ESQPT in systems with long-range interactions

Lea F. Santos
Yeshiva University, New York, NY, USA

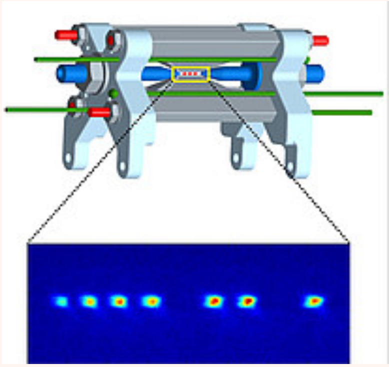
Francisco Pérez-Bernal
Universidad de Huelva, Spain

G. Luca Celardo and Fausto Borgonovi
Universita Cattolica del Sacro Cuore, Italy

- Consequences of the presence of an ESQPT (static and dynamics).
- Lipkin model [U(2)]: experiments with ion traps, BEC, NMR but valid also for U(n+1)

PRA **94,** 012113 (2016)
arXiv:1604.06851 (Fort. Physik)
PRA **92,** 050101R (2015)

Trapped ions: long-range interaction



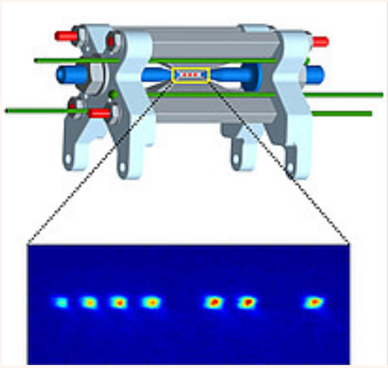
Ion Traps

$$H = B\sum_n \sigma_n^z + \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x$$

$$0 \sim \alpha \leq 3$$

P. Richerme et al, Nature **511**, 198 (2014)
P. Jurcevi et al, Nature **511**, 202 (2014)

Trapped ions: long-range interaction



$$H = B\sum_n \sigma_n^z + \sum_{n<m} \frac{J}{|n-m|^\alpha}\sigma_n^x \sigma_m^x$$

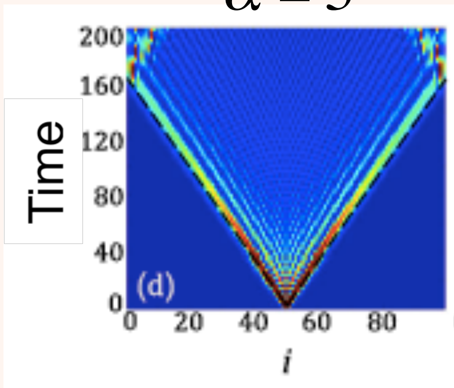
$0 \sim \alpha \leq 3$

P. Richerme et al, Nature **511**, 198 (2014)
P. Jurcevi et al, Nature **511**, 202 (2014)

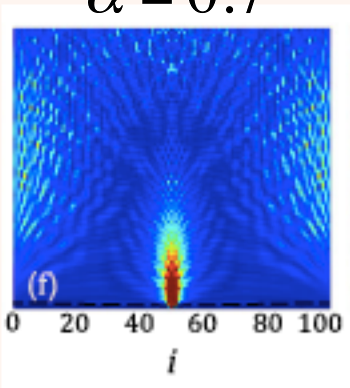
P. Hauke and L. Tagliacozzo, PRL **111**, 207202 (2013)

Magnetization in z of each site

$\alpha = 3$



$\alpha = 0.7$



Faster than Lieb-Robinson bound

N=100, excitation on 50

$\downarrow \downarrow \cdots \downarrow \downarrow \uparrow \downarrow \downarrow \cdots \downarrow \downarrow$ z

Lipkin Model: infinite-range interaction

$$H = B\sum_n \sigma_n^z + \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x$$

$\alpha = 0$

$$H = B\sum_n \sigma_n^z + J\sum_{n<m} \sigma_n^x \sigma_m^x$$

$$S_z = \sum_n \sigma_n^z \qquad S_x = \sum_n \sigma_n^x$$

$\text{dimension} = 2^N$

to

$\text{dimension} = \frac{N}{2} + 1$

N = number of sites

Lipkin Model: infinite-range interaction

$$H = B\sum_n \sigma_n^z + \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x$$

$\alpha = 0$

$$H = B\sum_n \sigma_n^z + J\sum_{n<m} \sigma_n^x \sigma_m^x$$

$$S_z = \sum_n \sigma_n^z \qquad S_x = \sum_n \sigma_n^x$$

dimension $= 2^N$

to

dimension $= \frac{N}{2} + 1$

N = number of sites

Lipkin-Meshkov-Glick model

$$H = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$

Control parameter

Ground state quantum phase transition

$\xi_c = 0.2$

Lipkin Model: U(2) algebraic structure

U(2) U(1) SO(2)

$$H = (1-\xi)\left(\frac{N}{2} + \mathcal{S}_z\right) - \frac{4\xi}{N}\mathcal{S}_x^2$$

Schwinger representation: $\mathcal{S}^+ = 2t^+ s$ $\mathcal{S}_z = \frac{1}{2}(t^+ t - s^+ s)$

U(2) U(1) SO(2)

Two species of scalar bosons

$$H = (1-\xi)n_t + \frac{\xi}{N}(t^+ s + s^+ t)^2$$

$$n_t = t^+ t$$

Ground state QPT

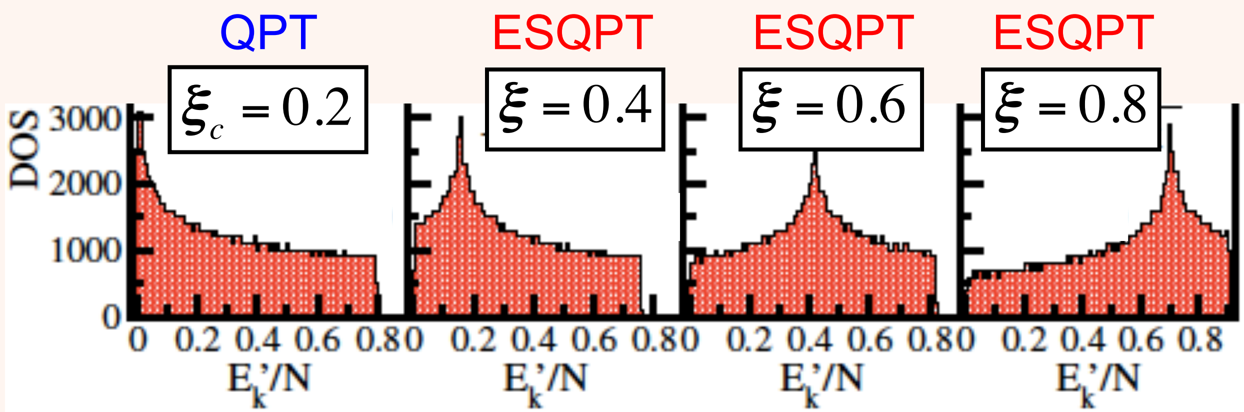
$$\xi_c = 0.2$$

In general:

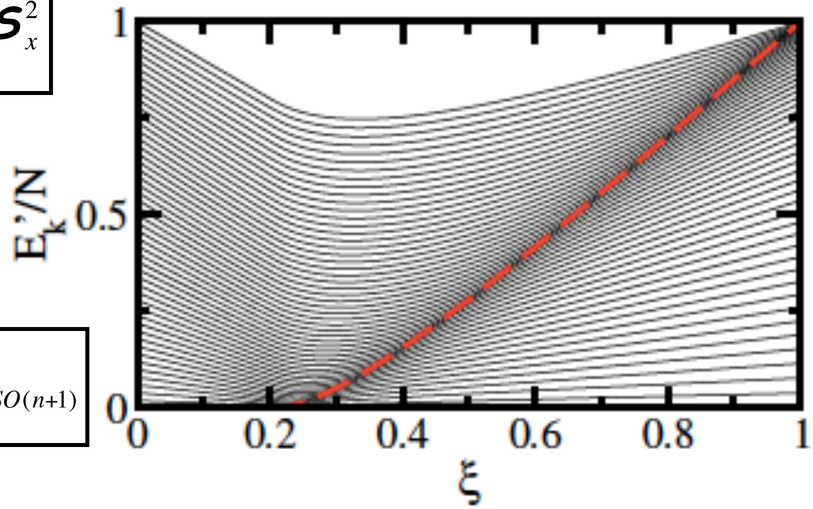
$$H_{U(n+1)} = (1-\xi)H_{U(n)} + \frac{\xi}{N}H_{SO(n+1)}$$

PRA **92**, 050101R (2015)

Excited State Quantum Phase Transition



$$H = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$



Separatrix
that marks the ESQPT

$$E_{ESQPT} = \frac{(1-5\xi)^2}{16\xi}$$

$$H_{U(n+1)} = (1-\xi) H_{U(n)} + \frac{\xi}{N} H_{SO(n+1)}$$

ESQPT: participation ratio in U(1) basis

$$H = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$

U(1)-basis

$$\left|\psi_{U(2)}^{(k)}\right\rangle = \sum_{s_z=-N/2}^{N/2} C_{s_z}^{(k)} \left|S\ m_z\right\rangle$$

Participation Ratio

$$PR^{(k)} \equiv \frac{1}{\sum_{s_z=-N/2}^{N/2} |C_{s_z}^{(k)}|^4}$$

Large PR: delocalized state

Small PR: localized state

LFS & Pérez-Bernal
PRA**92**, 050101R (2015).

