

The background of the slide is a Cosmic Microwave Background (CMB) fluctuation map, showing a complex pattern of temperature variations in shades of blue, orange, and yellow. A large, dark blue oval is superimposed over the map, containing the main title text.

# Dark Matter

## Lecture 4: Terrestrial Searches

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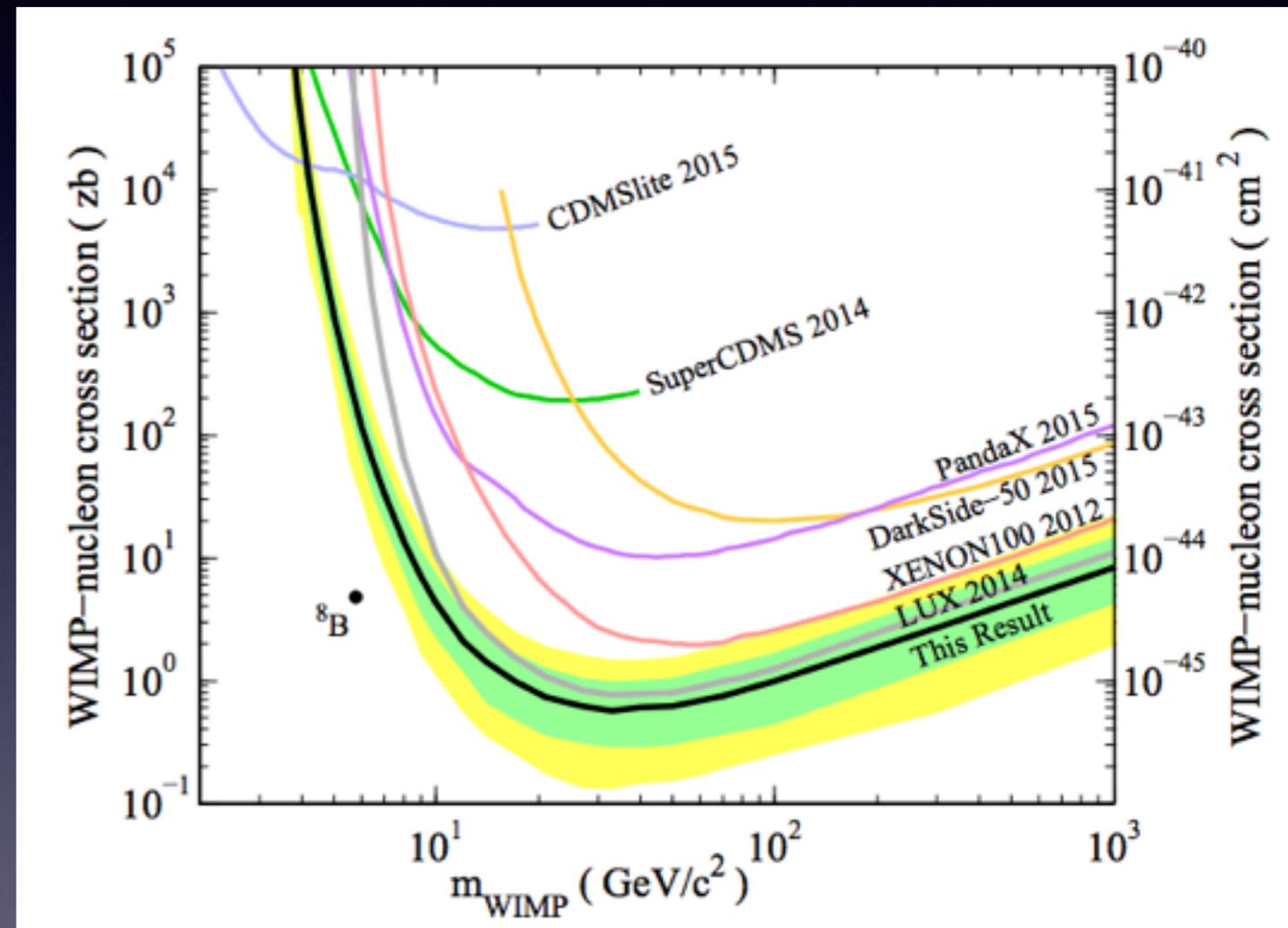
8 June 2016

# Goals (Lecture 4)

- Understand the principles of DM-nucleus scattering: the basis for WIMP direct detection
- Give overview of collider searches for DM
- Give overview of axion searches

# Direct detection in a nutshell

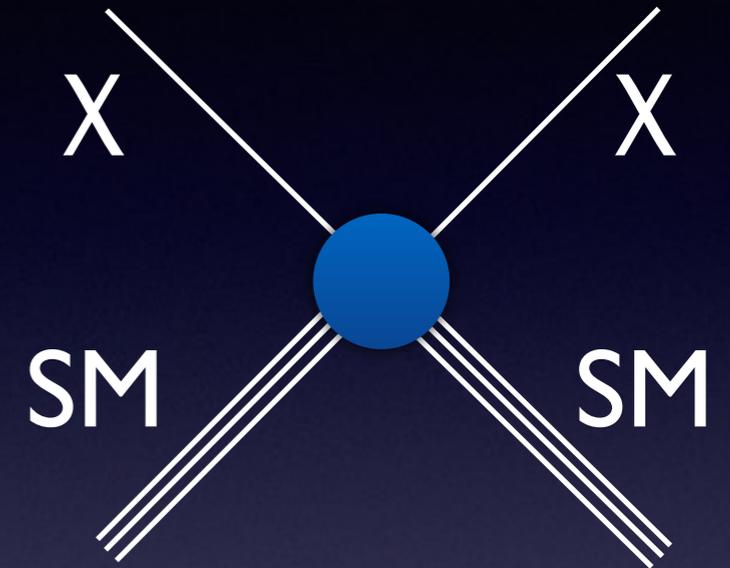
- Put sensitive detectors around large volume
- Bury it underground to reduce backgrounds
- Look for signs of nuclei “jumping”/recoiling with no apparent cause
- If other backgrounds can be shielded out, the cause must be something very weakly interacting - such as neutrinos or DM
- At present neutrino background is too faint to see - signal would be a sign of DM



LUX Collaboration 1512.03506

# DM-nucleus scattering

- Search for nuclei (of mass  $m_N$ ) recoiling due to scattering of dark matter particle (of mass  $m_\chi$ ).
- Observable:  $dR/dE_R$ , scattering rate for recoil energy in the range  $[E_R, E_R + dE_R]$
- Let's work out the classical kinematics in lab frame (nucleus initially at rest)



Before:

After:

conservation of energy and momentum:

$$\frac{1}{2}m_\chi v^2 = \frac{1}{2}m_\chi (v')^2 + E_R$$

$$m_\chi v = m_\chi v' \cos \theta' + \sqrt{2m_N E_R} \cos \theta$$

$$0 = m_\chi v' \sin \theta' + \sqrt{2m_N E_R} \sin \theta$$

Some algebra (eliminating  $v'$  and  $\theta'$ ) gives:  $E_R = \frac{2\mu^2 v^2 \cos^2 \theta}{m_N}$        $\mu = \frac{m_N m_\chi}{m_N + m_\chi}$

Note: this result depends only on kinematics of collision - needs to be modified for inelastic collisions, but else quite general

# Typical recoil energies

- We thus predict a spectrum of recoils extending from zero recoil energy to:

$$(E_R)_{\max} = 2\mu^2 v^2 / m_N$$

- Consequently, at a given recoil energy  $E_R$ , only DM particles with  $v > v_{\min} = \sqrt{m_N E_R / 2\mu^2}$  can contribute.
- Let's do some quick estimates: typical  $E_R \sim \mu^2 v^2 / m_N$ .
- Suppose the target nucleus is  $O(10-100)$  GeV (i.e. 10-100 protons+neutrons) and DM is similar mass or heavier, so  $\mu \sim m_N$ .
- Velocity dispersion of DM locally is  $v/c \sim 10^{-3}$  (determined by Galactic gravitational potential).
- Then typical recoil energies should be in the range  $\sim 10^{-6} m_N \sim 10-100$  keV.
- If DM is significantly lighter than nucleus,  $\mu \sim m_{\text{DM}}$ , and  $E_R$  suppressed by  $(m_{\text{DM}}/m_N)^2$  relative to  $O(10-100)$  keV scale.
  - e.g. for  $m_N \sim 100$  GeV,  $E_R \sim 100$  keV for  $m_{\text{DM}} > m_N$ , but only 0.01 keV for  $m_{\text{DM}} \sim 1$  GeV.
  - Detecting light DM this way requires light targets and very low energy thresholds.

