Oliver Hahn (OCA)

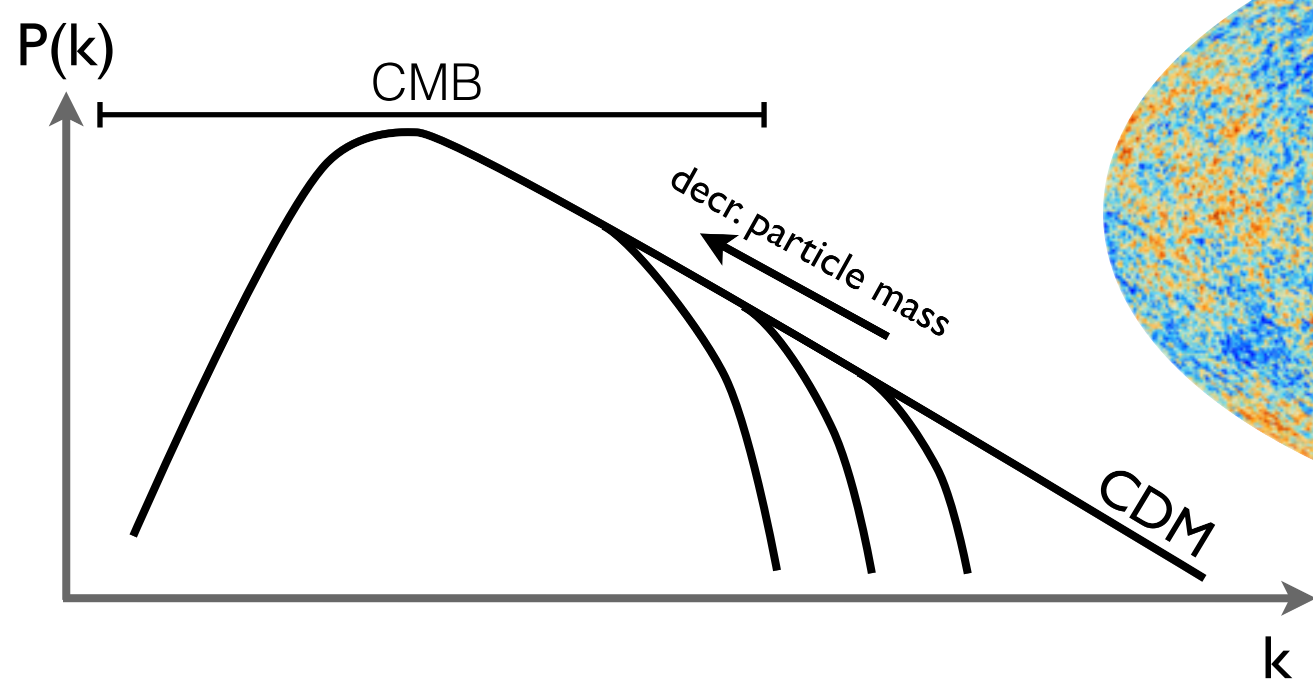
Dark matter properties and their impact on structure formation

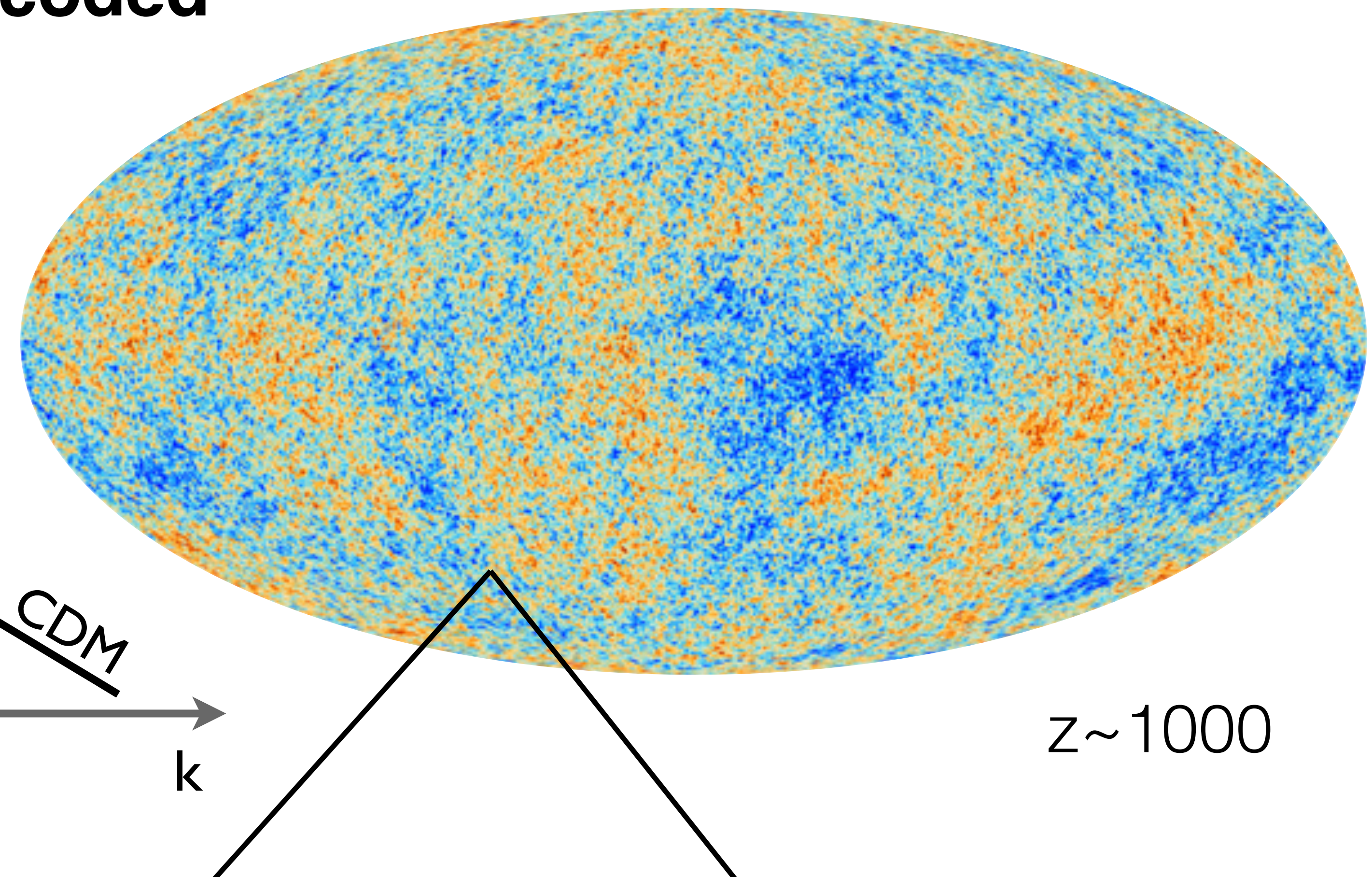
with Raul Angulo (CEFCA), Tom Abel (Stanford), Aaron Ludlow (Durham),
Silvia Bonoli (CEFCA), Aseem Paranjape (IUCAA) and others…

UNIVERSITÉ CÔTE D'AZUR
LAGRANGE
Observatoire de la CÔTE d'AZUR

What do we know about the properties of dark matter?

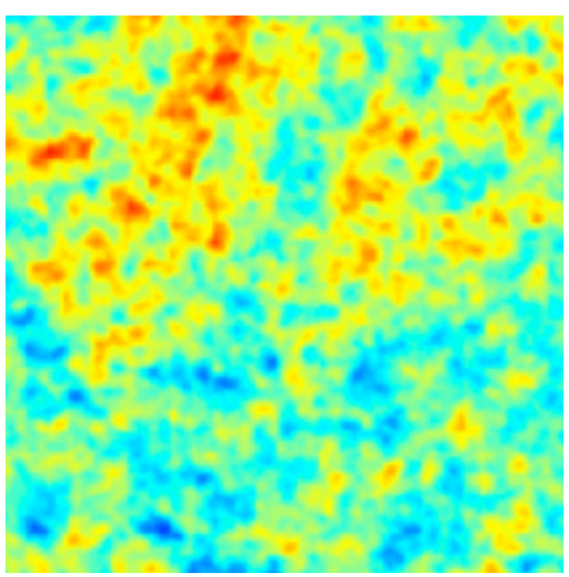
- **the kinetic temperature is encoded in the power spectrum**



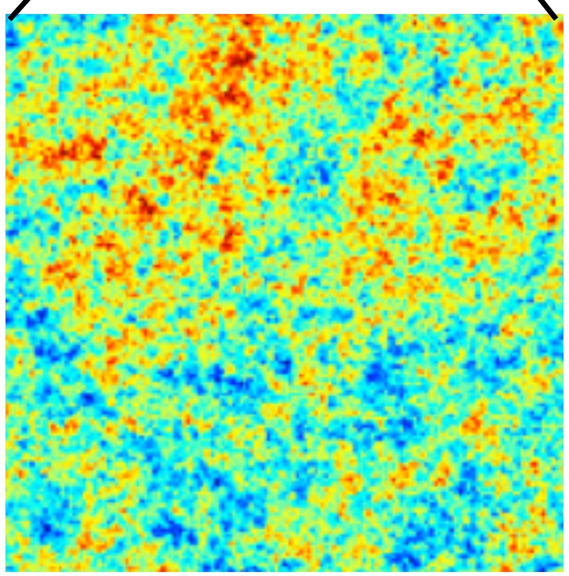


z~1000

Underlying DM density:



WDM

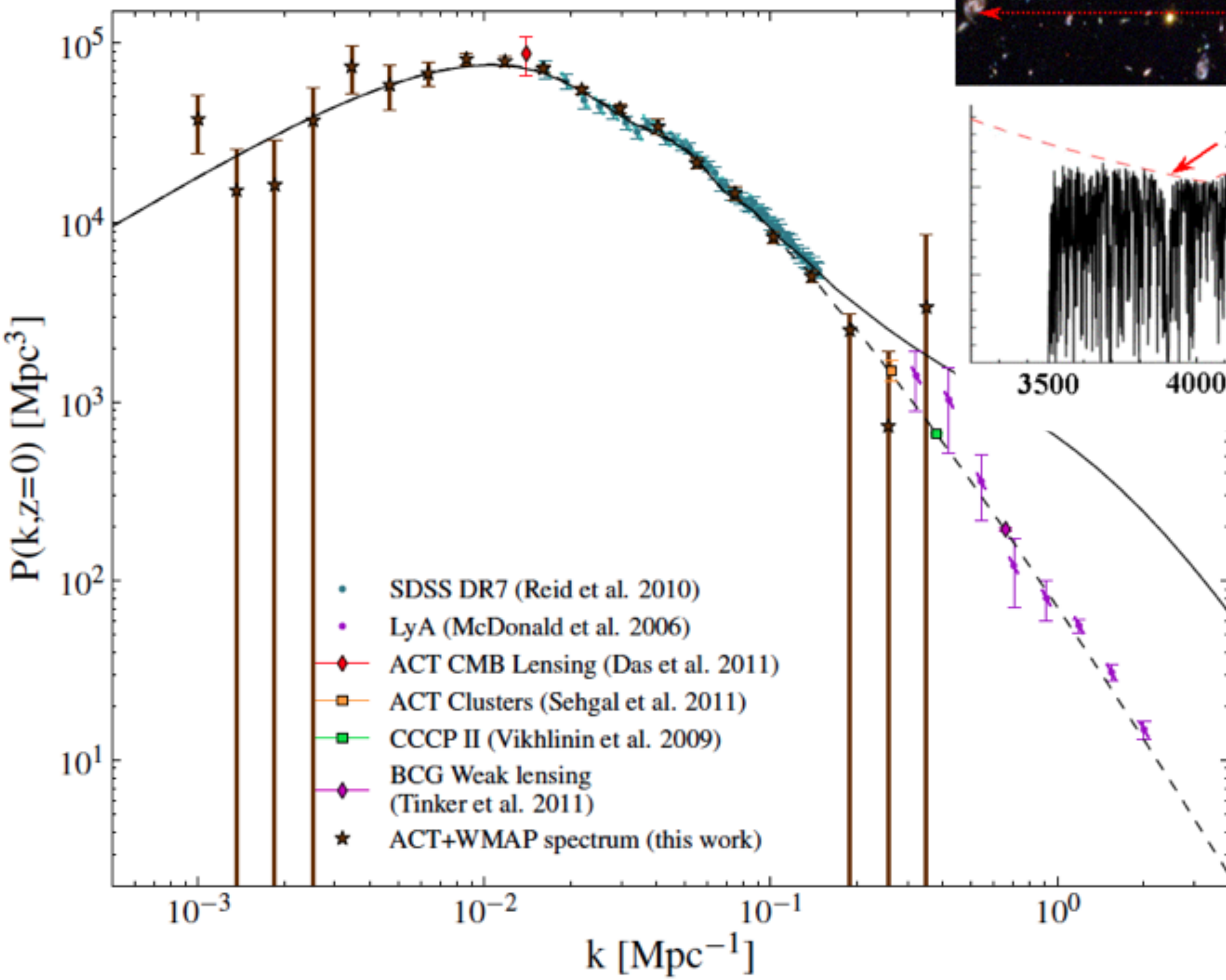


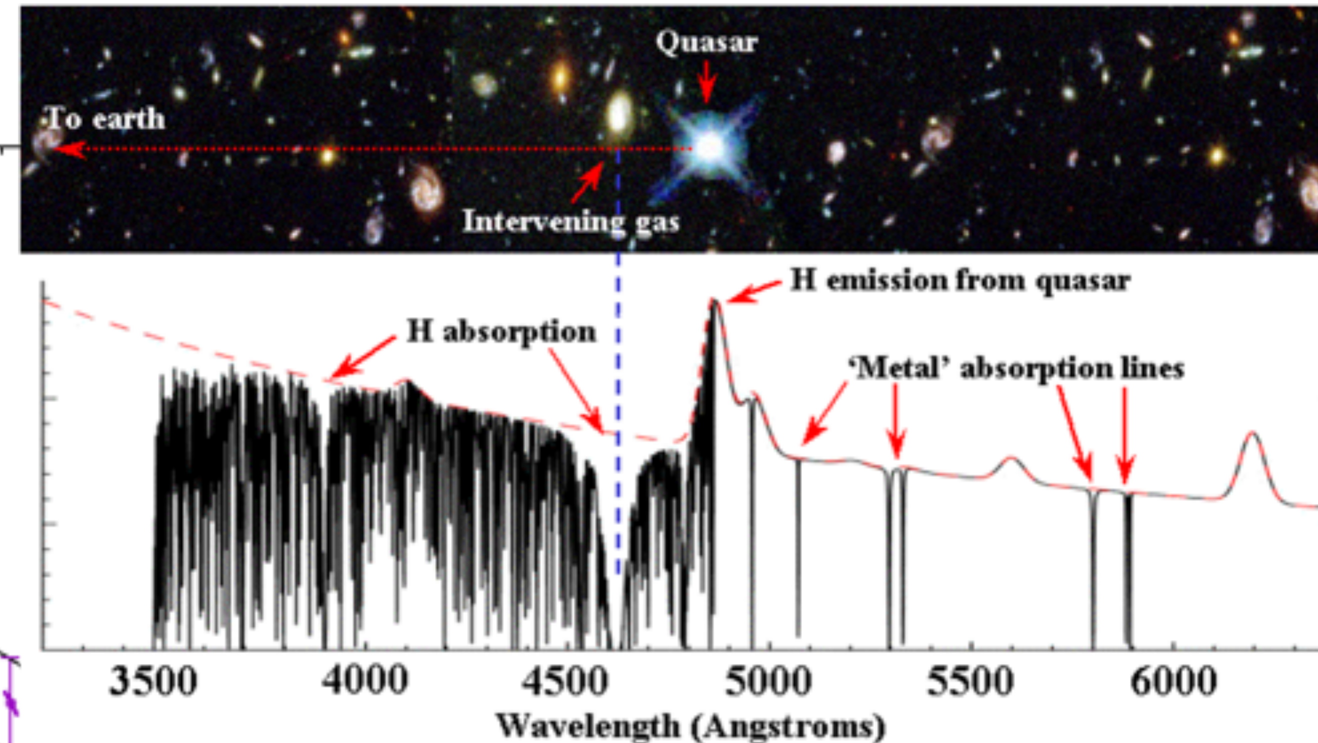
CDM

What do we know about the properties of dark matter?

