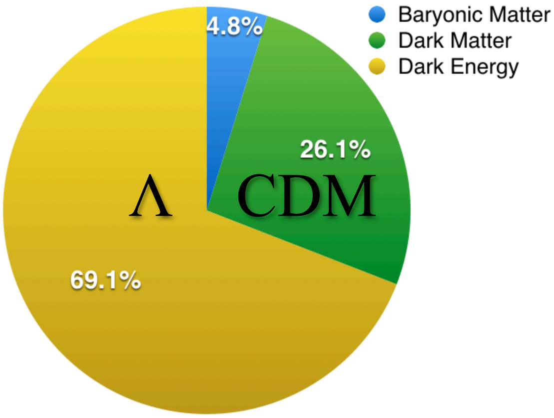

[CMB provides ~0.005% of today's energy density.]

Introduction to Cosmology Part 2

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If space is perfectly flat ($\kappa = 0$), then

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t)$$

For a given value of the Hubble parameter H,
there exists a critical density ε_c for which space is flat.

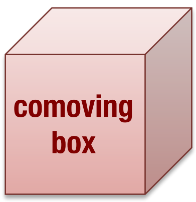
$$\varepsilon_{c,0} = \frac{3c^2 H_0^2}{8\pi G} = 4.9 \pm 0.3\,\mathrm{GeV\,m^{-3}}$$

$$\rho_{c,0} = \frac{\varepsilon_{c,0}}{c^2} = (1.3 \pm 0.1) \times 10^{11}\,\mathrm{M_\odot\,Mpc^{-3}}$$

Friedmann equation: 1 equation, 2 unknowns.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

We need another equation: the fluid equation.



$$V(t) \propto a(t)^3$$

$$E(t) = V(t)\varepsilon(t)$$

$$dE + PdV = dQ \quad [\text{but } dQ = 0]$$

$$\dot{E} + P\dot{V} = 0$$

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Friedmann equation + fluid equation: 2 equations, 3 unknowns.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}[\varepsilon(t) + P(t)] = 0$$

We need equations of state, relating the pressure $\boldsymbol{P}$ of each component to its energy density $\boldsymbol{\varepsilon}$.

$$P = P(\varepsilon)$$

$$P = w\varepsilon$$

where w is the dimensionless equation-of-state parameter.

Example: a gas of nonrelativistic particles (="matter")

matter:
DM particles
free electrons
(kT<0.5MeV)
free protons
(kT<1GeV)
atoms
molecules
stars
galaxies

$$P = nkT = \rho \frac{kT}{m} = \rho \frac{\langle \mathrm{v}^2 \rangle}{3}$$

$$\varepsilon = \rho c^2 + \frac{1}{2}\rho \langle \mathrm{v}^2 \rangle \approx \rho c^2$$

$$w = \frac{P}{\varepsilon} \approx \frac{\langle \mathrm{v}^2 \rangle}{3c^2} \ll 1$$

$w \sim 10^{-12}$ for N_2 at room temperature

Example: a gas of highly relativistic particles (="radiation")

$$\varepsilon = n\left\langle \frac{hc}{\lambda} \right\rangle = \alpha T^4 \text{ [blackbody]}$$

$$\alpha = \frac{\pi^2}{15} \frac{k^4}{\hbar^3 c^3}$$

$$P = \frac{1}{3} n\left\langle \frac{hc}{\lambda} \right\rangle = \frac{1}{3}\alpha T^4 \text{ [blackbody]}$$

radiation:
photons
neutrinos
(kT>$m_\nu c^2$)
electrons
(kT>0.5MeV)
quarks
(kT>1GeV)

$$w = \frac{P}{\varepsilon} = \frac{1}{3}$$

What's the value of w for **dark energy**?

A definition for "dark energy": a component of the universe that makes the expansion speed up ($\ddot{a} > 0$).