# Ingredients for the nuclear recoil spectrum

- Amplitude for scattering of DM on individual nucleons (function of  $v$ ,  $E_R$ ):
  - Particle physics: how does DM couple to quarks/gluons?
  - Nuclear physics: what is the quark content of the nucleon?
- Amplitude for nucleons  $\rightarrow$  amplitude for scattering on nucleus.
  - Particle physics: is the amplitude spin-dependent or not? More generally, how does it depend on the nucleon properties? Is it the same for protons and neutrons?
  - Nuclear physics: nuclear “form factor” (accounts for finite size of nucleus)
- Scattering amplitude  $\rightarrow$  scattering rate.
  - Astrophysics: number density & velocity distribution for dark matter.

# Standard simplifications

- Treat scattering as a contact interaction set by couplings  $f_n, f_p$  to neutrons and protons respectively.
  - Standard case: assume  $f_n, f_p$  are just constants, independent of e.g. velocity, momentum transfer, scattering angle, etc.
  - Often further assume that  $f_n = f_p$ .
- Consider the two cases of spin-independent and spin-dependent interactions:
  - Spin-independent interactions: nucleon amplitudes add coherently. Overall rate scales as  $(\text{atomic mass})^2$ .
  - Spin-dependent interactions: amplitudes from paired nucleons with opposite spins cancel exactly. Overall rate scales as  $(\text{net spin})^2$  - much weaker limit.
- Form factor: describes momentum dependence of interaction due to finite size of nucleus. Typically use simple parameterization “Helm form factor”.
- DM velocity distribution: typically just assume Maxwellian distribution.

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many “non-standard” DM models work by just changing one or more of these assumptions! Can substantially change comparisons between different experiments.

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# The Helm form factor

$$F(qr_n) = 3 \frac{j_1(qr_n)}{qr_n} e^{-(qs)^2/2}$$
$$= 3 \frac{\sin(qr_n) - qr_n \cos(qr_n)}{(qr_n)^3} e^{-(qs)^2/2}$$

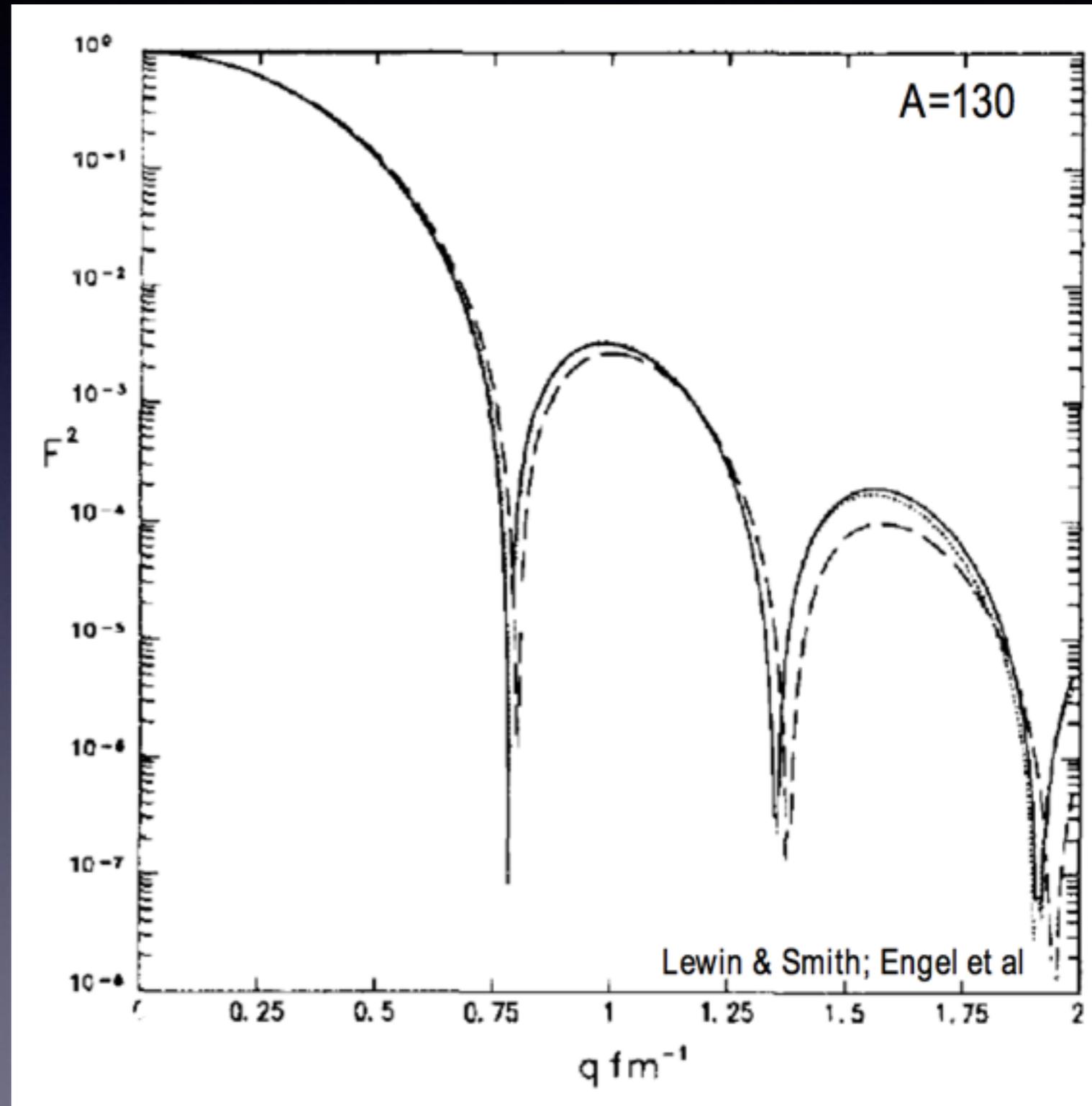
$$r_n^2 = c^2 + \frac{7}{3}\pi^2 a^2 - 5s^2$$

$$c = 1.23A^{1/3} - 0.60 \text{ fm}$$

$$a = 0.52 \text{ fm}$$

$$s = 0.9 \text{ fm}$$

Important effect for momentum transfers corresponding to scale  $\sim 1$  fm or smaller, i.e. momentum  $> 100$  MeV  
For  $v \sim 10^{-3}$ , relevant for  $m_{\text{DM}} \sim 100$  GeV+



# The standard calculation

- Now let's switch to the center-of-momentum (COM) frame. Let the scattering angle in this frame be labeled  $\theta$ .
- Why choose this frame? For simple models, rate is independent of scattering angle in COM frame.



- 3-momentum transfer  $q$  has magnitude given by:

$$q^2 = |\vec{q}|^2 = |\vec{p} - \vec{k}|^2 = p^2 + k^2 - 2pk \cos \theta = 2\mu^2 v_{\text{rel}}^2 (1 - \cos \theta)$$

- In LAB frame, nucleus gains momentum  $q = (2 m_N E_R)^{1/2}$

- So we can express lab-frame recoil energy (which we're interested in) in terms of COM-frame scattering angle:

$$E_R = q^2 / (2m_N) = \frac{\mu^2 v_{\text{rel}}^2}{m_N} (1 - \cos \theta)$$

- Thus the rate of events at a given  $E_R$  can be written in terms of the rate of events at a given COM scattering angle:

$$\frac{dR}{dE_R} = \frac{m_N}{\mu^2 v_{\text{rel}}^2} \frac{dR}{d \cos \theta}$$

# The standard calculation (II)

- Let us assume spin-dependent scattering, so contributions from different nuclei add coherently:

$$\mathcal{M}_{\text{nucleus}} = \mathcal{F}(q) [Z f_p + (A - Z) f_n]$$

- The cross section in the center-of-momentum frame is related to the matrix element  $\mathcal{M}$  here by:

$$\frac{d\sigma}{d\Omega} = \frac{\mu^2}{m_\chi^2 m_N^2} \frac{1}{64\pi^2} |\mathcal{M}_{\text{nucleus}}|^2$$

$$d\Omega = d\phi d(\cos \theta)$$

- To convert from cross section to rate, we have

$$\frac{dR}{d\Omega} = n N_T v_{\text{rel}} \frac{d\sigma}{d\Omega}$$

$$n = \text{DM \# density}$$

$$N_T = \text{\# target nuclei}$$

- Assuming no dependence on the angle  $\varphi$ , so we can trivially integrate over the possible values of  $\varphi$ , we can then finally write:

$$\rho = \text{DM mass density}$$

$$\frac{dR}{dE_R} = \frac{2\pi m_N}{\mu^2 v_{\text{rel}}^2} \frac{dR}{d\Omega} = \frac{2\pi n N_T m_N}{\mu^2 v_{\text{rel}}} \frac{d\sigma}{d\Omega} = \frac{m_N \rho N_T}{32\pi m_\chi^3 m_N^2 v_{\text{rel}}} |\mathcal{F}(q)|^2 |Z f_p + (A - Z) f_n|^2$$

# The standard calculation (III)

- Let's define an "effective cross-section" for scattering on a single nucleon:

$$\sigma_{\chi n} = \sigma_{\chi N} \Big|_{q=0} \frac{\mu_{\chi n}^2}{\mu^2} \frac{1}{A^2}$$

