$

with $\sum_{j^0=1}^{N} \frac{1}{\tilde{\epsilon}_\mu - E_j^0} = 0$.$C_\mu$ is a normalization factor, $|E_j^0\rangle$, E_{j^0} are the eigenstates and eigenvalues of the closed Anderson model.

$\tau \propto N/W$.

Anderson Localization

- Absence of diffusion
- Anderson Model:
  $H = \sum E_i|i><i| + \sum \Omega_{i,k}|i><k| + h.c.$
- localization only if Ω decays faster then D
- $|\psi| \sim e^{-|x-x_0|/\xi}$
- $< lnG > \propto -\xi L$: non ohmic!
- 1D: $\xi \propto l$,
  where l: elastic mean free path
- 2D: $\xi \propto le^{al}$
- 3D: Anderson transition



Participation Ratio:

$$PR = \frac{1}{\sum_{i=1}^{N} |\psi(i)|^4}.$$

EXTENDED: $PR \propto N$
LOCALIZED: $PR =$const.

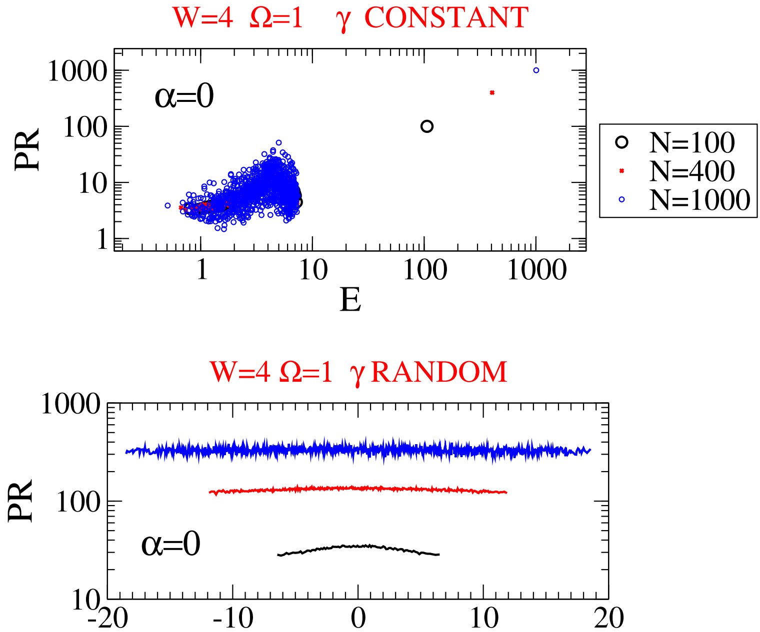
Random *vs* non-random long range

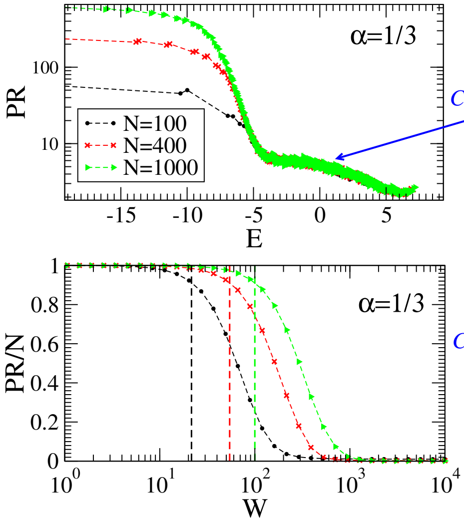
1. We consider a random coupling $\gamma_{i,j}$ is randomly distributed between -1 and $+1$.
2. Compute *PR vs E* for random and non random.

$$H = \sum_i E_i |i\rangle\langle i| - \Omega \sum_i |i\rangle\langle i+1| - \sum_{i,j} \frac{\gamma_{i,j}}{r_{i,j}^{\alpha}} |i\rangle\langle j|$$

$\gamma_{i,j} = \gamma$ or $\gamma_{i,j}$ random.

Random *vs* non random: localization





Cooperative Shielding

$PR = (\Sigma_k |\Psi_k|^4)^{-1}$

Cooperative Robustness

Experimental Relavance

- Ion Traps, valence electron can be in two states: spin $1/2$.
- Experimentally relevant: P. Jurcevic et al., Nature **511**, 202 (2014).
  $H = \sum_k h_k \sigma_k^z + J_{NN} \sum [\sigma_x^k \sigma_x^{k+1} + \sigma_y^k \sigma_y^{k+1}] + J_{LR} \sum \frac{[\sigma_x^l \sigma_x^m + \sigma_y^l \sigma_y^m]}{r_{lm}^\alpha}$
- long range hopping: coupling of electronic degrees of freedom with the ions collective modes of motion perpendicular to the string $0 \leq \alpha \leq 3$.

