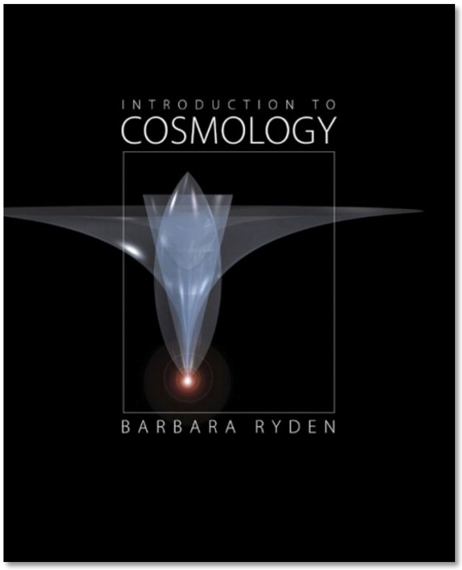

Introduction to Cosmology

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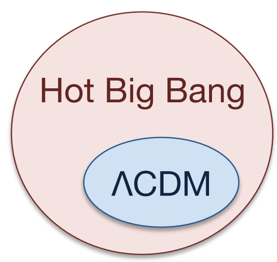
ICTP Summer School on Cosmology
2016 June 6

Today: Observational evidence for the "standard model" of cosmology
morning: the Hot Big Bang model
afternoon: the ΛCDM model



Tuesday: Special epochs of the universe (recombination, nucleosynthesis, inflation)

Wednesday: Structure formation

Danger: Astronomers at work!

Time: 1 Gyr = 10^9 yr ≈ 3.2×10^{16} s ~ 10^{60} t_{planck}

age of the Sun = 4.57 Gyr

time since Big Bang = 13.7 Gyr

Distance: 1 Mpc = 10^6 parsecs ≈ 3.1×10^{22} m ~ 10^{57} d_{planck}

distance to Andromeda Galaxy ≈ 0.75 Mpc

distance to Coma Cluster of galaxies ≈ 100 Mpc

Mass: 1 $M_\odot$ ≈ 2.0×10^{30} kg ~ 10^{38} m_{planck}

mass of Milky Way Galaxy ≈ 10^{12} $M_\odot$

mass of Coma Cluster ≈ 10^{15} $M_\odot$

Olbers' Paradox: The night sky is dark.

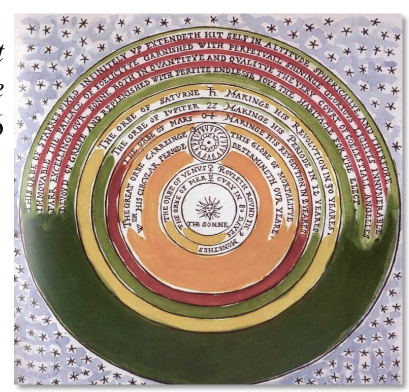
$\Sigma_{sky} \approx 5 \times 10^{-17}$ watts m^{-2} arcsec^{-2}

$\Sigma_\odot \approx 5 \times 10^{-3}$ watts m^{-2} arcsec^{-2}

Cunningham, *The Cosmological Glass*, 1559

Stars attached to celestial sphere: no paradox.

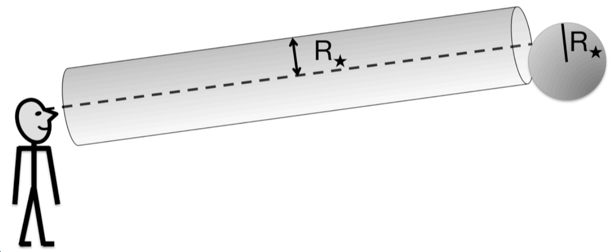
Digges, *A Perfect Description of the Celestial Orbs*, 1576



Infinite universe filled with stars: **PARADOX!**

Stars are opaque spheres, with typical radius
$R_* \sim R_\odot \sim 7 \times 10^8$ m $\sim 2 \times 10^{-14}$ Mpc.