- This is the actual quantity that's bounded on those limit plots.
- We can then write our observable spectrum in the form:

$$\frac{dR}{dE_R} = \frac{\sigma_{\chi n}}{\mu_{\chi n}^2} A^2 m_N N_T \frac{\rho}{2m_\chi v_{\text{rel}}} |\mathcal{F}(E_R)|^2$$

- In terms of the  $f_a, f_n$  parameters, we have:

$$\sigma_{\chi n} = \frac{1}{16\pi} \frac{\mu_{\chi n}^2}{m_\chi^2 m_N^2} |Z f_p + (A - Z) f_n|^2 / A^2$$

# The velocity distribution

$$\frac{dR}{dE_R} = \frac{\sigma_{\chi n}}{\mu_{\chi n}^2} A^2 m_N N_T \frac{\rho}{2m_\chi v_{\text{rel}}} |\mathcal{F}(E_R)|^2$$

- This result assumes we know the relative velocity of the DM and the nucleus - but in reality, the DM has a distribution of velocities.
- $\rho$  here should be understood to describe the mass density of DM particles with relative velocity  $v_{\text{rel}}$  - then need to integrate over this parameter.

$$\frac{d\rho}{dv_{\text{rel}}} = \rho_0 f(v_{\text{rel}})$$

↙ distribution function normalized to 1  
↘ overall density

$$\frac{dR}{dE_R} = \left[ A^2 m_N N_T |\mathcal{F}(E_R)|^2 \right]^{\text{target properties}} \left[ \frac{\sigma_{\chi n}}{2m_\chi \mu_{\chi n}^2} \right]^{\text{DM properties}} \rho_0 \int dv \frac{1}{v} f(v)^{\text{astrophysics}}$$

# The recoil spectrum

- Shape of the spectrum comes from two places:
  - Form factor dependence on  $E_R$  - suppresses spectrum at high recoil energies
  - Dependence of velocity integral on  $E_R$

astrophysical piece:  $\rho_0 \int \frac{1}{v} f(v) dv = \rho_0 \int_{v_{\min}}^{v_{\max}} \frac{1}{v} f(v) dv$

$$v_{\min} = \sqrt{\frac{m_N E_R}{2\mu^2}}$$

$v_{\max}$  set by Galactic escape velocity in frame of Earth  
 $\Rightarrow v_{\max}$  is (slightly) time-dependent!

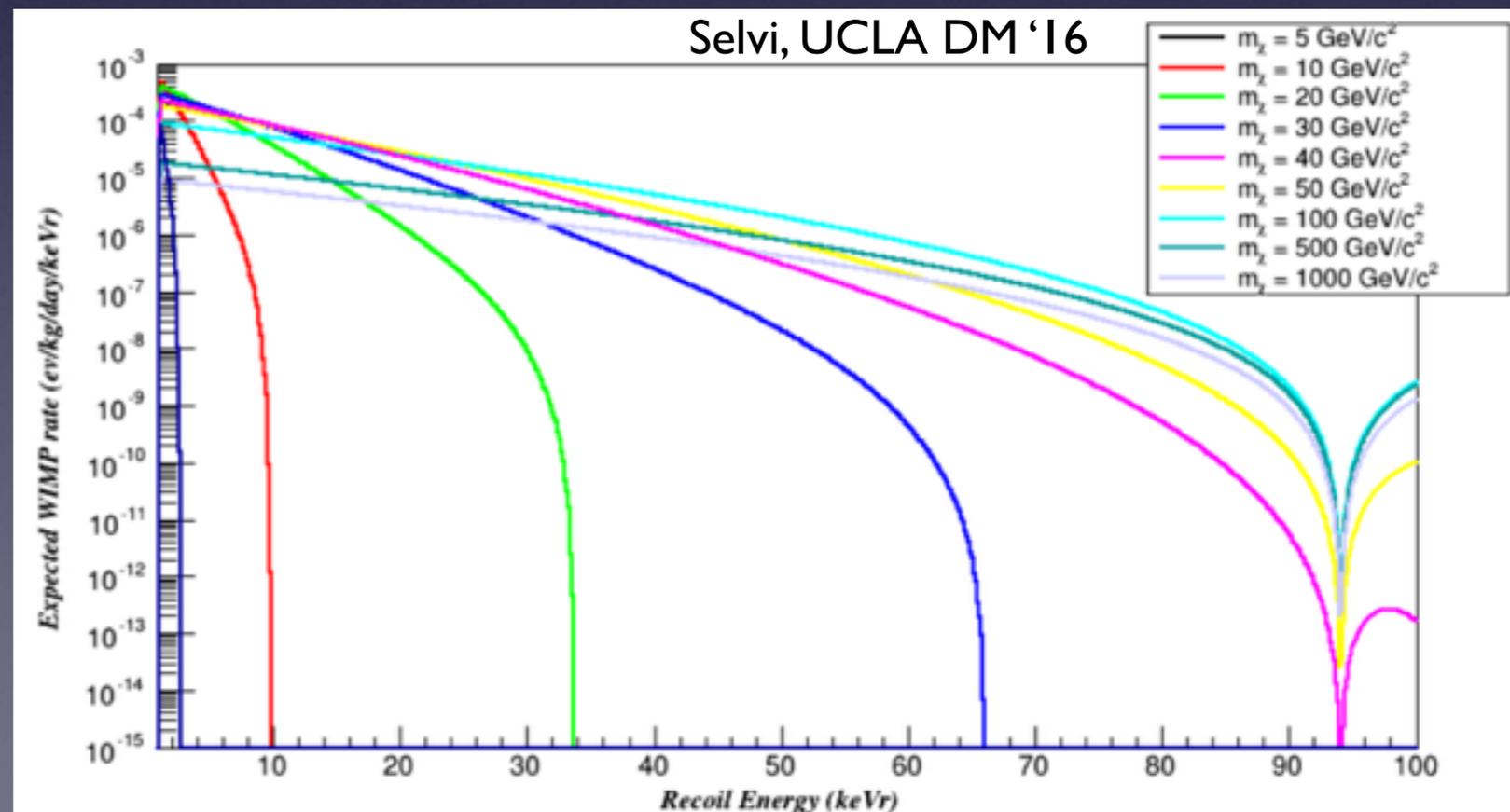
# A falling spectrum

- For the moment, treat  $v_{\max} \rightarrow$  infinite, and take  $f(v)$  to follow a Maxwellian speed distribution:

$$f(v) = \frac{4}{\sqrt{\pi}} \frac{1}{v_0^3} v^2 e^{-v^2/v_0^2}$$

- Then this integral becomes:  $\int_{v_{\min}}^{\infty} \frac{1}{v} f(v) dv = \frac{2}{\sqrt{\pi}} \frac{1}{v_0} e^{-E_R m_N / 2\mu^2 v_0^2}$
- Thus we expect to see a smooth, exponentially falling spectrum, multiplied by the form factor squared.
- Again we see low-energy sensitivity is critical, especially for light WIMPs.

- Note: for a time-dependent treatment, we would approximate  $f(v)$  as Maxwellian in the frame of the Galaxy, and include the motion of the Earth with respect to that frame.



# Estimating the total rate

- In the limit that the form factor can be ignored, we can integrate over  $E_R$  to get the total rate:

$$R = \frac{2}{\sqrt{\pi}} A^2 N_T \frac{\sigma_{\chi n} \mu^2}{m_\chi \mu_{\chi n}^2} \rho_0 v_0$$

- Consider a fiducial volume of 100 kg xenon (atomic mass 132 ~ 100).
- What WIMP-nucleon cross section do you need to see 1 event / year for a 100 GeV WIMP?

