

Intergalactic magnetic fields: constraints from gamma-ray observations

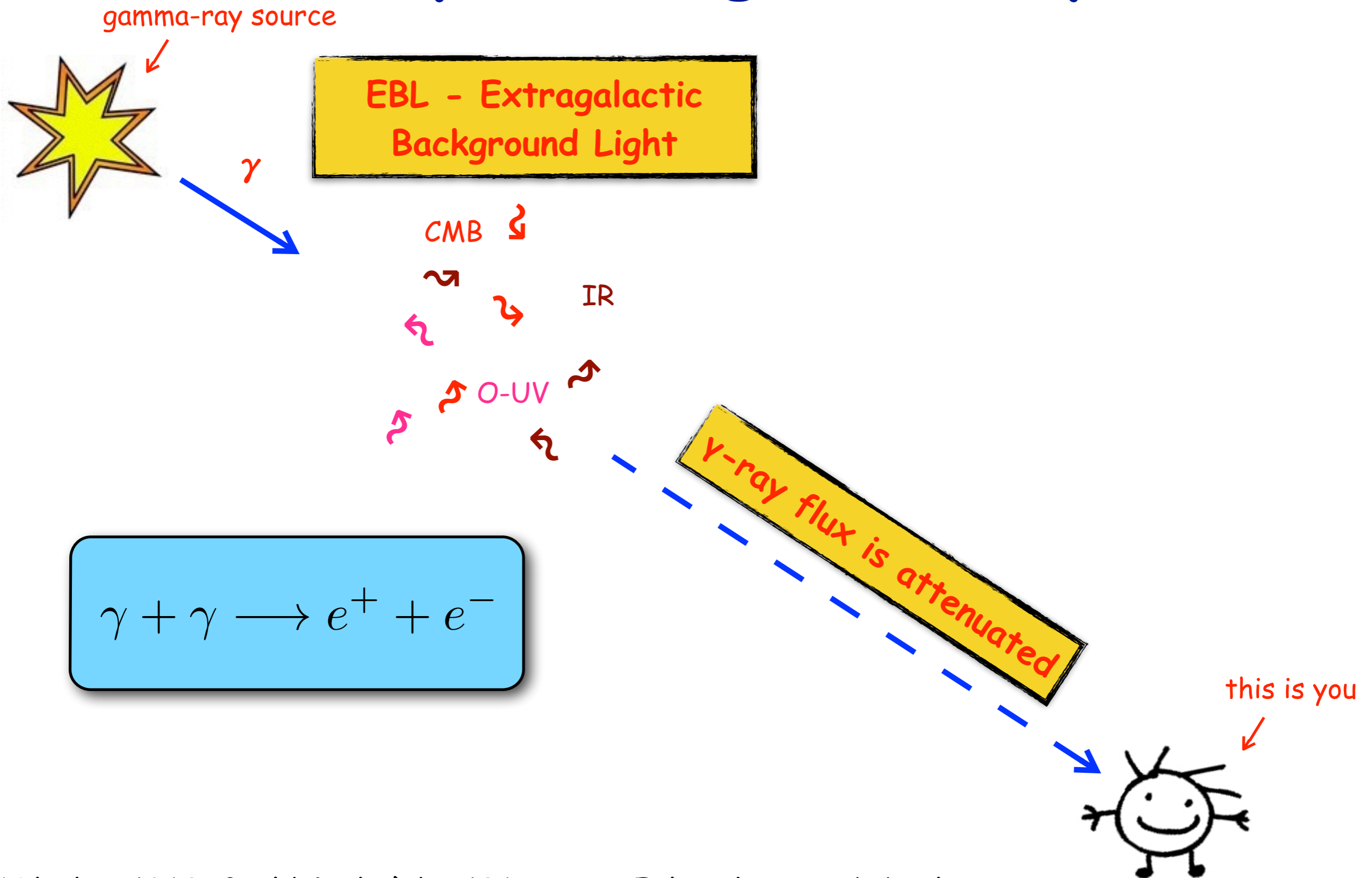


Stefano Gabici
APC, Paris

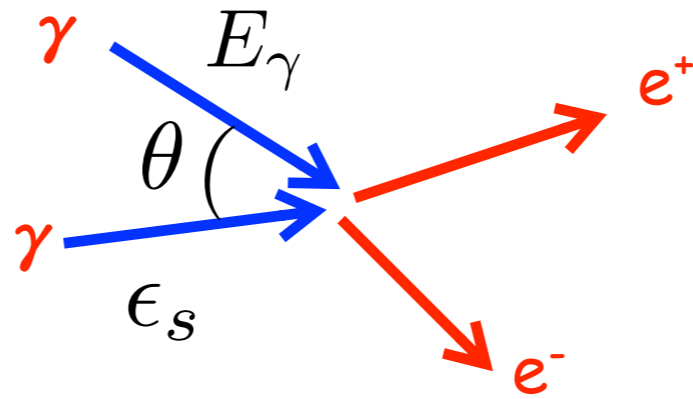
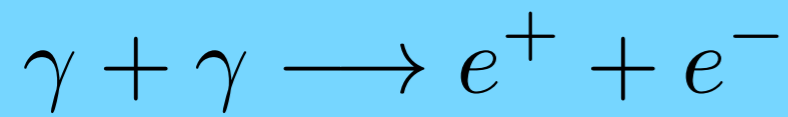


www.cnrs.fr

Absorption of gamma-rays



Photon-photon pair production

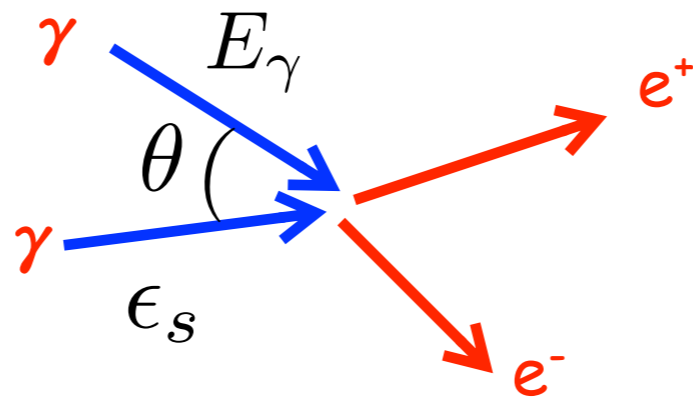


$$s_0 = \frac{E_\gamma \epsilon_s}{(m_e c^2)^2}$$

kinematic threshold $\rightarrow s_0 (1 - \cos \theta) > 2$

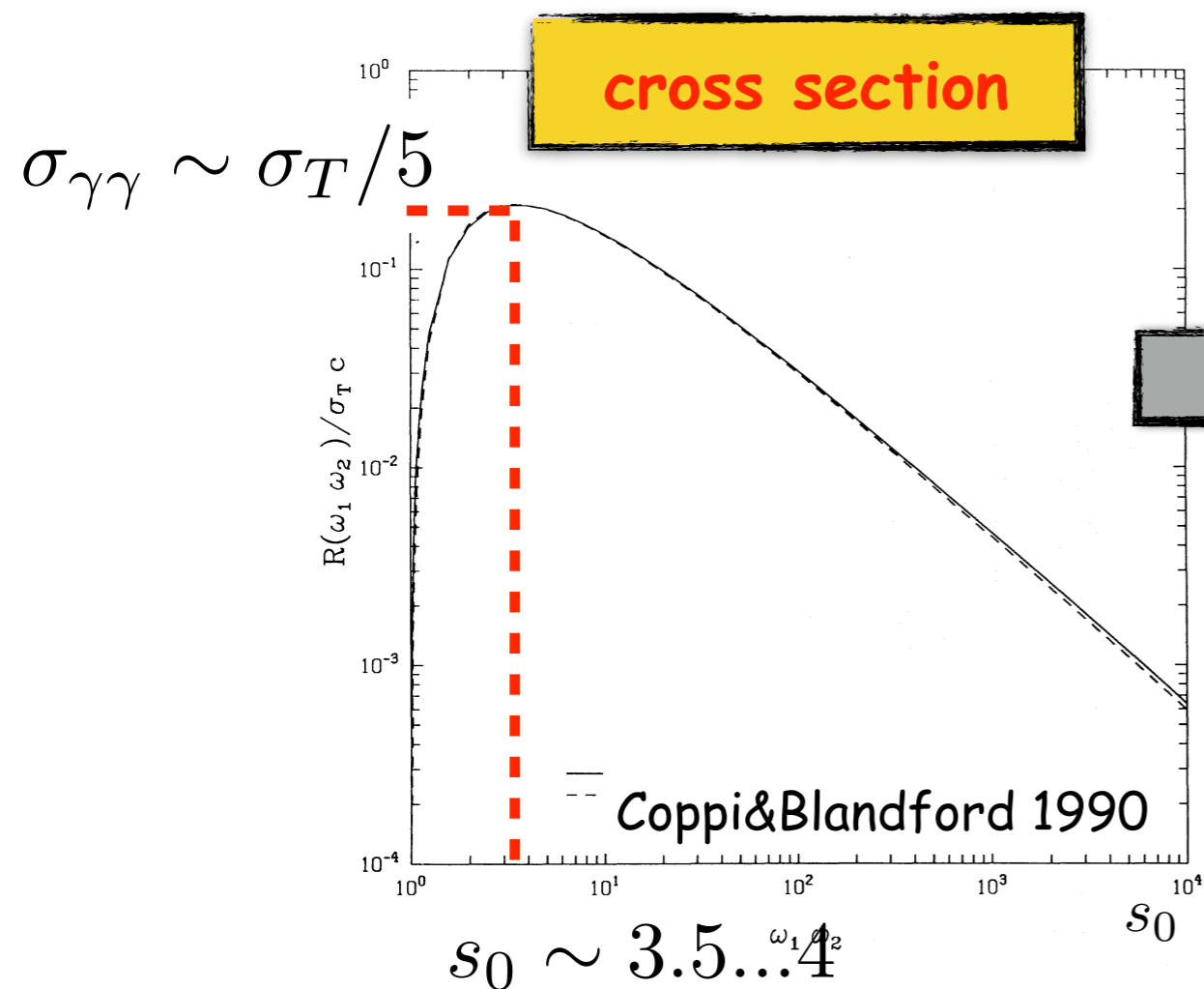
Photon-photon pair production

$$\gamma + \gamma \longrightarrow e^+ + e^-$$



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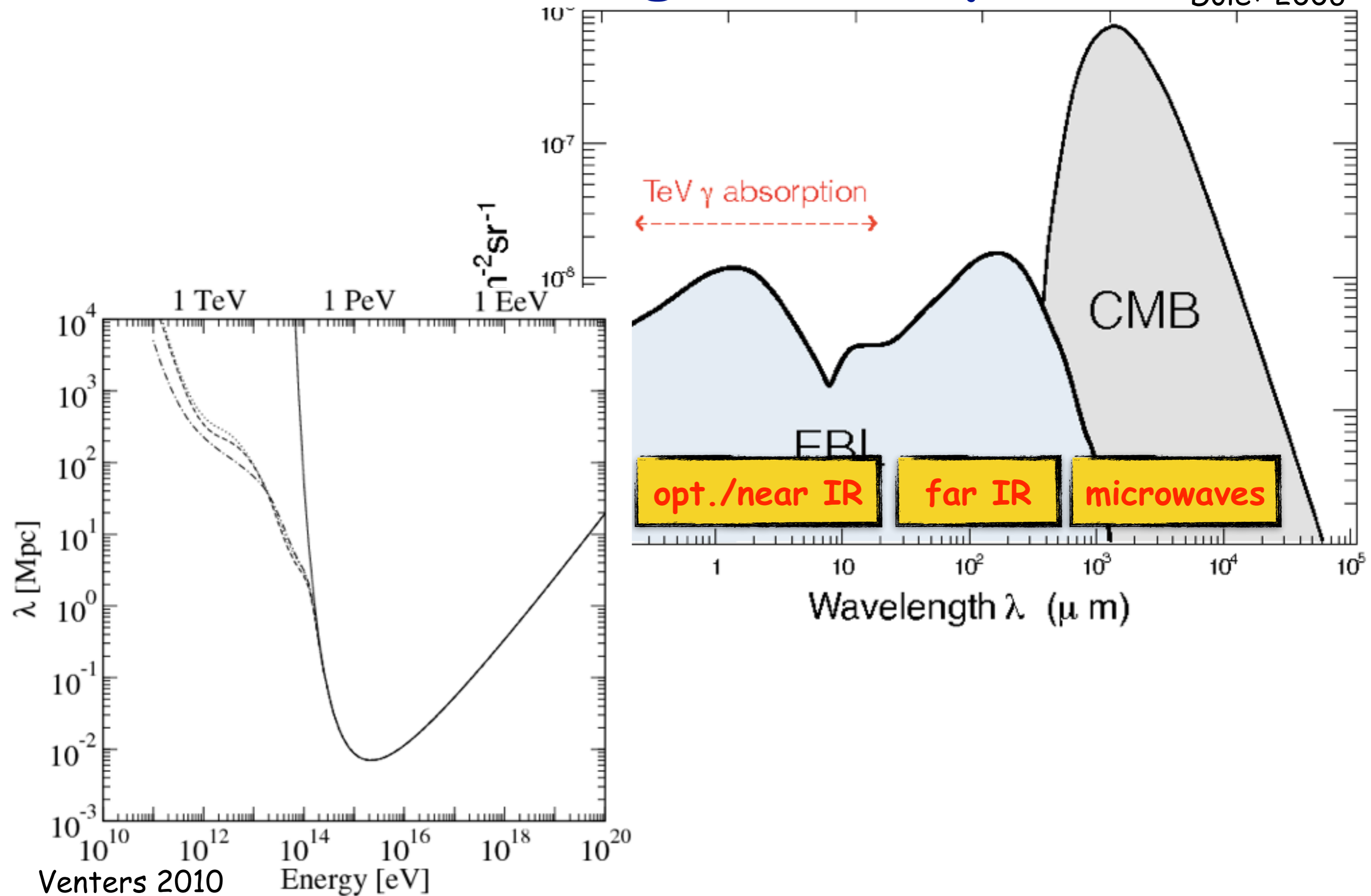


$$\lambda_s \sim 1.5 \left(\frac{E_\gamma}{1 \text{ TeV}} \right) \mu m$$

IR band \longleftrightarrow TeV domain

EBL and gamma rays

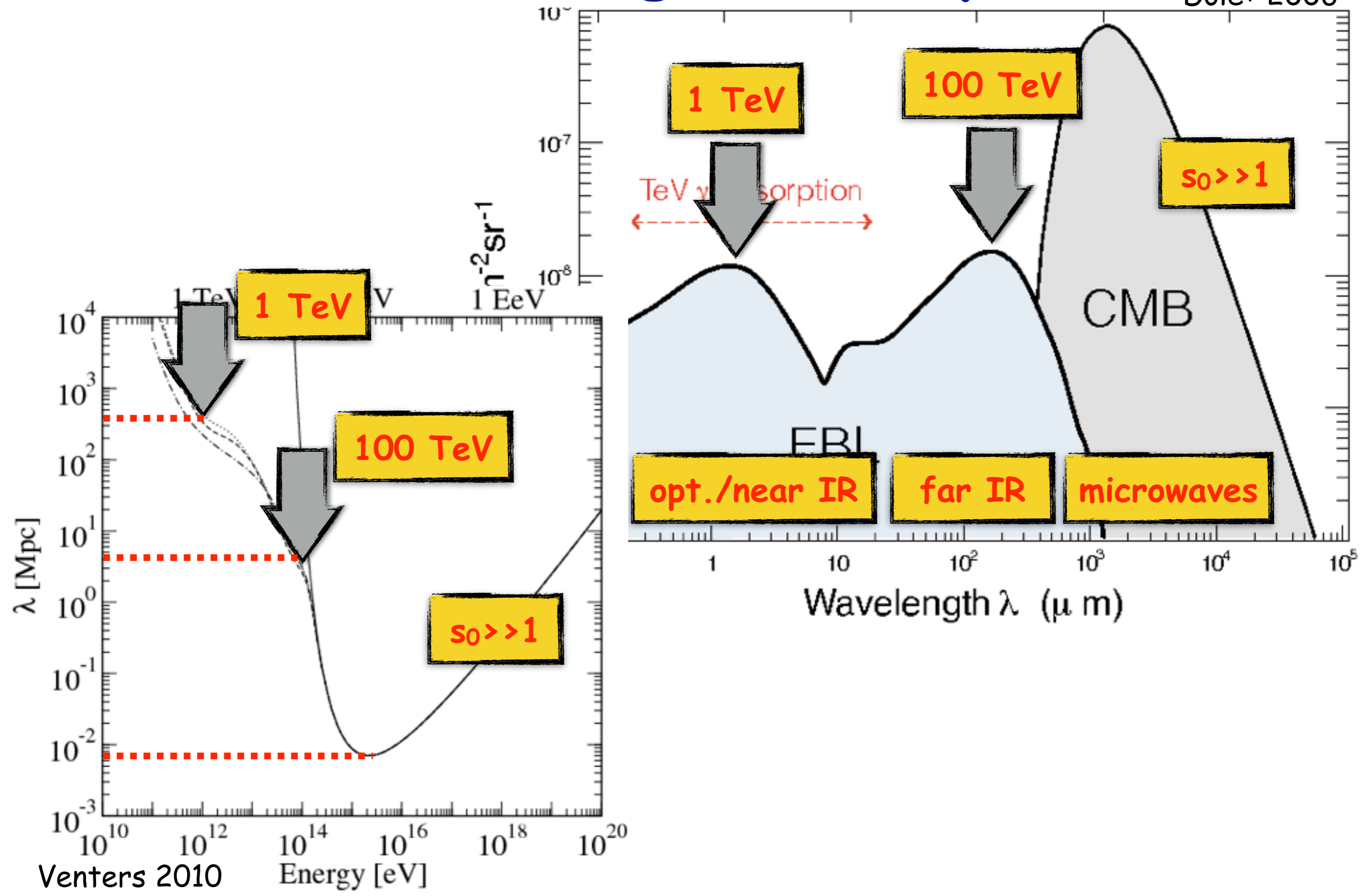
Dole+ 2006



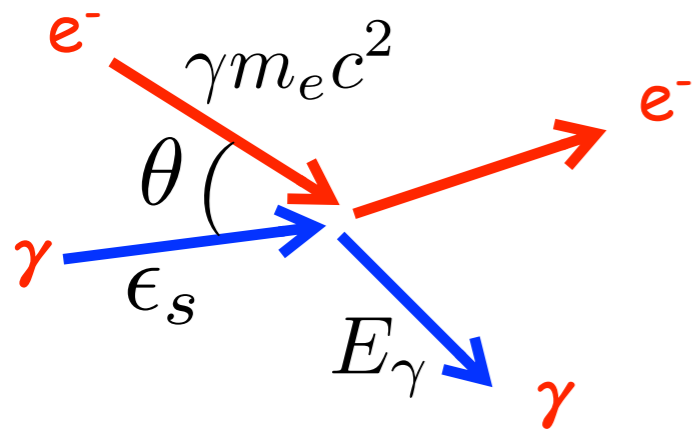
Venters 2010

EBL and gamma rays

Dole+ 2006

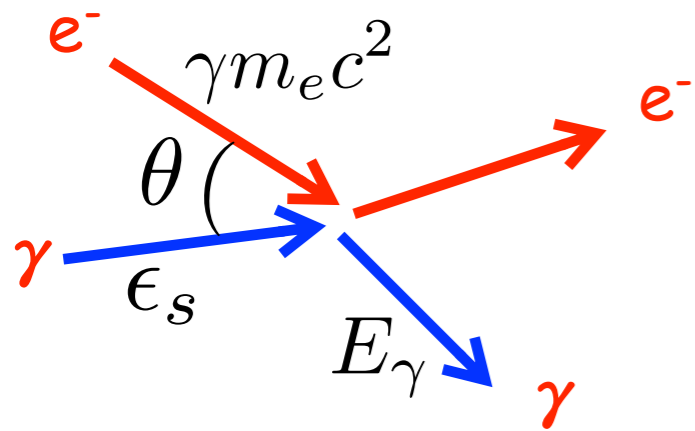


Inverse Compton scattering

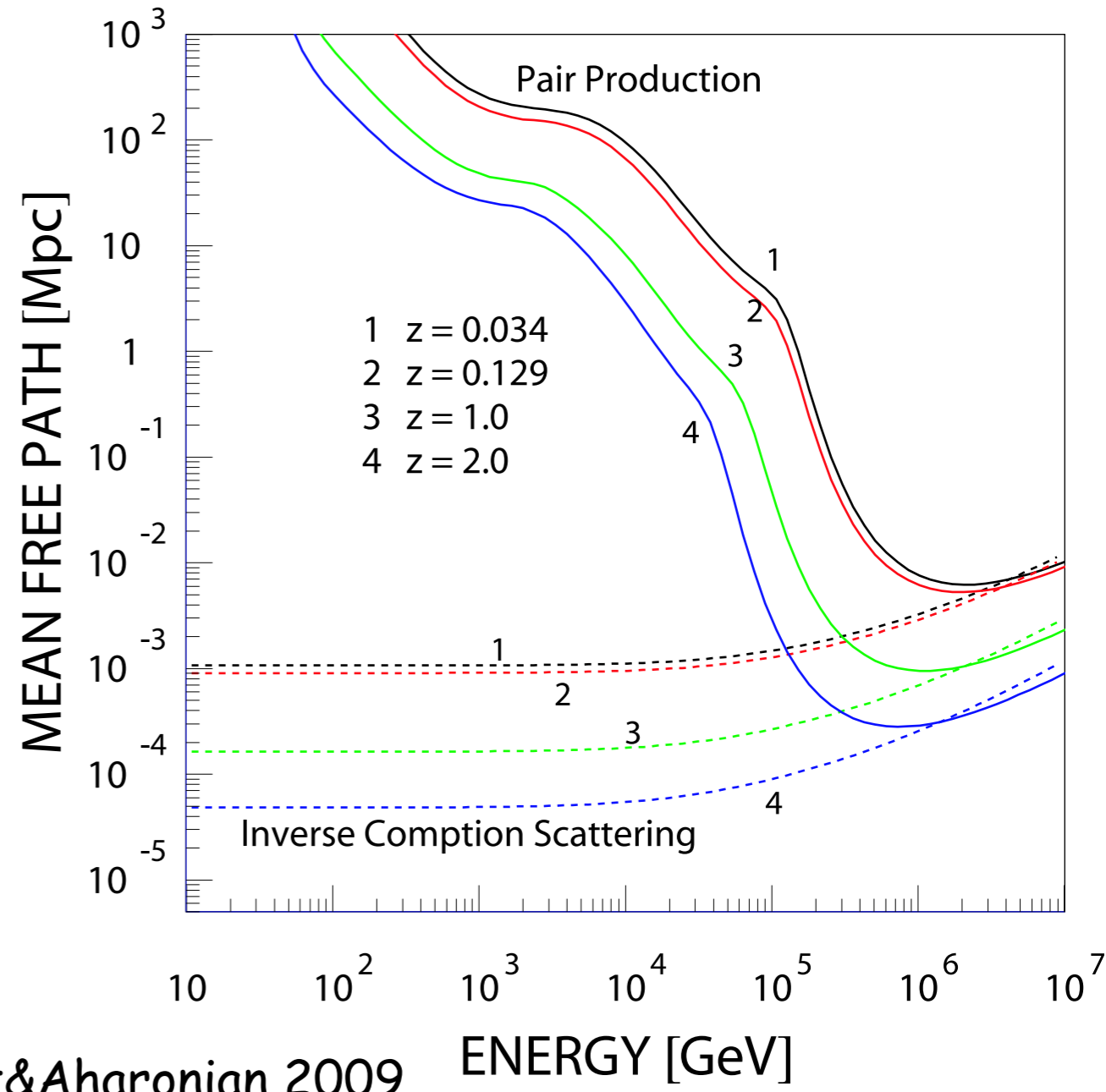
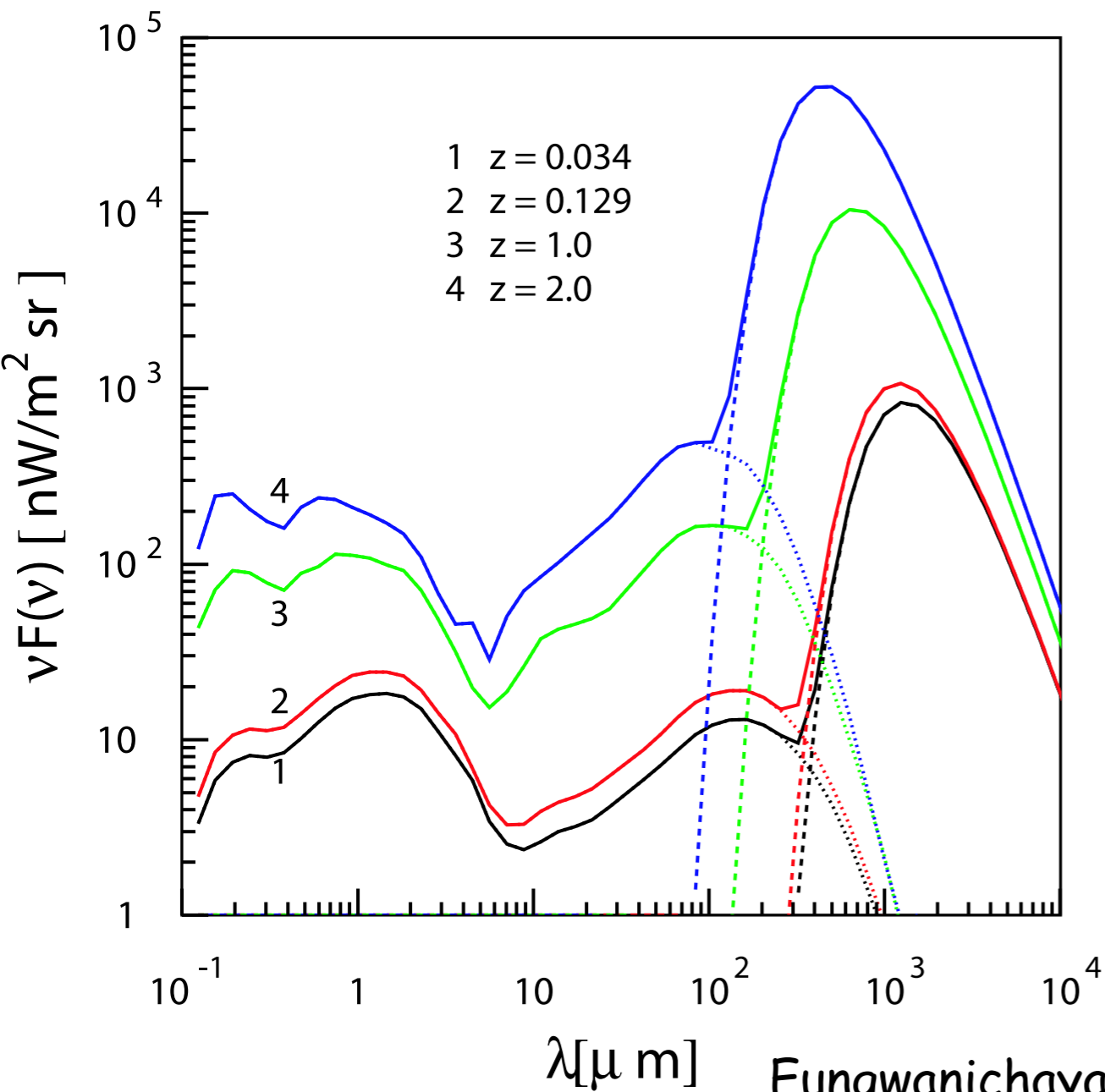


$$E_\gamma = \frac{4}{3} \gamma^2 \epsilon_s \longrightarrow E_e \sim 10 \left(\frac{E_\gamma}{0.4 \text{ TeV}} \right)^{1/2} \text{ TeV}$$

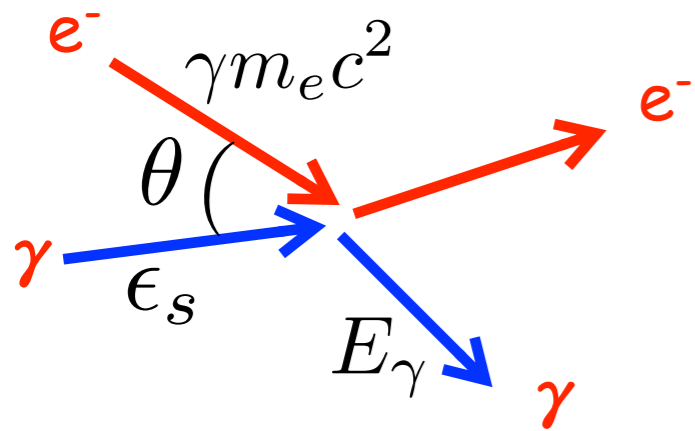
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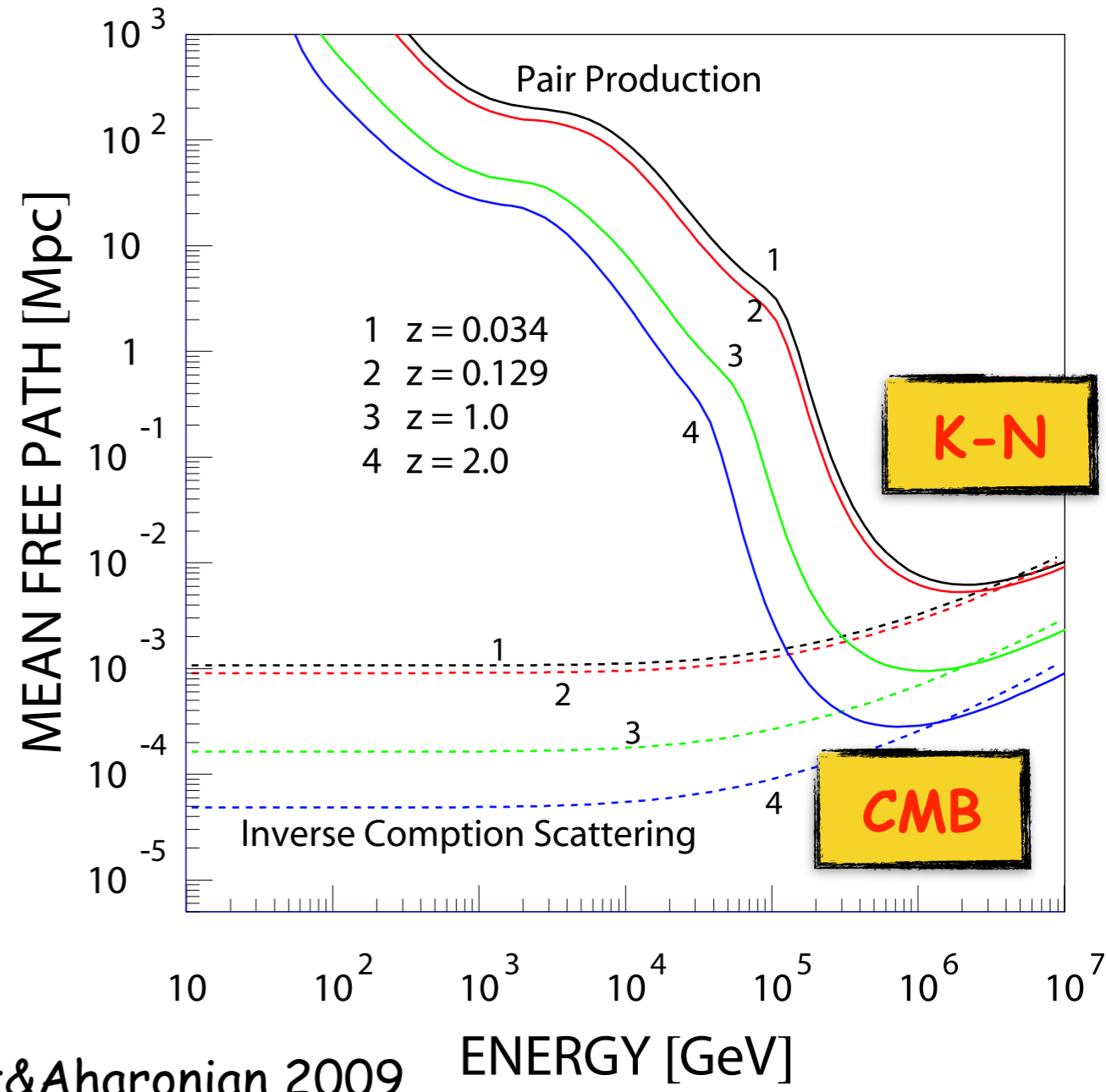
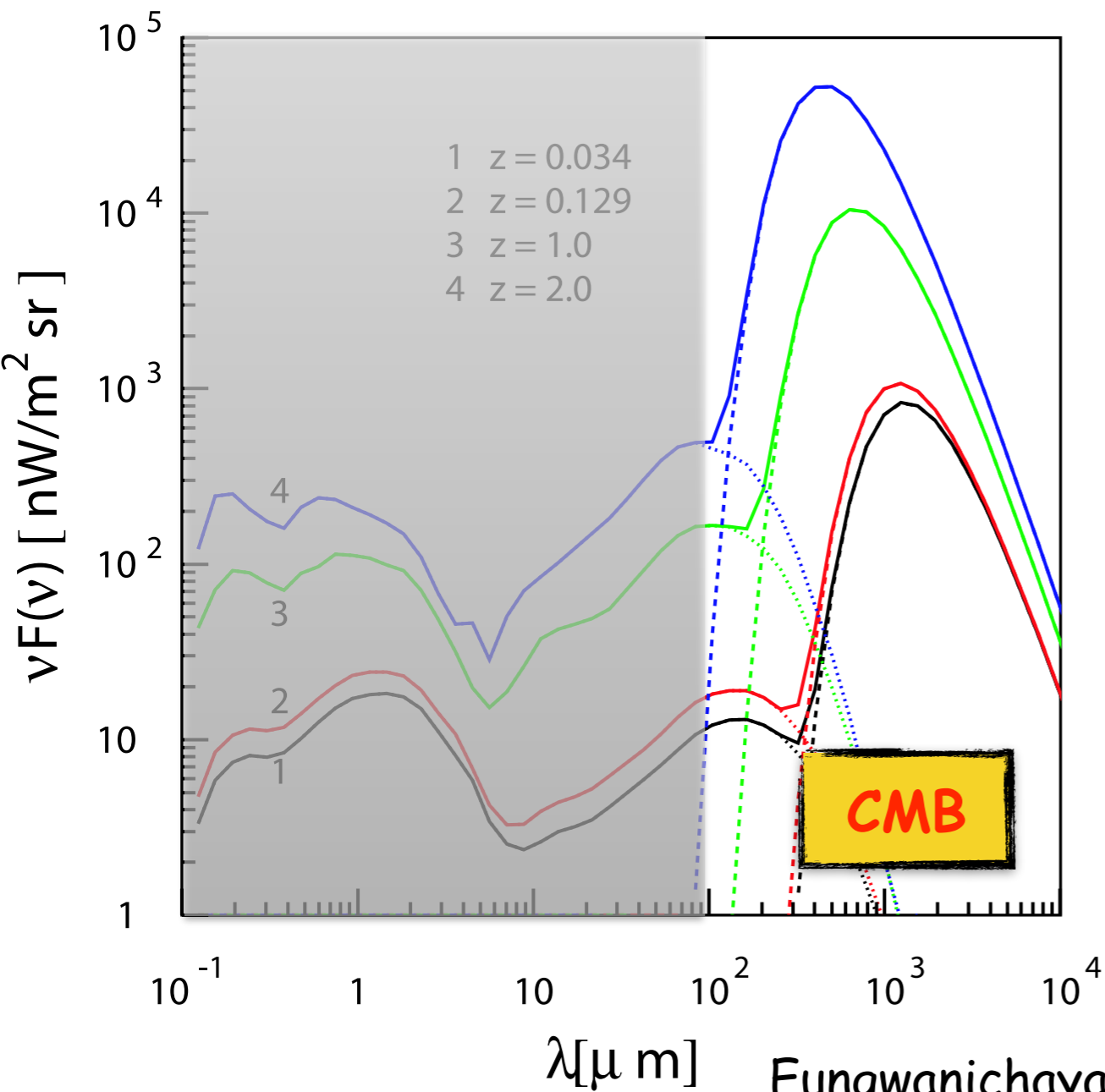
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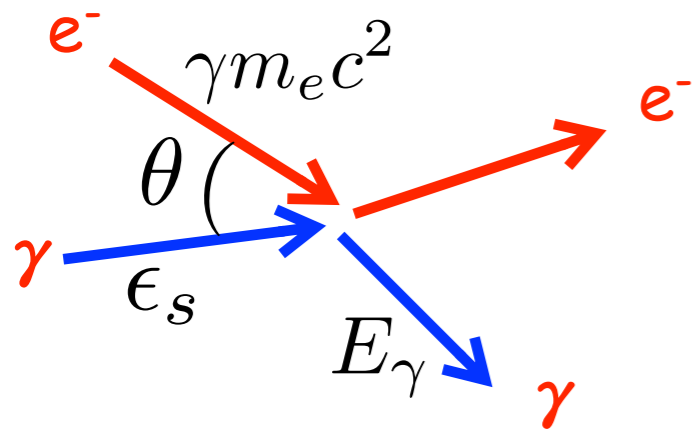
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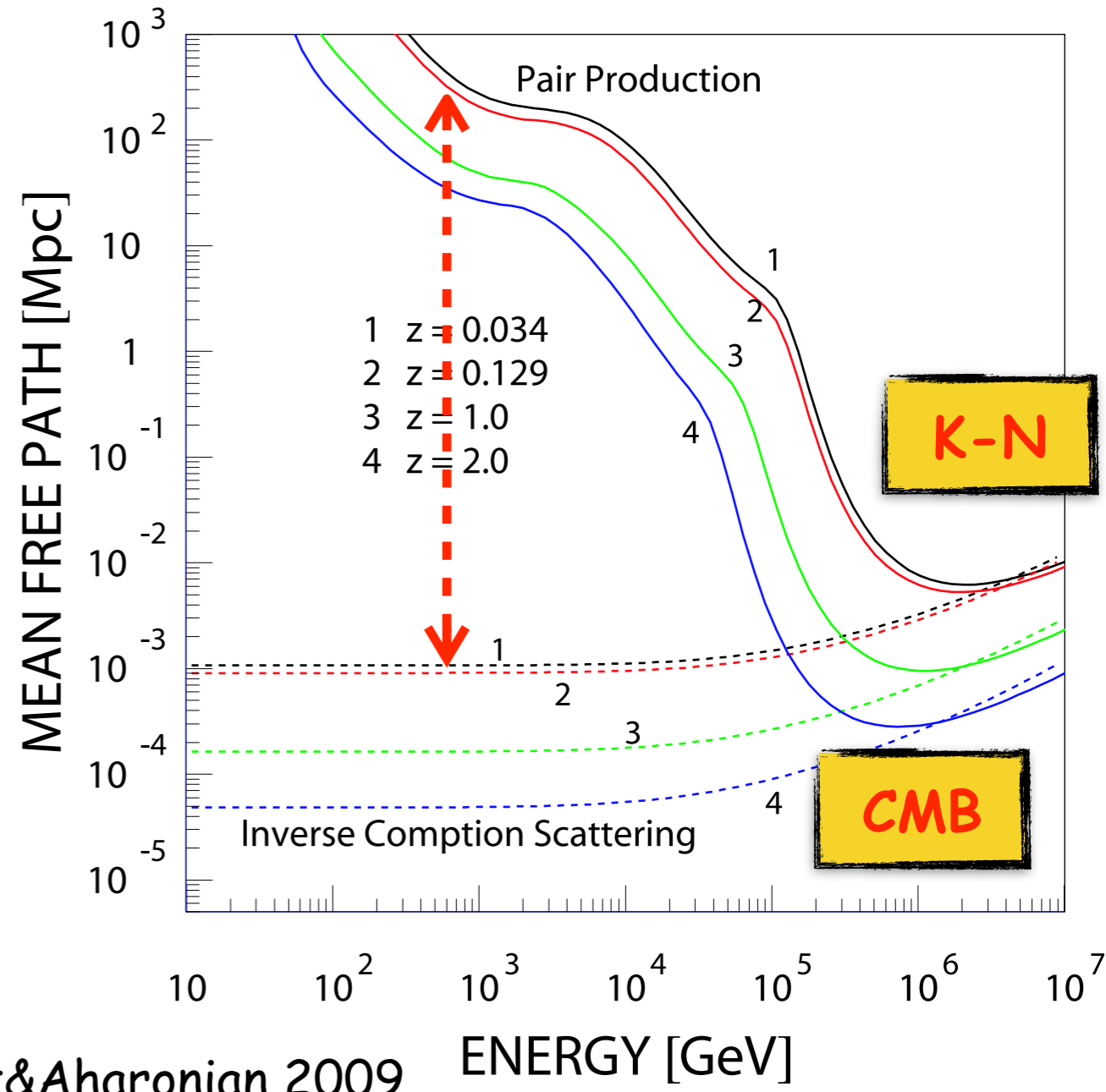
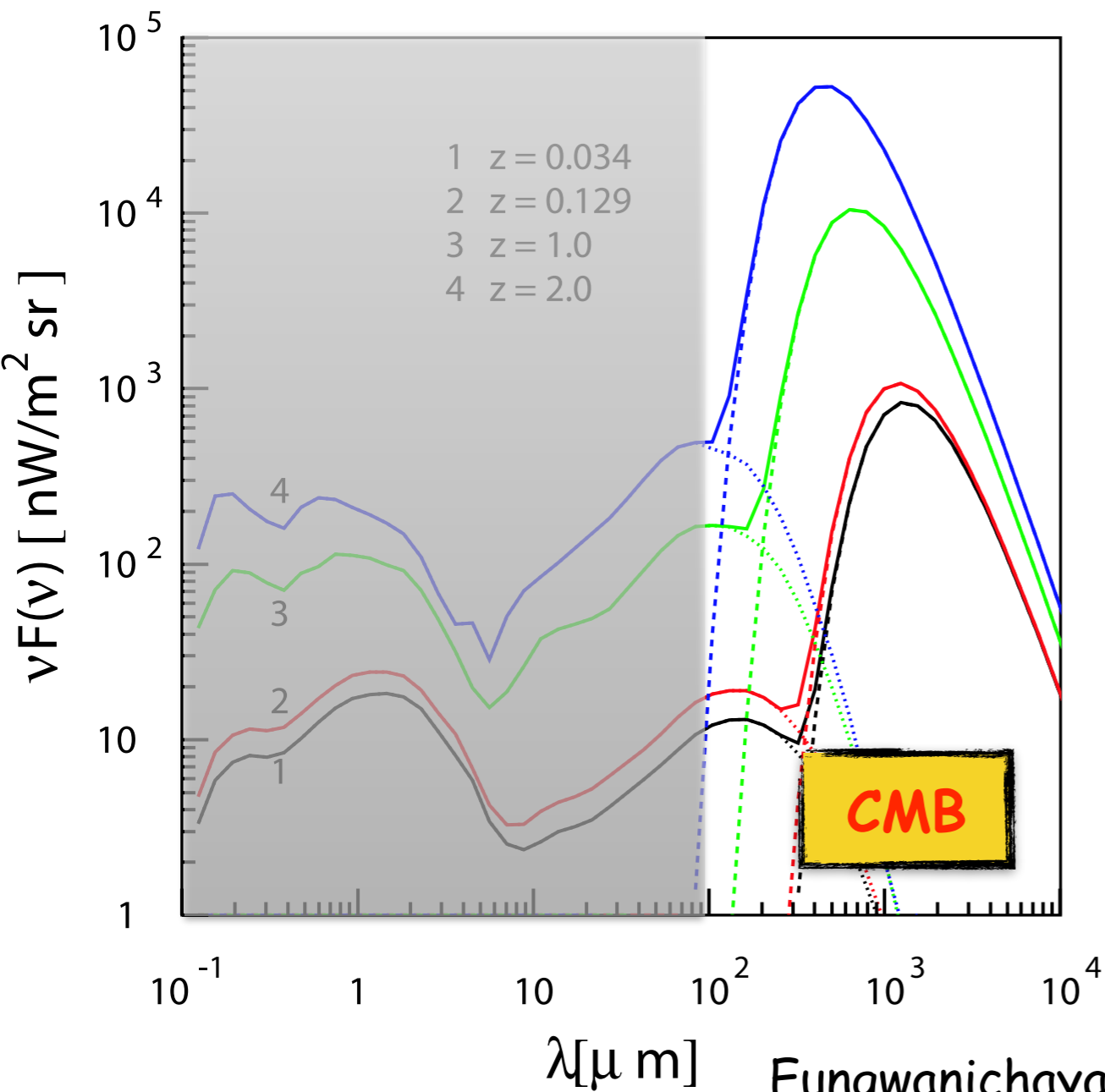
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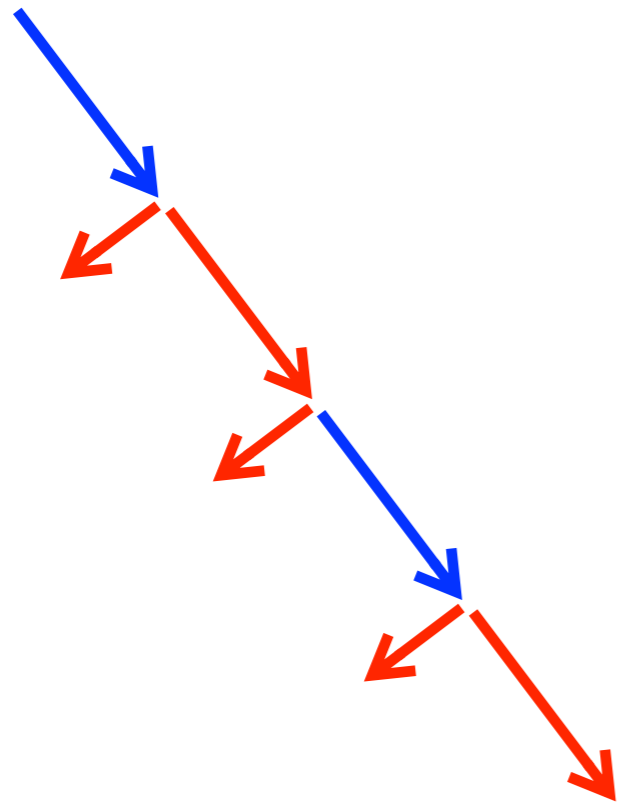
Electromagnetic cascade: development

$$s_0 = \frac{E_\gamma \epsilon_s}{(m_e c^2)^2}$$

Electromagnetic cascade: development

"Klein-Nishina"
 $s_0 \gg 1$

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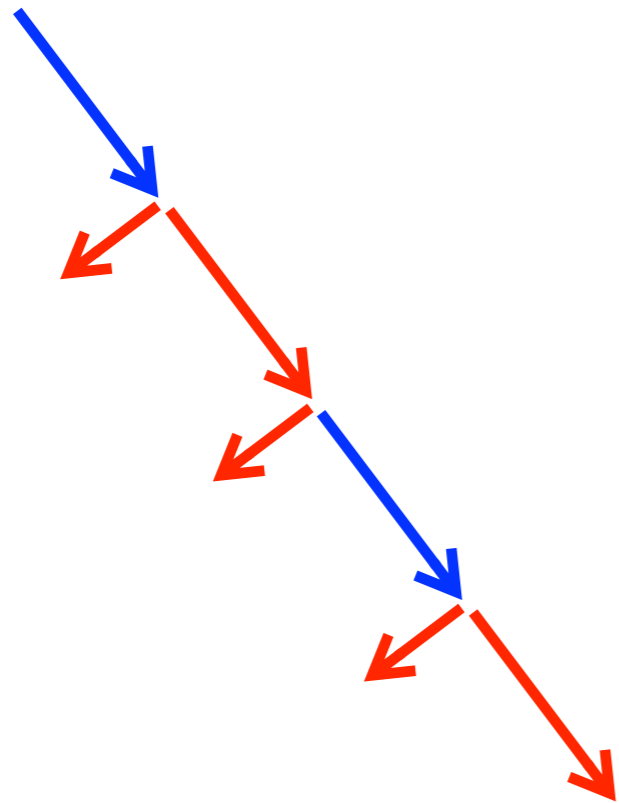
1 leading particle