The Friedmann equation $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}$$

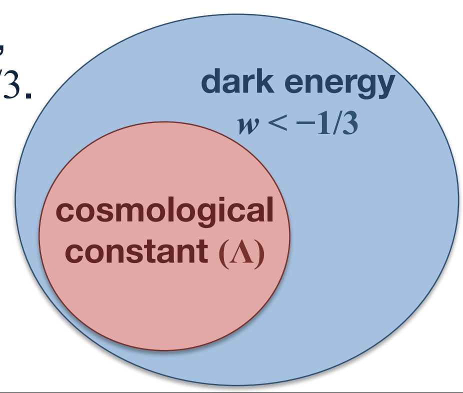
and the fluid equation $$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

can be combined into the acceleration equation: $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\varepsilon(1+3w)$$

Dark energy must have $$\varepsilon(1+3w) < 0.$$

If dark energy has $\varepsilon > 0$, then it must have $w < -1/3$.



Einstein introduced the cosmological constant Λ in 1917, to create a **static** universe.

matter only: $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) = -\frac{4\pi G}{3}\rho$$

Therefore, $\ddot{a}=0$ requires $\rho=0$.

matter + Λ: $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + \frac{\Lambda}{3}$$

Therefore, $\ddot{a}=0$ requires $\Lambda=4\pi G\rho$.

Friedmann equation with cosmological constant:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2} + \frac{\Lambda}{3}$$

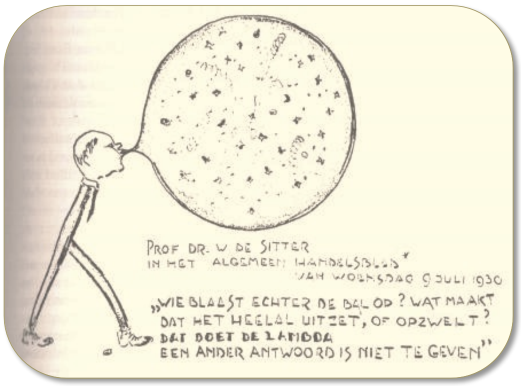
1917: de Sitter investigates a universe with nothing but Λ:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3}$$

This implies *exponential expansion*:

$$a(t) \propto \exp(H_0 t), \quad \text{where } H_0 = (\Lambda / 3)^{1/2}$$

What is the cosmological constant?



"What, however, blows up the ball? What makes the universe expand or swell up? That is done by the Lambda. Another answer cannot be given."

$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ modification of Einstein tensor [Einstein 1917]

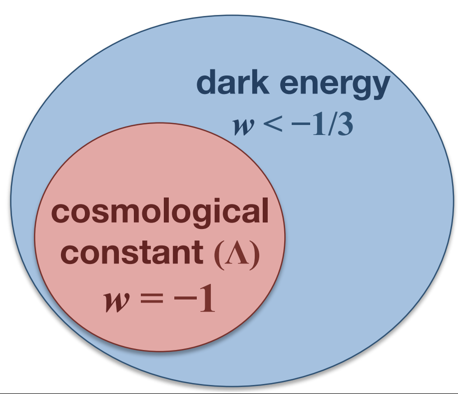
$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}$ contribution to energy density & pressure

Friedmann equation: $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2} + \frac{\Lambda}{3}$$

We can think of Λ as a component of the universe with constant energy density $\varepsilon_\Lambda = \frac{c^2}{8\pi G}\Lambda$

Fluid equation: $$\dot{\varepsilon} = -3\frac{\dot{a}}{a}(\varepsilon + P)$$

For ε_Λ to be constant, we need $P_\Lambda = -\varepsilon_\Lambda$, or $\boldsymbol{w = -1}$.