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$$N_T \approx N_A \times 1000 \approx 6 \times 10^{26} \quad \text{1 mole xenon} \sim 100\text{g}$$

$$\mu \approx 50\text{GeV}$$

$$\rho_0 \approx 0.4\text{GeV}/\text{cm}^3$$

$$\mu_{\chi n} \approx 1\text{GeV}$$

$$v_0 \approx 200\text{km}/\text{s} \approx 6 \times 10^{14}\text{cm}/\text{yr}$$

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$$R \approx 10^4 \times 6 \times 10^{26} \times 50^2 \times \frac{0.4}{100} / \text{cm}^3 \times 6 \times 10^{14} \text{cm}/\text{yr} \times \sigma_{\chi n}$$

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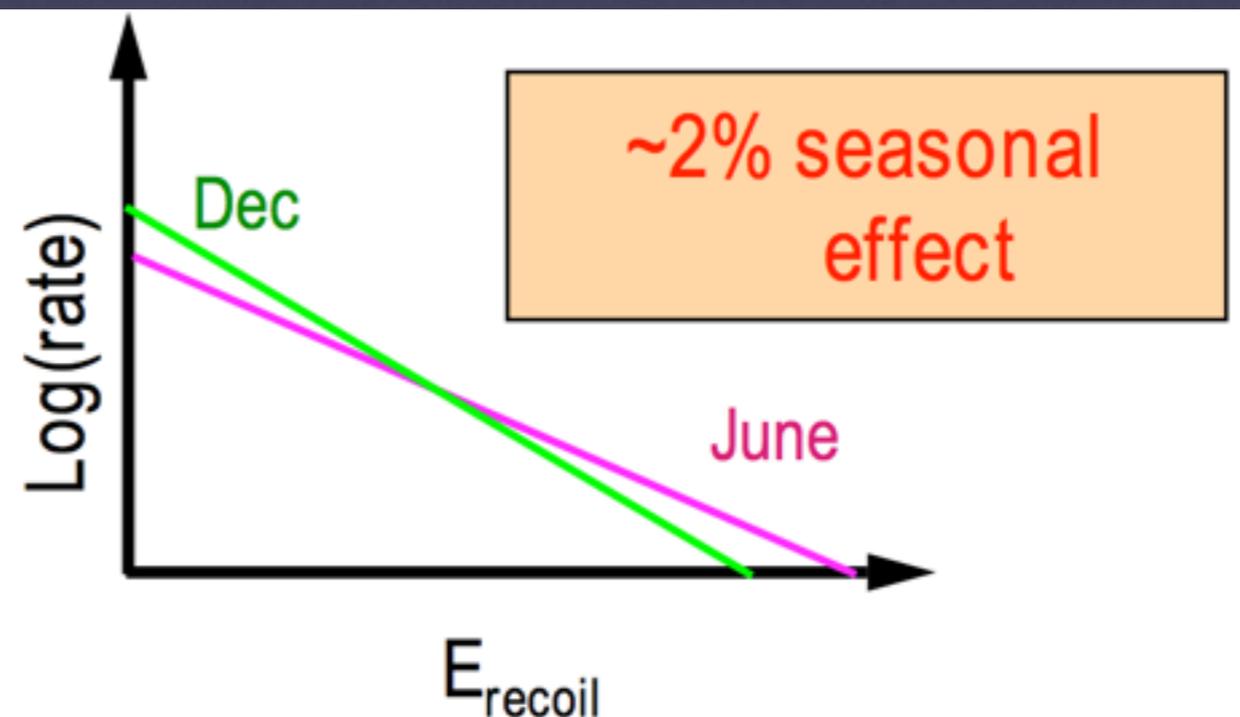
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$$\approx (10^{46} \sigma_{\chi n} / \text{cm}^2) / \text{yr}$$

# Modulation

- For more accurate treatment, need to include time dependence and asymmetry of velocity distribution as seen from Earth (even in this approximation, distribution is only isotropic and constant in Galactic frame)
- Finite escape velocity ( $\sim 500\text{-}600$  km/s) cuts off exponential distribution at large  $E_R$
- Time dependence induces  $\sim$ sinusoidal annual modulation
- If observed, could confirm cosmic origin of signal

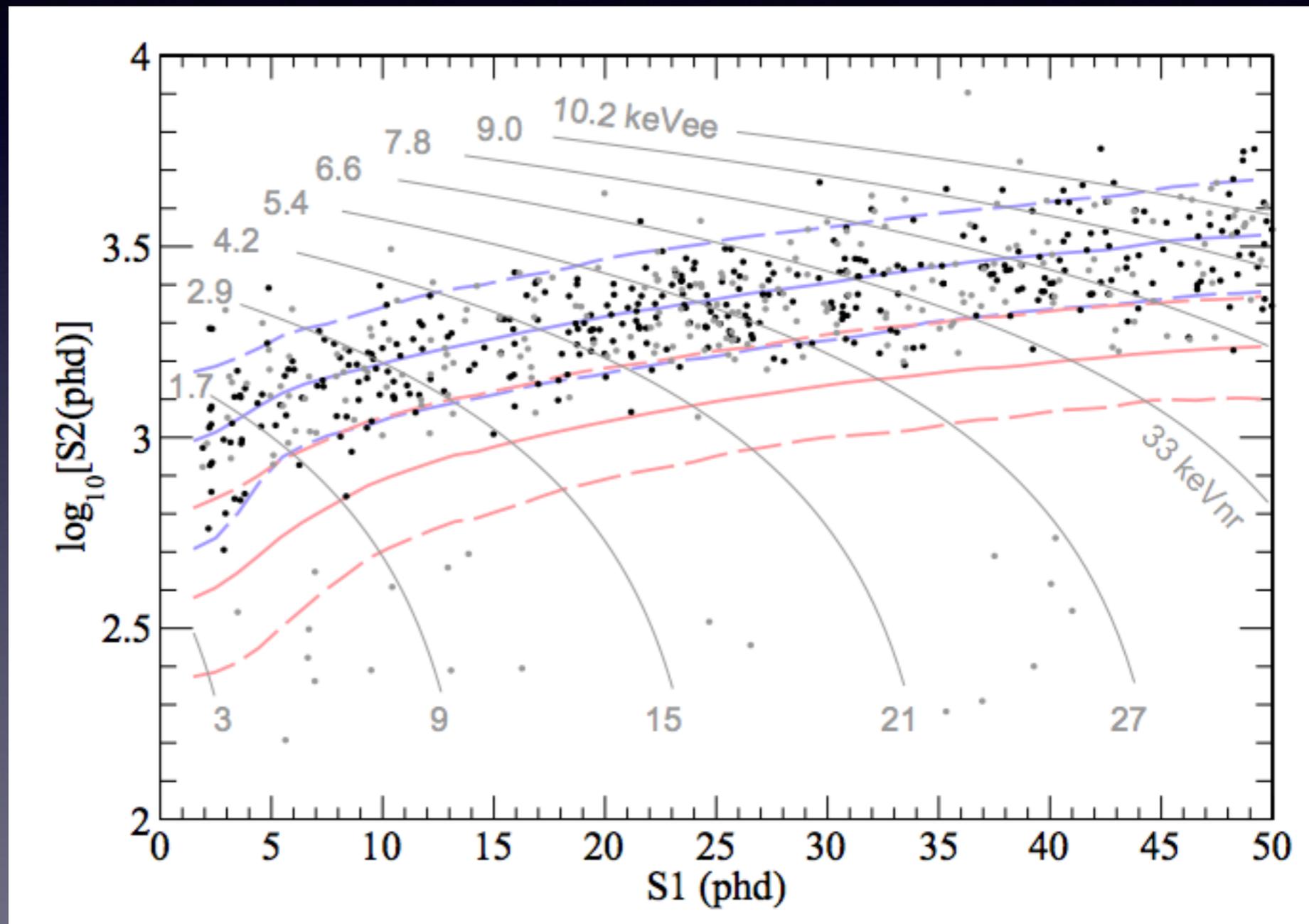


# Experimental strategies

- Want large volumes, high  $A$  (except for light DM), low backgrounds
- Backgrounds:
  - Neutron scatters: mimic nuclear recoils, but can be shielded
  - Photon/electron scatters: scatter dominantly off electrons (for kinematic reasons), need to distinguish from nuclear recoils
  - In the future: cosmic neutrino background (“neutrino floor”)
- Current flagship experiments focus on reducing background to zero, by identifying and rejecting electron scattering events. Key idea is to measure two observables, where behavior of electron/nuclear recoils differs.
  - LUX and XENON - liquid xenon, measure both ionization and scintillation light from recoil. Best limits over most of energy range.
  - SuperCDMS - silicon-germanium semiconductors, measure ionization + photons. Lower atomic mass = best limits for light DM.
- Worth mentioning: DAMA/LIBRA experiment has a long-standing claimed detection, based on annual modulation search - but not background-free, and difficult to reconcile this result with limits from other experiments.
- Also many other experiments using a range of different materials and techniques.