Ion Traps

Richerme et al., Nature Letter **511**, 198 (2014).

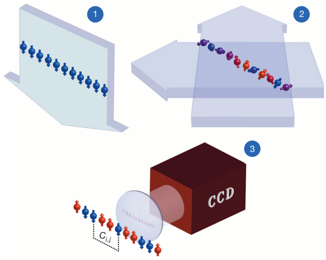


Figure 1 | Sketch of experimental protocol. Step (1): the experiment is initialized by optically pumping all 11 spins to the state $|\downarrow\rangle_z$. Step (2): after initialization, the system is quenched by applying laser-induced forces on the ions, yielding an effective Ising or *XY* spin chain (see text for details). Step (3): after allowing dynamical evolution of the system, the projection of each spin along the $\hat{z}$ direction is imaged onto a charge-coupled device (CCD) camera. Such measurements allow us to construct any possible correlation function $C_{i,j}$ along $\hat{z}$.

Shielding in cold atoms?

Our starting point is the effective Hamiltonian (E. Akkermans, A. Gero and R. Kaiser, PRL **101** 103602, 2008) for N two levels atoms system when only one photon is present

$$H_{eff} = \left(\hbar\omega_0 - \mathrm{i}\frac{\hbar\Gamma_0}{2}\right) S_z + \frac{\hbar\Gamma_0}{2}\sum_{i\neq j} V_{ij} S_i^+ \, S_j^- ,$$

The potential is a random and complex-valued quantity and in a scalar approximation can be written as:

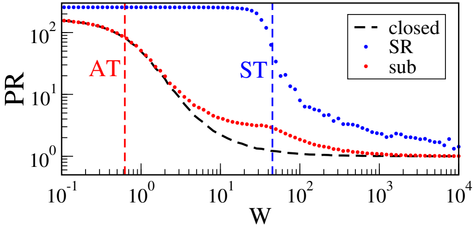
$$V_{ij} = -\frac{\cos(k_0 r_{ij})}{k_0 r_{ij}} - \mathrm{i}\,\frac{\sin(\mathrm{k_0 r_{ij}})}{\mathrm{k_0 r_{ij}}},$$

where $k_0 = 2\pi\lambda_0$ and $r_{ij} = |r_i - r_j|$. For $k_0 r_{i,j} \to 0$ Dicke limit, the same of Anderson Model!!

Cooperative Robustness to Noise

Open Anderson model with the addition of static (W) and dynamical (Γ^ϕ) disorder.

$$W_{cr} \propto \gamma N \quad \Gamma^\phi_{cr} \propto \gamma N$$



Fortschr. Phys. **61**, 250 (2013); EPL **103**, 57009 (2013); PRB **90**, 075113 (2014); PRB **90**, 085142 (2014); PRB **91**, 094301 (2015).

Long range interaction and energy gap at the origin of robustness:

$$H_{eff} = \begin{pmatrix} E_1 - i\gamma/2 & -i\gamma/2 \\ -i\gamma/2 & E_2 - i\gamma/2 \end{pmatrix}$$

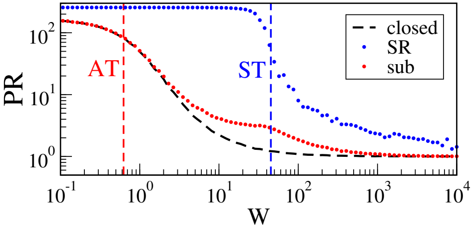
Superadiance *vs* Superconductivity
Distance independent coupling also present in discrete-BCS models (Jan von Delft, Ann. Phys. **3**, 219 (2001))

$$\Delta_{SR} = \Delta_{SC}$$

Shielding and Subradiance

1D and 3D Open Anderson model with static (W) and dynamical (Γ^{ϕ}) disorder.

$W_{cr} \propto \gamma N \quad \Gamma^{\phi}_{cr} \propto \gamma N$



Fortschr. Phys. **61**, 250 (2013); EPL **103**, 57009 (2013); PRB **90**, 075113 (2014); PRB **90**, 085142 (2014); PRB **91**, 094301 (2015).