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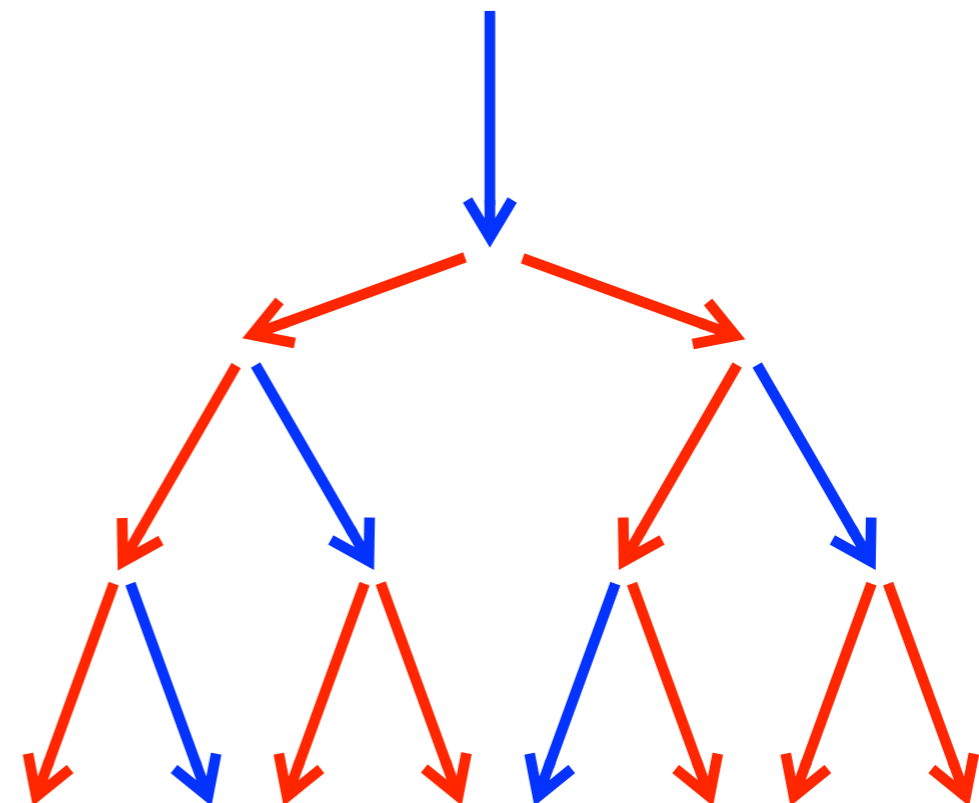
"Klein-Nishina"
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$$s_0 = \frac{E_\gamma \epsilon_s}{(m_e c^2)^2}$$

"Thomson"
 $s_0 \sim 1$

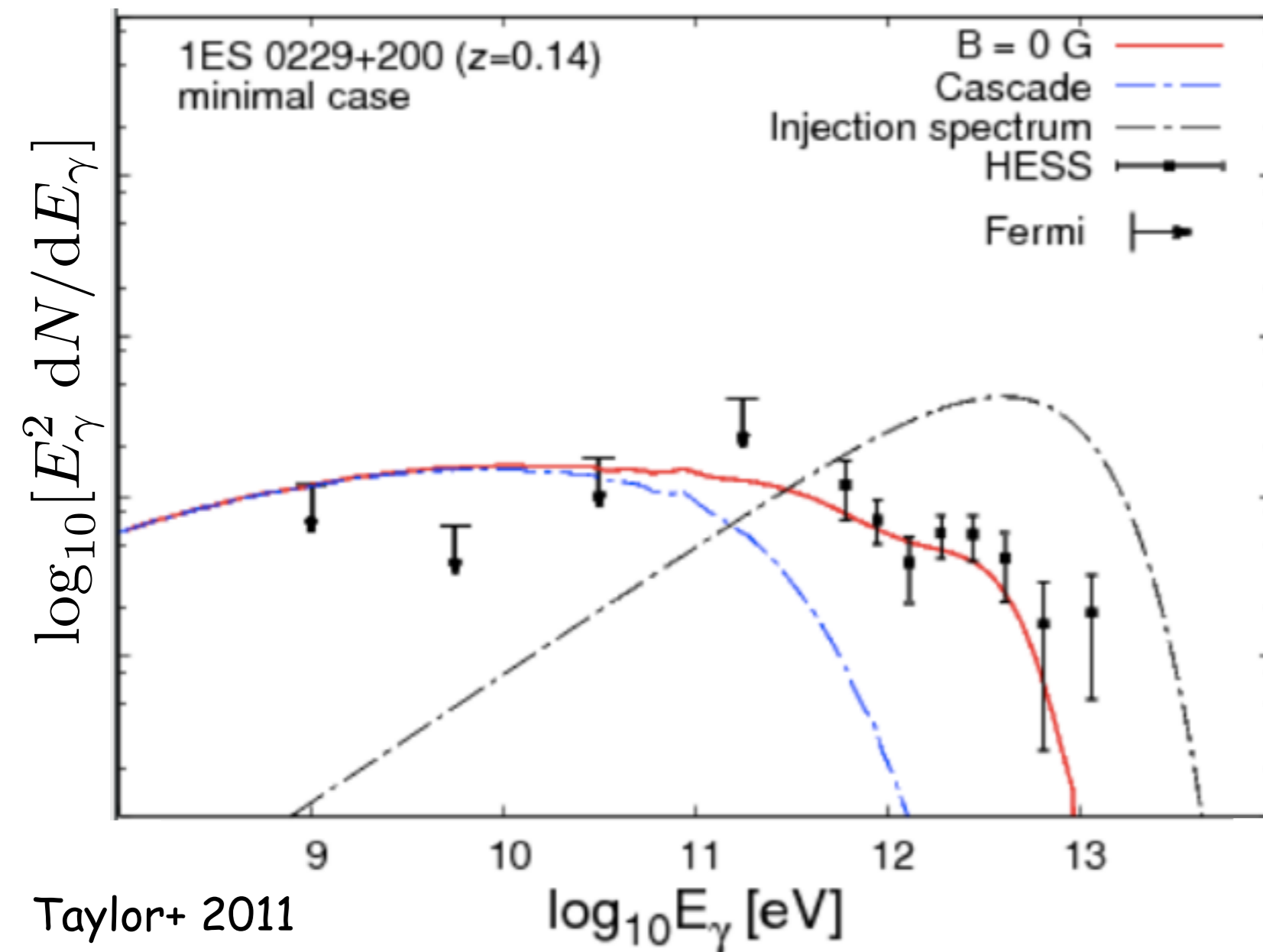


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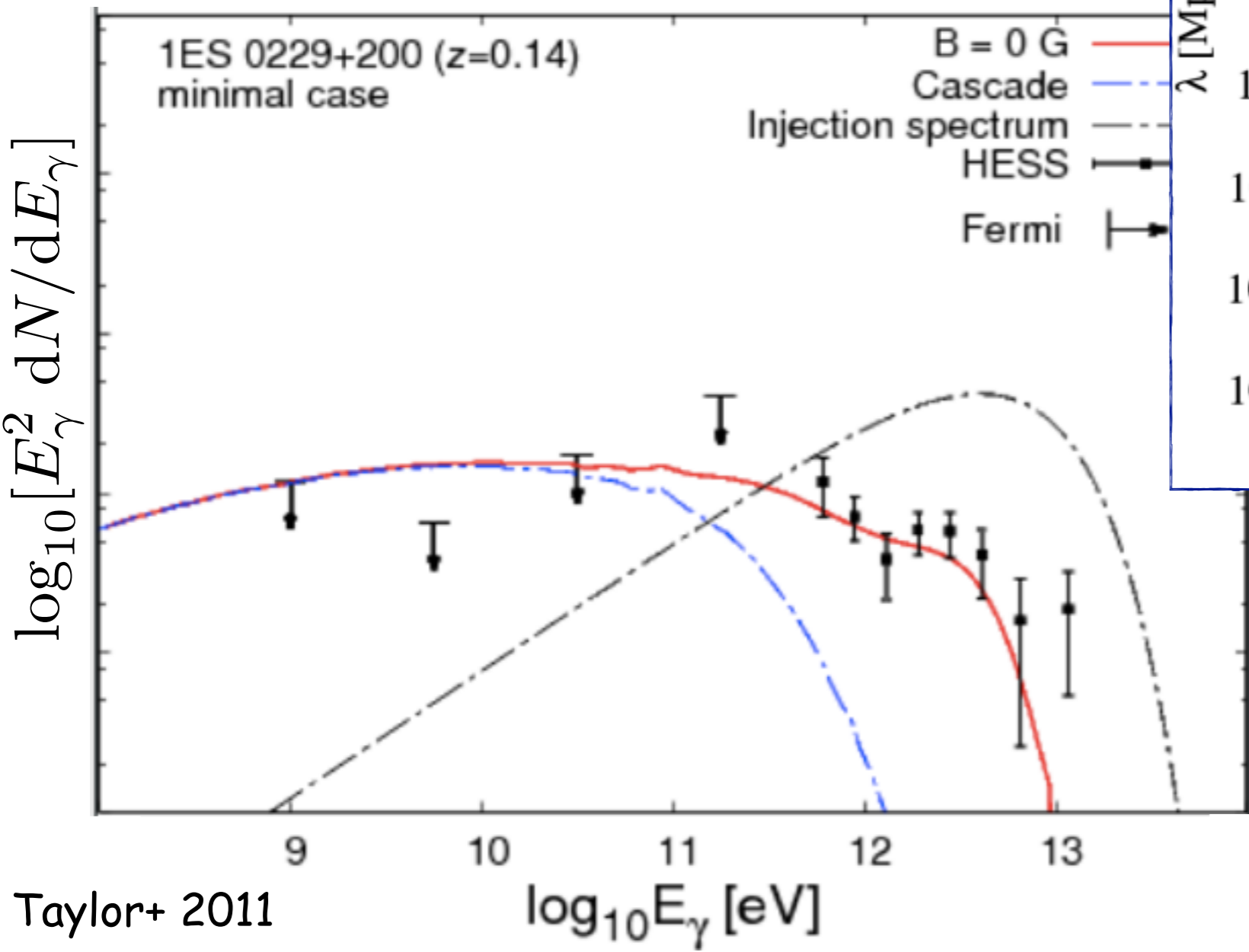
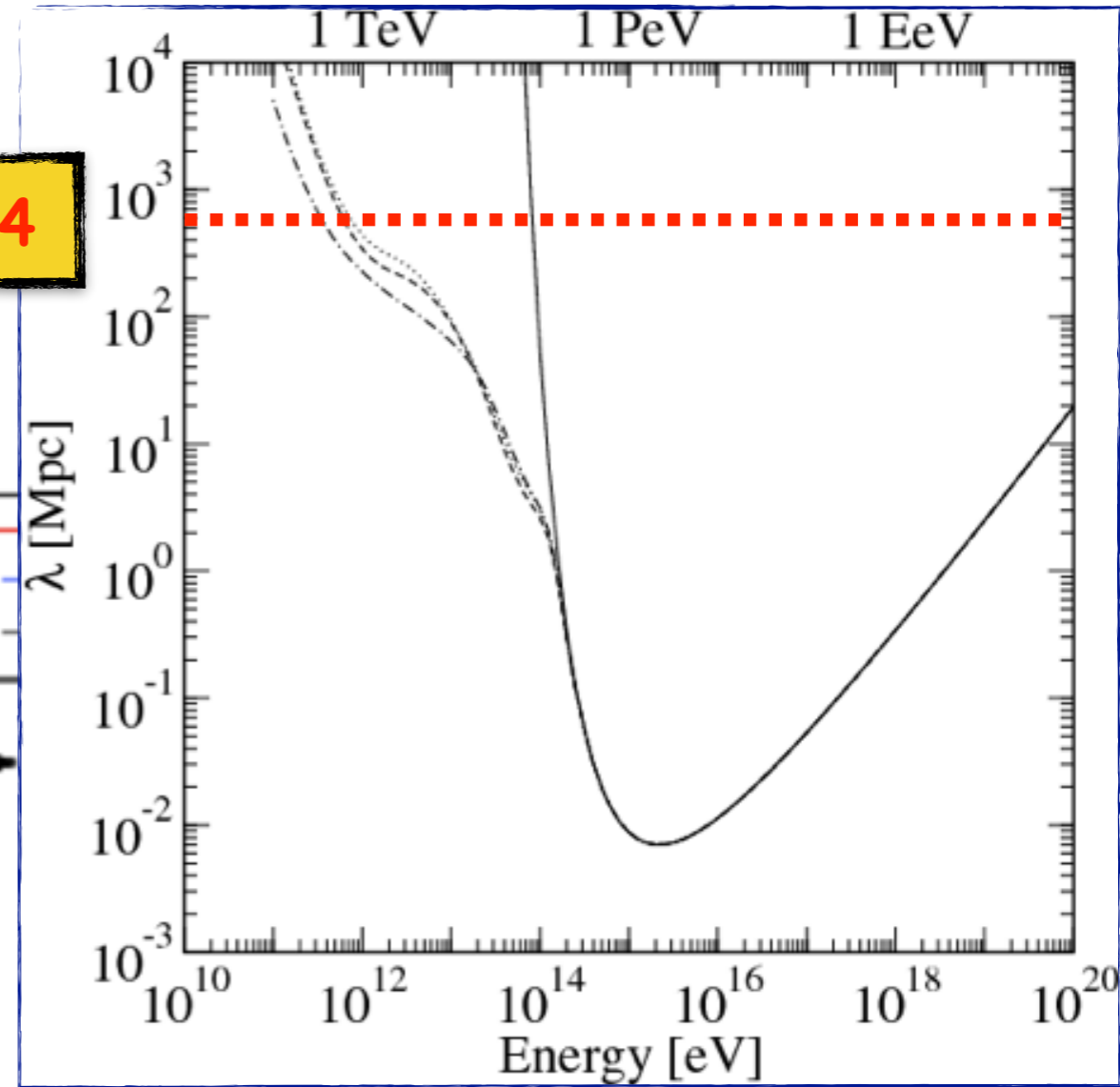
many particles

Electromagnetic cascade: spectrum

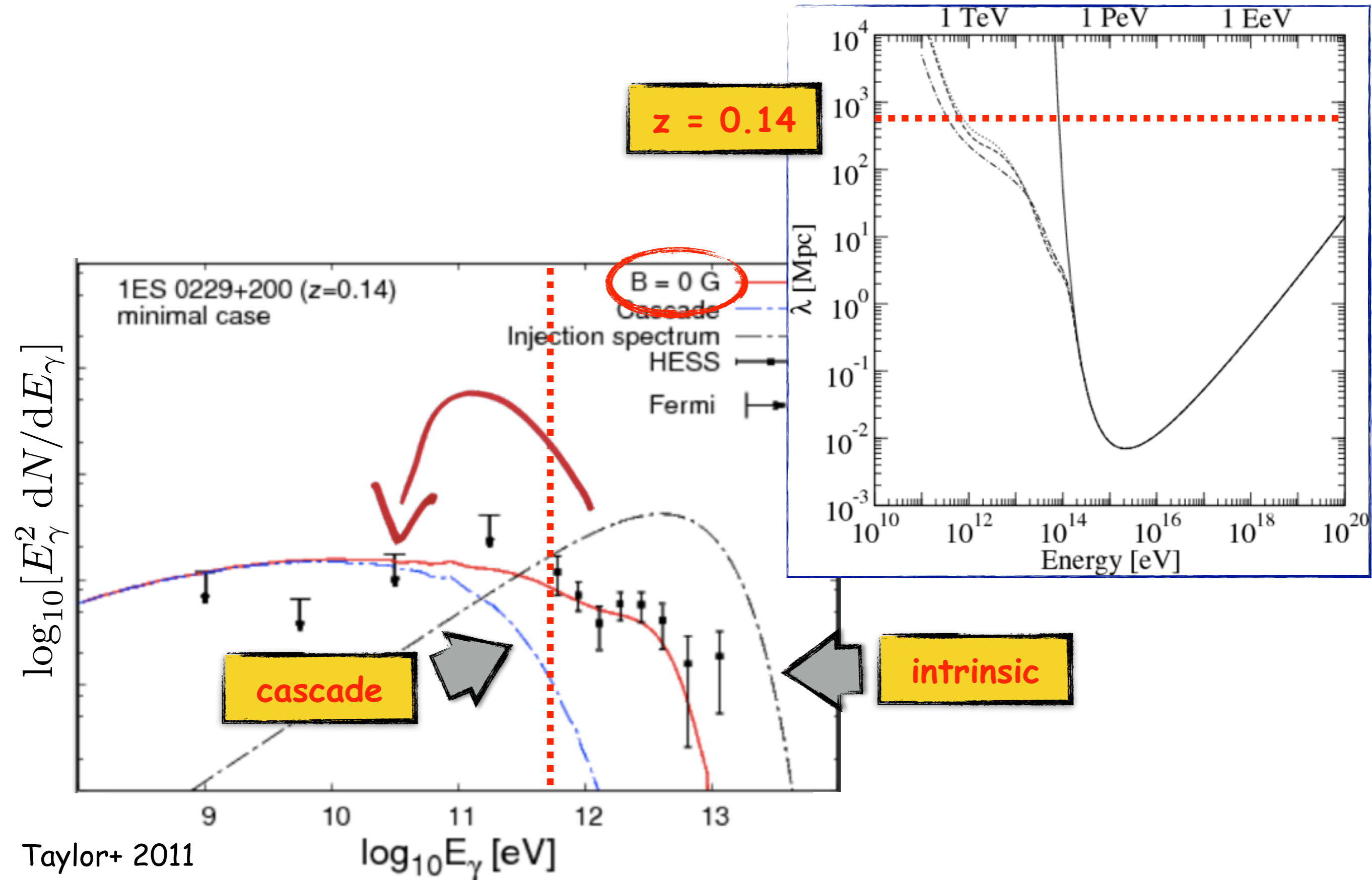


Electromagnetic cascade: spectrum

$z = 0.14$

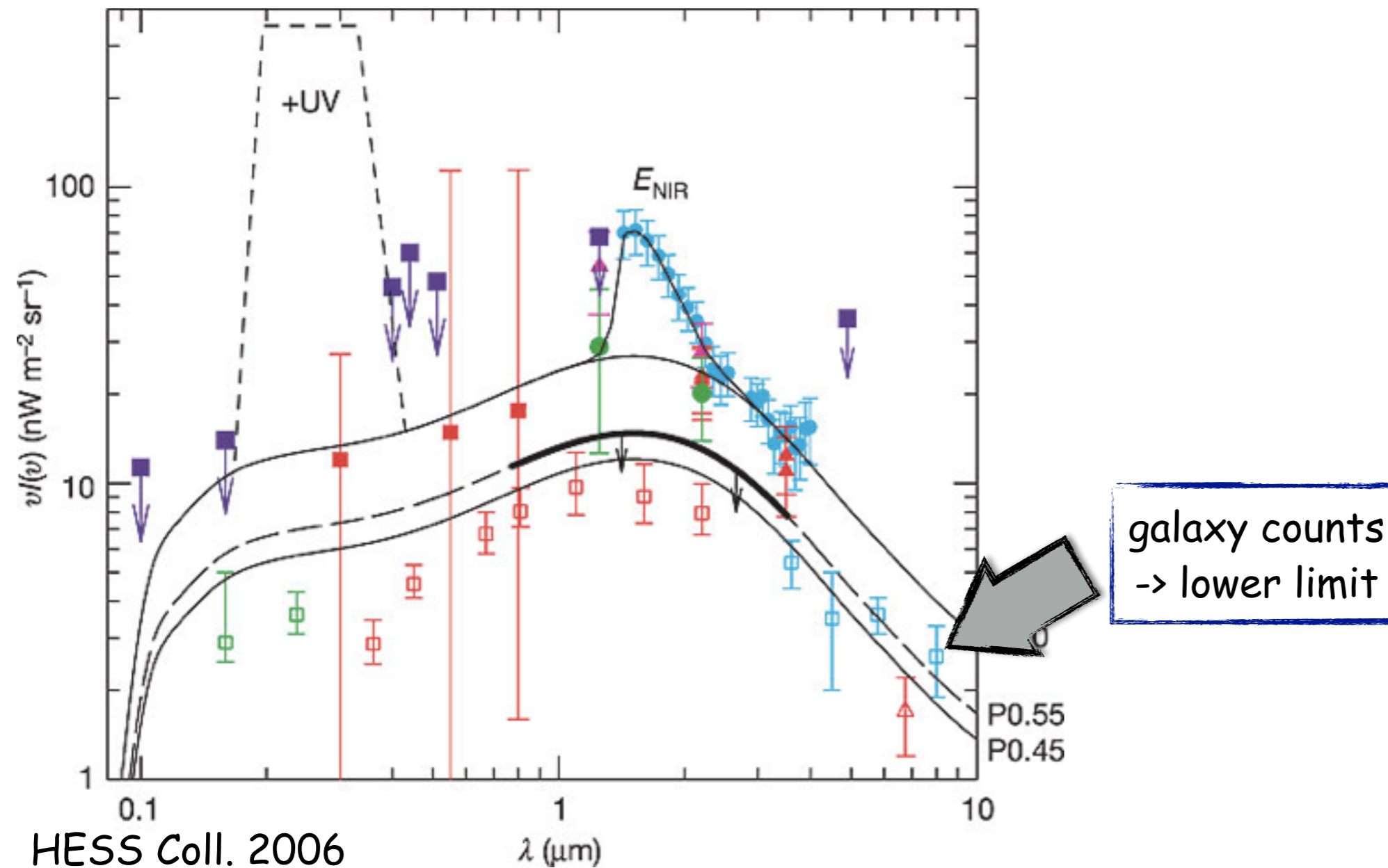


Electromagnetic cascade: spectrum



How well do we know the EBL?

for a review see
Costamante 2013

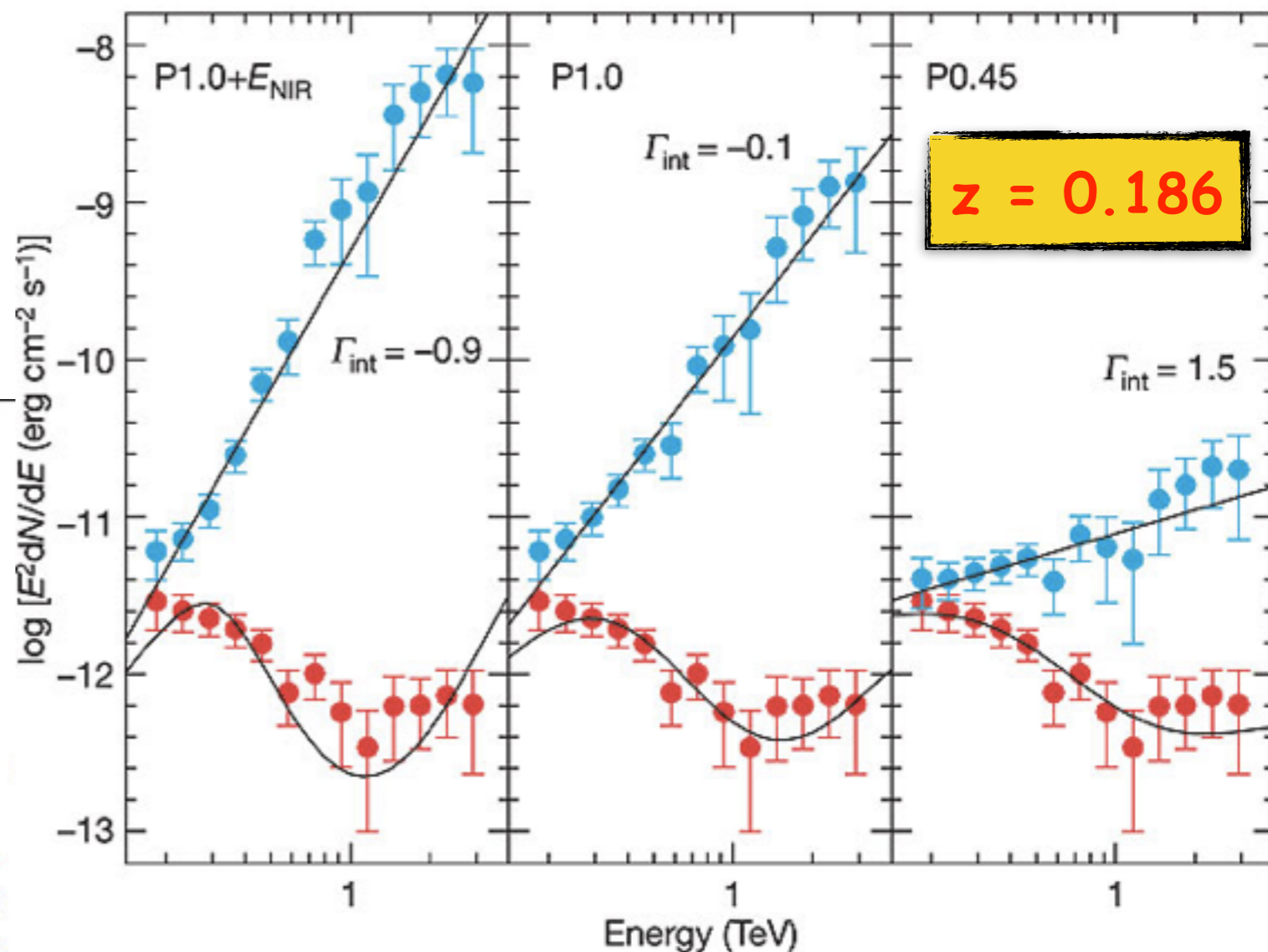
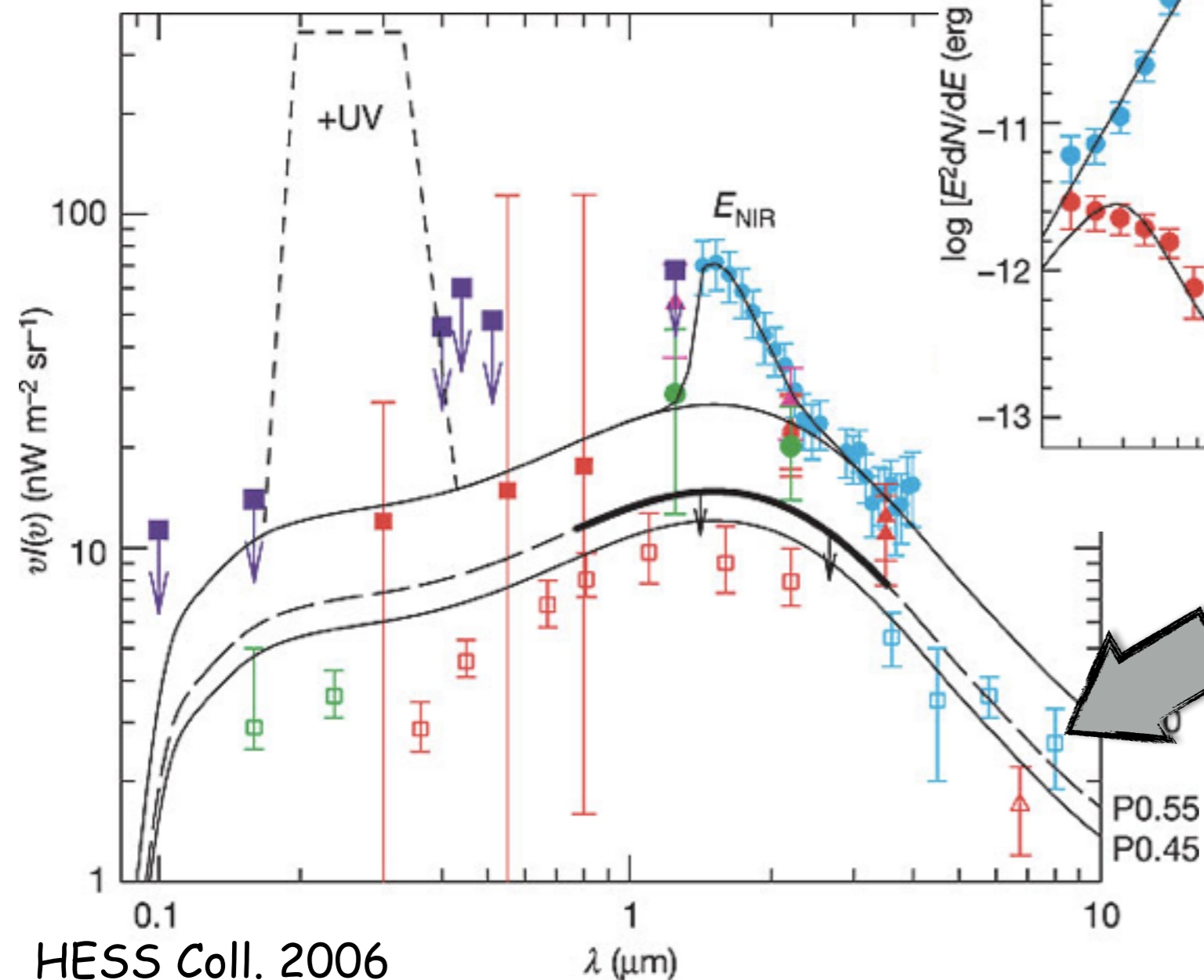


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for a review see
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1ES 1101-232

$z = 0.186$

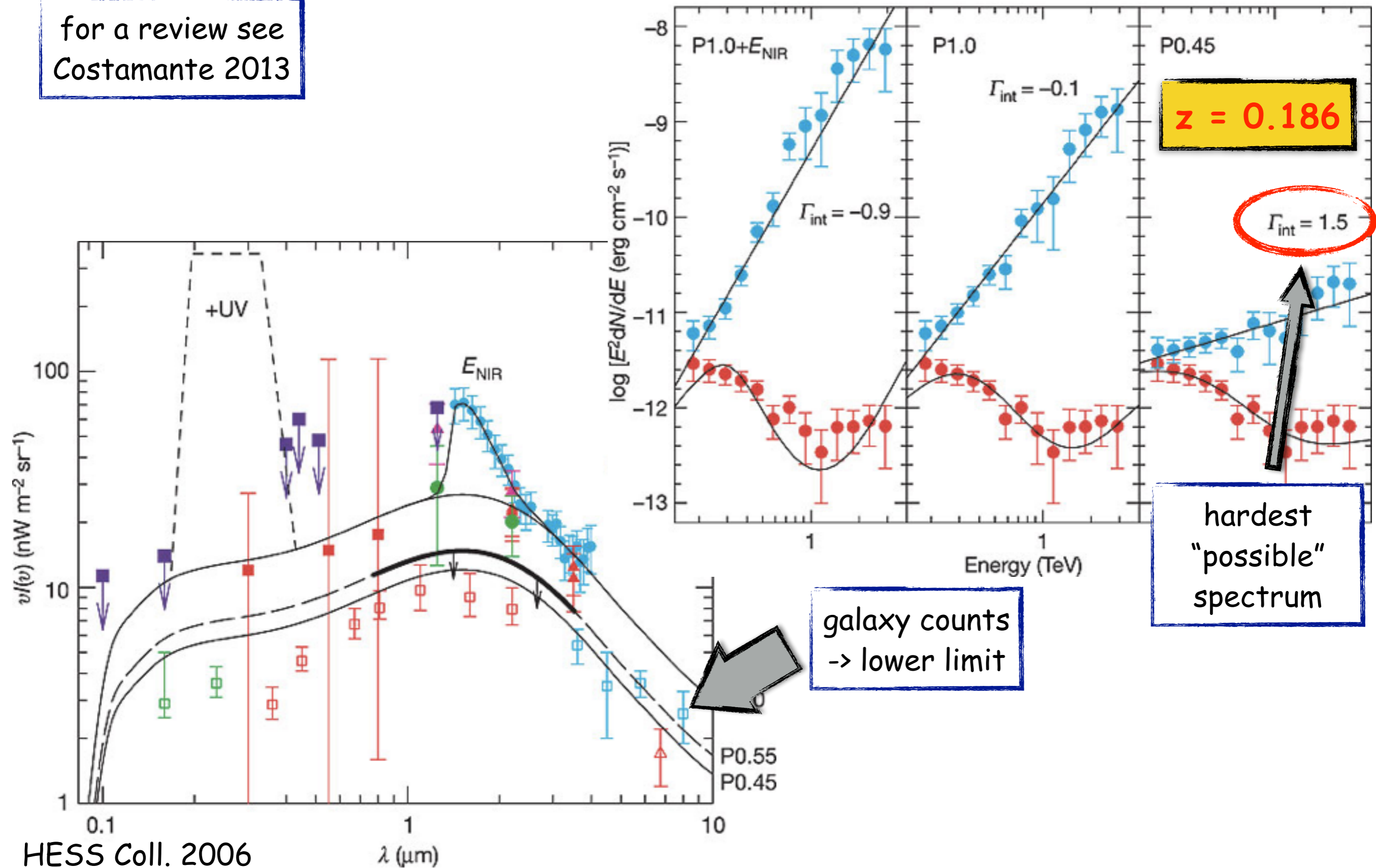


galaxy counts
-> lower limit

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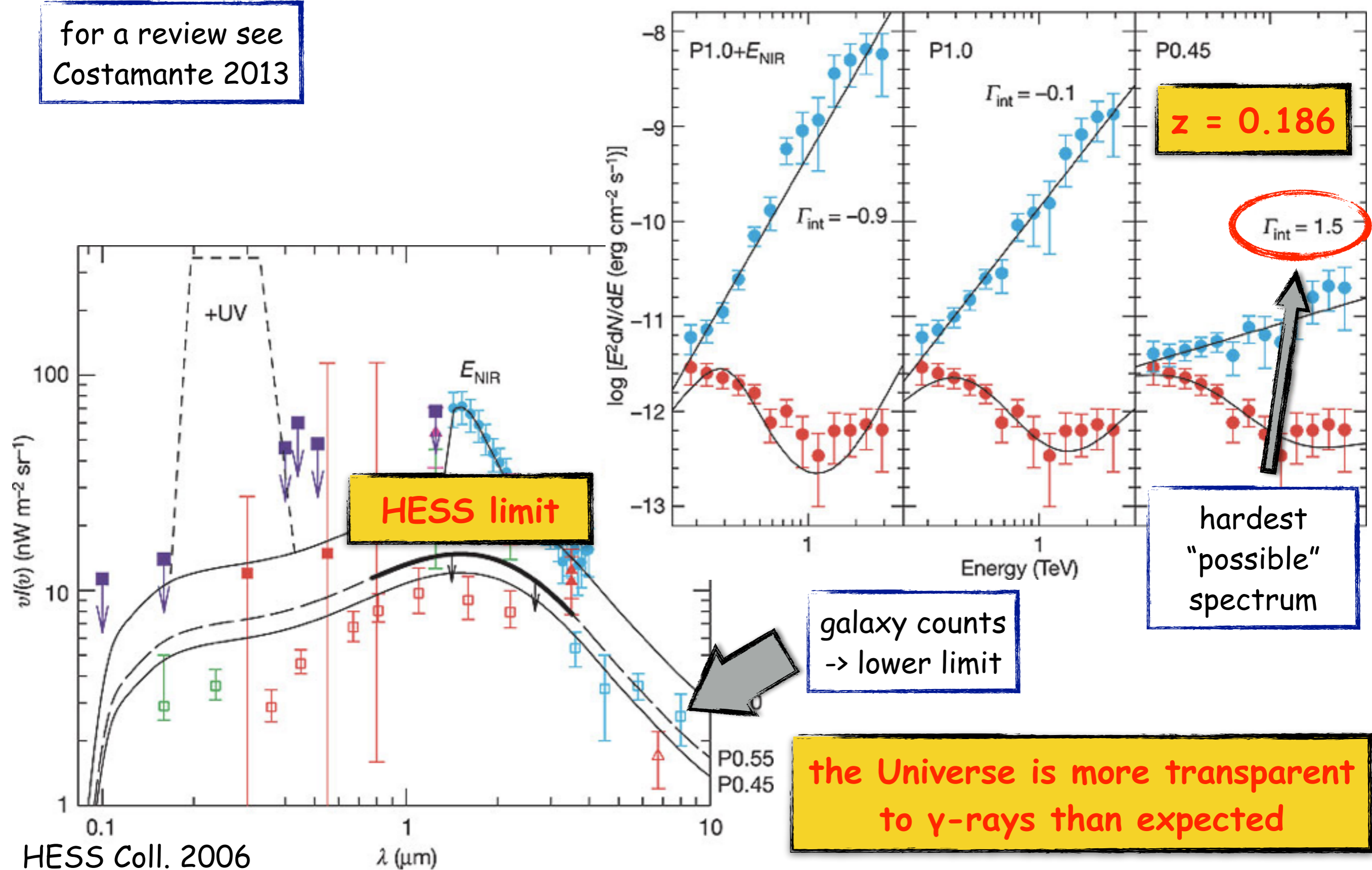
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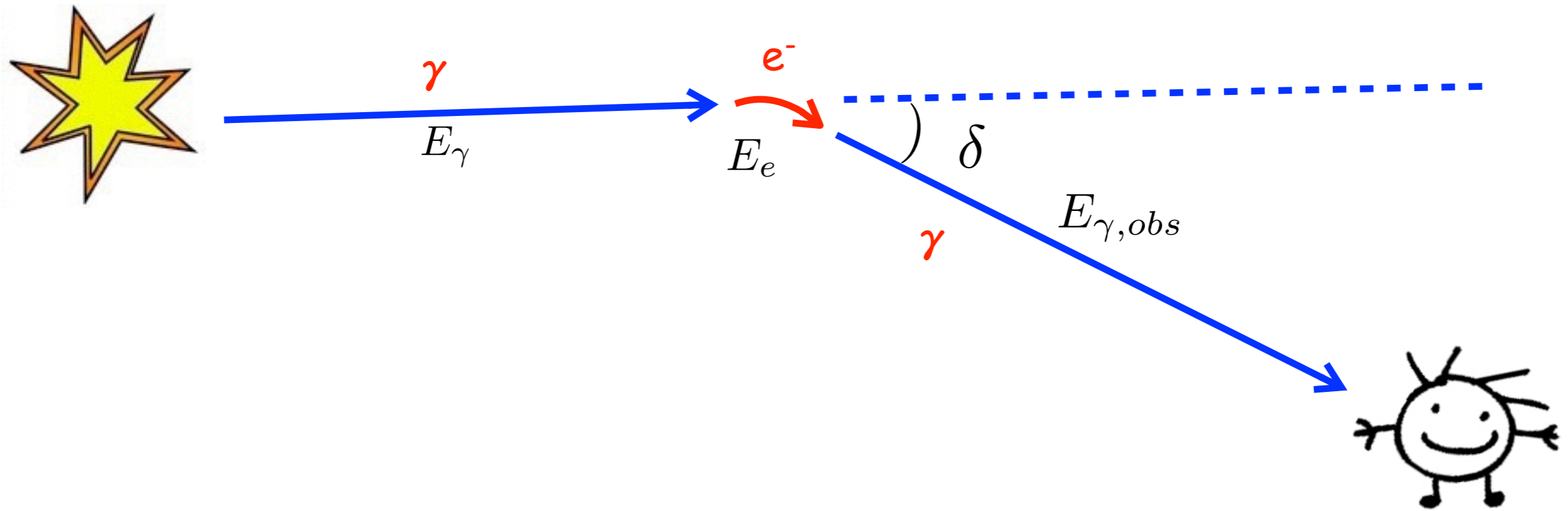
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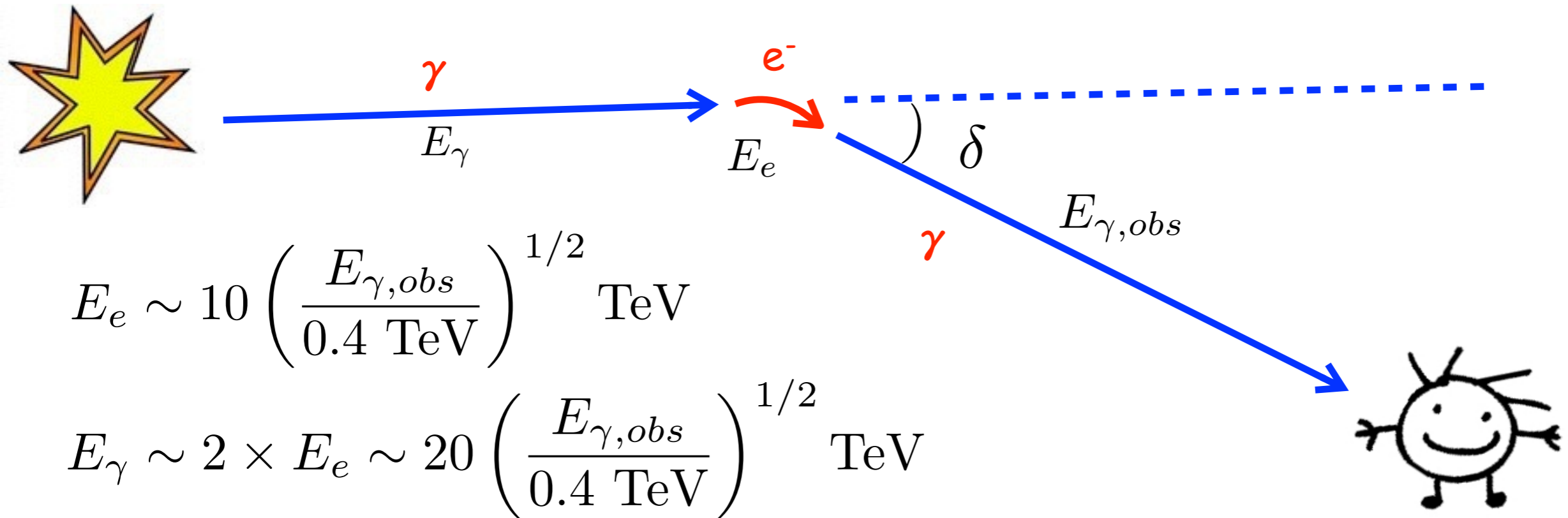
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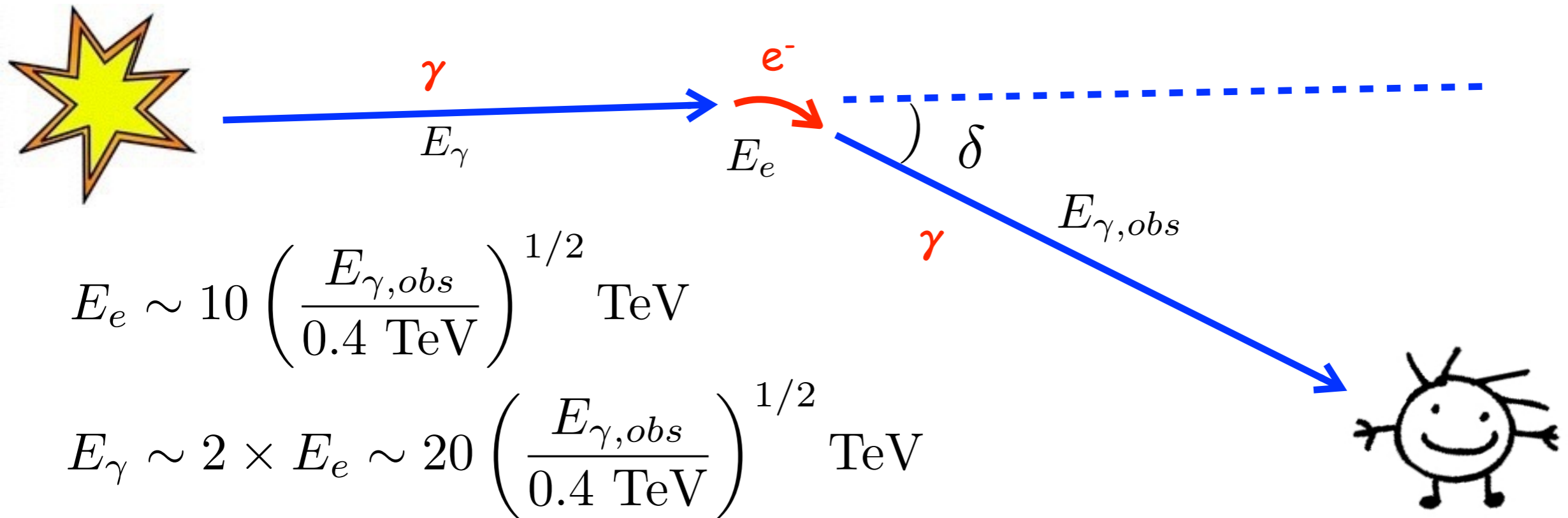
Magnetic fields: why should we care?