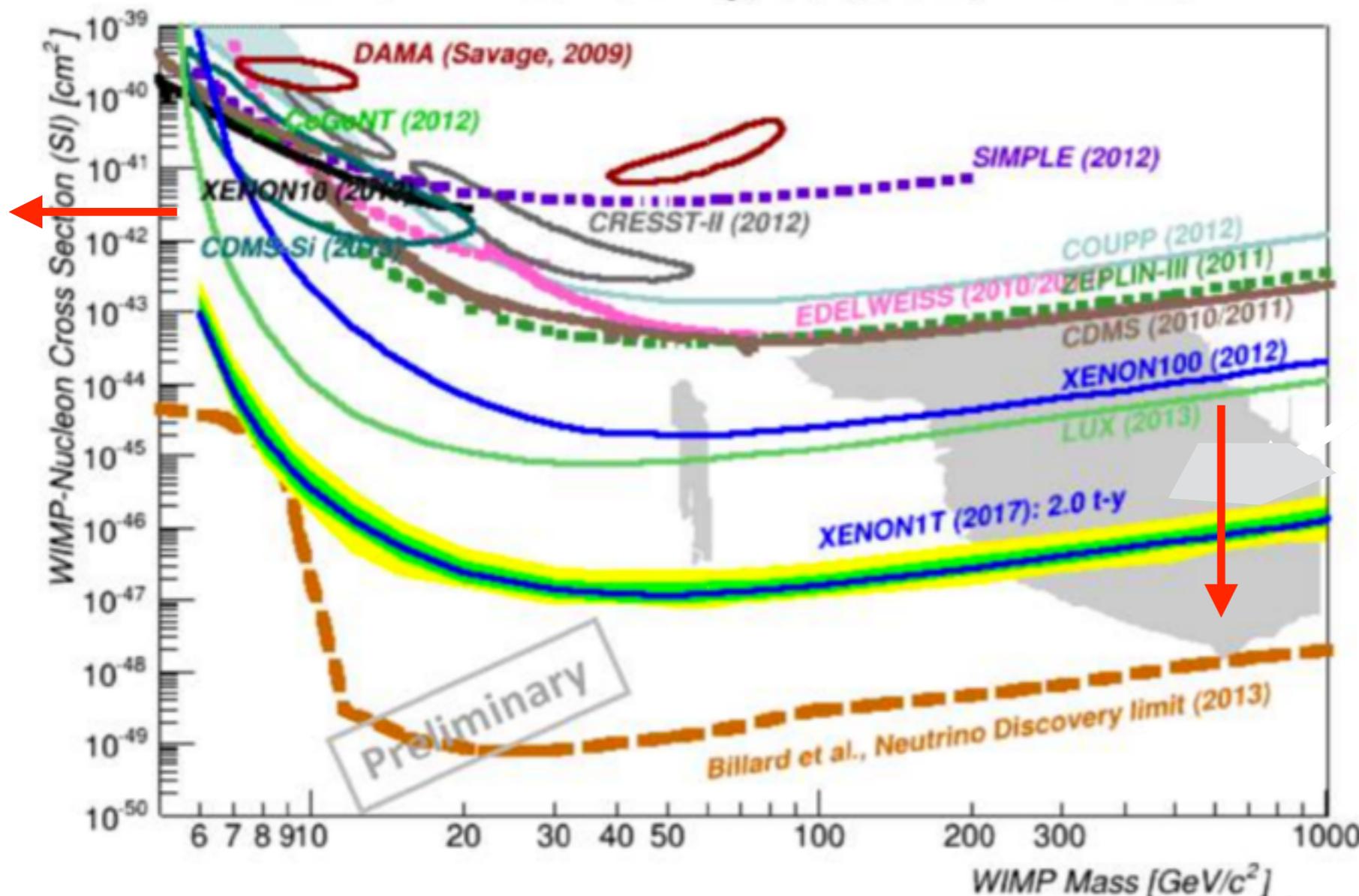
# Example: scintillation vs ionization

- S2 = ionization signal
- S1 = photon signal
- Blue bands = 80% containment for electron recoils
- Red bands = 80% containment for nuclear recoils



# The future

Experiments will continue to push the sensitivity curve downward - at least until neutrino “floor” is reached



Several proposals for clever techniques to probe light DM below the GeV scale - e.g. DM-electron scattering (1206.2644, 1509.01598), superconductors (1504.07237, 1604.06800), superfluids (1604.08206)

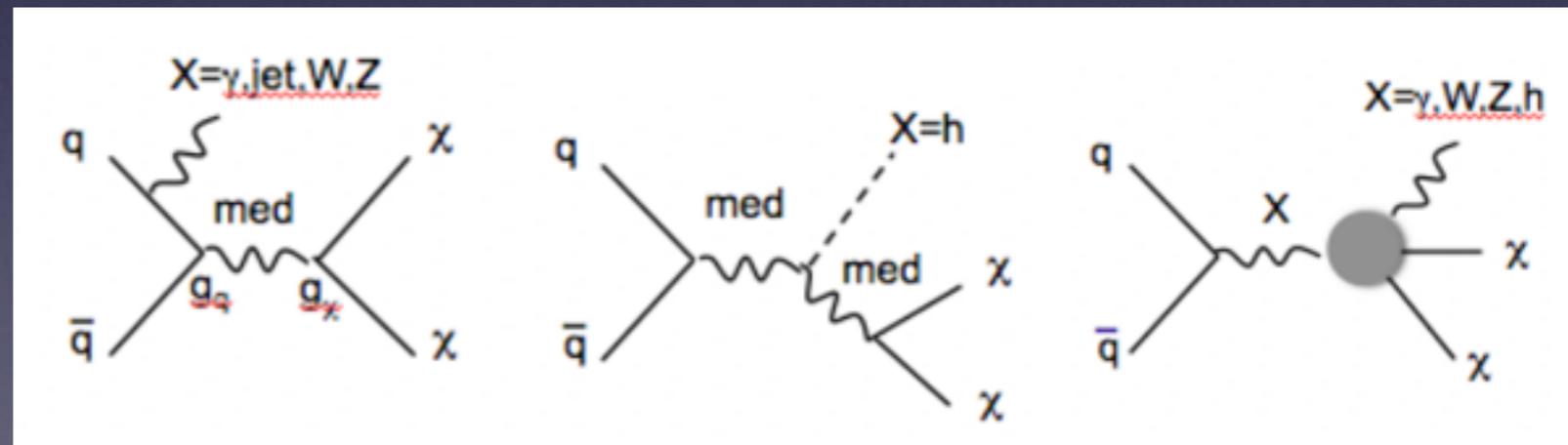
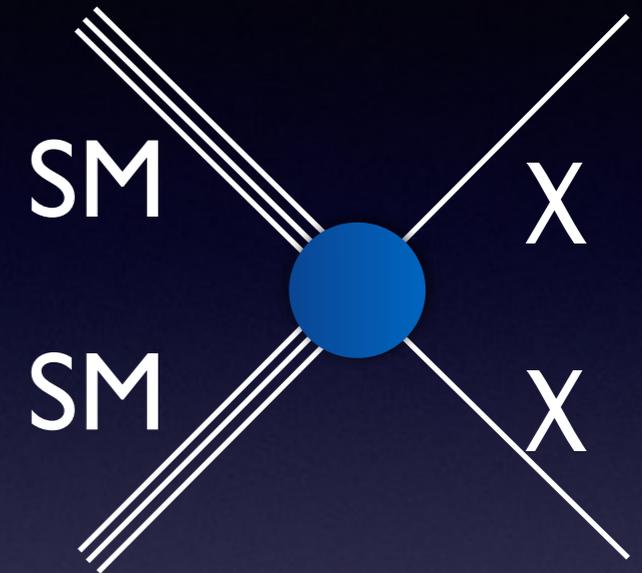
# Collider searches

# LHC searches in a nutshell

- If DM is produced at the LHC, it is stable => will escape the detector
- Cannot be detected directly, but will show up as missing energy/momentum
- Most direct DM searches at LHC are “mono-X” searches - look for visible particle recoiling off invisible partner

- e.g. mono-Higgs
- mono-jet
- mono-photon

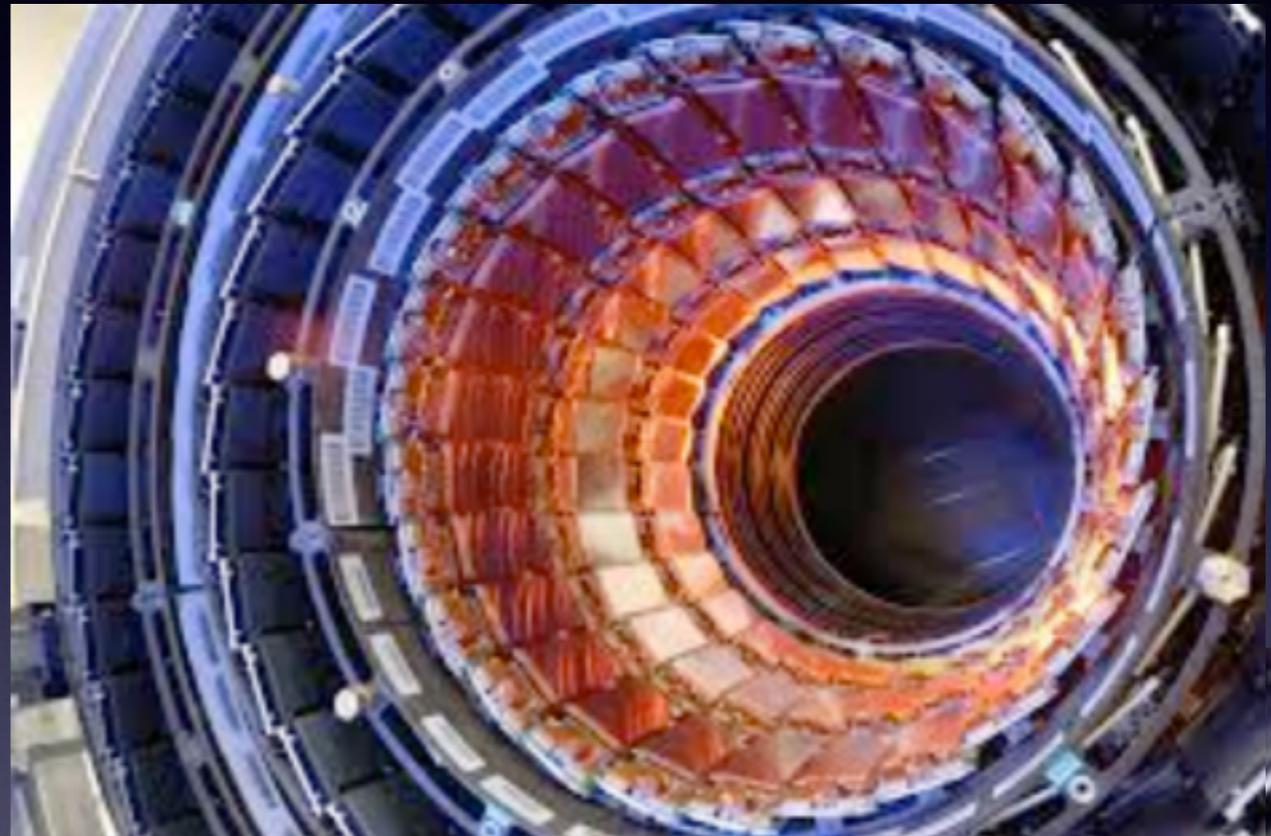
- Doesn't fundamentally need to be “mono” - could be more than one visible particle/jet in the event