Hybrid subradiant states

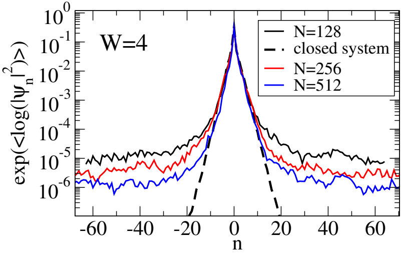


Figure: The averaged probability distribution of all eigenstates of the non-Hermitian Hamiltonian that are strongly peaked in the middle of the chain is shown. In all cases we fix $\Omega = 1, \gamma = 0.1$.

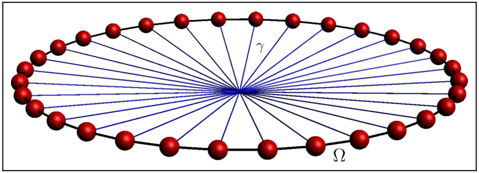
Common lore: no localization with long range..connection with CS

PART II: Cooperative Shielding. The Model

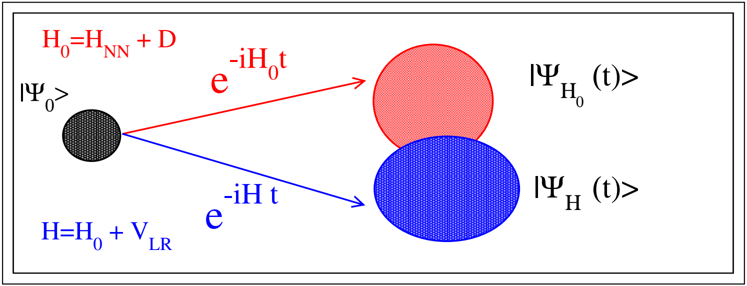
- 1d Anderson model with long range hopping:

$$H = D + H_{\rm NN} + V_{\rm LR} = \sum_i \epsilon_i^0 |i\rangle\langle i| - \Omega \sum_{\langle i,j\rangle} (|j\rangle\langle i| + |i\rangle\langle j|) - \gamma \sum_{i \neq j} \frac{|i\rangle\langle j|}{r_{\rm i,j}^{\alpha}}$$

- ϵ_j^0: are random energies $[-W/2, +W/2]$; $r_{i,j} = |i - j|$; long range for $\alpha < 1$. $\alpha = 0$: all to all.
- $\Omega > 0, \gamma > 0$: the tunnelling transition amplitude.
- Experimentally relevant in Ion Traps, P. Jurcevic et al., Nature **511**, 202 (2014).



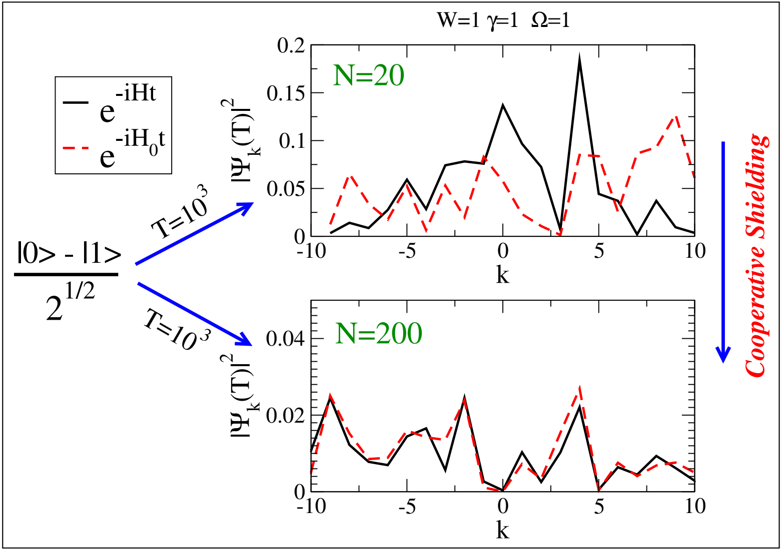
Loschmidt echo Fidelity



Fidelity: (Loschmidt echo)

$$F(t) = |\langle\psi_0|e^{iH_0t/\hbar}e^{-iHt/\hbar}|\psi_0\rangle|^2$$

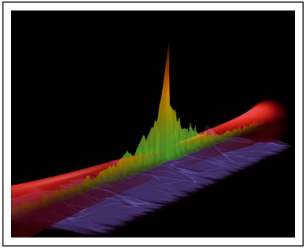
$|\psi_0\rangle$ is a random superposition of $N-1$ states with $Z=0$.