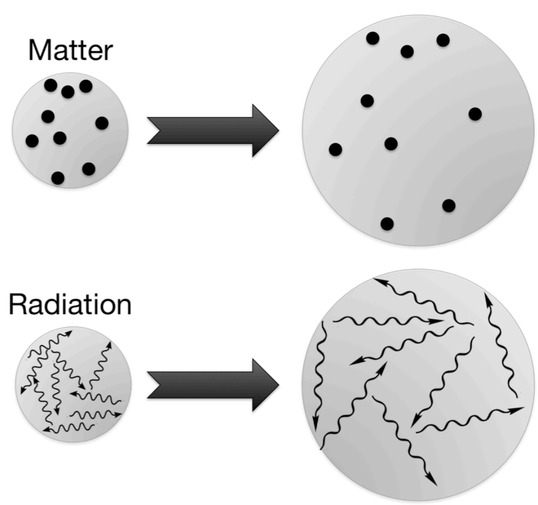
Let's consider a universe containing matter (w=0), radiation ($w = 1/3$), and Λ ($w = -1$).

It is possible that acceleration is due to dark energy with w ≠ -1, or to modified gravity, but Λ provides a useful parameterization.

Energy and pressure are additive:

$$\varepsilon = \sum_i \varepsilon_i \qquad P = \sum_i w_i \varepsilon_i$$

$$\frac{\dot{\varepsilon}_i}{\varepsilon_i} = -3\frac{\dot{a}}{a}(1+w_i) \quad \longrightarrow \quad \varepsilon_i = \varepsilon_{i,0}\, a^{-3(1+w_i)}$$



$$\varepsilon = (mc^2)n \propto a^{-3}$$

$$\varepsilon = (\frac{hc}{\lambda})n \propto a^{-1}a^{-3} \propto a^{-4}$$

Although photon number is not conserved, energy density of starlight is just 10% of the energy density of the CMB.

Relative densities are expressed in terms of the dimensionless density parameter Ω.

$$\Omega_i = \frac{\varepsilon_i}{\varepsilon_c}$$

where ε_c is the critical density at which the universe is spatially flat.

$$\varepsilon_{c,0} = \frac{3c^2 H_0^2}{8\pi G} = 4900 \pm 300 \text{ MeV m}^{-3}$$

Density parameter for background radiation

Cosmic microwave background

$$n_\gamma(E)dE \propto \frac{E^2 dE}{\exp(E/kT) - 1}$$

$$T_{\gamma,0} = 2.7255 \text{ K}$$

$$\varepsilon_{\gamma,0} = \alpha T_0^4 = 0.2606 \text{ MeV m}^{-3}$$

$$\Omega_{\gamma,0} = 5.35 \times 10^{-5}$$

Cosmic neutrino background

(*massless* neutrinos)

$$n_\nu(E)dE \propto \frac{E^2 dE}{\exp(E/kT) + 1}$$

$$\varepsilon_\nu = \frac{7}{8}\alpha T^4$$ *(for each of 3 species)*

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma,0} = 1.945 \text{ K}$$

$$\Omega_{\nu,0} = 0.681\,\Omega_{\gamma,0} = 3.65 \times 10^{-5}$$

Even if all species of neutrino were massless today, radiation would have $\Omega_{r,0} = 0.00009 << 1$.

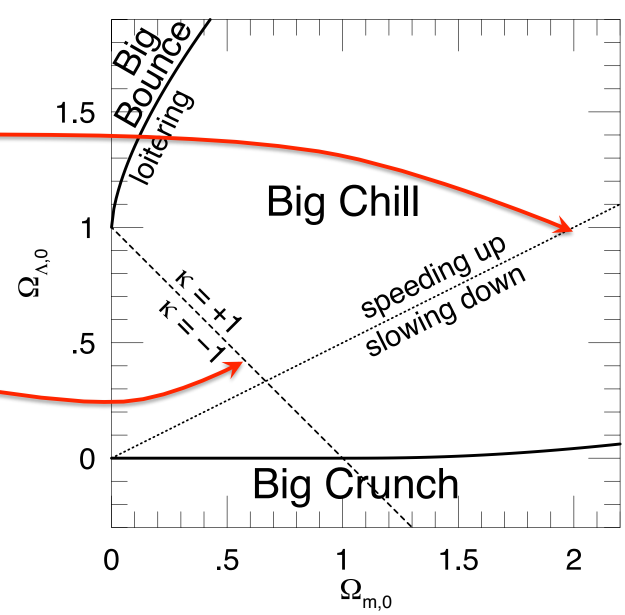
Recent expansion of the universe can be expressed in terms of $\Omega_{\Lambda,0}$ (cosmological constant) and $\Omega_{m,0}$ (matter, dark + baryonic).