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# Why the LHC?

- If DM couples to SM particles, especially quarks/gluons, it should be possible to produce DM in sufficiently energetic proton-proton collisions
- Time-reversal of annihilation process (broadly speaking)
- Producing DM under controlled conditions could allow us to probe DM-SM interactions in depth
- But we “see” particles at the LHC by observing their decay products - not going to happen for DM!

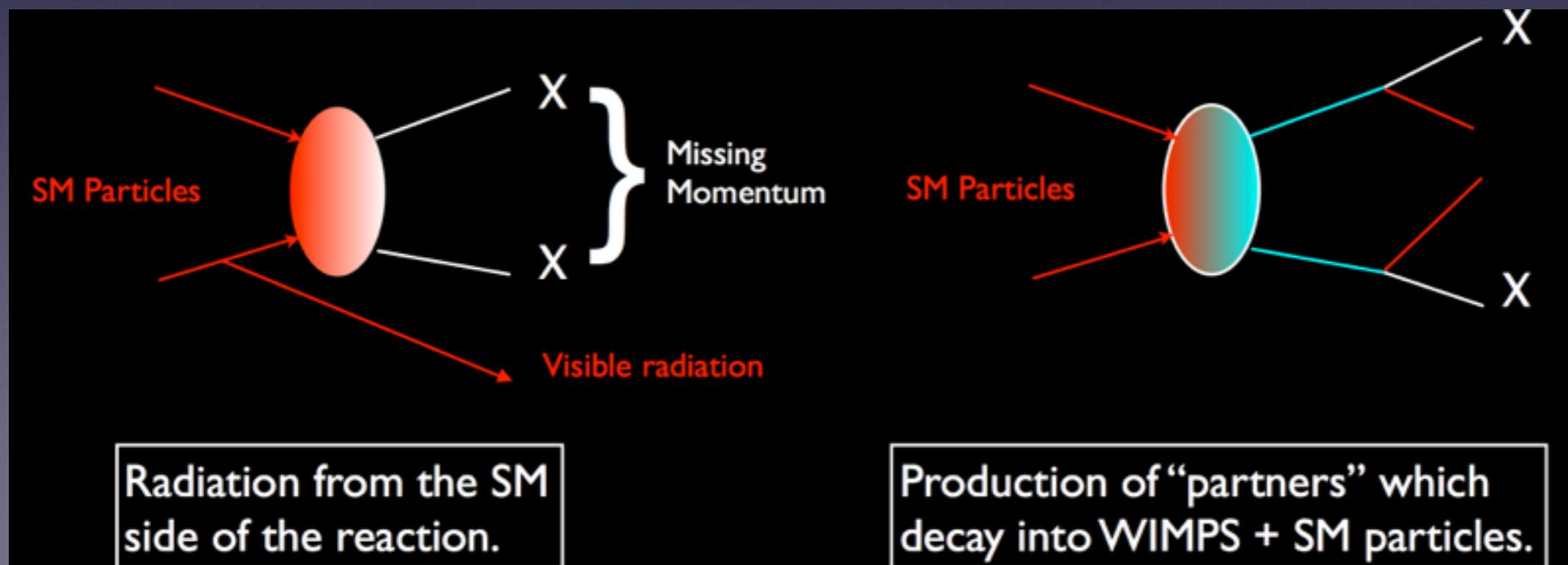


LHC detectors are great - but do not include a DM-spotting module



# Seeing the Dark Side

- Neutrinos are produced frequently at the LHC - but then pass out of the detectors invisibly, not decaying or interacting
- DM would likely behave the same way - invisible to detectors
- Two kinds of processes that could reveal them:

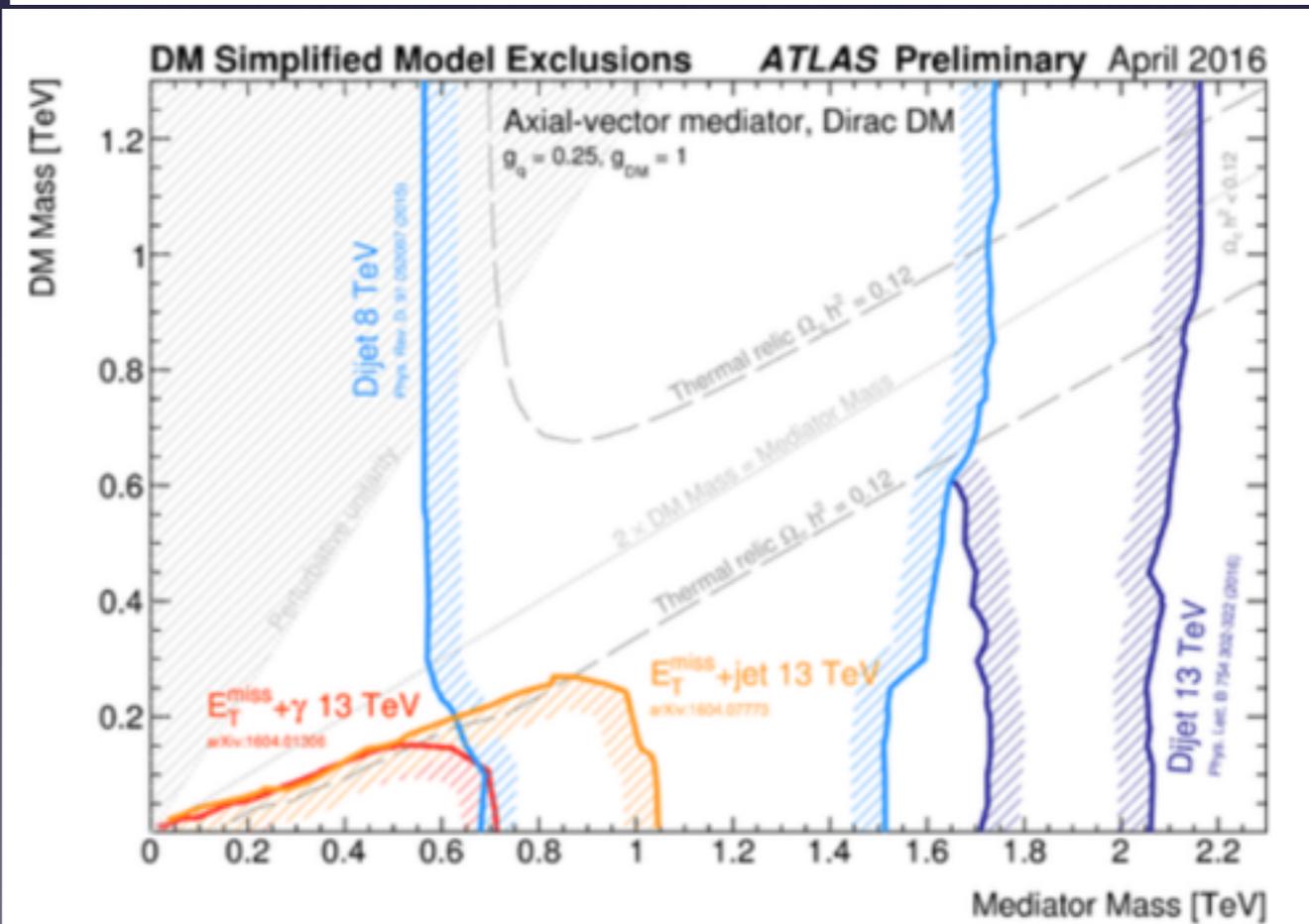
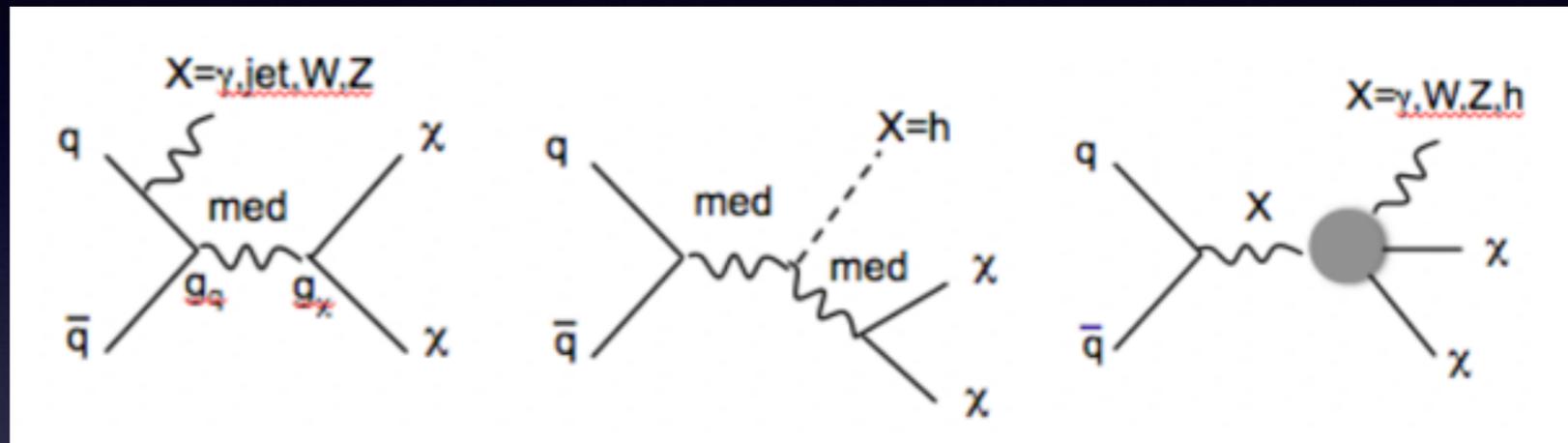


