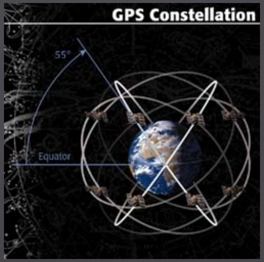
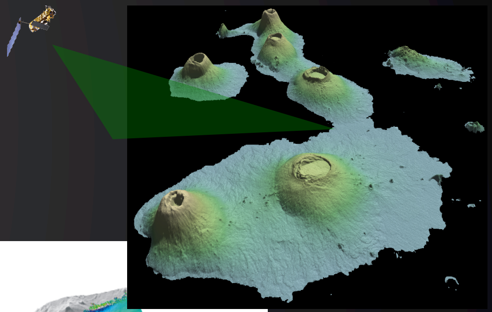
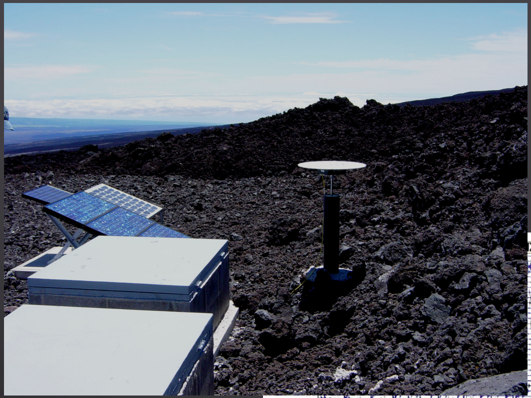
Volcano Deformation and Physics Based Eruption Models

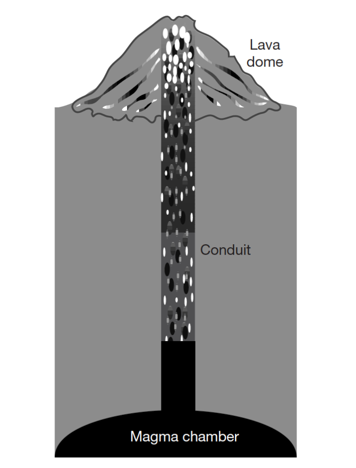
Paul Segall

Stanford University









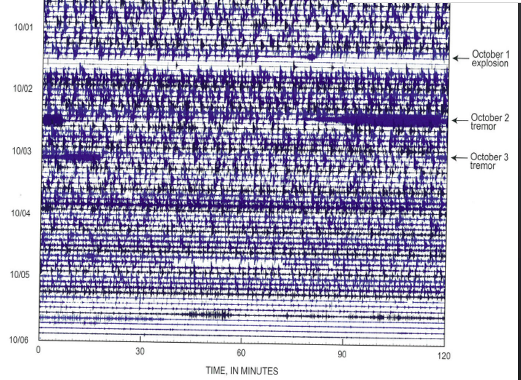
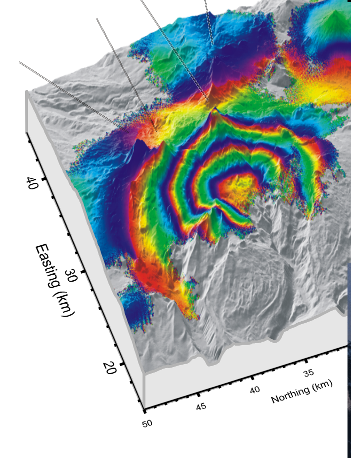
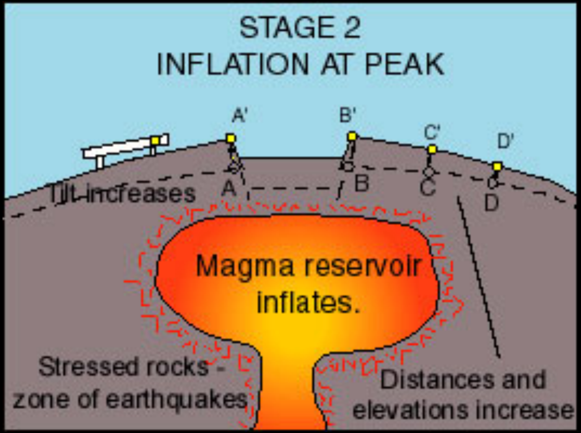
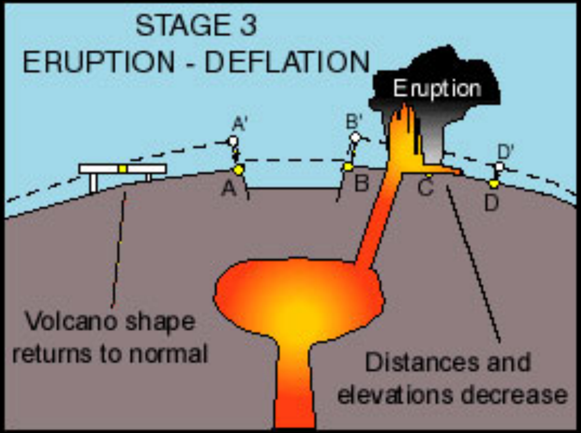


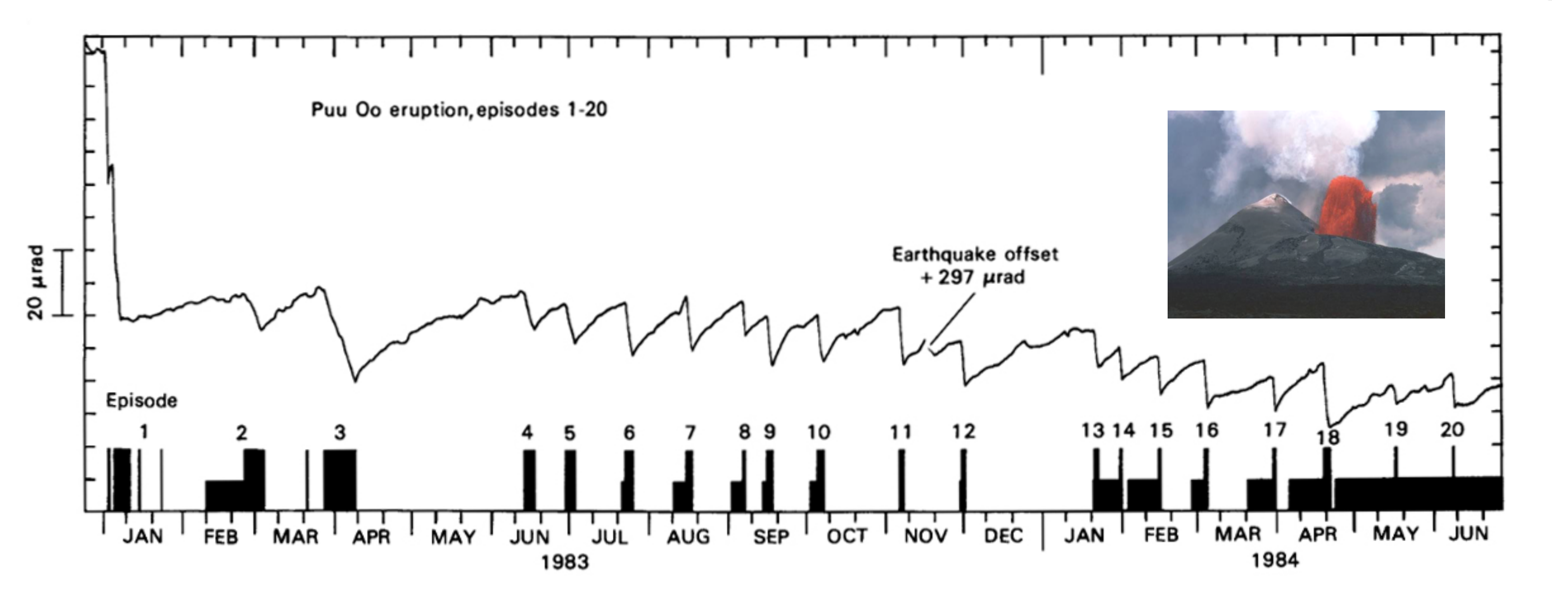
Figure 8. Seismic record from station JUN (6.5 km southeast of vent; fig. 1) from September 27 through October 5, 2004, showing seismicity trends during the vent-clearing phase.



Inflation Deflation Cycles

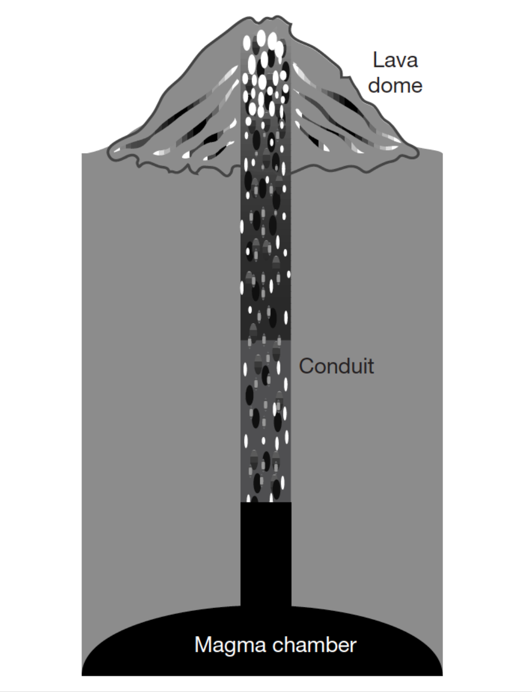


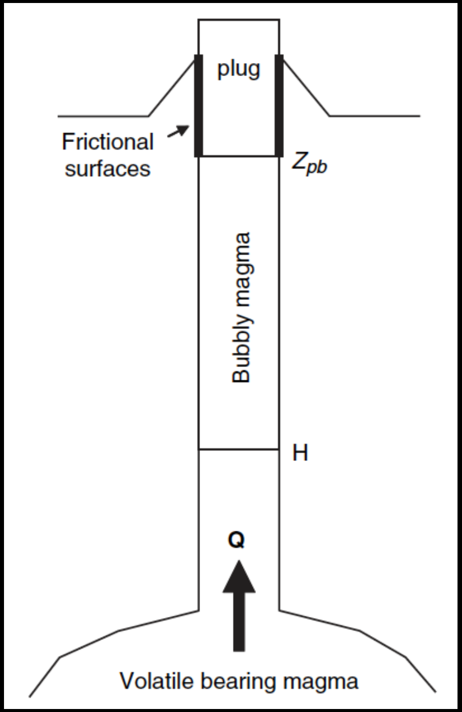




HVO website

Physics Based Eruption Models

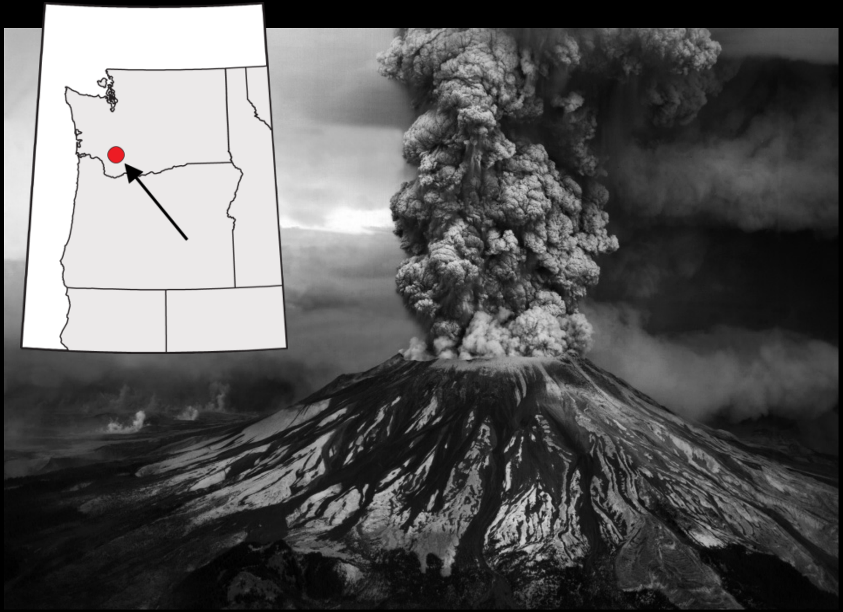




Melnik and Sparks, 1999, Nature   Lensky etal 2009, Geol Soc Lond

[Stasiuk et al., 1993; Ramos, 1995; Jaupart, 1996; Melnik and Sparks, 1999; 2002, 2005; 2006, Mastin and Ghiorso, 2000; Maeda, 2000; Massol et al., 2001; Huppert and Woods, 2002; Woods and Huppert, 2003; Barmin et al., 2002; Proussevitch and Sahagian, 2005; Starostin et al., 2005; Collier and Neuberg, 2006; Mason et al., 2006; Costa et al., 2007a, b; de' Michieli Vitturia et al., 2008; Lensky et al. 2008; Hautmann et al. , 2009; Mastin et al. , 2008; 2009]

Mount St. Helens, Washington

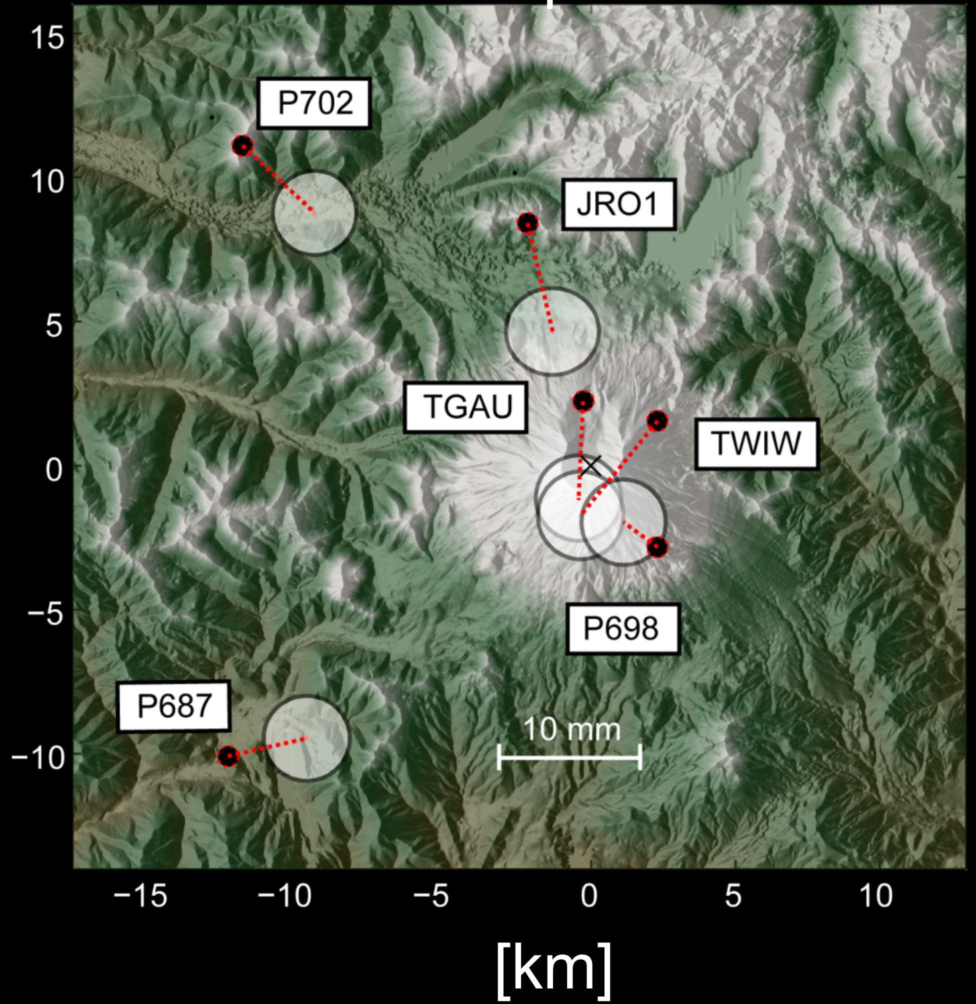



The “whaleback” 2/22/2005

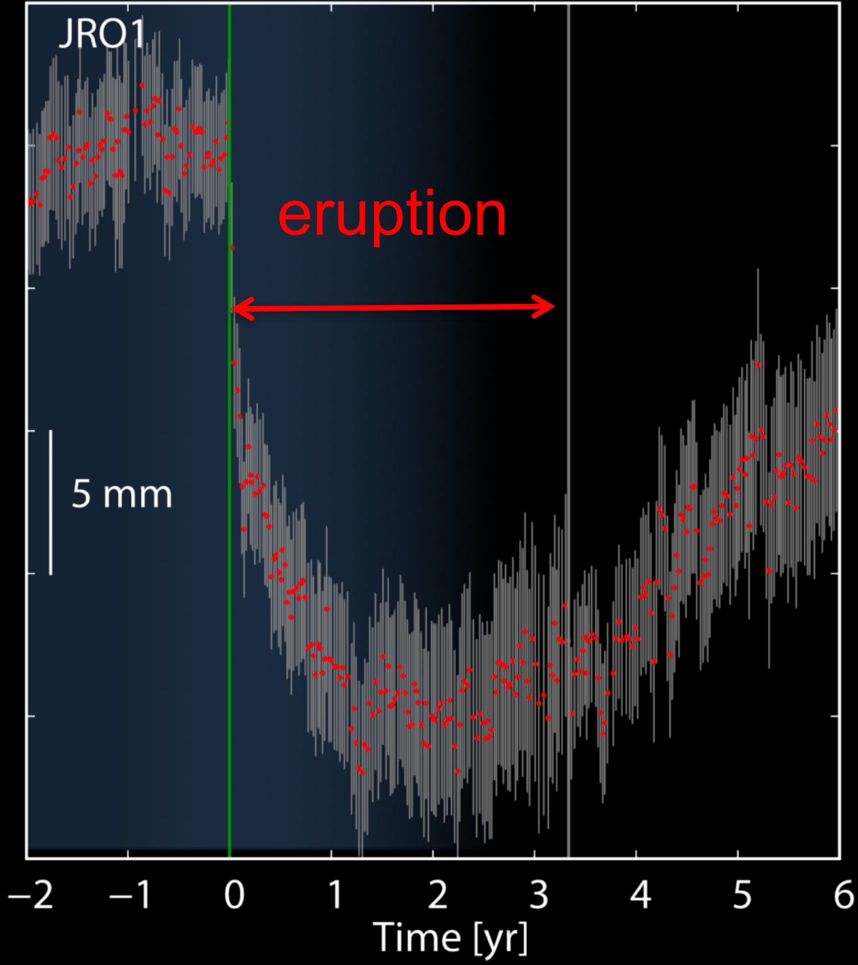
- Catastrophic eruption 1980
- Effusive eruption 2004-2008