The number density of stars is
$n_* \sim 10^9$ Mpc^{-3}.



How far can you see, on average,
before your line of sight intercepts a star?

$$\lambda = \frac{1}{n_*\left(\pi R_*^2\right)} \sim \frac{1}{(10^9\,\mathrm{Mpc}^{-3})(10^{-27}\,\mathrm{Mpc}^2)} \sim 10^{18}\,\mathrm{Mpc}$$

In an infinite universe (or one reaching to $r > 10^{18}$ Mpc),
the sky is paved with stars, with surface brightness
$\Sigma \sim \Sigma_\odot \sim 5 \times 10^{-3}$ W m^{-2} arcsec^{-2}.

The night sky in our universe has
a surface brightness smaller by
14 orders of magnitude.

Which of my assumptions was wrong?

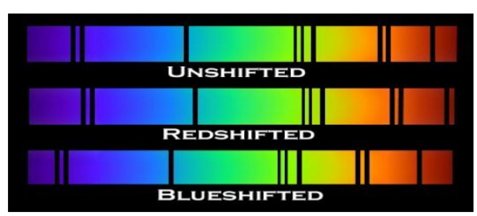
Possible resolutions of Olbers' Paradox:

1) Distant stars are hidden by opaque material. (This doesn't work in the long run.)
2) The universe has finite size: $r \ll 10^{18}$ Mpc. (Or stars occupy only a finite volume.)
3) The universe has finite *age*: $ct \ll 10^{18}$ Mpc. (Or stars have existed for a finite time.) ← Mostly this
4) Distant stars have low surface brightness. ← A little of this

Hubble's Law: Galaxies show a redshift proportional to their distance.

An emission (or absorption) line has wavelength λ_e in the light source's frame of reference, and wavelength λ_0 in the observer's frame of reference.

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e}$$



$z > 0$ → redshift

$z < 0$ → blueshift

1923: Arthur Eddington compiles a list of 41 galaxy wavelength shifts (mostly measured by Vesto Slipher).

RADIAL VELOCITIES OF SPIRAL NEBULAE

+ indicates receding, − approaching

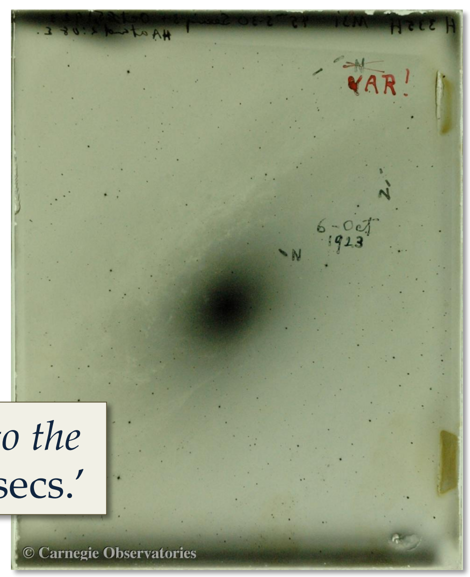
N.G.C.	R.A. h m	Dec. ° ′	Rad. Vel. km. per sec.	N.G.C.	R.A. h m	Dec. ° ′	Rad. Vel. km. per sec
221	0 38	+40 26	− 300	4151*	12 6	+39 51	+ 980
224*	0 38	+40 50	− 300	4214	12 12	+36 46	+ 300
278†	0 47	+47 7	+ 650	4258	12 15	+47 45	+ 500
404	1 5	+35 17	− 25	4382†	12 21	+18 38	+ 500
584†	1 27	− 7 17	+1800	4449	12 24	+44 32	+ 200
598*	1 29	+30 15	− 260	4472	12 25	+ 8 27	+ 850
936	2 24	− 1 31	+1300	4486†	12 27	+12 50	+ 800
1023	2 35	+38 43	+ 300	4526	12 30	+ 8 9	+ 580
1068*	2 39	− 0 21	+1120	4565†	12 32	+26 26	+1100
2683	8 48	+33 43	+ 400	4594*	12 36	−11 11	+1100
2841†	9 16	+51 19	+ 600	4649	12 40	+12 0	+1090
3031	9 49	+69 27	− 30	4736	12 47	+41 33	+ 290
3034	9 49	+70 5	+ 290	4826	12 53	+22 7	+ 150
3115†	10 1	− 7 20	+ 600	5005	13 7	+37 29	+ 900
3368	10 42	+12 14	+ 940	5055	13 12	+42 37	+ 450
3379*	10 43	+13 0	+ 780	5194	13 26	+47 36	+ 270
3489†	10 56	+14 20	+ 600	5195†	13 27	+47 41	+ 240
3521	11 2	+ 0 24	+ 730	5236†	13 32	−29 27	+ 500
3623	11 15	+13 32	+ 800	5866	15 4	+56 4	+ 650
3627	11 16	+13 26	+ 650	7331	22 33	+33 23	+ 500
4111†	12 3	+43 31	+ 800				