Eigenstate at ESQPT localized at U(1) basis with mz=-N/2

$$H = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$

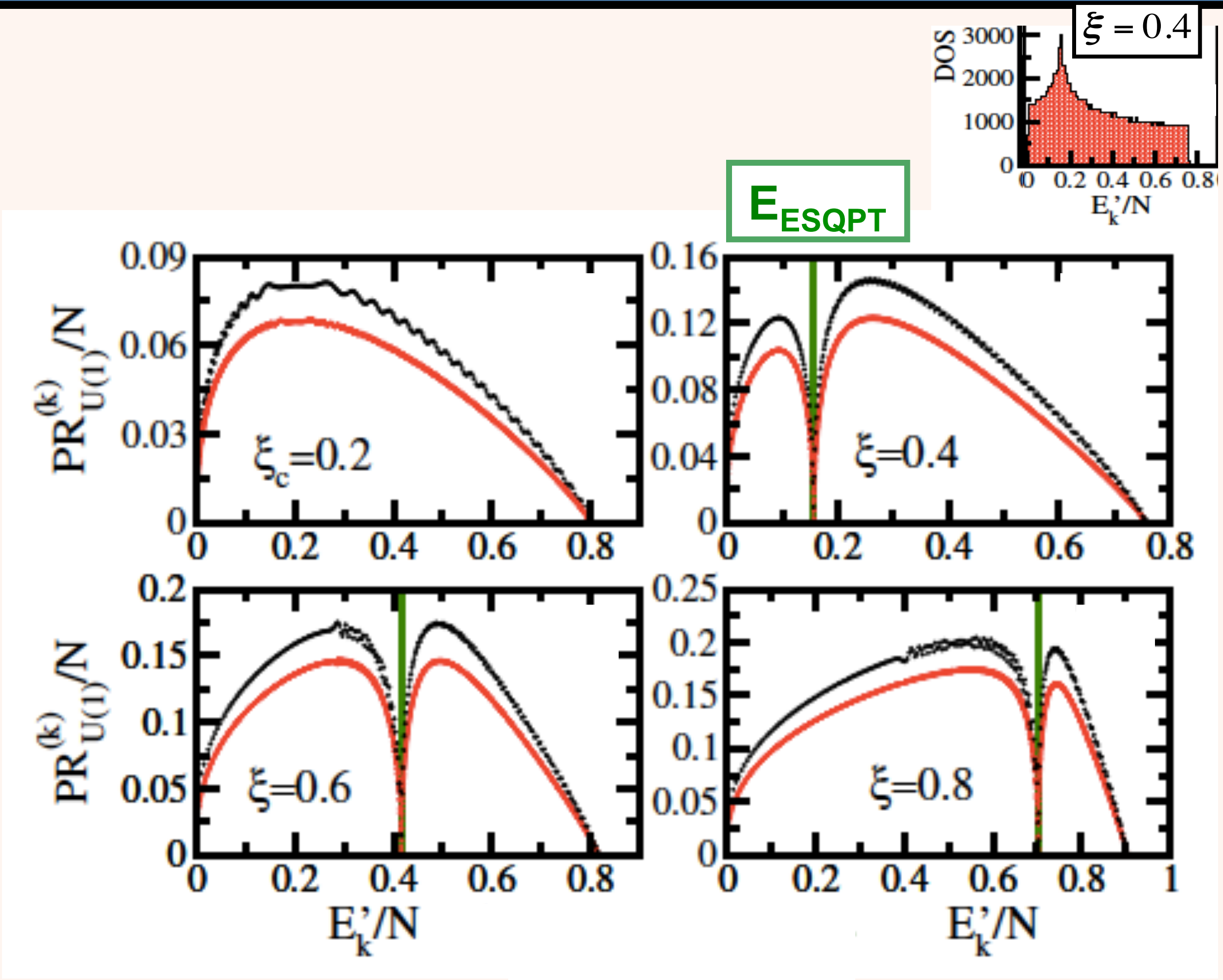
U(1)-basis

$$\left|\psi_{U(2)}^{(k)}\right\rangle = \sum_{s_z=-N/2}^{N/2} C_{s_z}^{(k)} \left|S\ m_z\right\rangle$$

Participation Ratio

$$PR^{(k)} \equiv \frac{1}{\sum_{s_z=-N/2}^{N/2} |C_{s_z}^{(k)}|^4}$$

N=600, 2000



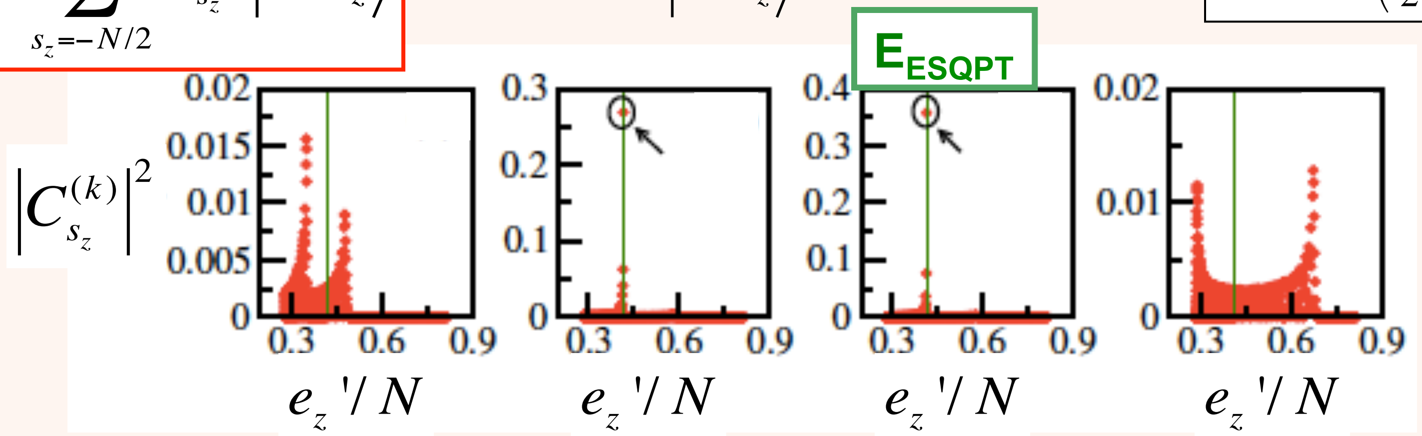
PRA**92**, 050101R (2015).

Eigenstate at ESQPT localized at U(1) basis with mz=-N/2

$$\left|\psi^{(k)}_{U(2)}\right\rangle = \sum_{s_z=-N/2}^{N/2} C^{(k)}_{s_z} \left|s\ m_z\right\rangle$$

U(1) basis $\left|s\ m_z\right\rangle$

$$H = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$



$\xi = 0.6$

N=600

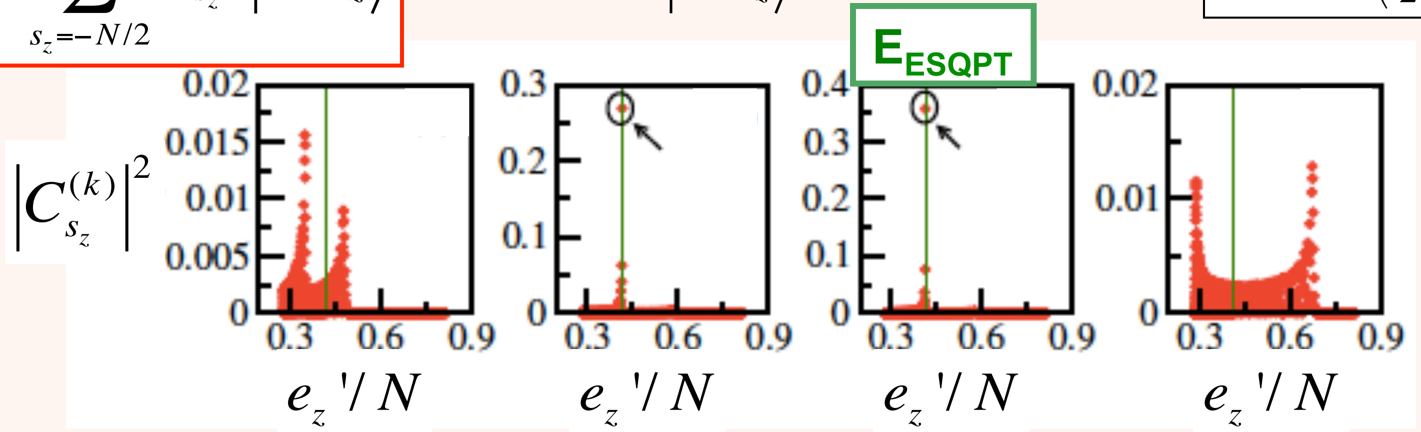
$$e_z = \left\langle s\ m_z \right| H_{U(2)} \left| s\ m_z \right\rangle$$

Eigenstate at ESQPT localized at U(1) basis with mz=-N/2

$$\left|\psi_{U(2)}^{(k)}\right\rangle = \sum_{s_z=-N/2}^{N/2} C_{s_z}^{(k)} \left|s\ m_z\right\rangle$$

U(1) basis $\left|s\ m_z\right\rangle$

$$H = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$



$\xi = 0.6$

N=600

$$e_z = \left\langle s\ m_z \right| H_{U(2)} \left| s\ m_z \right\rangle$$

Energy of the U(1) basis vectors

$$e_z = \left\langle s\ m_z \right| H_{U(2)} \left| s\ m_z \right\rangle$$

$$\downarrow\downarrow..\downarrow\downarrow\downarrow\downarrow..\downarrow\downarrow_Z$$

$$\left|m_z = -N/2\right\rangle$$

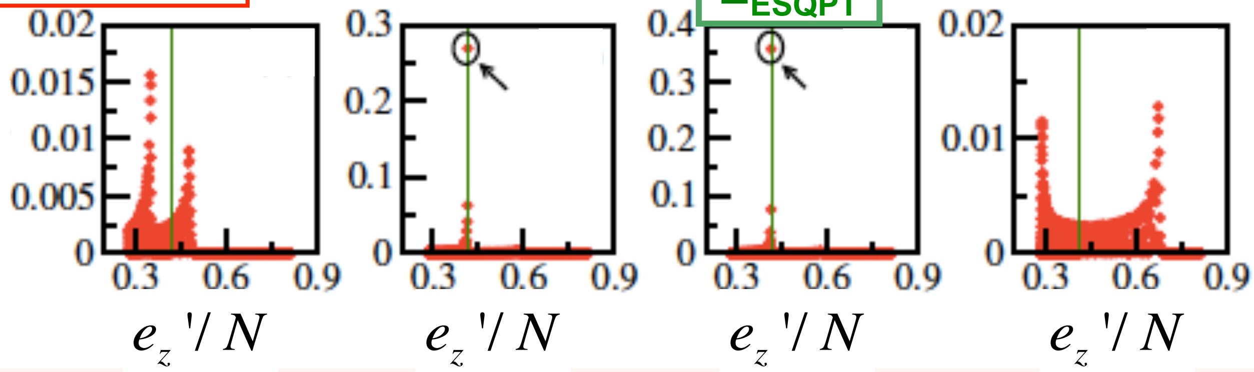
Eigenstate at ESQPT localized at U(1) basis with mz=-N/2

$$\left|\psi_{U(2)}^{(k)}\right\rangle = \sum_{s_z=-N/2}^{N/2} C_{s_z}^{(k)} \left| s\ m_z \right\rangle$$