Mount St Helens Dome Forming Eruption 2004-2008

Net Displacements

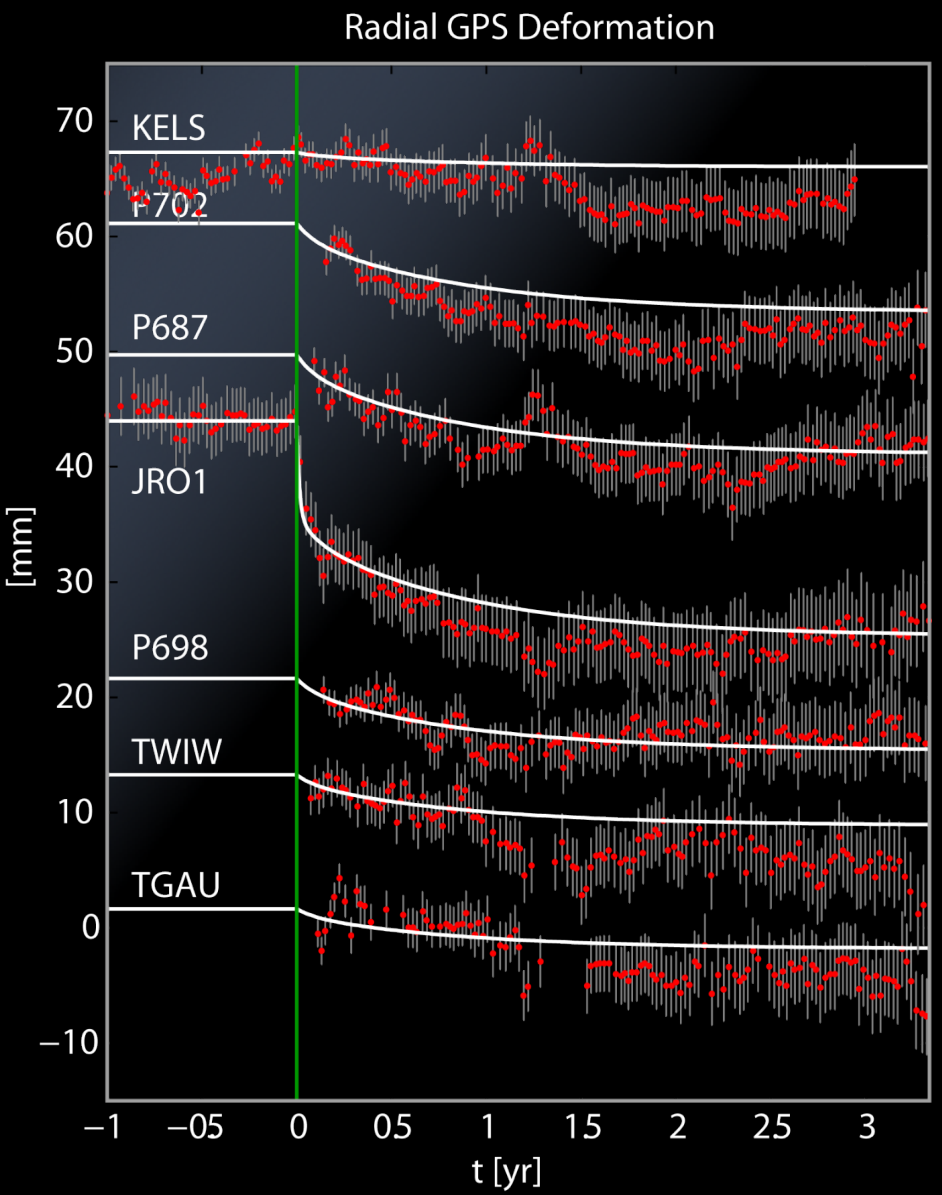


JRO1 Radial Time Series

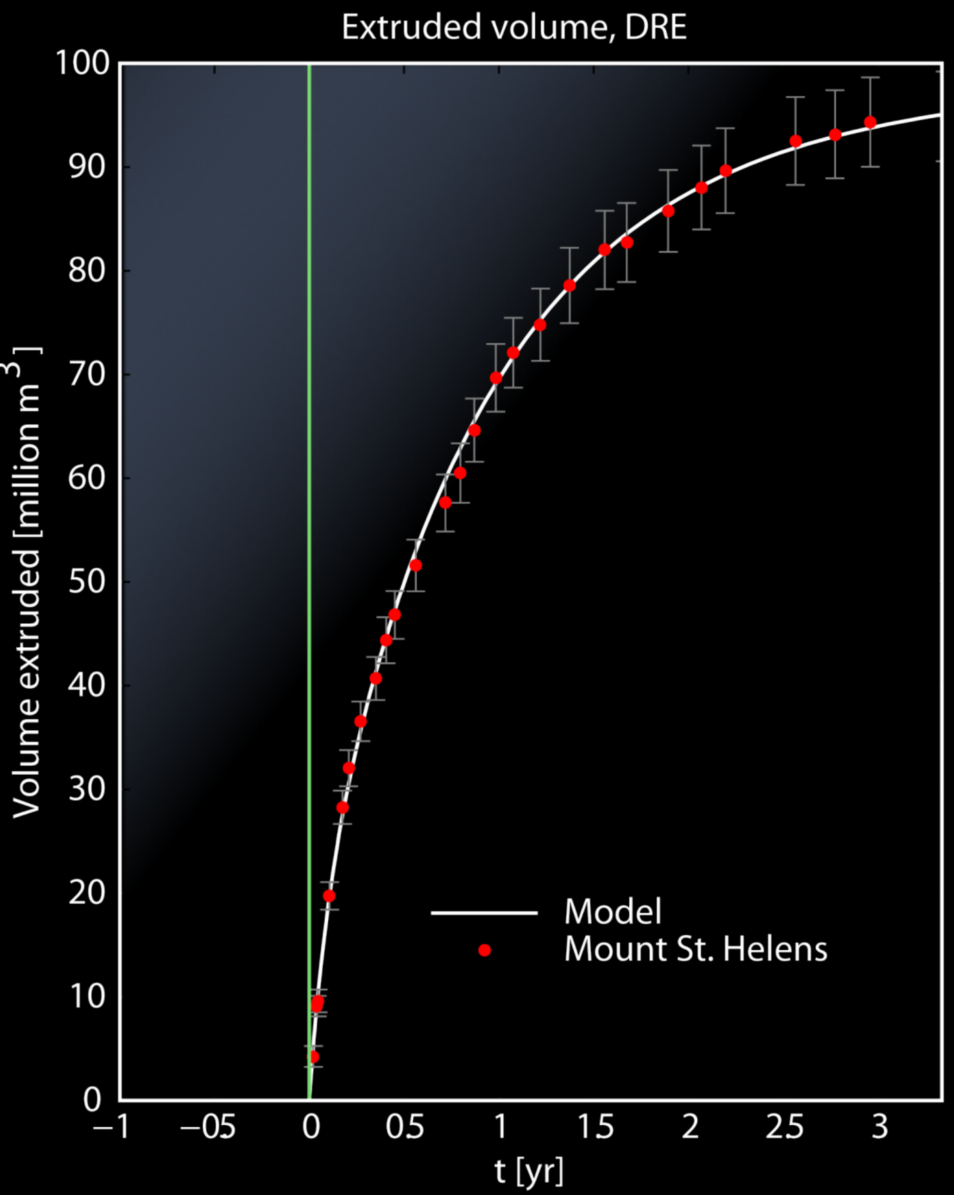


Lisowski et al. [2008]