- **kinetic temperature must be cold(ish)**

Hlozek et al. 2012





- best constraints from Ly-α forest (e.g. Viel et al. 2013) >~ 5keV
- constrains WDM

So, what is Dark Matter?

for our macroscopic purposes it suffices to assume that

microscopic → continuum limit

proton = 1GeV, WIMP 100GeV? -> 10^{21}/g

cold (or at most lukewarm) → $v_{\mathrm{thermal}} \ll v_{\mathrm{bulk}}$

e.g. thermally produced at very early times, cooled since then

negligible cross-section → $\sigma_{\mathrm{DM}} \ll \sigma_{\mathrm{em}}$ collisionless

weak-scale or even weaker

self-gravitating

(but in principle any of these can be dropped)

Kinetic description in terms of Vlasov-Poisson

Density of particles in phase space

= distribution function $f(\mathbf{x}, \mathbf{v}, t)$

Evolution governed by **Boltzmann equation**

$$\frac{\partial f}{\partial t} + \underbrace{\frac{\mathbf{v}}{a^2} \cdot \boldsymbol{\nabla}_\mathbf{x} f}_{\text{velocities advect in configuration space}} - \underbrace{\boldsymbol{\nabla}_\mathbf{x} \phi \cdot \boldsymbol{\nabla}_\mathbf{v} f}_{\text{grav. forces advect in momentum space}} = \underbrace{C[f]}_{\text{particle-particle interactions } \mathbf{= 0}}$$

+ Poisson equation for grav. potential

$$\boldsymbol{\nabla}^2_\mathbf{v} \phi = \frac{4\pi G}{a^3} \int (f - \bar{\rho})\, \mathrm{d}^n v$$

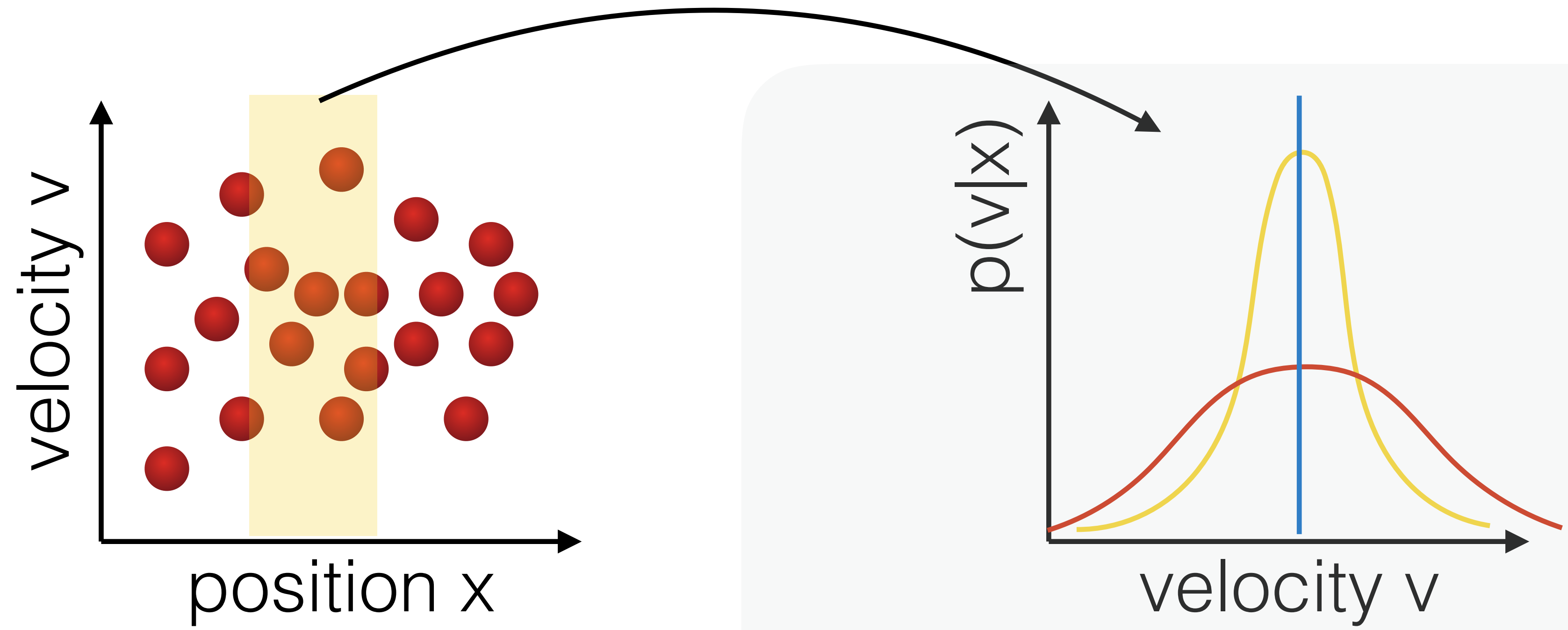
for C[f]=0: Vlasov-Poisson

Kinetic description of dark matter

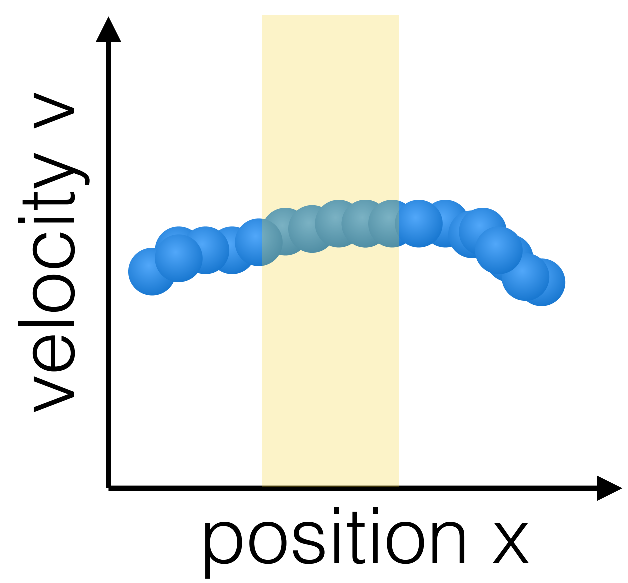
Density of particles in phase space given by

= distribution function $f(\mathbf{x}, \mathbf{v}, t)$

hot case



cold case



velocity distribution function

$$p(\mathbf{v}|\mathbf{x}) = f(\mathbf{x}, \mathbf{v}, t)/n(\mathbf{x})$$

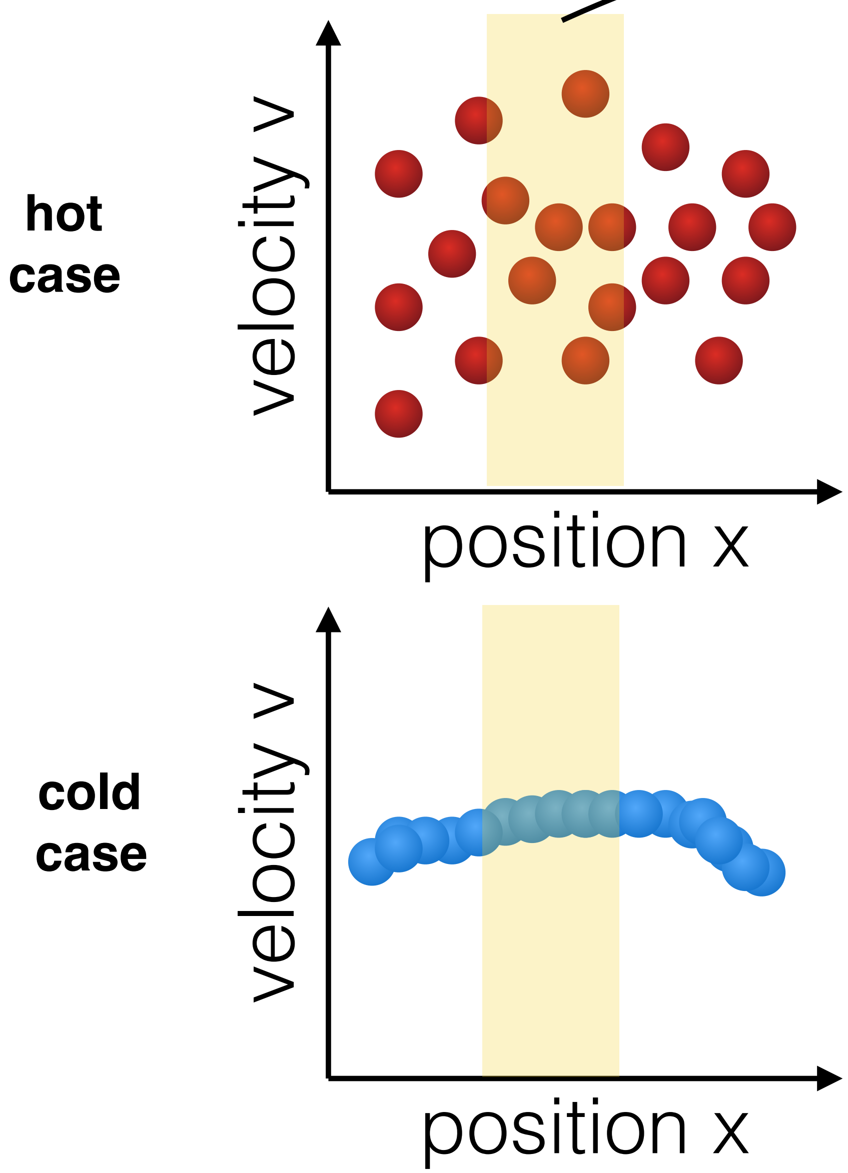
mass density is zeroth-moment

$$\rho(\mathbf{x}) = m_\chi \int f(\mathbf{x}, \mathbf{v}, t)\, \mathrm{d}^n v$$

Kinetic description of dark matter

Density of particles in phase space given by

= distribution function $f(\mathbf{x}, \mathbf{v}, t)$



phase space of n+n dim:

generally,
f is truly 2n-dimensional

in cold limit,
f is only n-dimensional
= **monokinetic**

What is special about a cold-collisionless system?

The 1D structure winds up but never tears or mixes! (neighbours stay neighbours!) topologically preserved

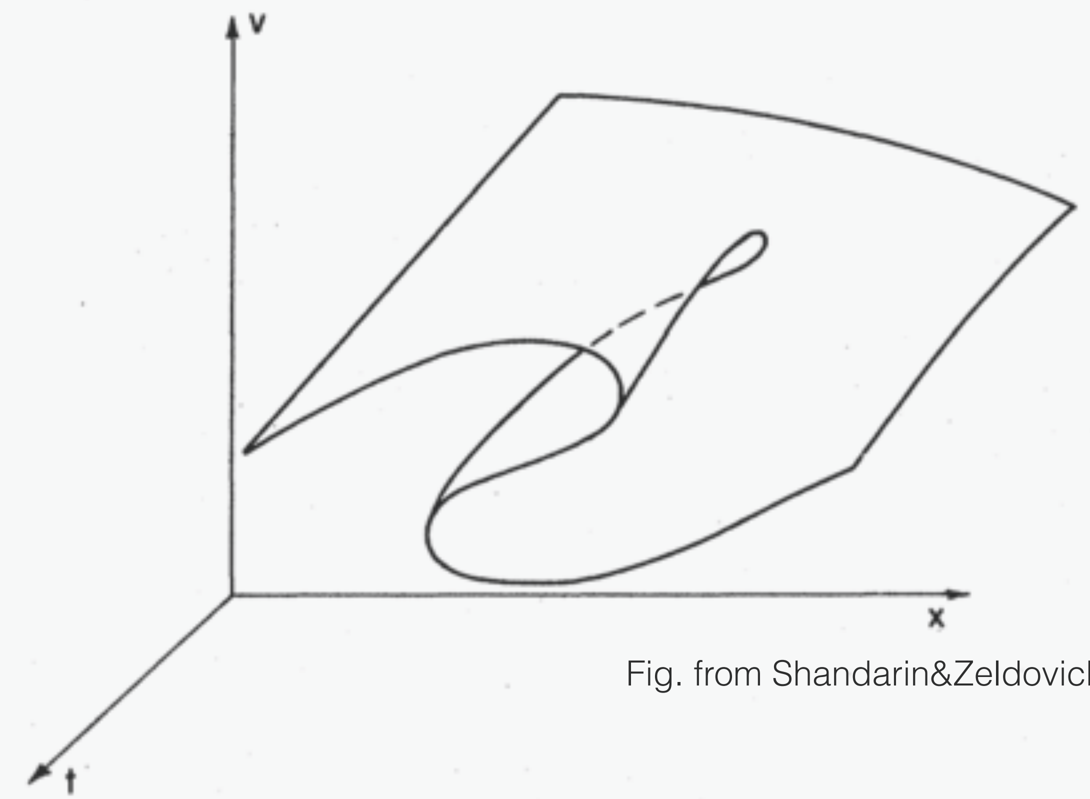


Fig. from Shandarin&Zeldovich 1989

Vanishing collision-term

⇒ not in hydro limit

⇒ velocity can be multi-valued

⇒ cannot stop at low order moments

⇒ have to discretize distribution function

⇒ singular caustics emerge (see later)

What is special about a cold-collisionless system?