36 redshifts, **5 blueshifts.**
Assuming classical Doppler shift, the mean radial velocity is

$$v_r = c\,z = +540 \text{ km s}^{-1}$$

Eddington: 'The great preponderance of positive (receding) velocities is very striking.'

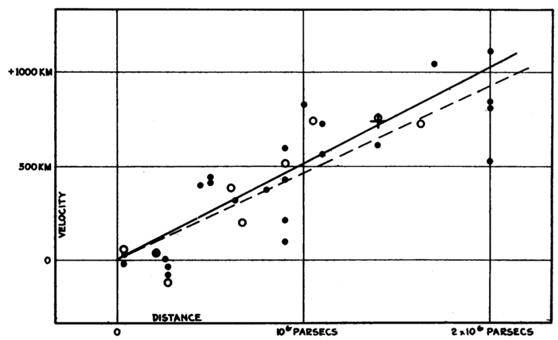
1923: Edwin Hubble estimates galaxy distances using Cepheid variable stars.



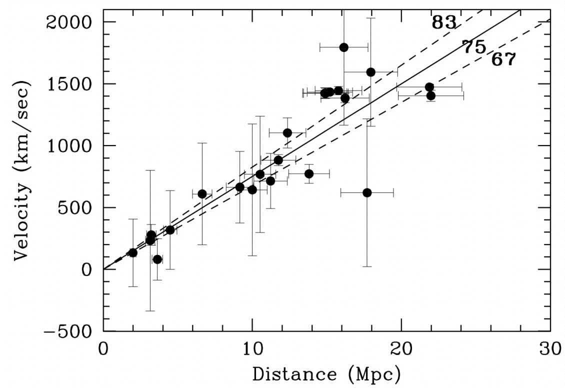
Hubble: 'The corresponding distance [*to the Andromeda Galaxy*] is about 285,000 parsecs.'

actually 0.75 Mpc

1929: Hubble shows that galaxies have a measured redshift proportional to estimated distance.



Hubble 1929



Freedman et al. 2001

Hubble's Law:

$$cz = H_0 r$$

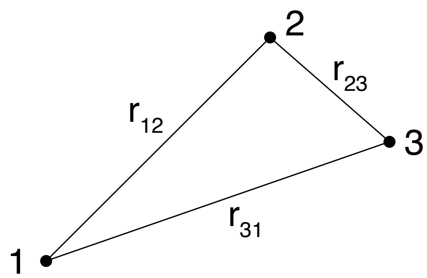
H_0 = 'Hubble constant' = 68 ± 2 km s^{-1} Mpc^{-1}

$1/H_0$ = 'Hubble time' = 14.4 ± 0.4 Gyr

$v_r = H_0 r$
$t = r / v_r = 1/H_0$
independent of r

c/H_0 = 'Hubble distance' = 4400 ± 100 Mpc

Hubble's Law: result of homogeneous, isotropic expansion.