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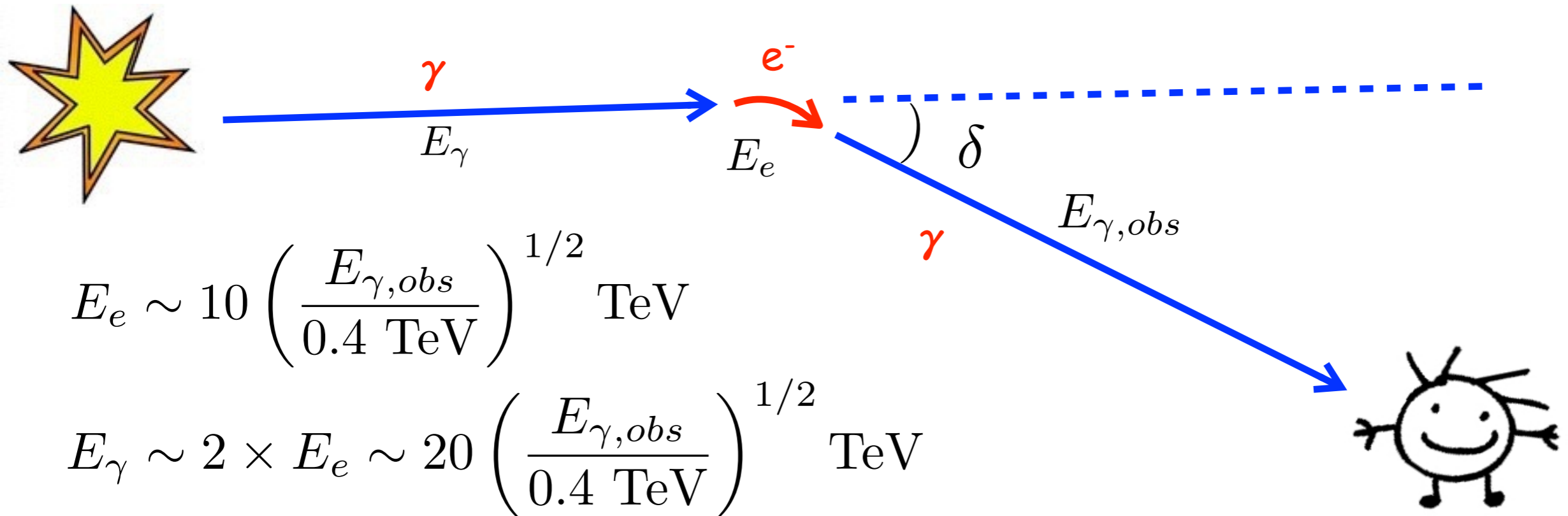
Magnetic fields: why should we care?



Compton cooling length $\rightarrow \lambda_e \sim 40 \left(\frac{E_{\gamma,obs}}{0.4 \text{ TeV}} \right)^{-1/2} \text{ kpc}$

Deflection $\rightarrow \delta = \frac{\lambda_e}{R_L} \sim 0.2^\circ \left(\frac{E_{\gamma,obs}}{0.4 \text{ TeV}} \right)^{-1} \left(\frac{B}{10^{-15} \text{ G}} \right)$

Magnetic fields: why should we care?



$$E_e \sim 10 \left(\frac{E_{\gamma,obs}}{0.4 \text{ TeV}} \right)^{1/2} \text{ TeV}$$

$$E_\gamma \sim 2 \times E_e \sim 20 \left(\frac{E_{\gamma,obs}}{0.4 \text{ TeV}} \right)^{1/2} \text{ TeV}$$

Compton cooling length $\rightarrow \lambda_e \sim 40 \left(\frac{E_{\gamma,obs}}{0.4 \text{ TeV}} \right)^{-1/2} \text{ kpc}$

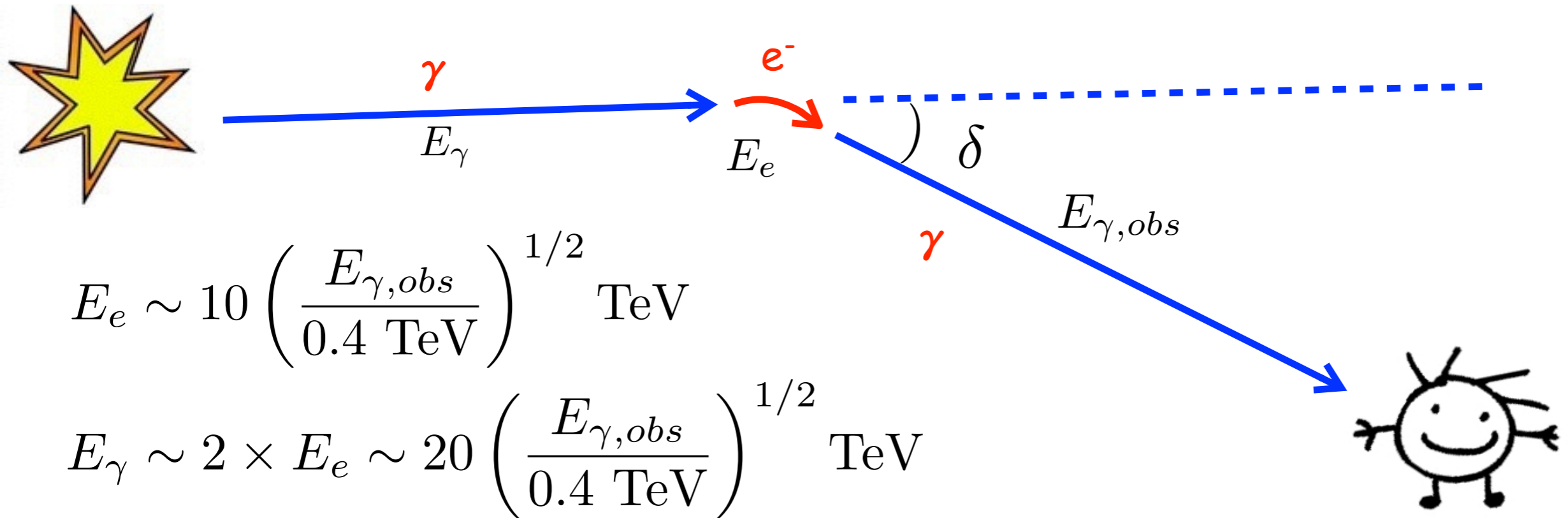
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if $\lambda_B \gg \lambda_e$

reduced by a random walk factor $\left(\frac{\lambda_B}{\lambda_e} \right)^{1/2}$

if $\lambda_B \ll \lambda_e$

Magnetic fields: why should we care?



upper limits from Faraday rotation of distant quasars and CMB fluctuations are at the **nanoGauss level** -> gamma-rays can be used to measure or constrain B!

Deflection ->

$$\delta = \frac{\lambda_e}{R_L} \sim 0.2^\circ \left(\frac{E_{\gamma,obs}}{0.4 \text{ TeV}} \right)^{-1} \left(\frac{B}{10^{-15} \text{ G}} \right)$$

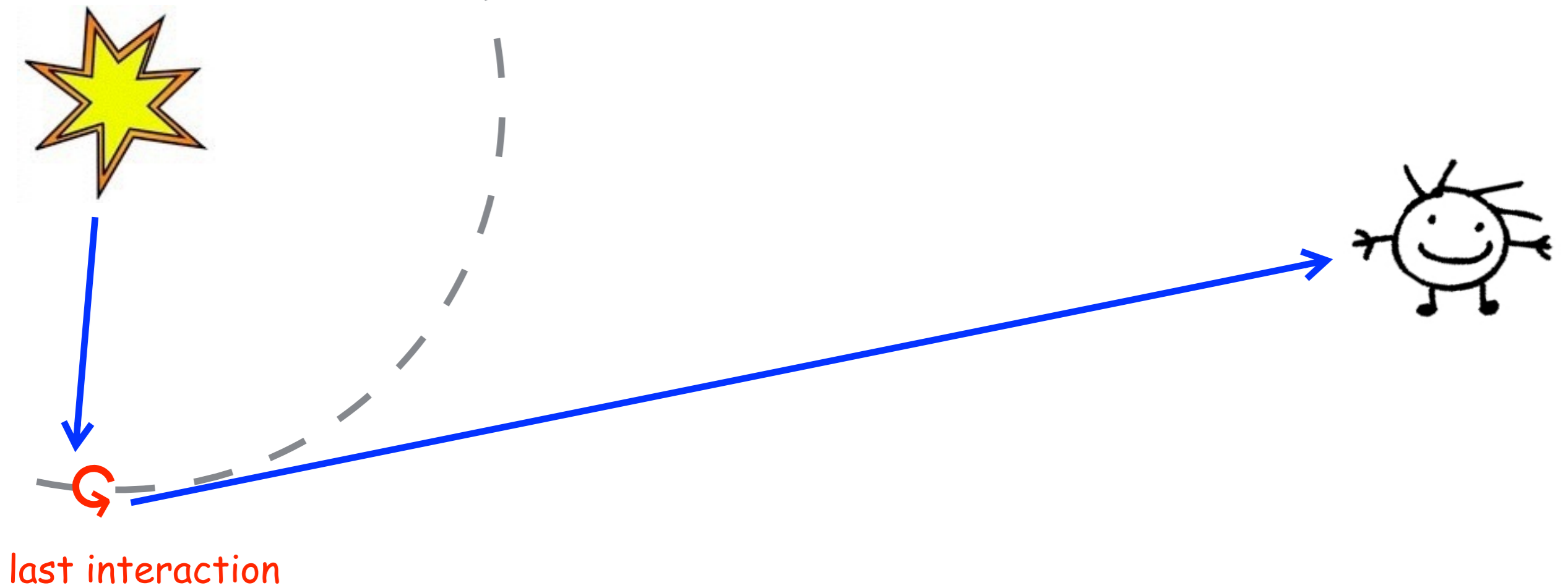
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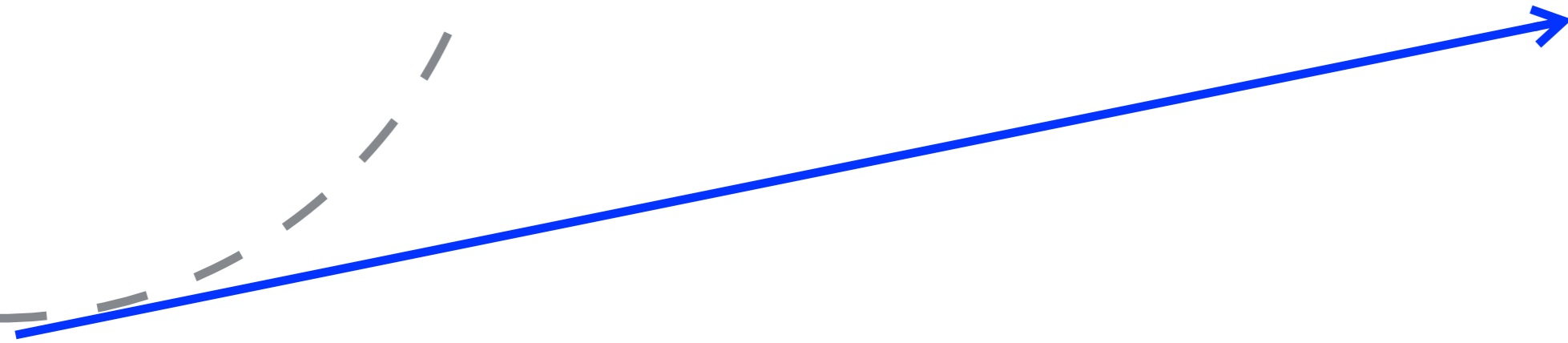
Full isotropization: pair haloes



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last interaction



pairs are isotropised when

$$B \gtrsim 2 \times 10^{-12} \left(\frac{E_{\gamma,obs}}{0.4 \text{ TeV}} \right) \text{ G}$$

Full isotropization: pair haloes



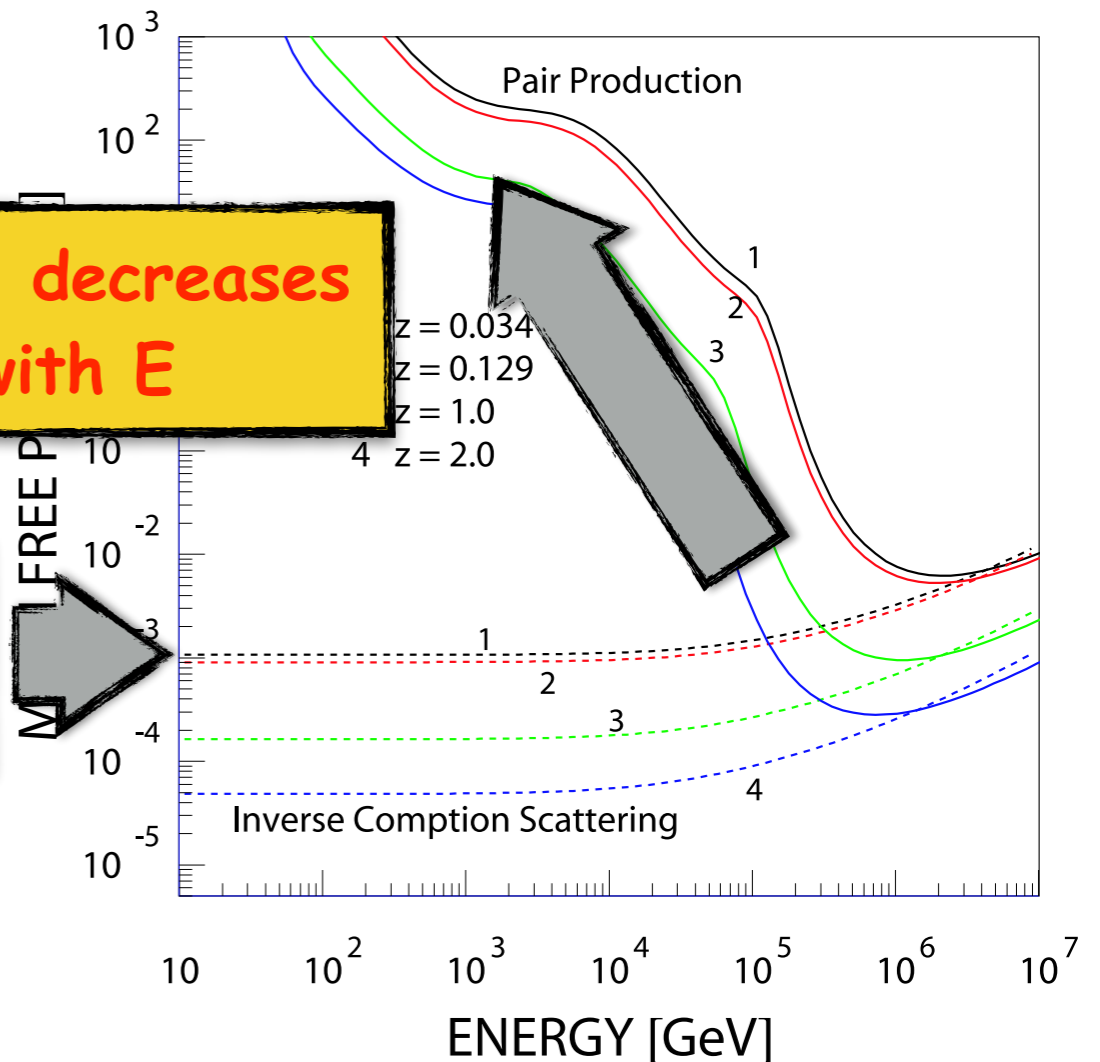
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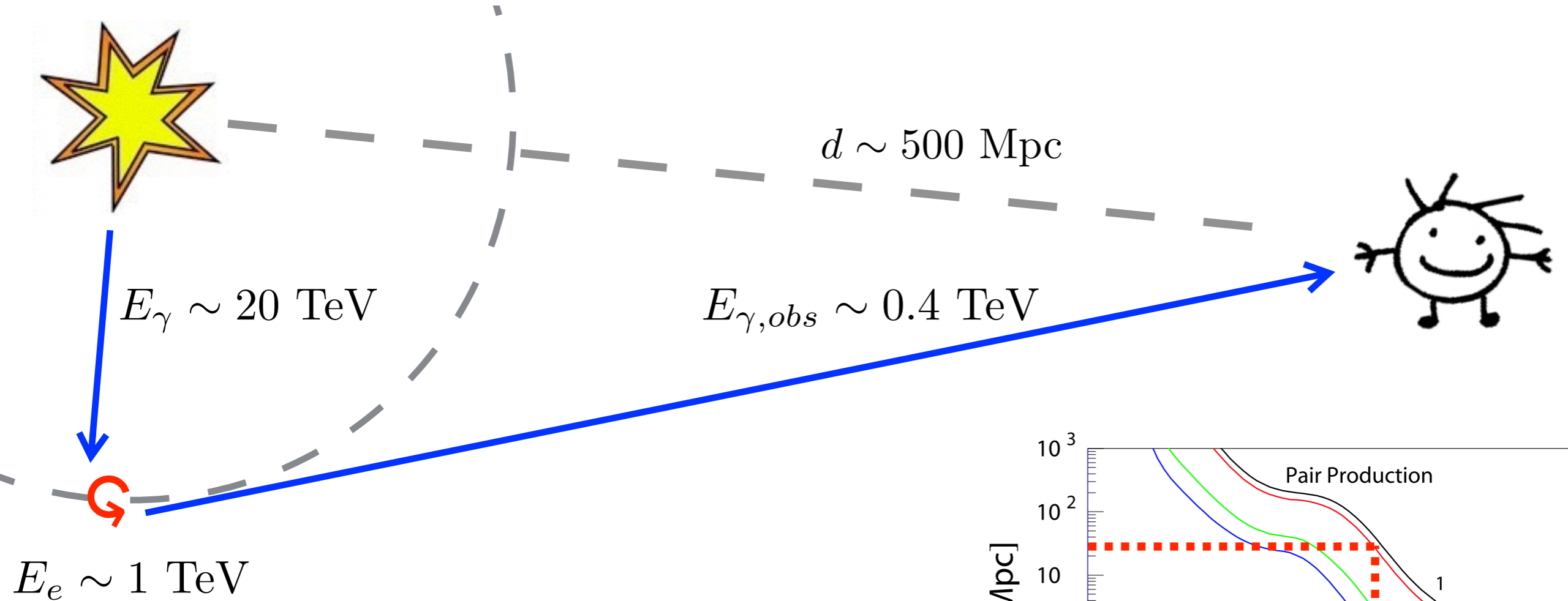
$$B \gtrsim 2 \times 10^{-12} \left(\frac{E_{\gamma,obs}}{0.4 \text{ TeV}} \right) \text{ G}$$

pairs do not move

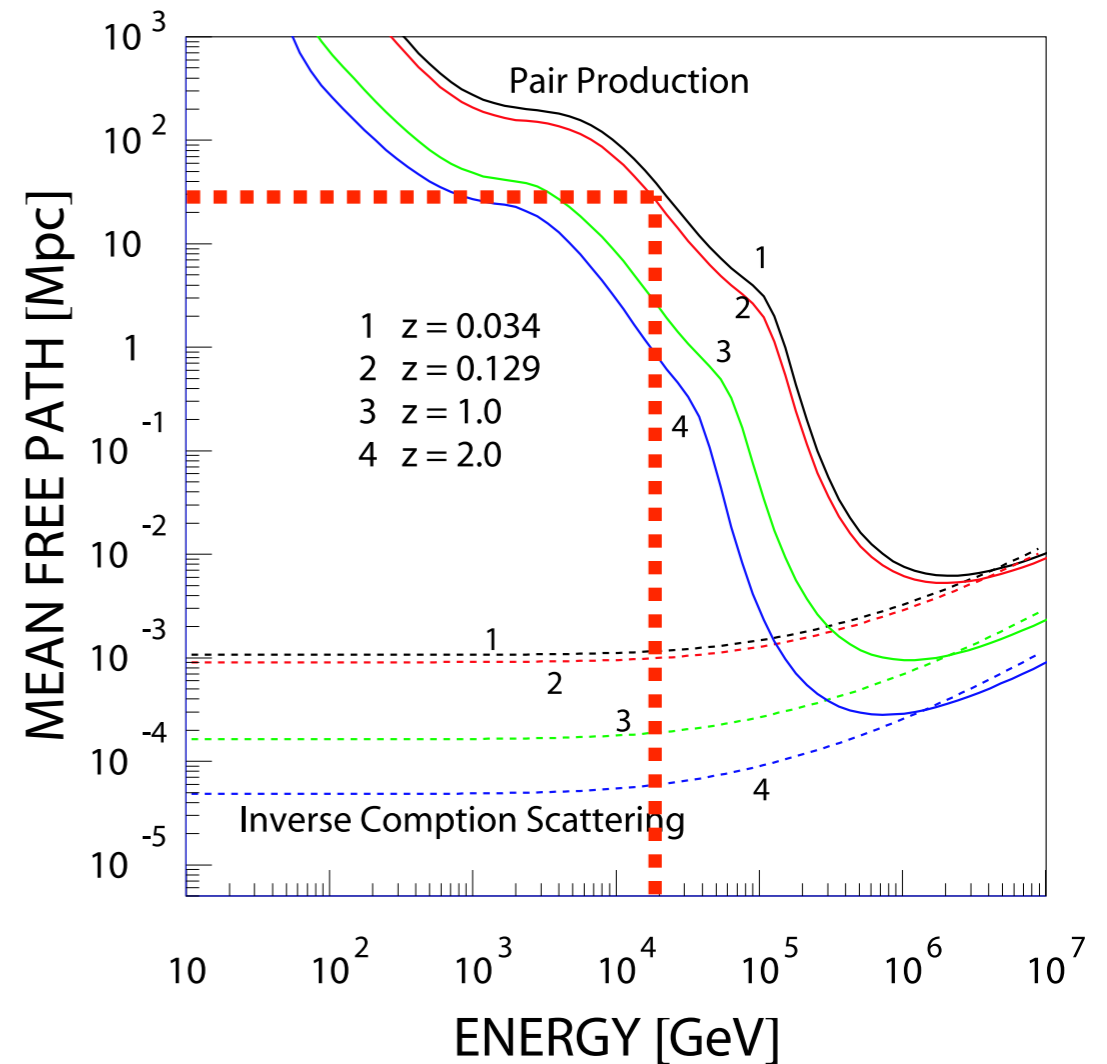
m.f.p. decreases with E



Full isotropization: pair haloes

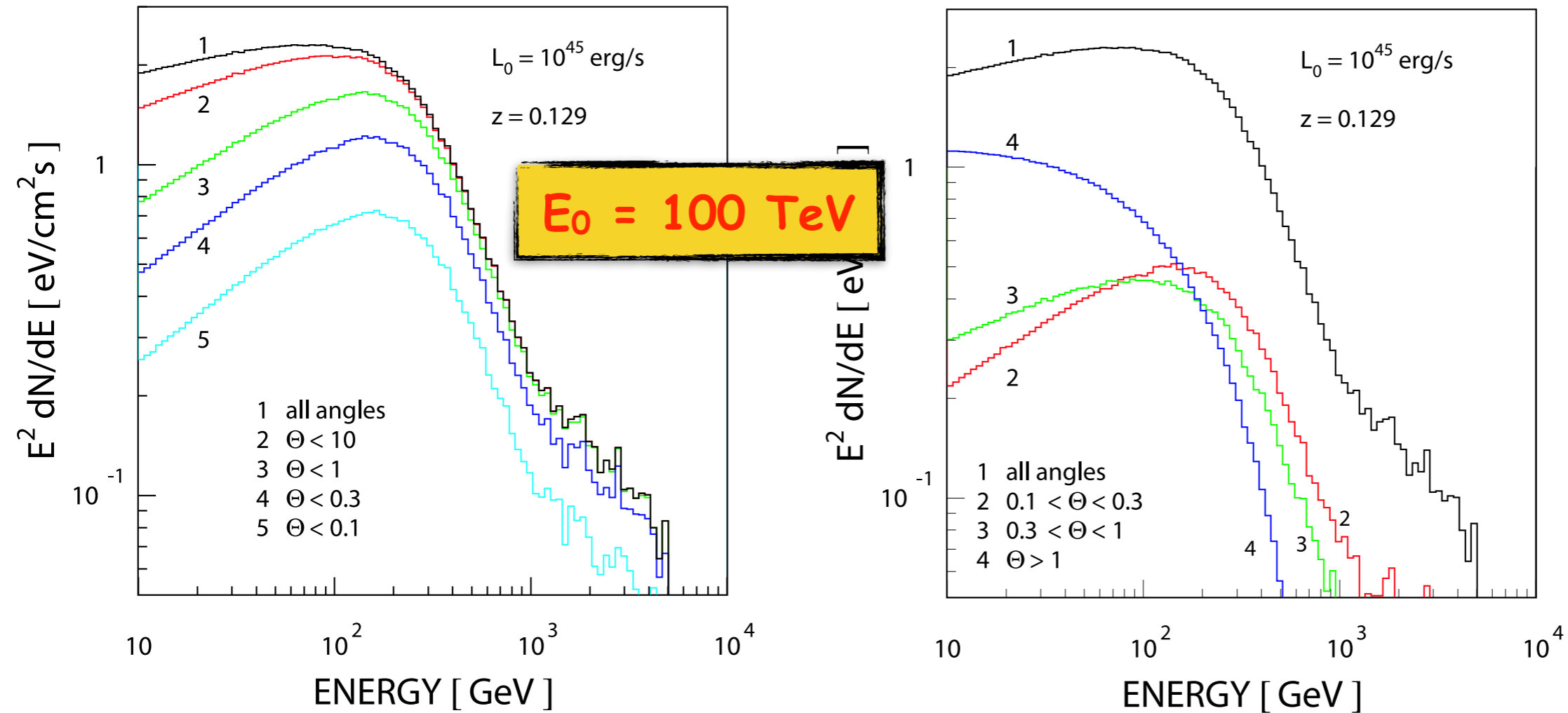


$$\vartheta_{halo} \sim \frac{\lambda_{\gamma\gamma}(E_\gamma)}{d} \approx 3^\circ$$



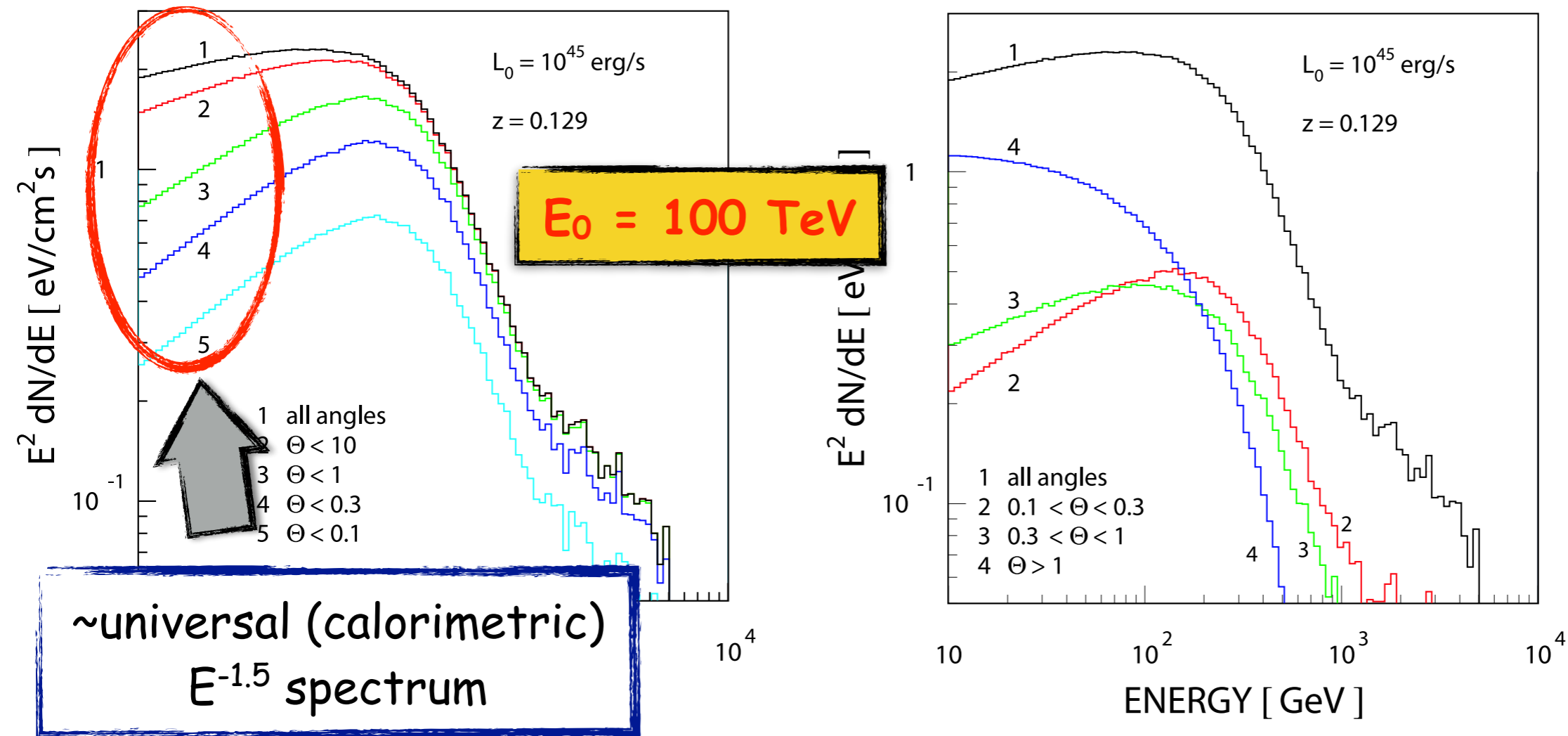
Pair haloes: spectra and morphology

Eungwanichayapant&Aharonian 09



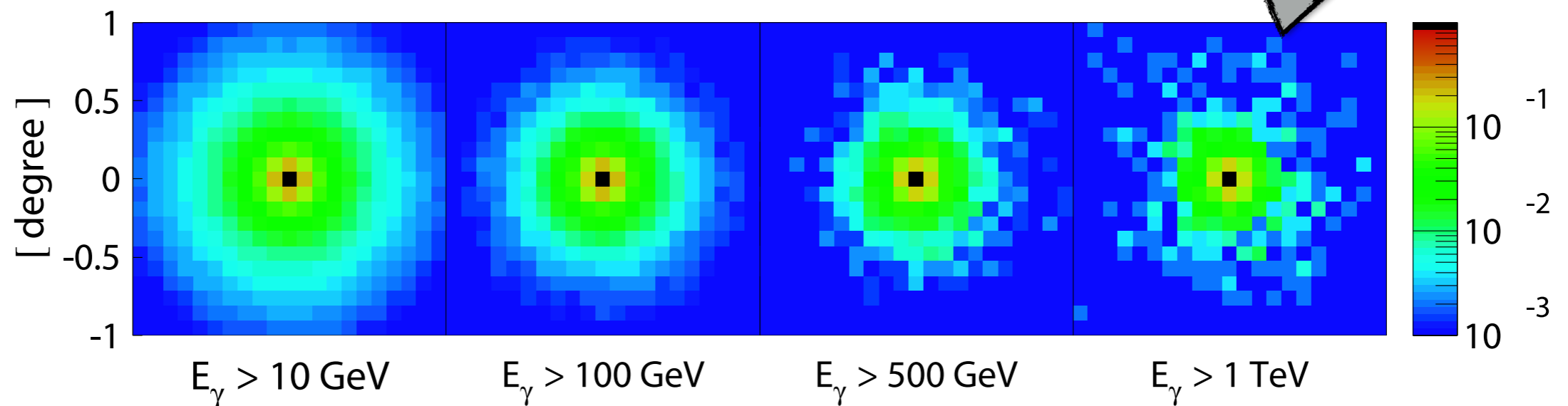
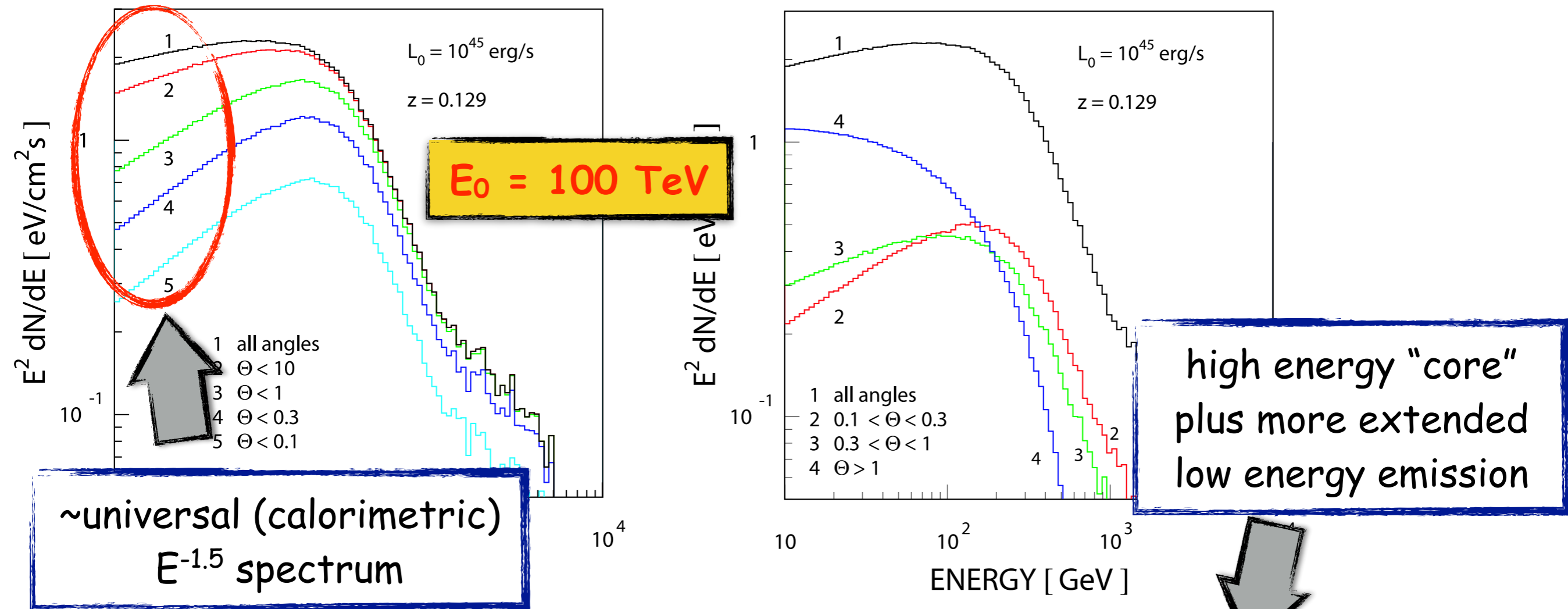
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Pair haloes: cosmological probes

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Pair haloes: cosmological probes

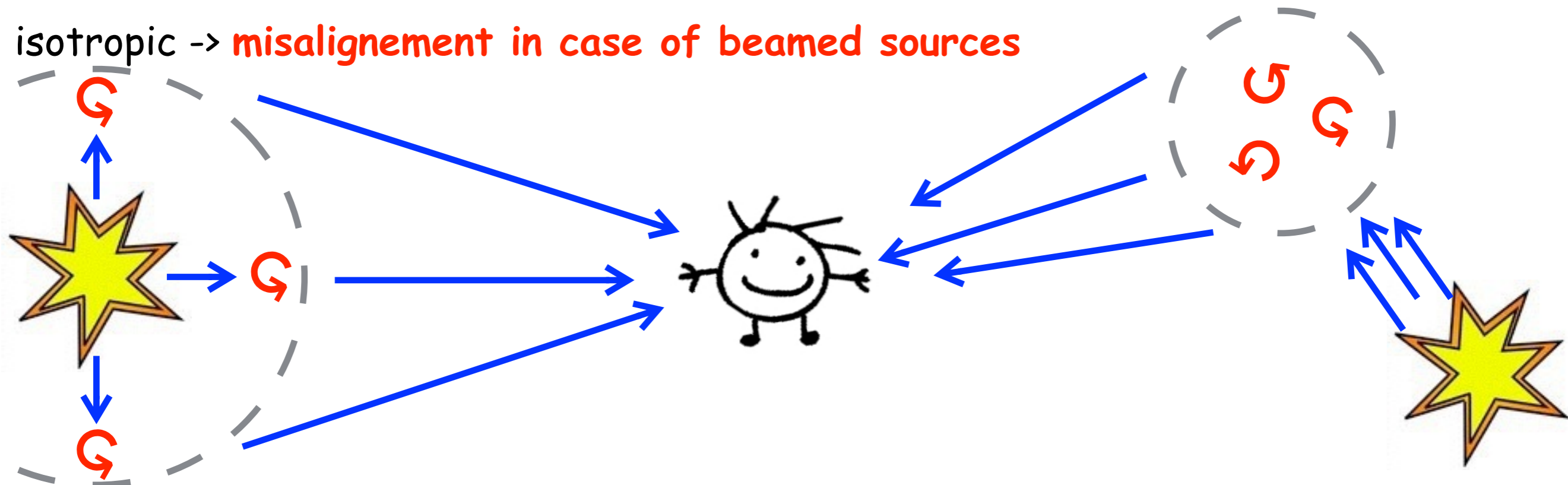
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- pair haloes -> **time integrated power in very high energy gamma rays** of the central source (Compton cooling time of electrons < 1 Myr)
- the halo emission is centered onto the central source if the primary emission is isotropic -> **misalignment in case of beamed sources**



Detection of pair halos/ γ -ray halos?