The 1D structure winds up but never tears or mixes! (neighbours stay neighbours!) topologically preserved

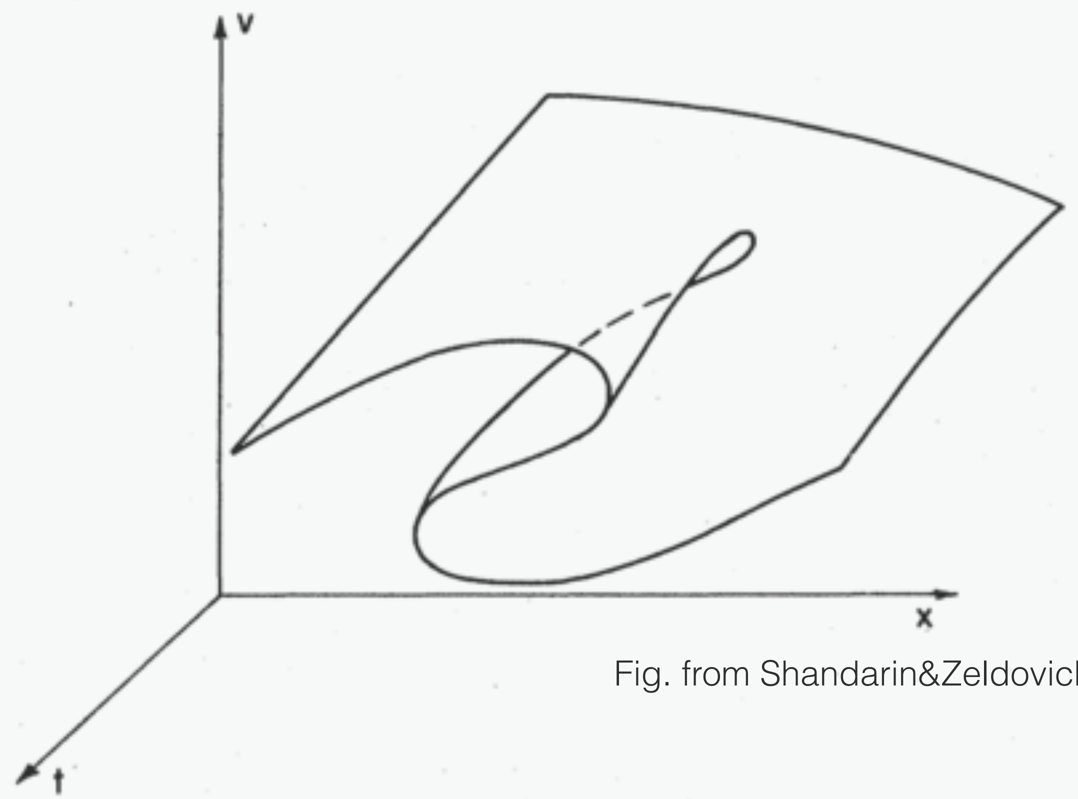


Fig. from Shandarin&Zeldovich 1989

Vanishing collision-term

⇒ not in hydro limit

⇒ velocity can be multi-valued

⇒ cannot stop at low order moments

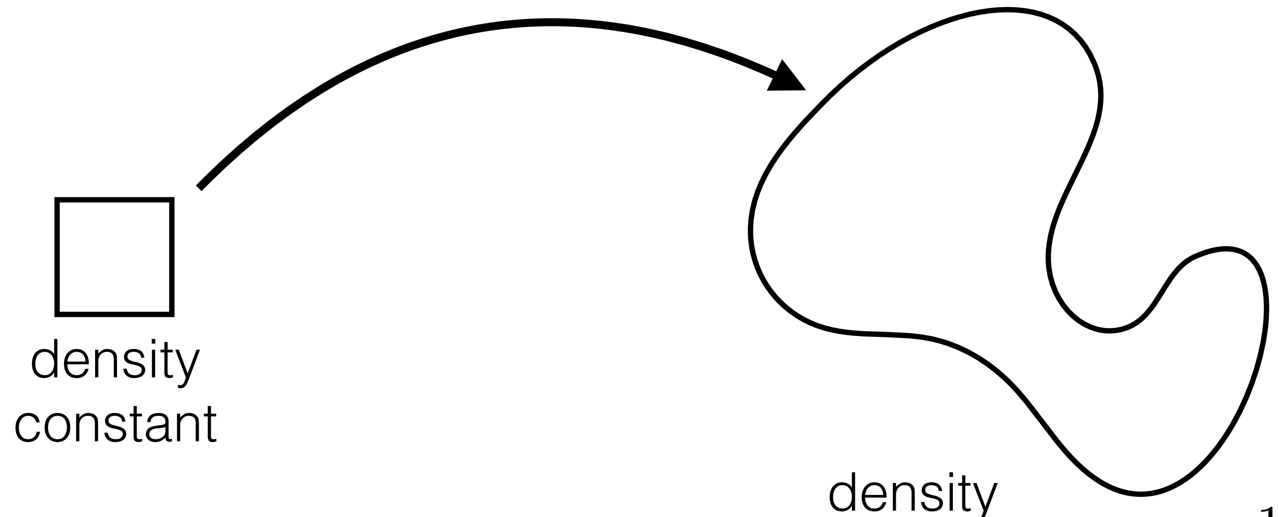
⇒ have to discretize distribution function

⇒ singular caustics emerge (see later)

Lagrangian description

Lagrangian description, evolution of fluid element

$$\mathbb{Q} \subset \mathbb{R}^3 \to \mathbb{R}^6 \; : \; \mathbf{q} \mapsto (\mathbf{x}_\mathbf{q}(t), \mathbf{v}_\mathbf{q}(t))$$



$$\rho = m_\mathrm{DM} \left| \frac{\partial x_i}{\partial q_j} \right|^{-1}$$

For DM, motion of any point **q** depends only on gravity

$$(\dot{\mathbf{x}}_\mathbf{q}, \dot{\mathbf{v}}_\mathbf{q}) = (\mathbf{v}_\mathbf{q}, -\boldsymbol{\nabla}\phi)$$

unlike hydro, no internal temperature, entropy, pressure

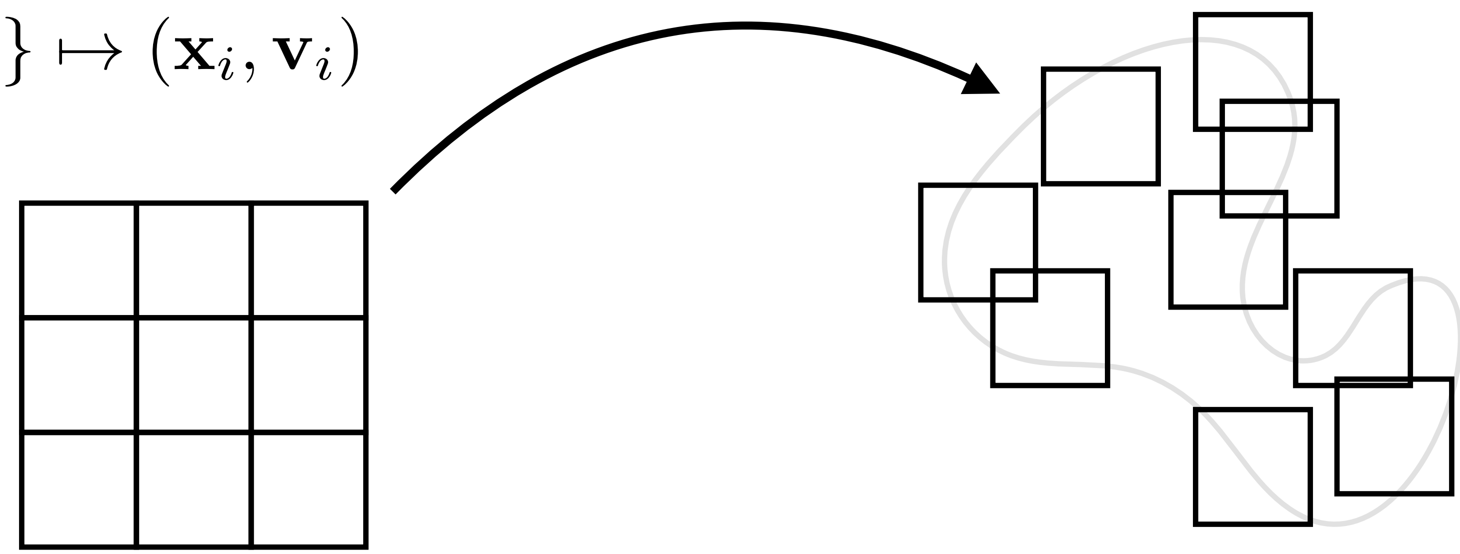
So the quest is to solve Poisson's equation

$$\Delta\phi = 4\pi G\rho$$

How to solve these systems: the N-body approach…

The N-body approximation:
cover distribution function with N 'coarse-graining' particles

$$i \in \{1 \ldots N\} \mapsto (\mathbf{x}_i, \mathbf{v}_i)$$



⇒ **EoM are just Hamiltonian N-body eq. (method of characteristics)**

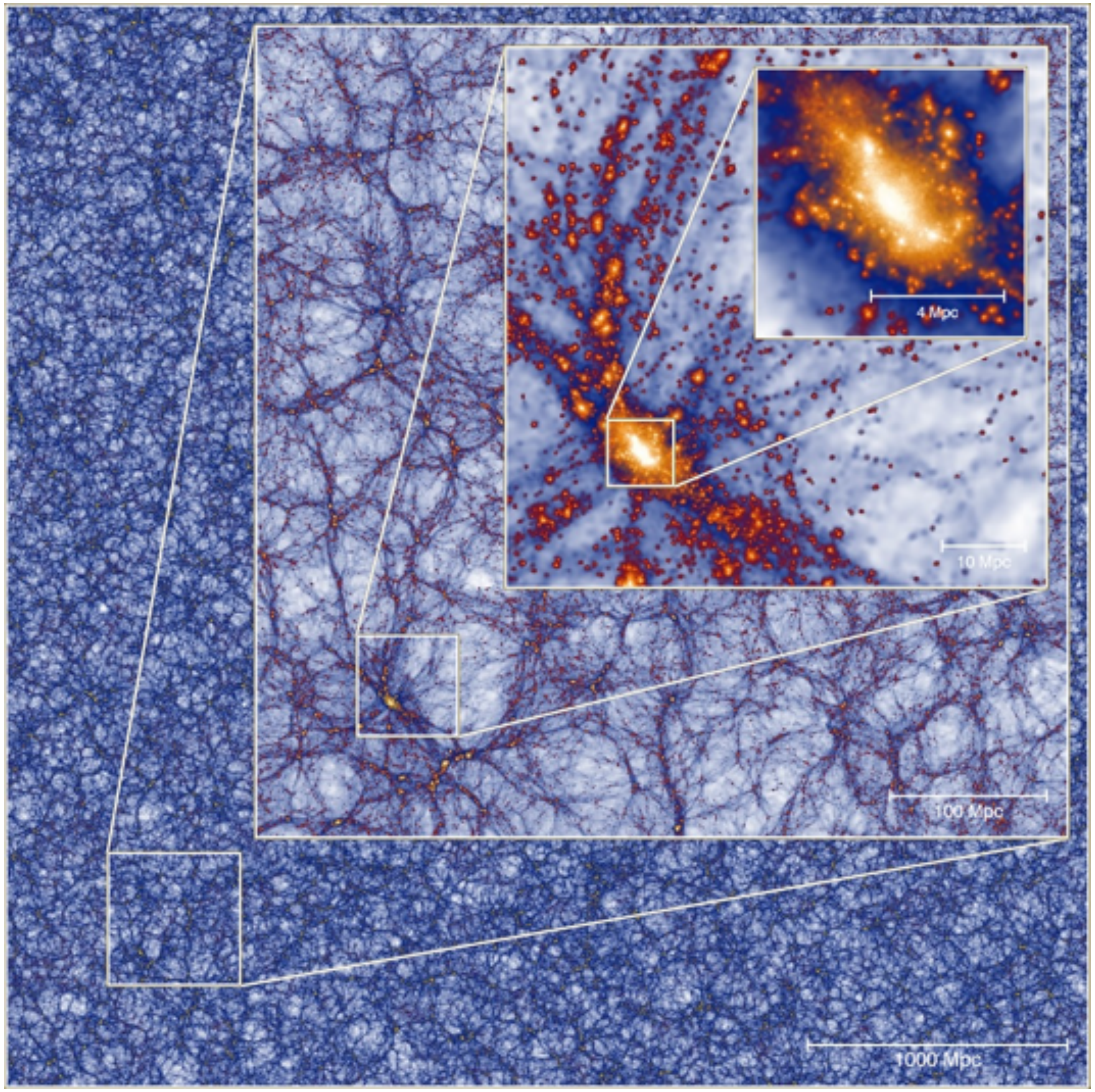
for small N, density field is poorly estimated,

$$\rho = m_p \sum \delta_D(x - x_i) \otimes W$$

continuum structure is given up, but 'easy' to solve for forces

hope that as N->very large numbers, approach collisionless continuum, but always ad hoc choice of W

Huge successes!
Predicting the distribution of matter of the Universe



Angulo et al. 2012

Input:
Powerspectrum of perturbations
+cosmological model

↓

mass functions of clusters
distributions of galaxies
evolution of structure over time
abundance of satellites
density profiles
…

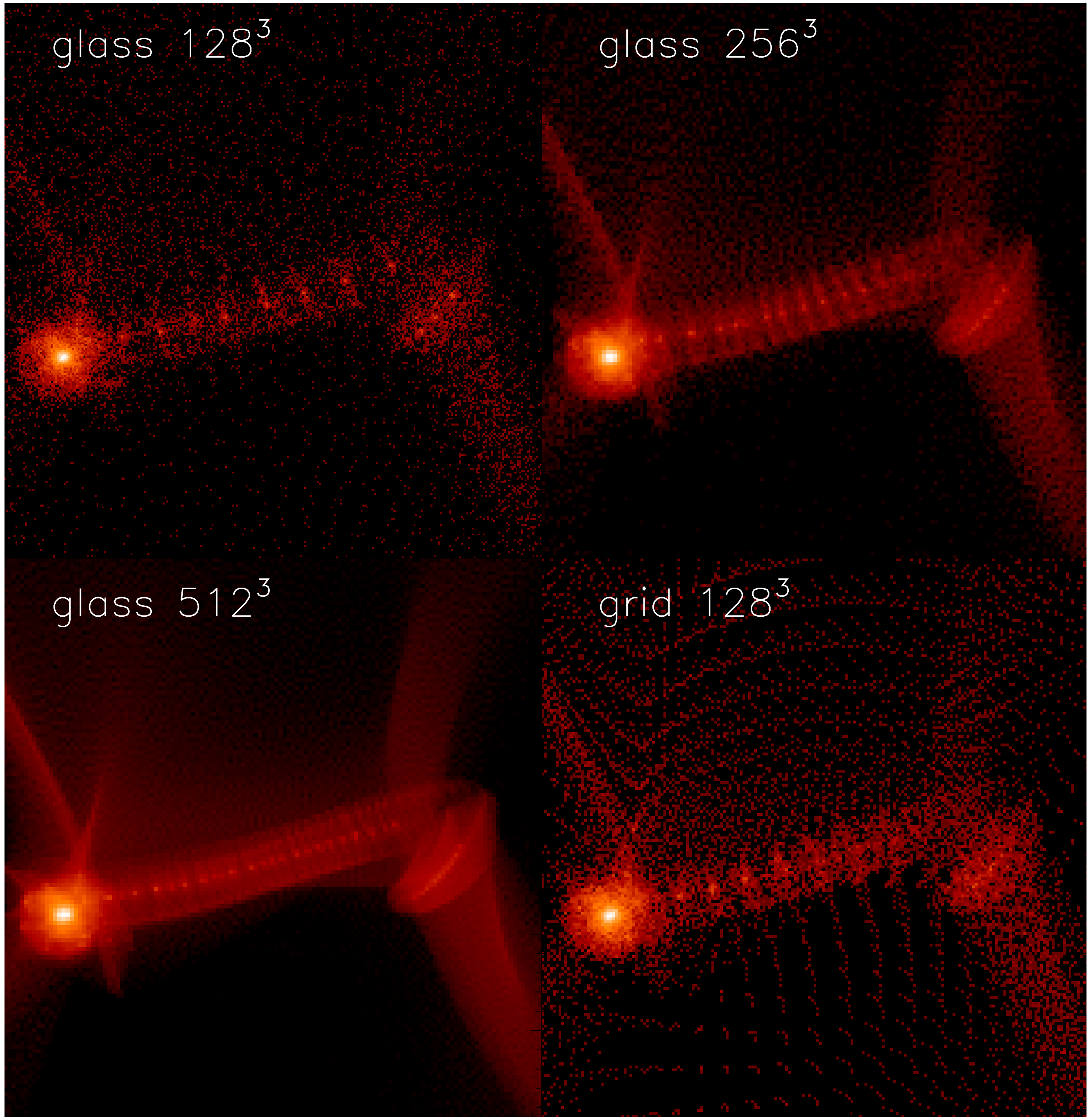
↓

the workhorse
of computational
cosmology

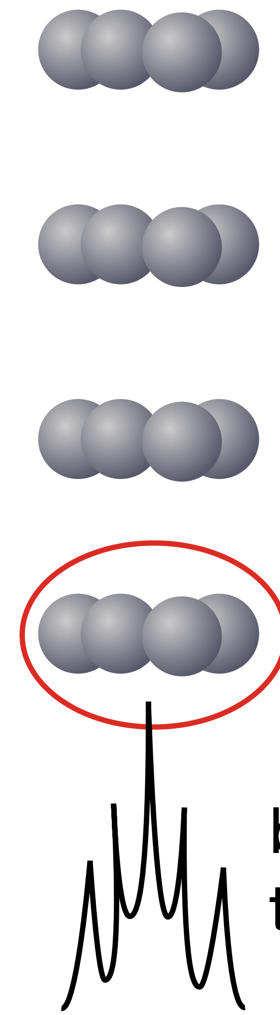
a lot is owed to this method!