$$r_{12}(t) = a(t) r_{12}(t_0)$$
$$r_{23}(t) = a(t) r_{23}(t_0)$$
$$r_{31}(t) = a(t) r_{31}(t_0)$$

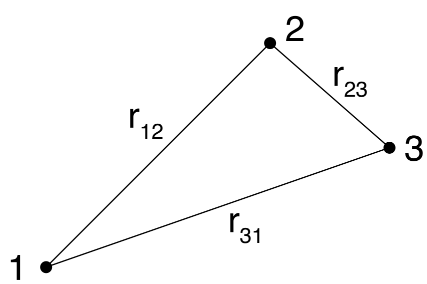
$a(t)$ = 'scale factor'

homogeneous: a is function of t, but not of $\vec{r}$

isotropic: a is scalar, not tensor

normalization: $a(t) = 1$ at $t = t_0 =$ now.

Hubble's Law: result of homogeneous, isotropic expansion.



$$\mathrm{v}_{12}(t) = \frac{dr_{12}}{dt} = \frac{da}{dt} r_{12}(t_0) = \frac{da}{dt}\frac{1}{a(t)} r_{12}(t)$$

$$\mathrm{v}_{12}(t) = H(t) r_{12}(t)$$

$$\text{where } H(t) \equiv \frac{\dot{a}}{a}$$

$H(t)$ = `Hubble parameter',
$H_0 = H(t_0)$ = 'Hubble constant'

Hubble's law is consistent with a Big Bang model, but does not require it.

Hot Big Bang	**Steady State** (Bondi, Gold, & Hoyle 1948)
Cosmological principle: universe is spatially homogeneous & isotropic (on large scales), but changes with time, becoming cooler & less dense.	***Perfect*** cosmological principle: universe is spatially homogeneous & isotropic (on large scales), and its global properties are constant with time.

Steady state model:

Hubble constant H_0 is constant with time.

$$\frac{dr}{dt} = H_0 r \quad \Rightarrow \quad r \propto e^{H_0 t}$$

exponential growth
r ➨ 0 as t ➨ $-\infty$

Mean density ρ_0 is constant with time.

$$V \propto r^3 \propto e^{3H_0 t} \quad \Rightarrow \quad \dot{M} = \rho_0 \dot{V} = \rho_0 3H_0 V$$

$$\frac{\dot{M}}{V} = \rho_0 3H_0 \sim 6\times10^{-28}\,\mathrm{kg\,m^{-3}Gyr^{-1}}$$

1963: "There are only 2½ facts in cosmology."

1) The sky is dark at night.

2) The galaxies are receding from each other as expected in a uniform expansion.

2½) The contents of the universe have *probably* changed as the universe grows older.

Radio galaxy 3C295:
$z = 0.464$, $z/H_0 = 6.7$ Gyr

Fact 3: The universe contains a cosmic microwave background (CMB), discovered by Penzias & Wilson in 1965.

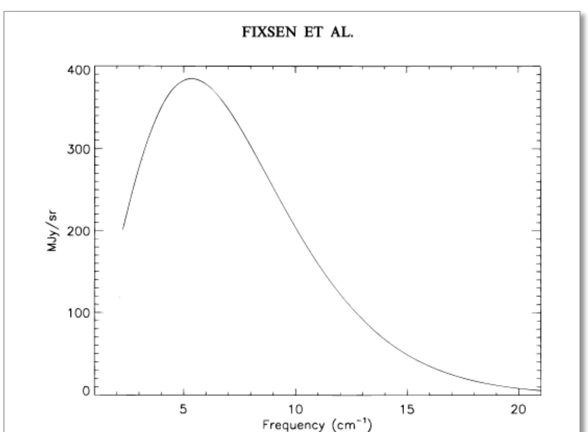


FIG. 4.—Uniform spectrum and fit to Planck blackbody (T). Uncertainties are a small fraction of the line thickness.