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) = -\frac{H^2}{2}(\Omega_m + \Omega_\Lambda - 3\Omega_\Lambda)$$

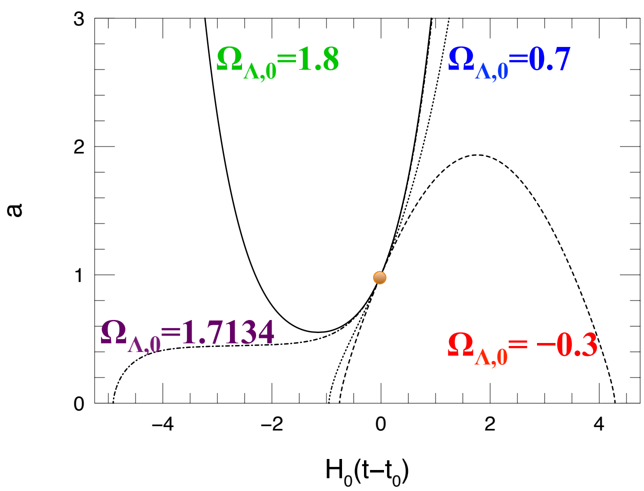
$$\frac{\ddot{a}}{a} = 0 \text{ today if } \Omega_{m,0} = 2\Omega_{\Lambda,0}$$

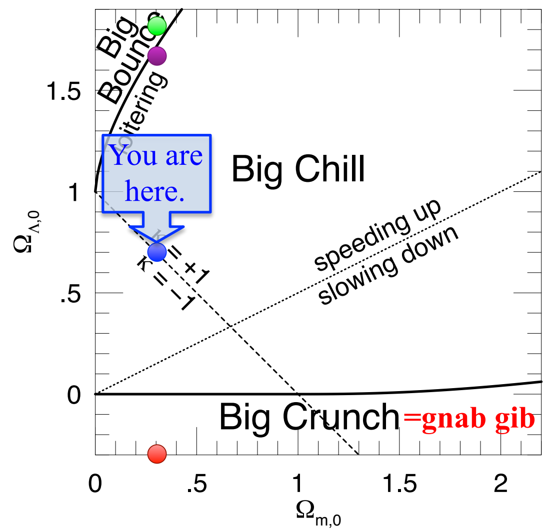
$$\left.\frac{\ddot{a}}{a}\right|_{t=t_0} = 0 \quad \text{if } \Omega_{m,0} = 2\Omega_{\Lambda,0}$$

$$\kappa = 0 \ \text{ if } \Omega_{m,0} + \Omega_{\Lambda,0} = 1$$



Consider 4 universes, all of which have $\Omega_{m,0} = 0.3$:





We know the universe contains matter ($\Omega_{m,0}$) and a little bit of radiation ($\Omega_{r,0} \ll 1$).

We permit the universe to have a cosmological constant ($\Omega_{\Lambda,0}$).

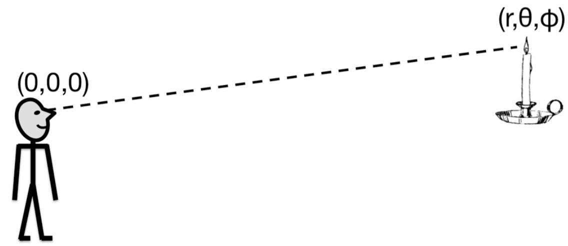
The total density parameter is $\Omega_0 = \Omega_{\Lambda,0} + \Omega_{m,0} + \Omega_{r,0}$.

If $\Omega_0 \neq 1$, the Friedmann equation tells us space is curved:

$$1 - \Omega_0 = -\kappa \left(\frac{c / H_0}{R_0} \right)^2$$

How can we use **observations** of the universe around us to constrain $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$?