Can we break it?

there is a regime where this method doesn't do well at all!
Universes with smooth primordial density fields (always at SOME scale)



Wang&White 2007



large softening needed.
these are no 'clumps',
just convergent points!

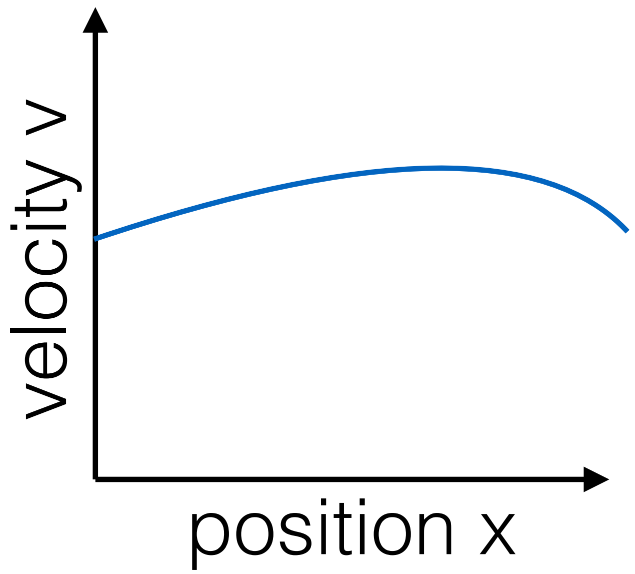
but want small softening
to get small scale structure!

it should not collapse
along vertical direction!
this info is not local!

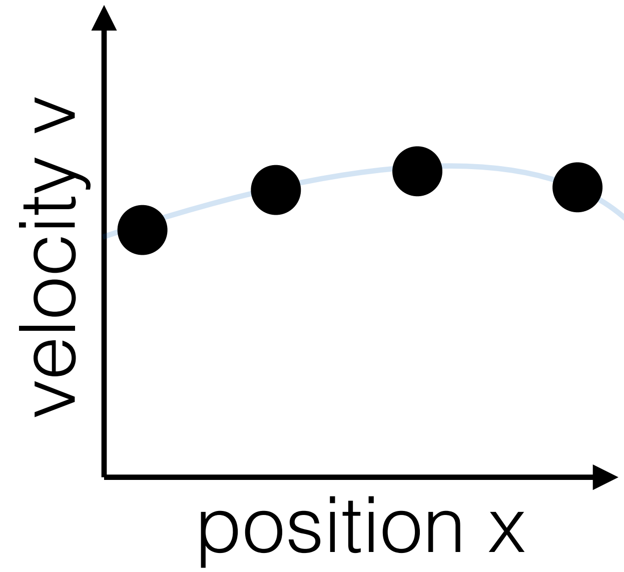
Most obvious for non-CDM simulations!
(e.g. Centrella&Melott 1983, Melott&Shandarin 1989, Wang&White 2007)

Evolving the fine-grained distribution function

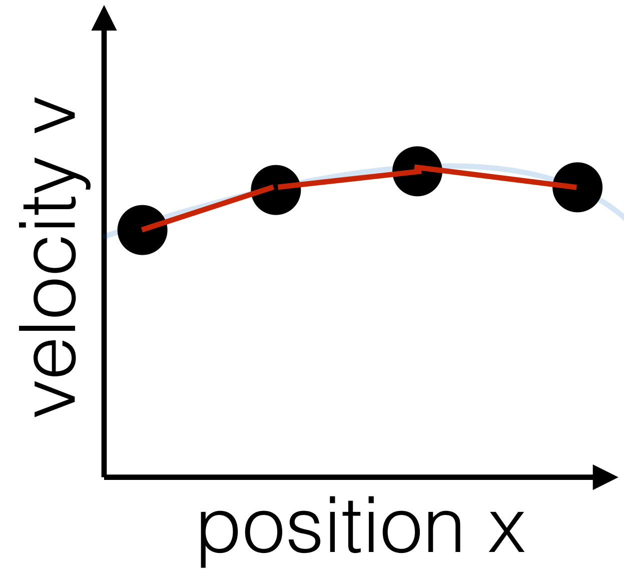
actual distribution function



N-body just have particles



now: connect particles by interpolating functions



density = 1 / projected length
put mass not at particles, but in-between

But need to split elements, when structure of distribution function becomes complicated -> costly!

Hahn&Angulo 2016
Sousbie&Colombi 2016

With refinement, it is possible to track very complicated orbits

orbit of square in chaotic potential…

movie by T. Sousbie, using ColDICE code (Sousbie&Colombi 2016)

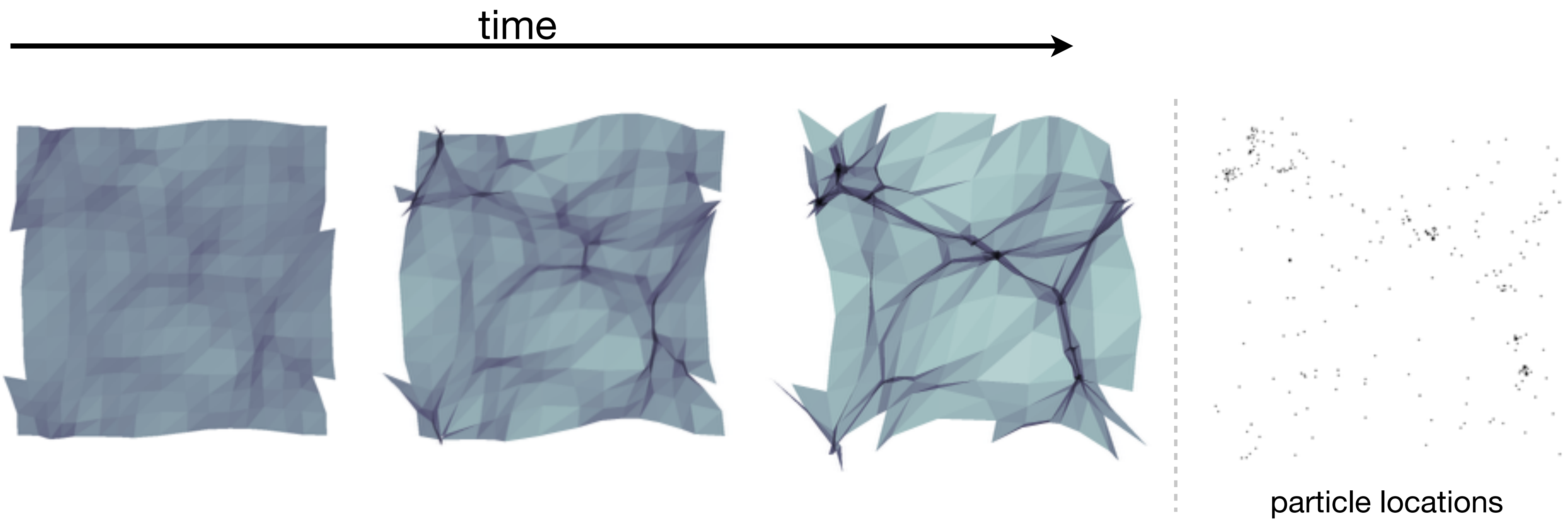
With refinement, it is possible to track very complicated orbits

orbit of square in chaotic potential…

movie by T. Sousbie, using ColDICE code (Sousbie&Colombi 2016)

So what do we gain for structure formation?

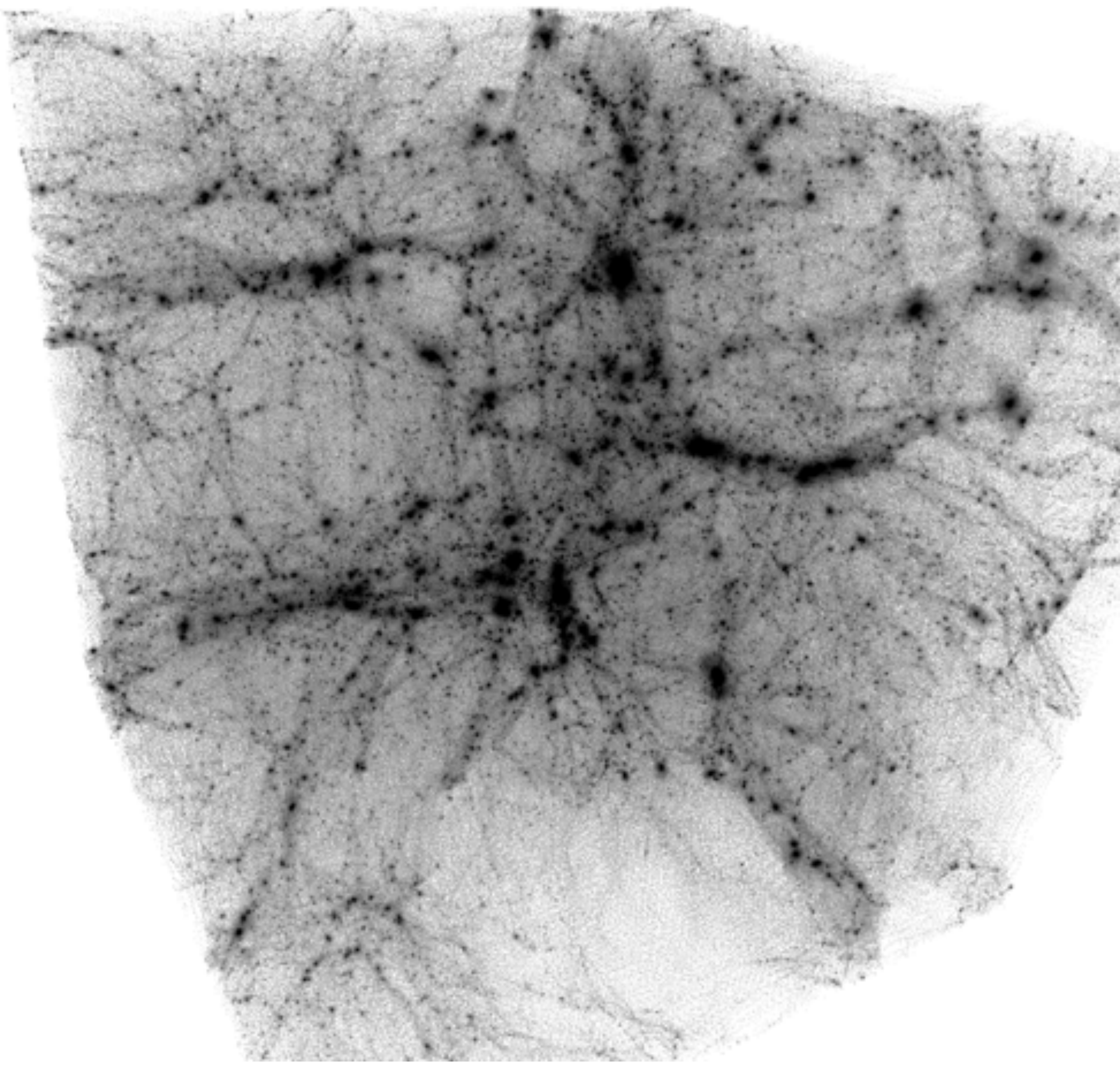
The real space density, velocity field, etc., at any given point can then be determined from **all** elements that contain that point (see also Shandarin et al. 2012).



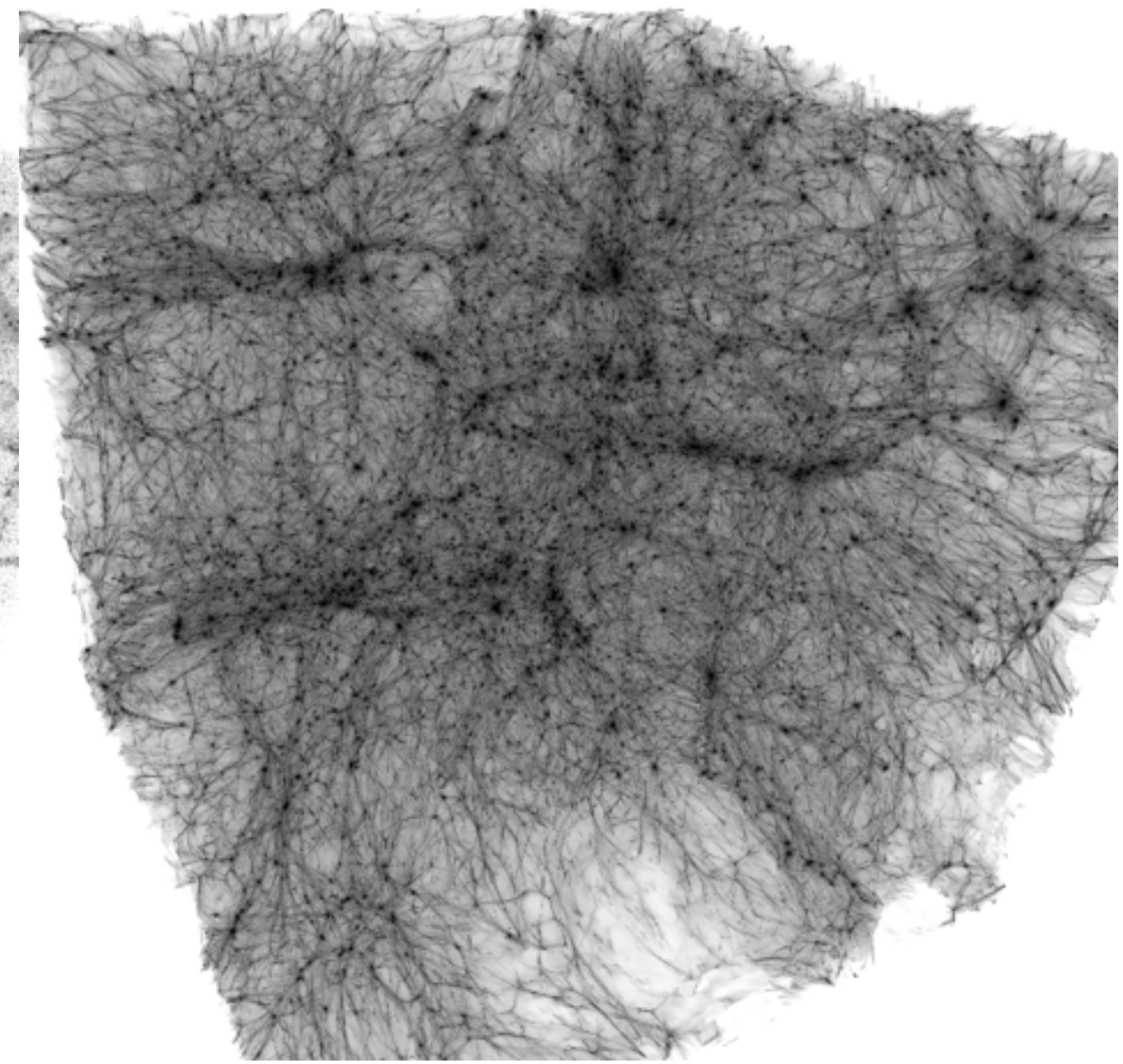
Structure formation is like high-dimensional origami: folding a n-dimensional sheet in 2n-dimensional space
(See also Neyrinck 2014, for the connection to mathematical origami).

each fold is a caustic

So what do we gain in 3+3D?