W=1 γ=1 Ω=1
e^{-iHt}
e^{-iH_0t}
$\frac{|0> - |1>}{2^{1/2}}$
$T=10^3$
$T=10^3$
N=20
N=200
$|\Psi_k(T)|^2$
$|\Psi_k(T)|^2$
k
k
Cooperative Shielding

Shielding and Transport: Anderson Localization

- Absence of diffusion
- Anderson Model:
  $H = \sum E_i |i><i| + \sum \Omega_{i,k} |i><k| + h.c.$
- localization only if Ω decays faster then D
- $|\psi| \sim e^{-|x-x_0|/\xi}$
- $< lnG > \propto -\xi L$: non ohmic!
- 1D: $\xi \propto l$,
  where l: elastic mean free path
- 2D: $\xi \propto le^{al}$
- 3D: Anderson transition



Participation Ratio:

$$PR = \frac{1}{\sum_{i=1}^{N} |\psi(i)|^4}.$$

EXTENDED: $PR \propto N$
LOCALIZED: $PR =$const.

Is Localization possible with long range?

- LONG RANGE HOPPING

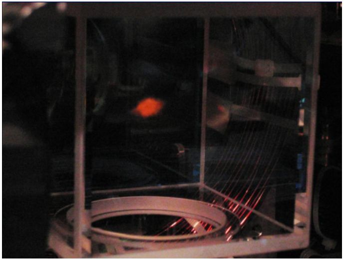
  Levitov, PRL **64**, 547 1990: "IT IS KNOWN THAT IN SYSTEMS WITH DIMENSION d WITH $r^{-\alpha}$ INTERACTION, LOCALIZATION CAN EXIST ONLY IF $\alpha > d$. FOR $\alpha \leq d$ A DIVERGING NUMBER OF RESONANCES DESTROYS LOCALIZED STATES".

  ANDERSON (1958): More distant sites are not important because the probability of finding one with the right energy increases much more slowly with distance than the interaction decreases

Experimental Relevance

Cold Atomic Clouds

Robin Kaiser (CNRS, France)



SESC in Exciton Wires

J. Feist and F. J. Garcia-Vidal

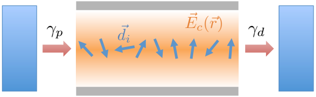
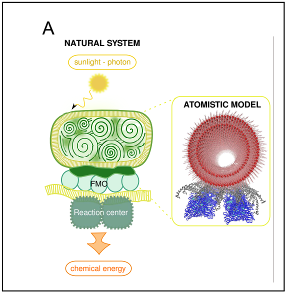
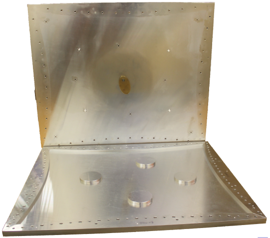


FIG. 1. Sketch of the model system. A 1D chain of (possibly disordered) quantum emitters with dipole moments $\vec{d}_i$ inside a cavity with cavity mode $\vec{E}_c(\vec{r})$. Excitons are pumped into the system from the left reservoir with rate γ_p. The exciton current is measured by the excitons reaching the sink reservoir on the right, coupled through incoherent decay of the last emitter with rate γ_d.

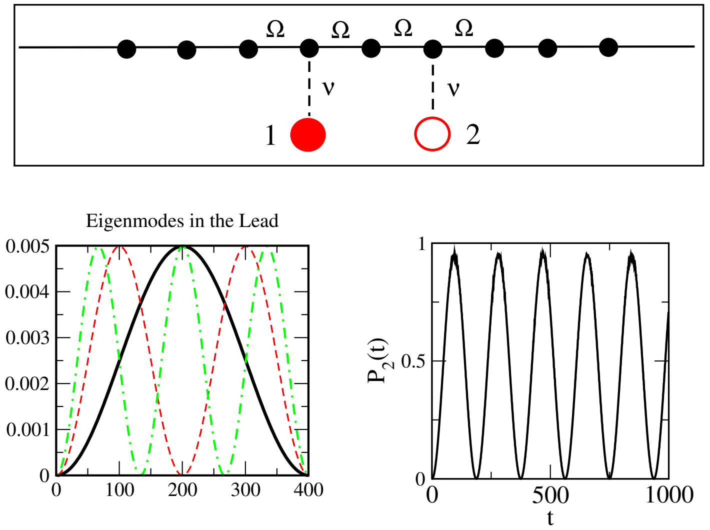
Transport Photosynthetic Complexes



Microwave Cavity, U. Kuhl (LPMC, France).