Learning about $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ from observing standard candles (objects of known luminosity L).



Current proper distance to the standard candle:

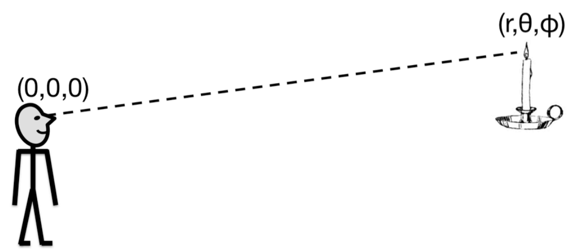
$$d_p(t_0) = r$$

Photons we observe at t_0 were emitted at t_e.
Photons follow a null geodesic: $ds^2 = -c^2dt^2 + a(t)^2dr^2 = 0$.

$$dr = \frac{cdt}{a(t)} \longrightarrow d_p(t_0) = r = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

Proper distance encodes expansion history:

$$d_p(t_0) = r = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$



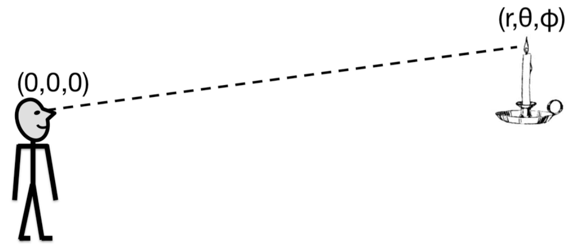
We can change our variable of integration from t to a:

$$\frac{da}{a} = \frac{\dot{a}\,dt}{a} = H\,dt \longrightarrow d_p(t_0) = c\int_{a_e}^{1} \frac{da}{a^2 H}$$

We can change our variable of integration from a to $1+z = 1/a$:

$$\frac{da}{a} = -\frac{dz}{1+z} \longrightarrow d_p(t_0) = c\int_{0}^{z} \frac{dz}{H(z)}$$

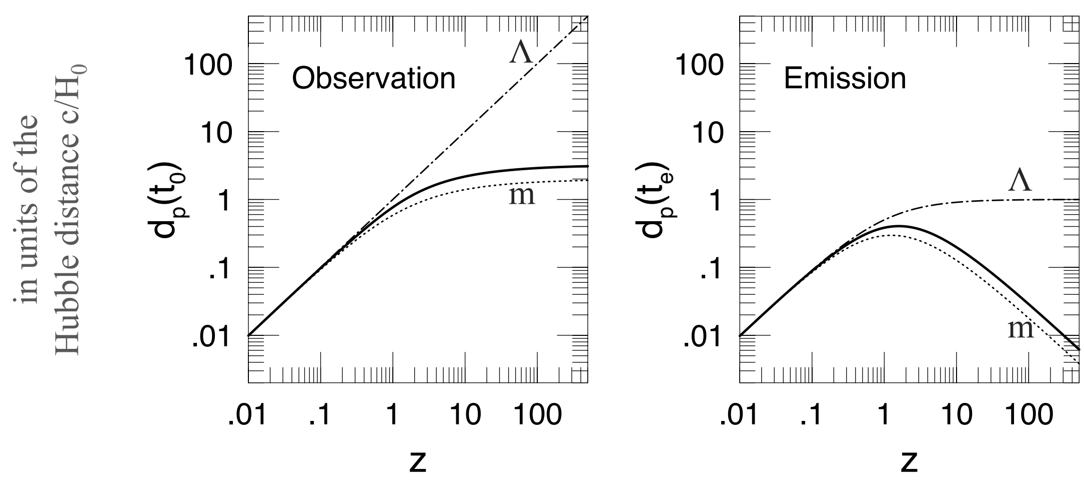
$$d_p(t_0) = c\int_0^z \frac{dz}{H(z)}$$