CMB is very well fitted by a blackbody spectrum (Planck function = Bose-Einstein distribution for massless bosons).

$$n(\nu)d\nu = \frac{8\pi}{c^3}\frac{\nu^2 d\nu}{\exp(h\nu / kT) - 1}$$

$$T_0 = 2.7255 \pm 0.0006\,\mathrm{K}$$

Blackbody spectra are produced by opaque objects:
CMB tells us that the early universe was opaque.

baryons leptons

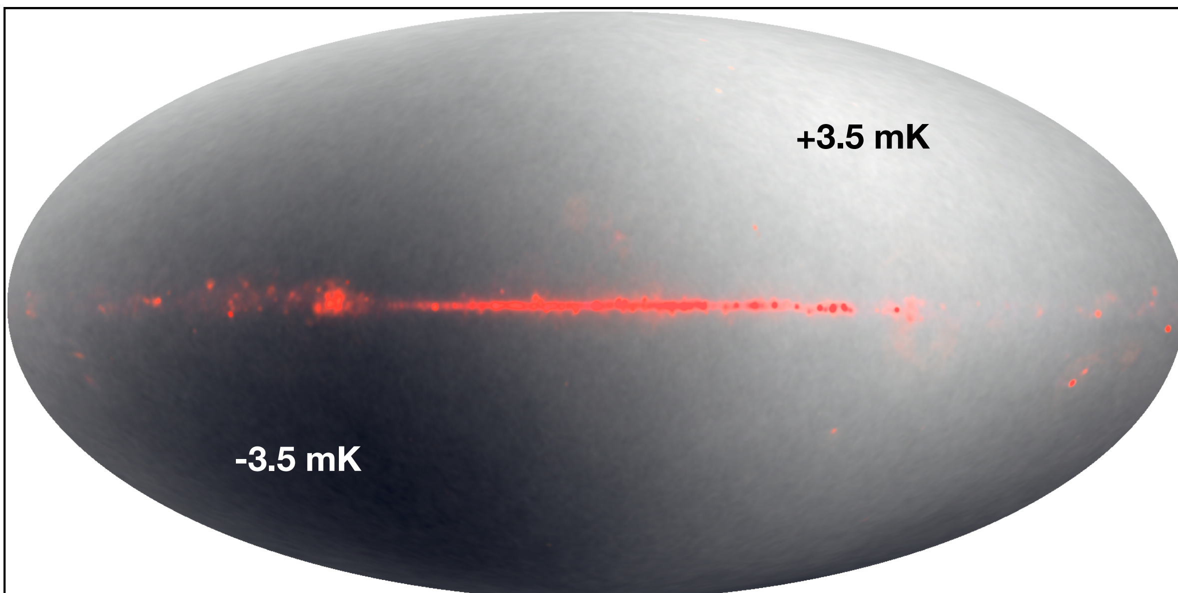
Baryonic matter (protons, neutrons) & (electrons) was *ionized*.

Rate at which photons scattered from free electrons was *greater than* the expansion rate of the universe ($\Gamma > H$).

Equivalently: mean free path for photons was *shorter than* the Hubble distance ($c/\Gamma < c/H$).