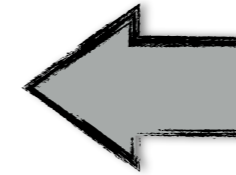
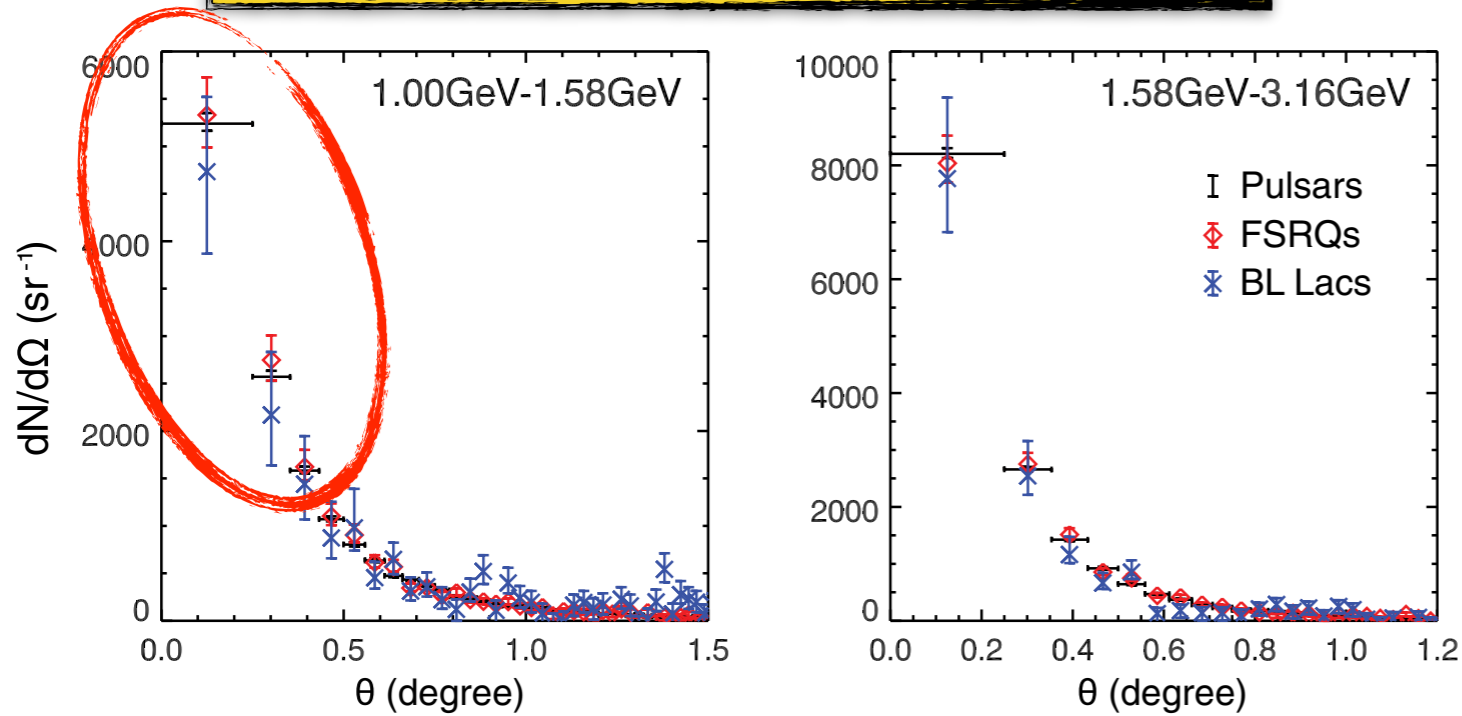
some confusion in the literature
about what a pair halo is

Detection of pair halos/ γ -ray halos?

stacking of 24 BL Lac objects

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Chen+ 2015



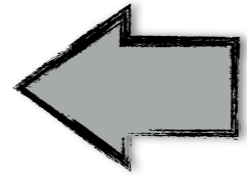
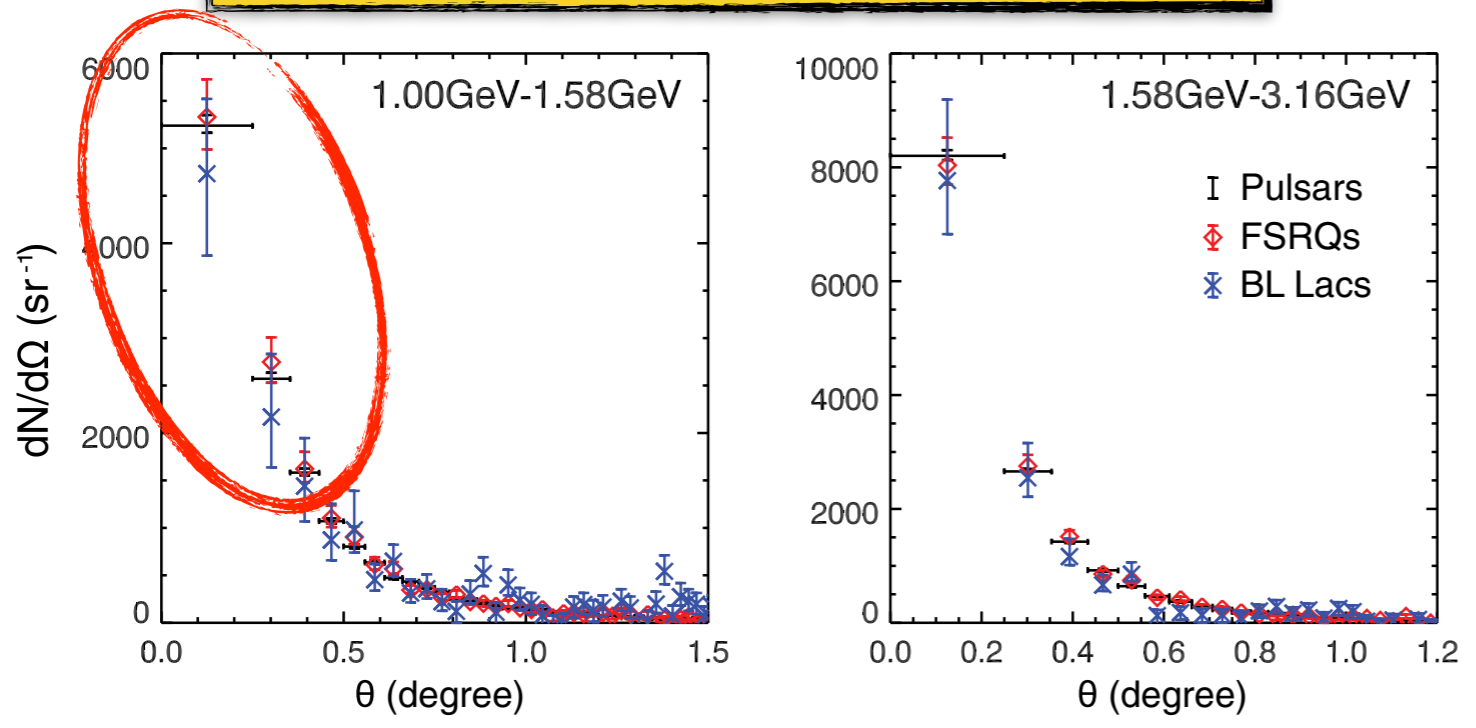
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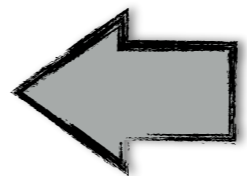
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Prokhorov&Moraghan 2016

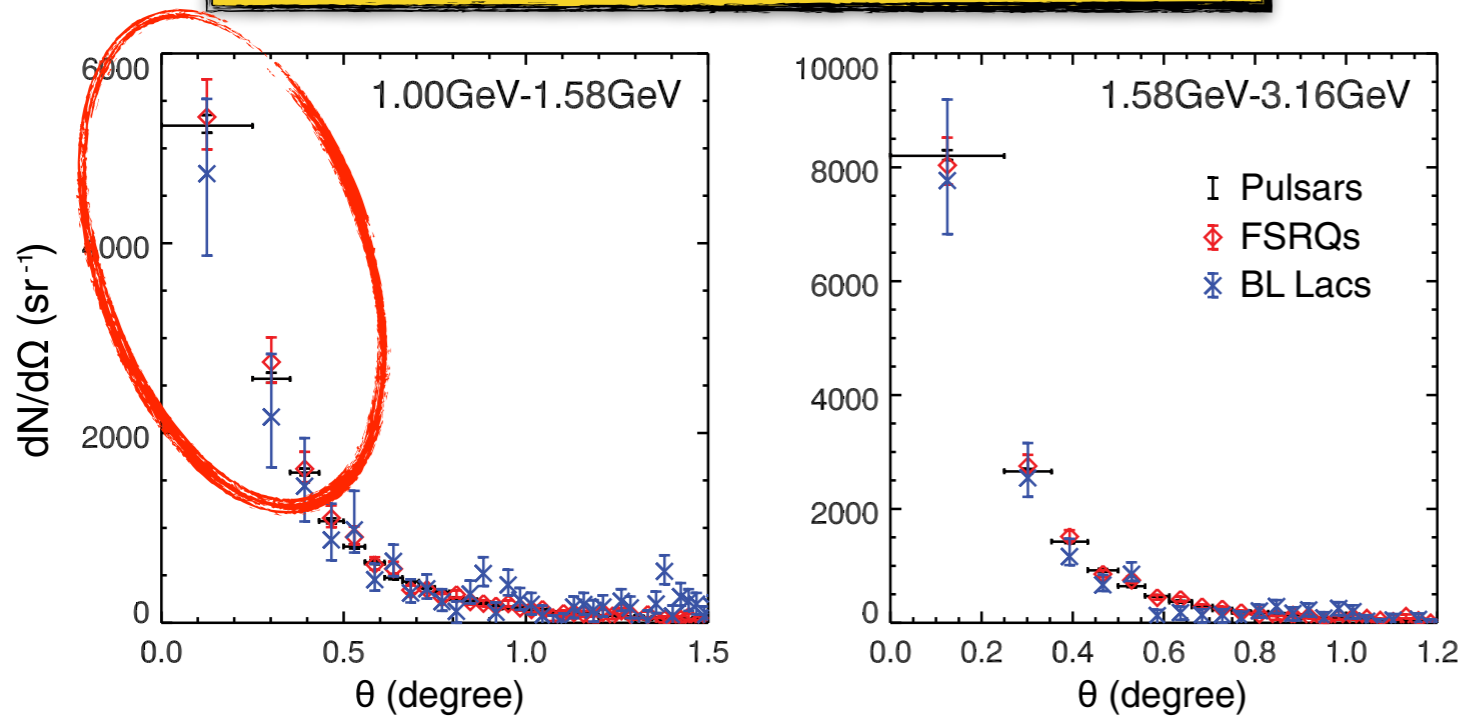
comparison of angular distribution of photons for quiescent and flaring (no halo contribution) blazars



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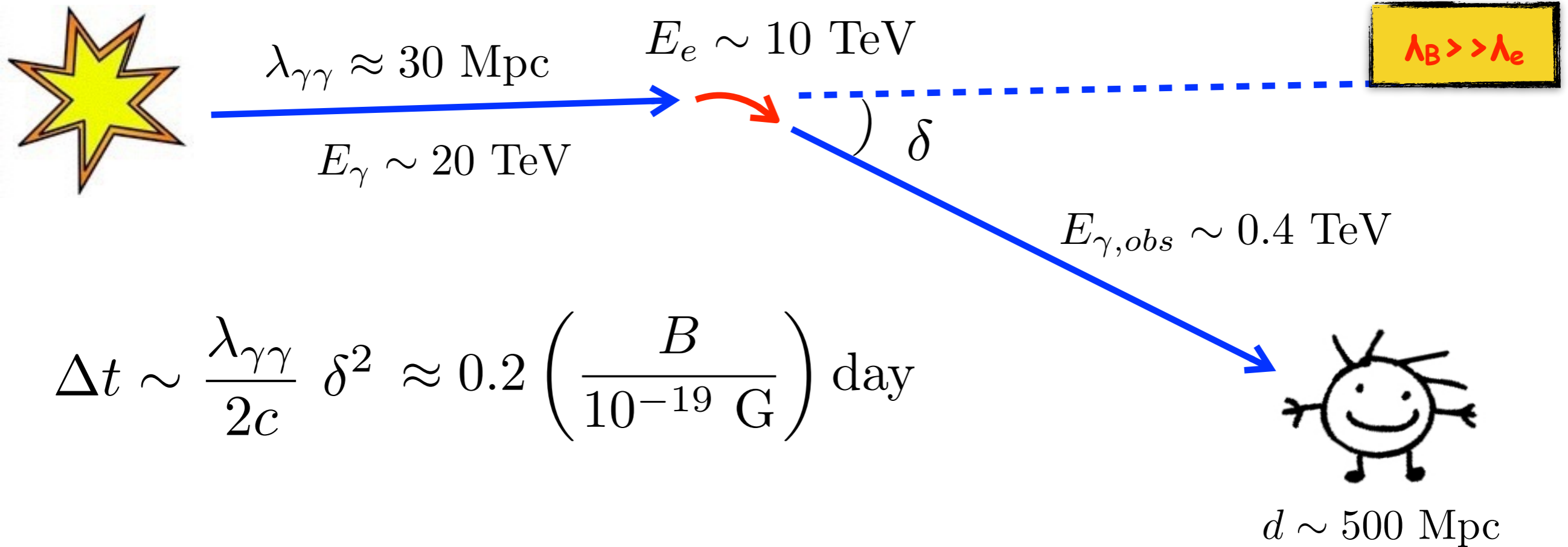
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no evidence for pair halos from HESS observations

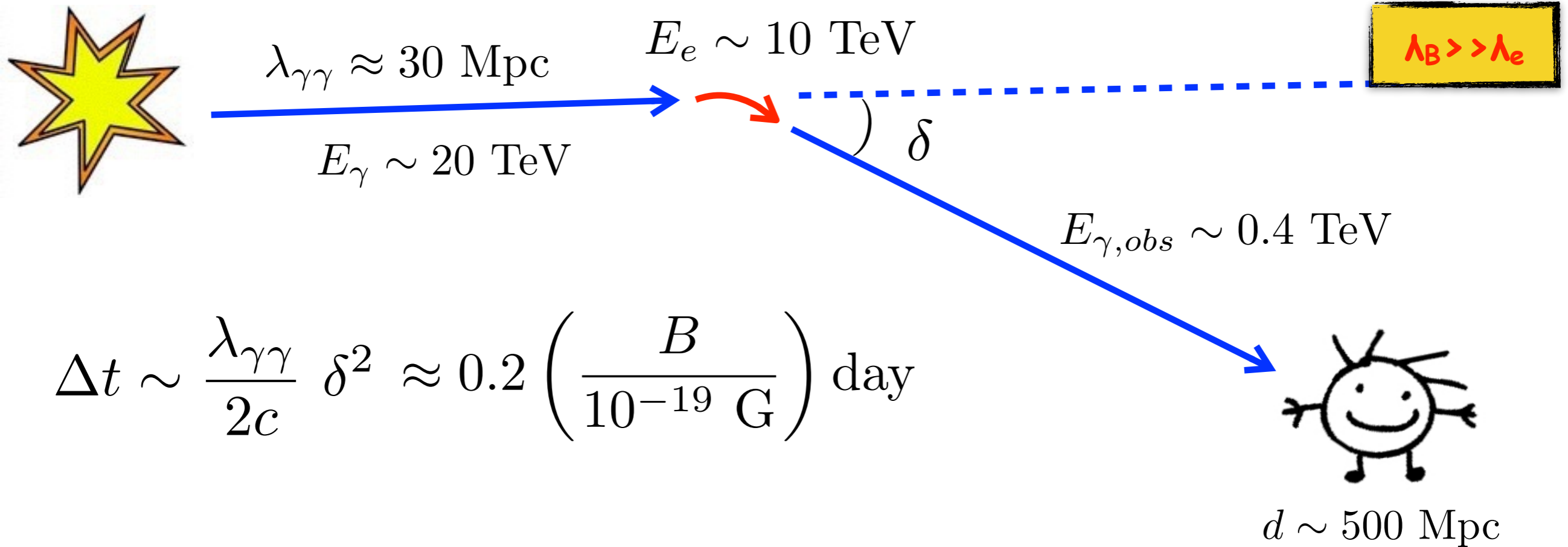
HESS Coll. 2014

Deflection: time delays in γ -ray pulses



$$\Delta t \sim \frac{\lambda_{\gamma\gamma}}{2c} \delta^2 \approx 0.2 \left(\frac{B}{10^{-19} \text{ G}} \right) \text{ day}$$

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intrinsic deflection (interaction) $\rightarrow 1/\gamma$

$$\delta \sim 2/\gamma \longrightarrow B_{min} \sim 3 \times 10^{-20} \text{ G}$$

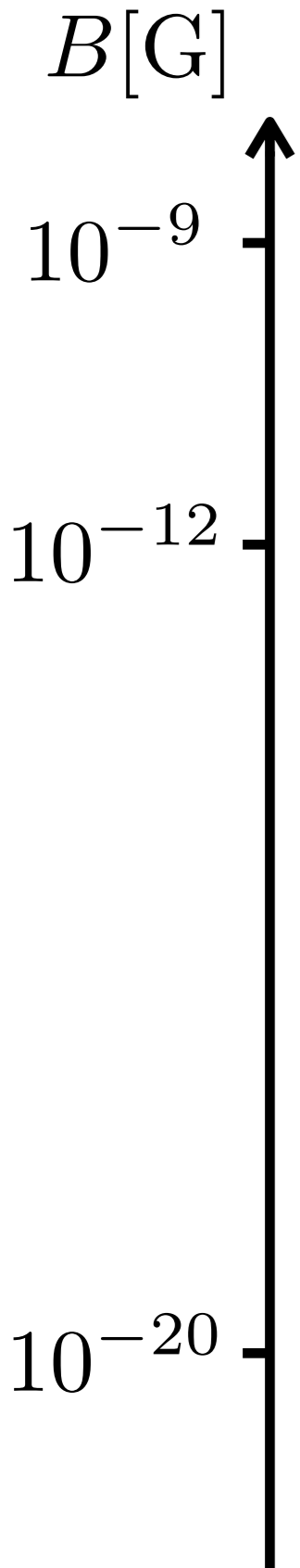
minimum field that can be constrained with time delays

Brief (and very rough) summary

$$E_{\gamma,obs} = 0.4 \text{ TeV}$$

$$\lambda_B \gg \lambda_e$$

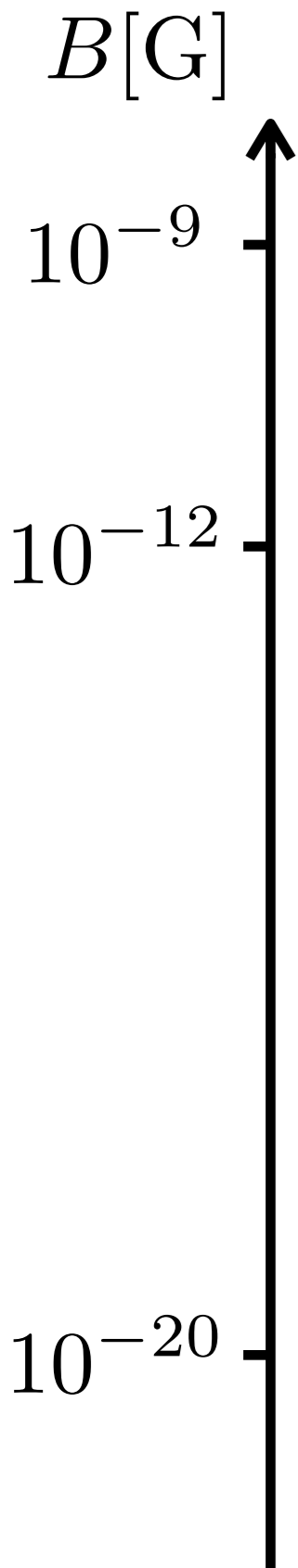
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isotropisation

pair halos

strong deflections

delayed & extended emission

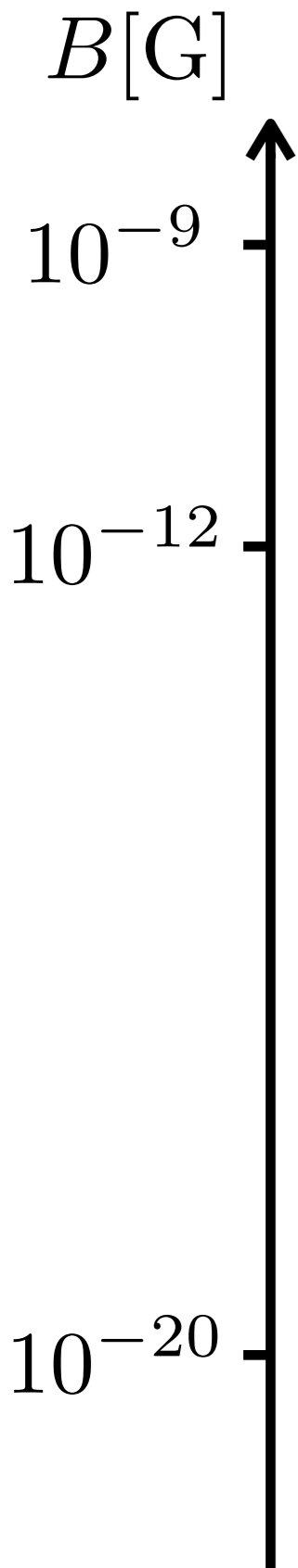
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$\approx \text{Myr}$
 $\lesssim \text{h}$

strong deflections

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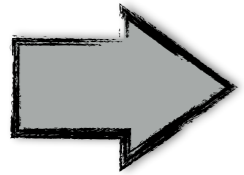
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Delayed emission from blazars

t_{var} -> typical time for blazar variability (days?)

time broadening of the signal: often assumed to be of the order of the delay time Δt



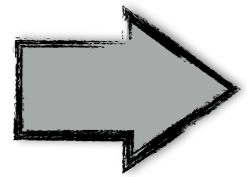
$$F_{delay} \approx \frac{t_{var}}{t_{var} + \Delta t} F_0$$

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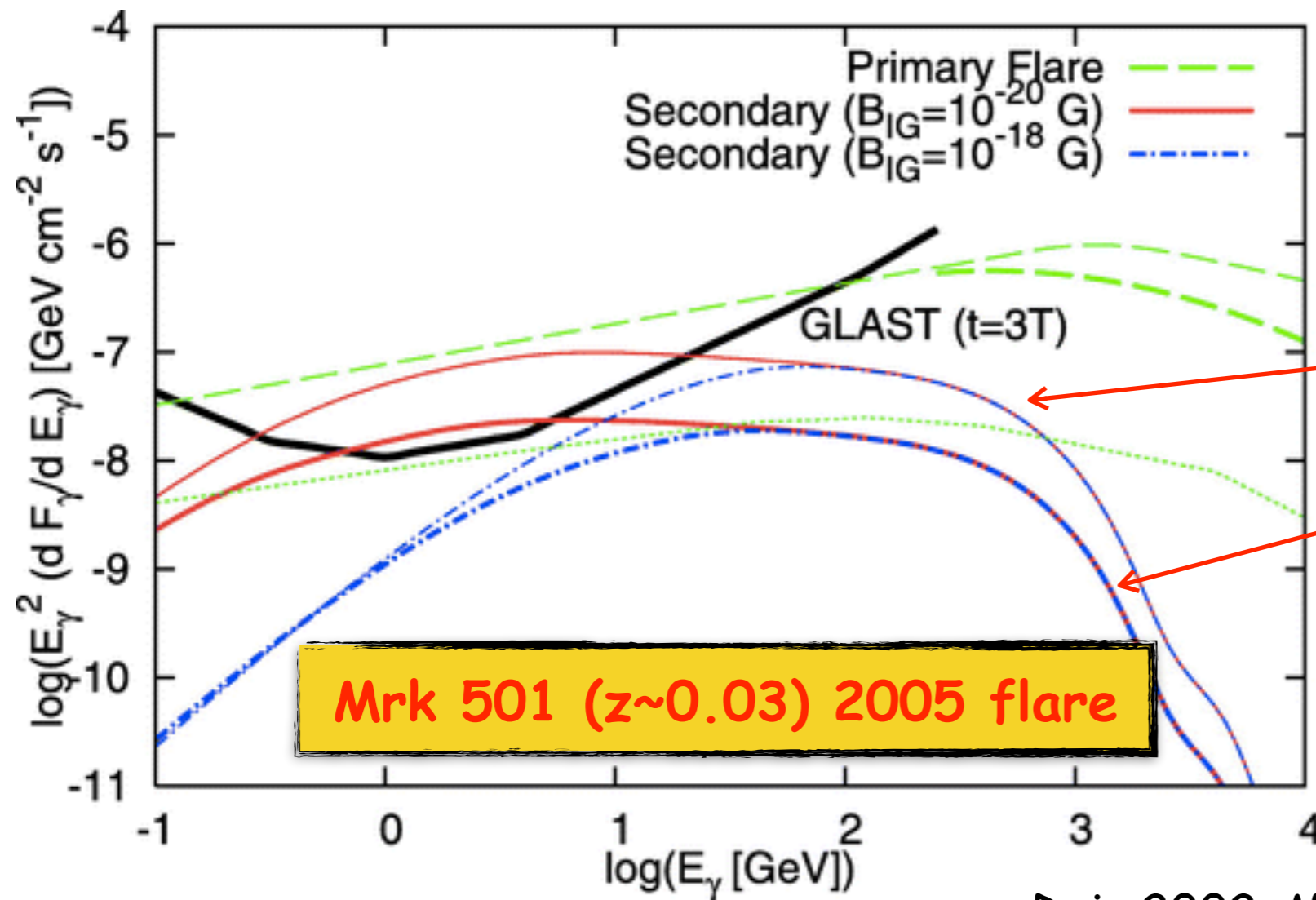
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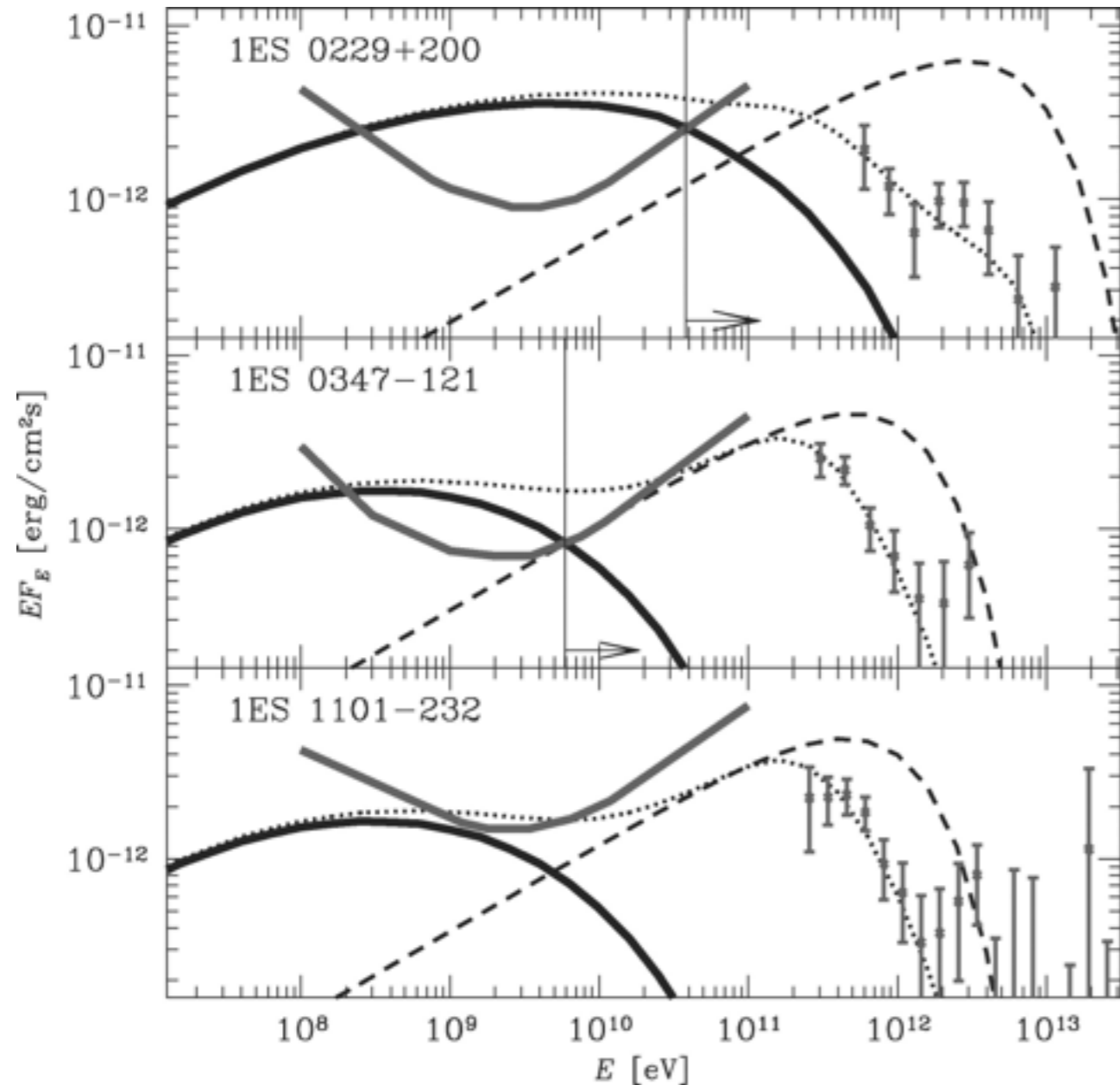
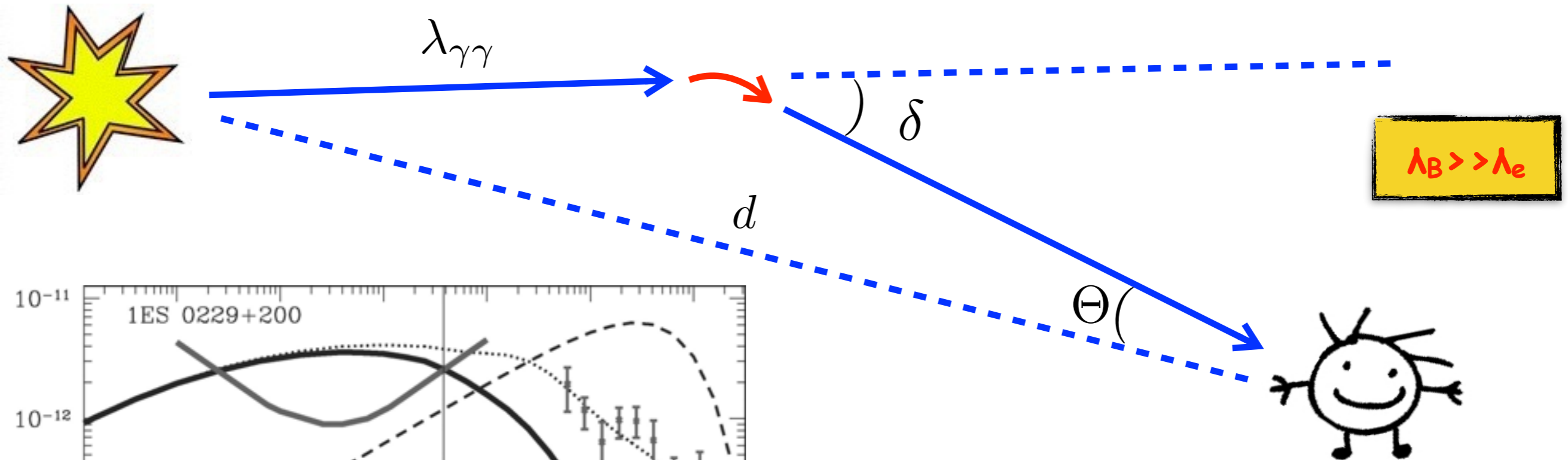
$t_{var} \approx 0.5 \text{ day}$

$t = t_{var}$

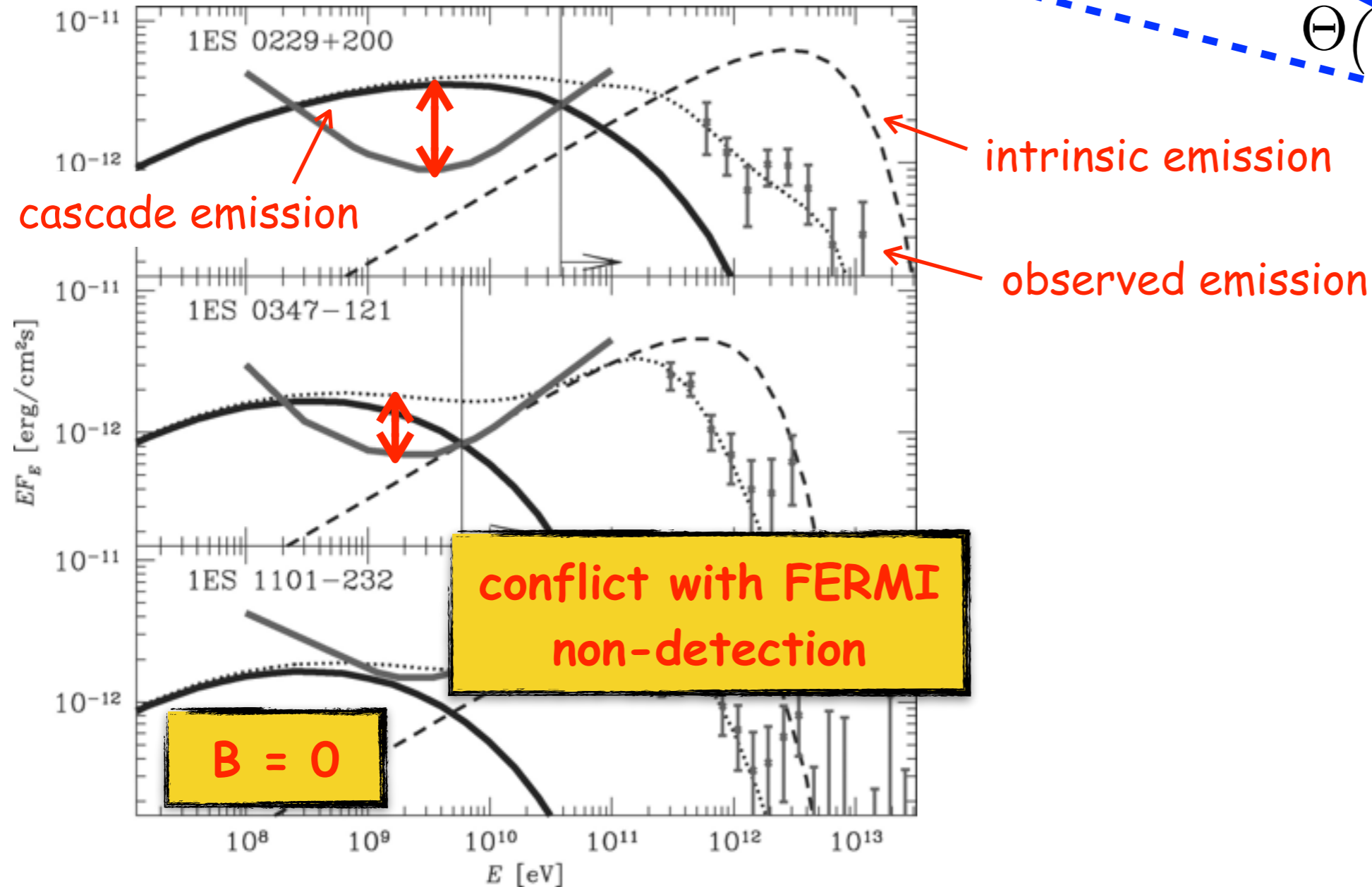
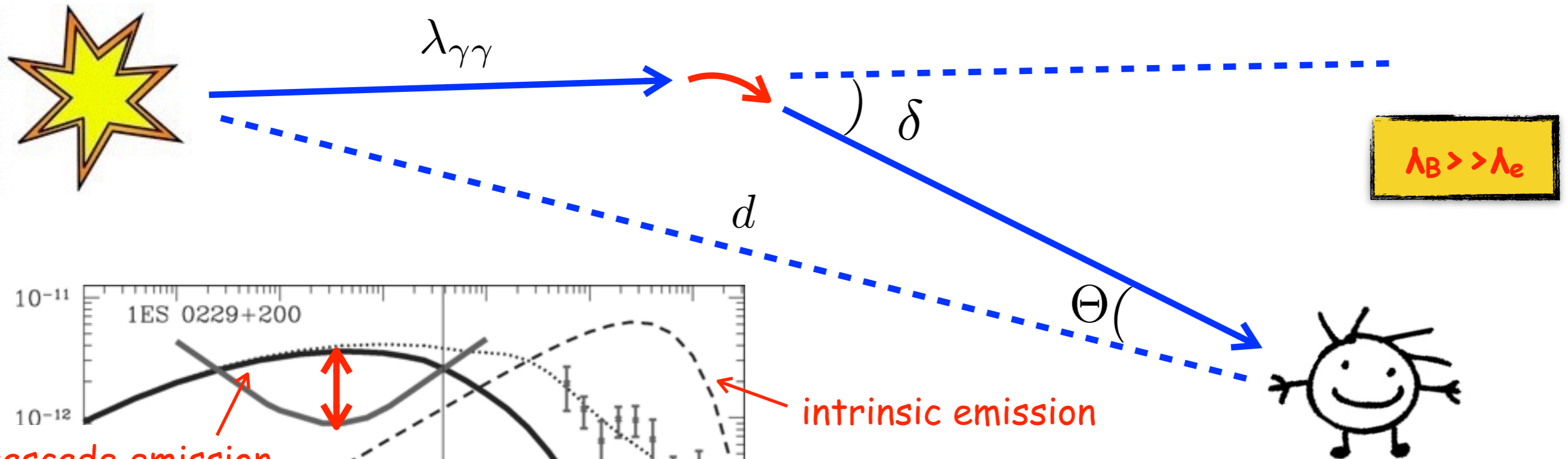
$t = 3 \times t_{var}$

within the reach of FERMI

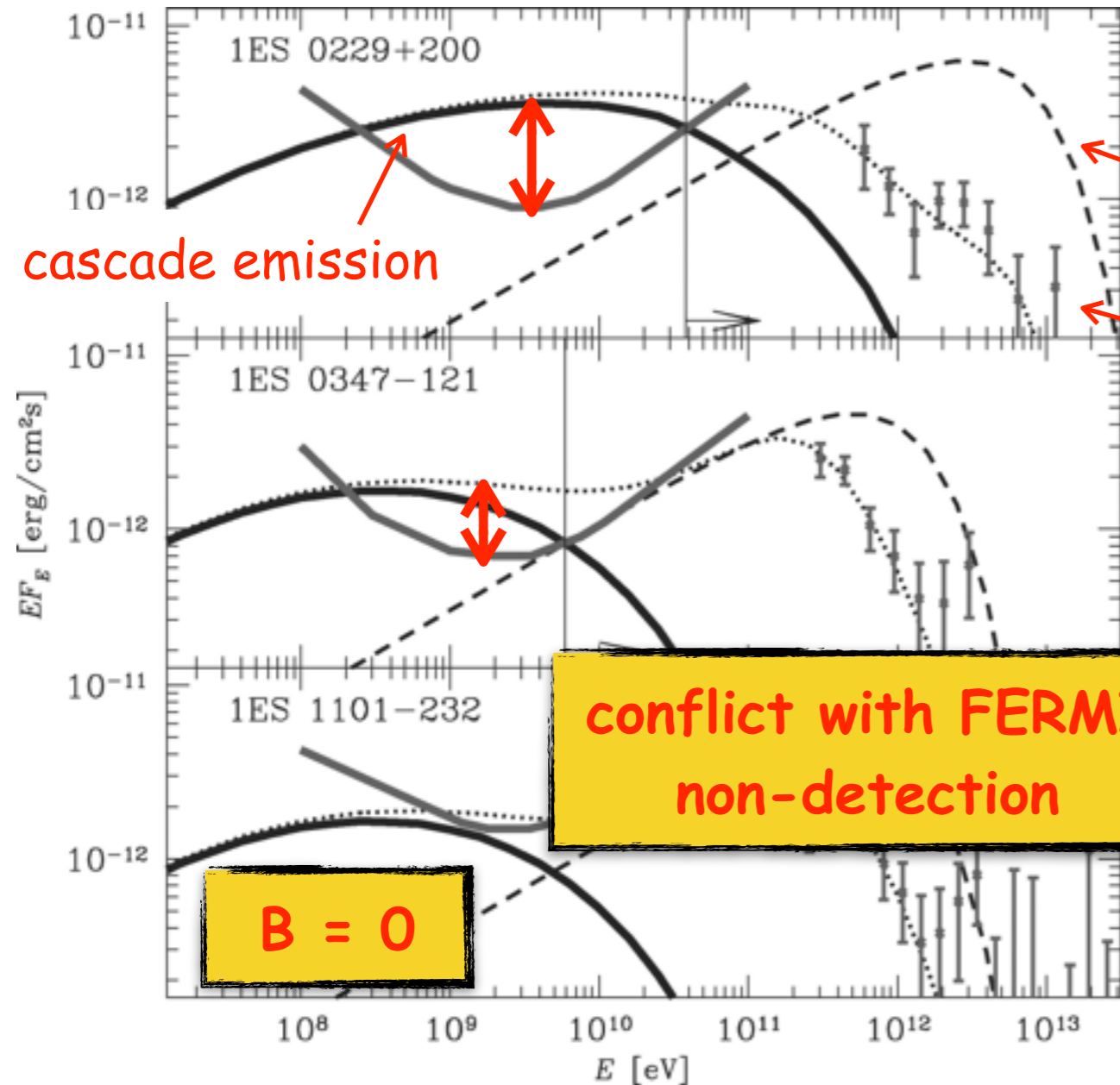
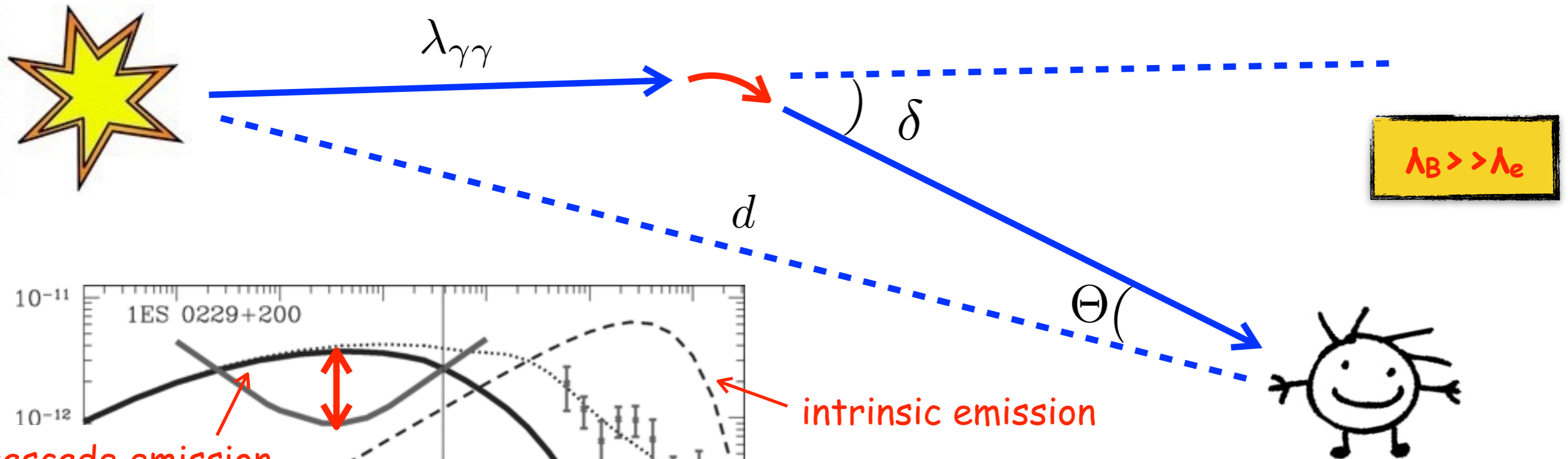
Constraints from GeV-TeV observations



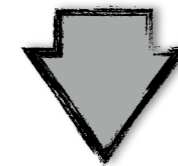
Constraints from GeV-TeV observations



Constraints from GeV-TeV observations



$$\Theta \sim \frac{\lambda_{\gamma\gamma}}{d} \delta > \Theta_{PSF}$$



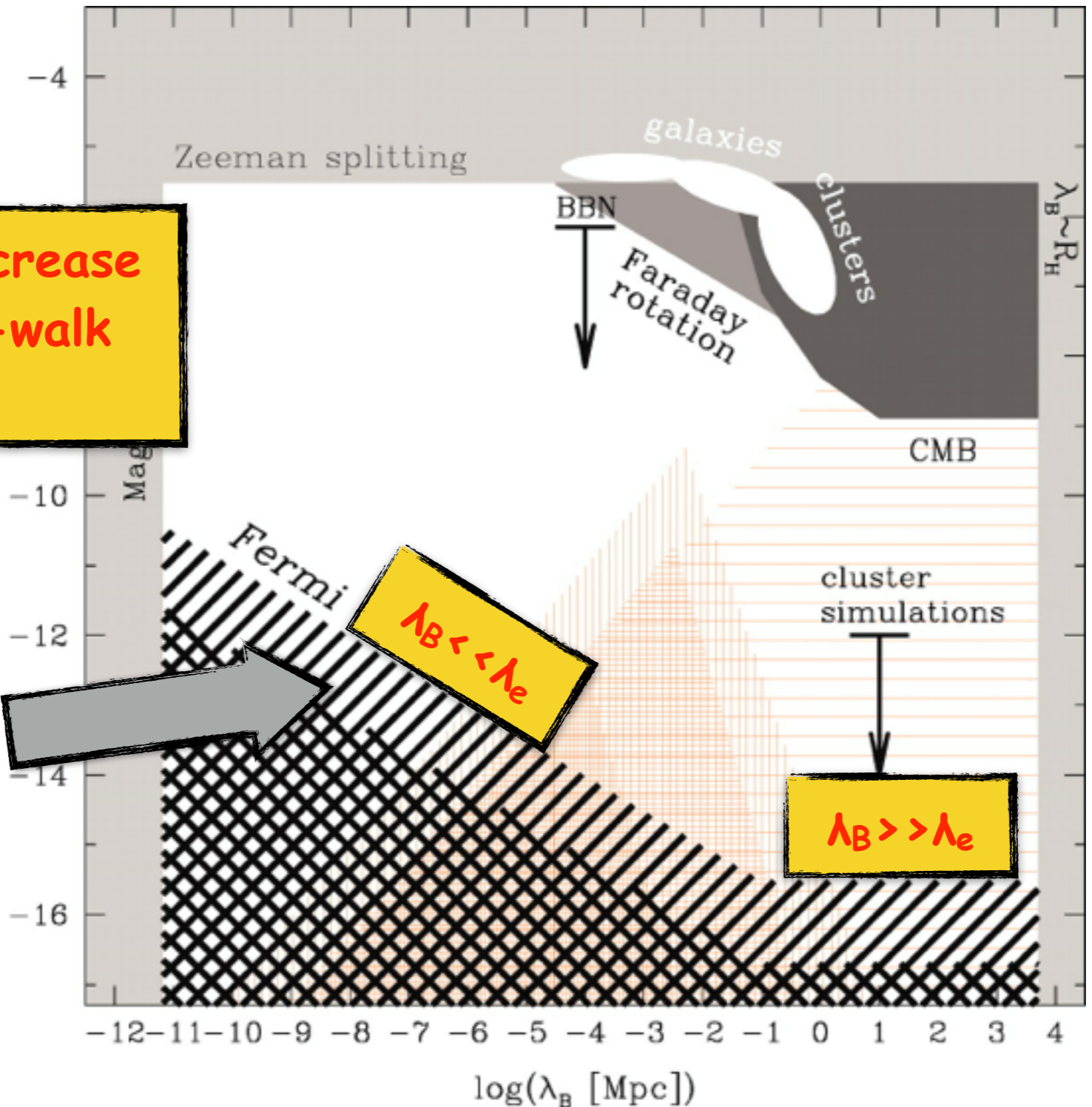
$$B \gtrsim 3 \times 10^{-16} \text{ G}$$