U(1) basis $\left| s\ m_z \right\rangle$

$$H = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$

$\left| C_{s_z}^{(k)} \right|^2$



$\xi = 0.6$

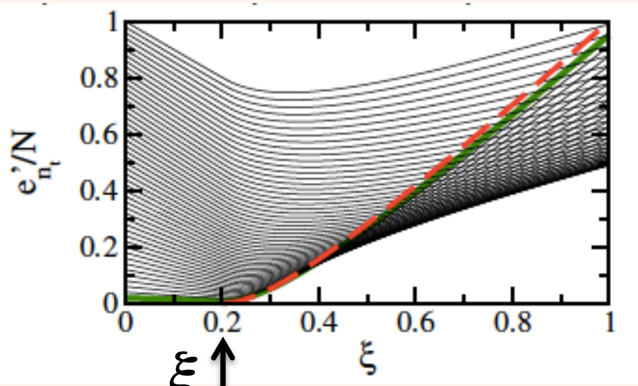
N=600

Energy of the U(1) basis vectors

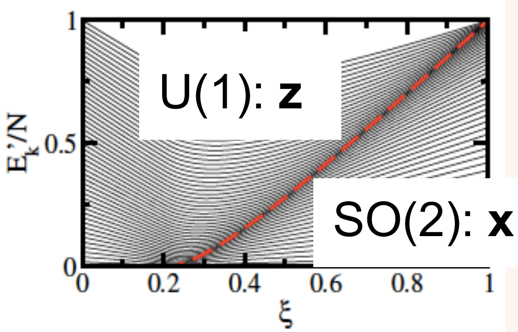
$e_z = \left\langle s\ m_z \right| H_{U(2)} \left| s\ m_z \right\rangle$

$\downarrow\downarrow..\downarrow\downarrow\downarrow\downarrow..\downarrow\downarrow_Z$

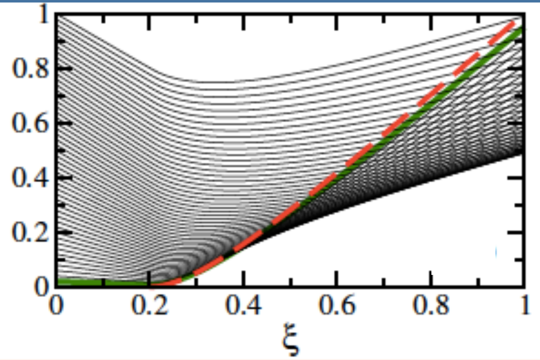
$\left| m_z = -N/2 \right\rangle$



Ground state QPT



Quench from U(n) to U(n+1)



U(1) ground state $\downarrow\downarrow\cdot\cdot\downarrow\downarrow\downarrow\downarrow\cdot\cdot\downarrow\downarrow_z$ $\longrightarrow H_{U(2)} = (1-\xi)\left(\frac{N}{2}+\boldsymbol{S}_z\right) - \frac{4\xi}{N}\boldsymbol{S}_x^2$

Initial state U(1)-basis

$$|\Psi(0)\rangle = |S\ m_z\rangle \longrightarrow H_{U(2)} = (1-\xi)\left(\frac{N}{2}+\boldsymbol{S}_z\right) - \frac{4\xi}{N}\boldsymbol{S}_x^2$$

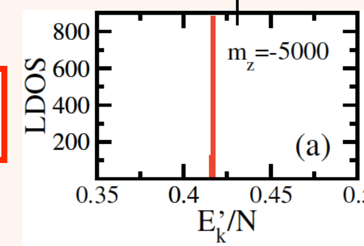
Survival Probability

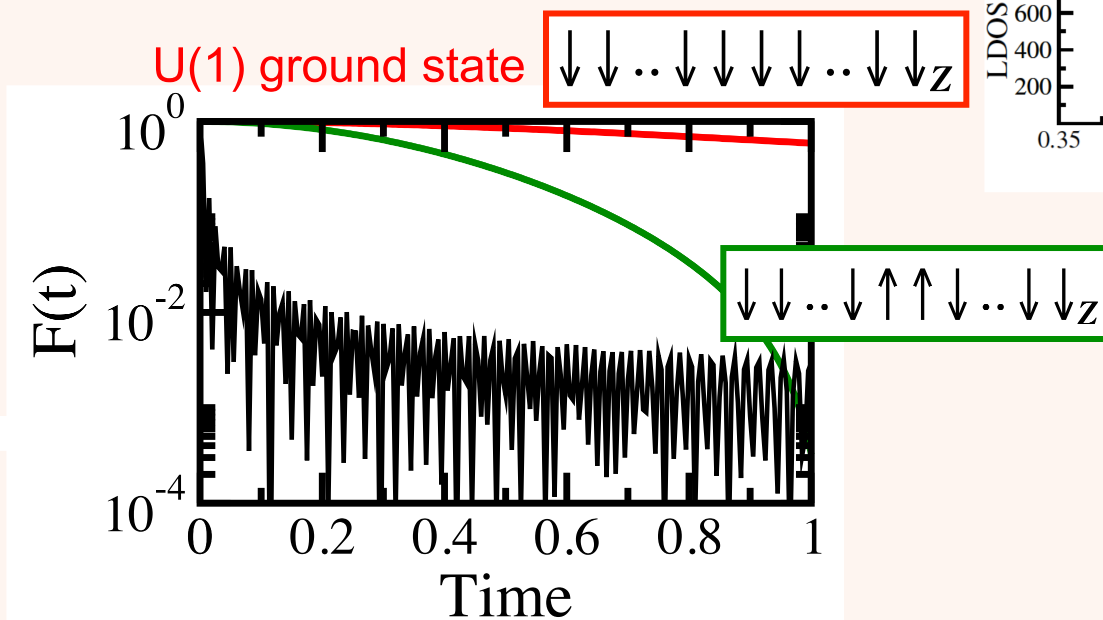
$$F(t) = \left|\langle\Psi(0)\,|\,\Psi(t)\rangle\right|^2$$

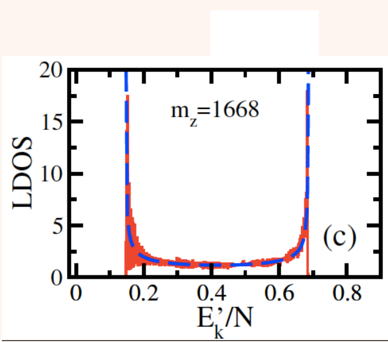
Initial state: U(1)-basis vector
Slow decay

Initial state $\left|\Psi(0)\right\rangle = \left|s\ m_z\right\rangle = \sum_k C_{s_z}^{(k)} \left|\psi_{U(2)}^{(k)}\right\rangle$

Survival Probability $F(t) = \left|\left\langle \Psi(0) | \Psi(t) \right\rangle\right|^2 = \left| \sum_k \left| C_{m_z}^{(k)} \right|^2 e^{-iE_k t} \right|^2 = \left| \int_{-\infty}^{\infty} \rho_{ini}(E) e^{-iEt}\, dE \right|^2$







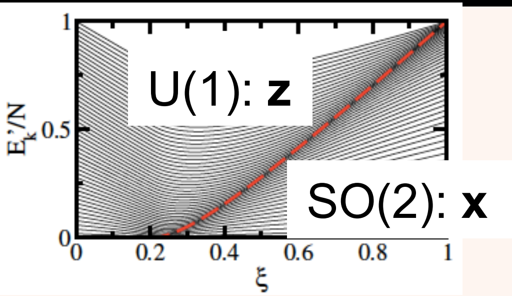
$\xi = 0.6$ N=1000

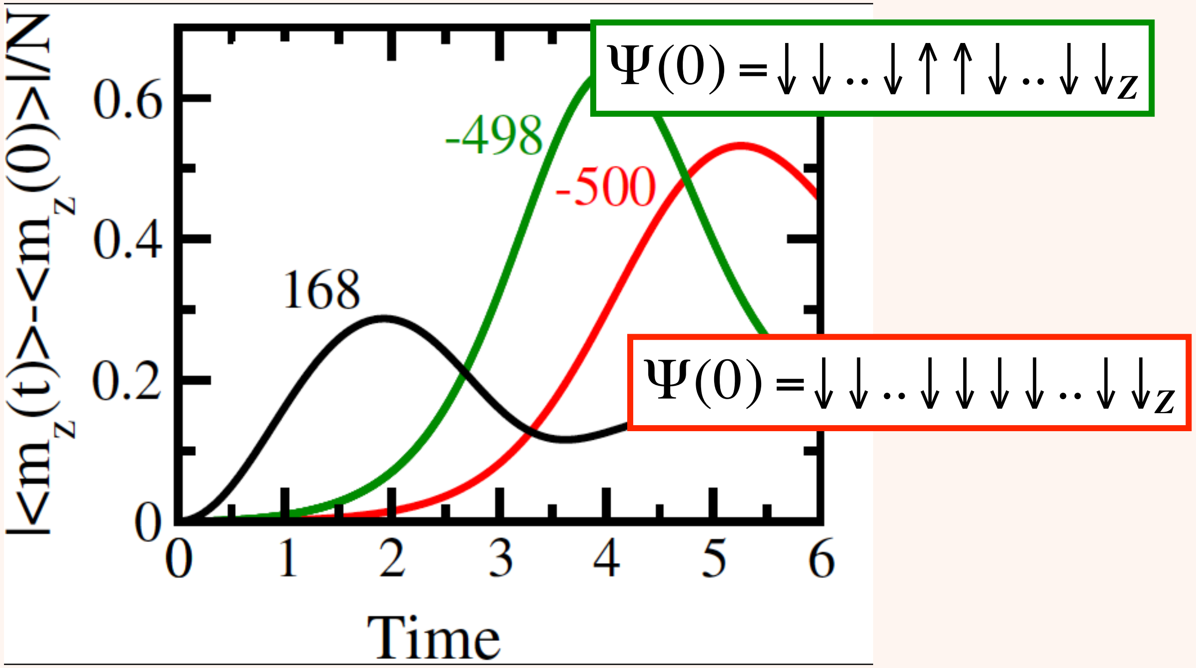
PRA **94,** 012113 (2016)