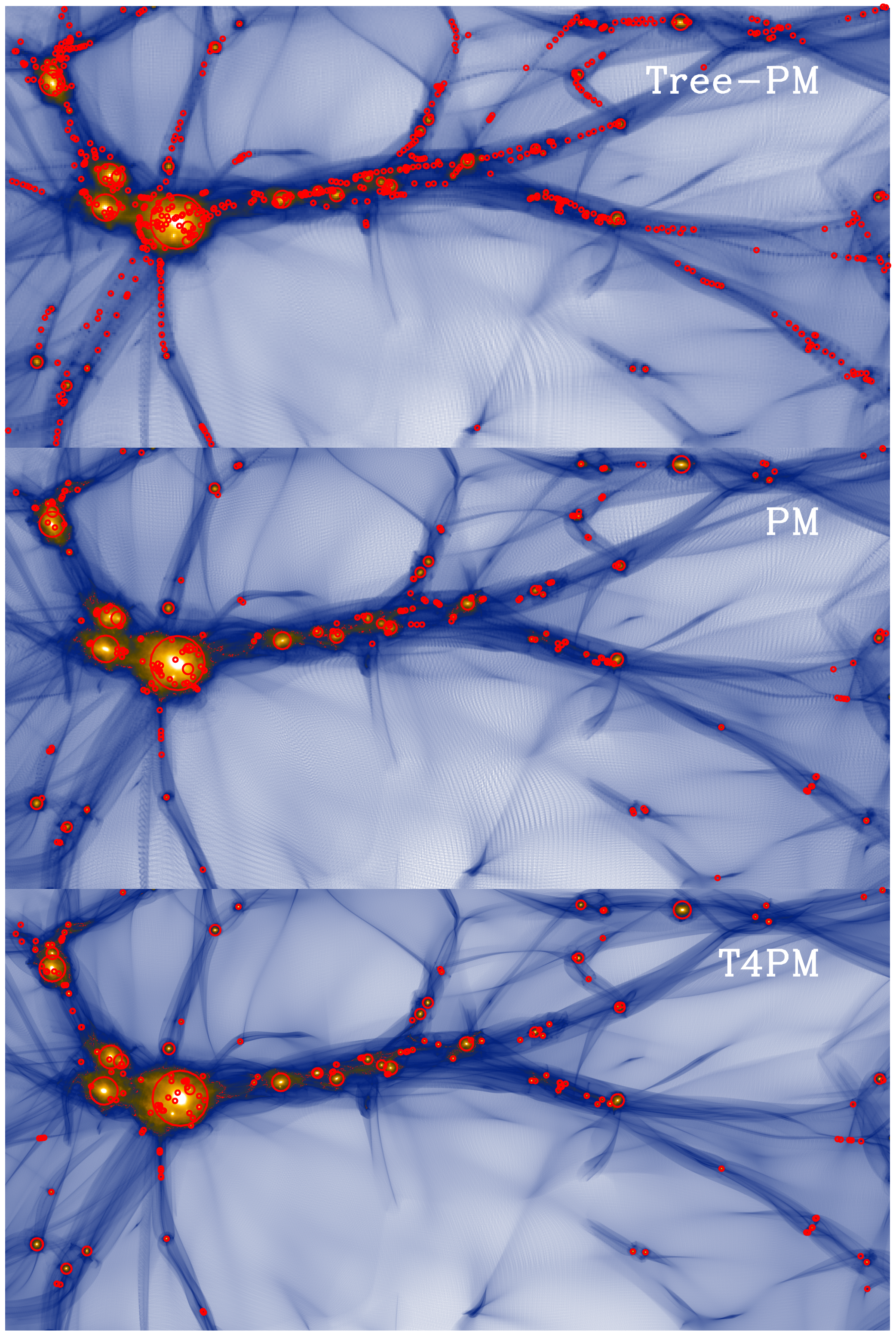
rendering points for particles.



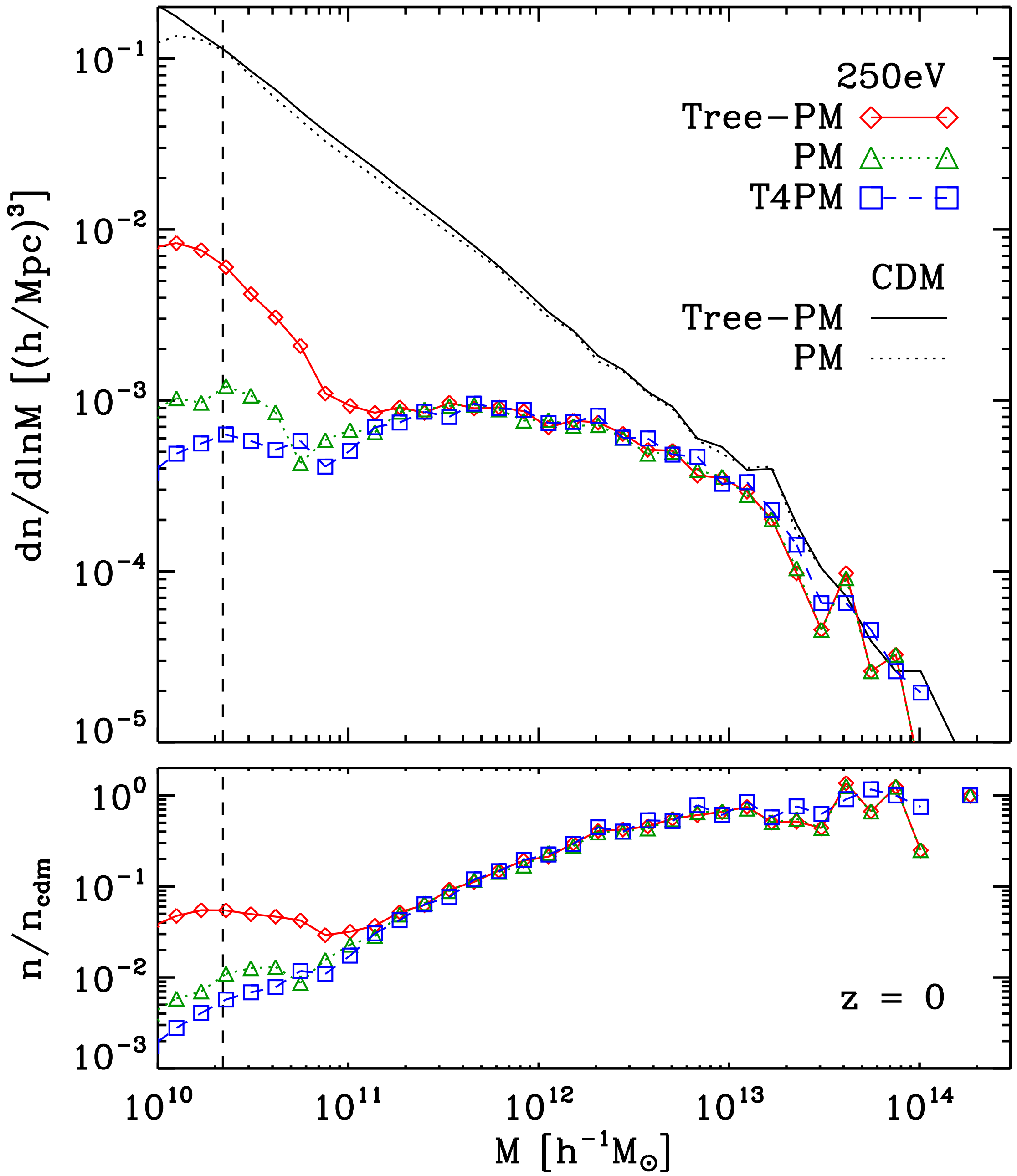
rendering tetrahedral phase space cells.

Same simulation data! (Abel, Hahn, Kaehler 2012)

If one uses this approach self-consistently, it cures the fragmentation problem of N-body

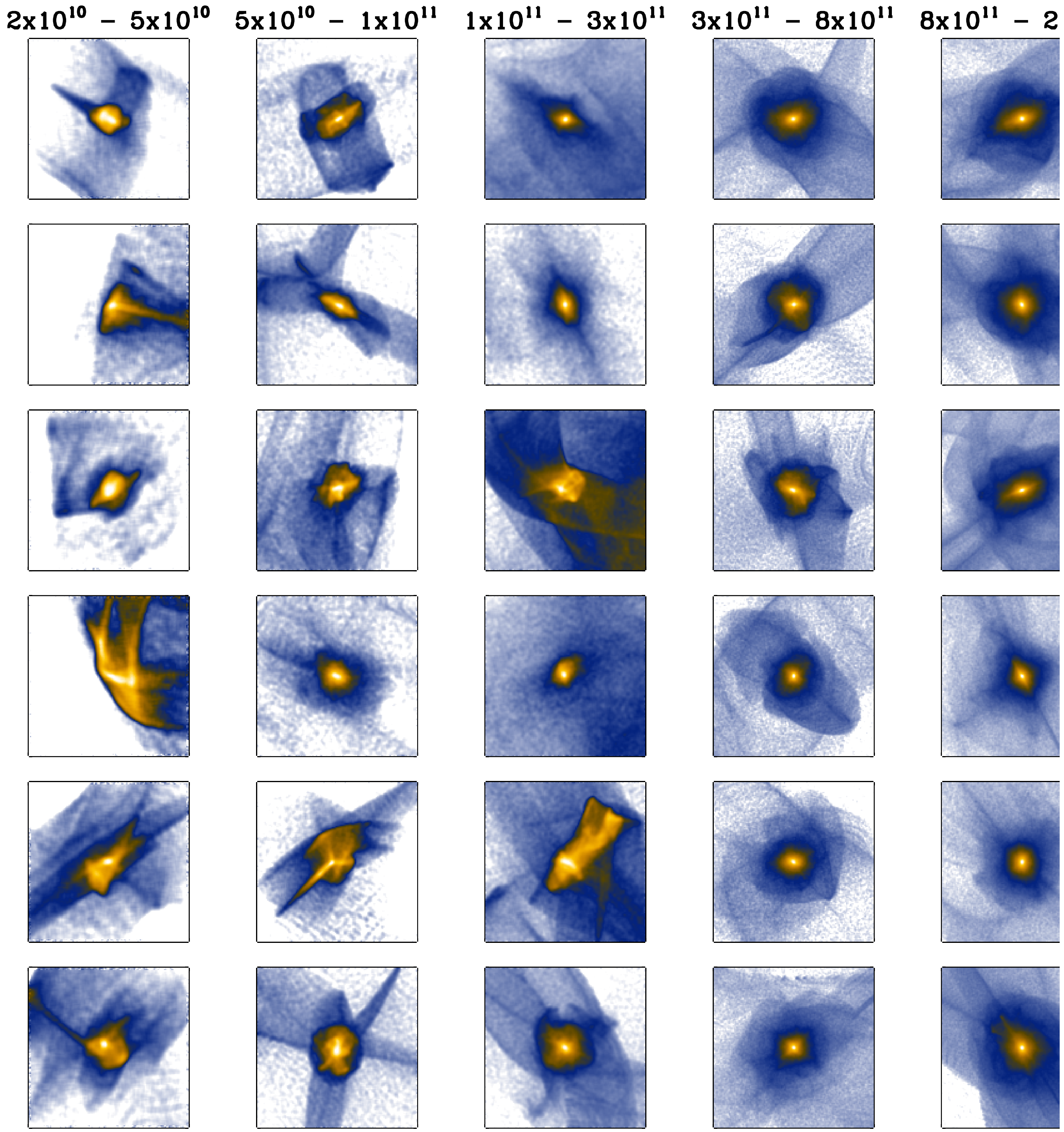


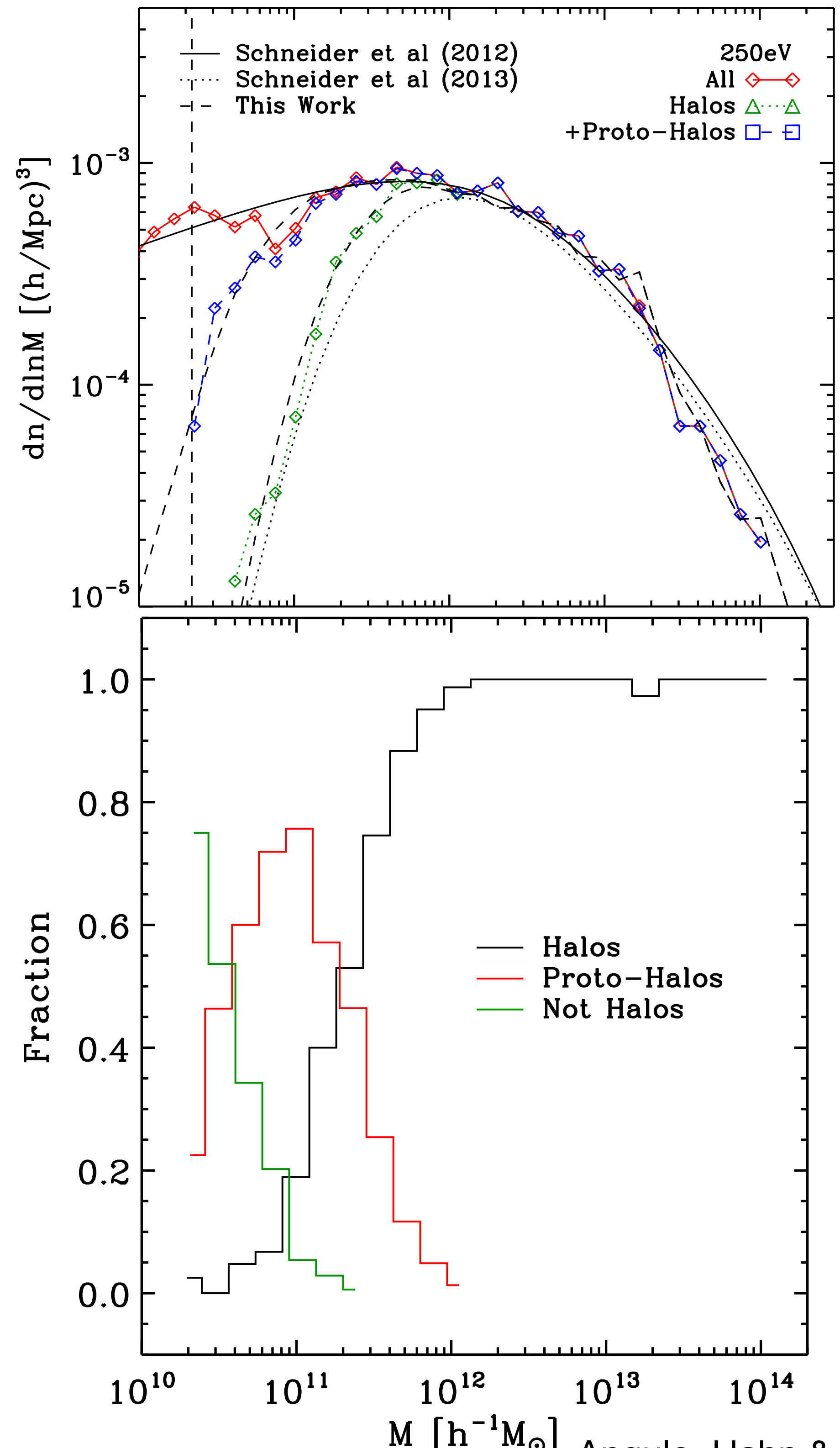
First determination of WDM halo mass function