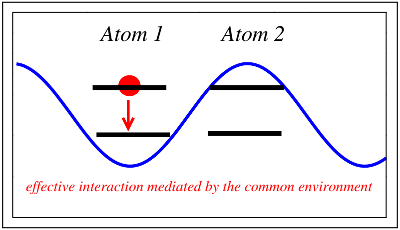


Effective Long Range interaction mediated by an external mode



The origin of cooperativity: effective coupling mediated by continuum

Dicke, PR **93**, 99 (1954).



One atom:

$$P(t) \propto e^{-\gamma t/\hbar}$$

with $\gamma/\hbar = \frac{2\pi}{\hbar}|A|^2\rho$ from FGR:
Two atoms: If I start with one atom

$$P_{1,2} \rightarrow 1/4$$

Single Excitation Superradiance: The Super of Superradiance Marlan O. Scully et al., Science, **325**, 1510 (2009). Single Atom:

$$e^{-\gamma t/\hbar}$$

$$|k\rangle = |0\rangle_1|0\rangle_2....|1\rangle_k....|0\rangle_N$$

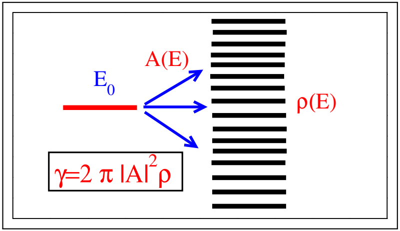
Cooperative Emission of N entangled atoms:

$$|Superradiant\rangle = \frac{1}{\sqrt{N}}\sum_{k=1,N}|k\rangle,$$

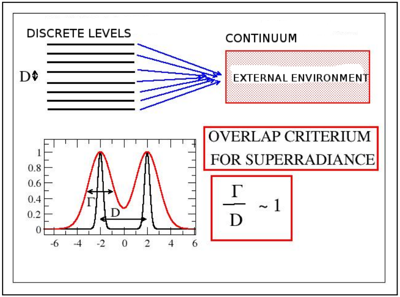
$$e^{-\Gamma_{SR}t/\hbar}, \quad \Gamma_{SR} = N\gamma$$

Subradiant, $\Gamma_{sub} = 0$

The Fermi Golden Rule and transition to Superradiance



What happens when we have many level?



$$P(t) = e^{-\gamma t/\hbar}, \gamma = 2\pi |A(E_0)|^2 \rho(E_0)$$

When exponential decay is valid? for $t < t_1$ $P(t) \approx 1 - \alpha t^2$ and for $t > t_2$ $P(t) \approx c/t^{\beta}$. S. Pascazio, H. Pastawski, A. Peres

Interference effects: transition to superradiance; cooperativity, deviations from FGR, Fano resonances.

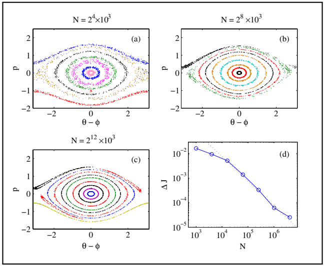
PART I: What does it mean long range?

- Statistical and Dynamical properties (S. Ruffo et al.). Coulomb, Gravitation (gravitational waves!), Magnetism..

$$V_{i,j} = \frac{J}{r_{ij}^{\alpha}}$$

- Non-Extensivity:
  $E \propto JN \int^{R} \frac{r^{d-1}}{r^{\alpha}} dr \propto JNR^{d-\alpha} \propto JN^{2-\alpha/d}$
- Non-Additivity:
  $E \neq E_1 + E_2$ even if $J \to J/N^{1-\alpha/d}$
- ensemble inequivalence
- Suppression of Chaoticity
- Non-Ergodicity (GLC, F Borgonovi, S Ruffo, J Barre')

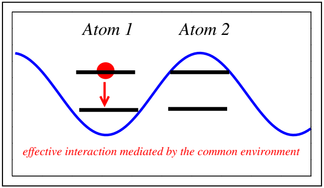
Abundance of Regular orbits

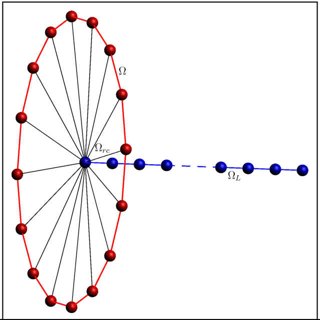


HMF, PRL **101** 260603 (2008)

The origin of robustness: effective coupling mediated by continuum

Dicke, PR **93**, 99 (1954).