Constraints from GeV-TeV observations

Neronov & Vovk 2010

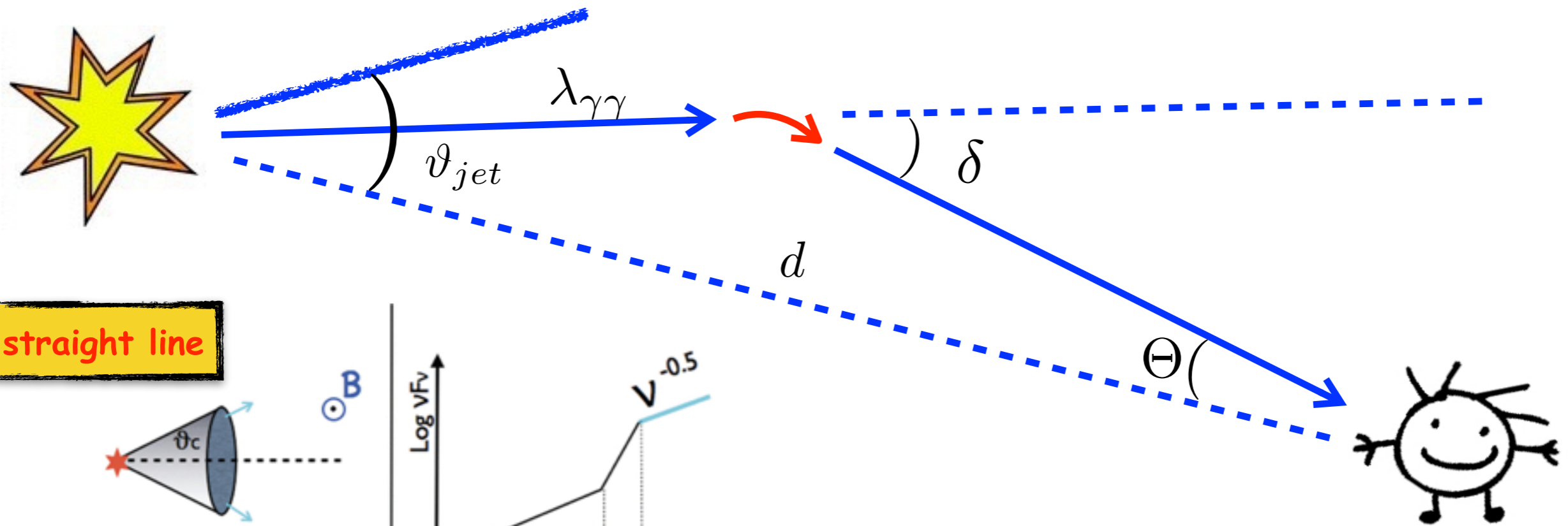
lower limits increase
by a random-walk
factor

$$\left(\frac{\lambda_B}{\lambda_e}\right)^{1/2}$$

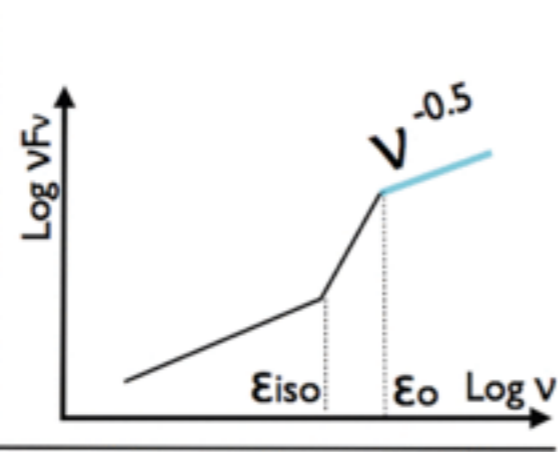
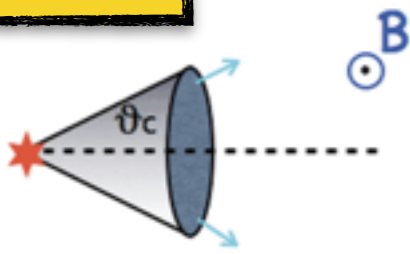


implicit assumption: steady and isotropic emission from the blazars

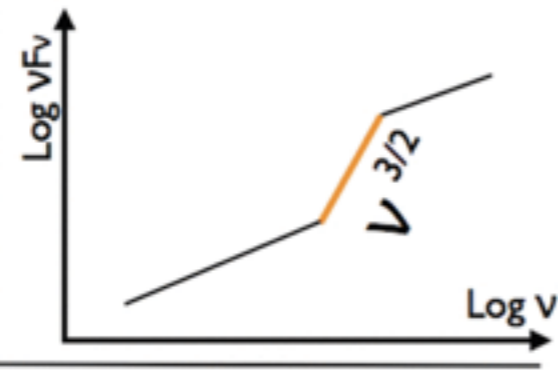
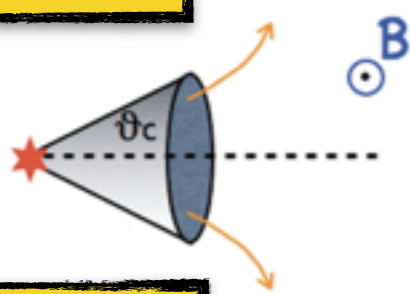
On the isotropy assumption



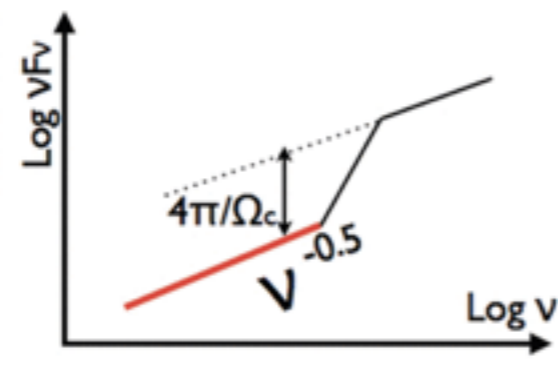
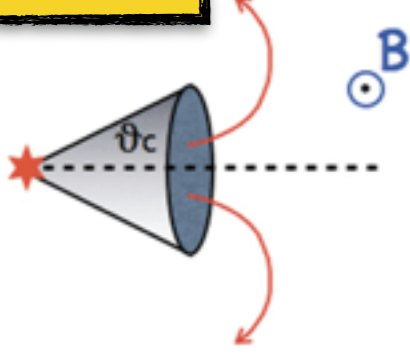
straight line



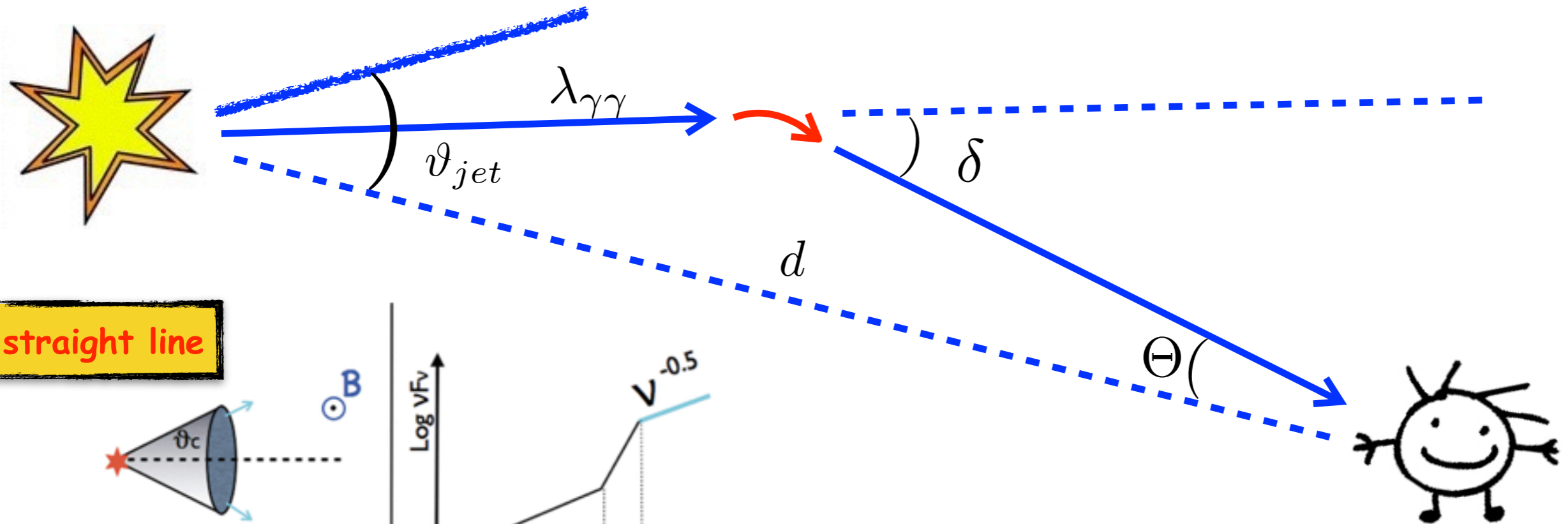
deflection



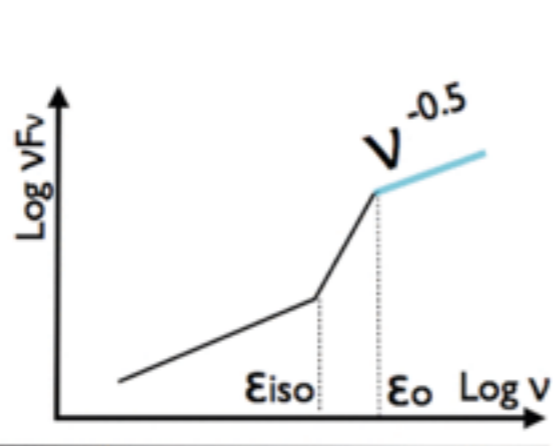
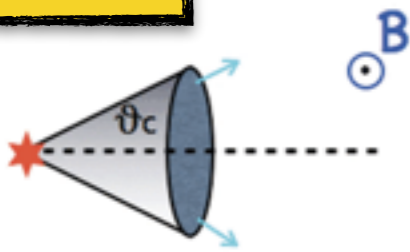
isotropization



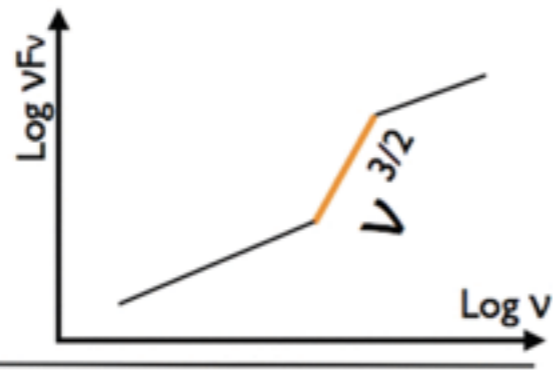
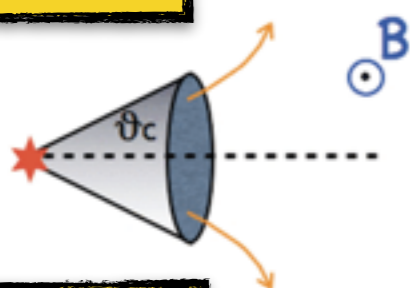
On the isotropy assumption



straight line



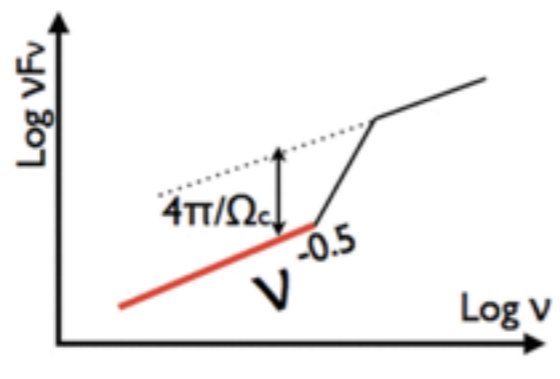
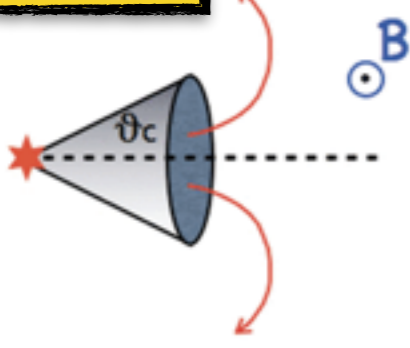
deflection



isotropic source

$$\Theta \sim \frac{\lambda_{\gamma\gamma}}{d} \delta$$

isotropization

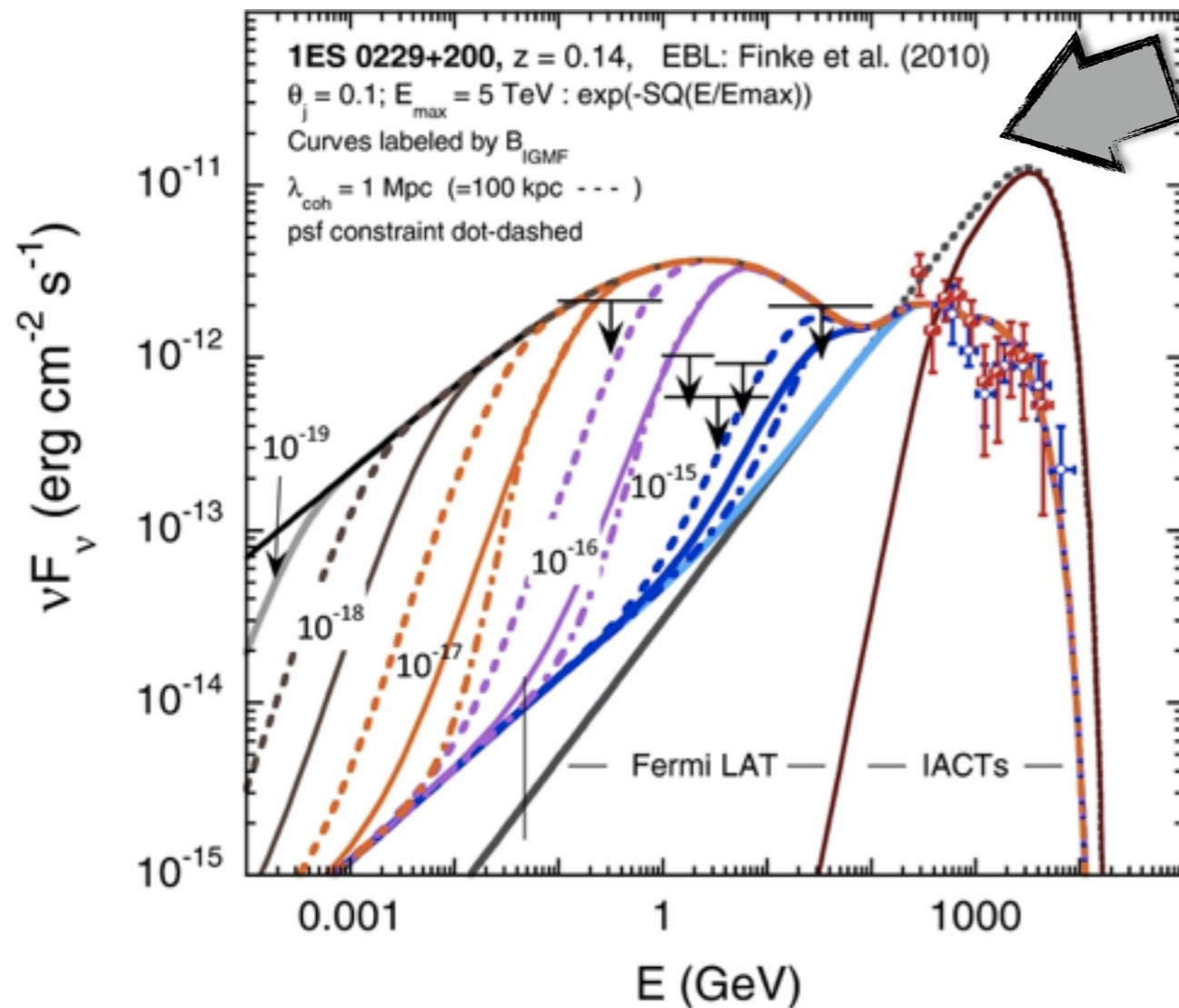


beamed source

$$\Theta \leq \frac{\lambda_{\gamma\gamma}}{d} \vartheta_{jet}$$

On the steady state assumption

how do we know whether the emission from a blazar is steady or not?



steady source

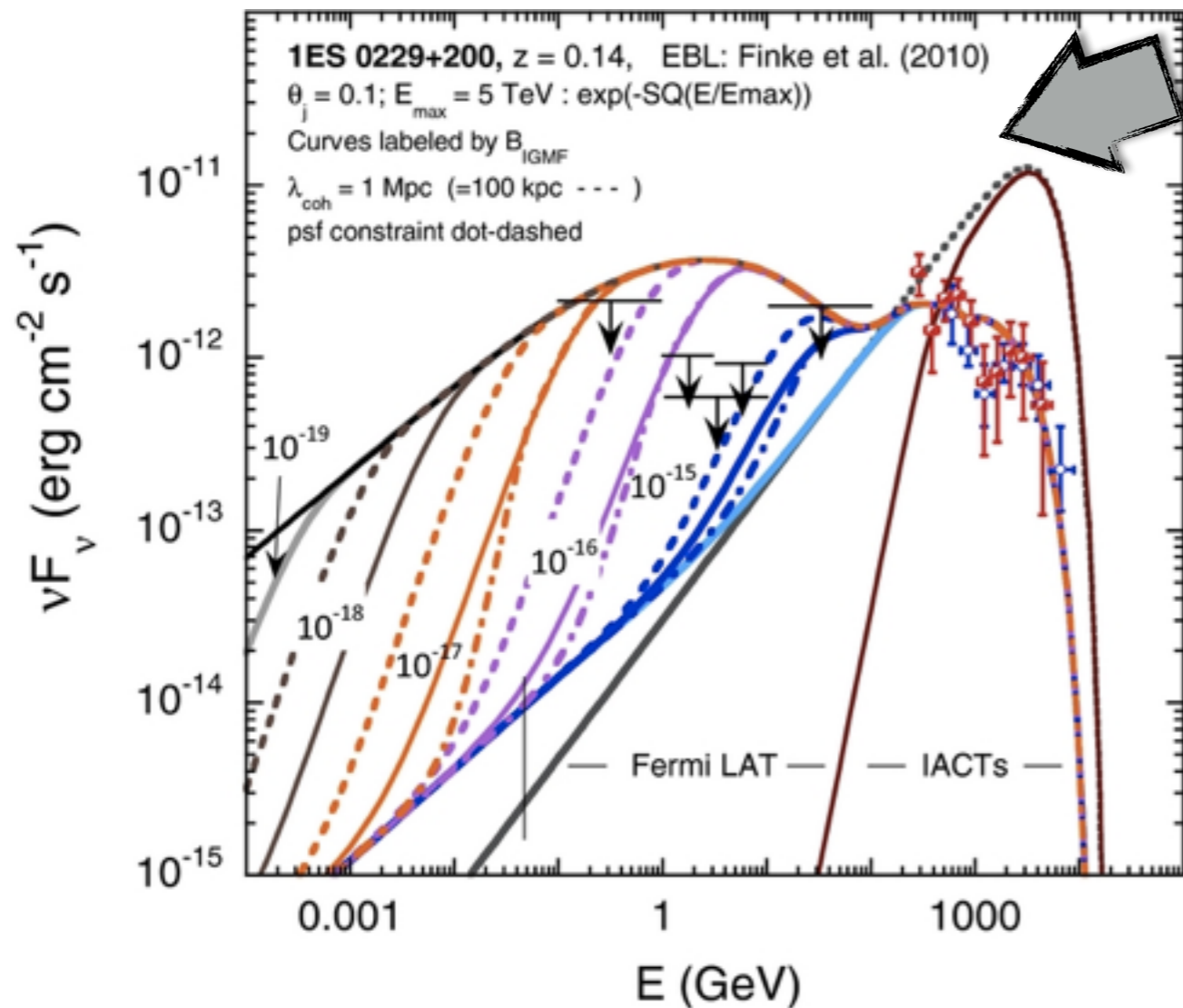
$$B \gtrsim 3 \times 10^{-16} \text{ G}$$

On the steady state assumption

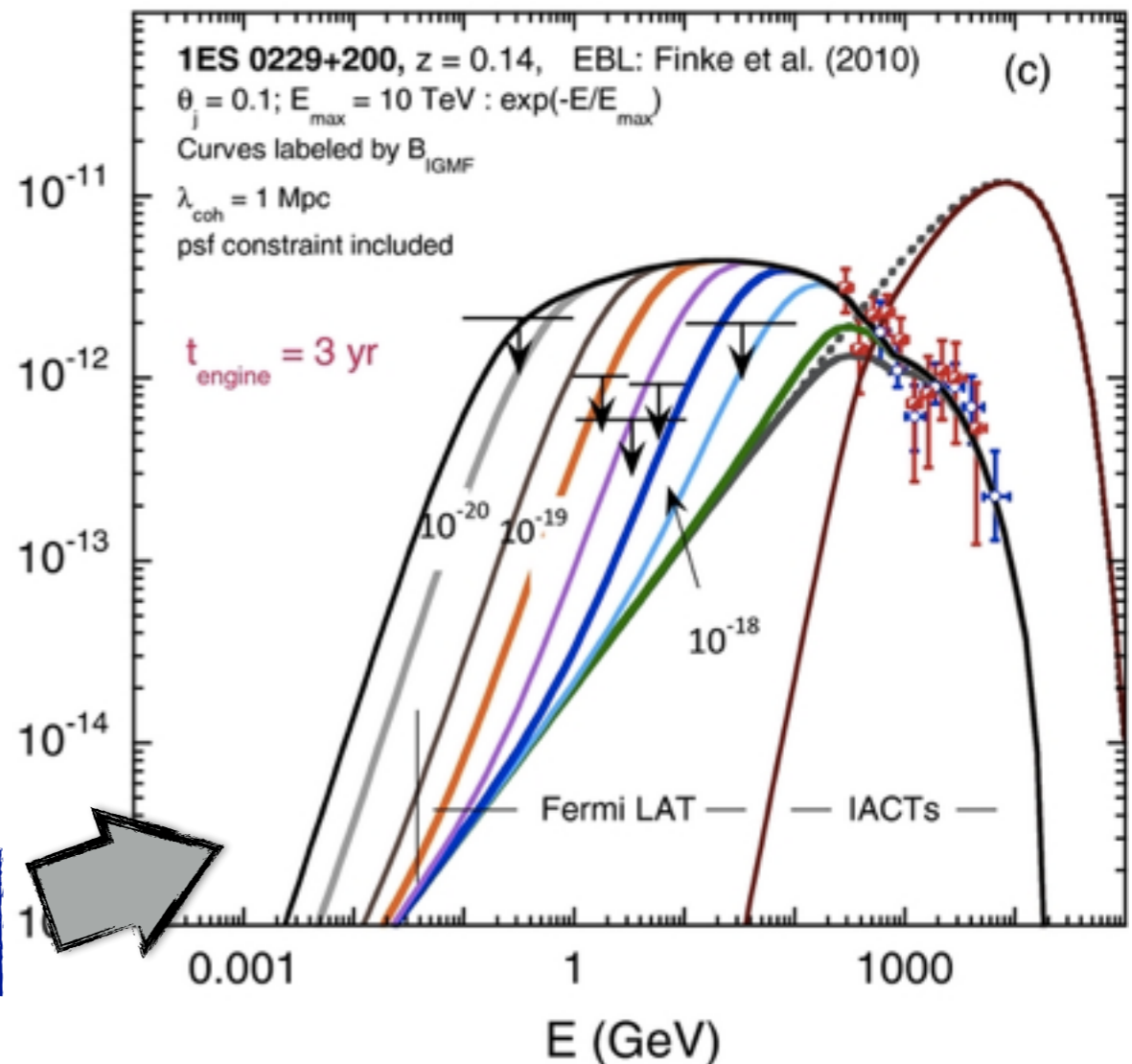
how do we know whether the emission from a blazar is steady or not?

steady source

$$B \gtrsim 3 \times 10^{-16} \text{ G}$$



νF_{ν} (erg cm $^{-2}$ s $^{-1}$)



$$B \gtrsim 10^{-18} \text{ G}$$

3 yr activity

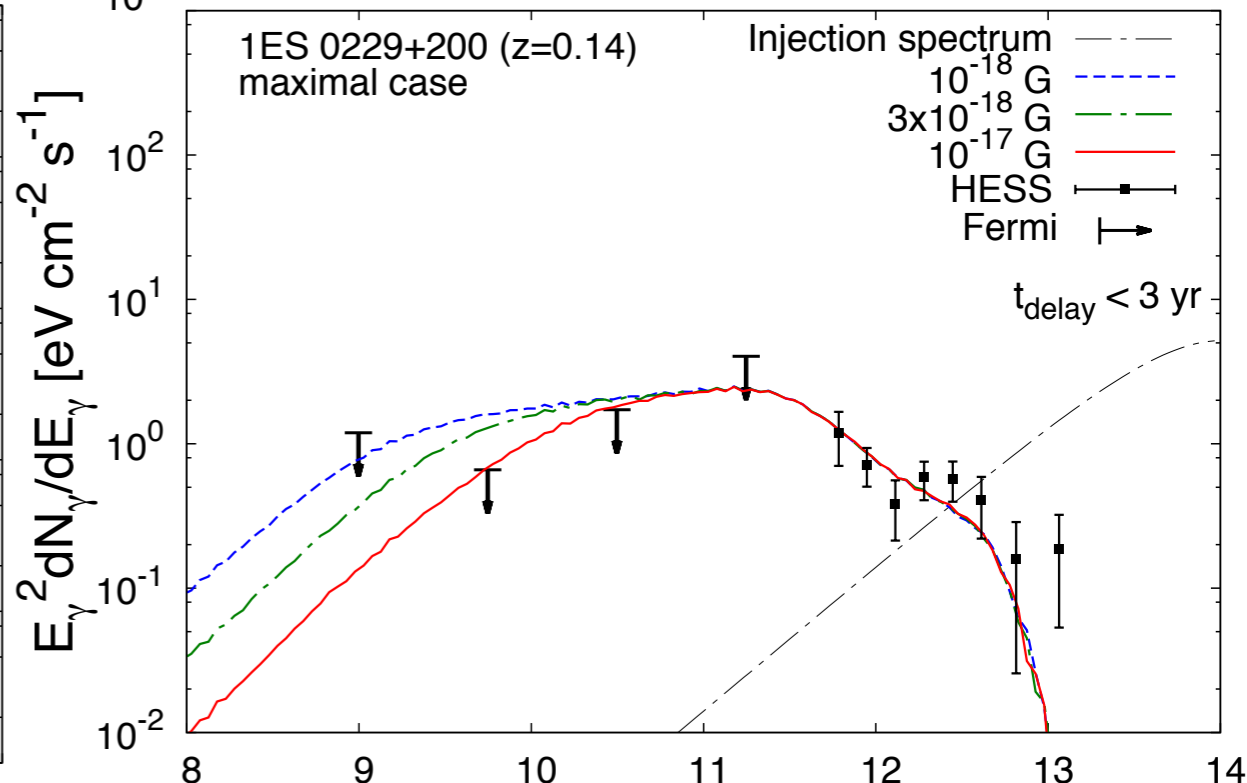
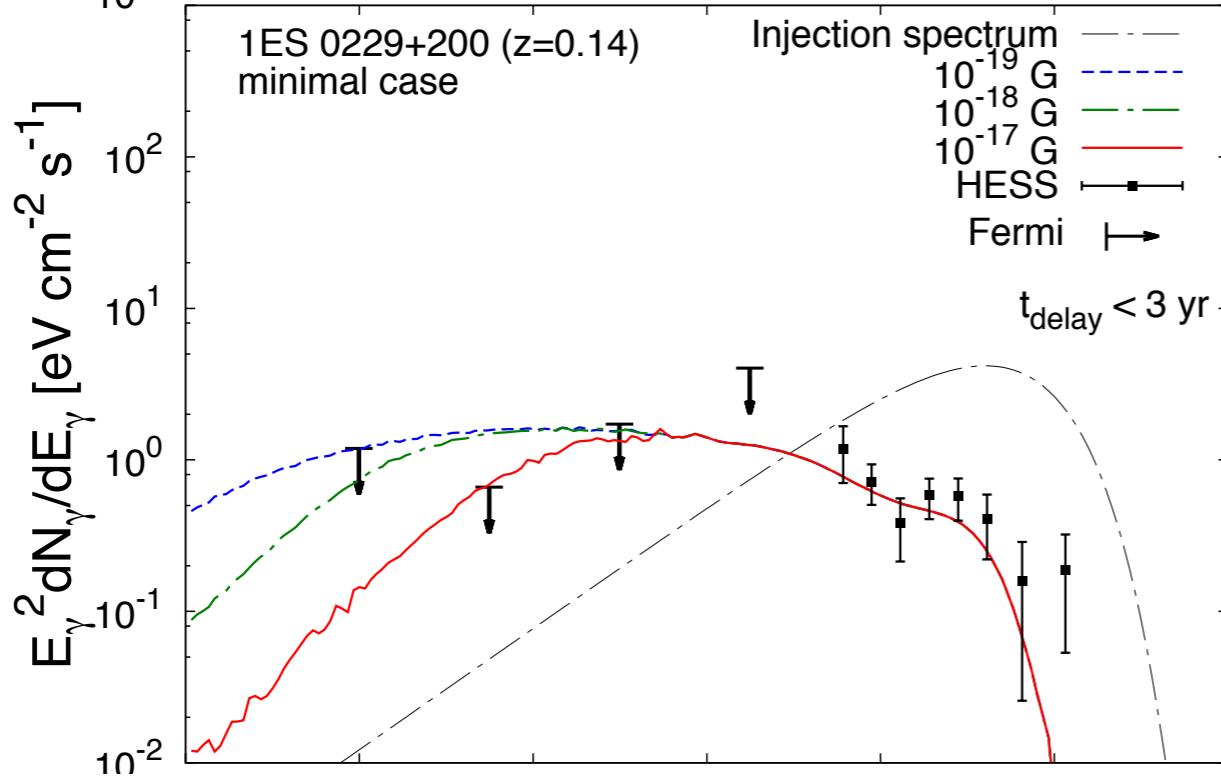
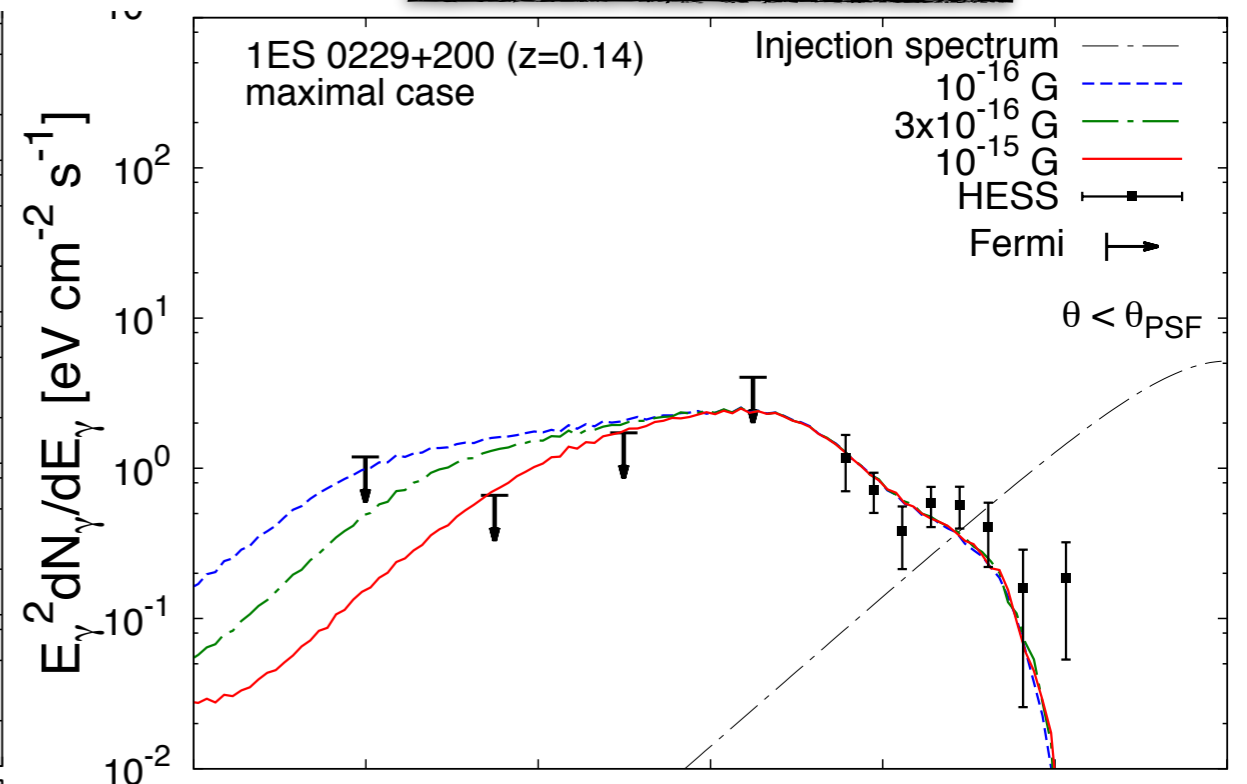
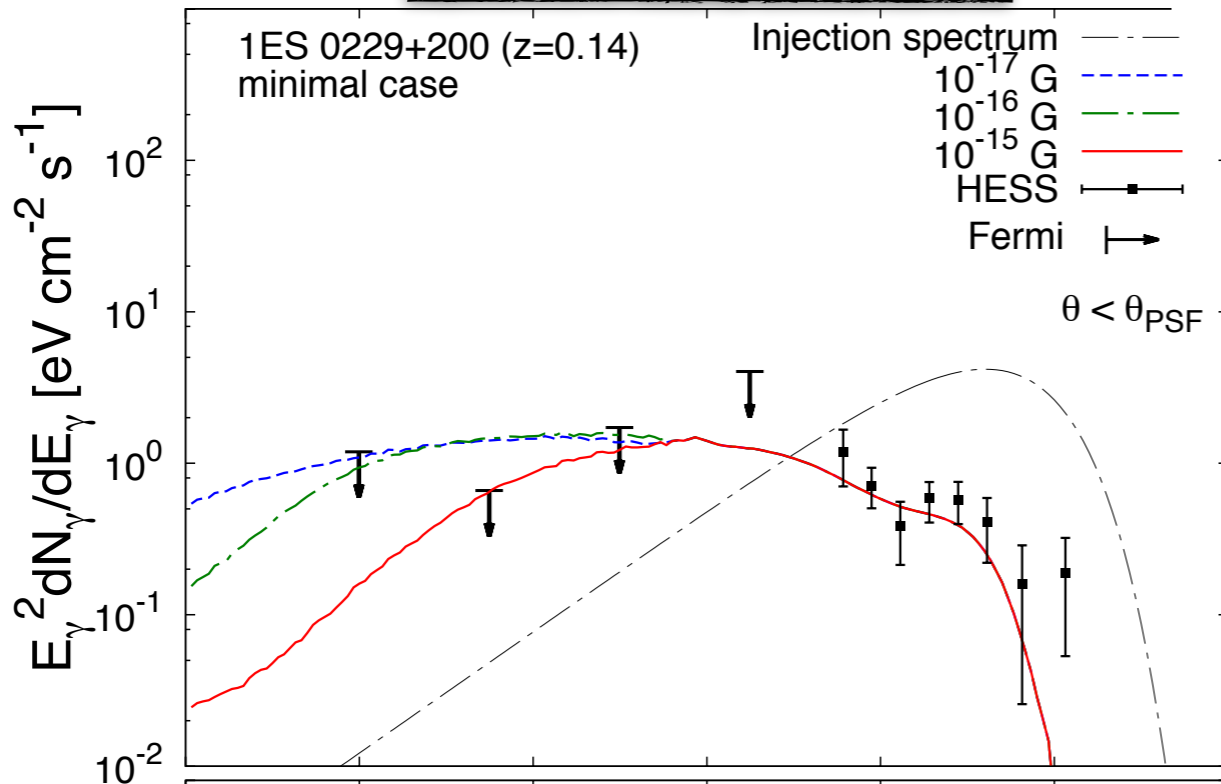
Monte Carlo simulations

minimal cascade

maximal cascade

PSF limit

time delay limit



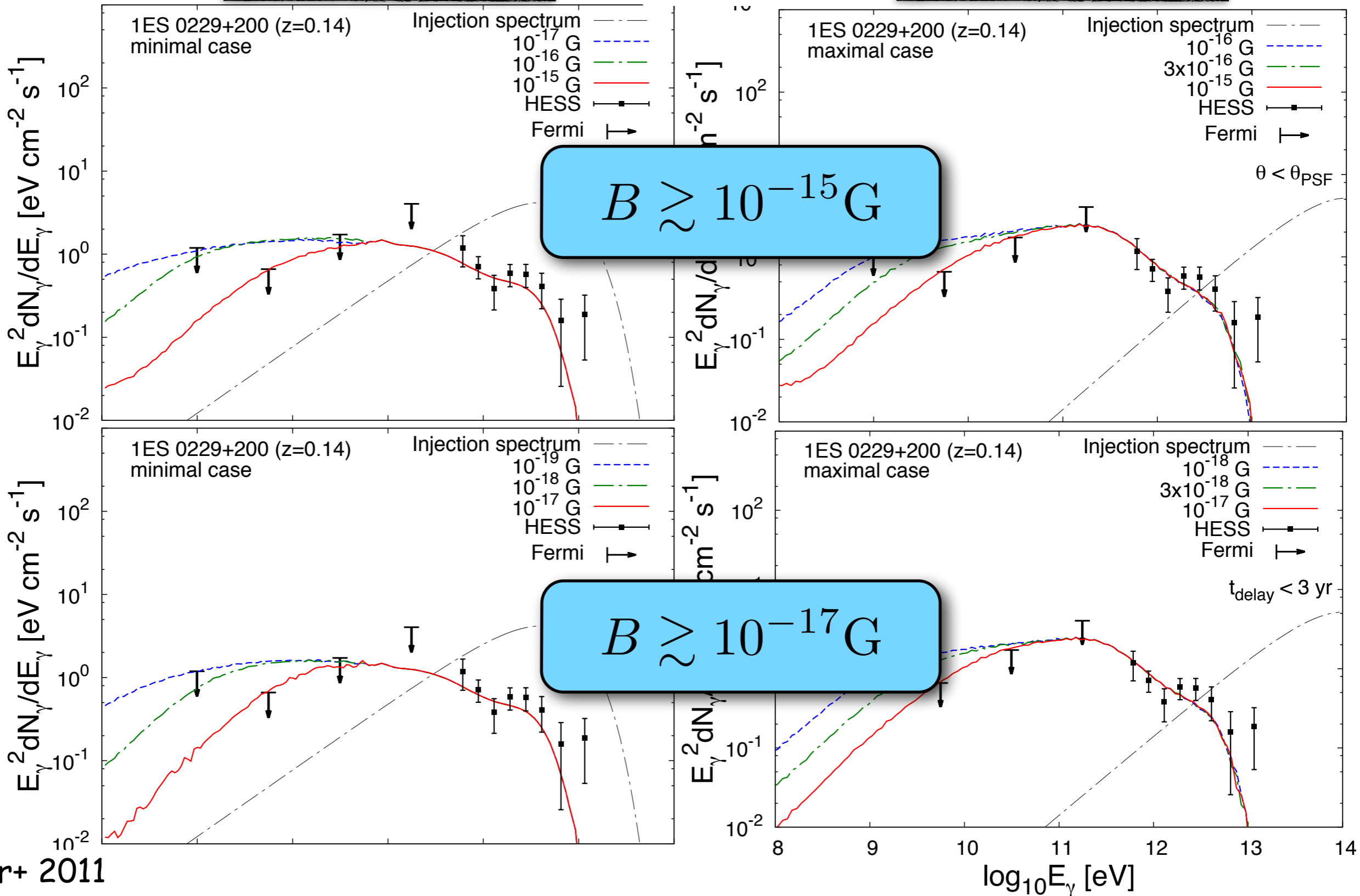
Monte Carlo simulations

minimal cascade

maximal cascade

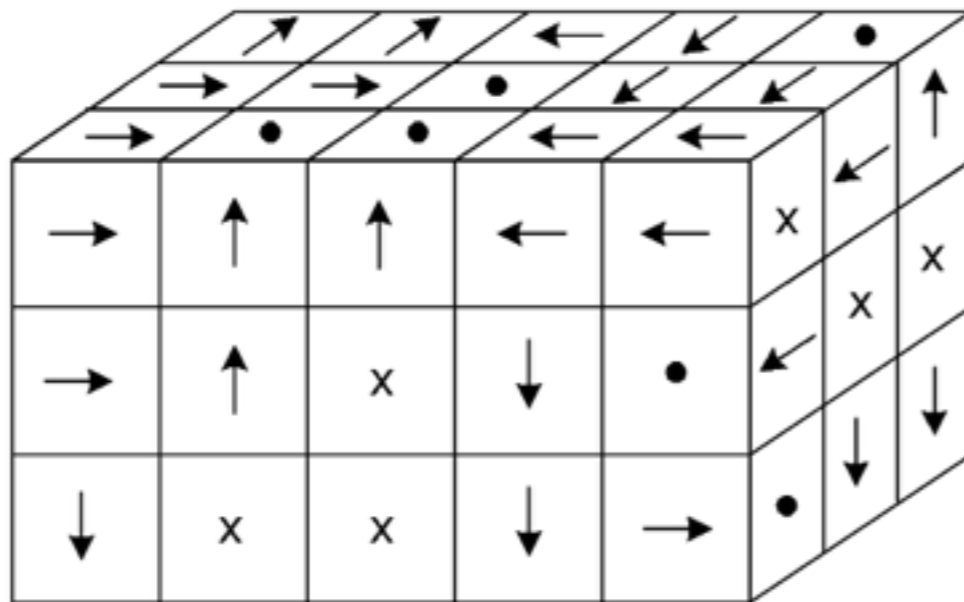
PSF limit

time delay limit



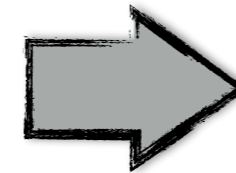
The role of the power spectrum

many works assumed cells with uniform B randomly oriented



I

λ_B

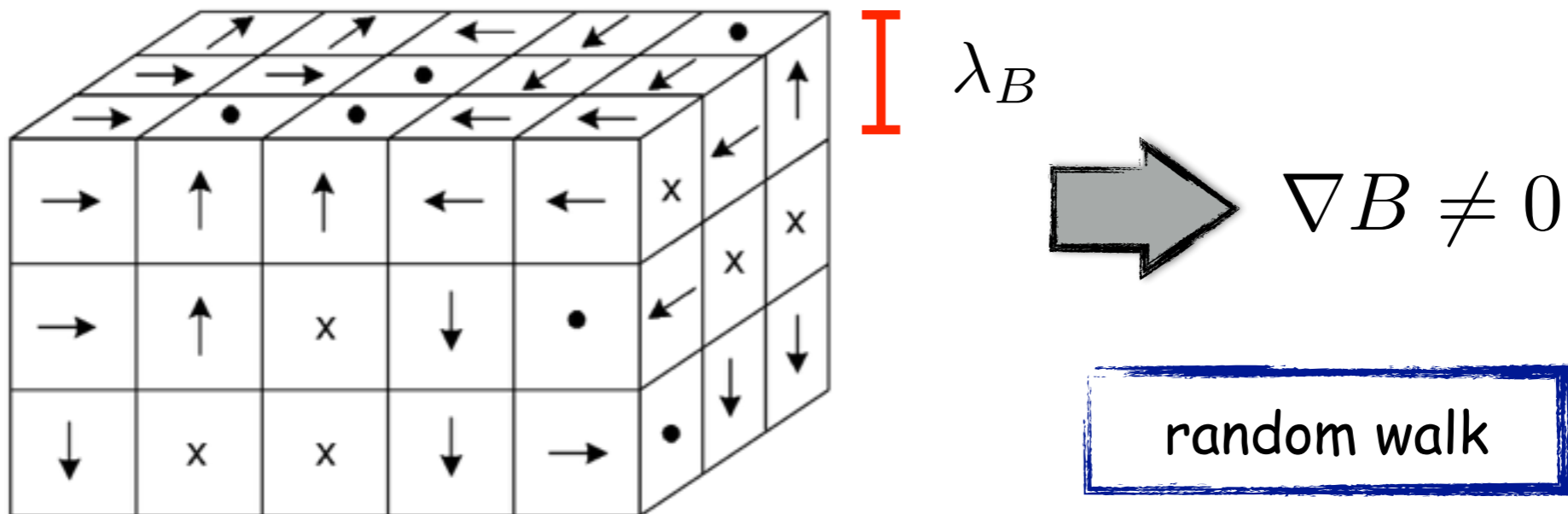


$\nabla B \neq 0$

random walk

The role of the power spectrum

many works assumed cells with uniform B randomly oriented



more realistically...

$$\nabla B = 0$$

and

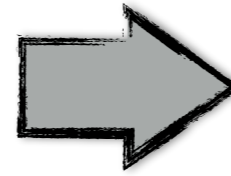
$$P_B(k) = A \begin{cases} \left(\frac{k}{k_0}\right)^{n_B} & \text{if } k < k_0 \\ \left(\frac{k_0}{k}\right)^{m_B} & \text{if } k > k_0 \end{cases}$$

$n_B > -3$ and $m_B > 3$ to avoid divergencies

Coherence length and deflections

$$P_B(k) = A \begin{cases} \left(\frac{k}{k_0}\right)^{n_B} & \text{if } k < k_0 \\ \left(\frac{k_0}{k}\right)^{m_B} & \text{if } k > k_0 \end{cases}$$