Angulo, Hahn & Abel 2013

Structure formation in WDM very different than in CDM





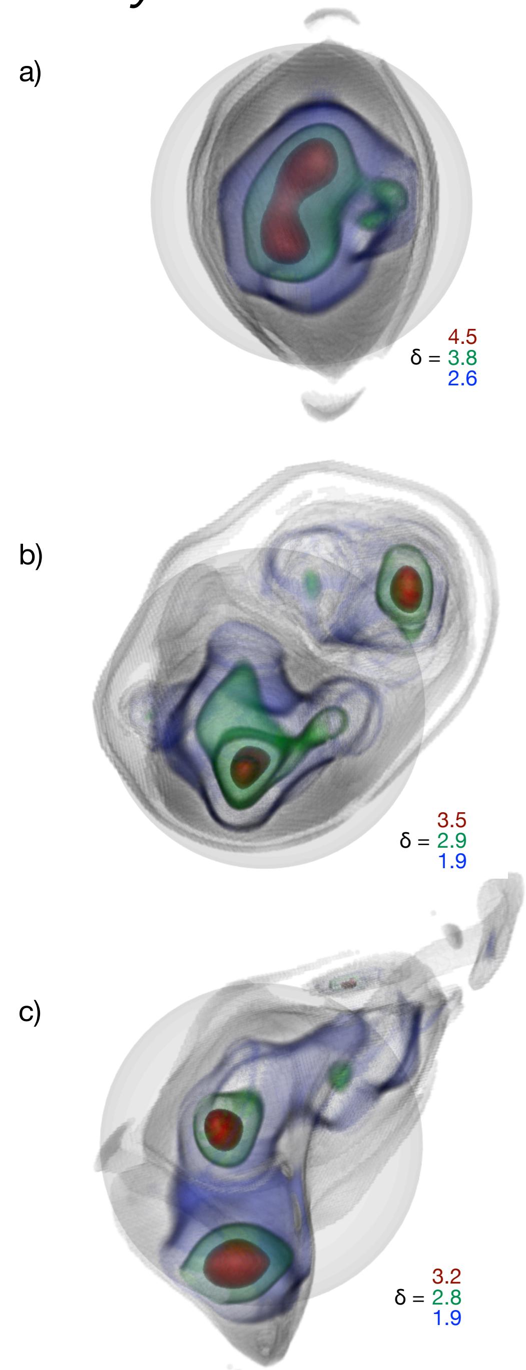
Angulo, Hahn & Abel 2013

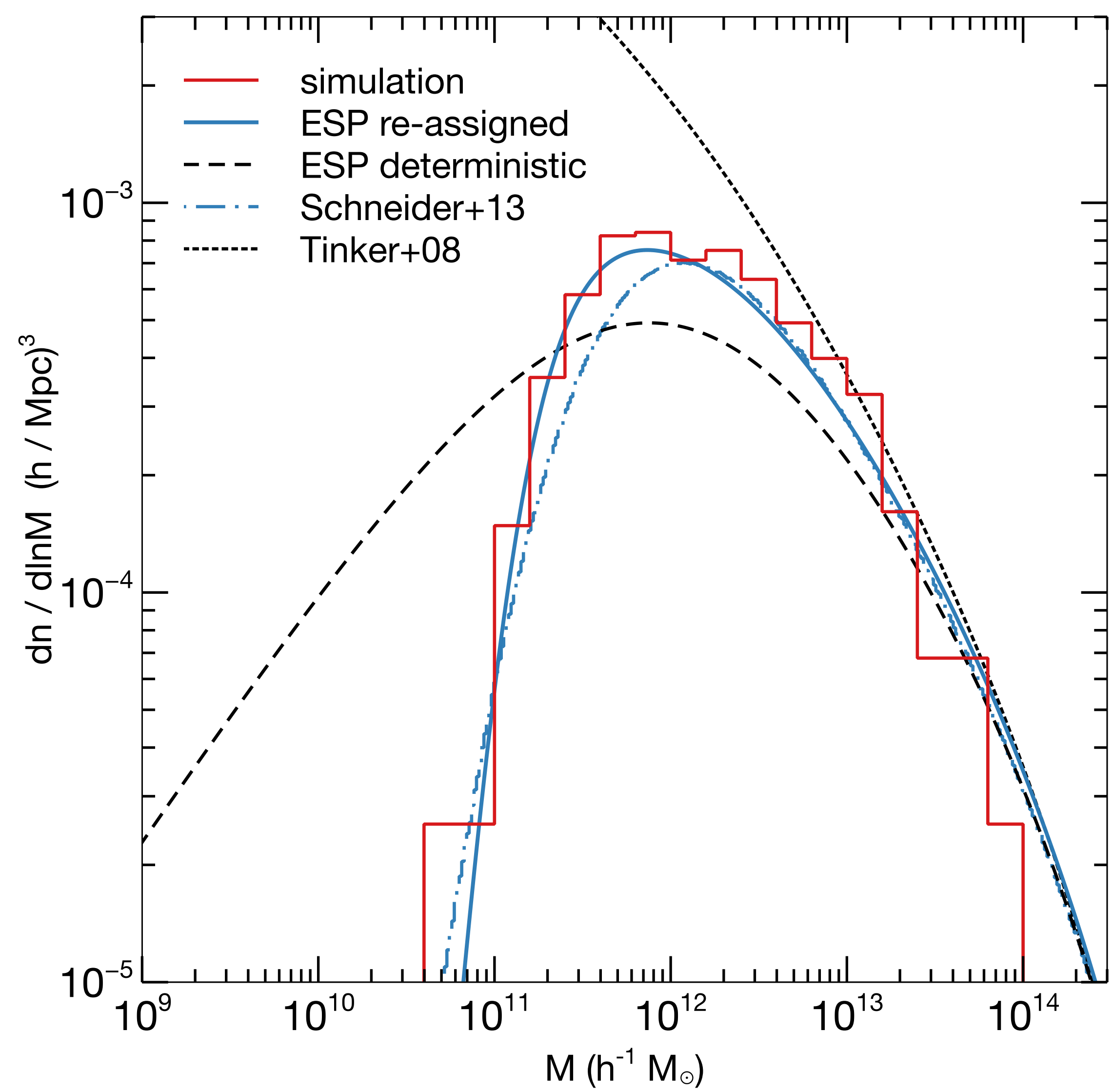
Mass=evolutionary stage,
no progenitors below some mass

Incorporating this into mass-function calculations

From peaks in the initial density field to mass functions

non-trivial with cut-off, requires more careful treatment of collapse than vanilla perfectly cold CDM





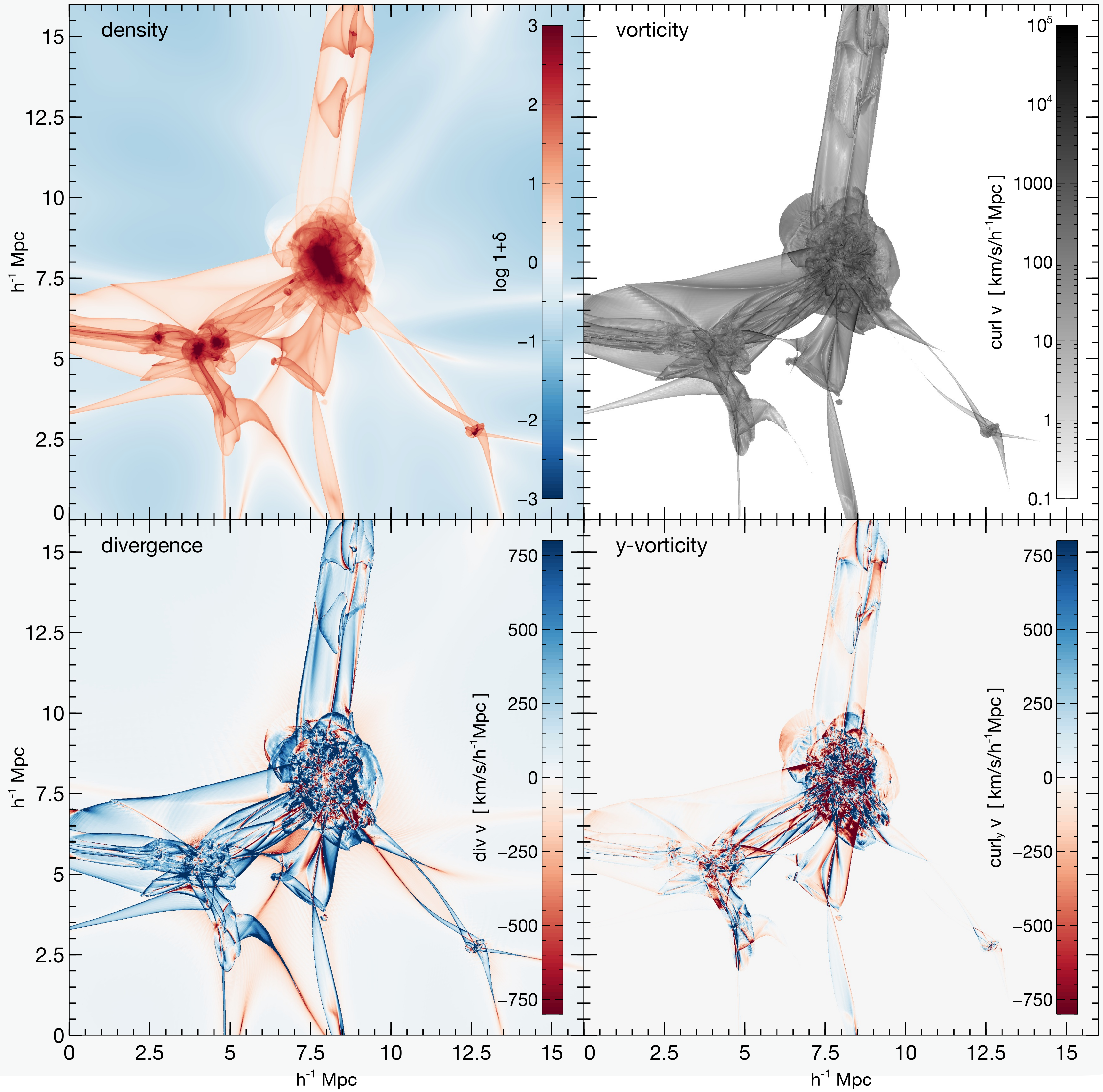
Hahn & Paranjape 2013

New insights from mean field dynamics

It is possible to investigate moments of the Boltzmann hierarchy a-posteriori

Measurements impossible from N-body

New insights into DM dynamics



Structure formation in WDM very different than in CDM

collapse from initially smooth field

no progenitors below certain mass

caustics everywhere

Structure formation in WDM very different than in CDM

collapse from initially smooth field

no progenitors below certain mass

caustics everywhere

Mass distribution in halos follows very simple functional form

Navarro, Frenk & White (1995,1996)