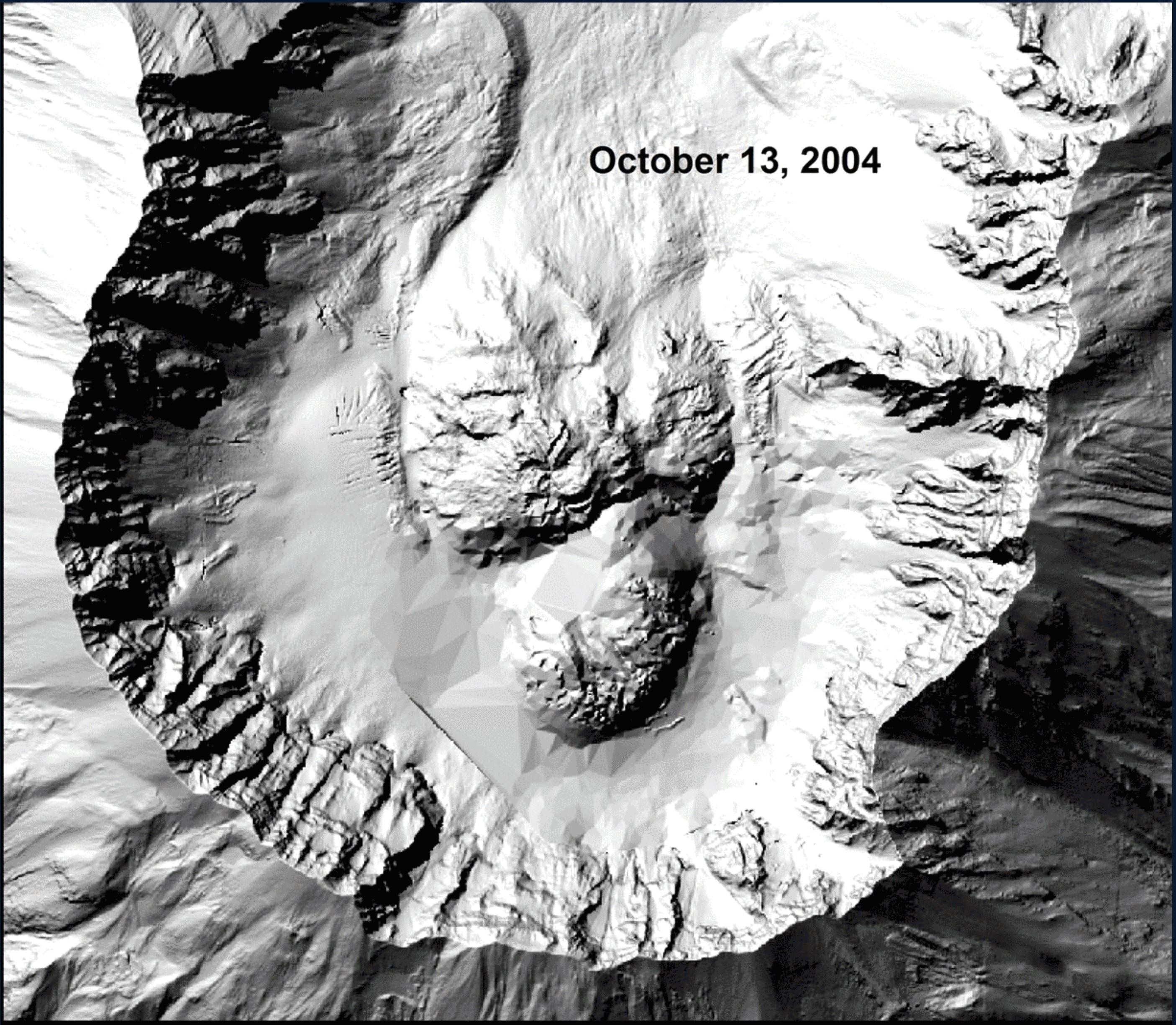
Model Fits both GPS and Extrusion Data



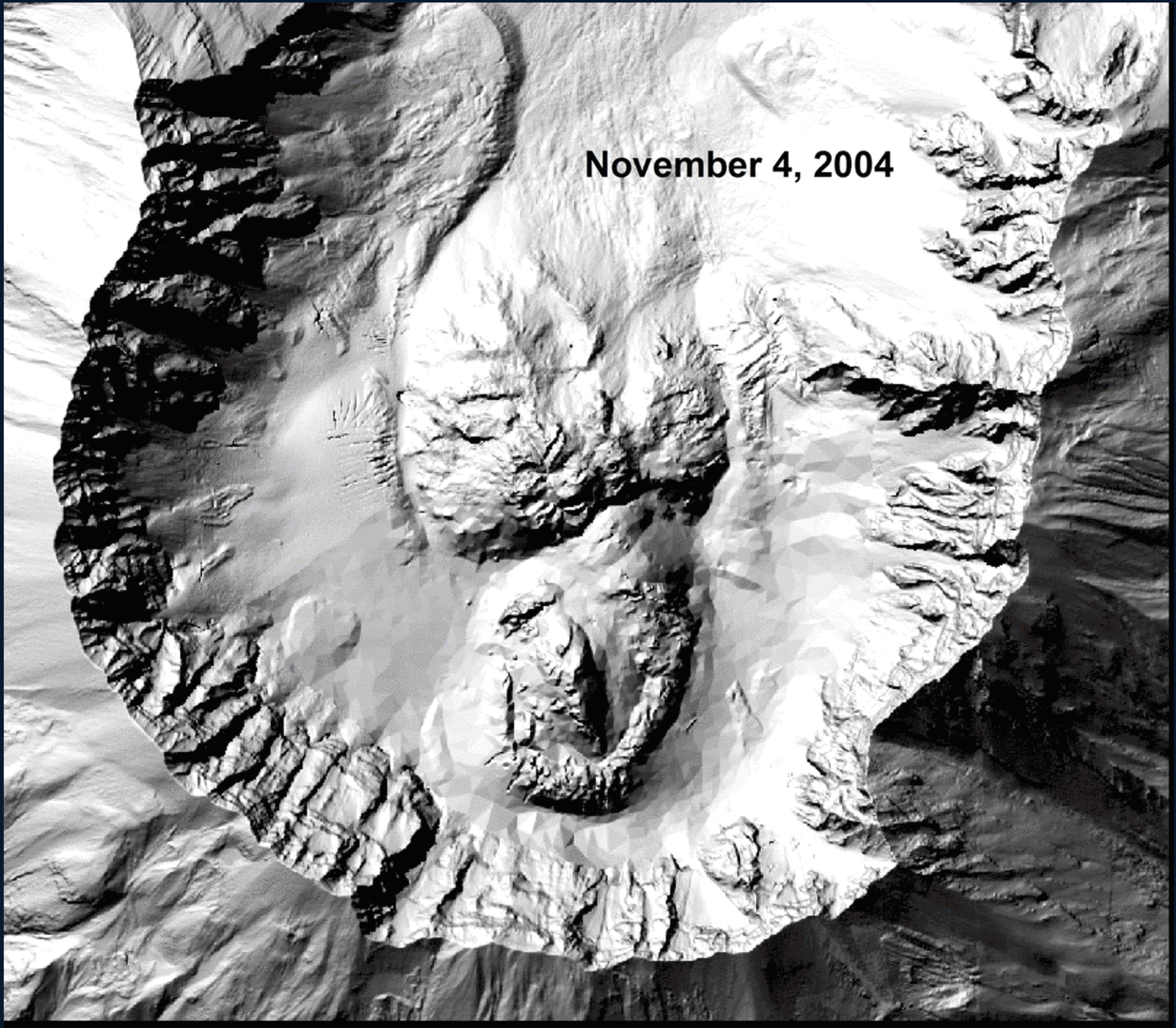
Radial GPS Deformation



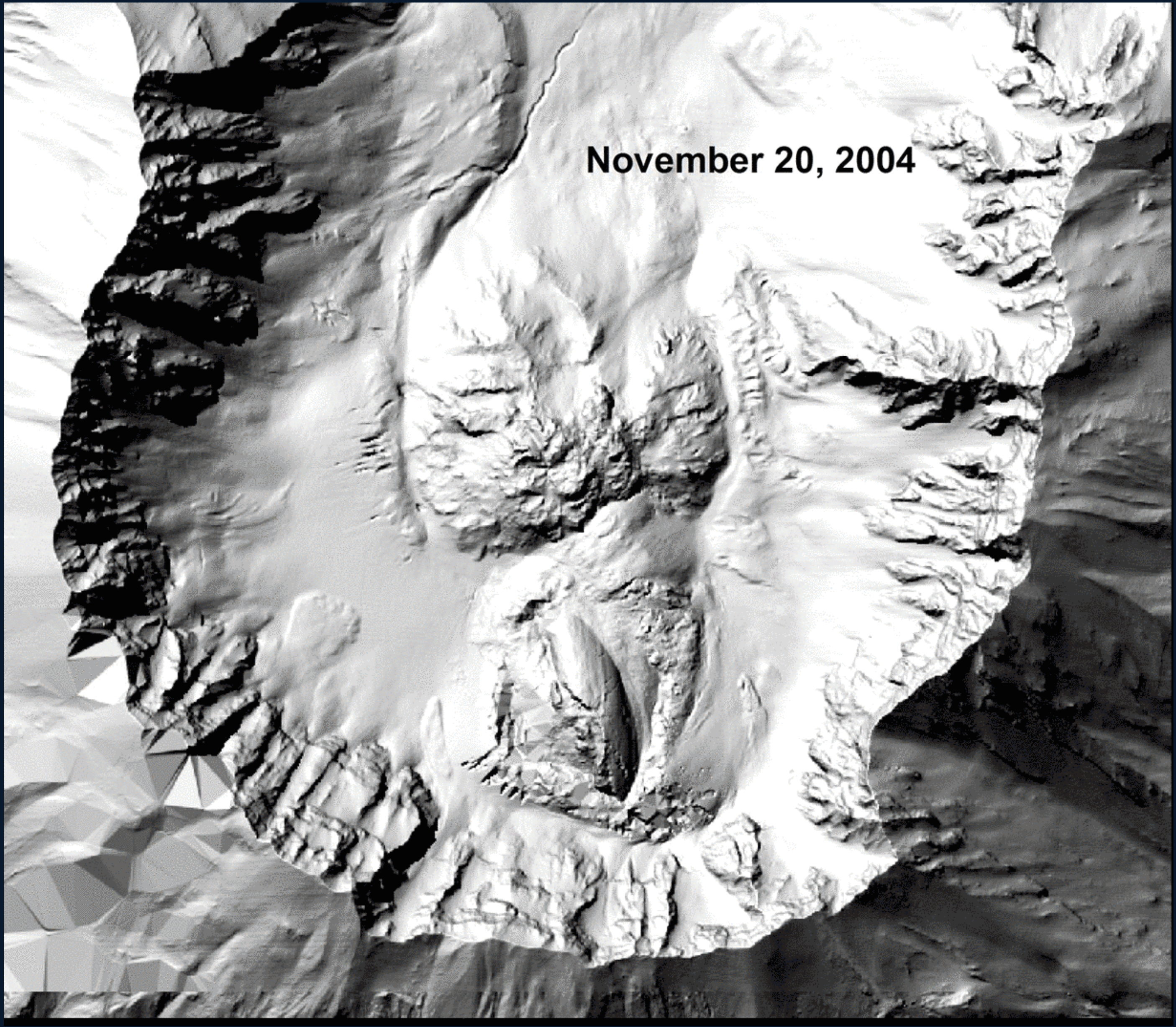
Extruded volume, DRE



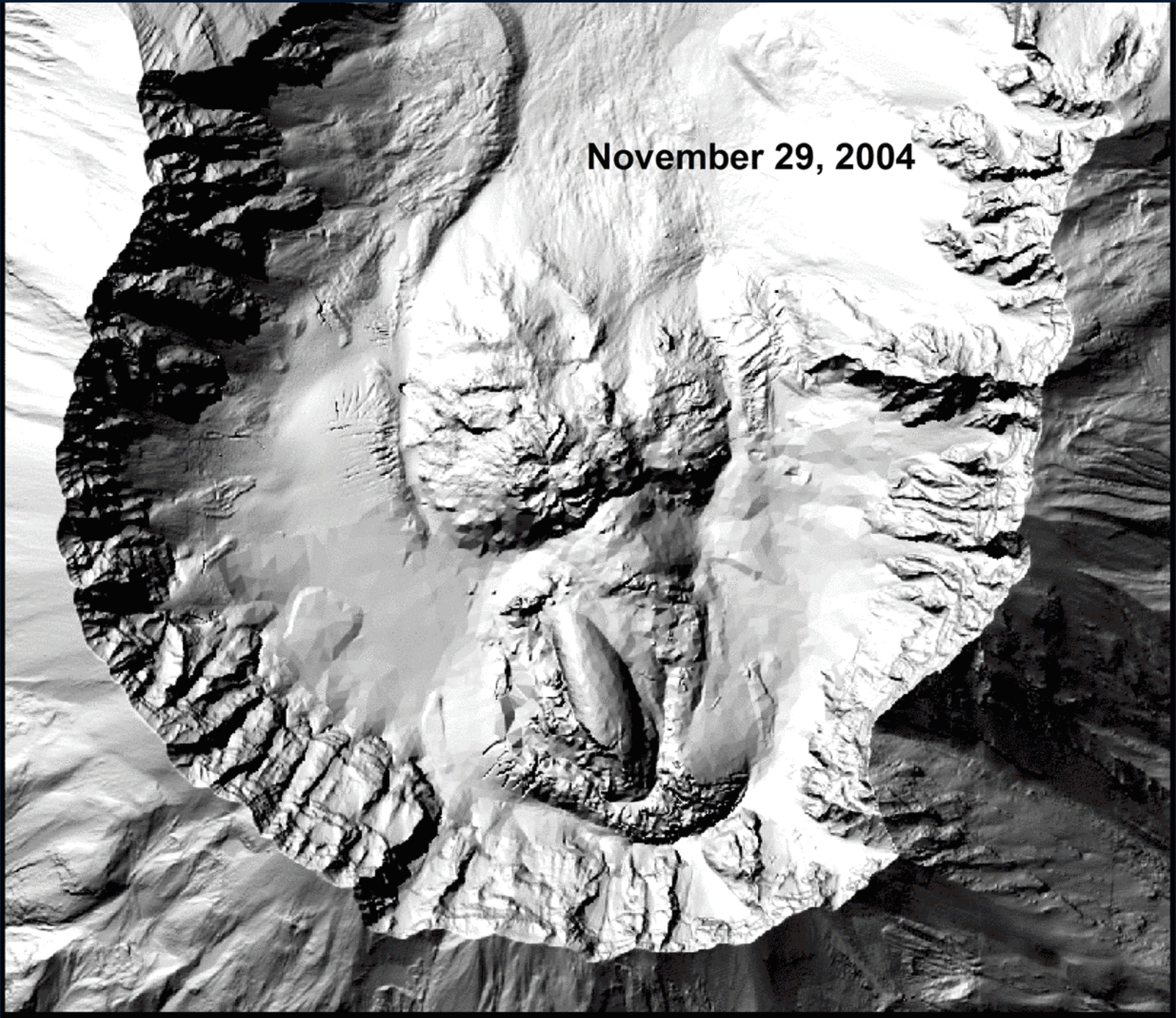
October 13, 2004

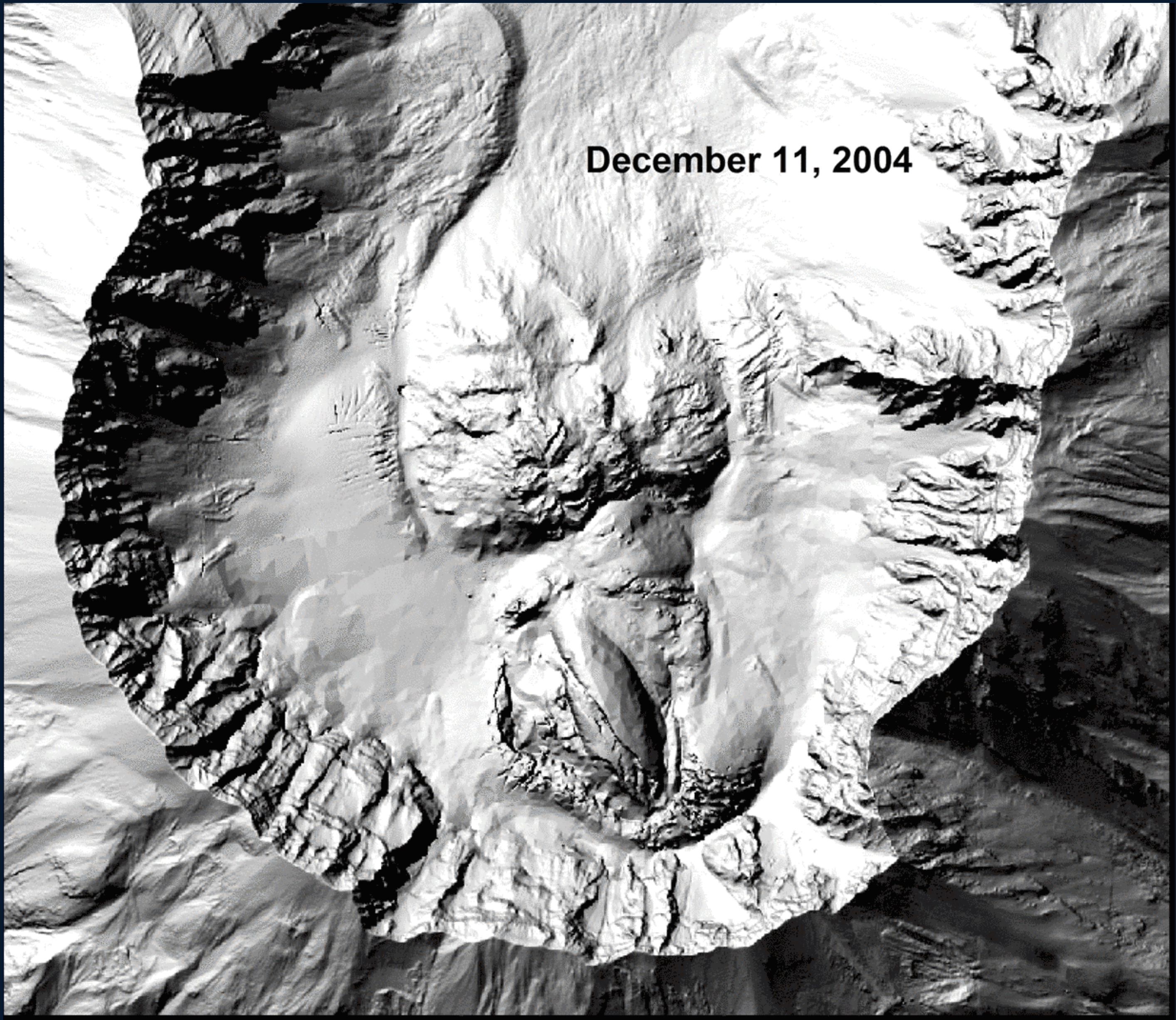


November 4, 2004

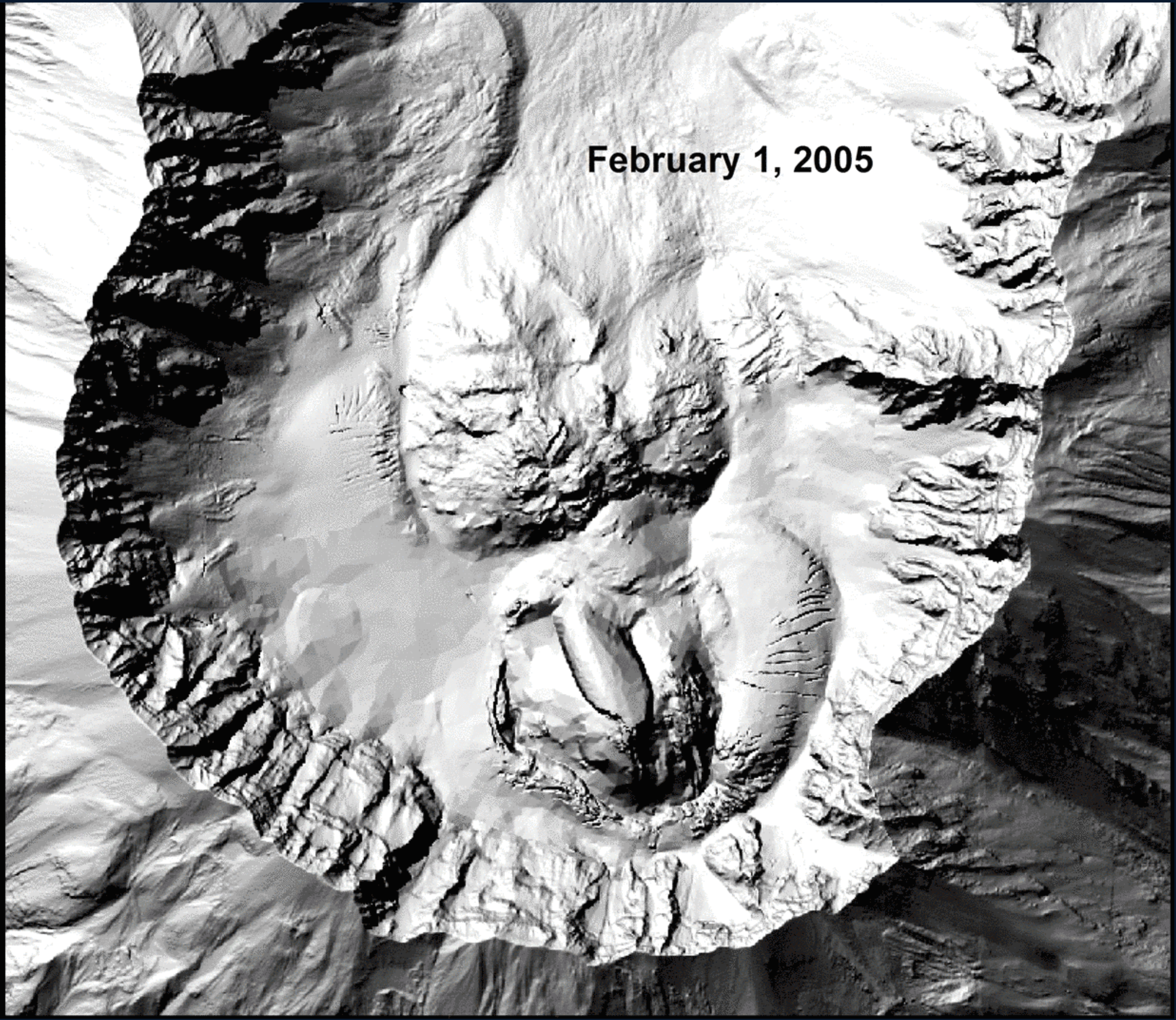


November 20, 2004

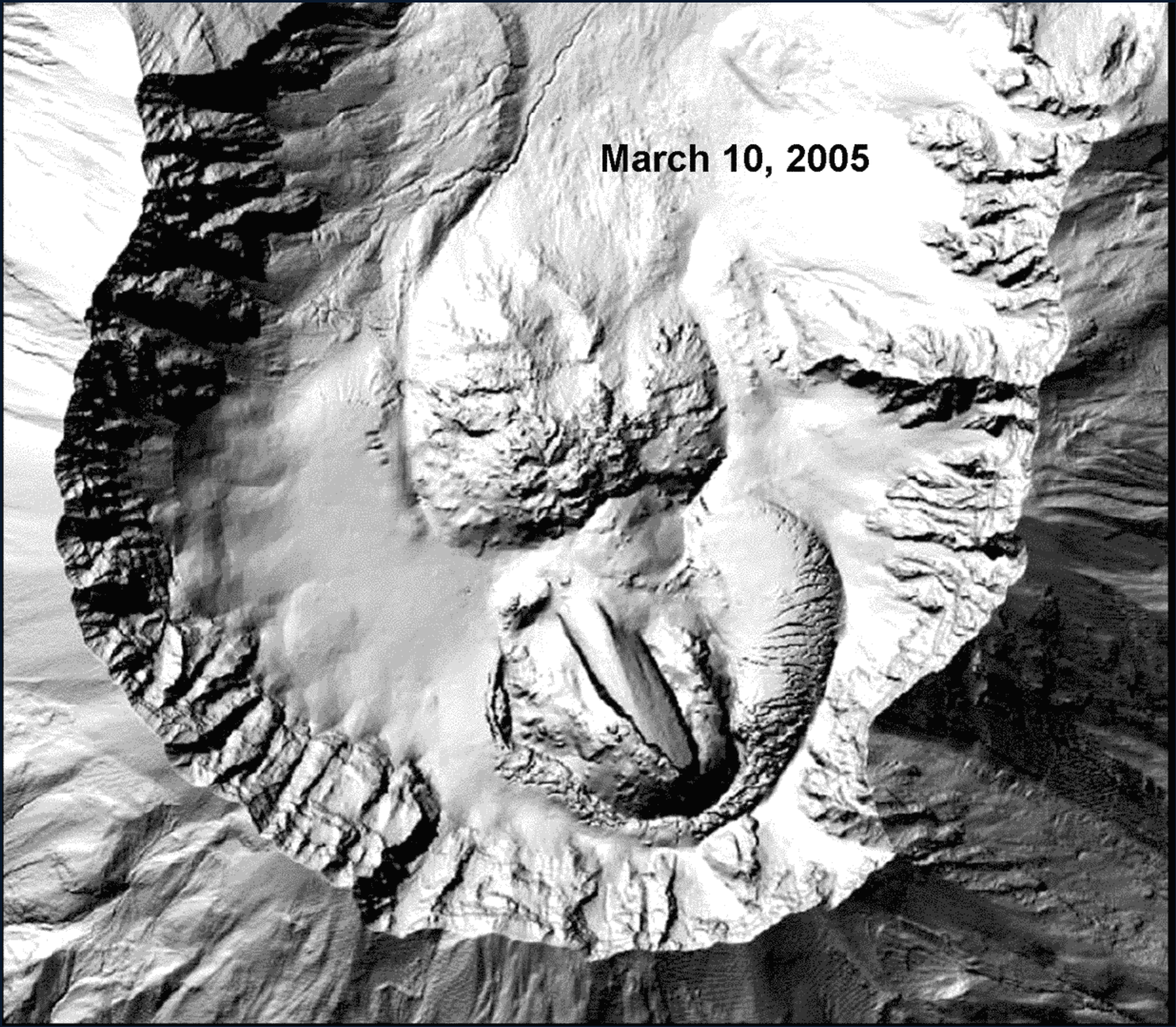

November 29, 2004



December 11, 2004



February 1, 2005

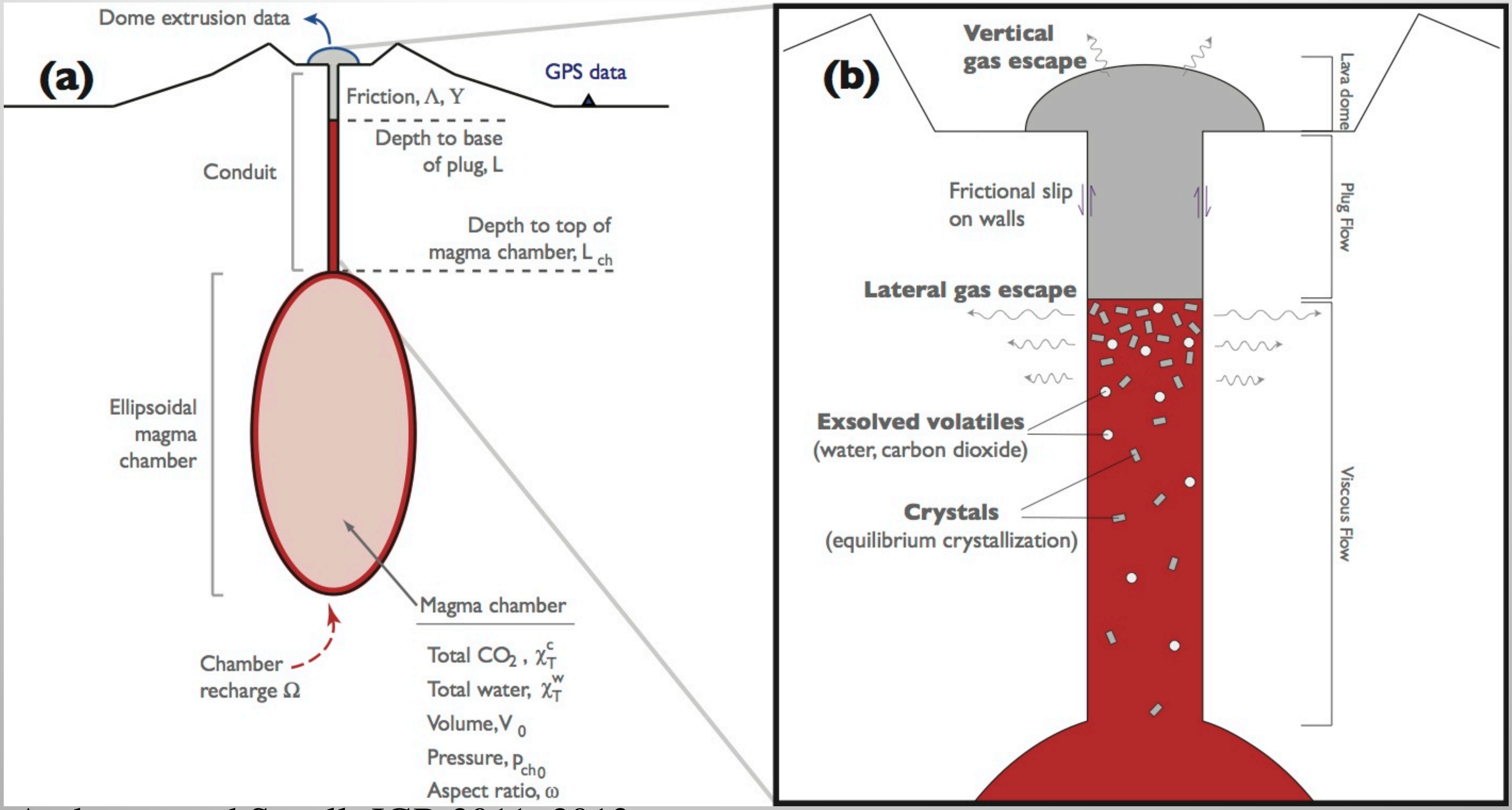


March 10, 2005

Physics-based Volcano Deformation

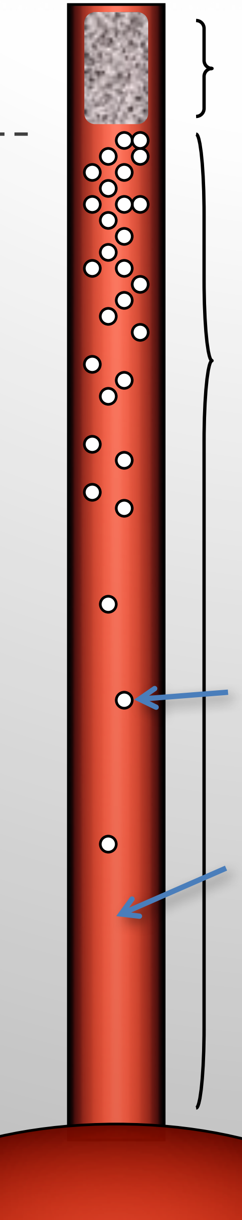
Kyle Anderson

YingQi Wong



Anderson and Segall, JGR 2011; 2013

Governing equations



Crystallization depth (fixed)

Solid plug

Compressible, multiphase 1D Newtonian flow

No gas loss

Bubbles (α)

Melt (λ), Phenocrysts (φ), Dissolved H20χ_d^w *and CO2(Xd)*

Chamber Equations

Compressibility

$$\beta_m = \frac{1}{\rho_{cc}} \frac{\partial \rho_{cc}}{\partial p_{cc}}$$

$$\beta_{ch} = \frac{1}{V} \frac{\partial V}{\partial p_{cc}}$$

Evolution of pressure

$$\frac{dp_{ch}}{dt} = \frac{q_{in} - q_{out}}{V_0 \left(\beta_m + \beta_{ch}\right)}$$

Chamber influx

$$q_{in} = \Omega \left(p_{deep} - p_{ch}\right)$$

Conduit Equations

Momentum & mass conservation

$$0 = \frac{\partial p}{\partial z} + \rho g + \frac{8\eta v}{R^2}$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial z}(\rho v)$$

Magma viscosity

$$\eta = \eta_\chi \left(\chi_d^w, T\right) \eta_\phi \left(\phi^v\right)$$

Plug friction

$$\tau_p = a\bar{\sigma} \operatorname{arcsinh}\left(\frac{v}{2v_r} \exp \frac{f_0}{a}\right)$$