11/3 (Kolmogorov)



operative definition

$$\lambda_B \equiv \frac{2\pi}{k_0}$$

Coherence length and deflections

$$P_B(k) = A \begin{cases} \left(\frac{k}{k_0}\right)^{n_B} & \text{if } k < k_0 \\ \left(\frac{k_0}{k}\right)^{m_B} & \text{if } k > k_0 \end{cases}$$

11/3 (Kolmogorov)

operative definition

$$\lambda_B \equiv \frac{2\pi}{k_0}$$

coherence length $\rightarrow \lambda_B \sim \int dk P_B(k) k$

not well defined
for $n_B > -2$

**-3 < n_B < -2 \rightarrow red spectrum \rightarrow no causal origin
 \rightarrow might be generated during inflation**

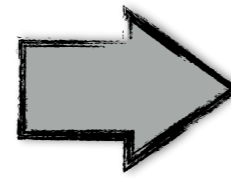
Coherence length and deflections

$$P_B(k) = A \begin{cases} \left(\frac{k}{k_0}\right)^{n_B} & \text{if } k < k_0 \\ \left(\frac{k_0}{k}\right)^{m_B} & \text{if } k > k_0 \end{cases}$$

11/3 (Kolmogorov)

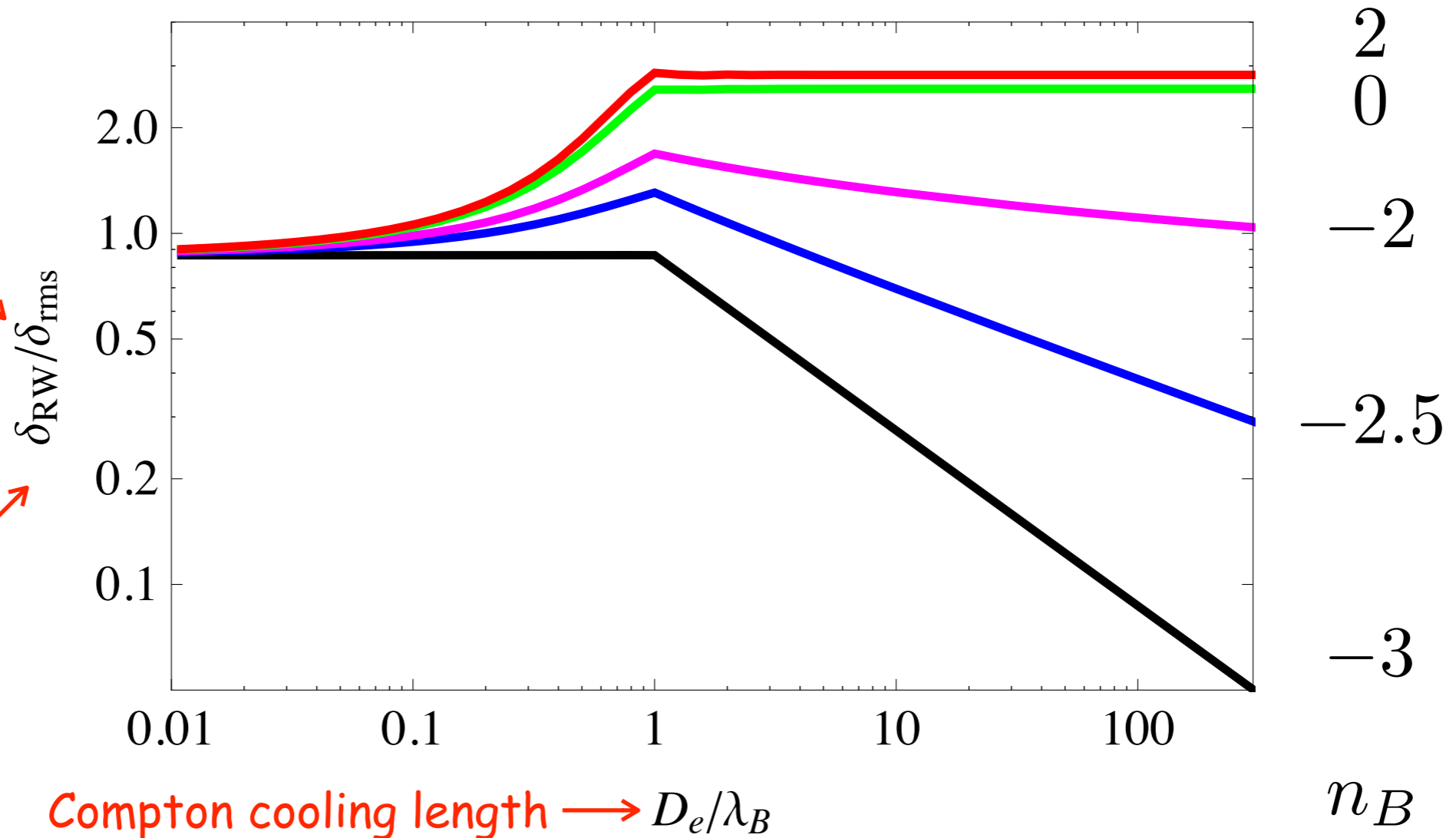
operative definition

$$\lambda_B \equiv \frac{2\pi}{k_0}$$



"this work"
deflection

"random walk"
deflection



Coherence length and deflections

$$P_B(k) = A \begin{cases} \left(\frac{k}{k_0}\right)^{n_B} & \text{if } k < k_0 \\ \left(\frac{k_0}{k}\right)^{m_B} & \text{if } k > k_0 \end{cases}$$

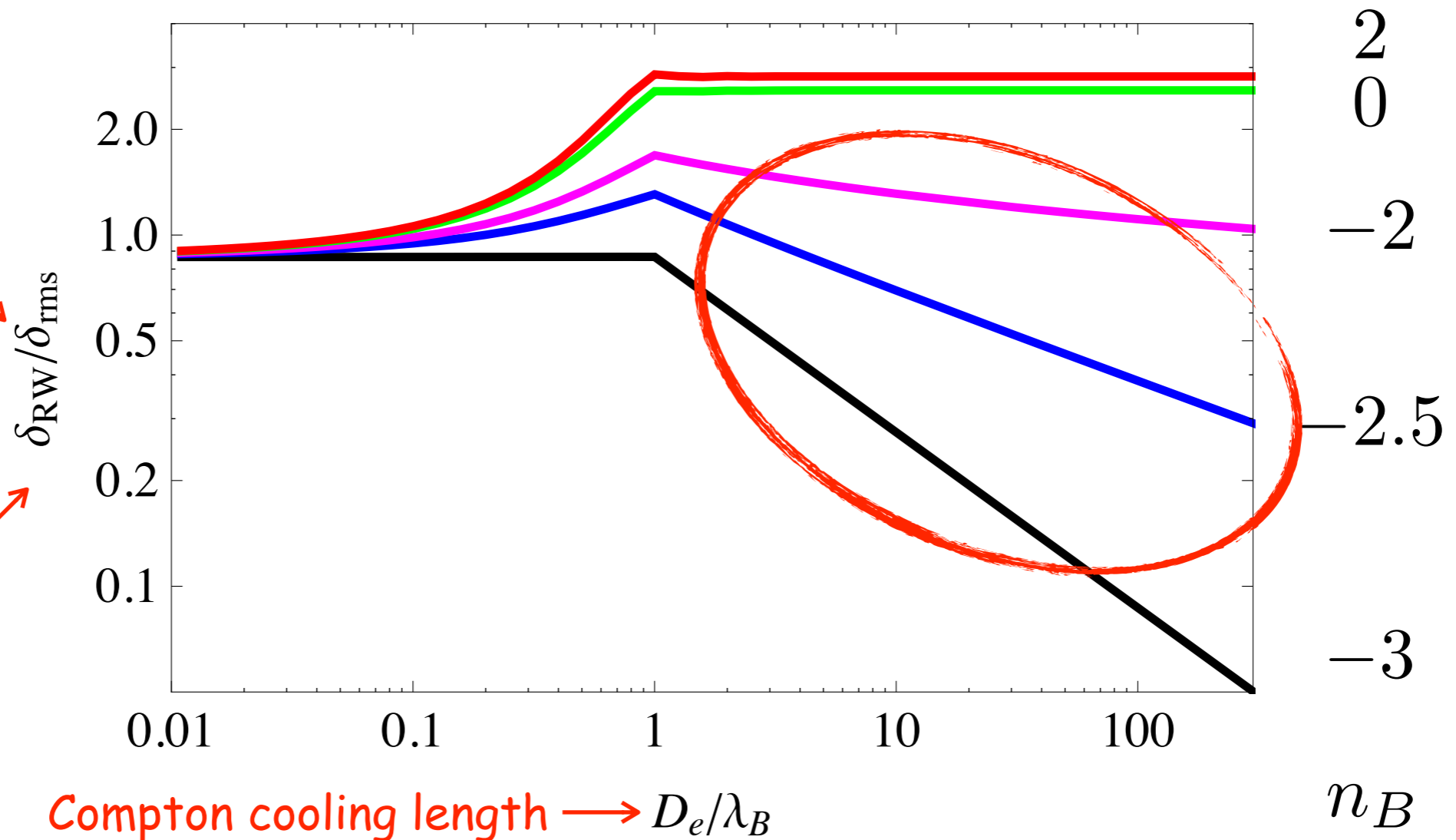
11/3 (Kolmogorov)

operative definition

$$\lambda_B \equiv \frac{2\pi}{k_0}$$

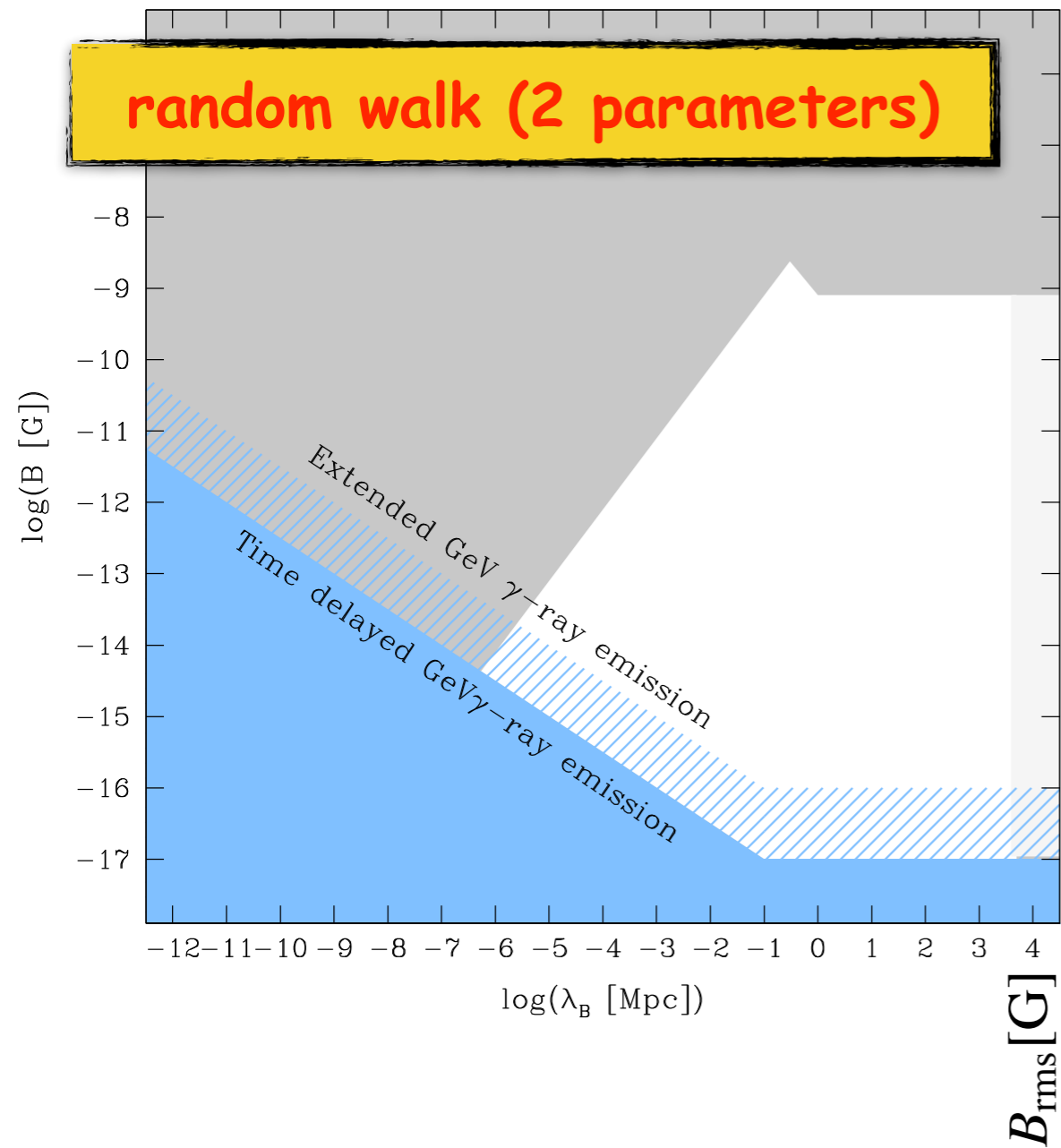
"this work"
deflection

"random walk"
deflection



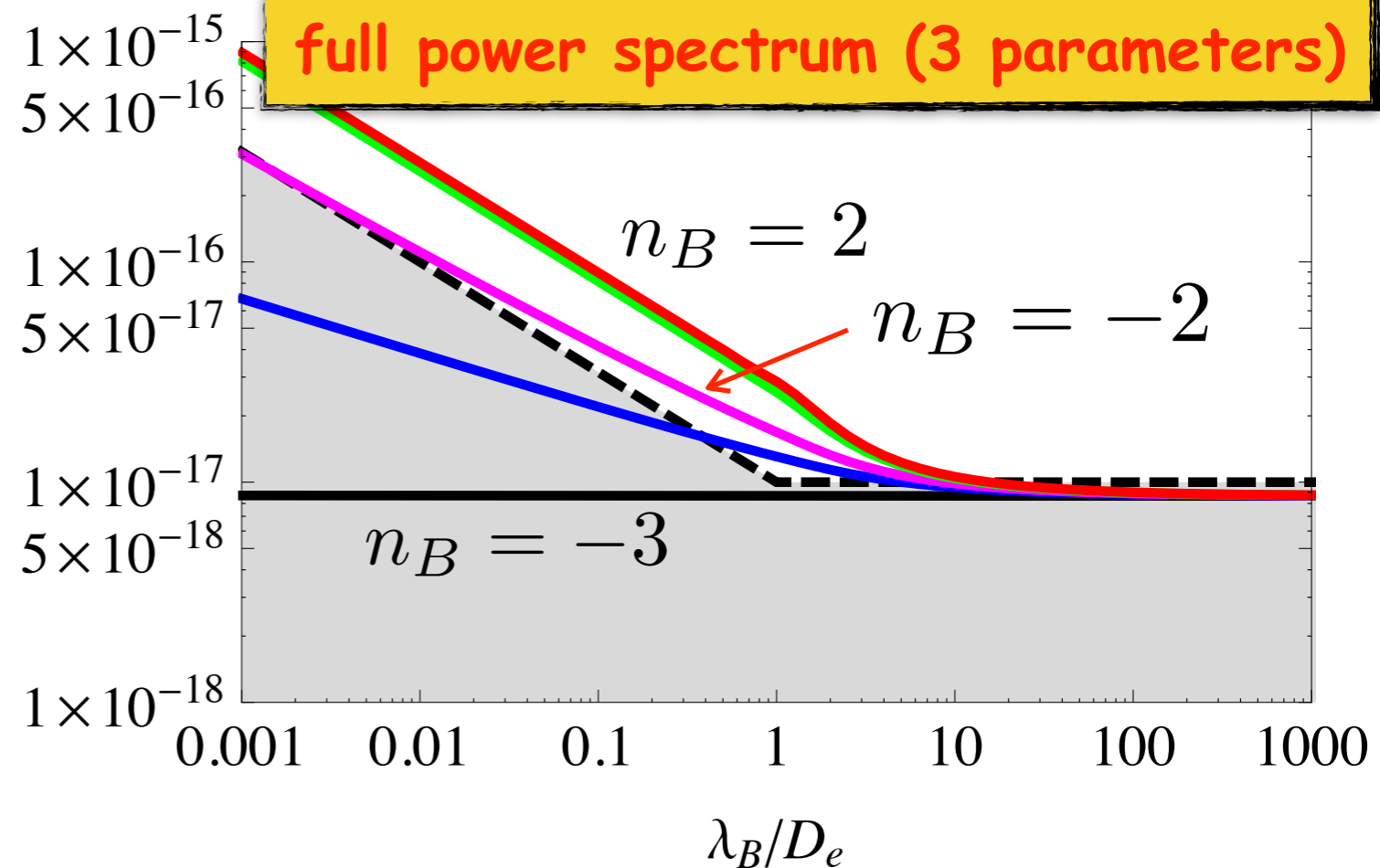
The role of the power spectrum

random walk (2 parameters)



Neronov&Durrer 2013

full power spectrum (3 parameters)

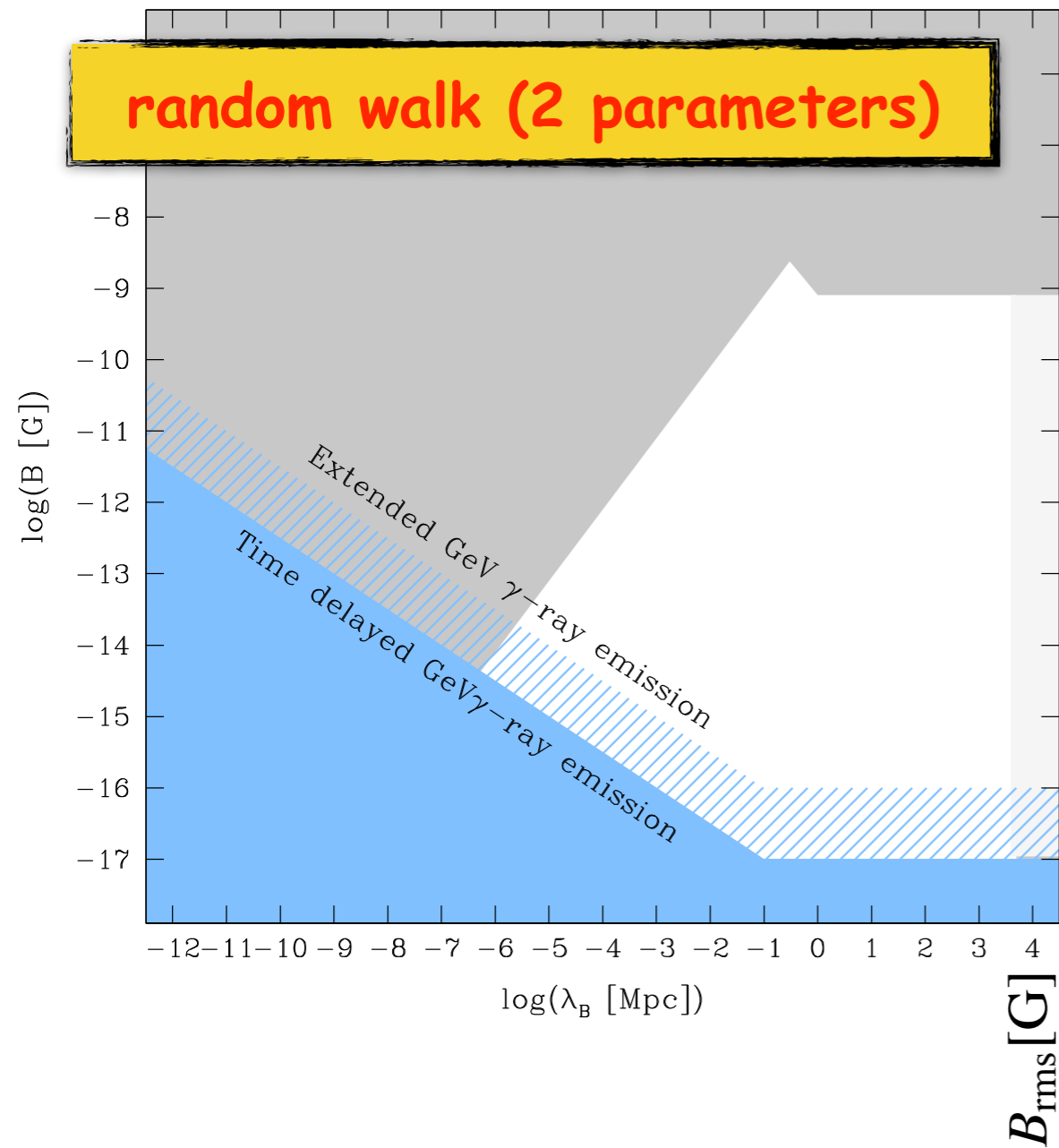


Caprini&Gabici 2015



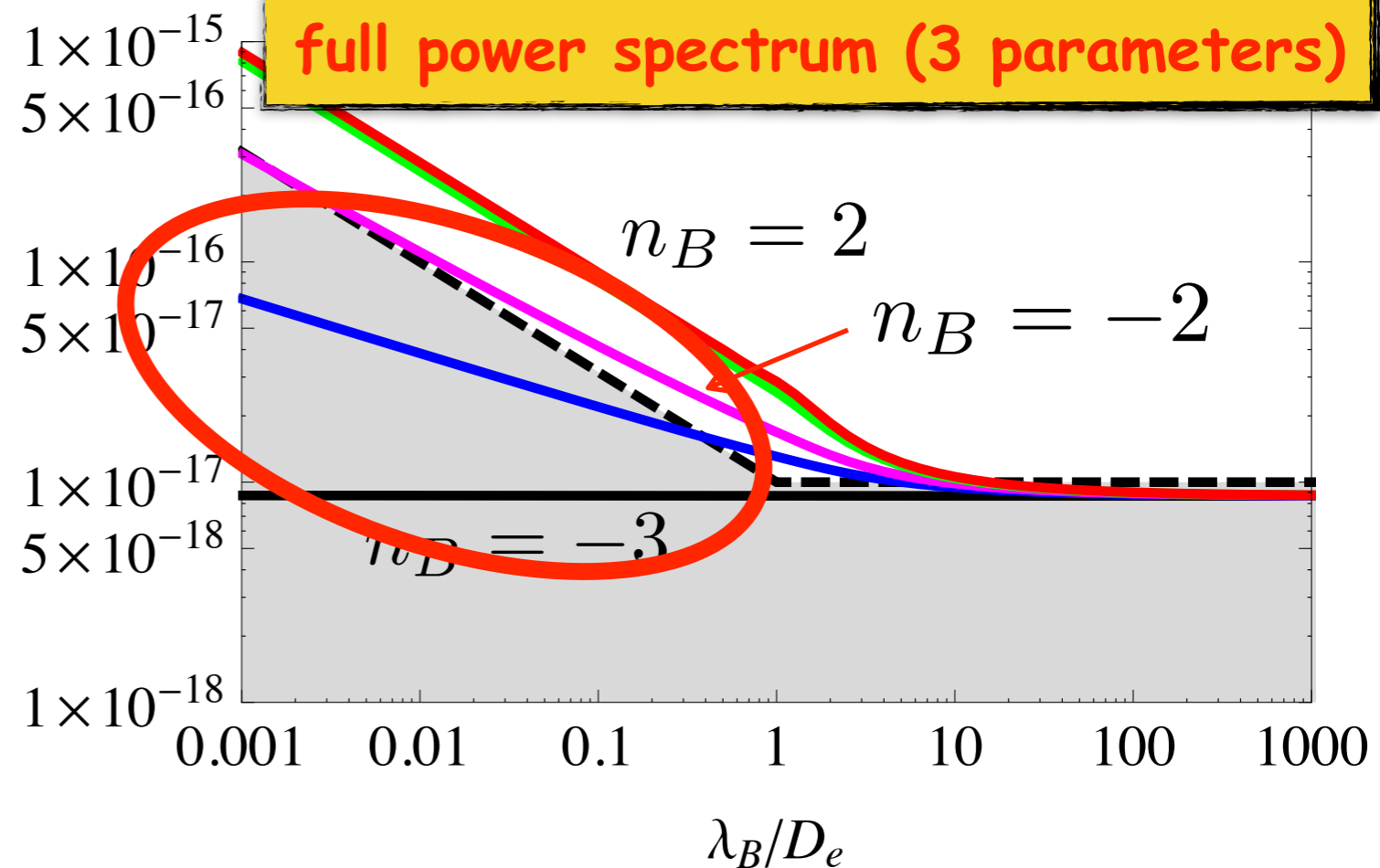
The role of the power spectrum

random walk (2 parameters)



Neronov&Durrer 2013

full power spectrum (3 parameters)



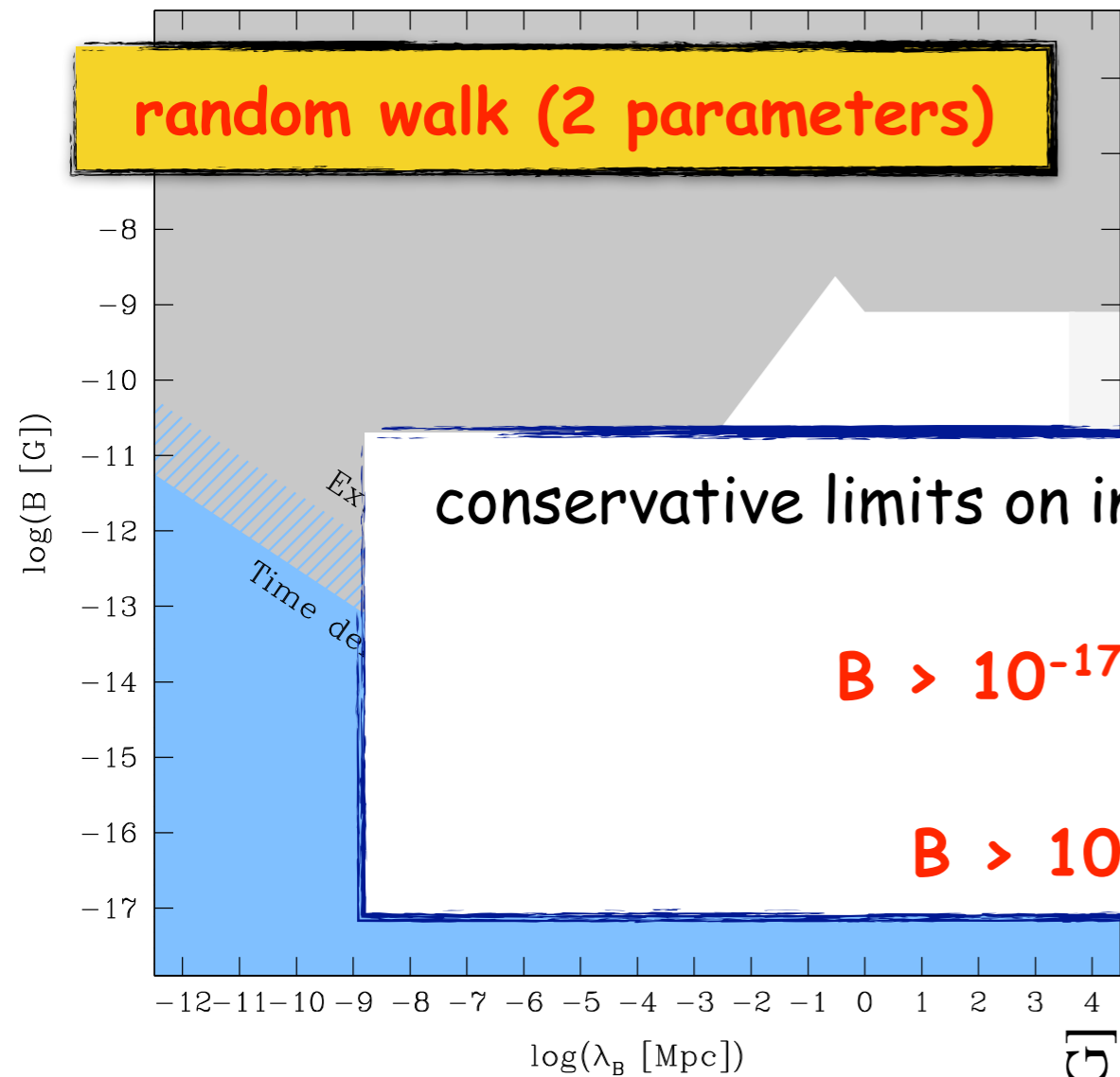
Caprini&Gabici 2015



The role of the power spectrum

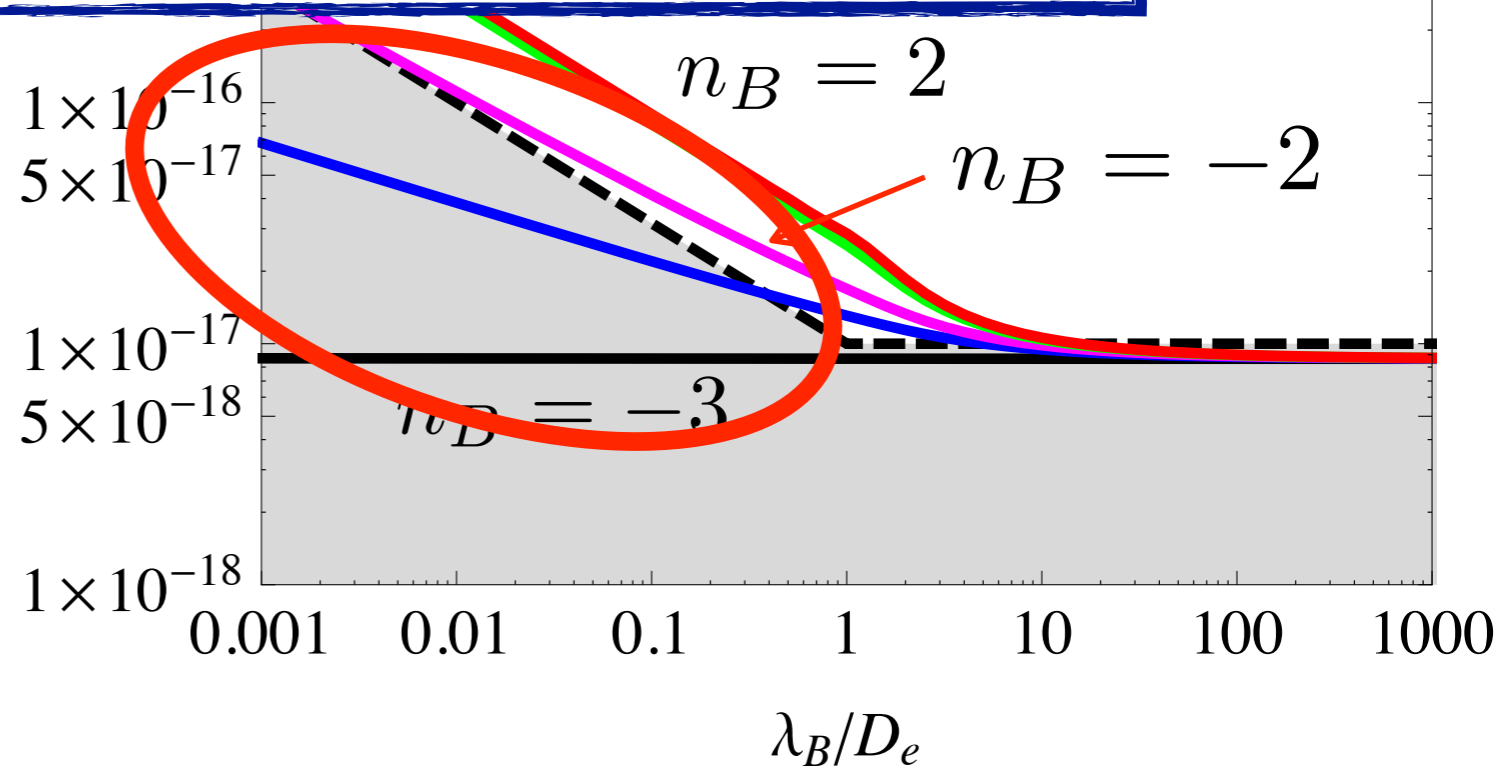
← Neronov&Durrer 2013

random walk (2 parameters)



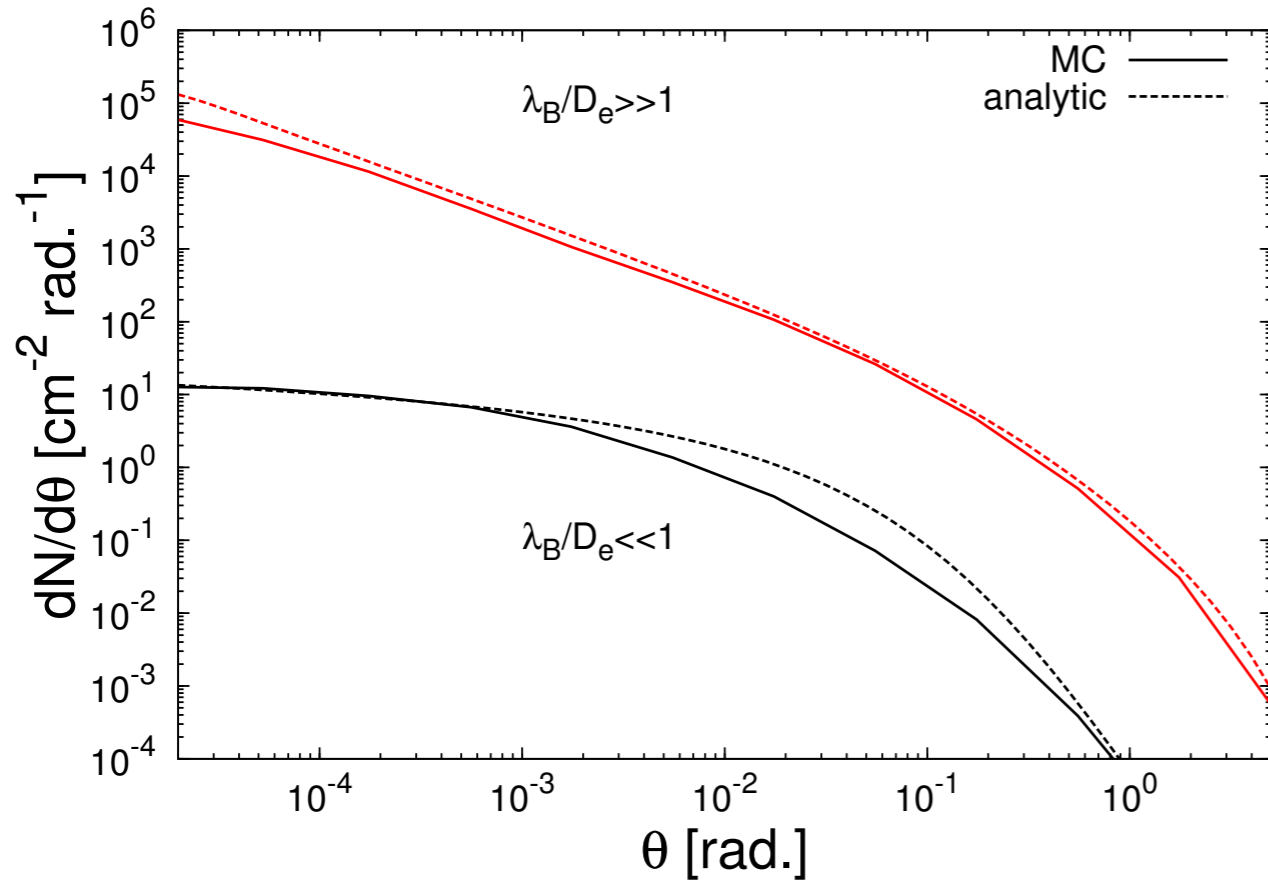
parameters)

Caprini&Gabici 2015 →



Helicity, coherence length...

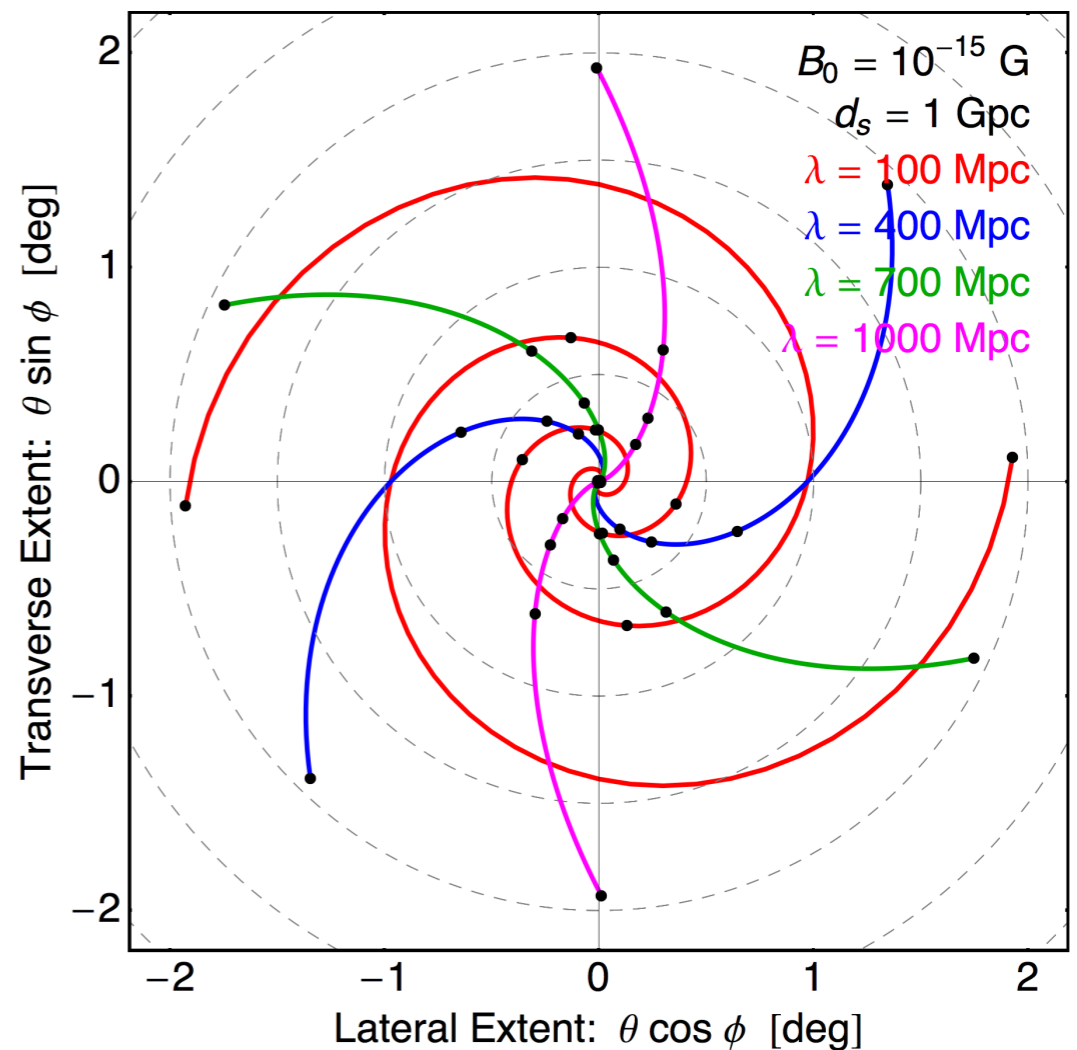
Neronov+ 2013



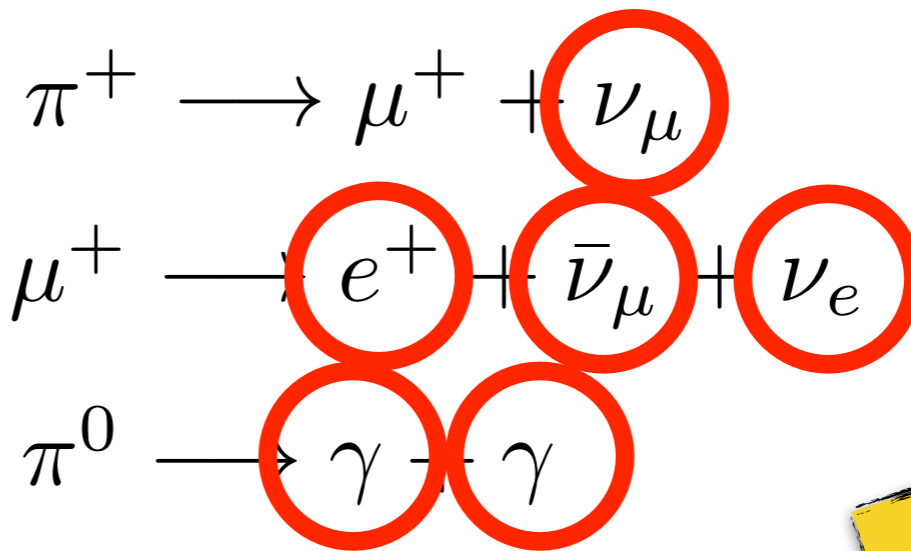
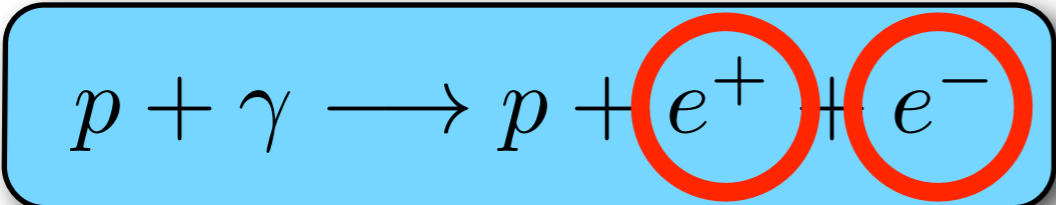
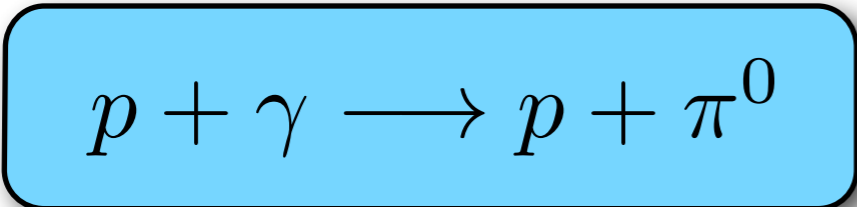
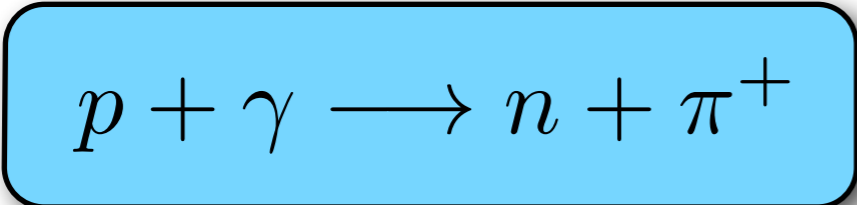
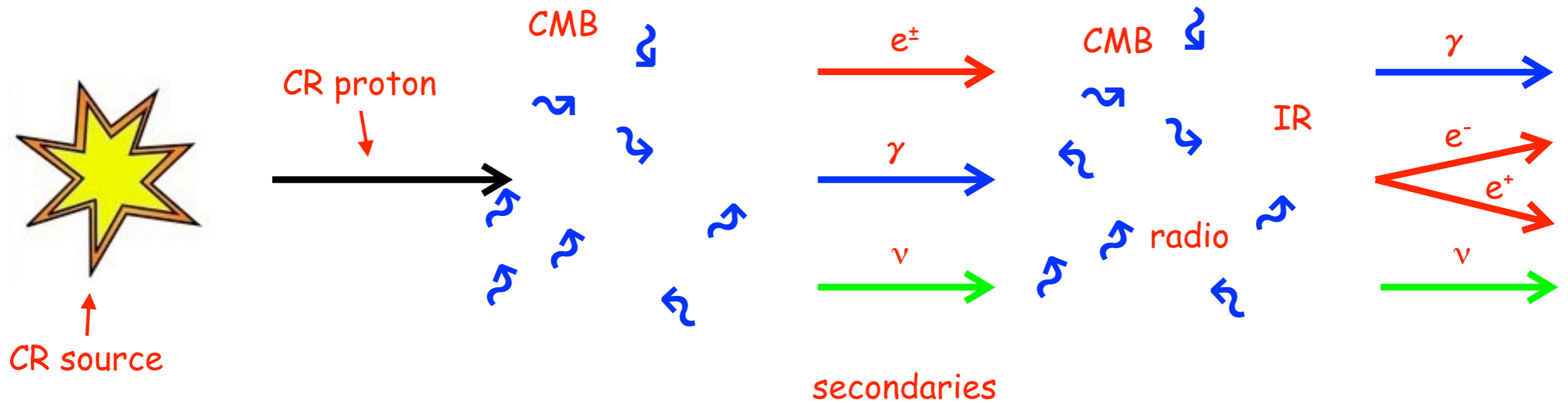
brightness profile of cascade emission for different coherence lengths

signatures of magnetic helicity in the morphology of the cascade emission

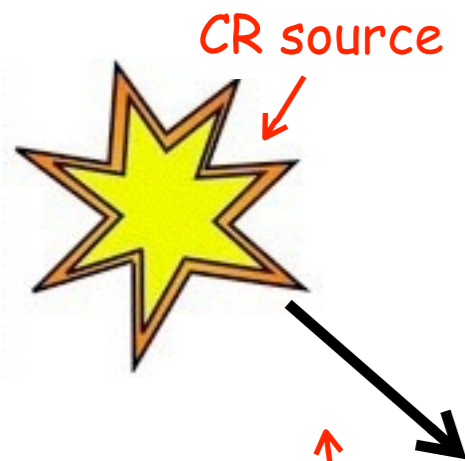
e.g. Long & Vachaspati 2015



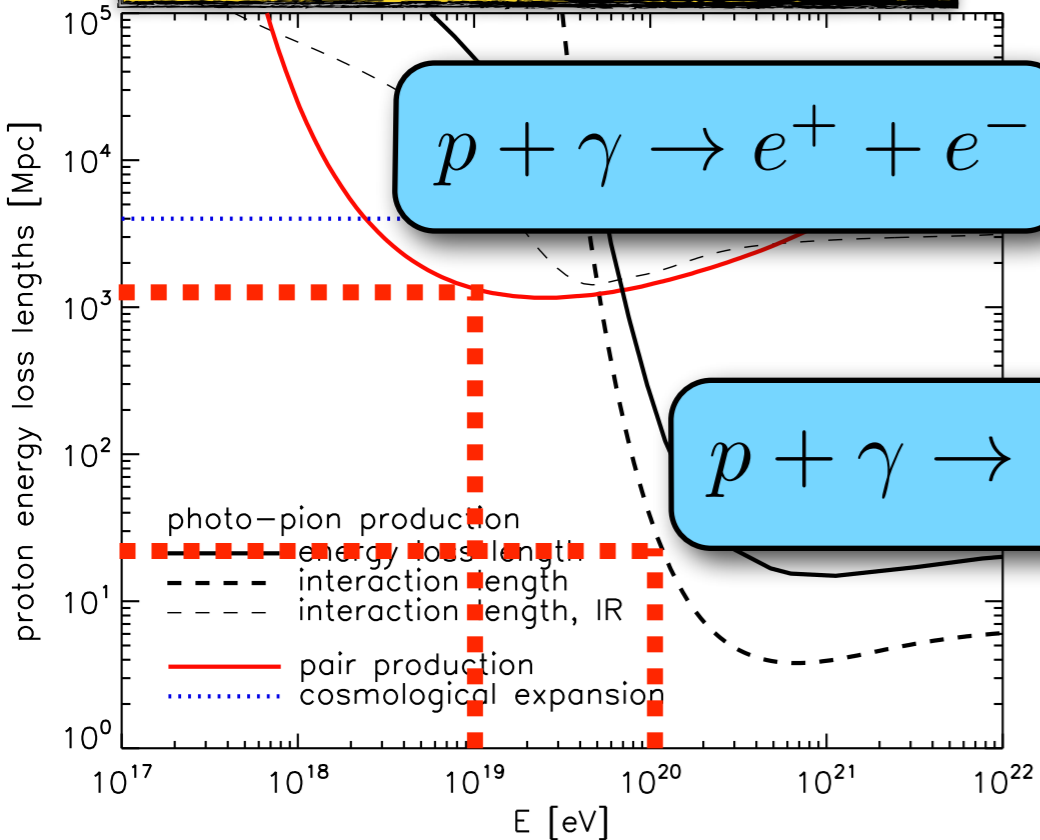
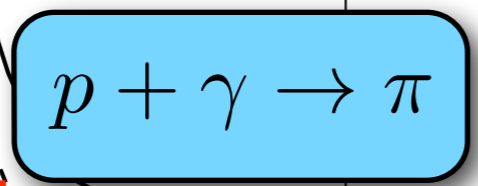
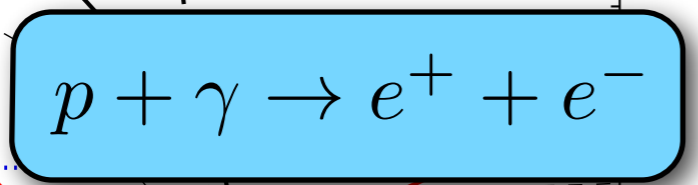
Cascade initiated by Ultra High Energy Cosmic Rays