$$\rho(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$$

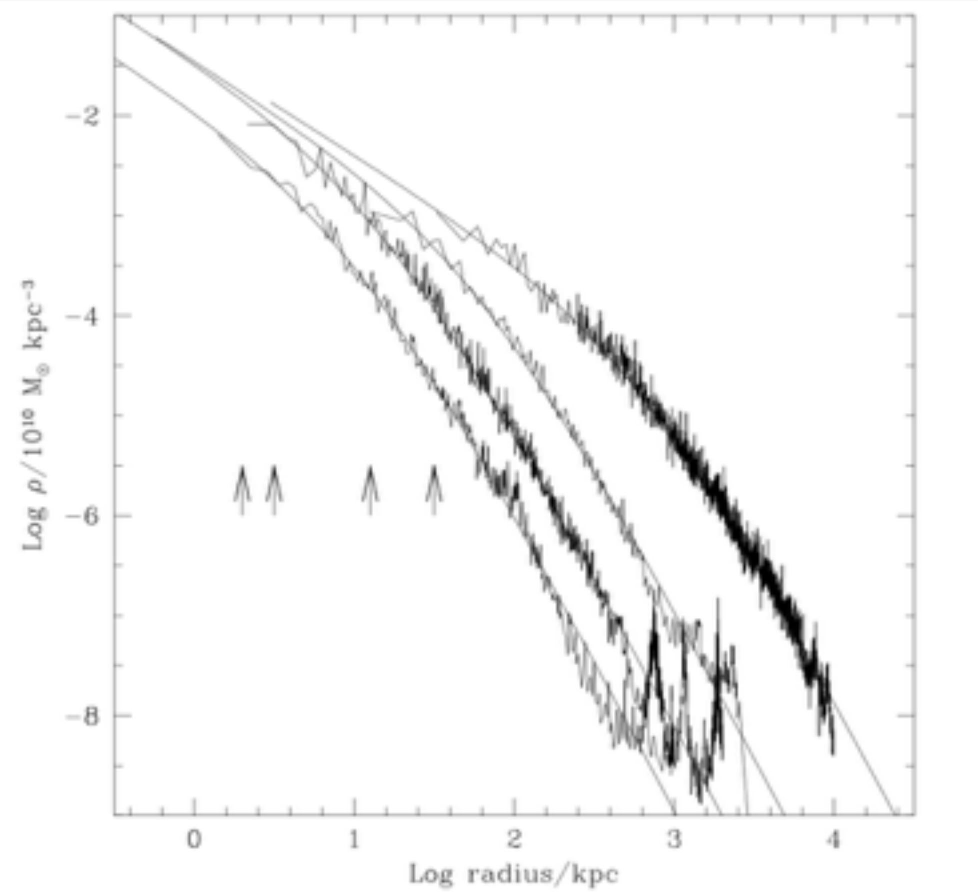
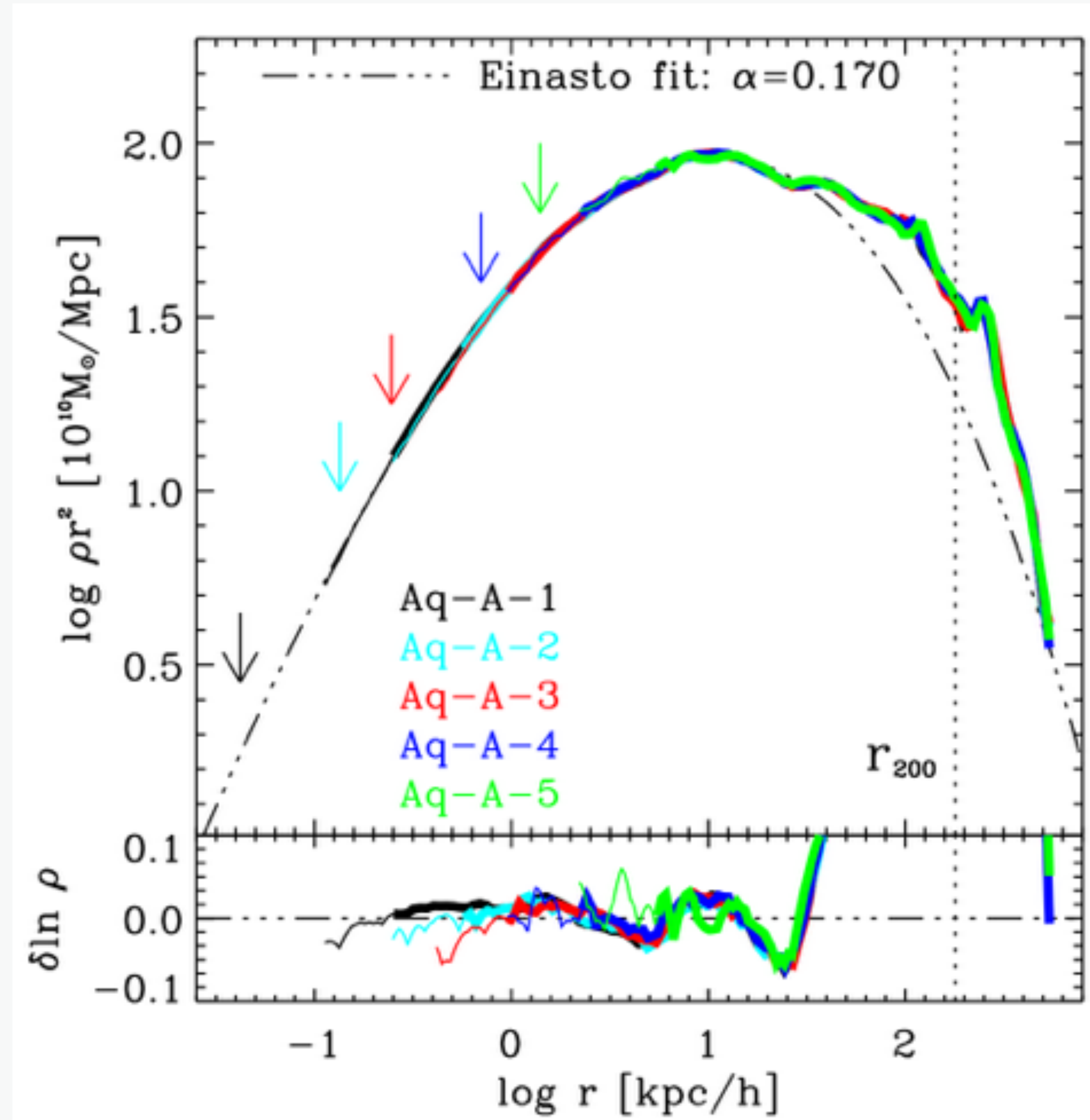
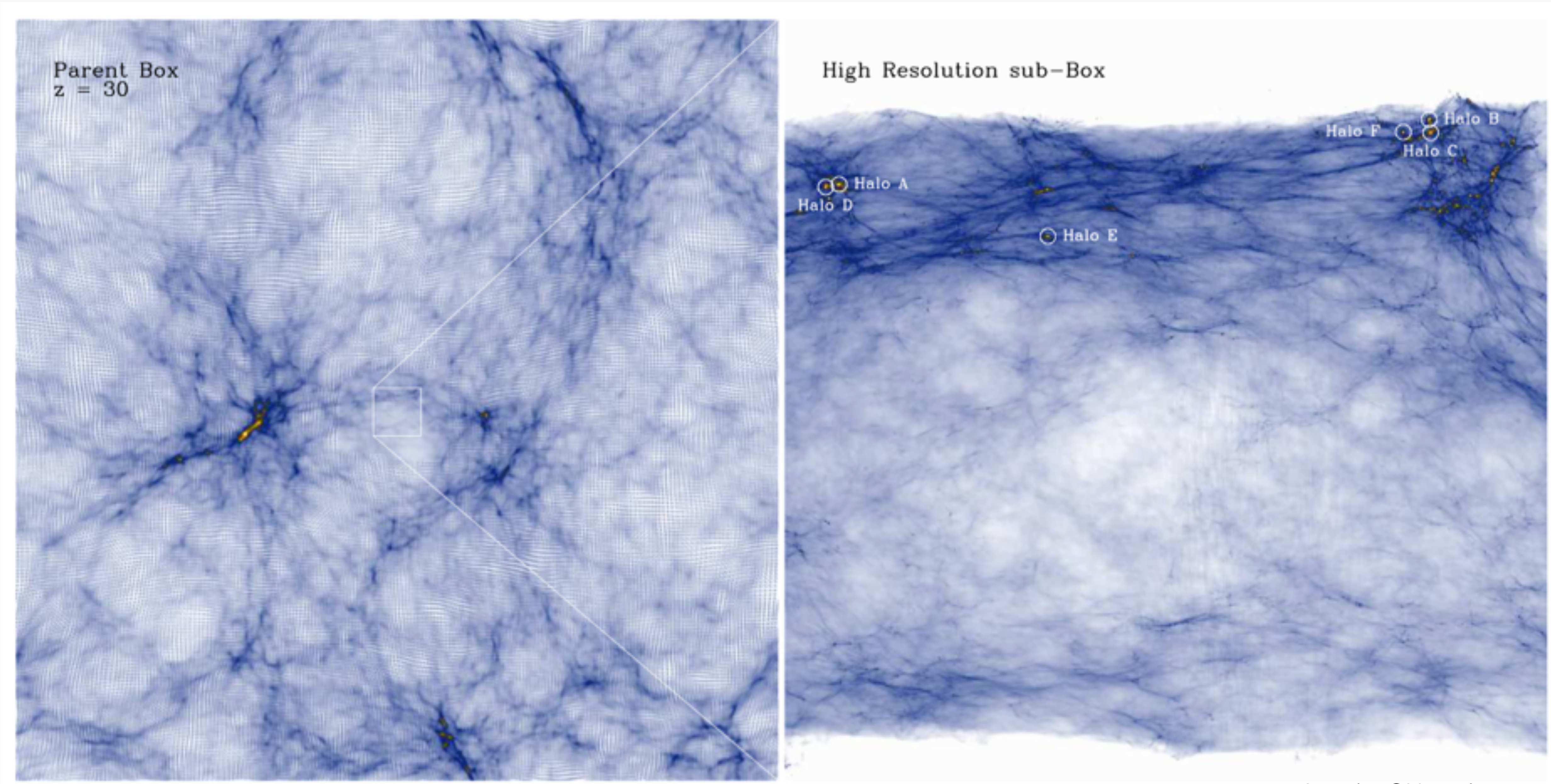


Fig. 3.— Density profiles of four halos spanning four orders of magnitude in mass. The arrows indicate the gravitational softening, h_g, of each simulation. Also shown are fits from eq.3. The fits are good over two decades in radius, approximately from h_g out to the virial radius of each system.



Navarro 2009

How do the first CDM micro halos form?



Angulo, OH et al. 2016

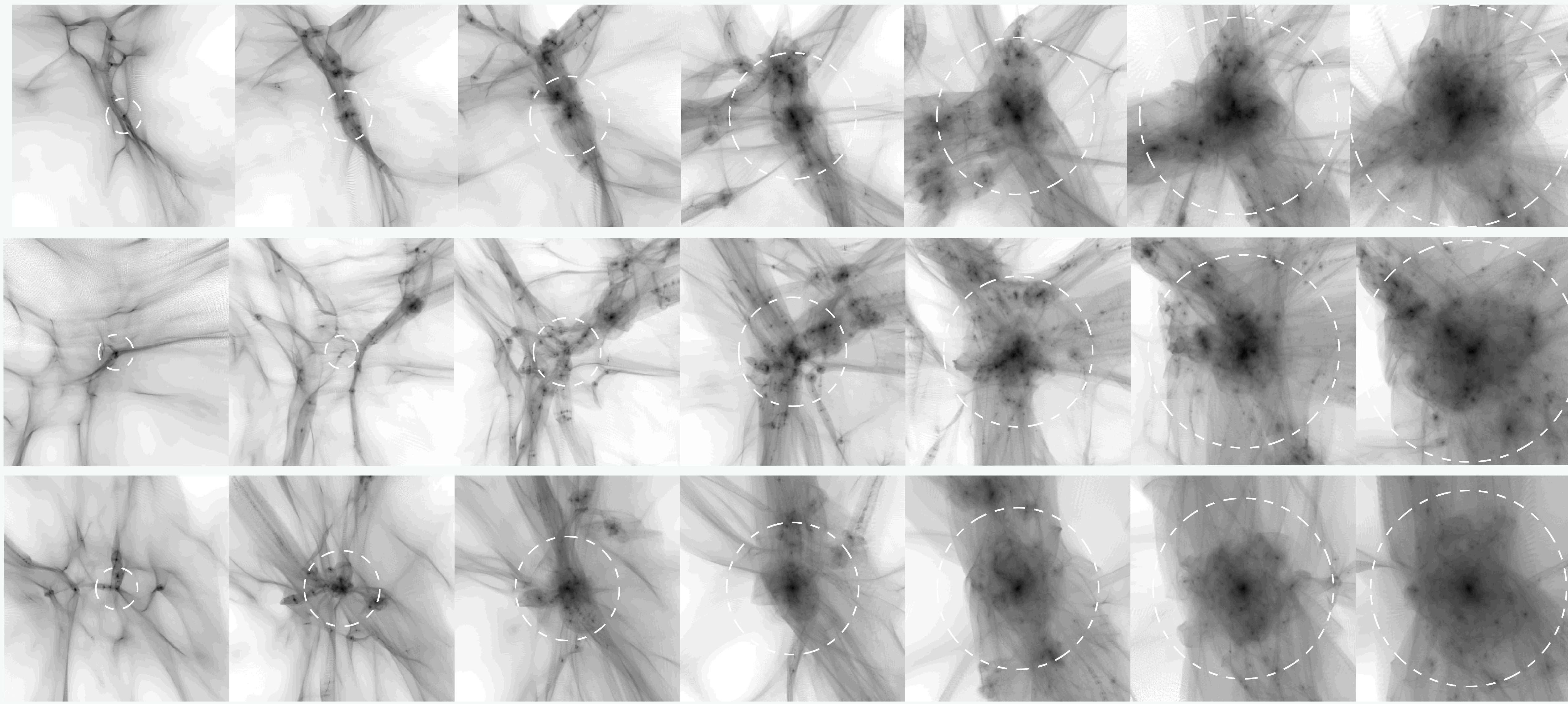
23kpc

“zoom simulations” of 5 haloes, effective resolution 131 072^3

How do the first CDM micro halos form?

collapse from smooth field

phase rapid mass growth by many major mergers



Angulo, OH et al. 2016

How do the profiles of first haloes evolve?

Ishiyama 2014

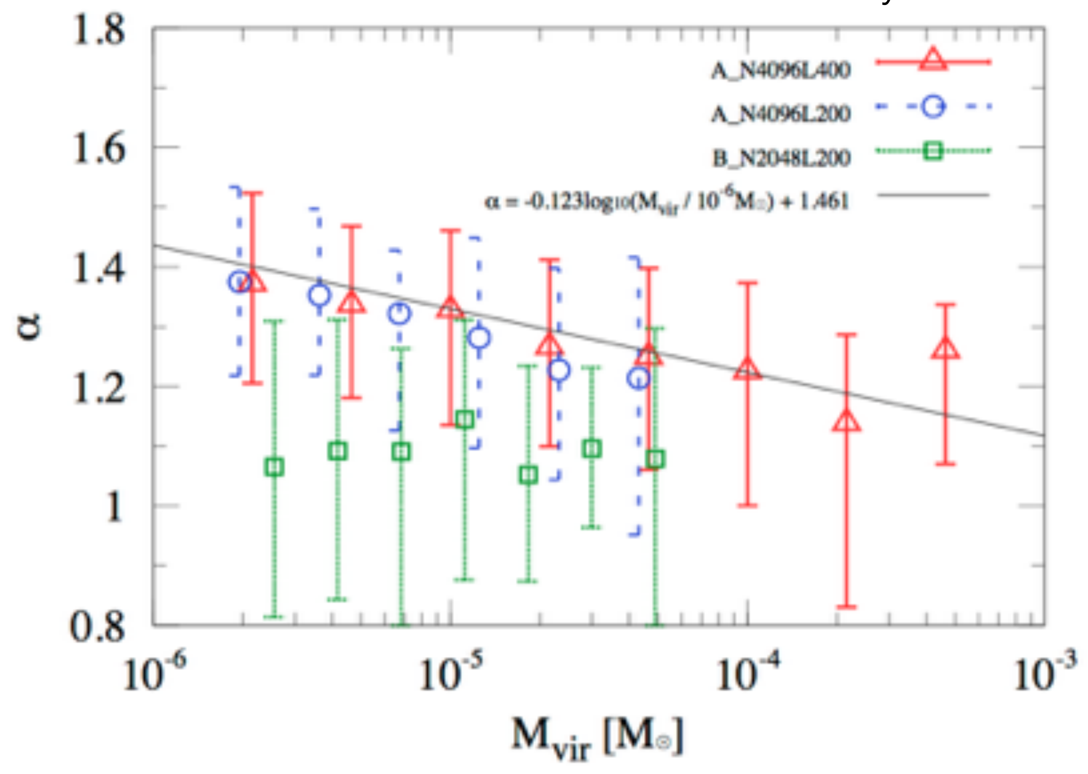
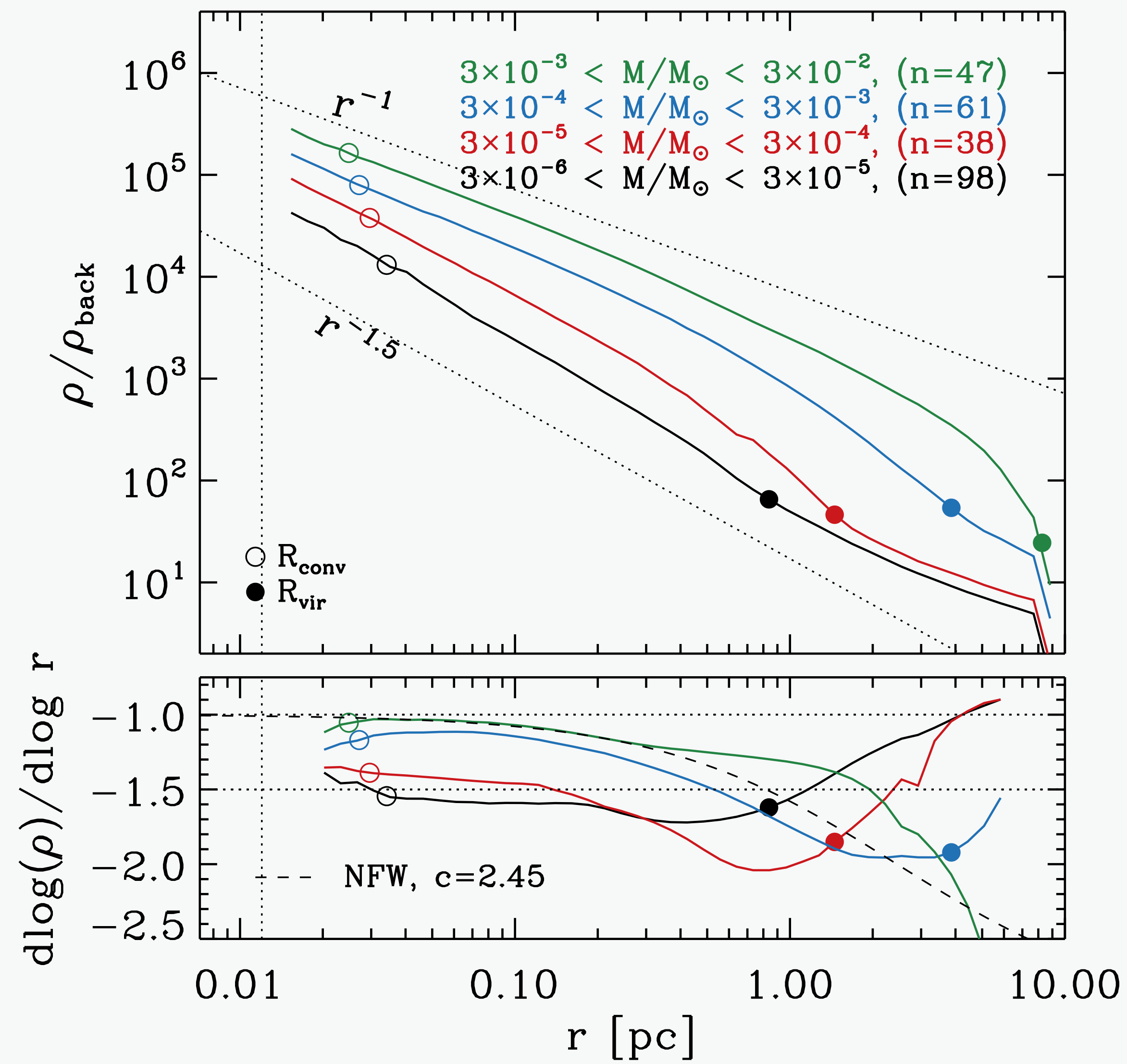