Long range interaction and energy gap:

$$H_{eff} = \begin{pmatrix} E_1 - i\gamma/2 & -i\gamma/2 \\ -i\gamma/2 & E_2 - i\gamma/2 \end{pmatrix}$$

$$\Gamma_{SR} \propto N\gamma; \qquad \Gamma_{sub} \ll \gamma$$

Superadiance *vs* Superconductivity

Distance independent coupling also present in discrete-BCS models (Jan von Delft, Ann. Phys. **3**, 219 (2001))

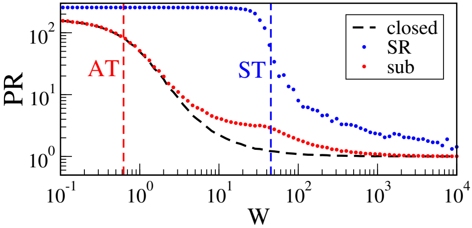
$$\Delta_{SR} = \Delta_{SC}$$

Robustness: Quantum *vs* Classical?

Shielding and Subradiance

1D and 3D Open Anderson model with static (W) and dynamical (Γ^{ϕ}) disorder.

$W_{cr} \propto \gamma N \quad \Gamma^{\phi}_{cr} \propto \gamma N$



Fortschr. Phys. **61**, 250 (2013); EPL **103**, 57009 (2013); PRB **90**, 075113 (2014); PRB **90**, 085142 (2014); PRB **91**, 094301 (2015).

Hybrid subradiant states

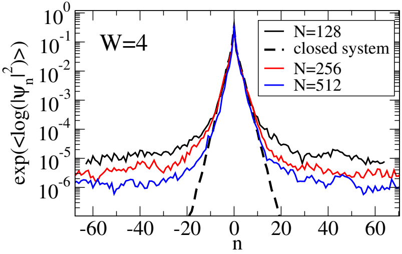


Figure: The averaged probability distribution of all eigenstates of the non-Hermitian Hamiltonian that are strongly peaked in the middle of the chain is shown. In all cases we fix $\Omega = 1, \gamma = 0.1$.

Common lore: no localization with long range..connection with CS

Cooperative Shielding in many-body.

Experimentally accessible spin 1/2 Hamiltonian:

$$H = H_0 + V, \tag{4}$$
$$H_0 = \sum_{n=1}^{L} (\mathcal{B} + h_n)\sigma_n^z + \sum_{n=1}^{L-1} J_z \sigma_n^z \sigma_{n+1}^z,$$
$$V = \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x.$$

- transverse field: $h_n \in [-W/2, W/2]$.
- $\alpha < 1$: long range. $\alpha > 1$: short range.

The case $\alpha = 0$:

$$V = J \sum_{n<m} \sigma_n^x \sigma_m^x = \frac{JM_x^2}{2} - \frac{JL}{2} \quad \text{where} \quad M_x = \sum_n \sigma_n^x$$

$$V_b = J(L/2 - b)^2/2 - JL/2, \quad \text{where} \quad b = 0, 1, \ldots L/2$$

Zeno Shielding in Many body Systems

In Such subspaces long range does not affect the dynamics: *Cooperative Zeno Shielding*.

- External Field:

$$\sum_{n=1}^{L}(\mathcal{B}+h_n)\sigma_n^z \to \sum_{n=1}^{L}(\mathcal{B}+h_n)(\sigma_n^+ + \sigma_n^-)/2$$

  does not connect states inside the bands.

- NN interaction:

$$J_z\sum_{n=1}^{L-1}\sigma_n^z\sigma_{n+1}^z \to (J_z/4)\sum_{n=1}^{L-1}(\sigma_n^+ + \sigma_n^-)(\sigma_{n+1}^+ + \sigma_{n+1}^-)$$

  The projection leaves only the terms $\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+$: **effective NN interaction** which conserves the number of excitation inside each band *b*.