EM cascade
neutrinos

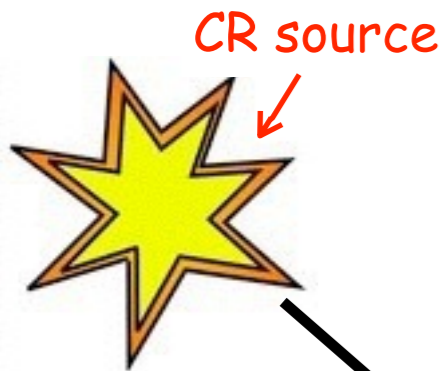


proton interaction length

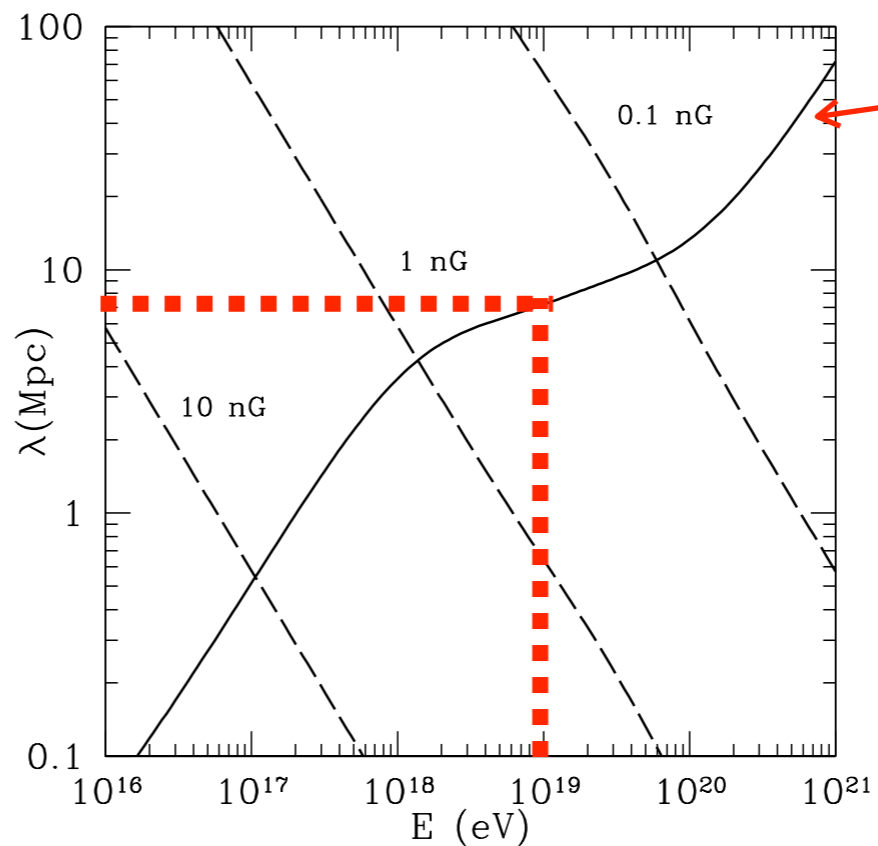


Kotera&Olinto 2011



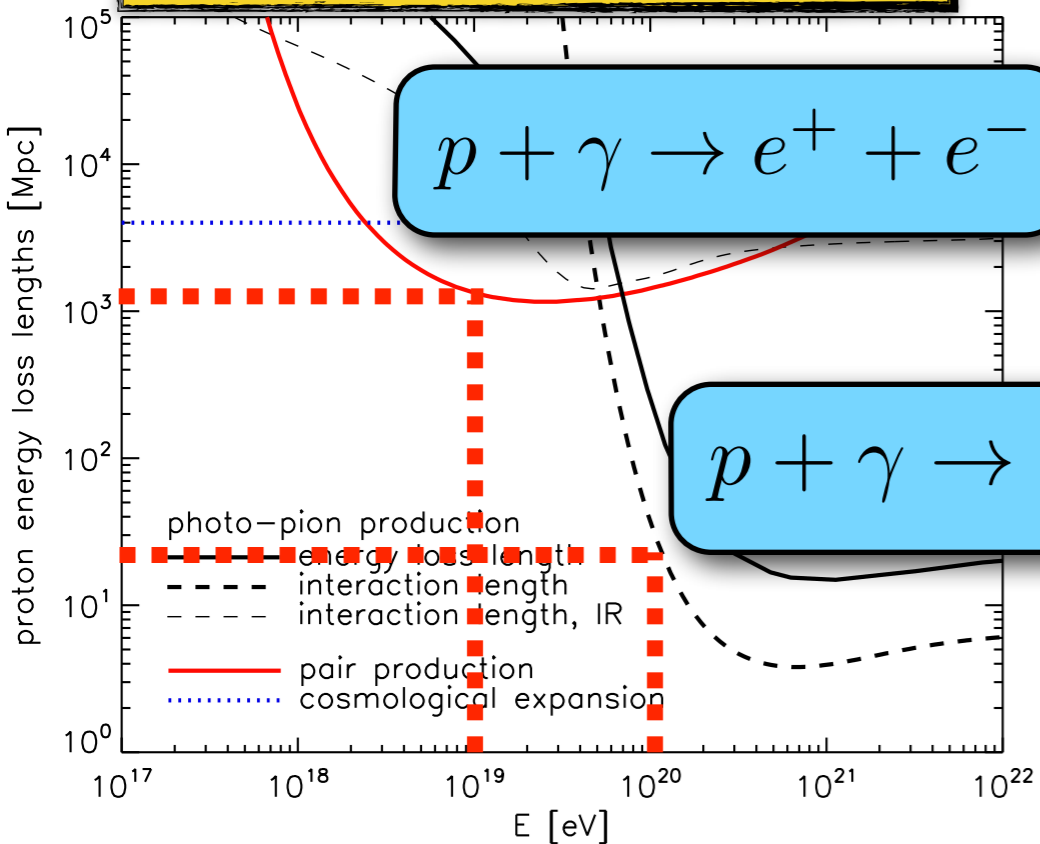
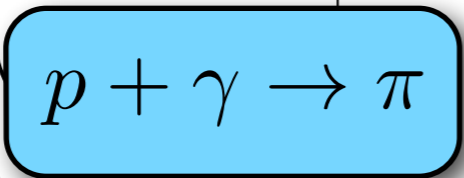
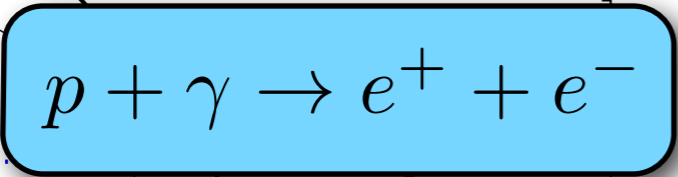


Gabici&Aharonian 2005



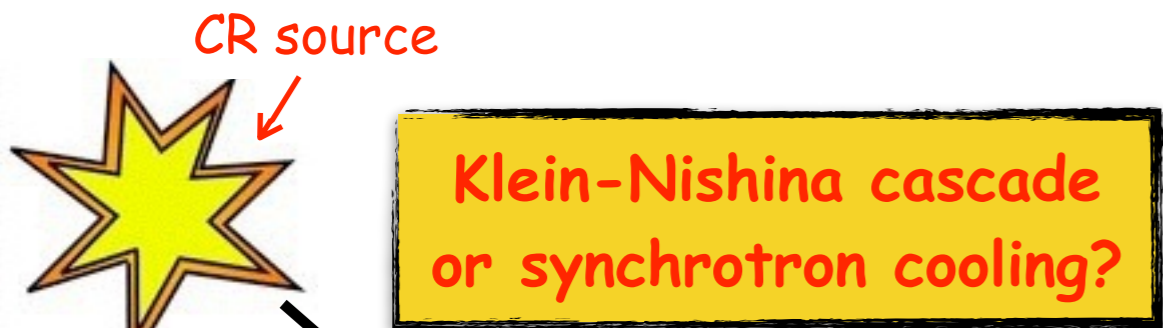
inverse Compton + pair production losses

proton interaction length

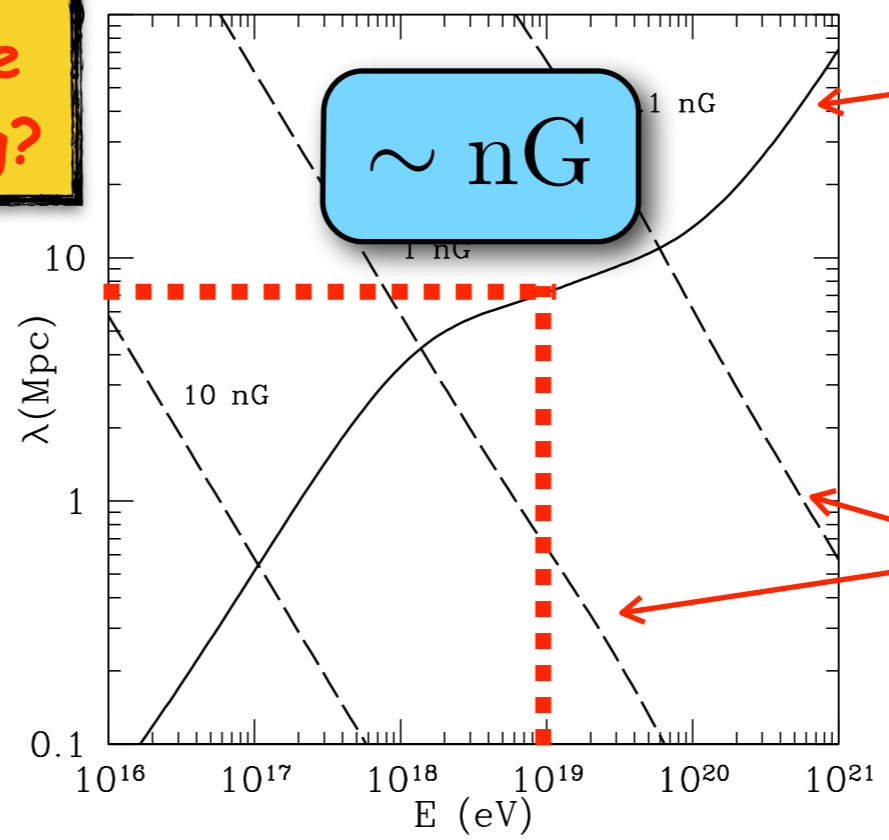


Kotera&Olinto 2011





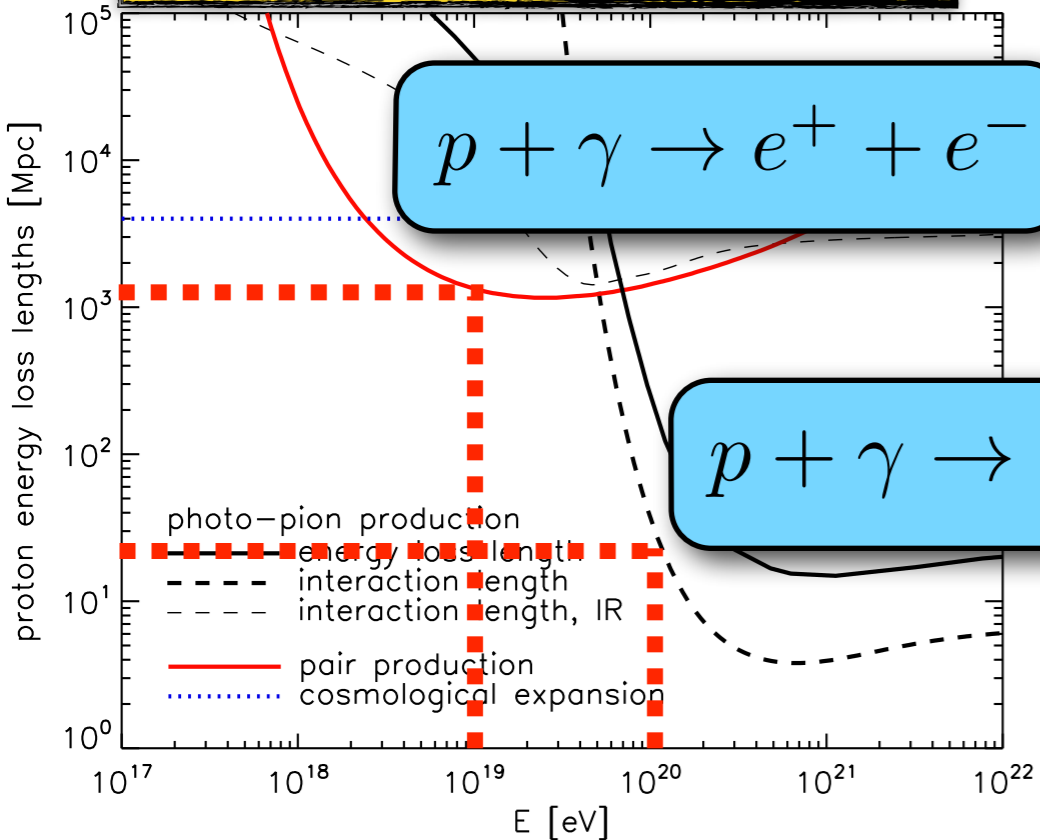
Gabici&Aharonian 2005



inverse Compton + pair production losses

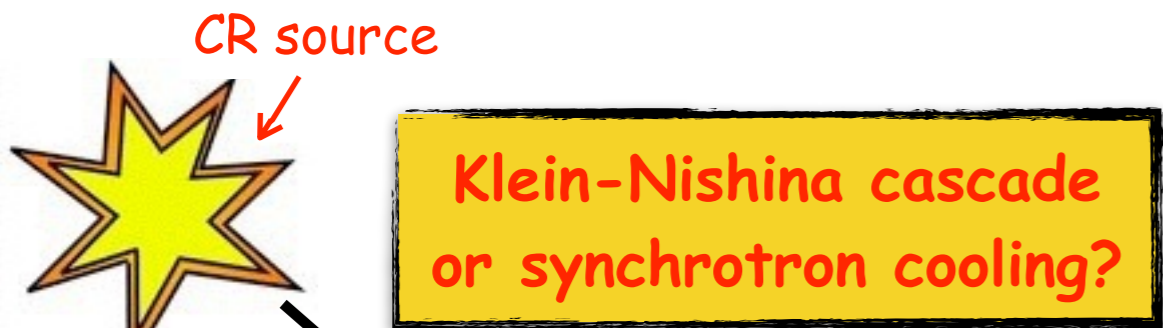
synchrotron losses

proton interaction length

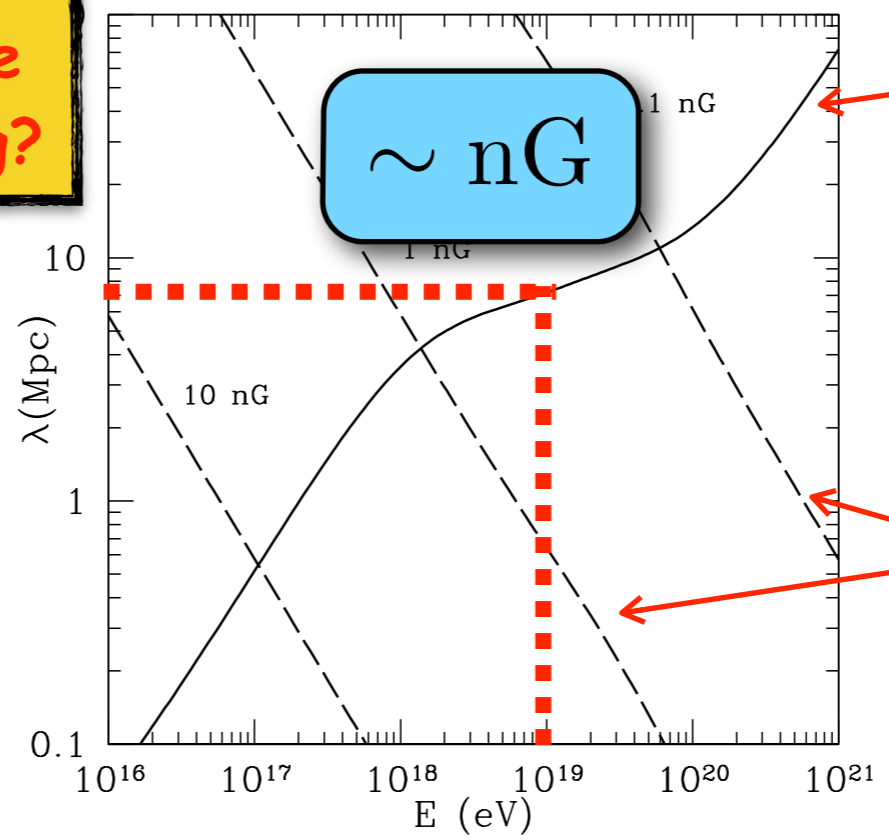


Kotera&Olinto 2011





Gabici&Aharonian 2005

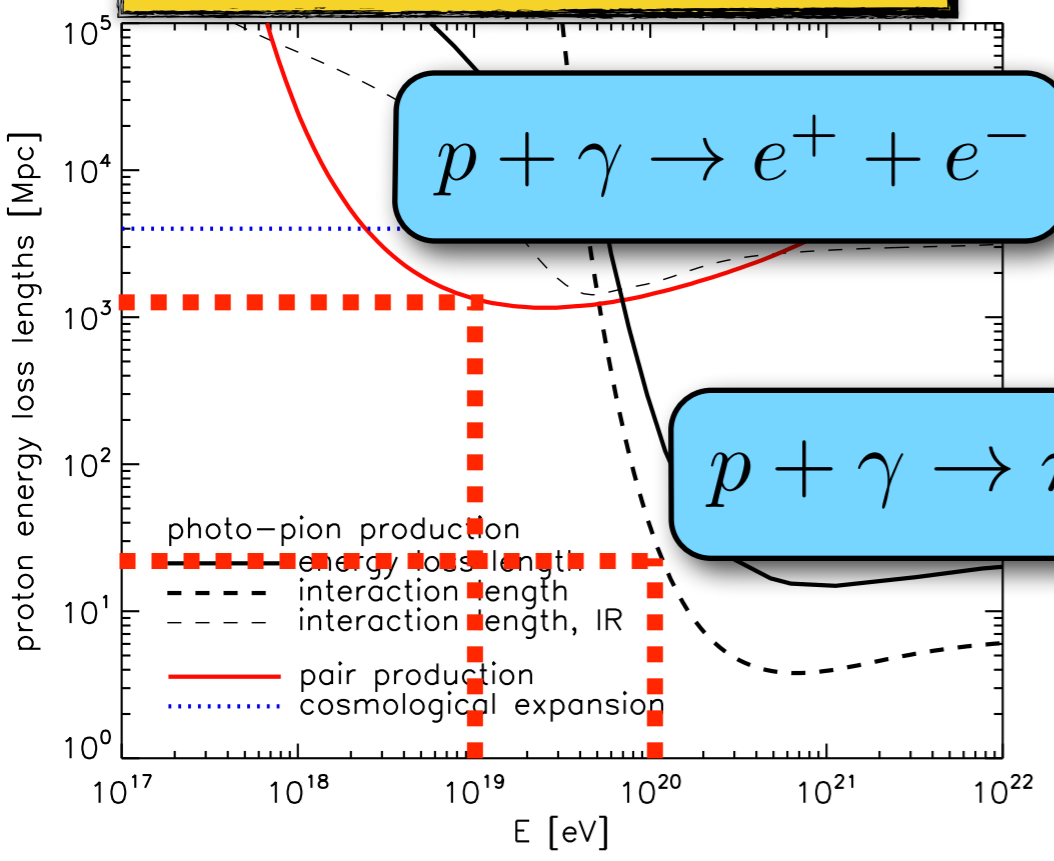


inverse Compton
+ pair production
losses

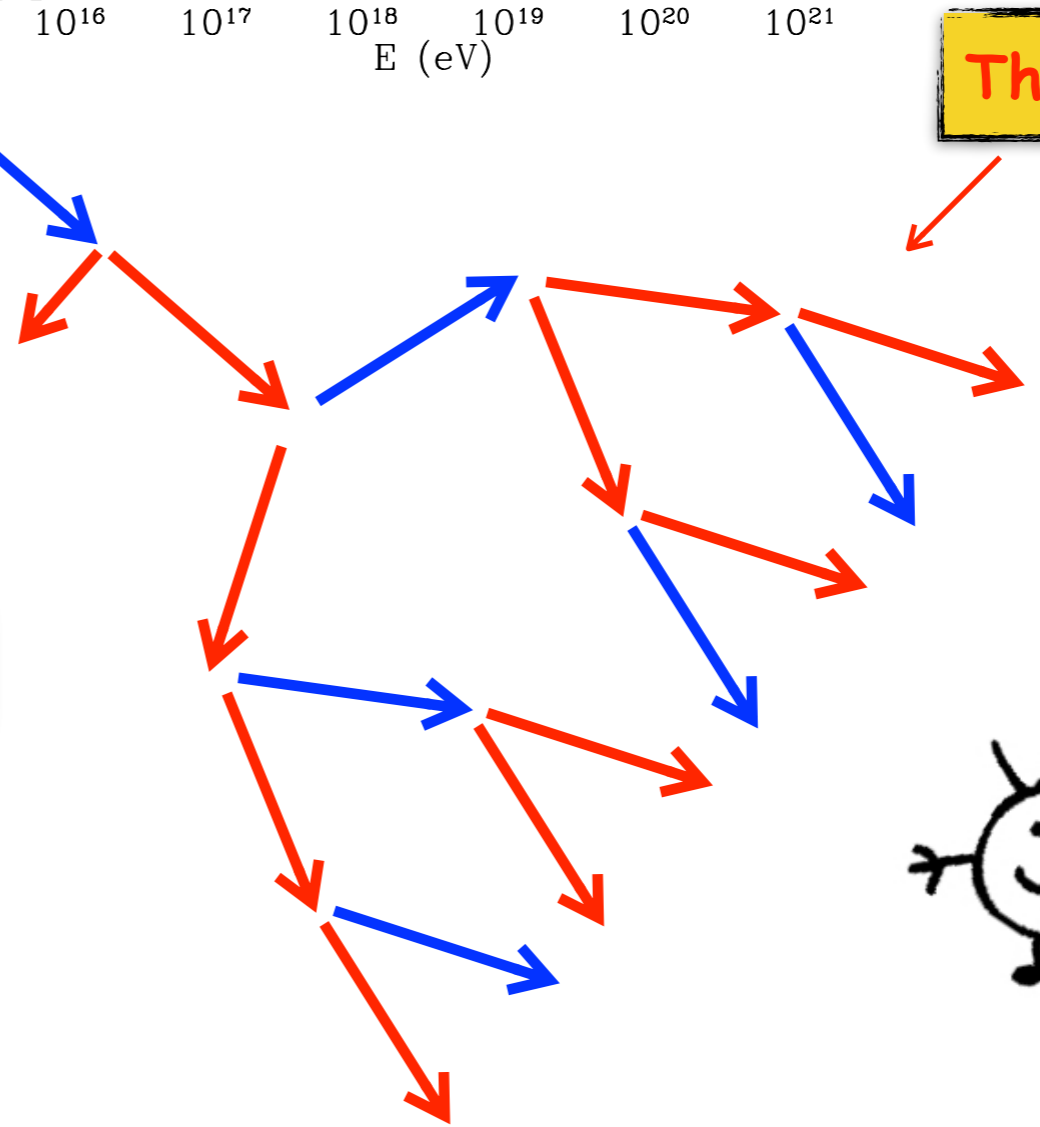
synchrotron losses

proton interaction length

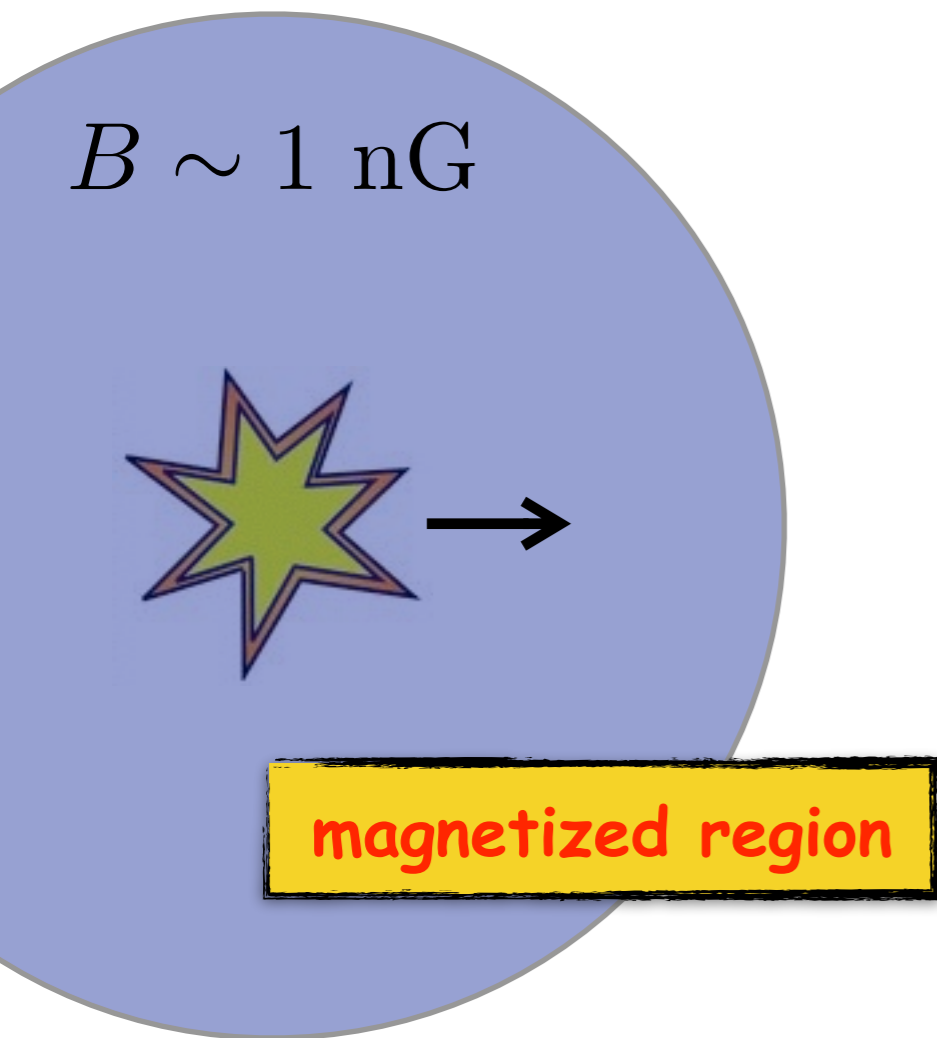
Thompson cascade



Kotera&Olinto 2011



A new channel: synchrotron gamma rays



$B \sim 1 \text{ nG}$

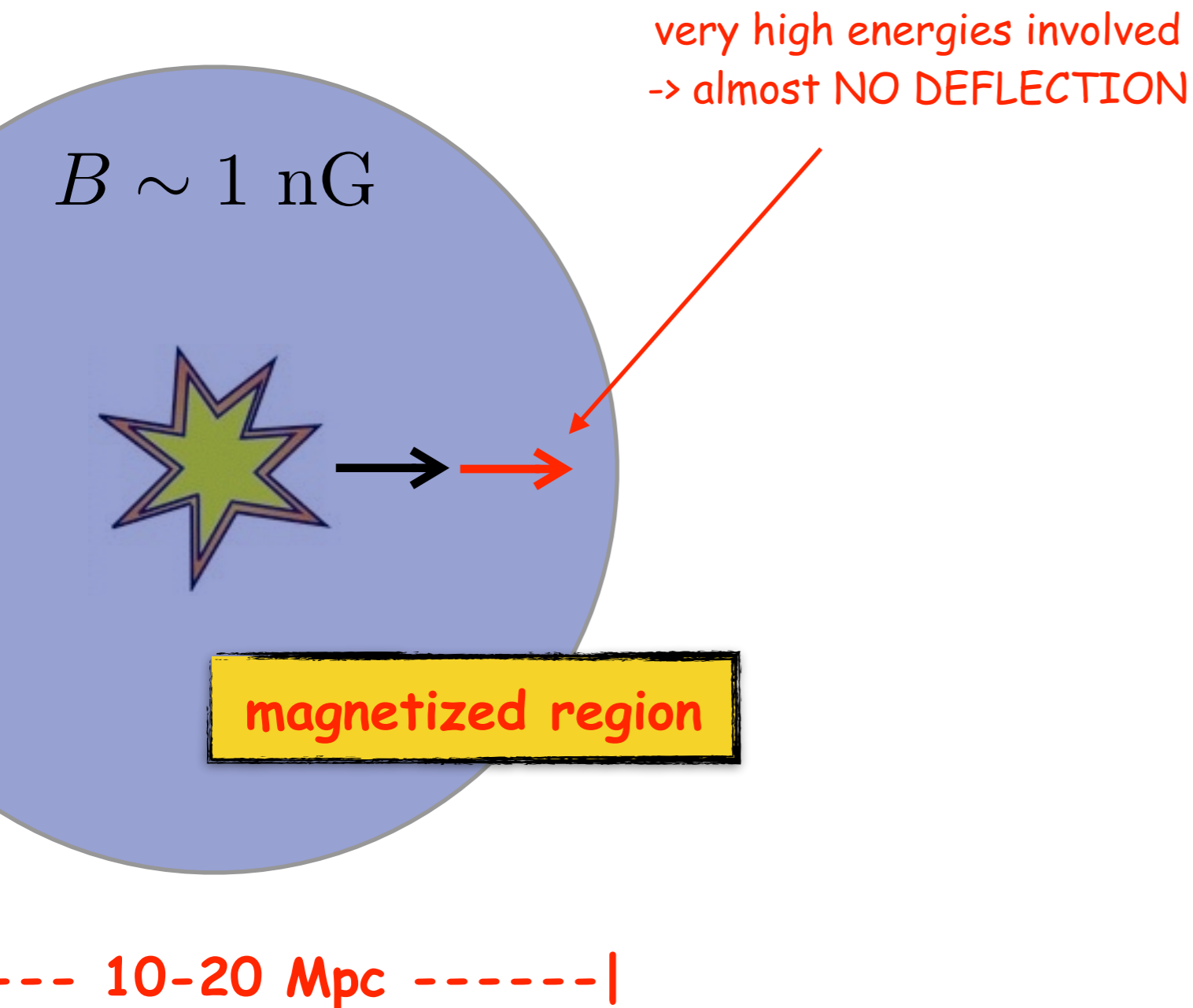
magnetized region



--- 10-20 Mpc -----|

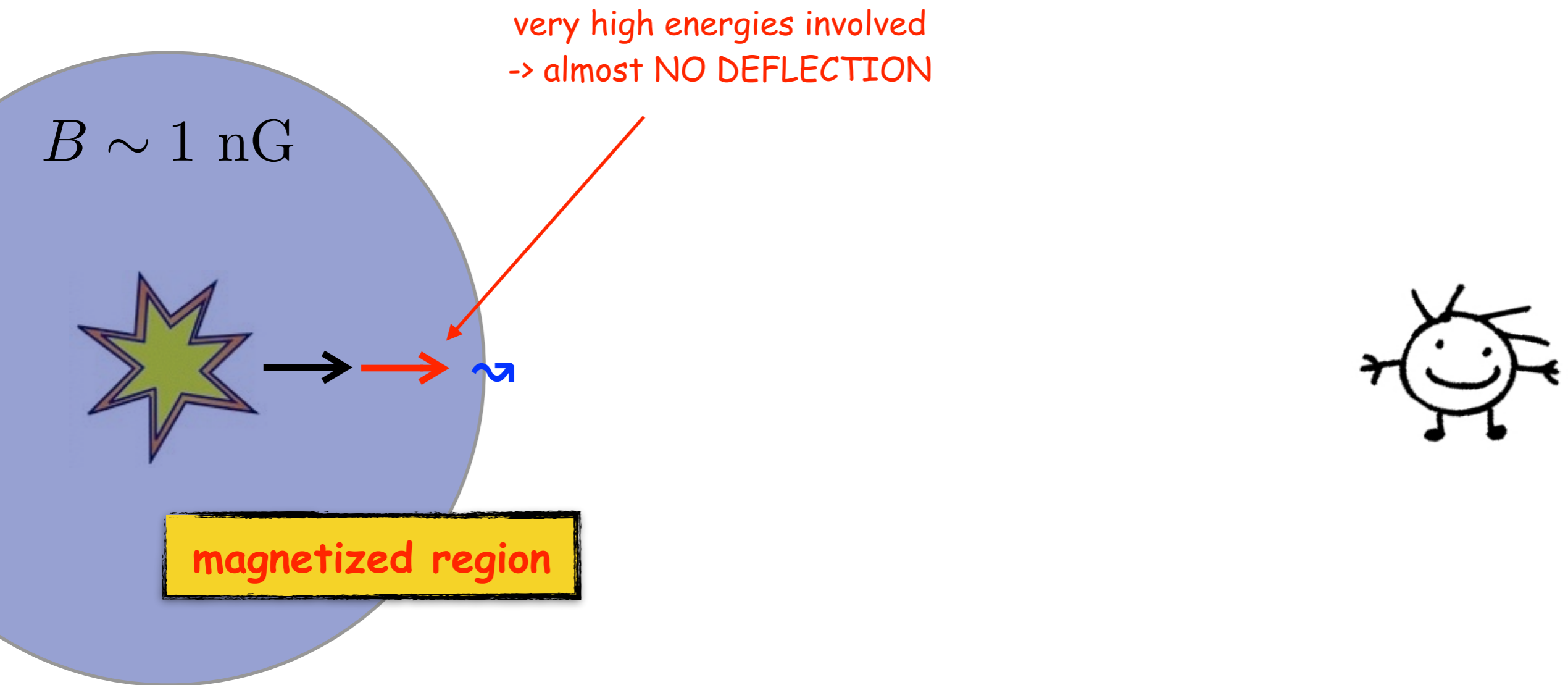
supercluster?

A new channel: synchrotron gamma rays



supercluster?

A new channel: synchrotron gamma rays



very high energies involved
-> almost NO DEFLECTION

$B \sim 1 \text{ nG}$

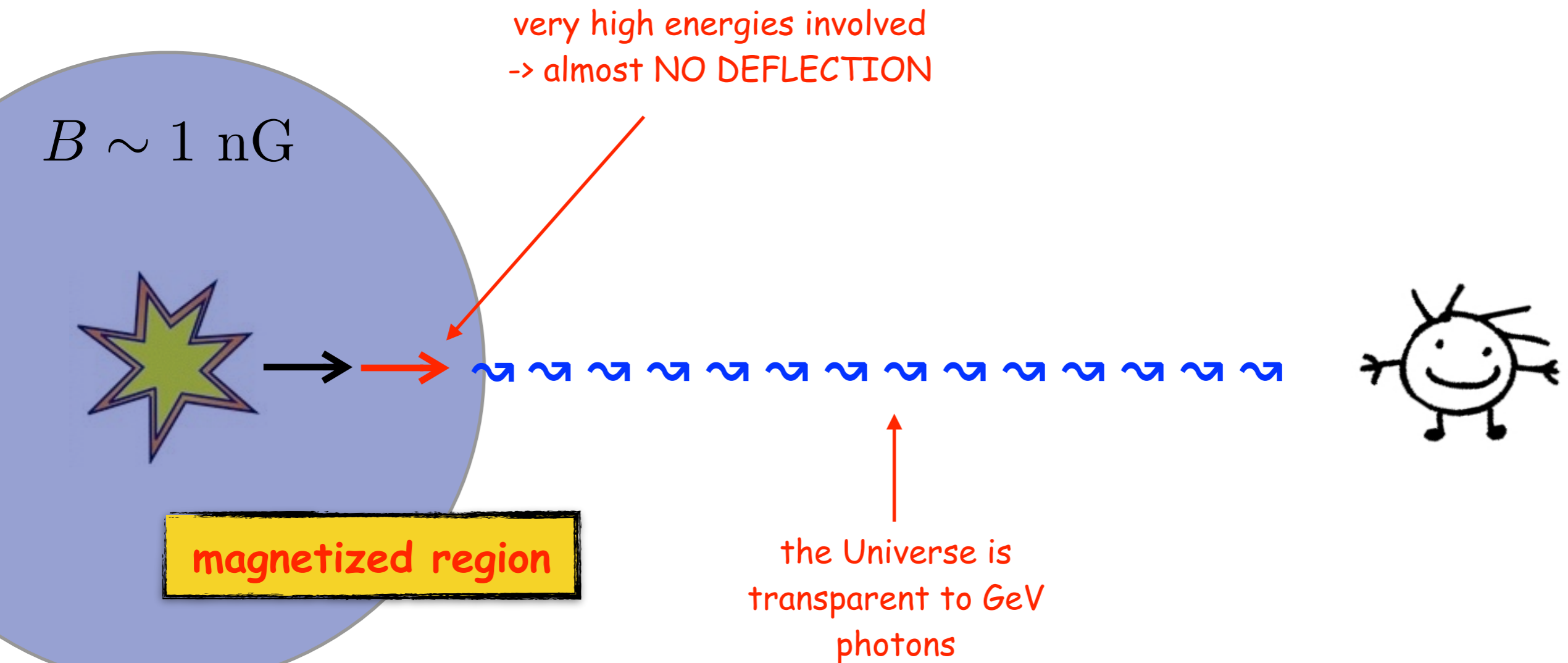
magnetized region

----- 10-20 Mpc -----|

supercluster?

$$E_{syn} \approx 2 \left(\frac{B}{\text{nG}} \right) \left(\frac{E_e}{10^{19} \text{ eV}} \right)^2 \text{ GeV}$$

A new channel: synchrotron gamma rays

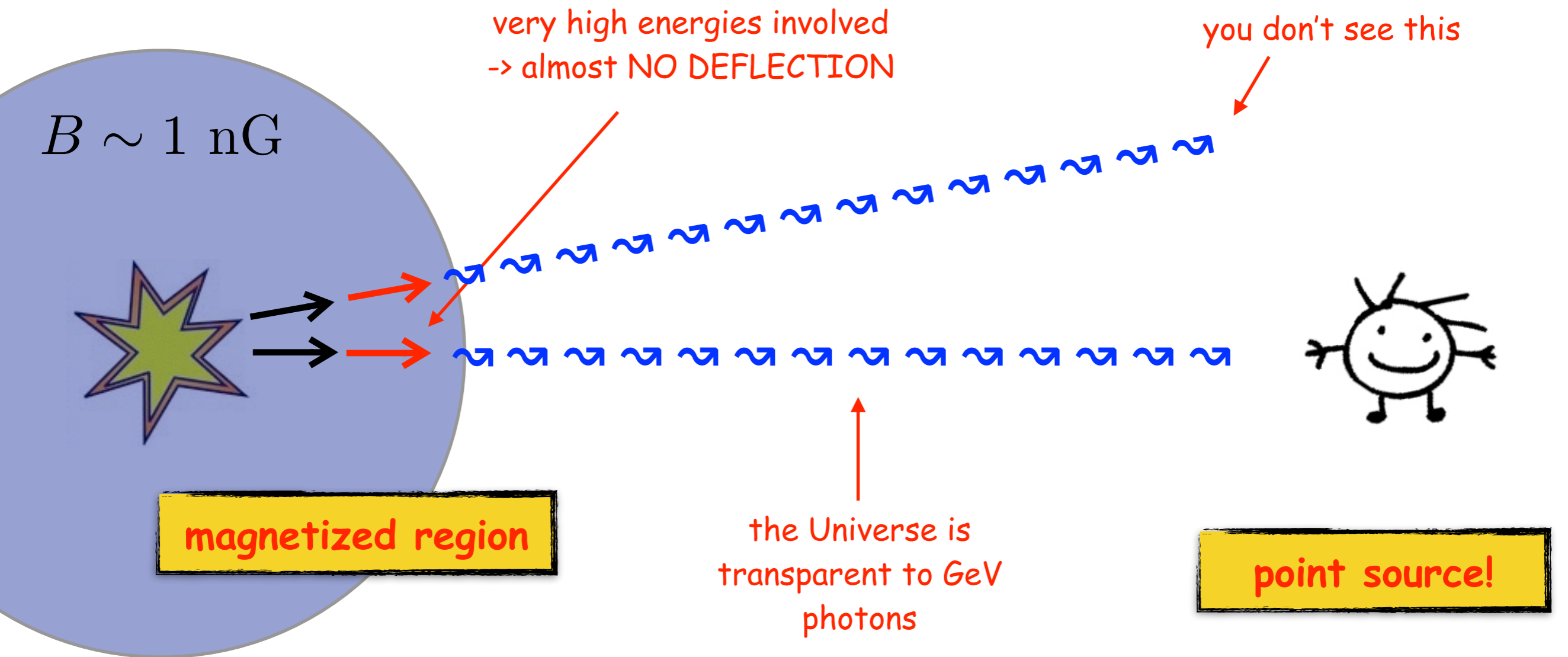


--- 10-20 Mpc ---|

supercluster?