Magnetization in z: slow dynamics

$$\left\langle m_z^{(k)} \right\rangle / N = \left\langle \psi_k \left| S_z \right| \psi_k \right\rangle / N$$

$$H_{U(2)} = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$





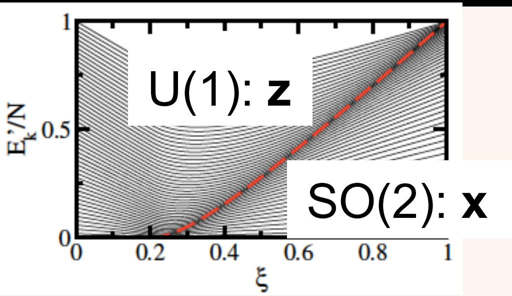
Same initial states studied in **ion traps**

PRA **94,** 012113 (2016)

Magnetization in z: dip

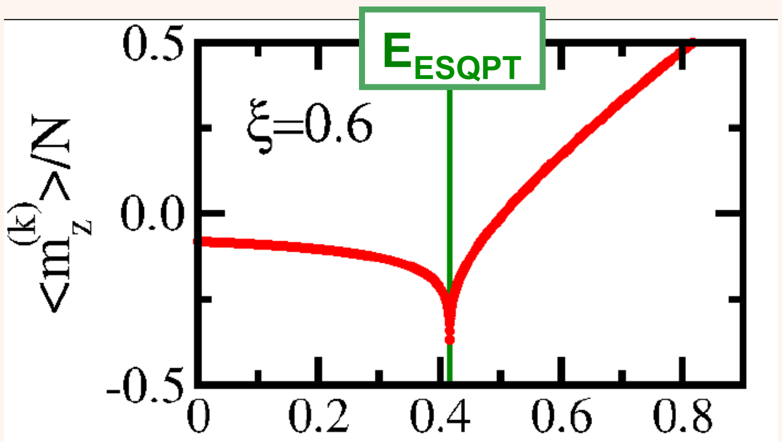
$$\langle m_z^{(k)} \rangle / N = \langle \psi_k | S_z | \psi_k \rangle / N$$

$$H_{U(2)} = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$



At the separatrix $|\psi_k\rangle$ is localized at $|m_z = -N/2\rangle$

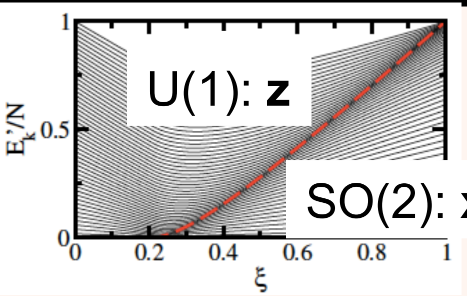
$$\downarrow\downarrow\cdots\downarrow\downarrow\downarrow\downarrow\cdots\downarrow\downarrow_Z$$



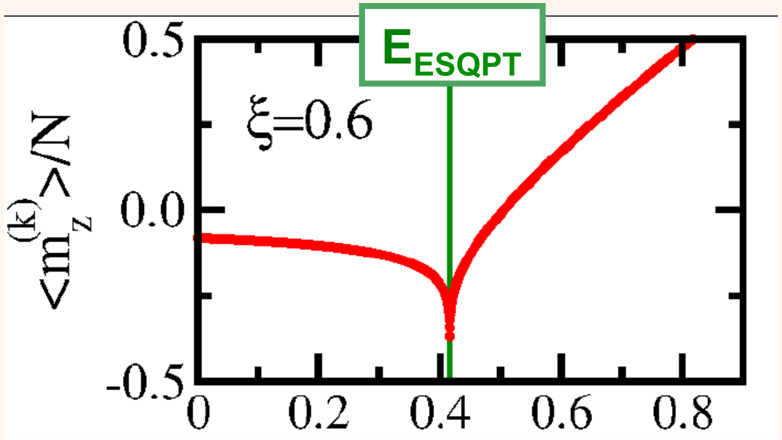
E'_k / N

Magnetization in x: bifurcation

$$H_{U(2)} = (1-\xi)\left(\frac{N}{2} + \mathcal{S}_z\right) - \frac{4\xi}{N}\mathcal{S}_x^2$$

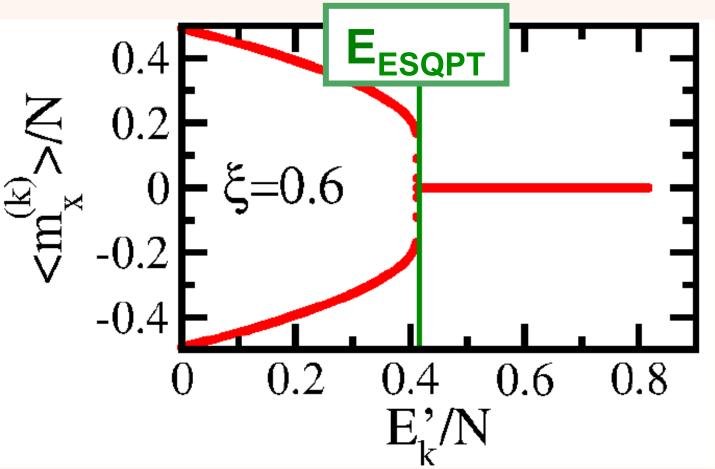


$$\langle m_z^{(k)} \rangle / N = \langle \psi_k | \mathcal{S}_z | \psi_k \rangle / N$$



E'_k / N

$$\langle m_x^{(k)} \rangle / N = \langle \psi_k | \mathcal{S}_x | \psi_k \rangle / N$$



PRA **94,** 012113 (2016)

QPT with parity-symmetry breaking

$$H = \frac{\Lambda}{2} \boldsymbol{S}_z^2 - \boldsymbol{S}_x \Rightarrow \frac{\Lambda}{2} z^2 - \sqrt{1-z^2} \cos\zeta$$

$$H(z,\phi) = \frac{\tilde{g} z^2}{2} - \sqrt{1-z^2} \cos\phi$$

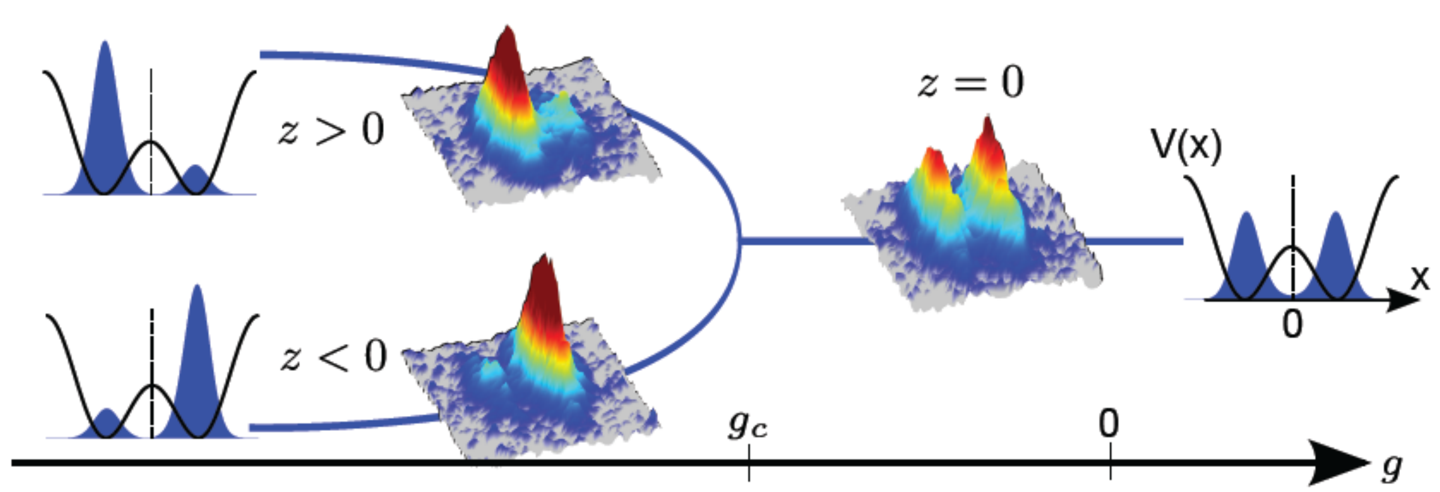
Trenkwalder … Inguscio, Fattori
arXiv: 1603.02979

Imbalance

$$z = (N_L - N_R)/N,$$

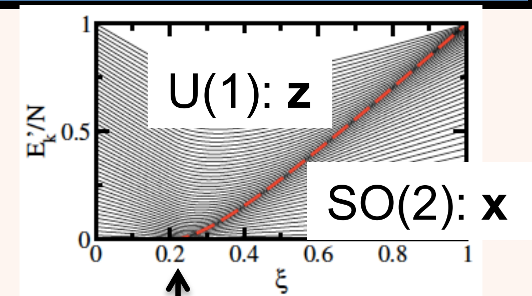
$$\phi = \phi_L - \phi_R$$

Tuning g to large negative values, the ground state of the system goes from a gapped symmetric state (z = 0) to two degenerate asymmetric states (|z| > 0). The system undergoes a second-order QPT where the spatial parity symmetry is broken.