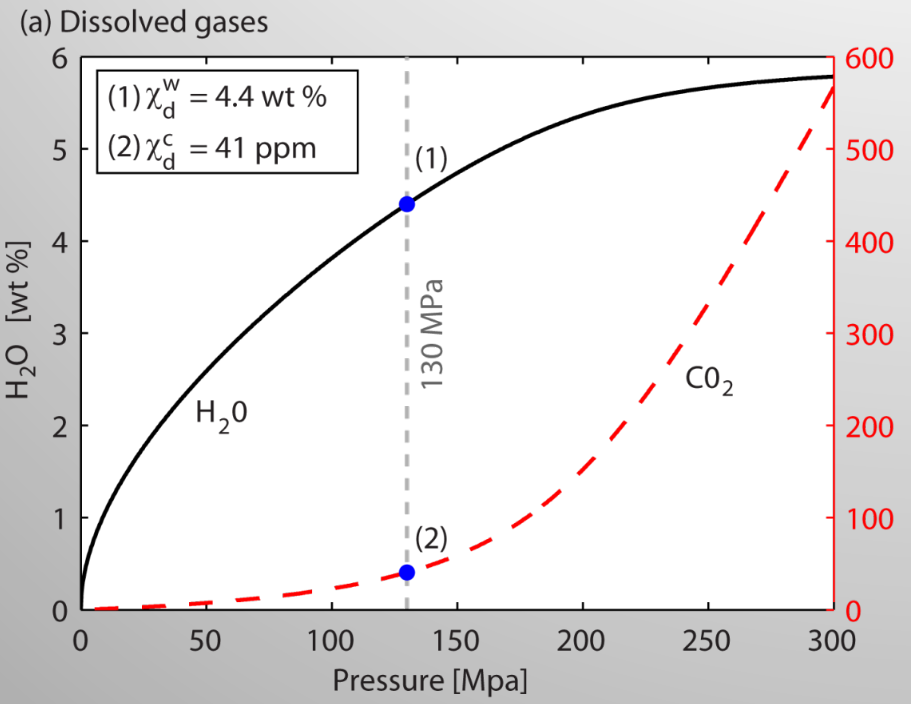
Density

$$\rho^* = \left[\frac{1-\phi}{1+\chi_e+\chi_d}\left(\frac{\chi_e}{\rho_\alpha} + \frac{\chi_d}{\rho_c} + \frac{1}{\rho_\lambda}\right) + \frac{\phi}{\rho_\phi}\right]^{-1}$$

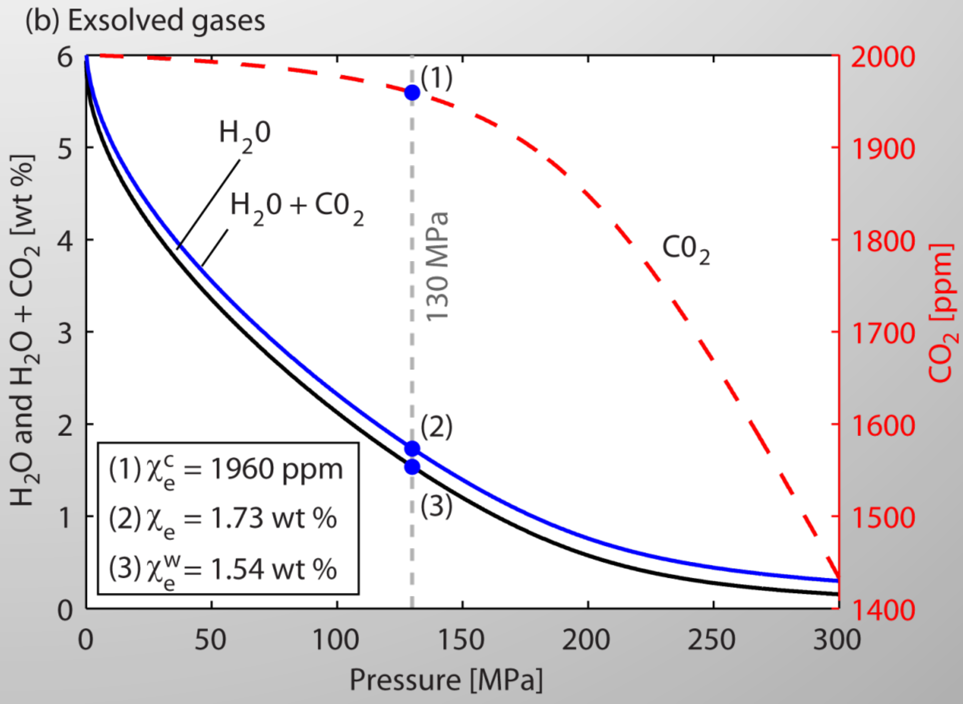
Solubility of H_2O and CO_2 in melt

$$\text{H}_2\text{O:}\quad \chi_d^w = \chi_T^w - \chi_e^w = S^w\left(\overbrace{m^w, m^c}^{\text{Mole fraction H}_2\text{O and CO}_2\text{ in gas phase}}, \overbrace{p, T}^{\text{Pressure and temperature}}\right)$$

$$\text{CO}_2\text{:}\quad \underbrace{\chi_d^c}_{\text{Dissolved volatiles}} = \underbrace{\chi_T^c}_{\text{Total volatiles}} - \underbrace{\chi_e^c}_{\text{Exsolved volatiles}} = \underbrace{S^c\left(m^w, m^c, p, T\right)}_{\text{Empirical solubility relationship from } \textit{Liu et al.}\text{ [2005]}}$$



(a) Dissolved gases



(b) Exsolved gases

Bayes' Theorem

$$P(m|d) \propto P(d|m)\,P(m)$$

Posterior Probability

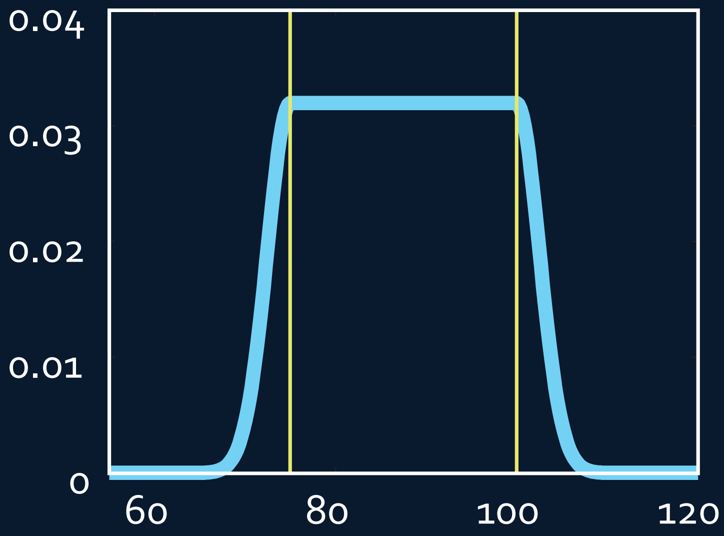
Data Likelihood

Prior Probability

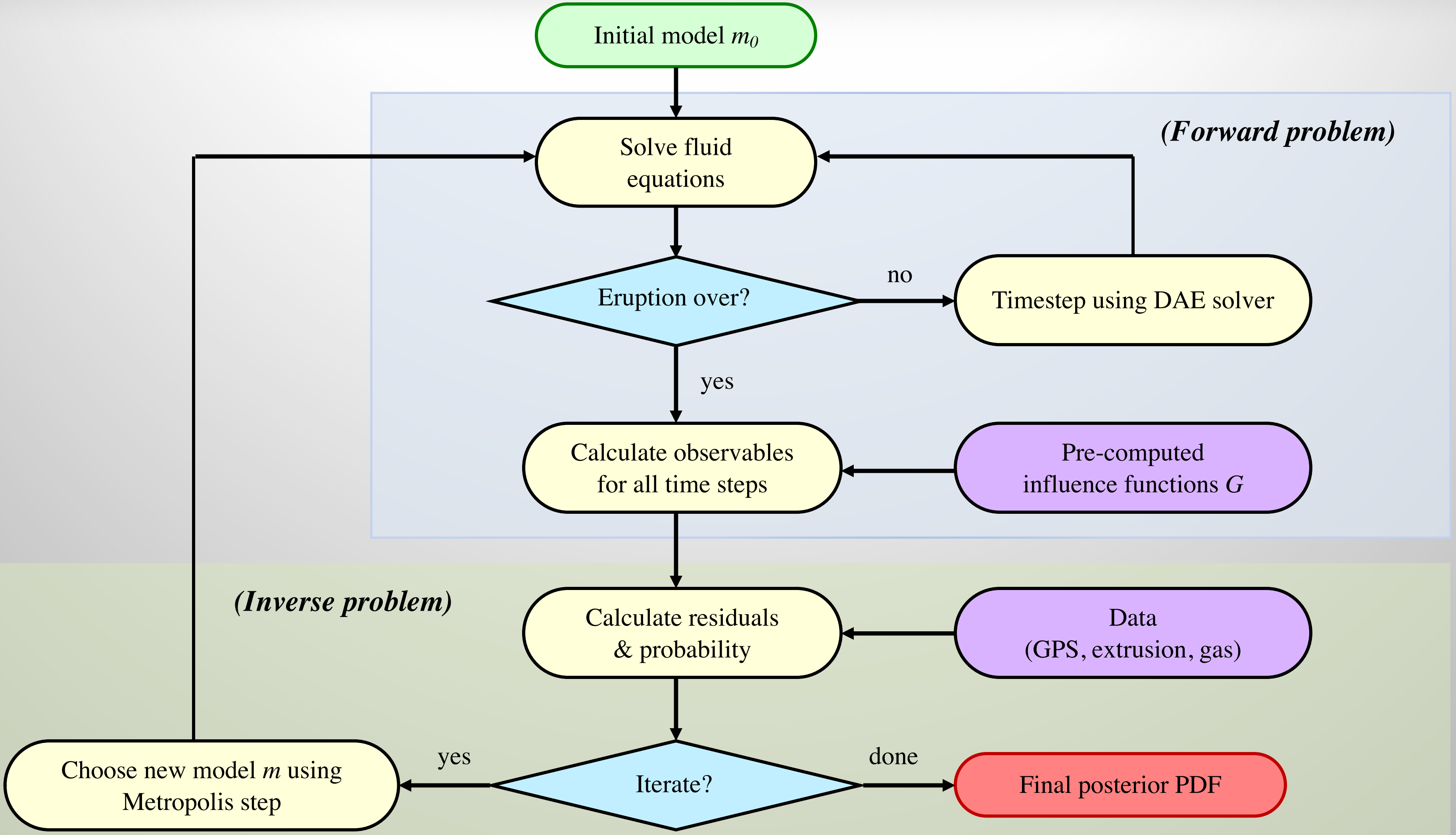
m: model
d: data

Use Markov Chain Monte Carlo (MCMC) method to characterize posterior probability distributions

Chamber Pressure
MPa



Incorporation into Markov-Chain Monte Carlo inversion



Initial model m_0

(Forward problem)

Solve fluid equations

Eruption over?

no

Timestep using DAE solver

yes

Calculate observables for all time steps

Pre-computed influence functions G

(Inverse problem)

Calculate residuals & probability

Data (GPS, extrusion, gas)

Iterate?

yes

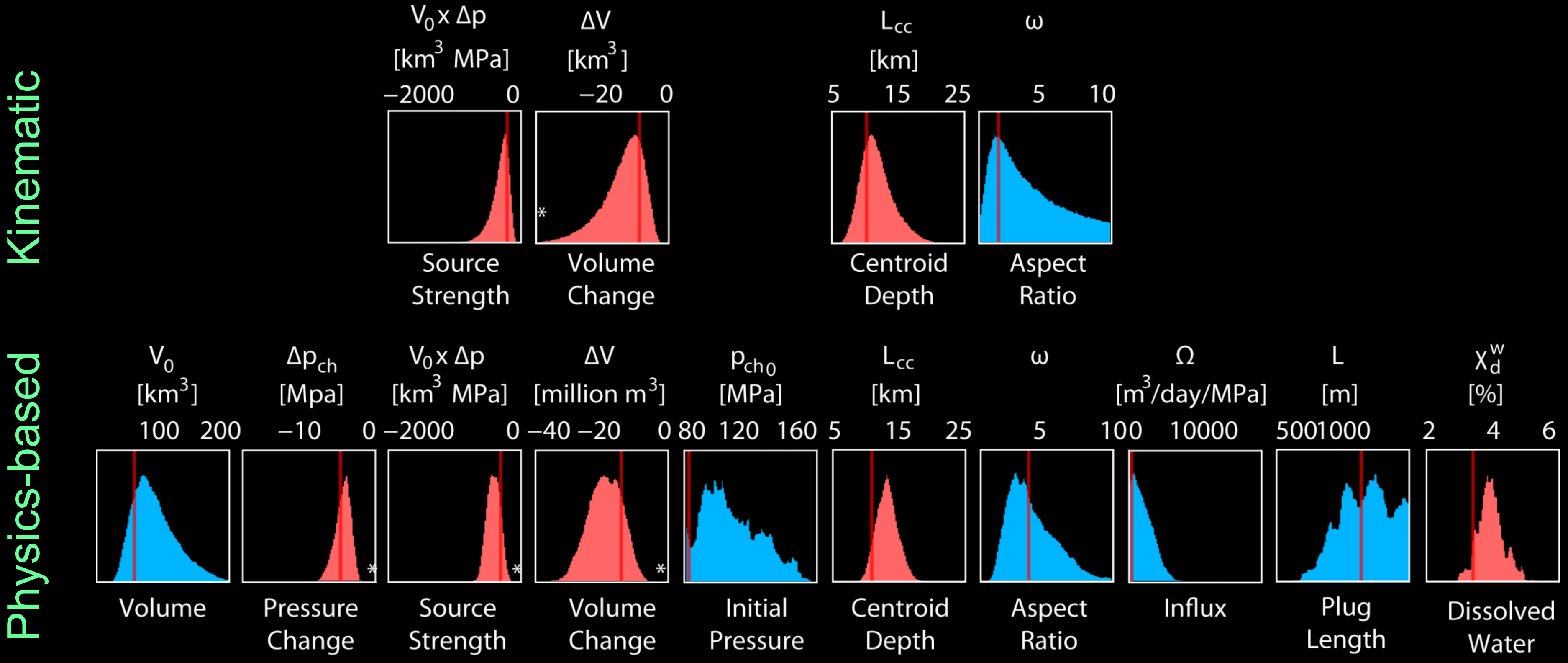
Choose new model m using Metropolis step

done

Final posterior PDF

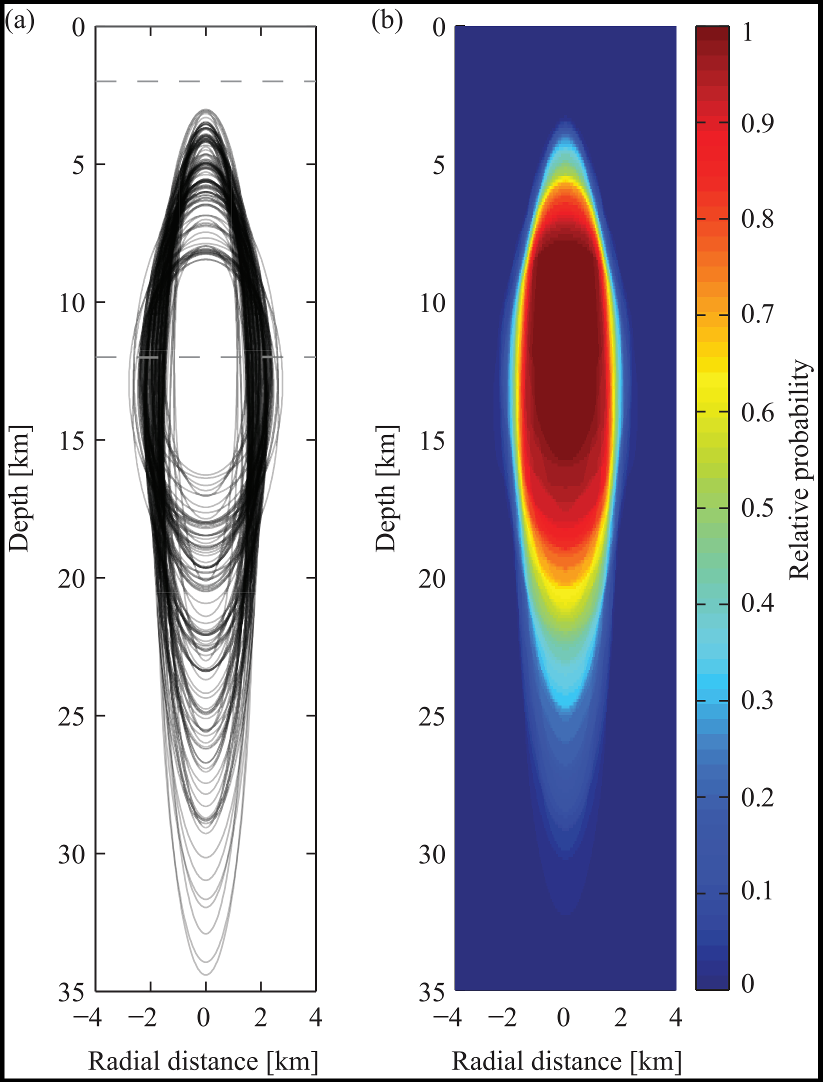
$$P\left(\mathbf{d_1}, \mathbf{d_2}, \ldots, \mathbf{d_K} | \mathbf{m}\right) = \prod_{k=1}^{K} \left\{ \left(2\pi\gamma_k^2\right)^{-N_k/2} \left|\Sigma_k\right|^{-1/2} \exp\left(-\frac{1}{2\gamma_k^2} \mathbf{r_k^T} \Sigma_k^{-1} \mathbf{r_k} \right) \right\}$$