# Approaches for dark matter

- Construct detailed model of high-energy physics (e.g. SUSY model), search for resulting signatures
  - Upside: since there are many non-DM particles, can have striking effects. Characteristic signatures can include cascades producing many particles, with large “MET” (missing momentum transverse to the beam direction) - since all SUSY partners decay to the LSP eventually
  - Downside: not easy to translate constraints on one model into bounds on another model. (Not “model-independent”.) Makes interpretation more challenging.
- Construct simplified model with only a few ingredients, develop generic searches
  - Upside: easy to translate to many models, reduces the risk of missing a signal due to searching too narrowly
  - Downside: sometimes effects of extra ingredients are important! No guarantee that simplified model can be embedded into reasonable high-energy theory.
- Approaches are complementary

# ATLAS simplified model results

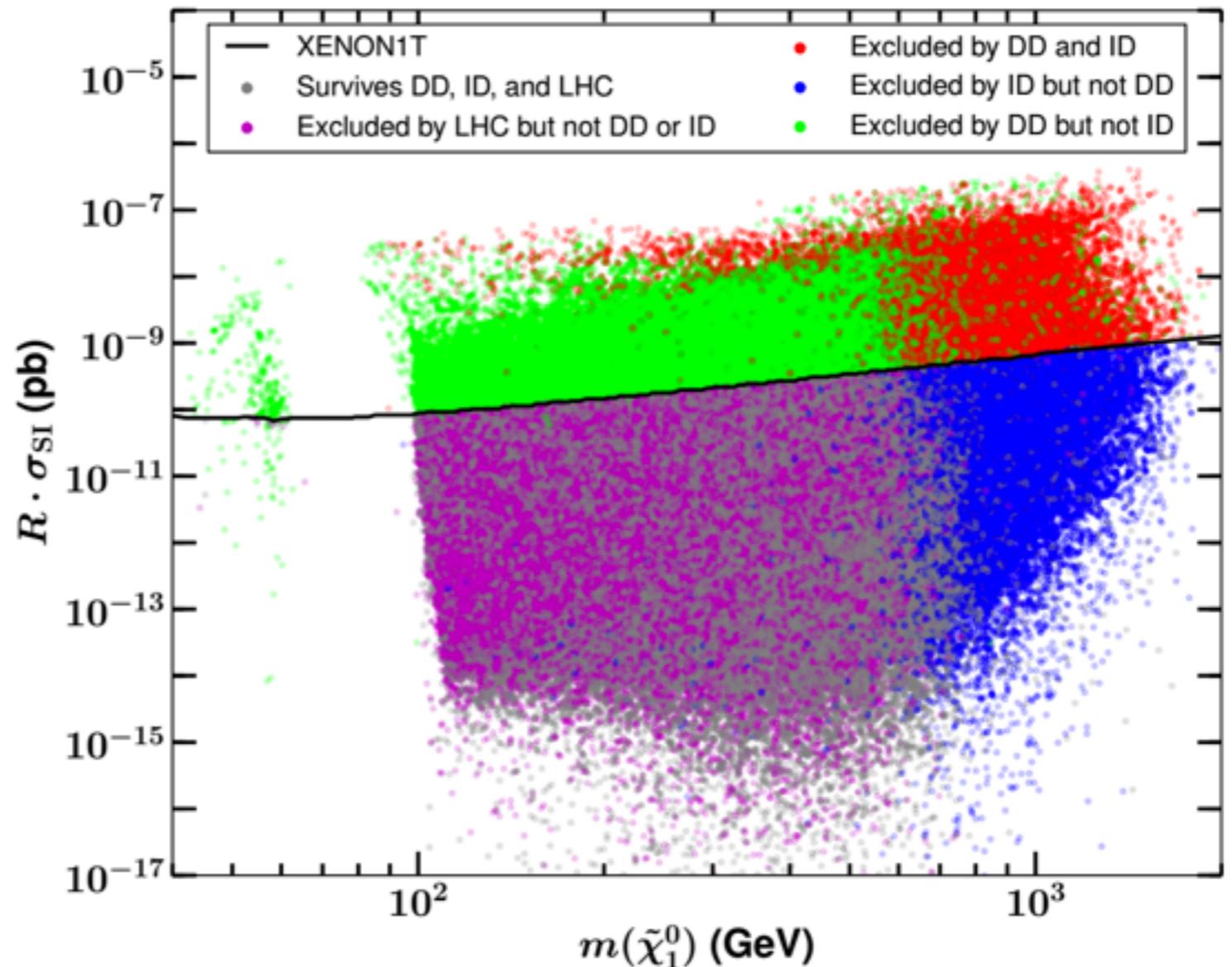
- Example of a simplified model approach: suppose DM couples to some heavy mediator, which also couples to quarks
- Exchange of this mediator allows pair production of DM, along with other particles
- Can consider different possibilities for the spin of the mediator - vector, axial vector, scalar, pseudoscalar, etc.



Constraints based on 13 TeV ATLAS data

# Complementarity

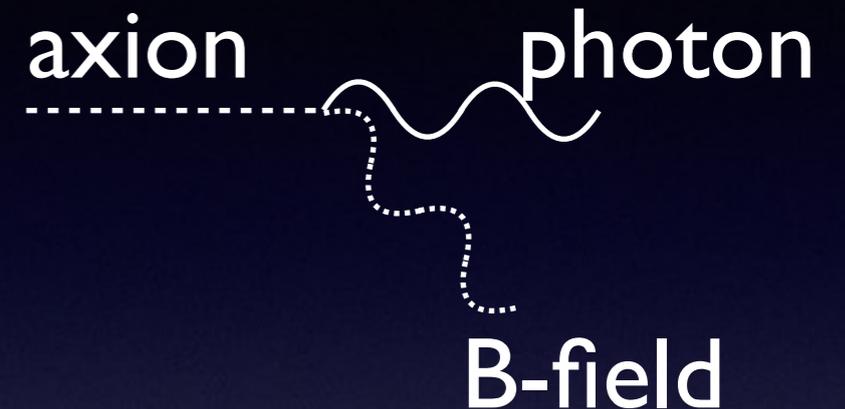
- To fully combine direct, indirect and collider constraints we do need some complete model - significant model-dependence in mapping constraints from one to the other
- One example: the pMSSM (phenomenological MSSM), simplified SUSY model
- Different searches probe different regions of parameter space; ideally we hope to see a signal in two or more