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A new channel: synchrotron gamma rays



--- 10-20 Mpc ---|

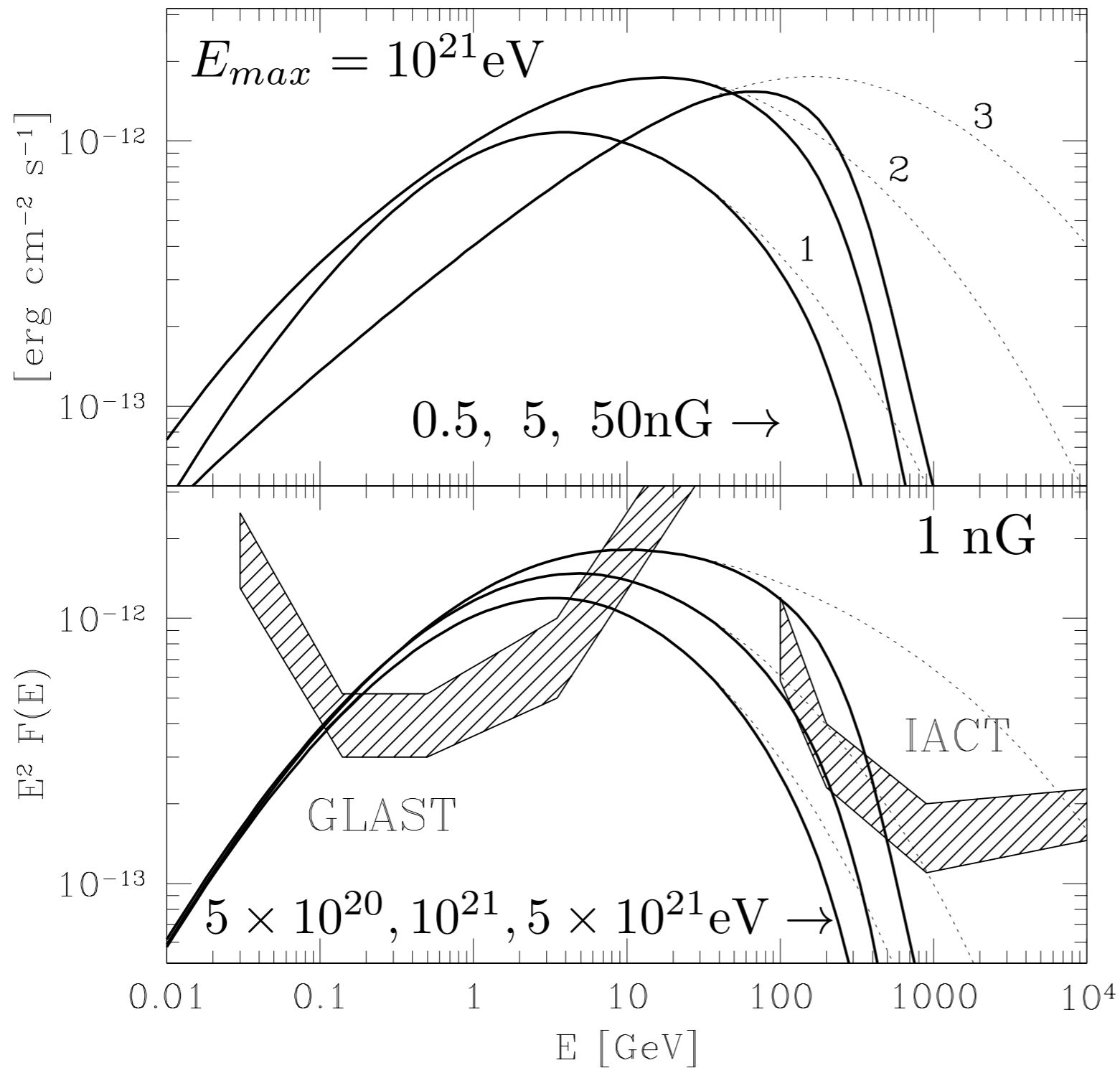
supercluster?

$$E_{syn} \approx 2 \left(\frac{B}{\text{nG}} \right) \left(\frac{E_e}{10^{19} \text{ eV}} \right)^2 \text{ GeV}$$

Detectability condition

LUHECR = 10^{46} erg/s, $d = 1$ Gpc

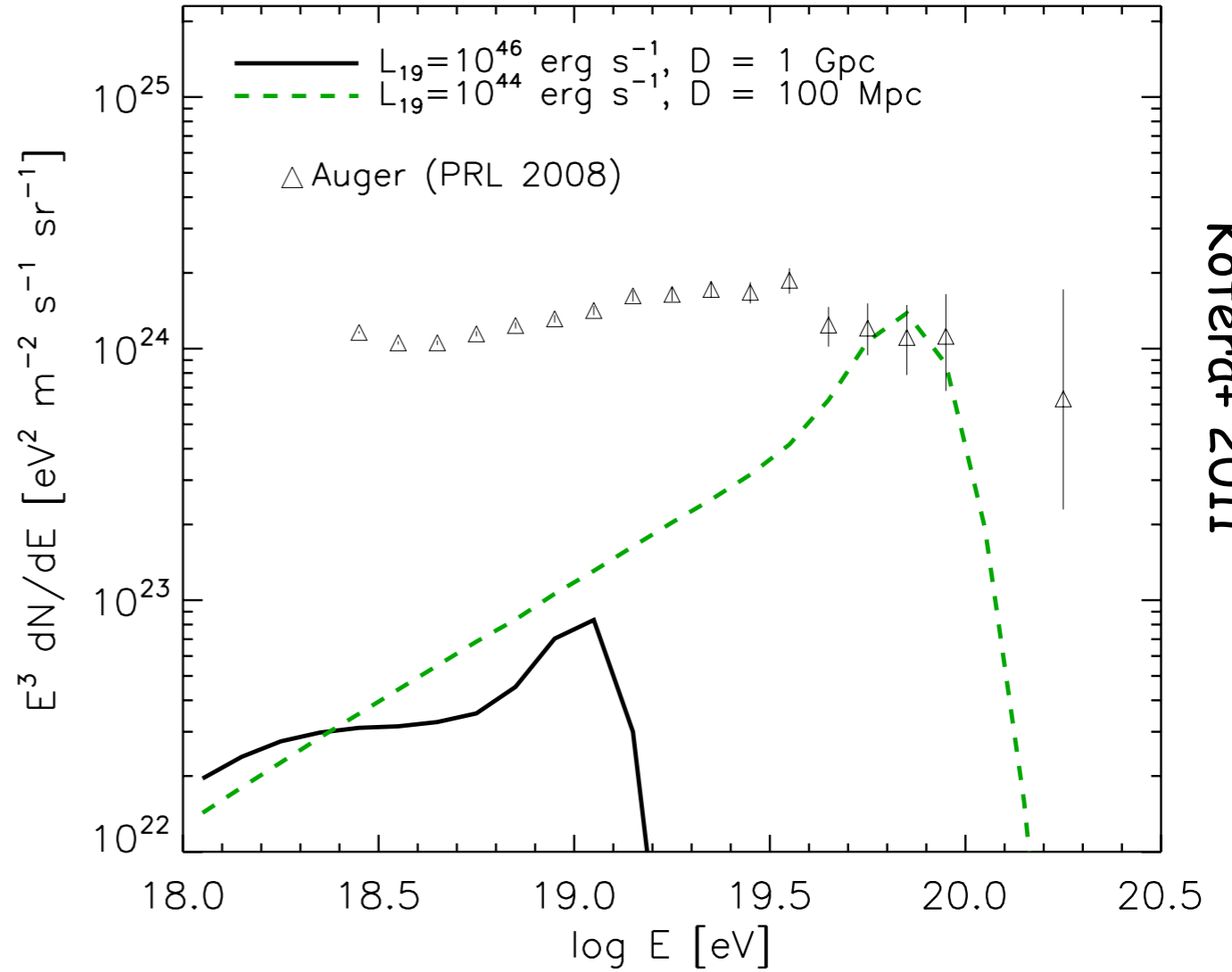
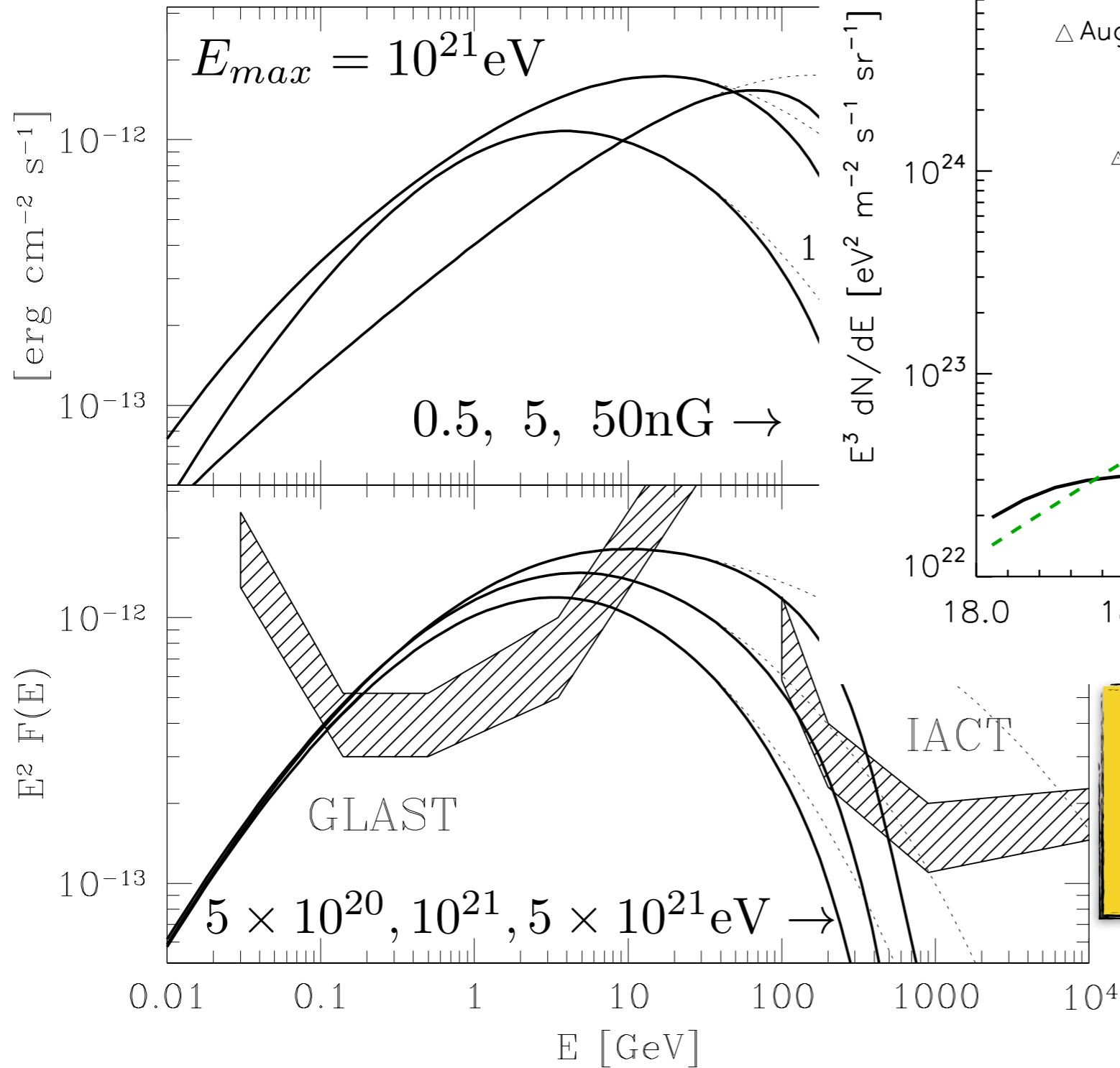
Gabici&Aharonian 2005, 2007



Detectability condition

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Gabici&Aharonian 2005, 2007



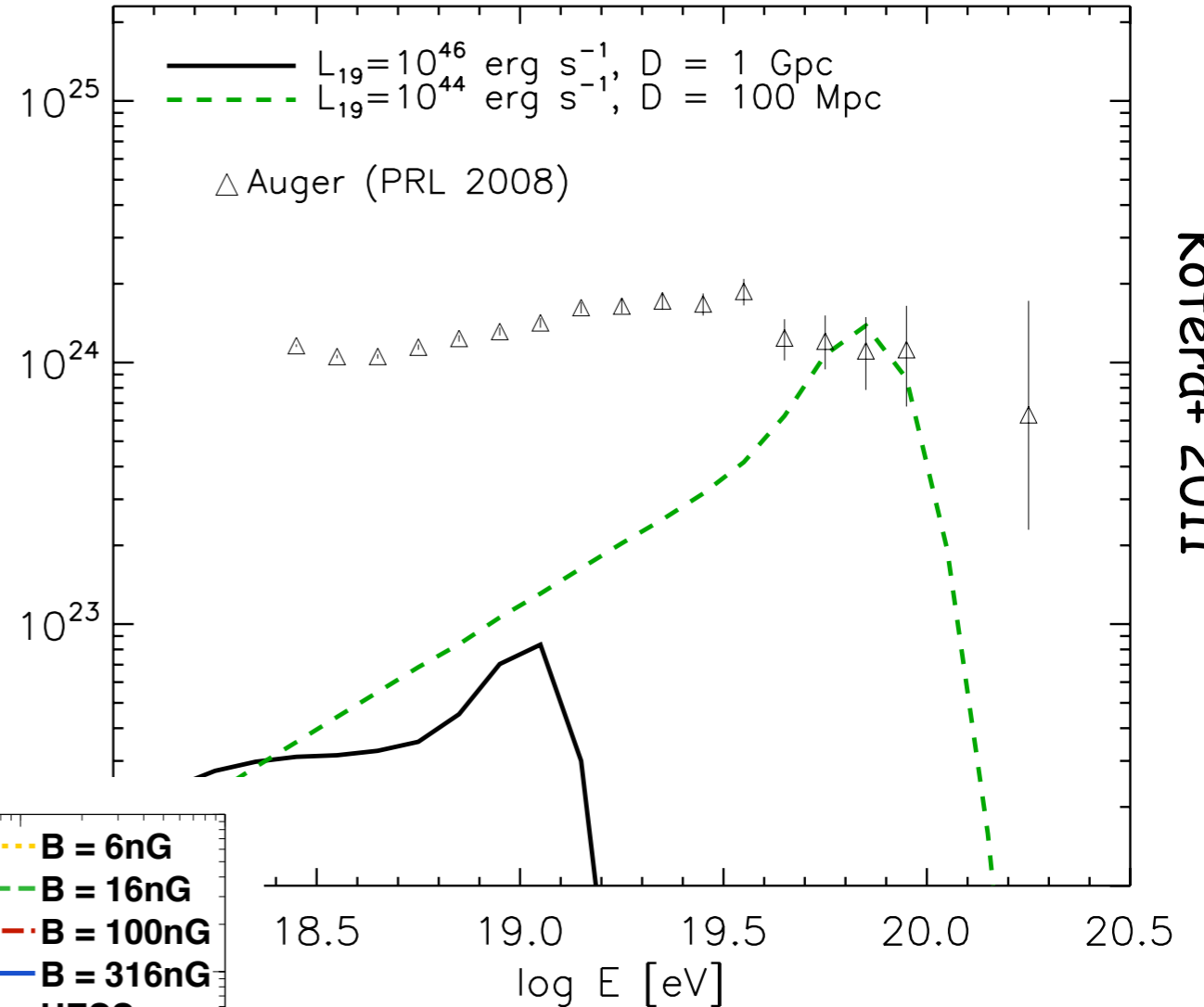
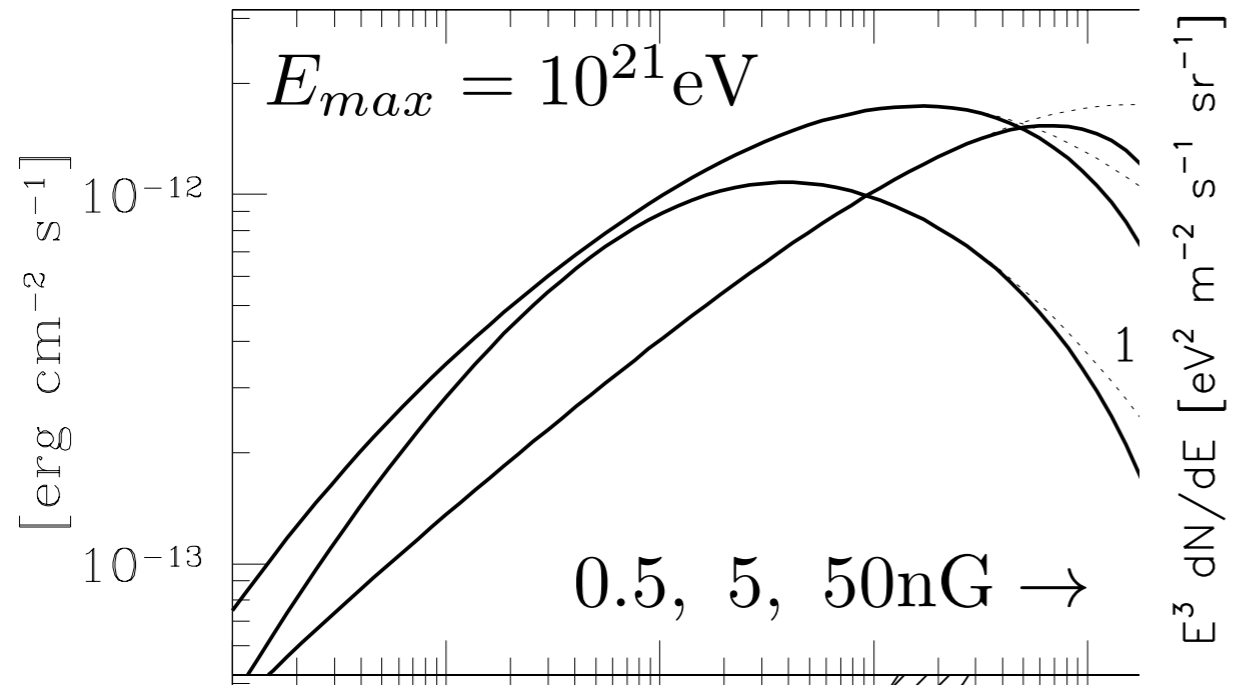
Kotera+ 2011

caveats: must be protons and must not overshoot UHECR diffuse spectrum

Detectability condition

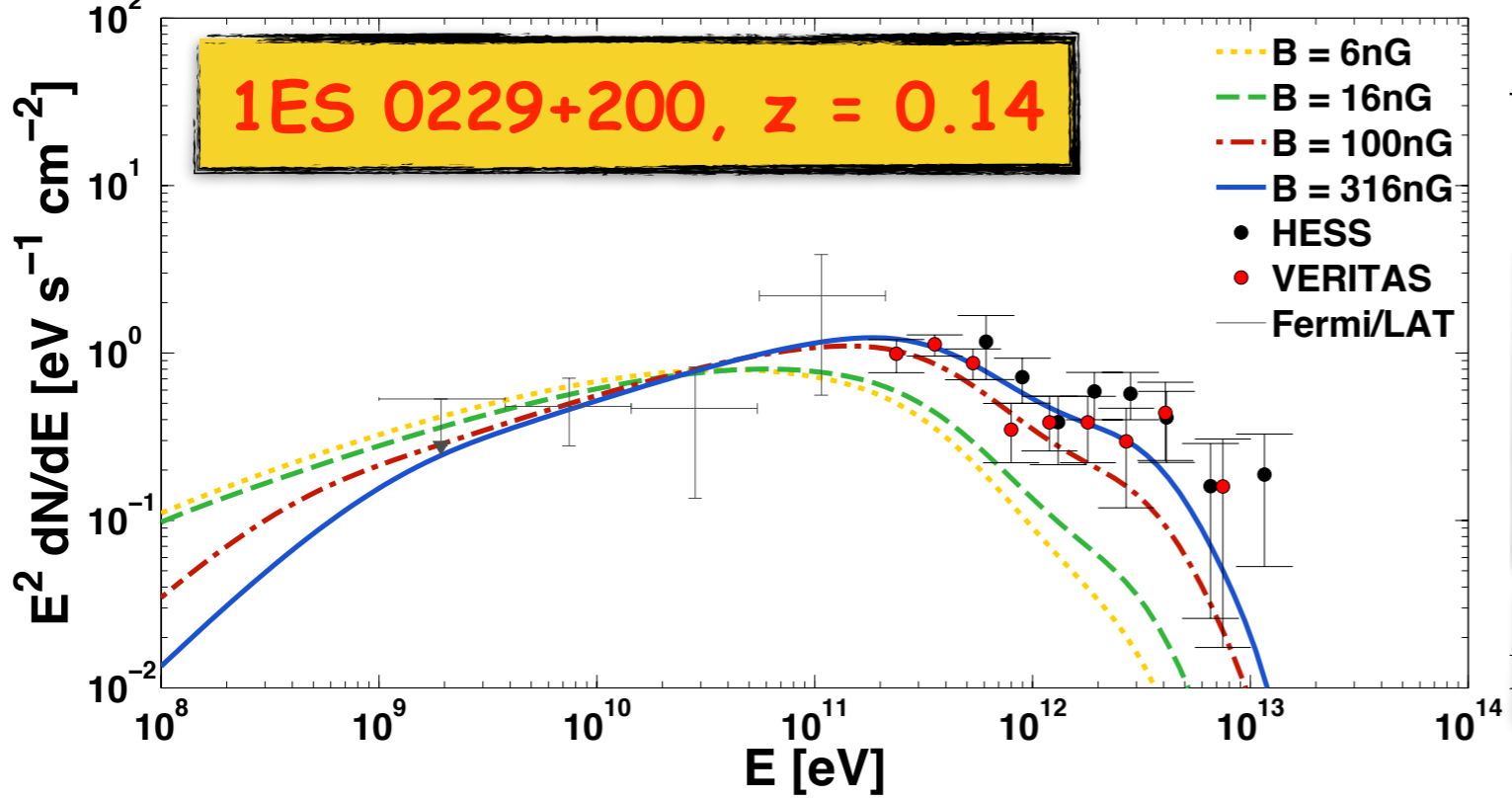
LUHECR = 10^{46} erg/s, $d = 1$ Gpc

Gabici&Aharonian 2005, 2007



Kotera+ 2011

1ES 0229+200, $z = 0.14$



caveats: must be protons and must not overshoot UHECR diffuse spectrum

caveat: steady emission

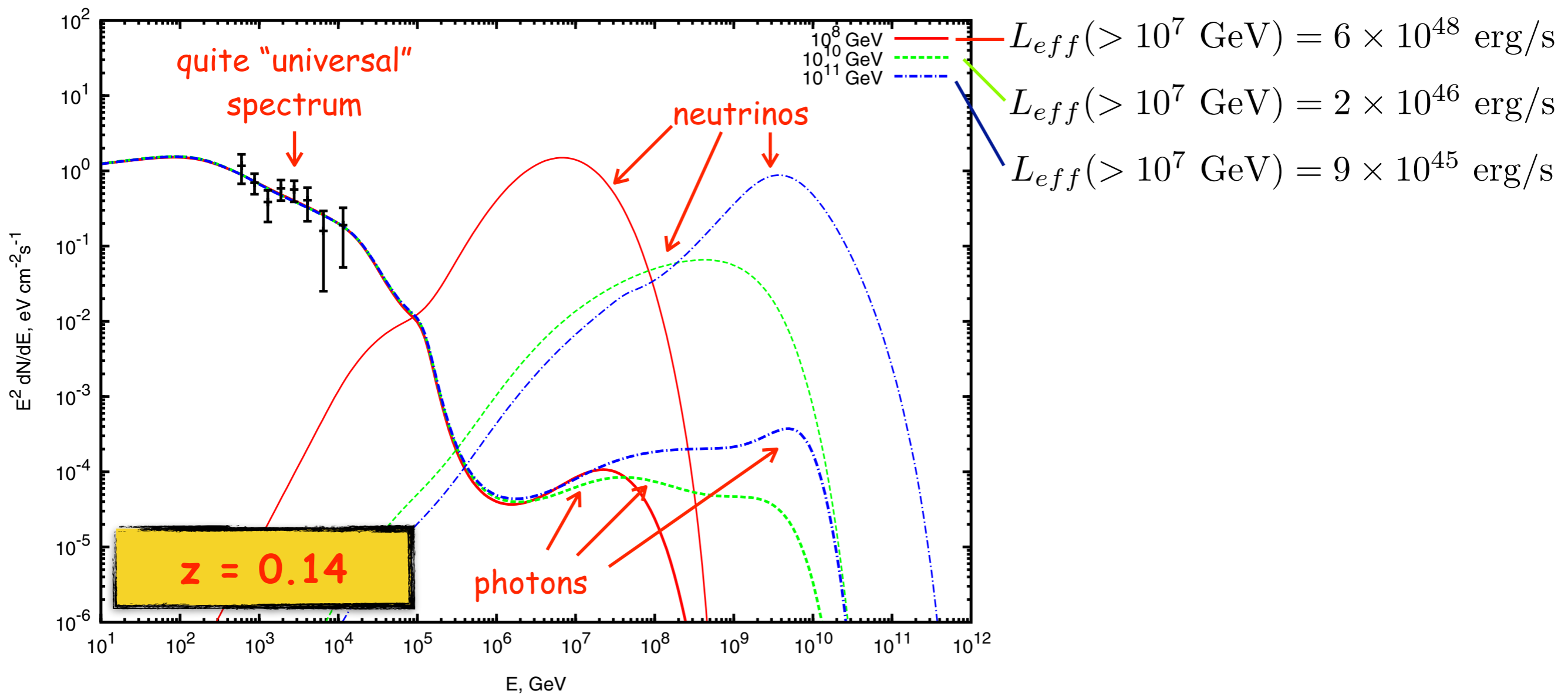
Oikonomou+ 2014

see also Aharonian+2010 Prosekin+ 2011

γ -ray blazar: an alternative scenario

Essey & Kusenko 2010, Essey+ 2010, 2011

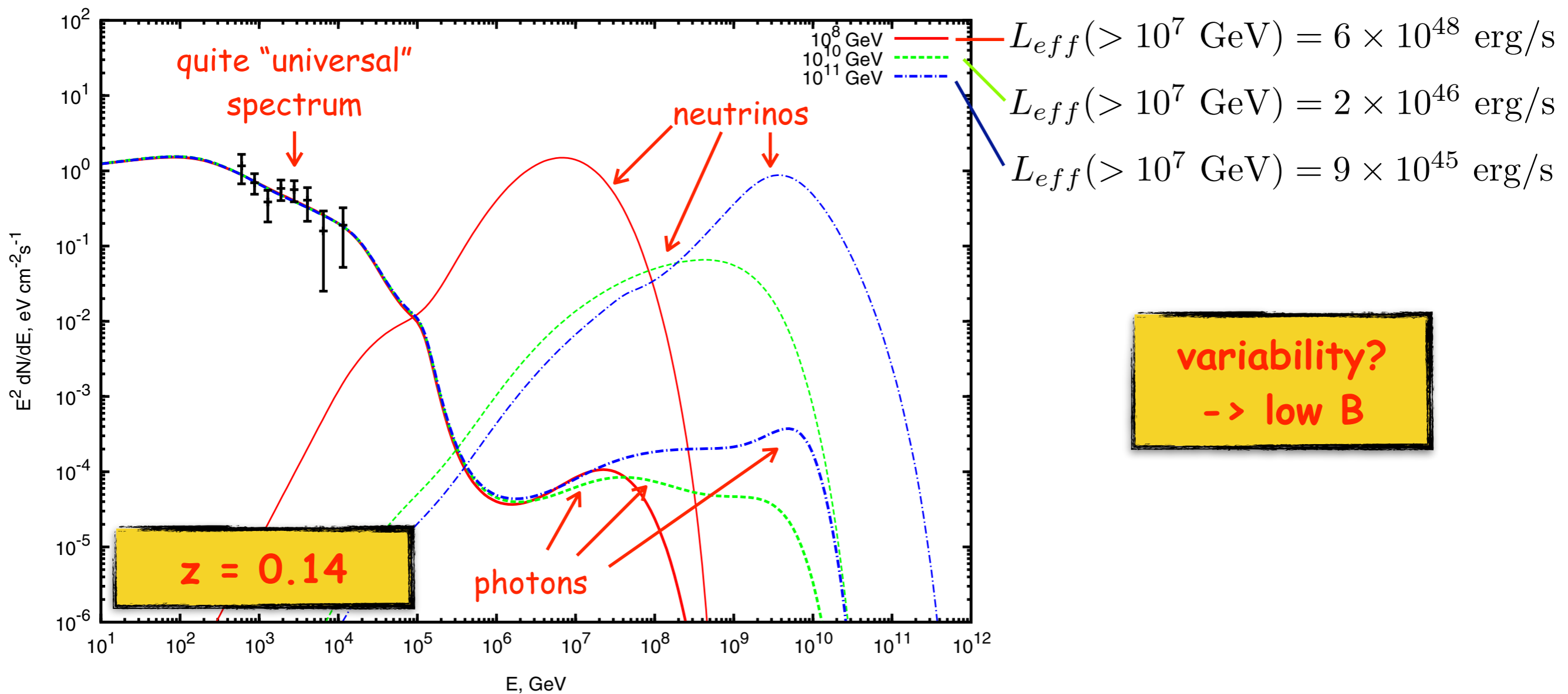
all the γ -ray emission comes from the cascade initiated by UHECR interactions



γ -ray blazar: an alternative scenario

Essey & Kusenko 2010, Essey+ 2010, 2011

all the γ -ray emission comes from the cascade initiated by UHECR interactions



Fermi non detection + HESS detection ->

$$10^{-17} \text{ G} < B < 3 \times 10^{-14} \text{ G}$$

γ -rays from high z blazar

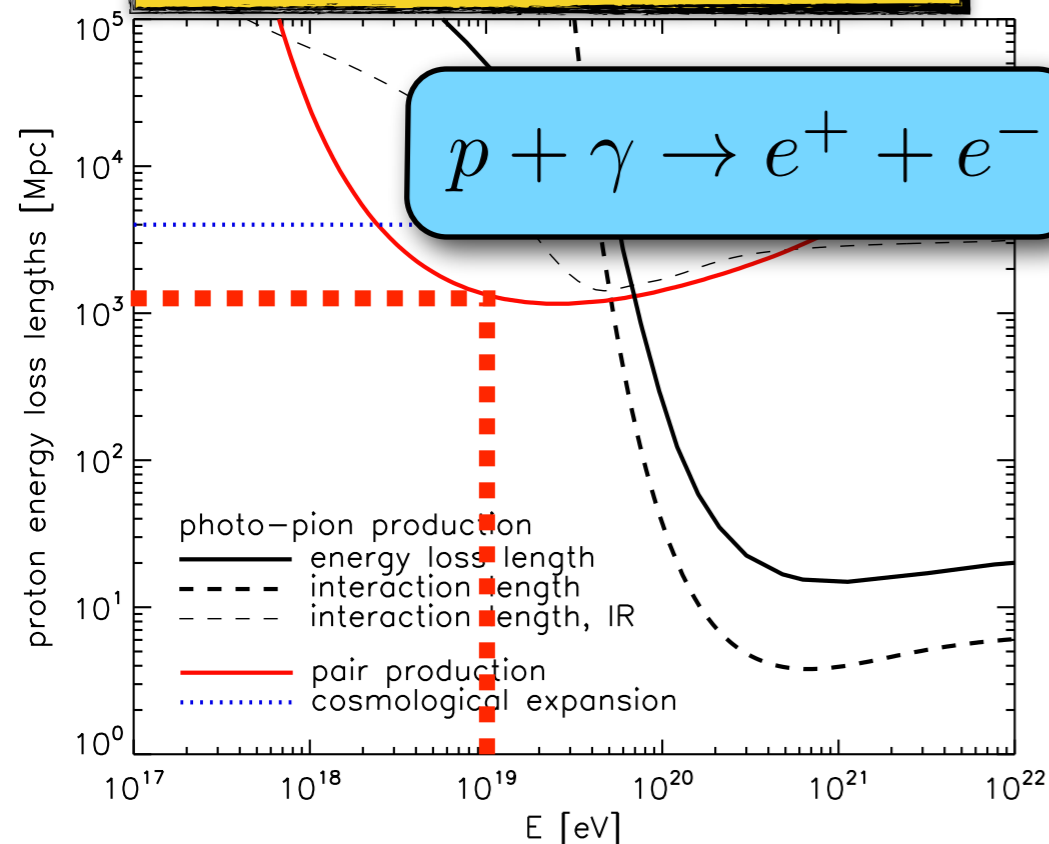
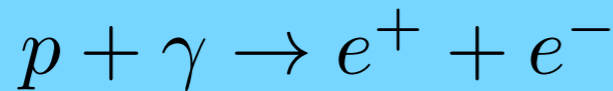
rationale: blazars at $z \sim 0.2$ \rightarrow EBL close to minimum possible level
 \rightarrow we shouldn't see gamma rays from blazars at $z > 0.2$



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proton interaction length

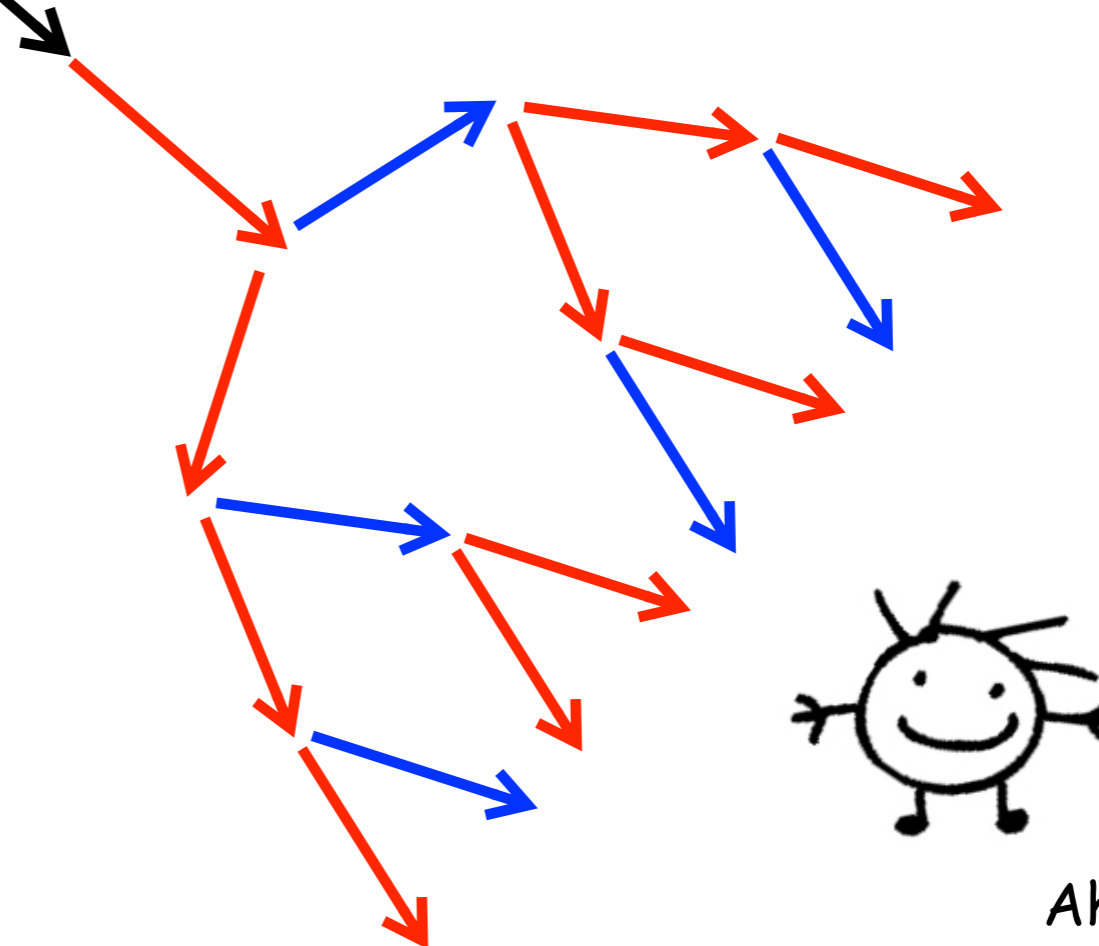
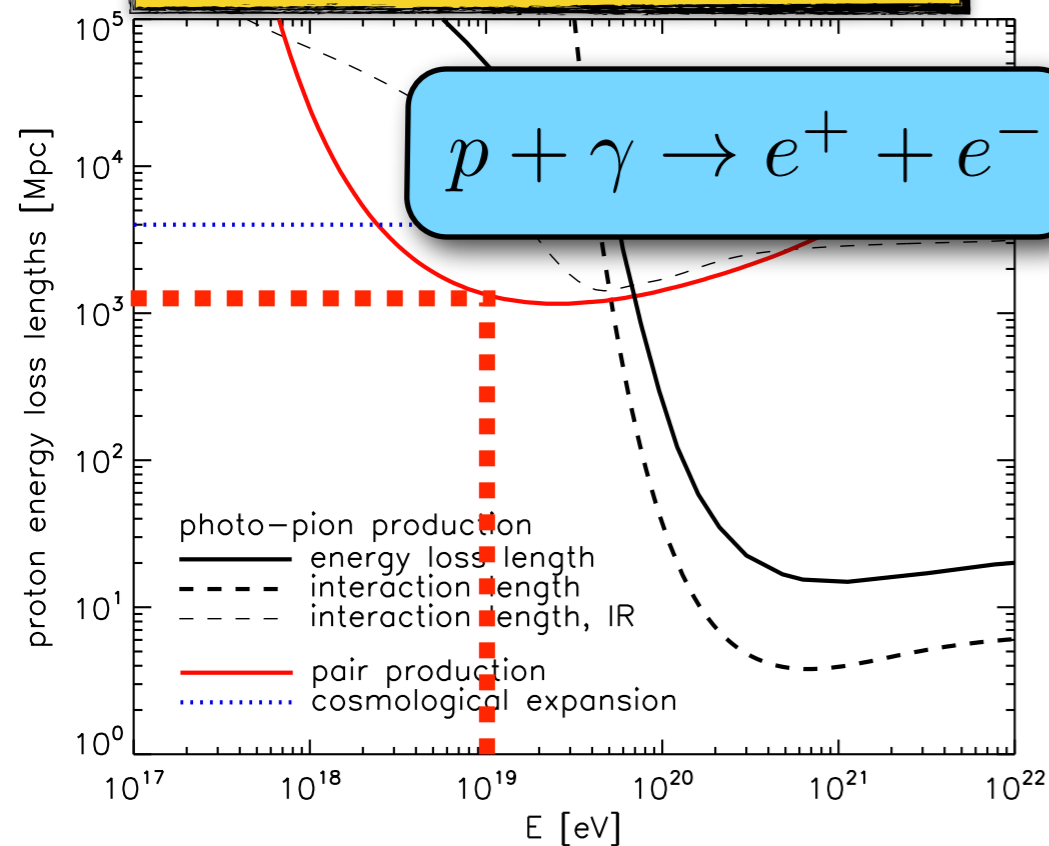
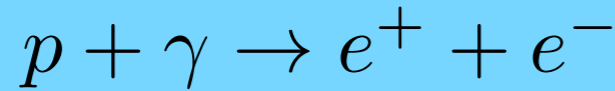


γ -rays from high z blazar

rationale: blazars at $z \sim 0.2$ \rightarrow EBL close to minimum possible level
 \rightarrow we shouldn't see gamma rays from blazars at $z > 0.2$

$$E_e \sim \left(\frac{m_e}{m_p} \right) E_p = 10^{15} \left(\frac{E_p}{2 \times 10^{18} \text{ eV}} \right) \text{ eV}$$

proton interaction length

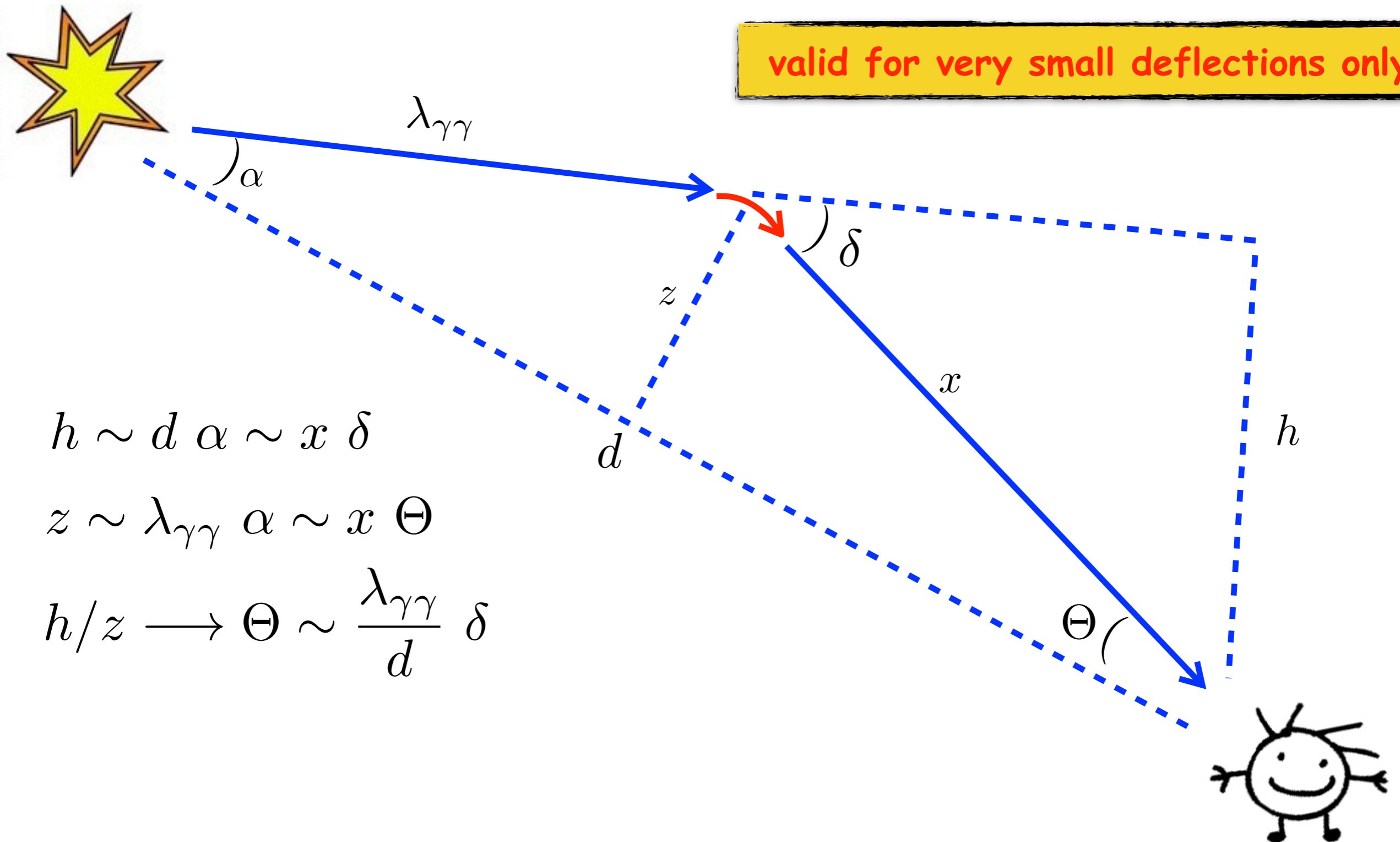


Conclusions

- **gamma ray observations of blazars** -> EM cascade in EBL -> **constrain the extragalactic magnetic fields** (especially in voids)
- PSF constrain: **$B > 10^{-15} \text{ G}$**
- time delay constrain: **$B > 10^{-17} \text{ G}$**
- **alternative scenarios** to the pure electromagnetic cascade exist: cascades initiated by **UHECR** interactions. this typically works well for distant and very powerful sources (remember that powerful sources are indeed expected to be distant!)
- extreme synchrotron scenario -> accelerator immersed in a nG field (steady emission)
- UHECR scenario -> gamma rays from very distant blazars without axions and/or violation of Lorentz invariance

Simple sketch

valid for very small deflections only!



$$h \sim d \alpha \sim x \delta$$

$$z \sim \lambda_{\gamma\gamma} \alpha \sim x \Theta$$

$$h/z \longrightarrow \Theta \sim \frac{\lambda_{\gamma\gamma}}{d} \delta$$