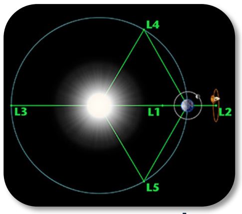
Then: opaque
Now: transparent

Violation of the perfect cosmological principle



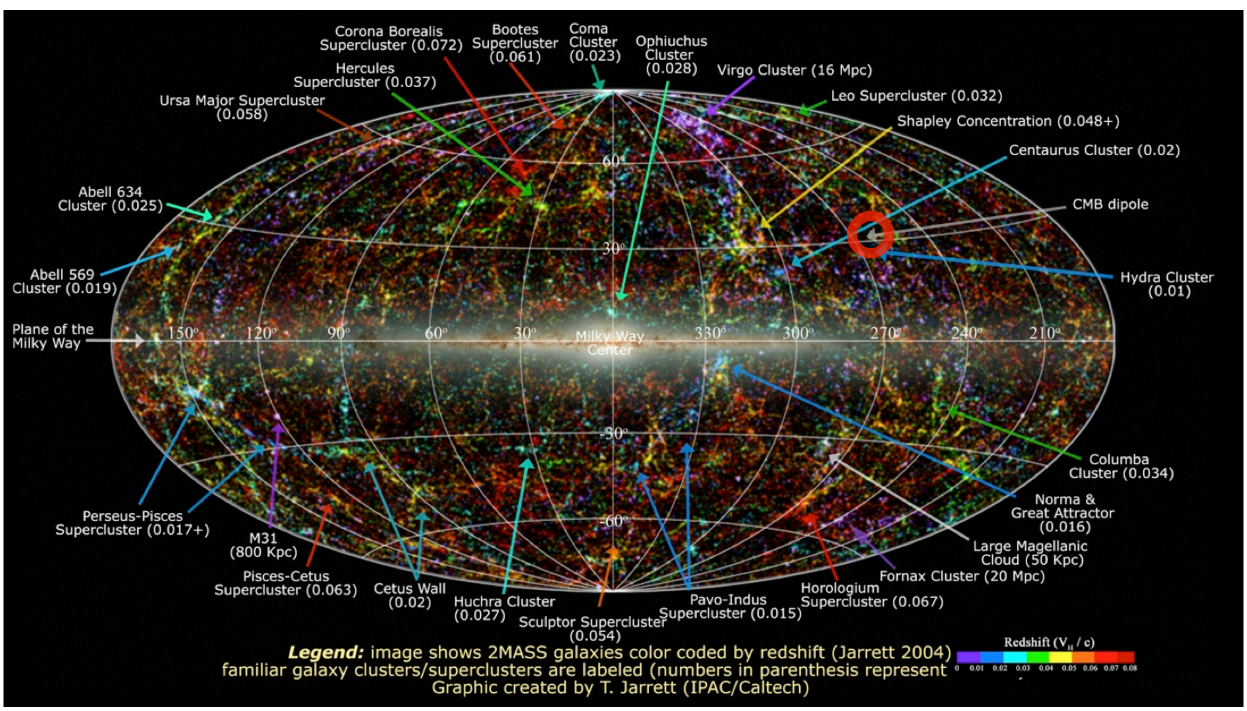
CMB temperature dipole
(red = foreground synchrotron emission in our galaxy) [NASA/*WMAP*]

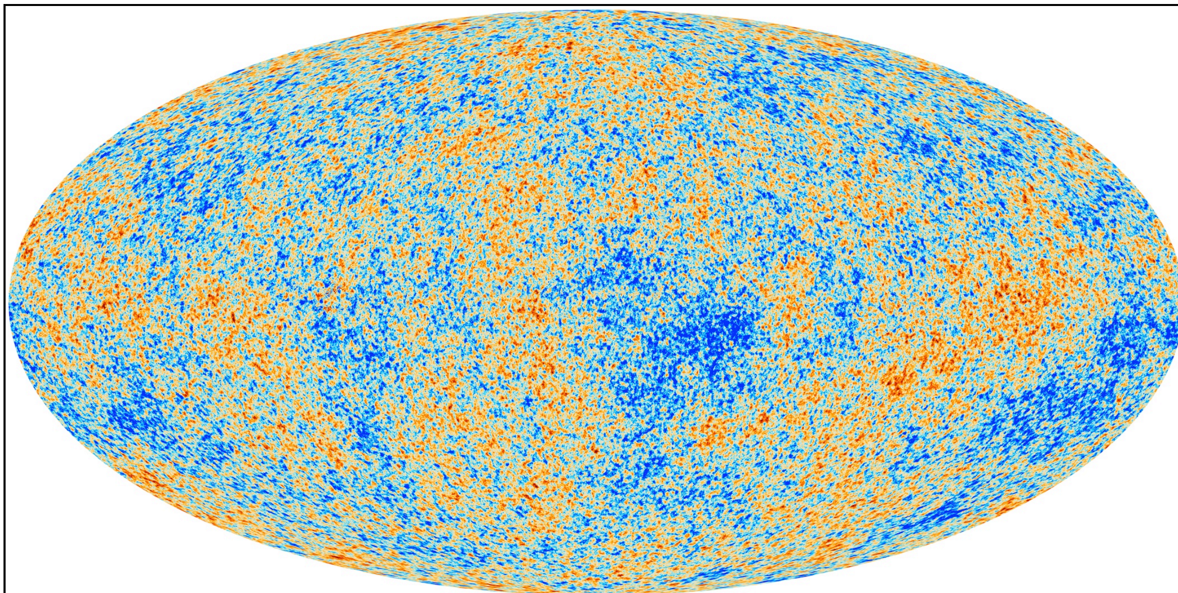
CMB *dipole* anisotropy: due mainly to a Doppler shift from our motion through space.