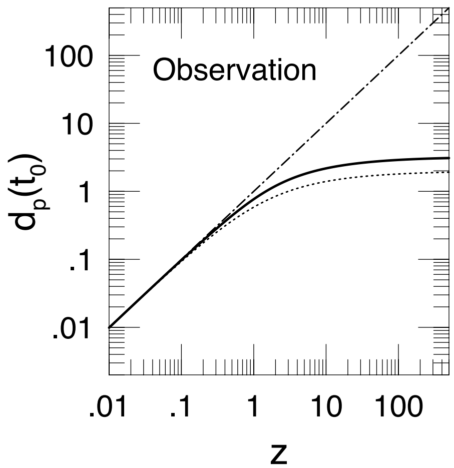
The redshift z is observable.
The Hubble parameter is given by the Friedmann equation:

$$\frac{H(z)^2}{H_0^2} = \Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0}-\Omega_{\Lambda,0})(1+z)^2 + \Omega_{\Lambda,0}$$

Thus, the relation between proper distance and redshift is a (fairly) simple integral.



dot-dash: flat, lambda-only ($\Omega_{\Lambda,0}$=1, $\Omega_{m,0}$=0)
dotted: flat, matter-only ($\Omega_{\Lambda,0}$=0, $\Omega_{m,0}$=1)
solid: Benchmark Model ($\Omega_{\Lambda,0}$=0.69, $\Omega_{m,0}$=0.31)



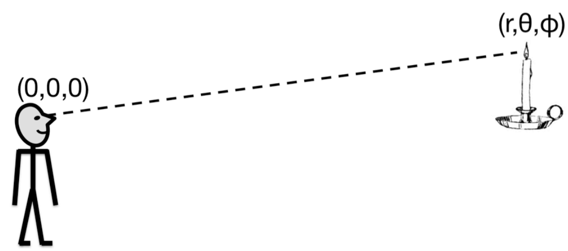
In the limit that z goes to infinity, $d_p(t_0)$ approaches the particle horizon distance $d_{hor}(t_0)$.

Benchmark model:

$d_{hor}(t_0) = 3.2\ c/H_0 = 14{,}000$ Mpc

Stars more than 14,000 Mpc away haven't had time to send us light yet.

Alas! Proper distance isn't directly measurable. How can we estimate the distance from observable properties?



We can measure the standard candle's redshift z.
We can (ideally) measure its bolometric flux f.

We can compute a function called the luminosity distance

$$d_L \equiv \left(\frac{L}{4\pi f} \right)^{1/2}$$

This is how Edwin Hubble estimated distances.

$d_L = d_p$ if space is static and Euclidean.