Kinematic vs. Physics-Based Inversion

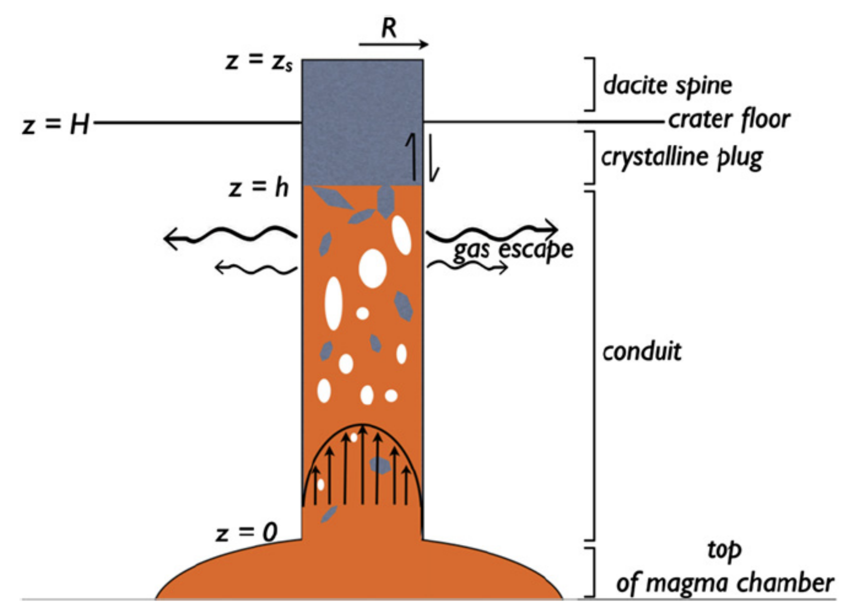


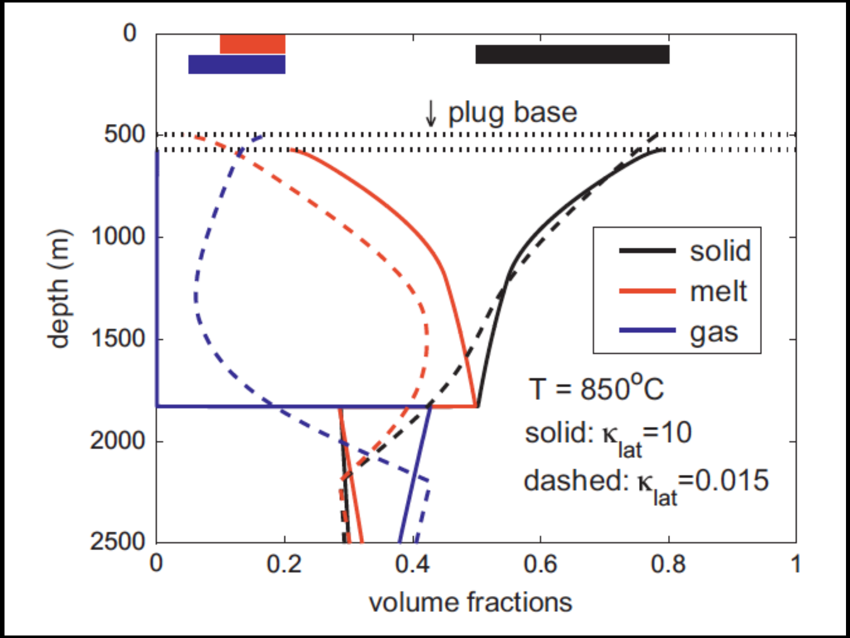
Anderson and Segall, JGR 2013

Summary of Magma Chamber Geometry



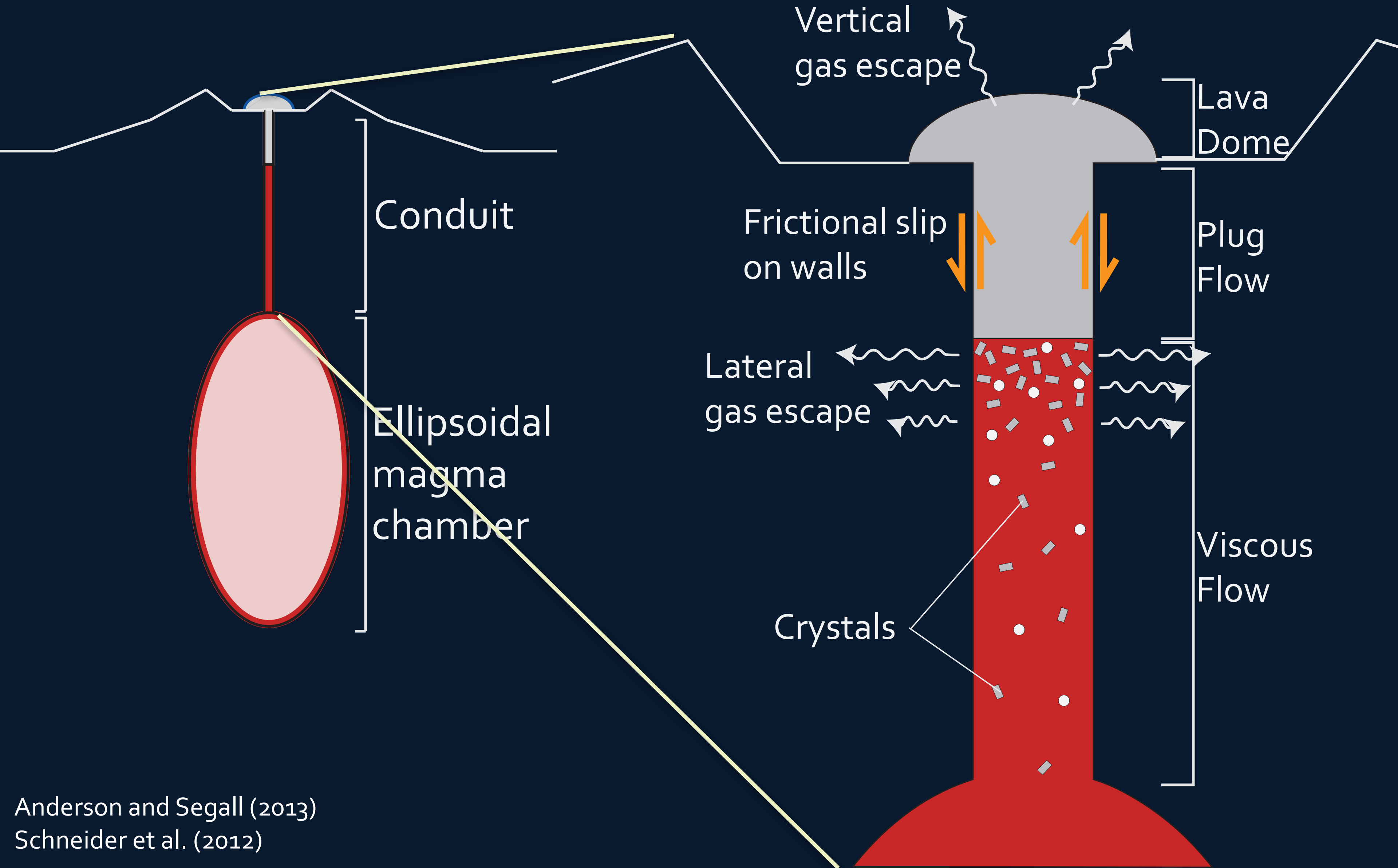
Schneider, Rempel, Cashman (2012)





- Steady state eruption model
- Equilibrium crystallization based on MELTS
- “Membrane diffusion” gas loss

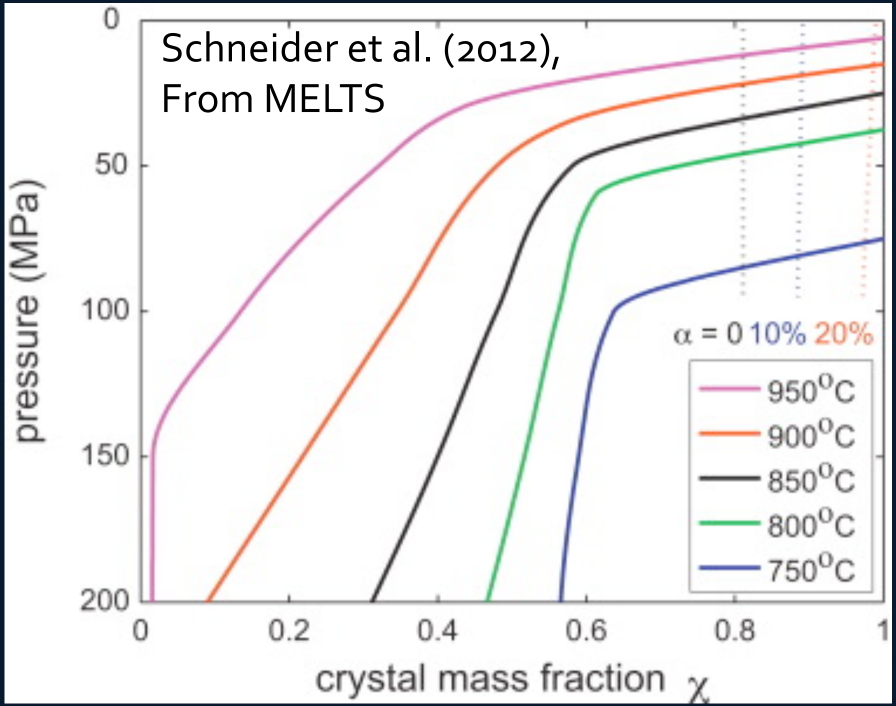
Physics-based model: Geometry and Setup

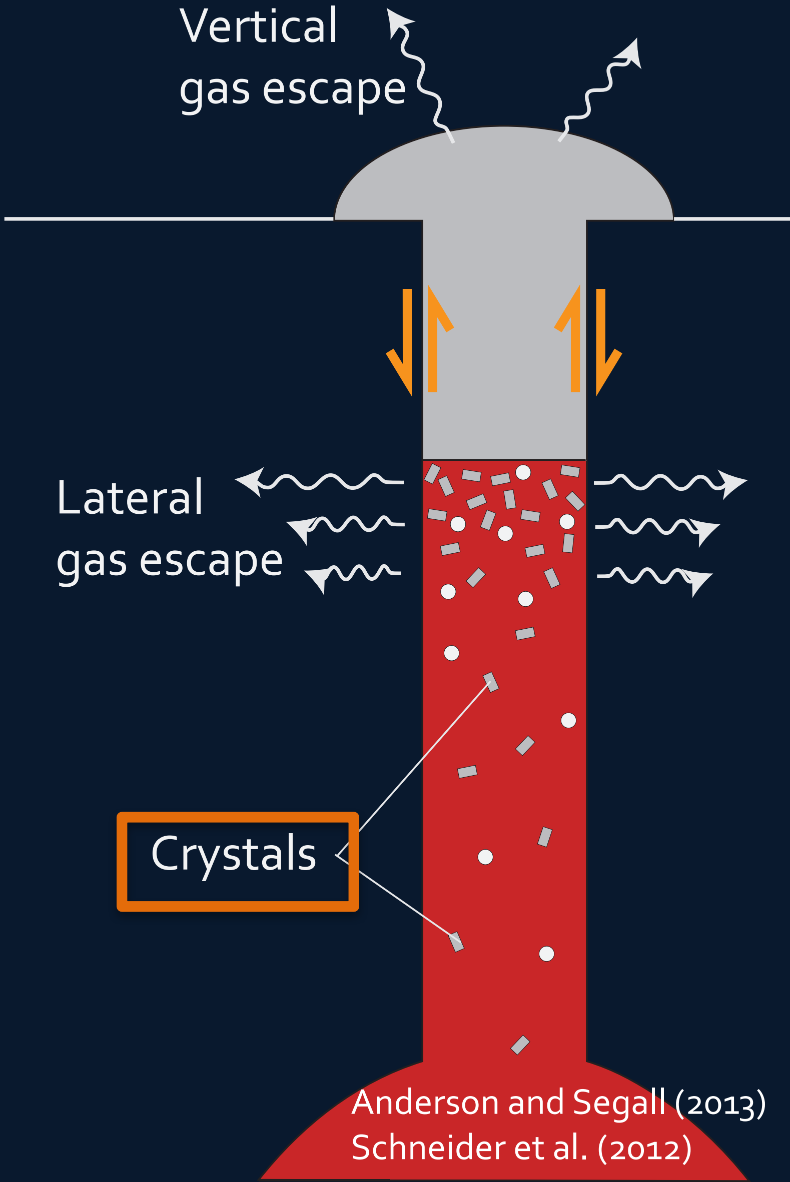


Anderson and Segall (2013)
Schneider et al. (2012)

Equilibrium crystallization

Depressurization
↓
Crystallization
(Assumed isothermal)


Schneider et al. (2012),
From MELTS


Vertical
gas escape

Lateral
gas escape

Crystals

Anderson and Segall (2013)
Schneider et al. (2012)

Equilibrium crystallization

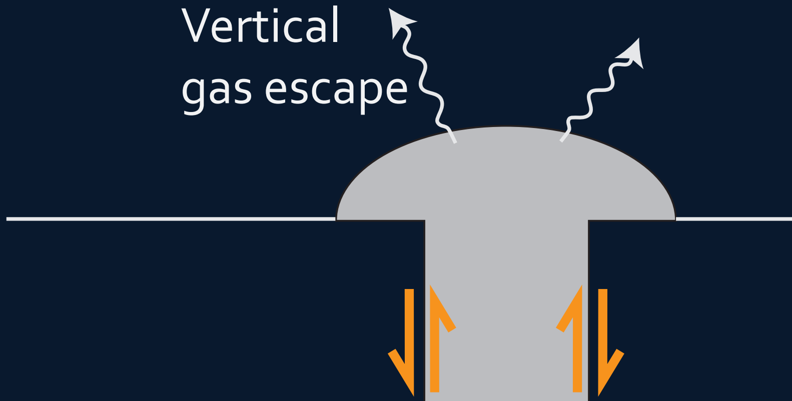
Depressurization

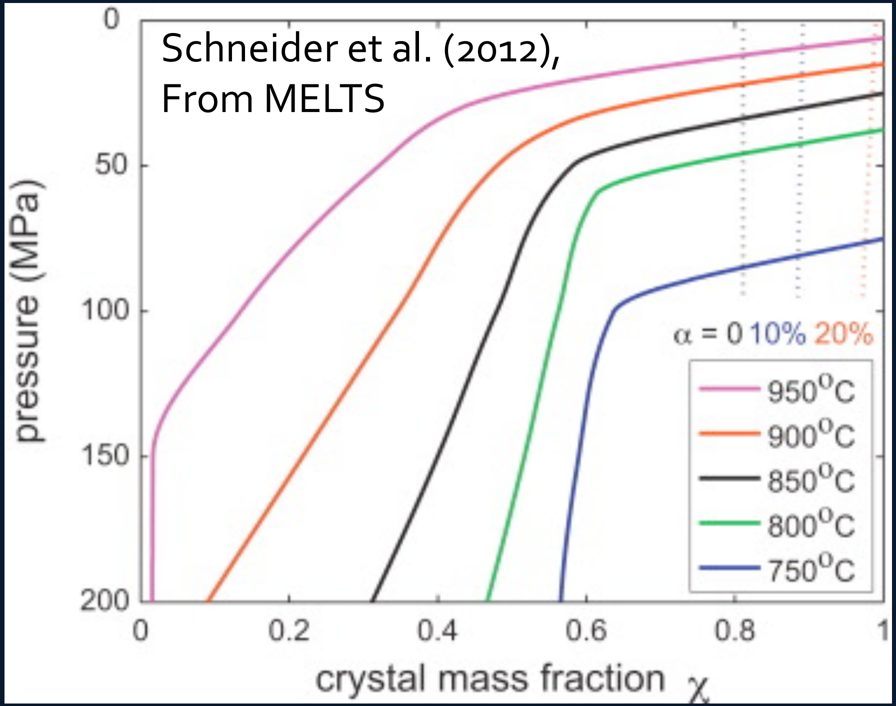
↓

Crystallization

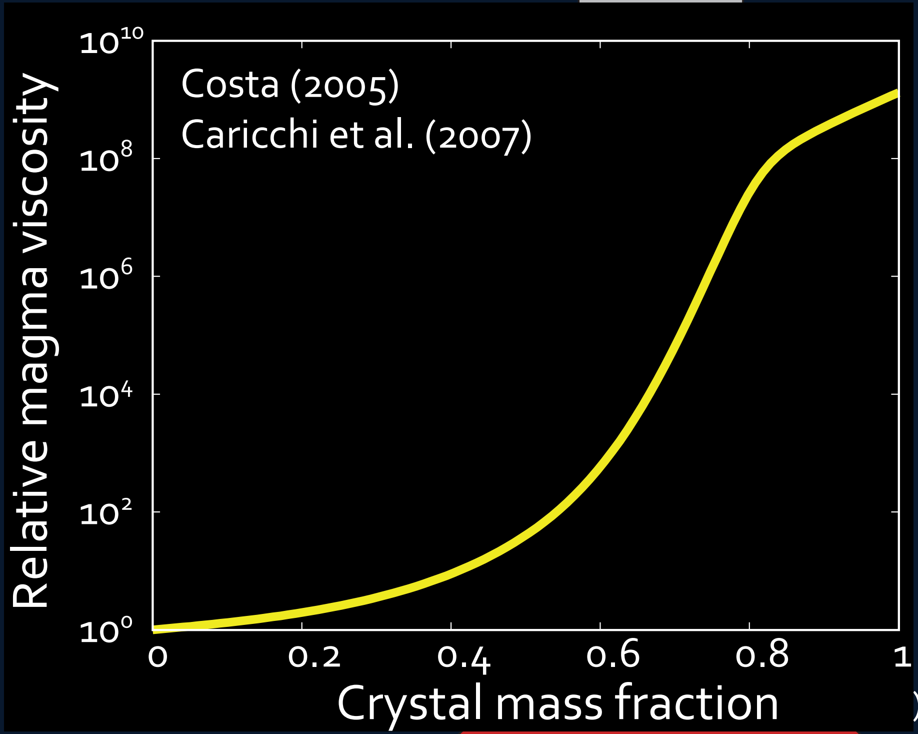
↓

Increase in viscosity





Schneider et al. (2012),
From MELTS



Costa (2005)
Caricchi et al. (2007)

Schneider et al. (2012)

Viscous flow to plug flow transition

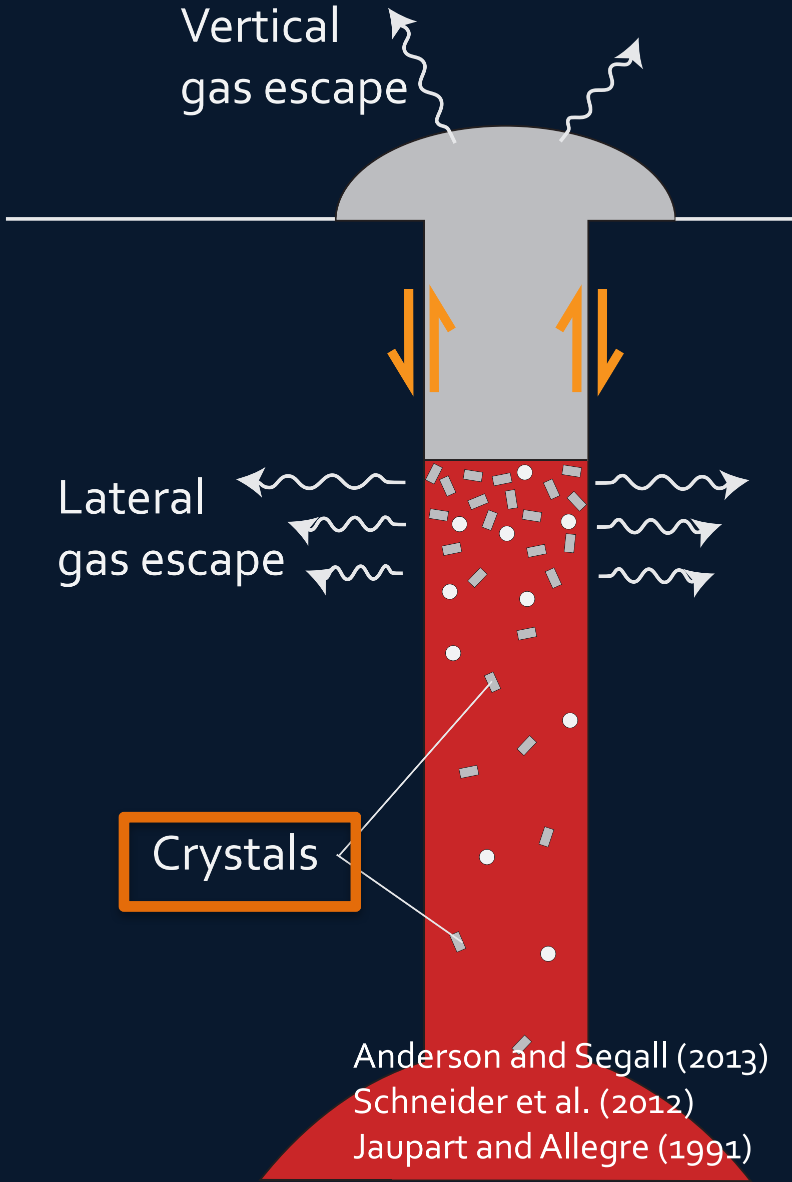
Unknown parameters

$$v = v_{\text{visc}} + v_{\text{fric}}$$

$$= \frac{\tau_R R}{4\eta} + 2v_{\text{ref}}\exp\left(\frac{\tau_R}{a\sigma} - \frac{f_0}{a}\right),$$

where

$$\tau_R = -\frac{R}{2}\left(\frac{\partial p}{\partial z} - \rho g\right)$$



Anderson and Segall (2013)
Schneider et al. (2012)
Jaupart and Allegre (1991)