subtract *WMAP*'s orbital motion about the Sun (~30 km s^{-1})
Sun's orbital motion about the center of our galaxy (~220 km s^{-1})
our galaxy's motion relative to Andromeda (~80 km s^{-1})

Local Group of galaxies is moving toward Hydra, with $v \approx 630$ km $s^{-1} \sim 0.002c$





CMB temperature anisotropy after dipole subtraction [*Planck*/ESA]

CMB *small-scale* anisotropy: due to inhomogeneity at the time when photons last scattered.

$$\left(\frac{\delta T}{T}\right)_{\mathrm{rms}} \approx 10^{-5} \quad \left[5^\circ < \theta < 180^\circ\right]$$

At the time of last scattering, density and potential fluctuations were low in amplitude ($\delta\varphi/\varphi \sim 10^{-5}$).

Then: smooth
Now: lumpy

Violation of the perfect cosmological principle

Olbers' paradox+Hubble's law+CMB ➔
A universe described by a Hot Big Bang model
(*began in a hot, dense state a finite time ago*).

The cosmological principle (*homogeneous & isotropic*) applies only on large scales today (>100 Mpc). In the past, the universe was more nearly homogeneous & isotropic.

Expansion of a homogeneous & isotropic universe is described by the Robertson-Walker metric and the Friedmann equation.

Expansion of the universe is regulated by gravity.

Newtonian gravity

$$\nabla^2 \varphi = 4\pi G \rho$$

potential **mass density**

$$\frac{d^2\vec{r}}{dt^2} = -\vec{\nabla}\varphi$$

Non-zero acceleration in regions where $\rho=0$.

General Relativity

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein tensor (spacetime curvature)

stress-energy tensor (energy density ε, pressure P, etc...)

Non-zero curvature in regions where $\varepsilon=P=0$.

Propagating wave solutions.

We describe the local curvature of spacetime with a metric.

Metric = relation that gives the shortest distance between two neighboring points.

Example: 2-dimensional Euclidean (flat) space

(x,y) • - - - • $(x+dx, y+dy)$

$$ds^2 = dx^2 + dy^2$$

$dx^2 = (dx)^2 \neq d(x^2)$

or, in polar coordinates,

$$ds^2 = dr^2 + r^2\, d\theta^2$$

Example: 3-d Euclidean (flat) space

(x,y,z) • - - - • $(x+dx, y+dy, z+dz)$

$$ds^2 = dx^2 + dy^2 + dz^2$$

or, in spherical coordinates,

$$ds^2 = dr^2 + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2)$$
$$= dr^2 + r^2\, d\Omega^2$$

Example: 4-d Minkowski spacetime [metric of special relativity]

(t,x,y,z) • - - - • $(t+dt, x+dx, y+dy, z+dz)$

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

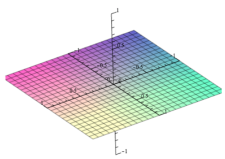
or, with spherical coordinates,

$$ds^2 = -c^2dt^2 + dr^2 + r^2\, d\Omega^2$$

Spacetime curvature
can be complicated.