In an expanding, spatially curved universe,

$$f = \frac{L}{4\pi S_\kappa(r)^2(1+z)^2}$$

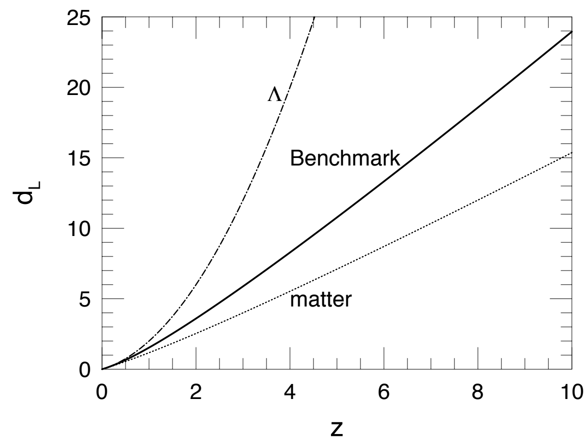
and thus

$$d_L = S_\kappa(r)(1+z).$$

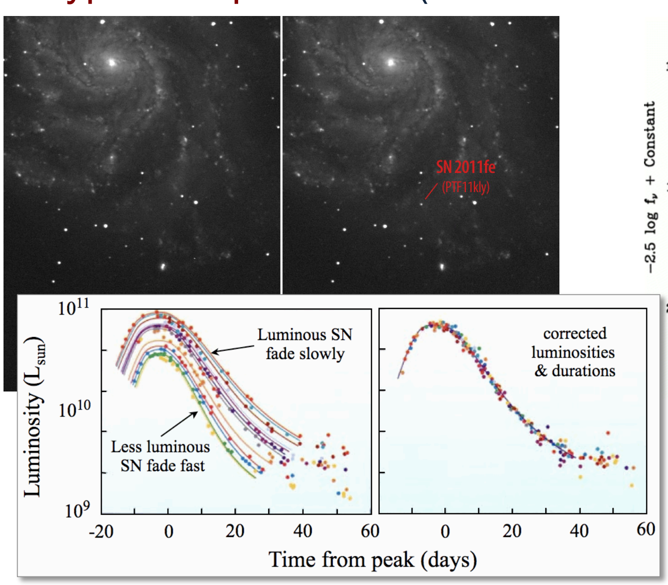
If space is nearly flat, then

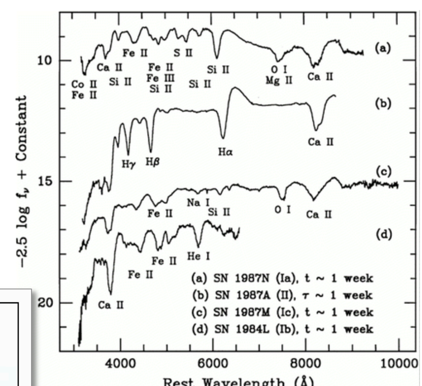
$$d_L \approx d_p(t_0)(1+z) > d_p(t_0)$$

& luminosity distance is an *overestimate* of proper distance.

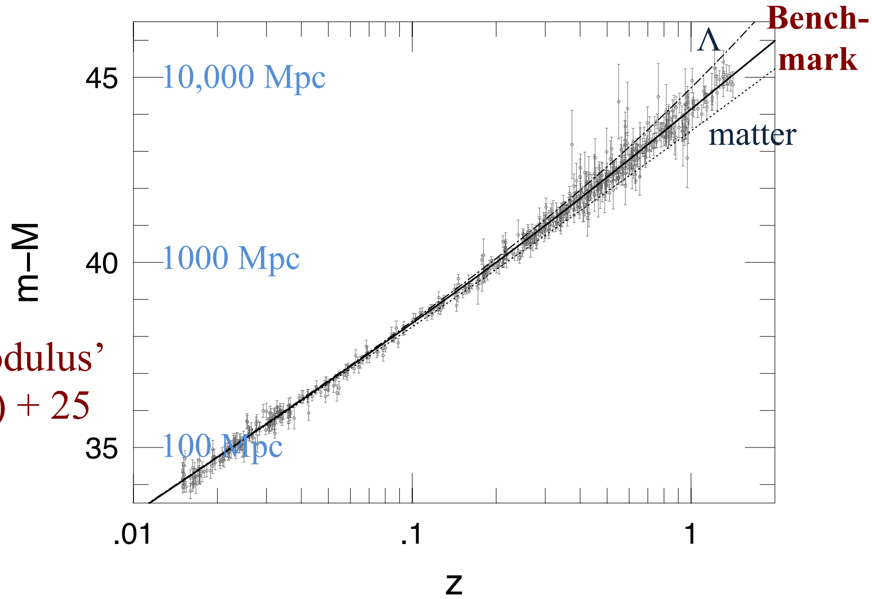


A preferred standard candle of cosmologists:
Type Ia supernovae (alias thermonuclear supernovae).



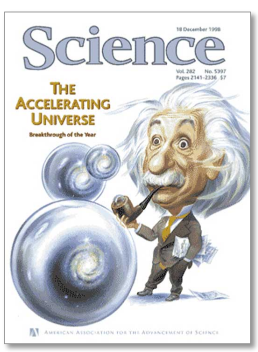


Luminosity distance vs. z for 580 type Ia supernovae



m-M = 'distance modulus' = $5 \log (d_L / 1\ \mathrm{Mpc}) + 25$

95% confidence interval for the type Ia supernova results:



('Accelerating' result is robust: doesn't depend on the Λ assumption.)