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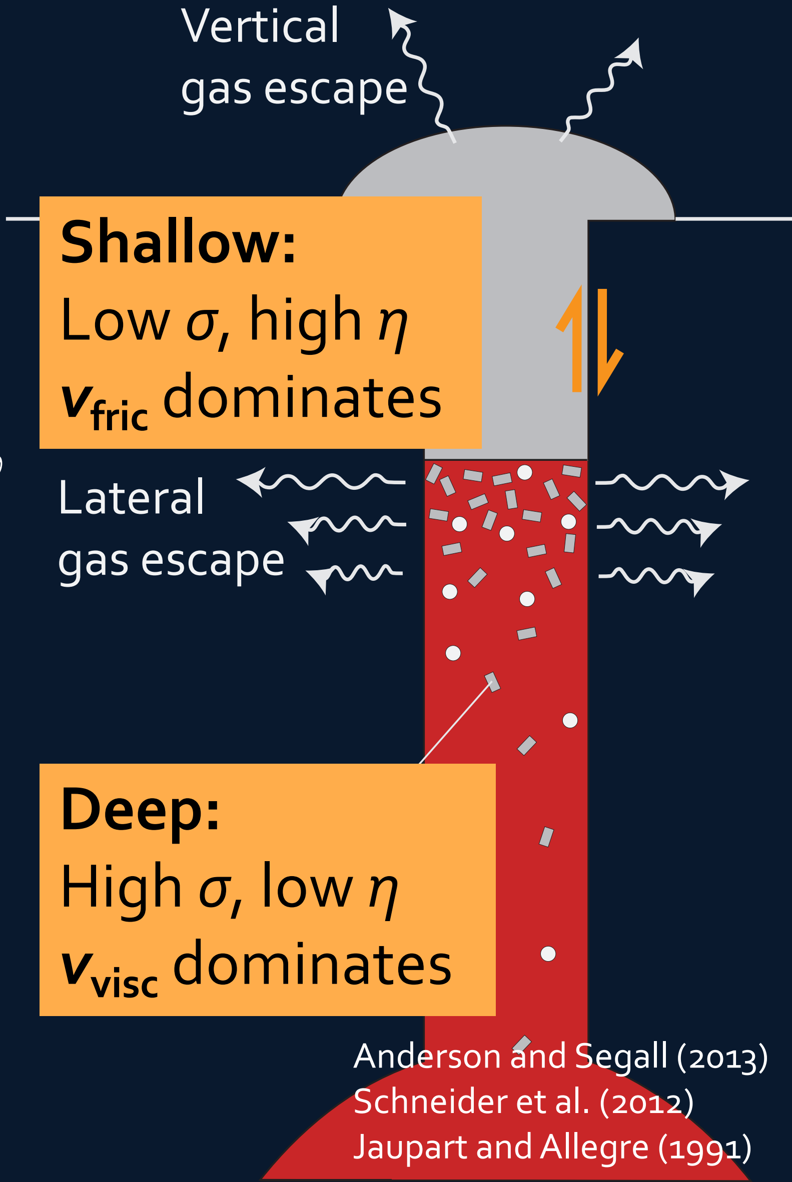
where

$$\tau_R = -\frac{R}{2}\left(\frac{\partial p}{\partial z} - \rho g\right)$$



Vertical gas escape

Shallow:
Low σ, high η
$\boldsymbol{v}_{\textbf{fric}}$ dominates

Lateral gas escape

Deep:
High σ, low η
$\boldsymbol{v}_{\textbf{visc}}$ dominates

Anderson and Segall (2013)
Schneider et al. (2012)
Jaupart and Allegre (1991)

Continuity Equations

Liquid + Solid: $\frac{\partial}{\partial t}\left(\rho_l\phi_l + \rho_s\phi_s\right) + \frac{\partial}{\partial z}\left[\left(\rho_l\phi_l + \rho_s\phi_s\right)v\right] = 0$

Water:

$$\frac{\partial}{\partial t}\left(\chi_h^d\rho_l\phi_l + \frac{1}{1+\Gamma}\phi_g\rho_g\right) + \frac{\partial}{\partial z}\left[\left(\underbrace{\chi_h^d\rho_l\phi_l}_{\text{Dissolved}} + \underbrace{\frac{1}{1+\Gamma}\phi_g\rho_g}_{\text{Exsolved}}\right)v + \frac{1}{1+\Gamma}\phi_g\rho_g\underbrace{(v_g - v)}_{\text{Vertical Flow}}\right] = -\underbrace{\frac{2}{R}\frac{\rho_g u_{lat}}{1+\Gamma}}_{\text{Lateral Gas Loss}}$$

Analogous for CO2:

$$u_{lat} = \phi_g\frac{k}{\eta_g}\frac{(p - p_{hyd})}{\lambda} \qquad v_g - v = \frac{k}{\eta_g}\frac{\partial p}{\partial z}$$

Magma Permeability $\quad k_m = k_c\phi^3,\ \phi > \phi_{gc}$

Vertical gas escape through magma

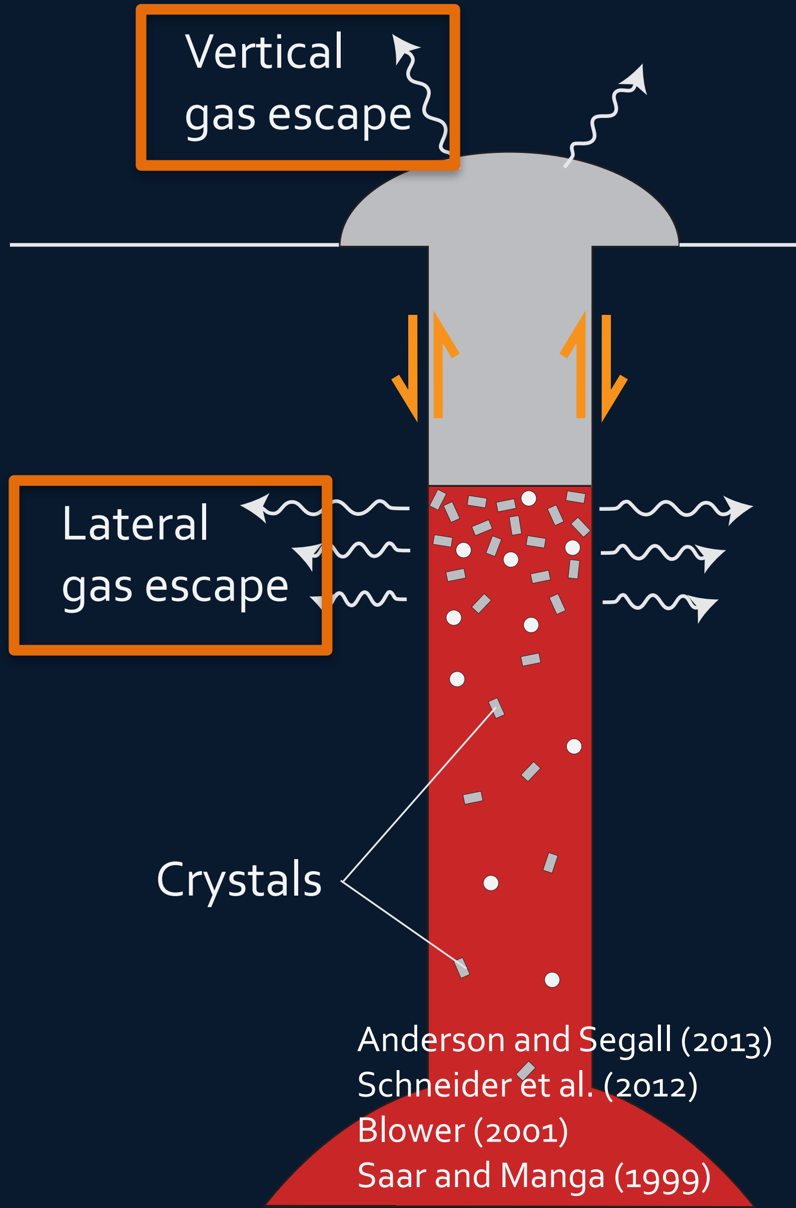
Percolation Theory:

$$k_m = k_c\phi^3,\ \phi > \phi_{gc}$$

Permeability scaling constant (unknown)

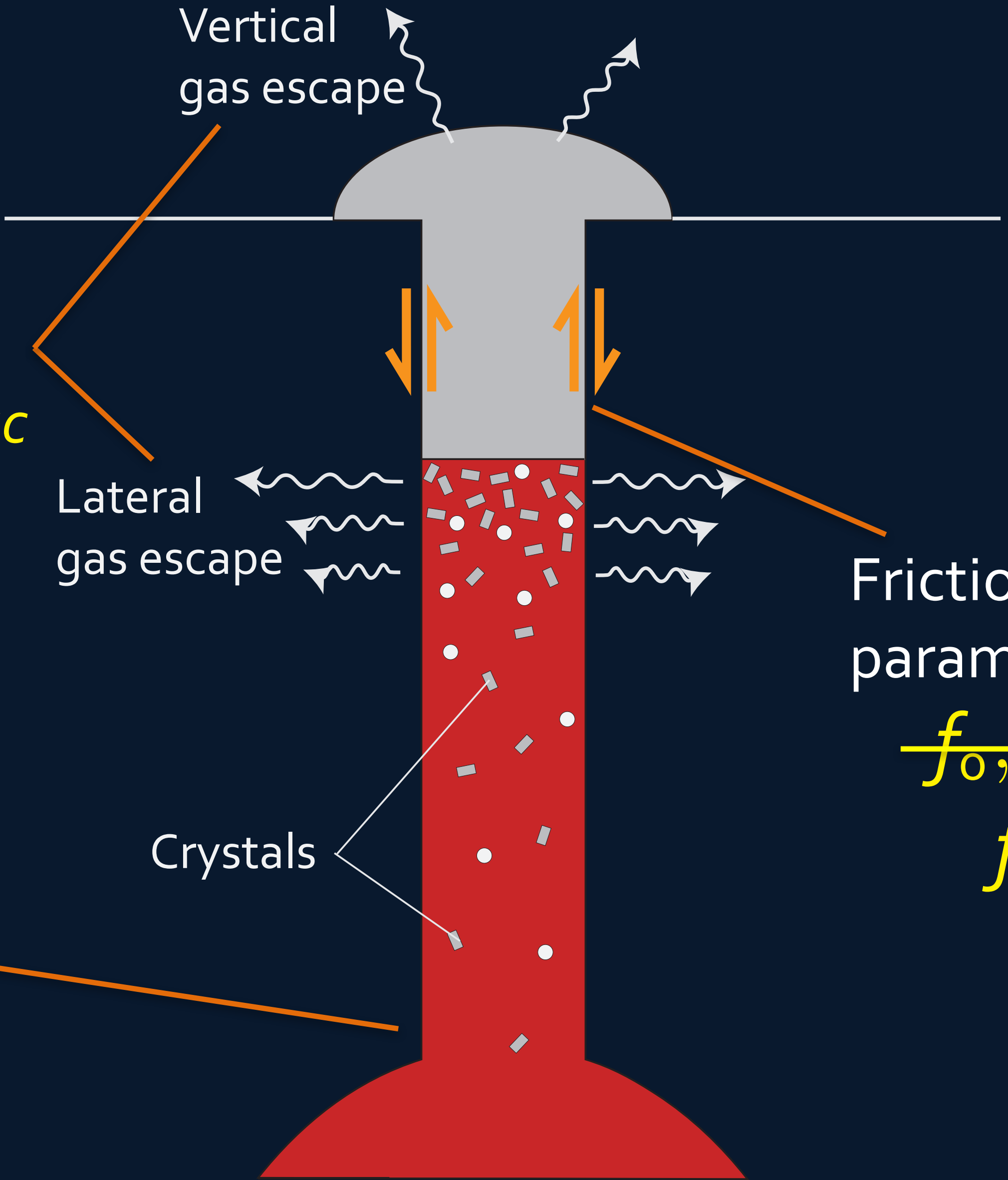
Porosity

Percolation threshold (unknown)



Anderson and Segall (2013)
Schneider et al. (2012)
Blower (2001)
Saar and Manga (1999)

Summary of model parameters



Permeability Constant k_c

Percolation Threshold φ_{gc}

Friction parameters

~~f_0, a~~

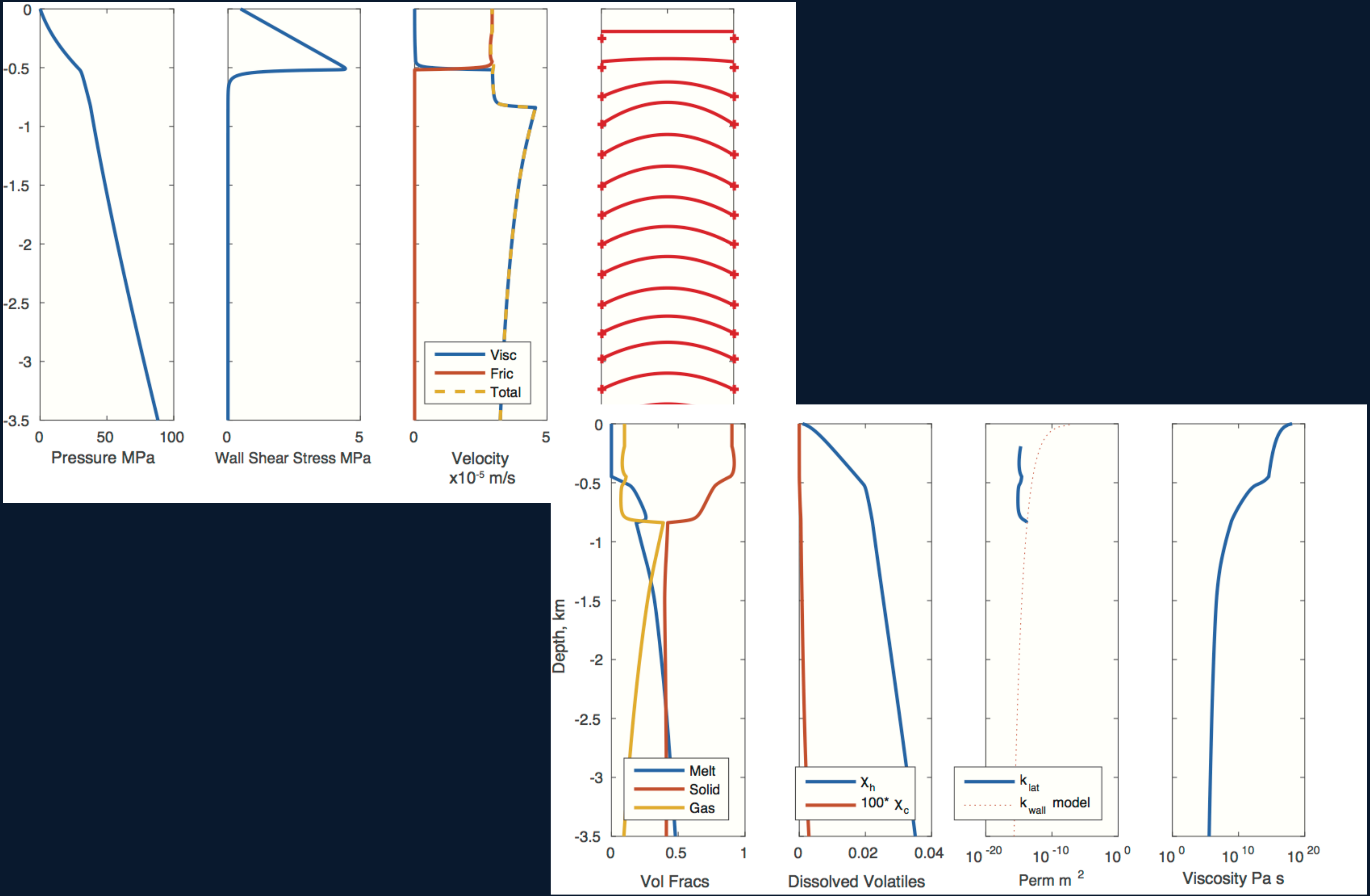
f

Boundary conditions at base of conduit:

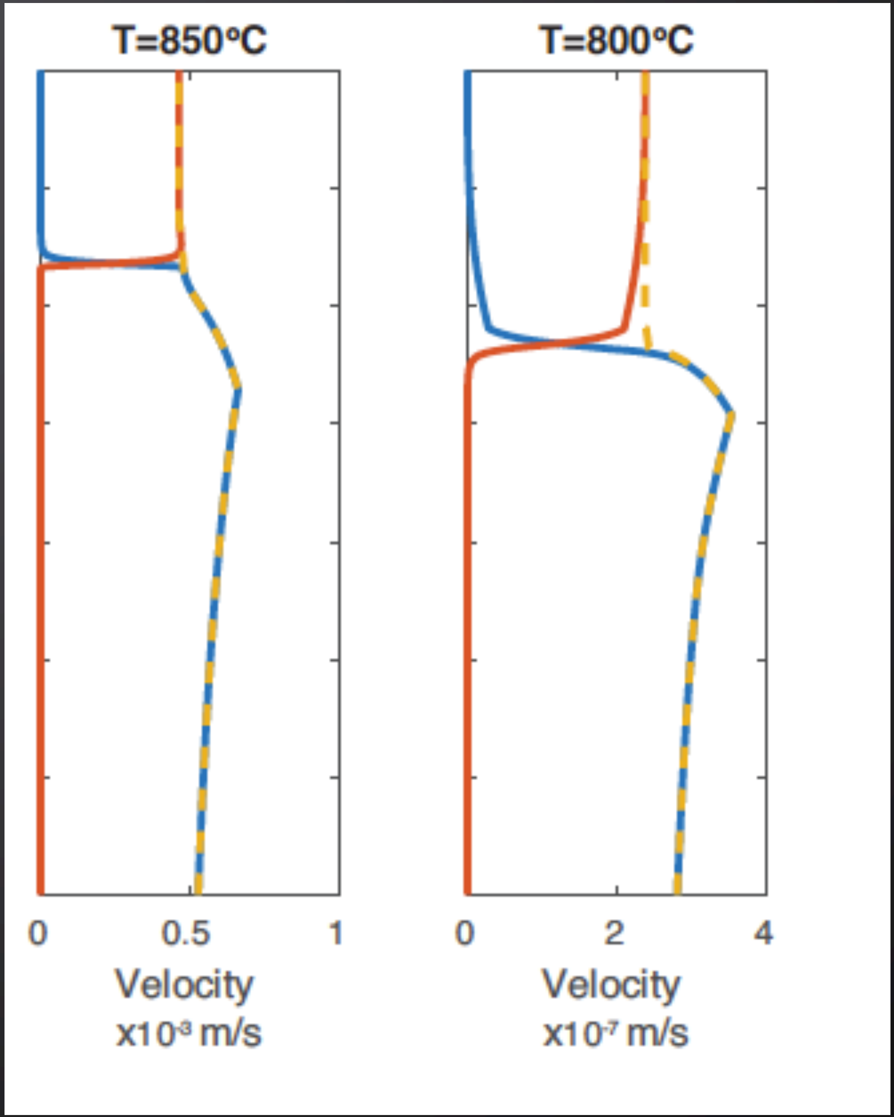
Pressure p_{ch}

Total water content χ_h

Steady state solution



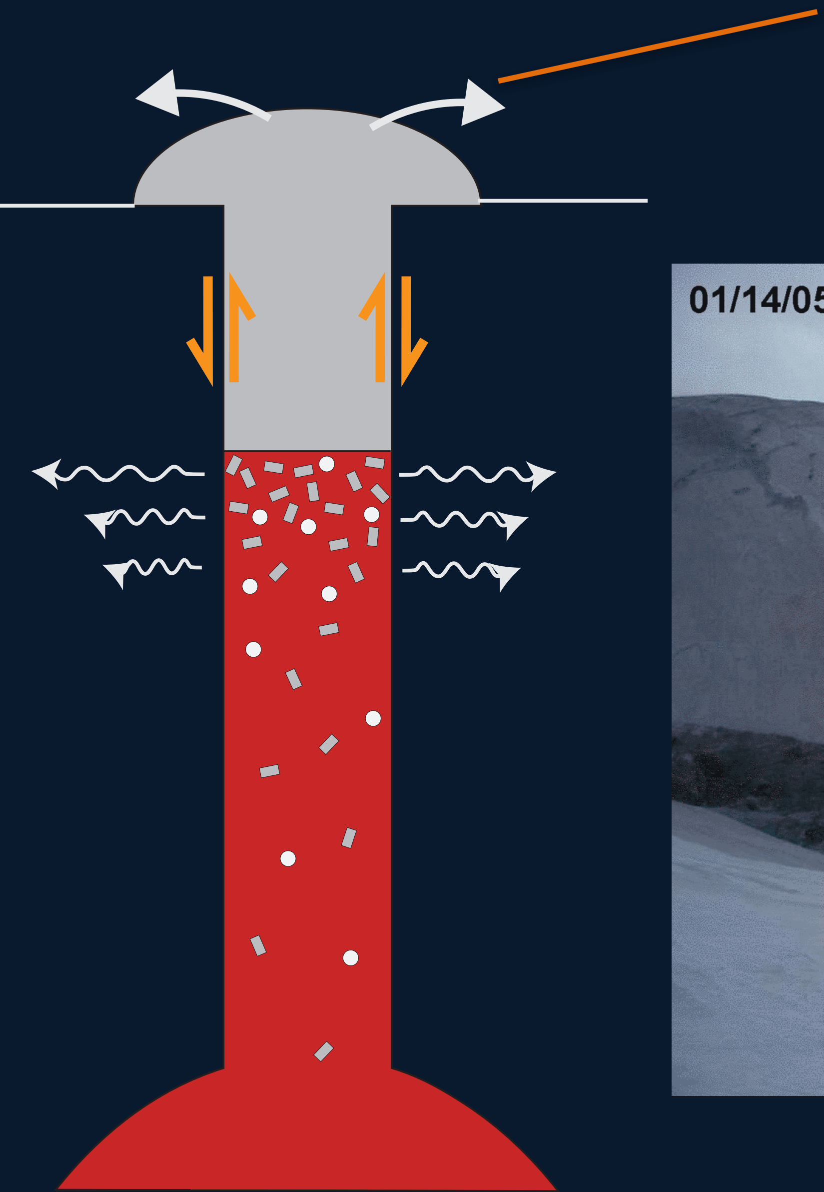
Temperature Dependence

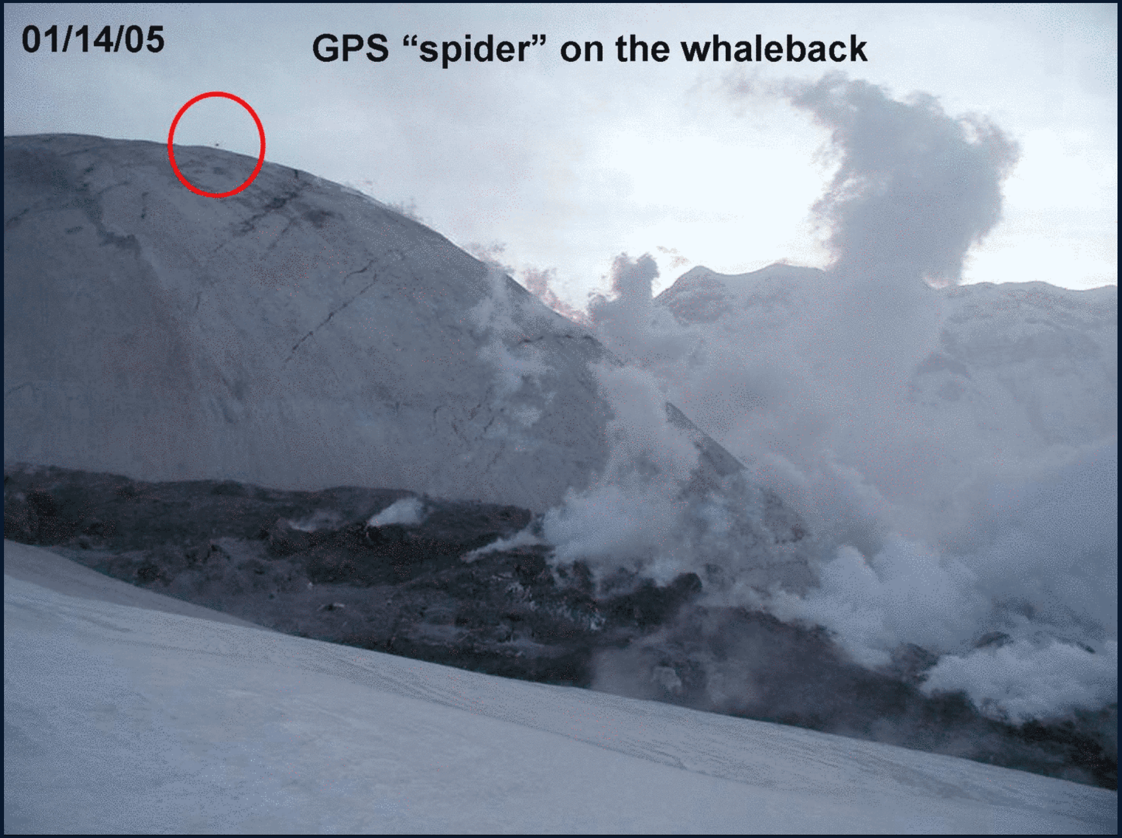


Quasi-steady state system: data

1) Exit velocity of plug

$3 - 7 \times 10^{-5}$ m/s

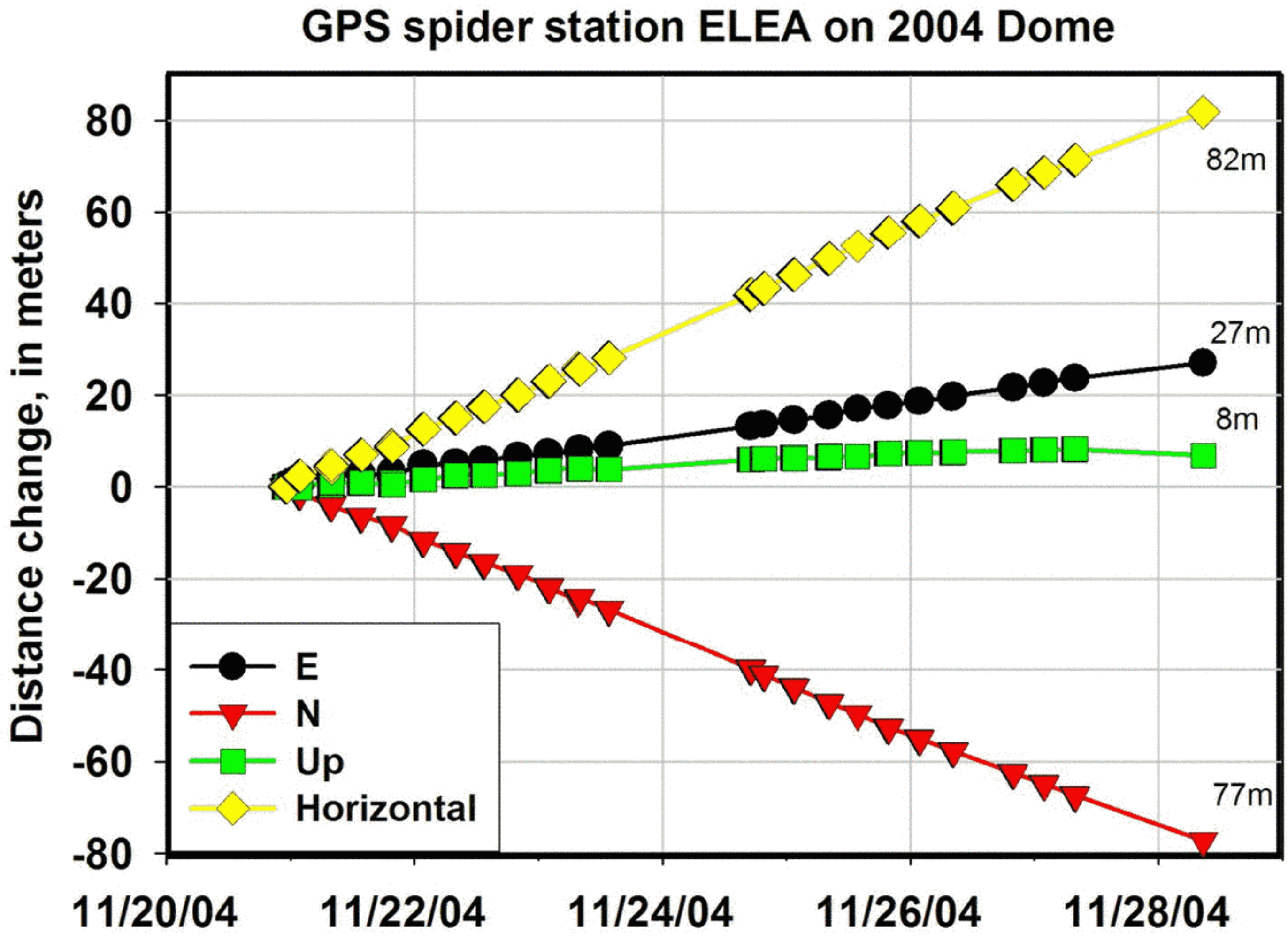




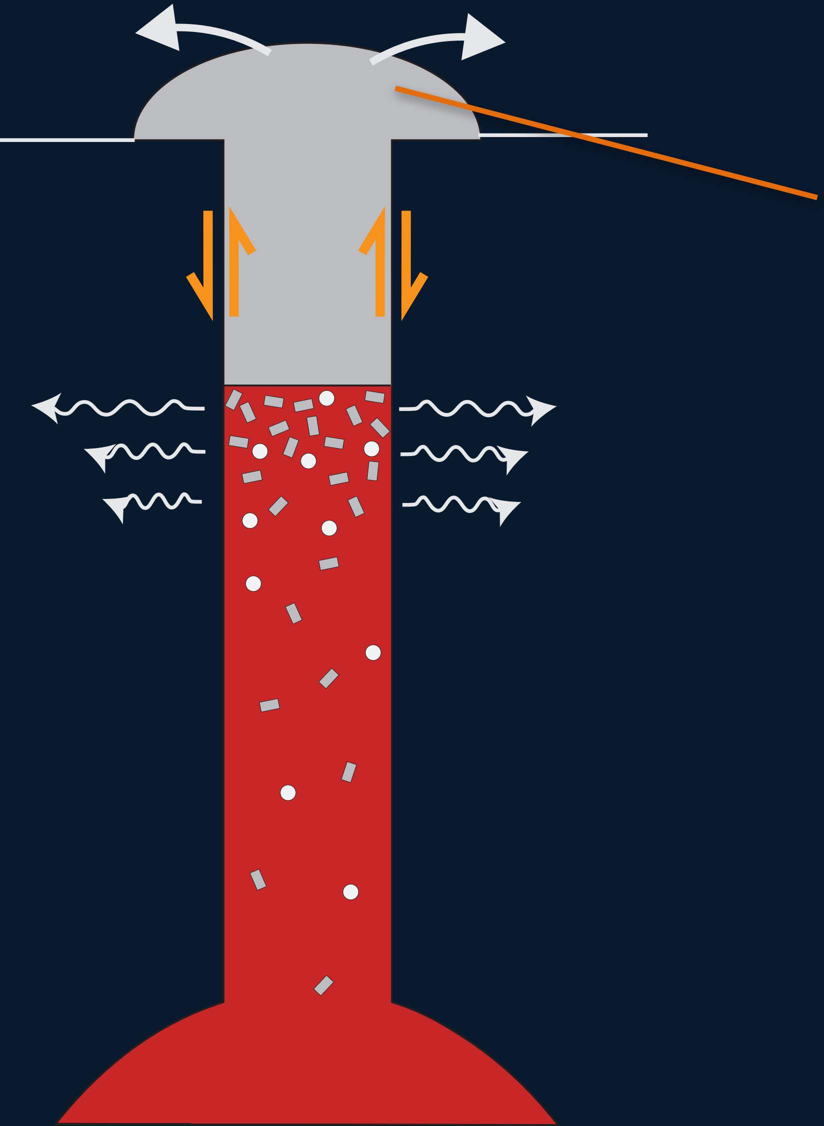
01/14/05

GPS “spider” on the whaleback

Whaleback motion 11/21/04 – 11/29/04



Quasi-steady state system: data

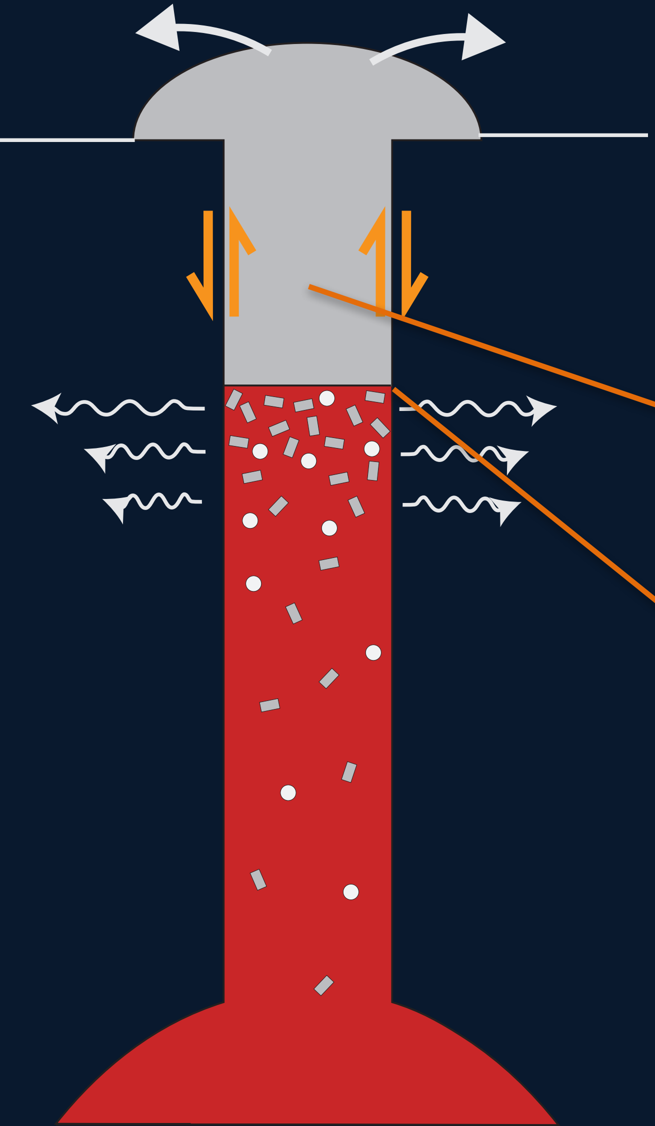


1) Exit velocity of plug
   $3 - 7 \times 10^{-5}$ m/s

2) Dome rock porosity
   5 – 10%

Thornber et al. (2008), Cashman et al. (2008)

Quasi-steady state system: Data



1) Exit velocity of plug
   $3 - 7 \times 10^{-5}$ m/s

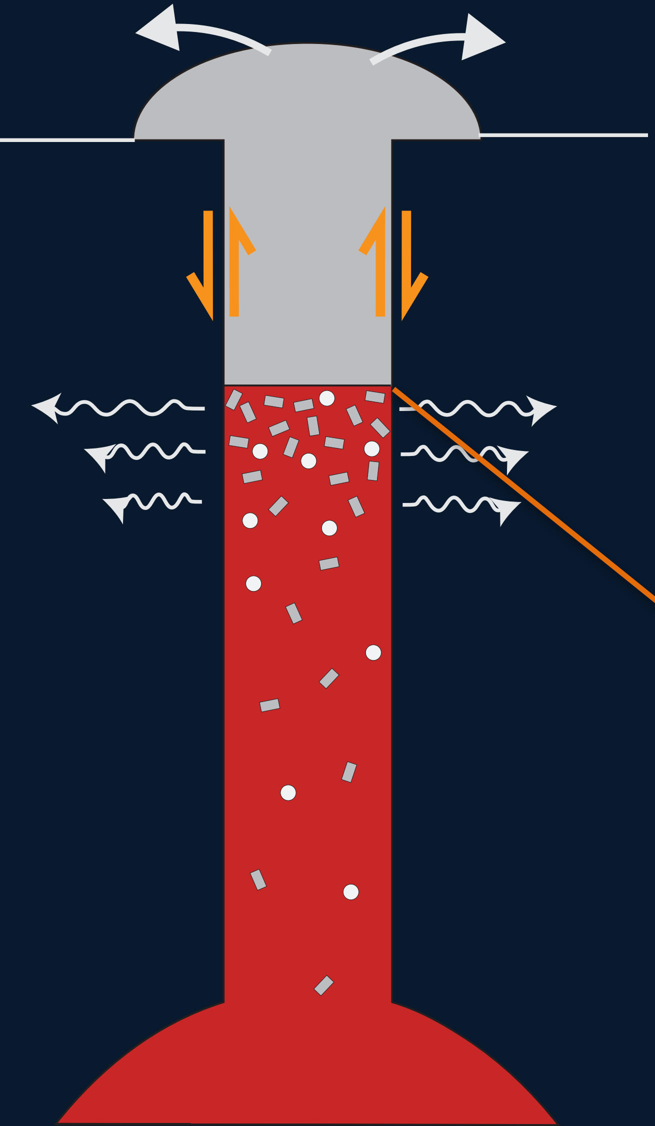
2) Dome rock porosity
   5 – 10%

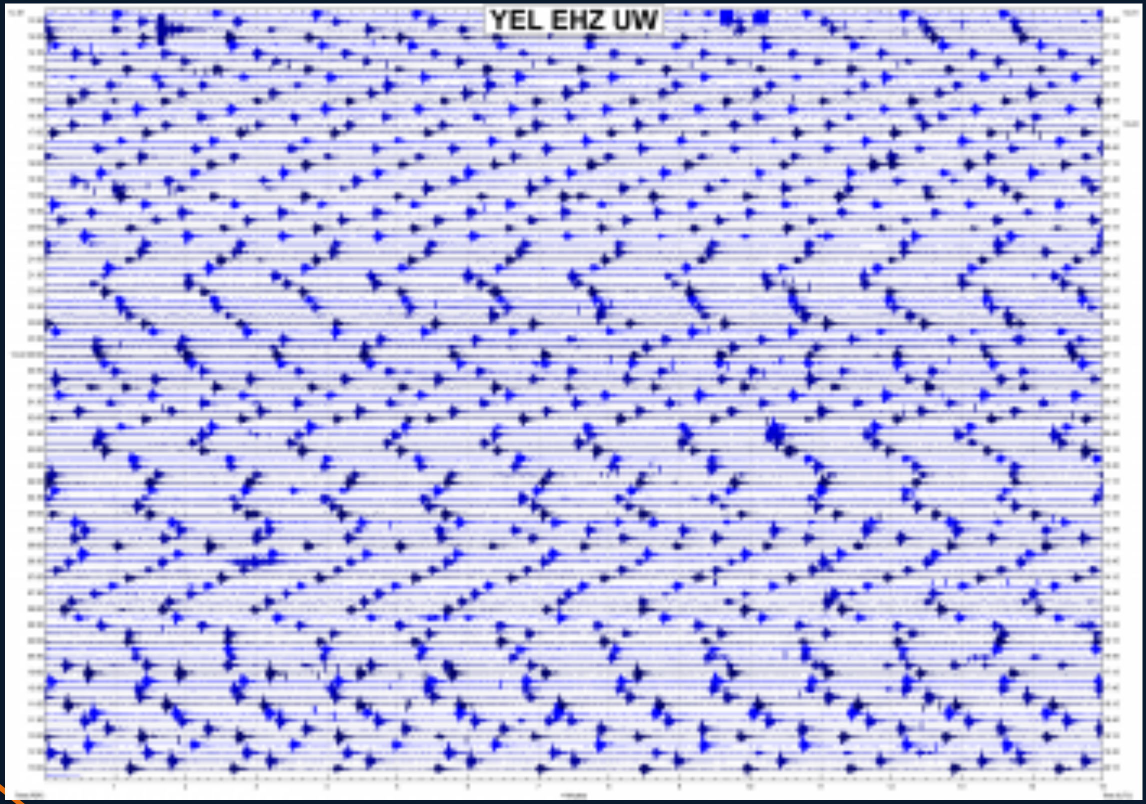
3) Crystallization depth
   0.5 – 1 km

4) Plug depth
   0.5 – 1 km

Vallance et al. (2008), Schilling et al. (2008)
Thornber et al. (2008), Cashman et al. (2008)
Pallister et al. (2008)
Iverson et al. (2006), Moore et al. (2008)