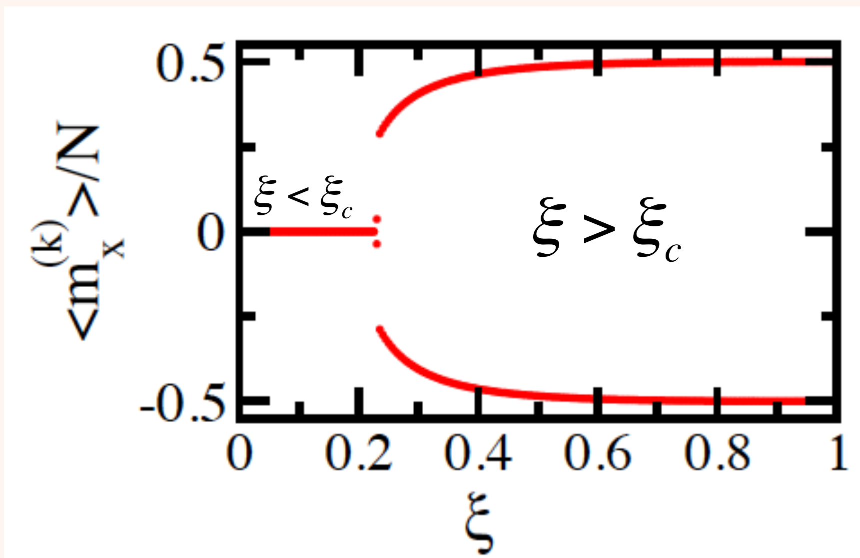
Magnetization in x: bifurcation

$$H_{U(2)} = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$



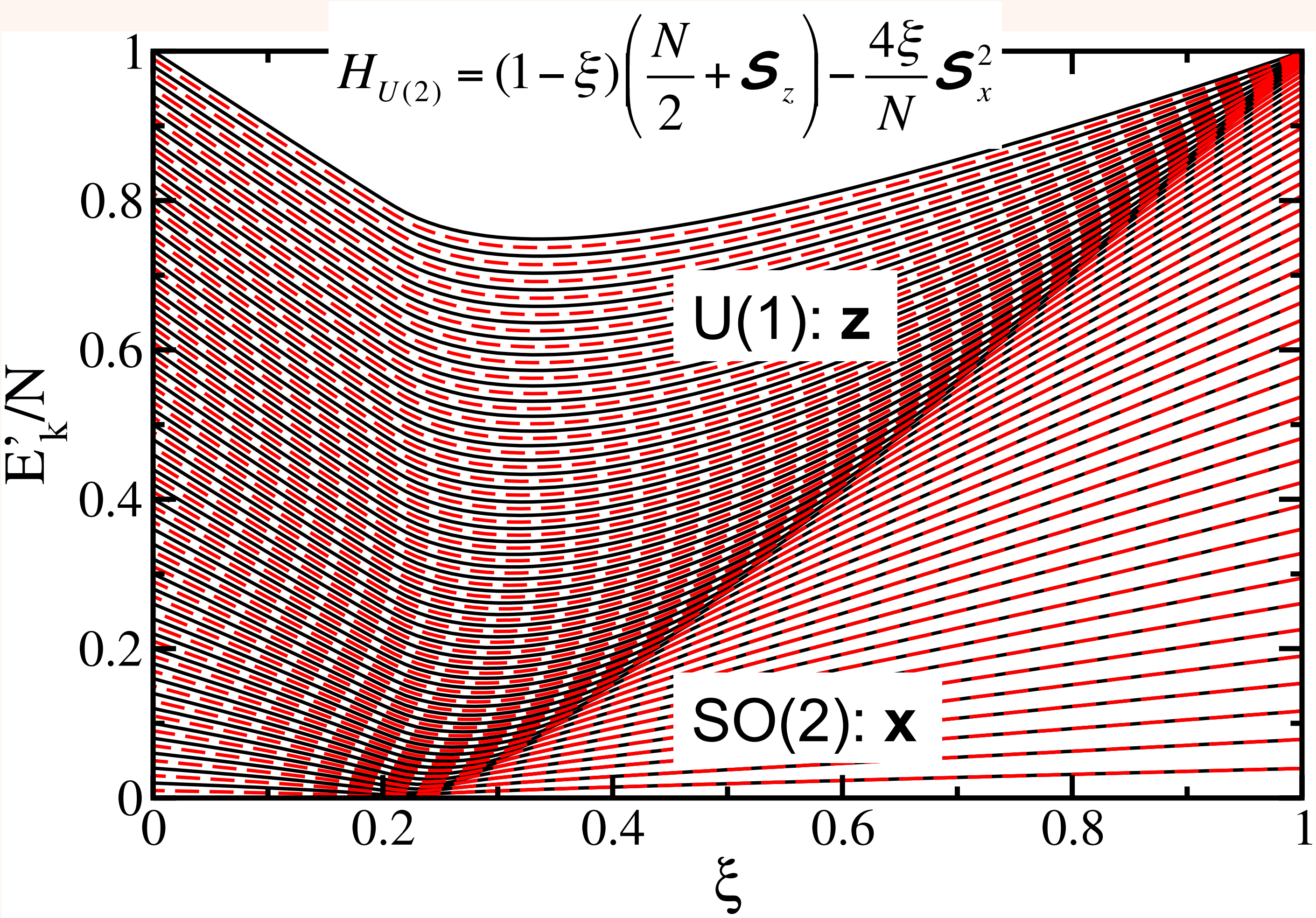
$\xi_c = 0.2$
Ground state
QPT

$$\langle m_x^{(k)} \rangle / N = \langle \psi_k | S_x | \psi_k \rangle / N$$



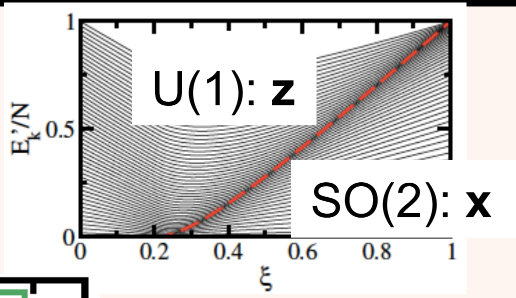
Bifurcation of m_x for the **ground state** as ξ increases

Magnetization in x: bifurcation

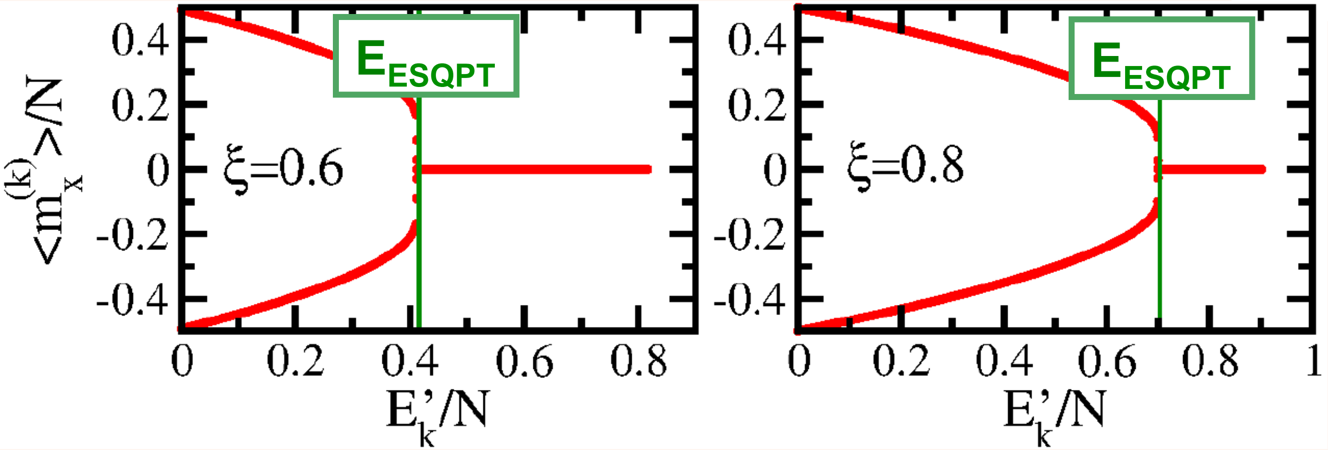


Magnetization in x: bifurcation

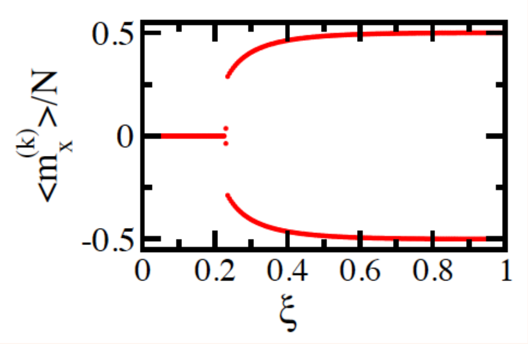
$$H_{U(2)} = (1-\xi)\left(\frac{N}{2} + S_z\right) - \frac{4\xi}{N} S_x^2$$



$$\left\langle m_x^{(k)} \right\rangle / N = \left\langle \psi_k \left| S_x \right| \psi_k \right\rangle / N$$



Bifurcation of m_x at the **ESQPT**



Bifurcation of m_x for the **ground state** as ξ increases

Lea F. Santos, Yeshiva University

Classical bifurcation

$$H=(1-\xi)\left(\frac{N}{2}+S_z\right)-\frac{4\xi}{N}S_x^2$$

$$H=\frac{\Lambda}{2}J_z^2-J_x \Rightarrow \frac{\Lambda}{2}z^2-\sqrt{1-z^2}\cos\phi$$

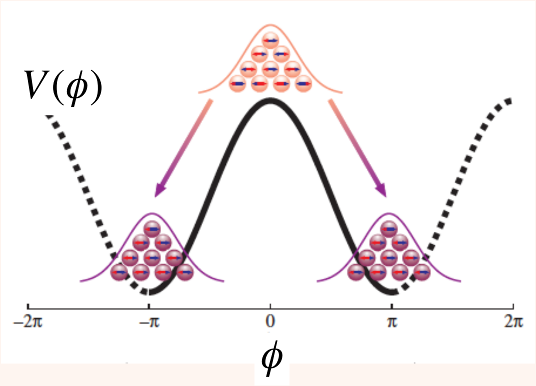
Oberthaler's group
PRL **105** (2010)
BEC

$$z=(N_a-N_b)/N$$

Oliveira's group
PRA **87** (2013)
NMR

$z =$ Temporal mean magnetization

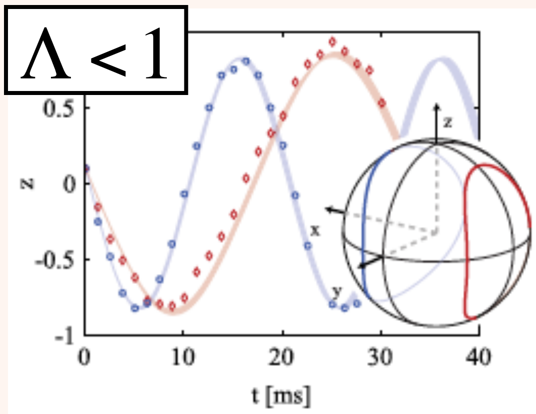
$$V(\phi)=\sqrt{1-z^2}\cos\phi$$

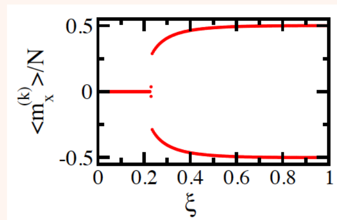


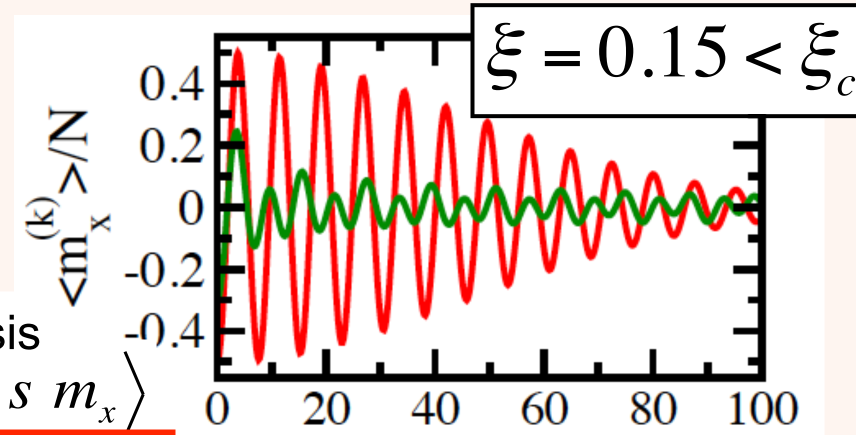
Self-trapping: depending on ξ

$$H=\frac{\Lambda}{2}\boldsymbol{J}_z^2-\boldsymbol{J}_x \Rightarrow \frac{\Lambda}{2}z^2-\sqrt{1-z^2}\cos\phi$$

$$H=(1-\xi)\left(\frac{N}{2}+\boldsymbol{S}_z\right)-\frac{4\xi}{N}\boldsymbol{S}_x^2$$