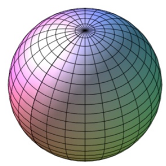
However, if the curvature of 3-d space is
homogeneous & isotropic, there are only 3 possibilities.

Flat (Euclidean)



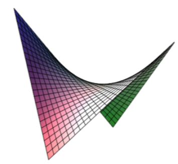
$ds^2 = dr^2 + r^2\, d\Omega^2$

Positive curvature



$ds^2 = dr^2 + R^2 \sin^2(r/R)\, d\Omega^2$

Negative curvature



$ds^2 = dr^2 + R^2 \sinh^2(r/R)\, d\Omega^2$

Combine homogeneous & isotropic expansion (or contraction)
with homogeneous & isotropic curvature of space.

The result is the Robertson-Walker metric:

$$ds^2 = -c^2 dt^2 + a(t)^2 [dr^2 + S_\kappa(r)^2 d\Omega^2]$$

$$S_\kappa = \begin{cases} R_0 \sin(r/R_0) & [\kappa = +1] \\ r & [\kappa = 0] \\ R_0 \sinh(r/R_0) & [\kappa = -1] \end{cases}$$

Also known as the Friedmann-Robertson-Walker (FRW) metric
or the Friedmann-Lemaître-Robertson-Walker (FLRW) metric.

The assumption of homogeneity & isotropy
is *extremely* powerful!

With this assumption, all you need to know
about spacetime curvature is:

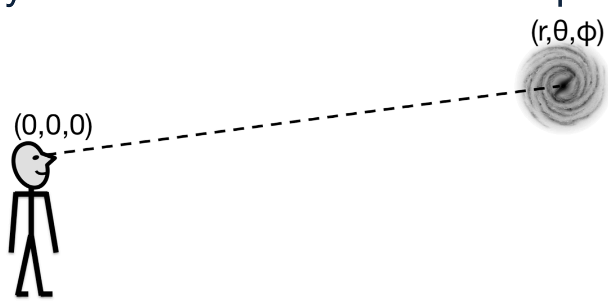
curvature constant $\kappa = +1, 0, \text{or } -1$

radius of curvature R_0 (if $\kappa \neq 0$)

scale factor $a(t)$

Robertson-Walker metric: $ds^2 = -c^2 dt^2 + a(t)^2 [dr^2 + S_\kappa(r)^2 d\Omega^2]$

Time coordinate t = cosmological proper time [or *cosmic time*]
(measured by an observer who sees isotropic expansion).



Radial coordinate r = proper distance at time t_0.
(length of a spatial geodesic when $a(t_0)=1$).

Proper distance increases as

$d_p(t) = a(t)\, d_p(t_0) = a(t)\, r$

Radius of curvature increases as

$R(t) = a(t)\, R_0$

r,θ,ϕ are ***comoving*** coordinates

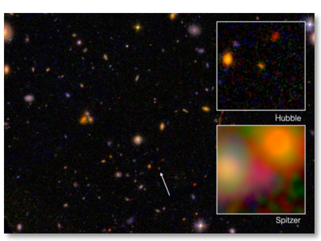
Wavelength of light moving freely through space increases as

$\lambda(t) = a(t)\, \lambda_0$

Light was emitted with wavelength λ_e at time t_e, and observed with wavelength λ_0 at time t_0. The redshift is...

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{\lambda_0 - a(t_e)\lambda_0}{a(t_e)\lambda_0} = \frac{1}{a(t_e)} - 1$$

$z = 0.464; \quad a(t_e) = 1/(1+z) = 0.68$



$z = 8.68; \quad a(t_e) = 1/(1+z) = 0.103$

Monotonically expanding Big Bang model:

larger z ➔ smaller $a(t_e)$ ➔ earlier t_e ➔ greater $r = d_p(t_0)$

The curvature of spacetime is related to its energy content by Einstein's field equation:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

If space is homogeneous & isotropic, this reduces to the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

Александр Фридман