Figure 7. Slope of the density profile of each halo α plotted against the halo virial mass $M_{\rm vir}$. Circles, triangles and squares show the median value in each mass bin. Whiskers are the first and third quantiles. Black solid line is the best fit power law function (Equation (2) in the text).

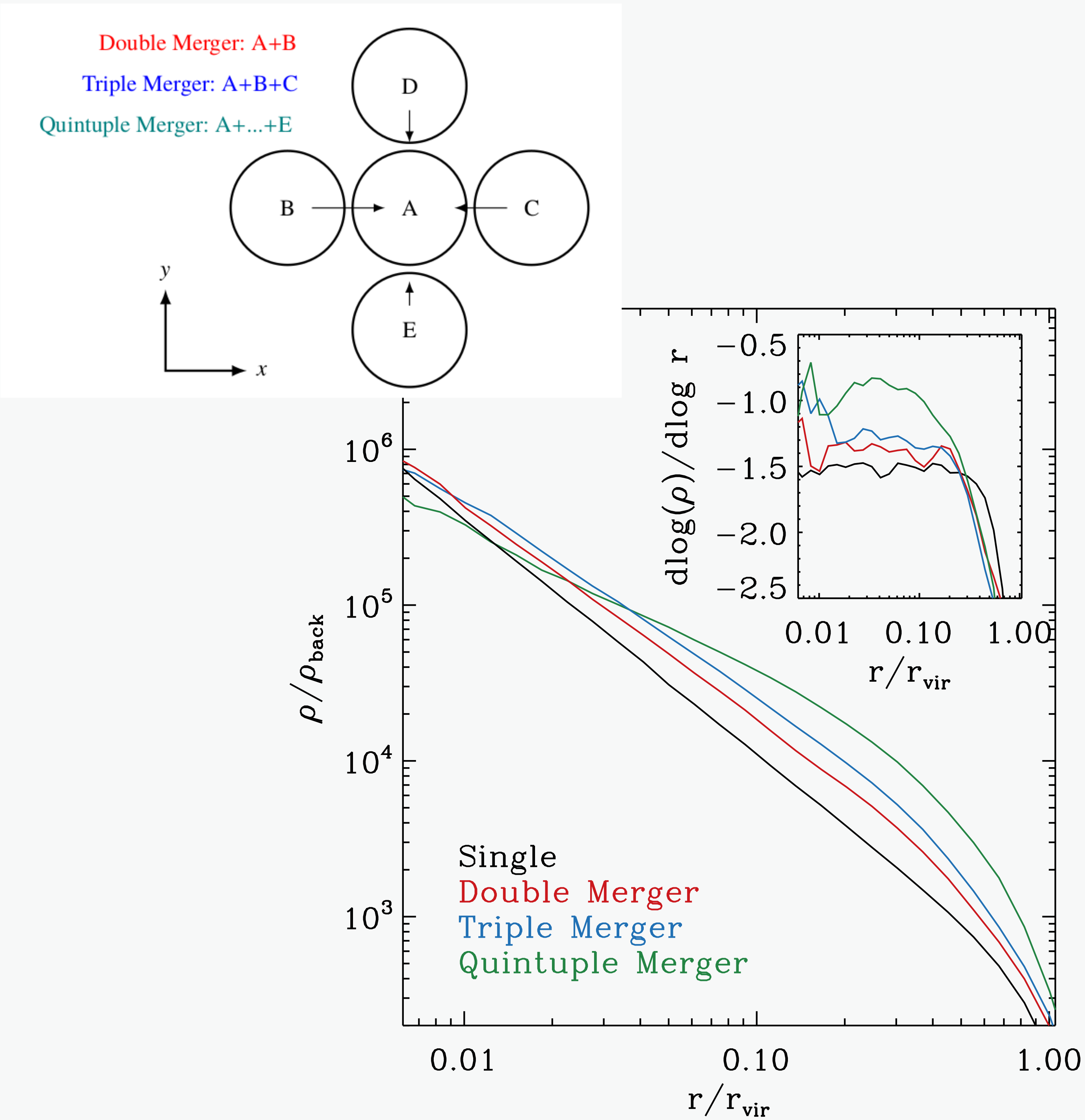
(see also e.g. Ishiyama et al. 2010, Anderhalden & Diemand 2013)

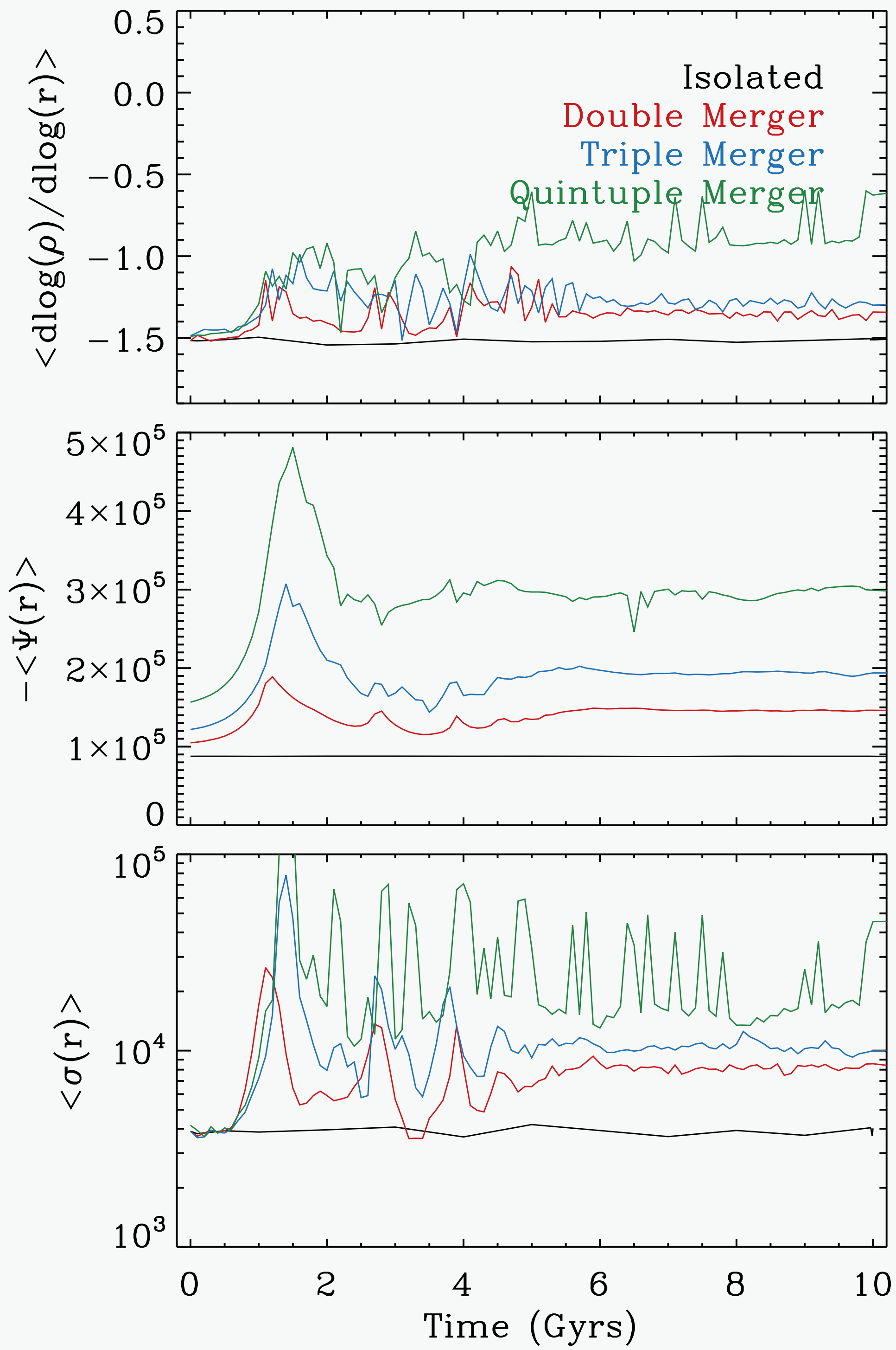


Do microhaloes become slowly NFW-like as they grow? How?

Evidence that mergers drive a transformation towards NFW

Good evidence that violent relaxation (Lynden-Bell 1967) is driving this
this is a collective relaxation process (much like Landau damping)



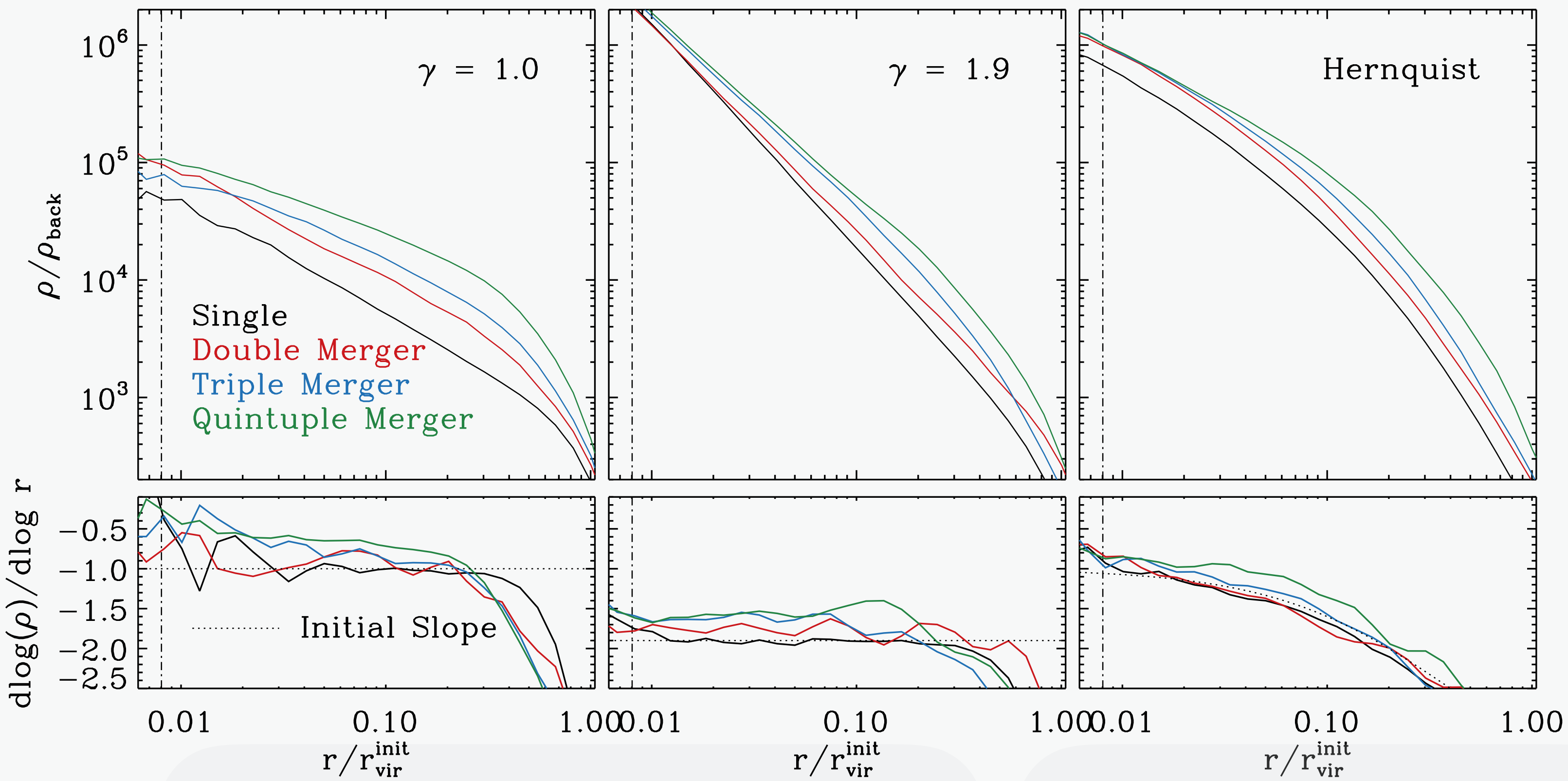


Angulo, OH et al. 2016

see also Ogiya et al. 2016

Why do we always see NFW in most simulations?

Angulo, OH et al. 2016



for power law profiles, result depends on slope

Hernquist/NFW shows no evolution in slope before quintuple merger!

NFW/Hernquist particularly resilient to perturbations

see also work by El Zant, claiming that perturbations are efficiently spread through such haloes

How many of these steep haloes survive?
What is the contribution to an annihilation signal?
Do all of the microhaloes form like single power law profiles?

Summary

Dark matter is collisionless and rather cold -> challenges for modelling

New tessellation methods overcome important limitations of N-body method (virtually noise-free but more costly)

Allow to study wealth of additional properties of collisionless systems

New angle on studying small scale properties of dark matter (improved constraints on particle nature..)