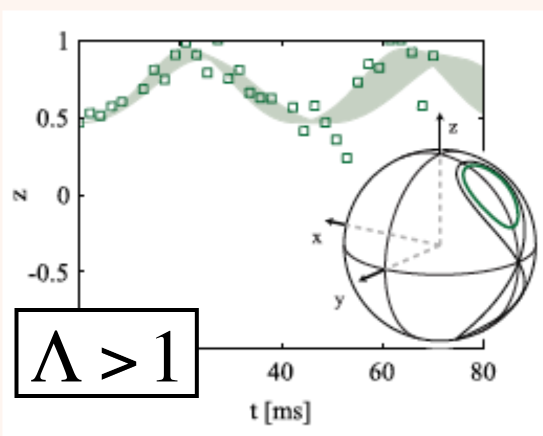


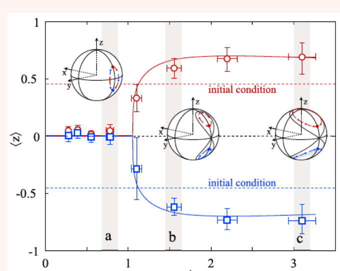


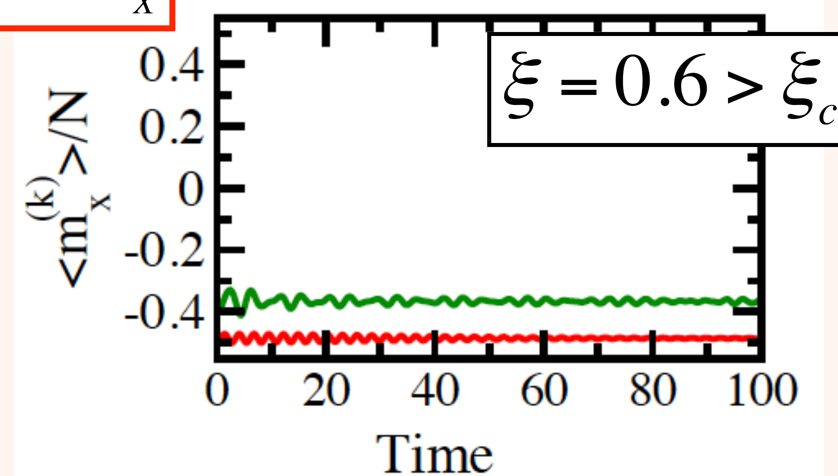
SO(2) basis

$$|\Psi(0)\rangle=|s\ m_x\rangle$$

$$++..+-++..-+_X$$

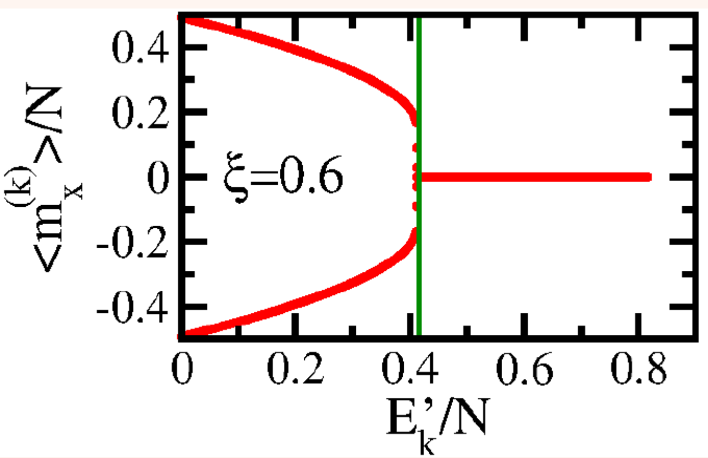






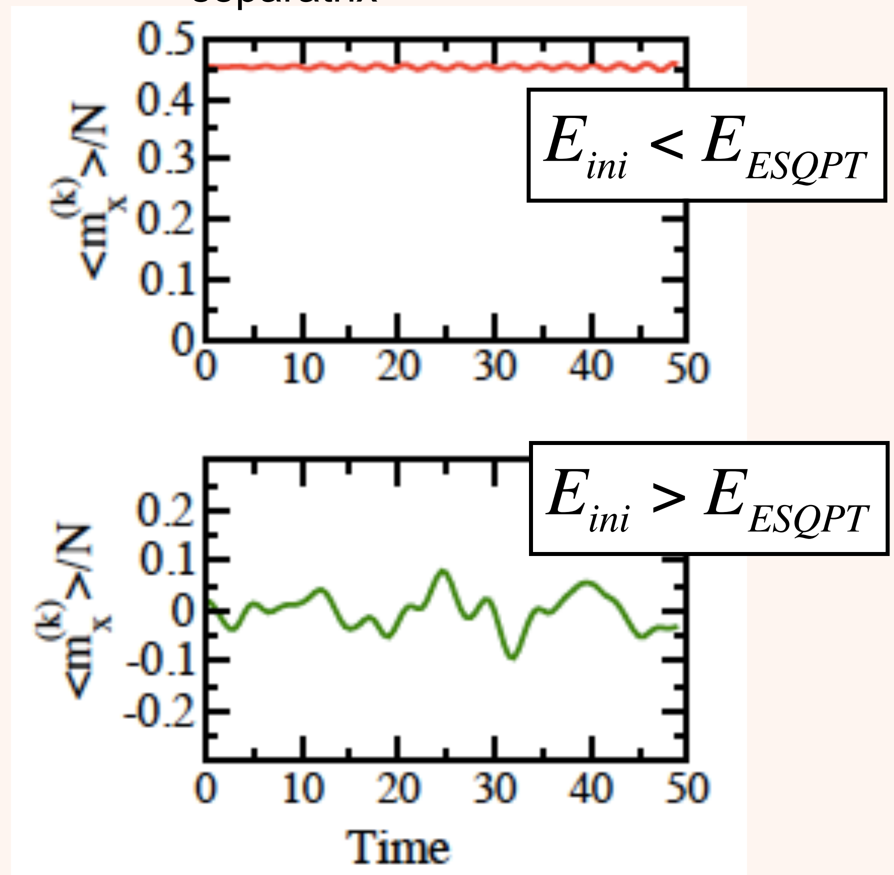
Self-trapping: depending on energy

$$\langle m_x^{(k)} \rangle / N = \langle \psi_k | S_x | \psi_k \rangle / N$$



Bifurcation of m_x
at the
ESQPT

$|\Psi(0)\rangle$ Superposition of eigenstates only below or only above the separatrix



Conclusions

- Different ways **<u>to capture an ESQPT</u>**:

  (1) Structure of the Hamiltonian matrix.

  (2) Level of delocalization of the eigenstates.

  (3) Magnetization in x and z.

  (4) Dynamics.

  Evolution of

  $\downarrow\downarrow..\downarrow\downarrow\downarrow\downarrow..\downarrow\downarrow_z$

  $\downarrow\downarrow..\downarrow\uparrow\uparrow\downarrow..\downarrow\downarrow_z$

  as done in experiments with **<u>ion traps</u>**.

- The dynamics of the system depends on the interplay between the initial state and the final Hamiltonian.

PRA **94,** 012113 (2016)

arXiv:1604.06851 (Fort. Physik)

PRA **92,** 050101R (2015)

ESQPT: participation ratio

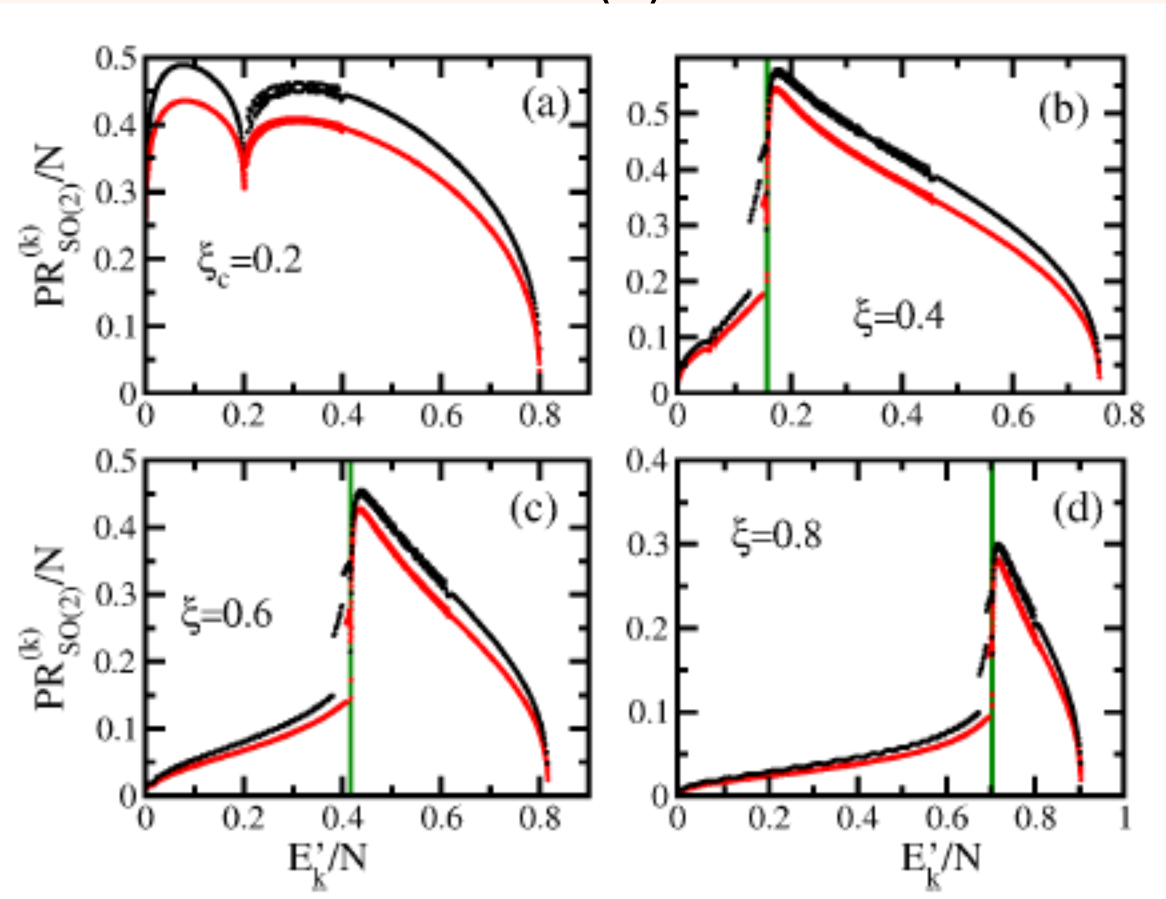
$$\left|\psi^{(k)}_{SO(2)}\right\rangle = \sum_{m_x=-N/2}^{N/2} C_n^{(k)} \left| s\ m_x \right\rangle$$