Quasi-steady state system: Data

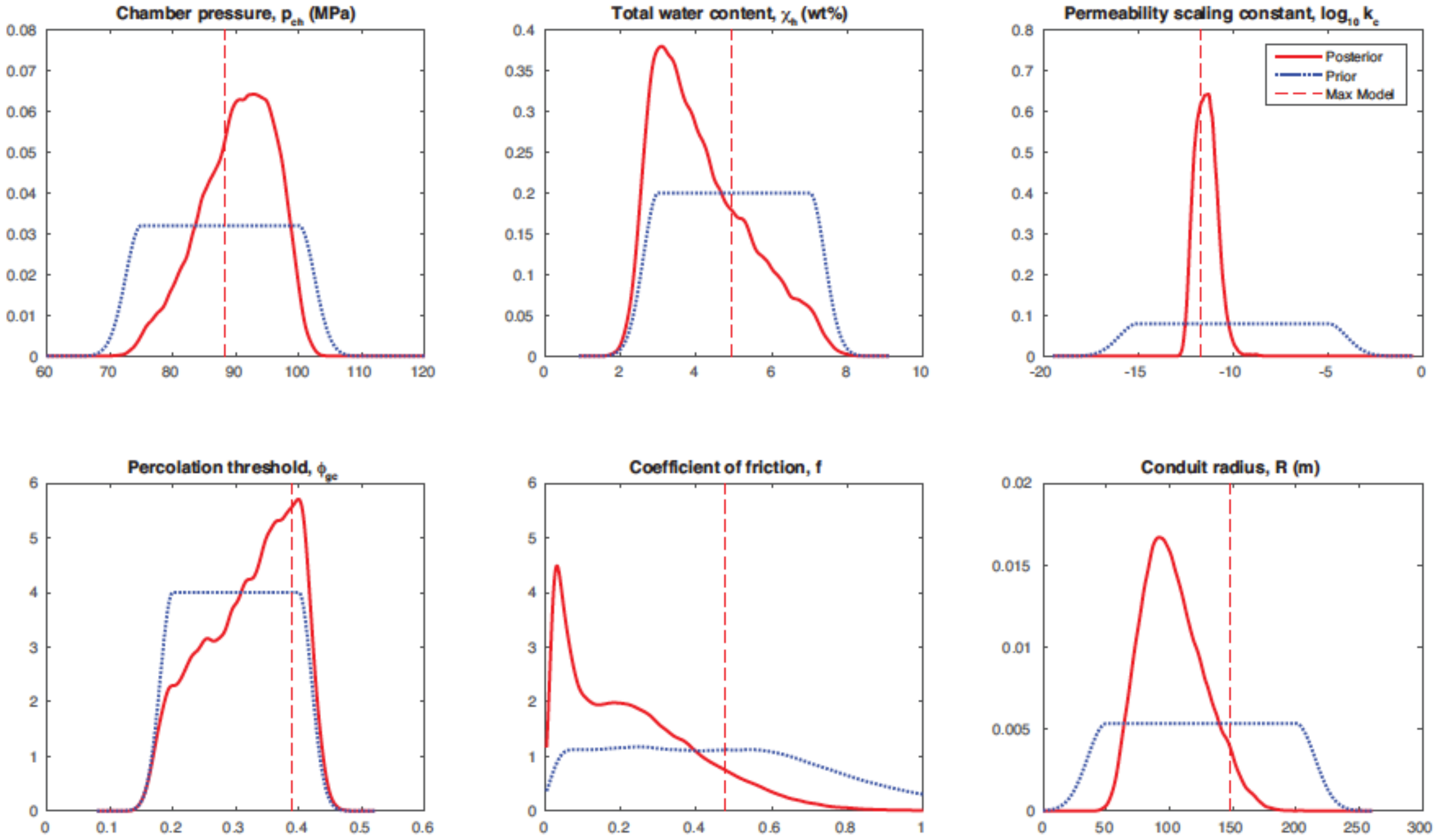




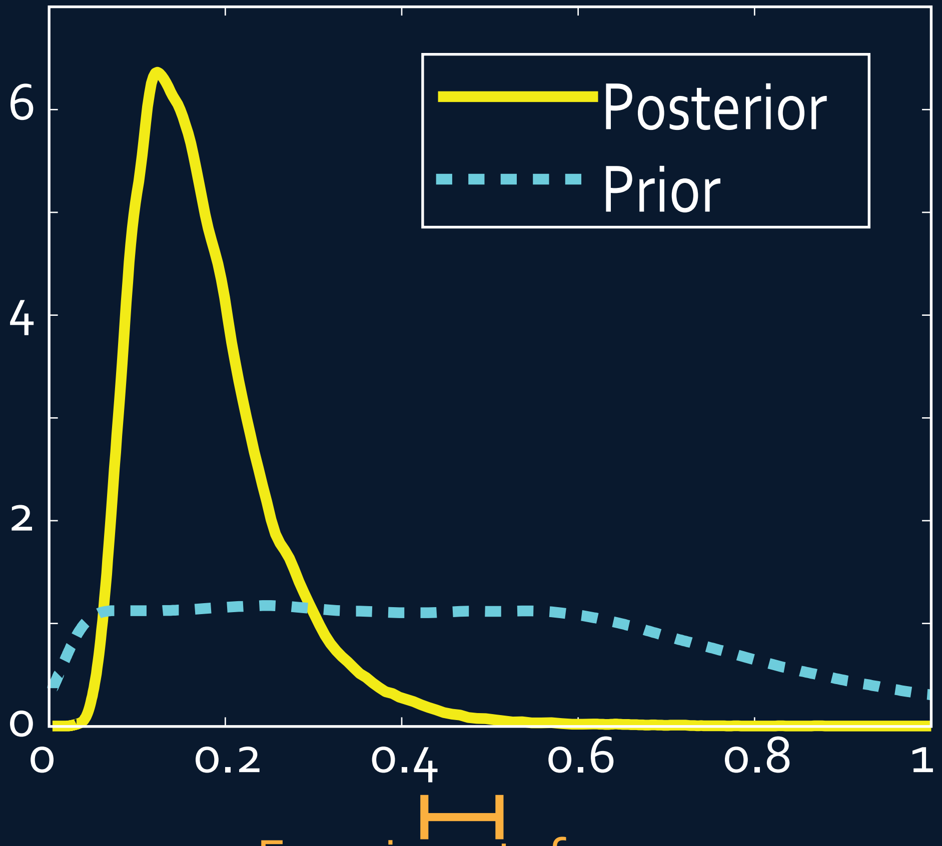
4) Plug depth
0.5 – 1 km

Vallance et al. (2008), Schilling et al. (2008)
Thornber et al. (2008), Cashman et al. (2008)
Pallister et al. (2008)
Iverson et al. (2006), Moore et al. (2008)

Results: Posterior PDFs



Low friction along conduit wall



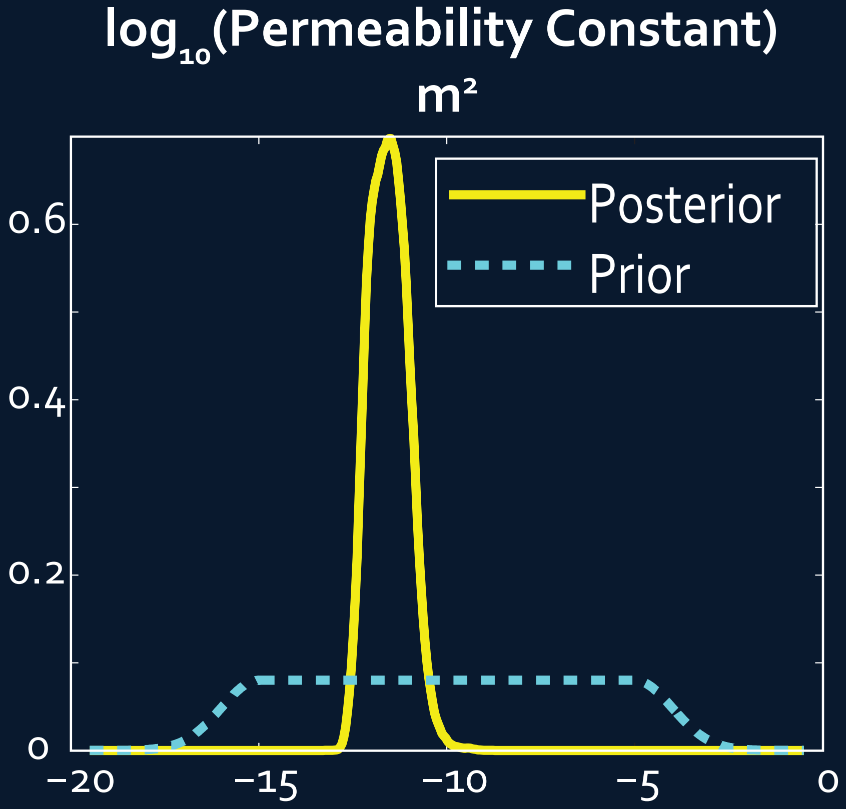
Experiments from
Moore et al. (2008)

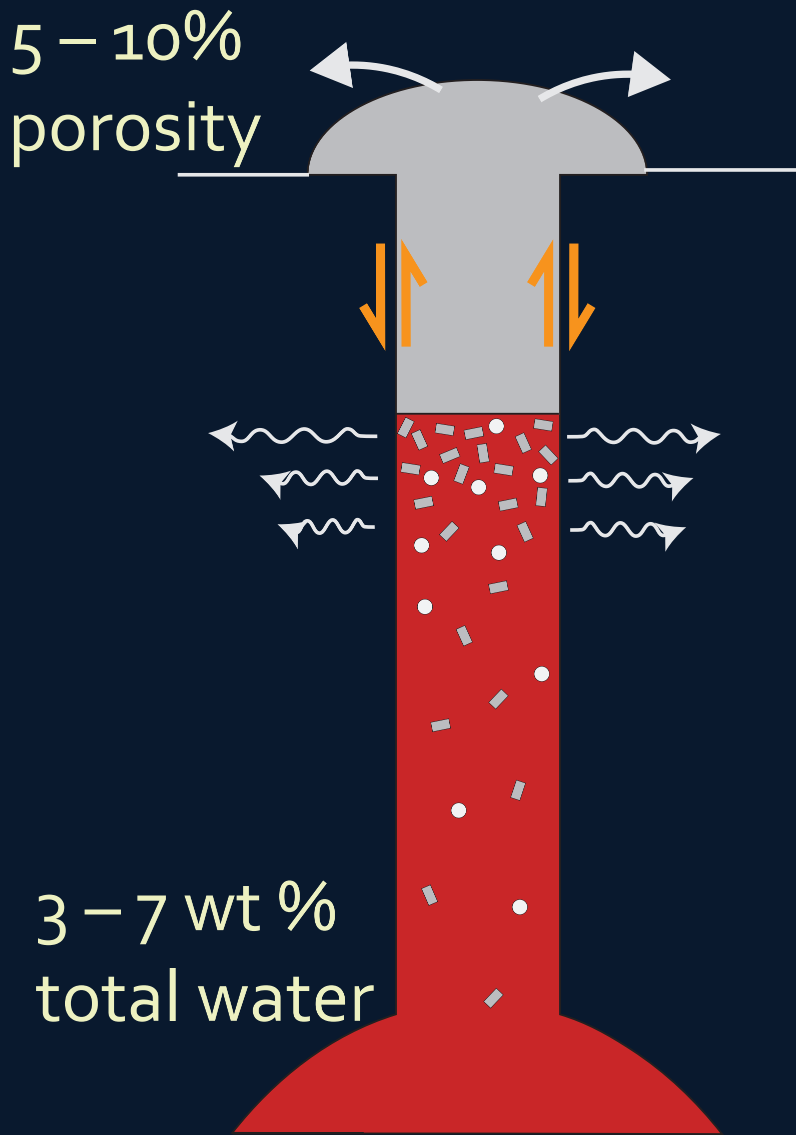
- Most accepted models $f < 0.3$

$$f = \frac{\tau_R}{\sigma - p} = \frac{\tau_R}{\sigma_{\text{eff}}}$$

- Possible explanations
  – High pore pressure
  – Reduced normal stress
  – Bias in viscosity

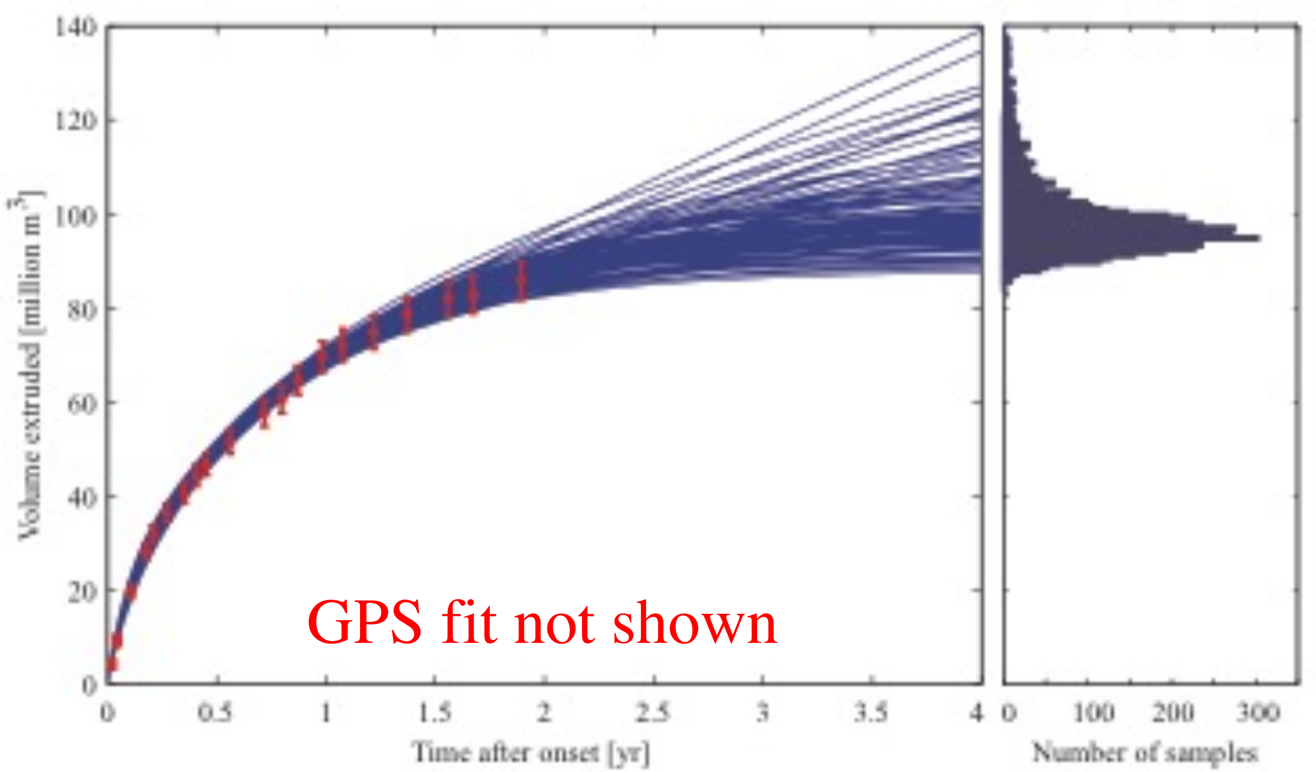
Well-constrained magma permeability constant





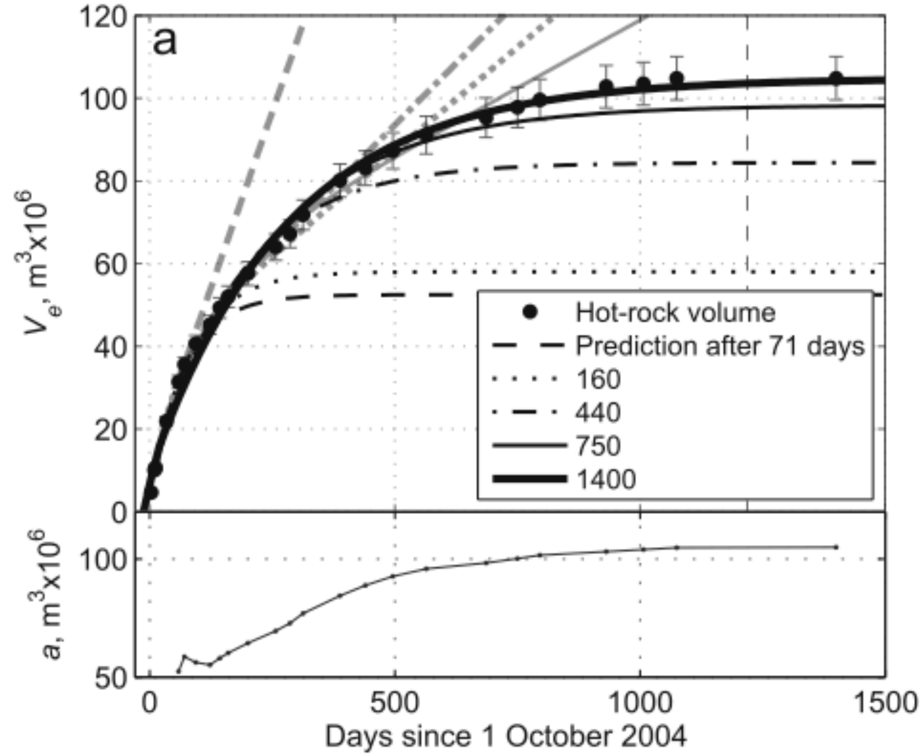
Can We Forecast the Size and Duration of an Eruption?

Monte Carlo Forecasting



- Forecast based on knowledge of the system and all existing data.
- Yields probabilistic forecast including uncertainties in the underlying parameters (not “epistemic” uncertainty).
- Uncertainty increases with time.

"Real Time" Forecasting at Mount St Helens



$$\frac{V_{ex}(t)}{V_0\bar{\beta}(p_0-\rho gL)} = \left(\frac{\alpha}{\Omega+\alpha}\right)\left[\left(\frac{\alpha}{\Omega+\alpha}\right)\left(1-e^{-t/t_c}\right)+\frac{t}{t_c}\left(\frac{\Omega}{\Omega+\alpha}\right)\right]$$

Mastin et al, GRL 2009

Conclusions

- Combining geodetic data and physics-based models constrains more properties of magmatic systems than geodetic data alone.
- It is possible to include other data types (gas emission, gravity, ….) to better constrain systems.
- *May* be possible to use in forecasting mode, but open question.
- Model components need to be well understood.