Participation Ratio

$$PR^{(k)} \equiv \frac{1}{\sum_{n=L}^{N} |C_n^{(k)}|^4}$$

N=500, 2000

In the SO(2)-basis

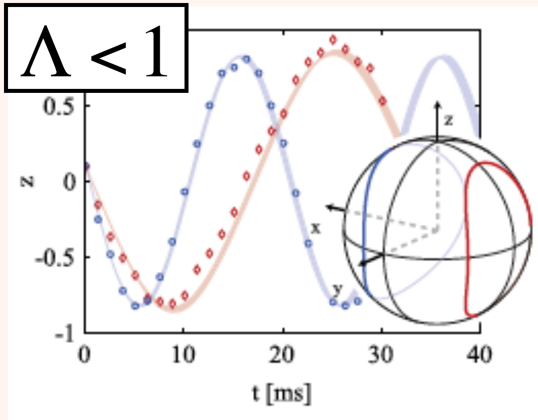


LFS, Távora, Bernal
arXiv:1604.04289

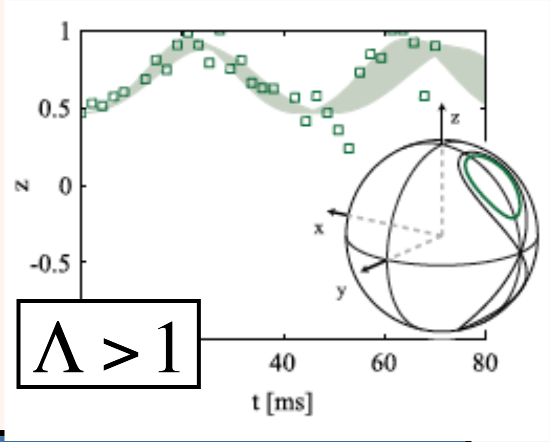
Classical bifurcation

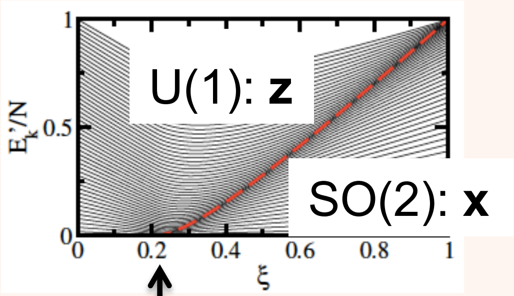
$$H = \frac{\Lambda}{2}\boldsymbol{J}_z^2 - \boldsymbol{J}_x \Rightarrow \frac{\Lambda}{2} z^2 - \sqrt{1-z^2}\cos\phi$$

$\Lambda < 1$

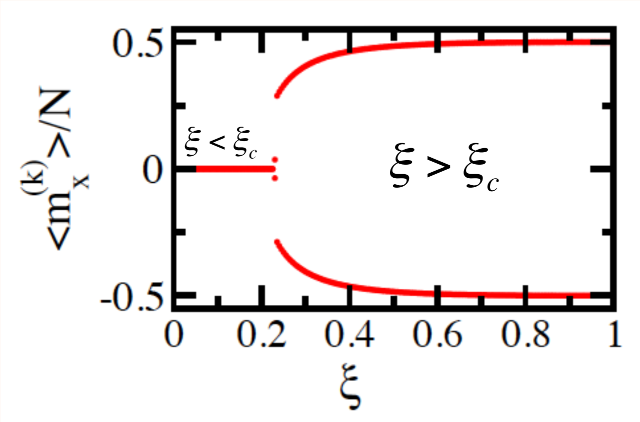


$\Lambda > 1$

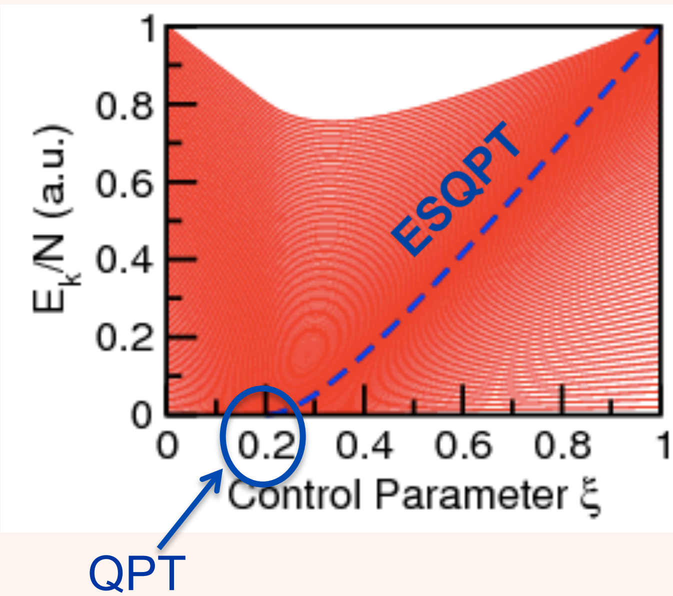




$\xi_c = 0.2$
Ground state QPT



Structure of the eigenstates



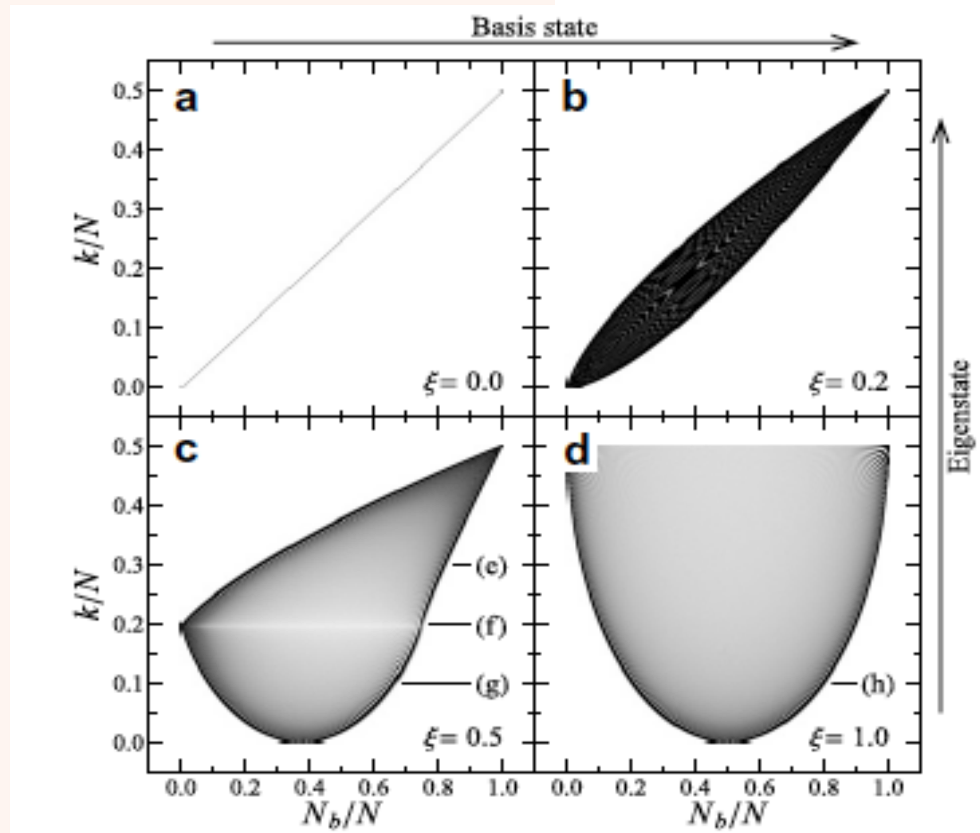
Caprio, Cejnar, and Iachello
Ann. Phys. **323**, 1106 (2008).

$$H_{U(3)} = (1-\xi)H_{U(2)} + \frac{\xi}{N} H_{SO(3)}$$

U(3) model
Eigenstates in the U(2)-basis

Structure of the eigenstates above and below the separatrix.

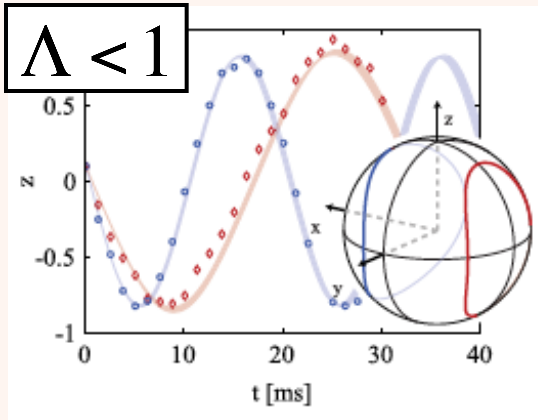
$$\left|\psi_{U(3)}^{(k)}\right\rangle = \sum_{n=L}^{N} C_n^{(k)} \left|[N]\ n\ L\right\rangle_k$$



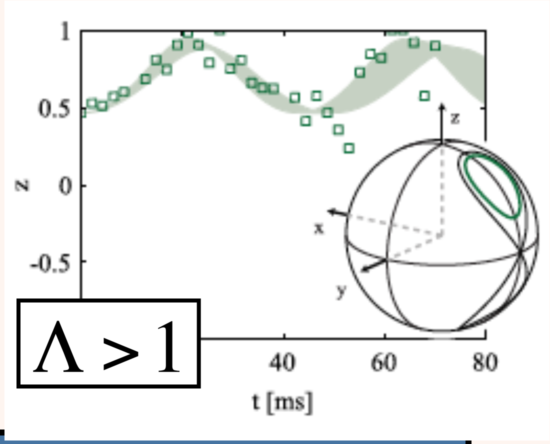
Classical bifurcation

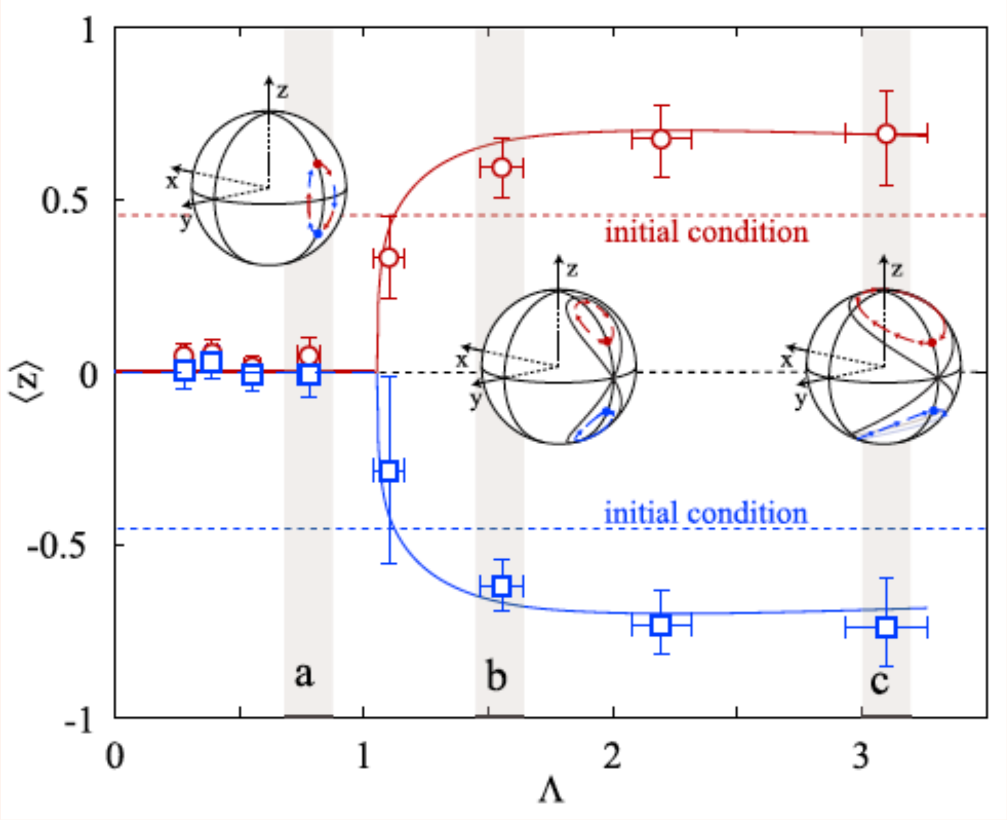
$$H = \frac{\Lambda}{2} \boldsymbol{J}_z^2 - \boldsymbol{J}_x \Rightarrow \frac{\Lambda}{2} z^2 - \sqrt{1 - z^2} \cos\phi$$

$\Lambda < 1$



$\Lambda > 1$





Initial state: U(1)-basis vector
Slow decay

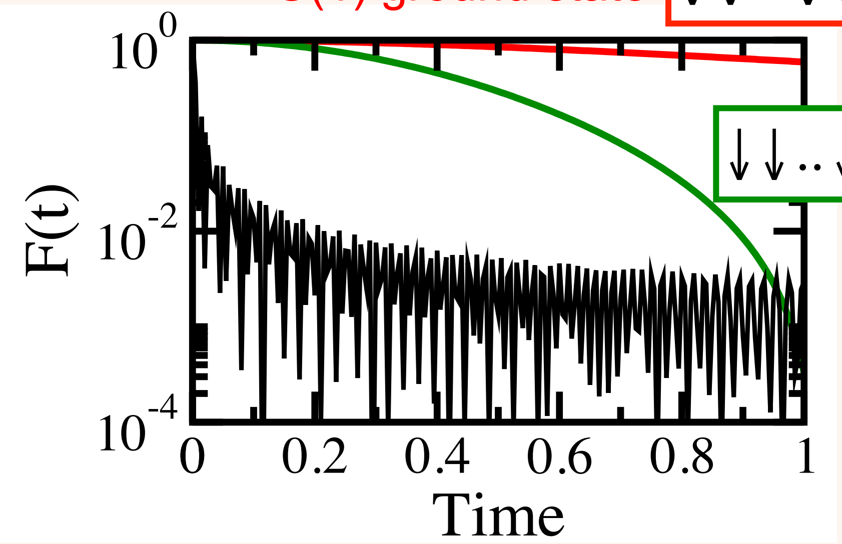
Initial state $|\Psi(0)\rangle = |s\ m_z\rangle = \sum_k C_{m_z}^{(k)} |\psi_{U(2)}^{(k)}\rangle$

Survival Probability $F(t) = |\langle \Psi(0) | \Psi(t)\rangle|^2 = \left| \sum_k |C_{m_z}^{(k)}|^2 e^{-iE_k t} \right|^2 = \left| \int_{-\infty}^{\infty} \rho_{ini}(E) e^{-iEt}\, dE \right|^2$

$$\rho_{ini}(E) = \sum_k |C_{m_z}^{(k)}|^2 \delta(E - E_k)$$

LDOS (local density of states)

U(1) ground state $\downarrow\downarrow..\downarrow\downarrow\downarrow\downarrow..\downarrow\downarrow_Z$



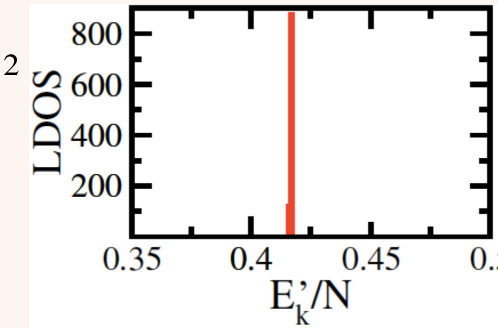
$\xi = 0.6$ N=1000

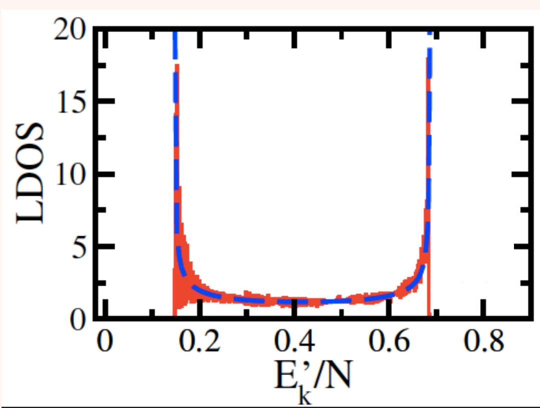
arXiv:1604.04289

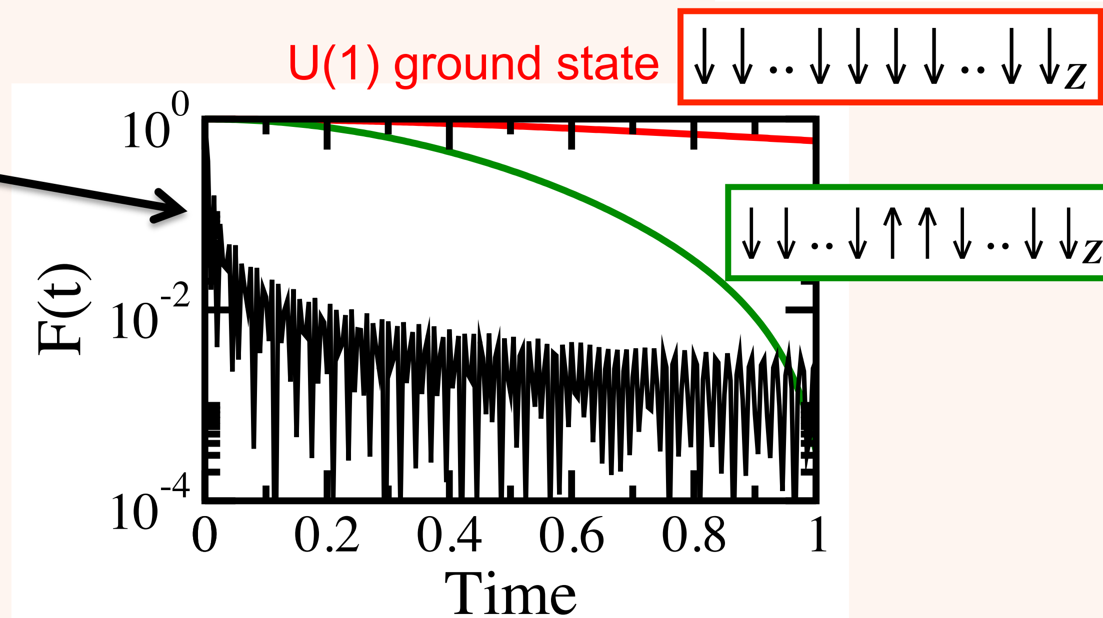
Initial state: U(1)-basis vector
Slow decay

Initial state $\left|\Psi(0)\right\rangle = \left|S\ m_z\right\rangle = \sum_k C_{s_z}^{(k)} \left|\psi_{U(2)}^{(k)}\right\rangle$

Survival Probability $F(t) = \left|\left\langle \Psi(0) \mid \Psi(t) \right\rangle\right|^2 = \left|\sum_k \left|C_{s_z}^{(k)}\right|^2 e^{-iE_k t}\right|^2$







$\xi = 0.6$ N=1000

arXiv:1604.04289