# Axion searches

# Axion searches in a nutshell

Good review by Graham et al 1602.00039



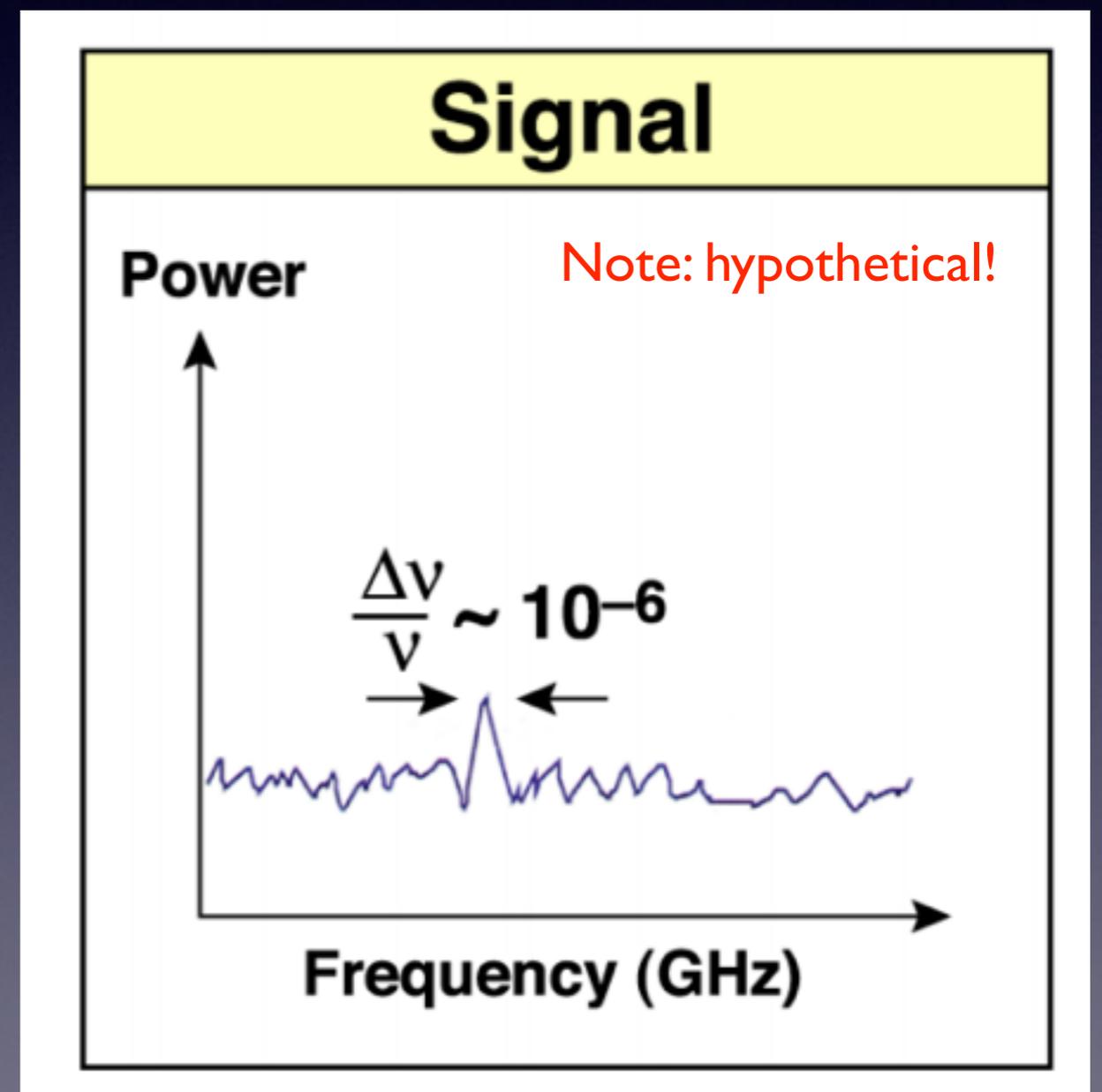
- In the presence of a magnetic field an axion can convert into a photon (or vice versa)
- This means:
  - photons can travel through regions that should be opaque to them, by converting into axions and then back
  - it might be possible to “catch” cosmological axions using magnetic fields, turning them into visible photons / inducing electromagnetic fields
- Axions can also induce nuclear electric dipole moments (CASPer experiment, 1306.6089), change the proton-neutron mass splitting and so affect nucleosynthesis (1401.6460), and otherwise have interesting QCD effects.

# Axion probes

- photons can travel through regions that should be opaque to them, by converting into axions and then back
  - high-energy photons traveling from high redshifts (to be discussed)
  - “light shining through a wall” (e.g. 1009.4875)
  - stellar cooling - axions escape more easily than photons (e.g. 0806.2807)
- it might be possible to “catch” cosmological axions using magnetic fields, turning them into visible photons / inducing electromagnetic fields
  - ADMX experiment (to be discussed)
  - CAST - using the magnetic field of the Sun! (Arik et al Phys. Rev. Lett. 112, 091302)
  - ABRACADABRA (1602.01086)

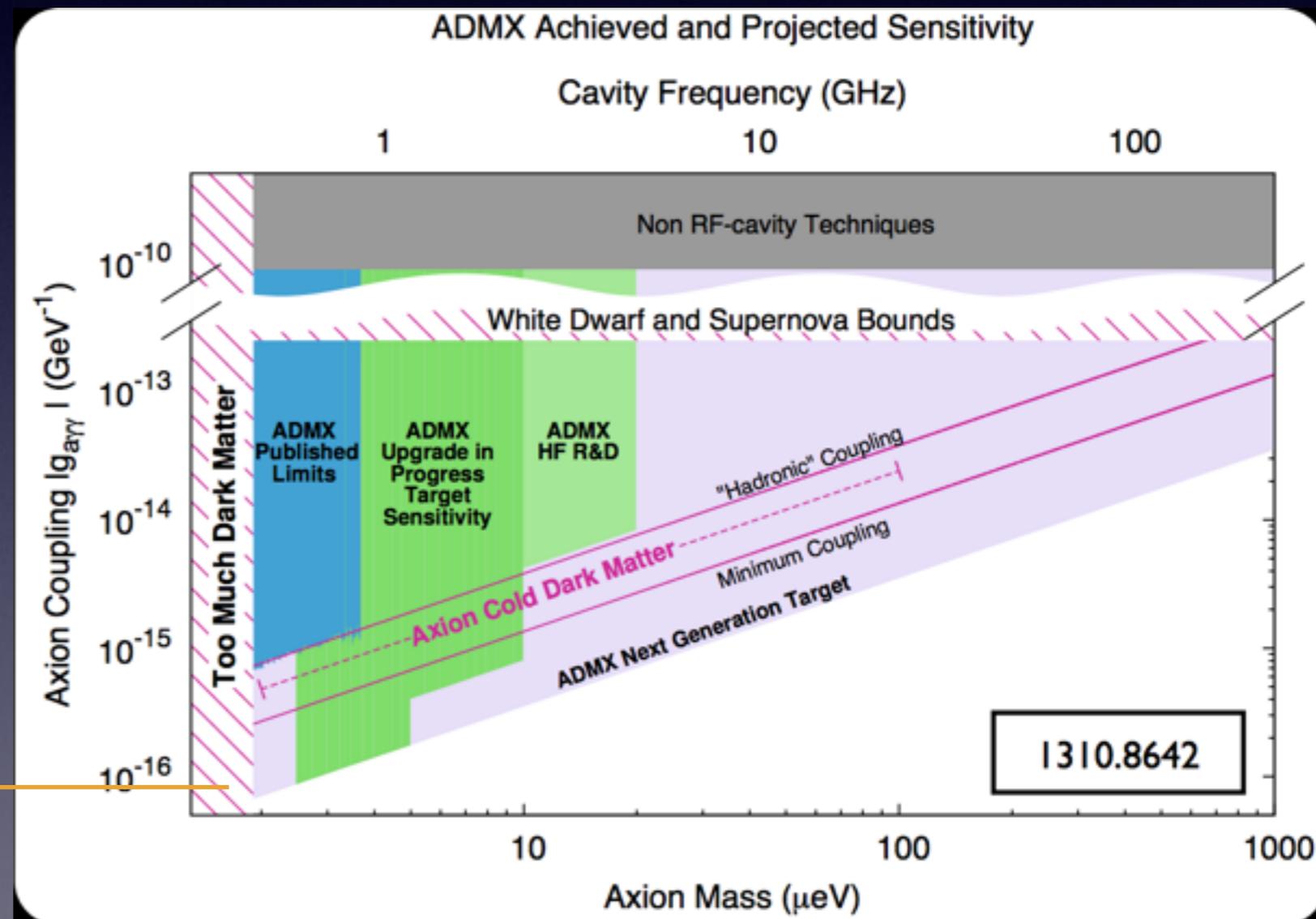
# The Axion Dark Matter Experiment

- Idea: build a resonant microwave cavity containing a strong magnetic field
- Measure output power from cavity
- The axion-photon conversion will only occur if the frequency of the magnetic field matches with the axion energy (i.e. axion mass - DM axions are very cold)
- Vary cavity frequency, look for a bump in power. Detection would also measure axion mass.



# ADMX sensitivity

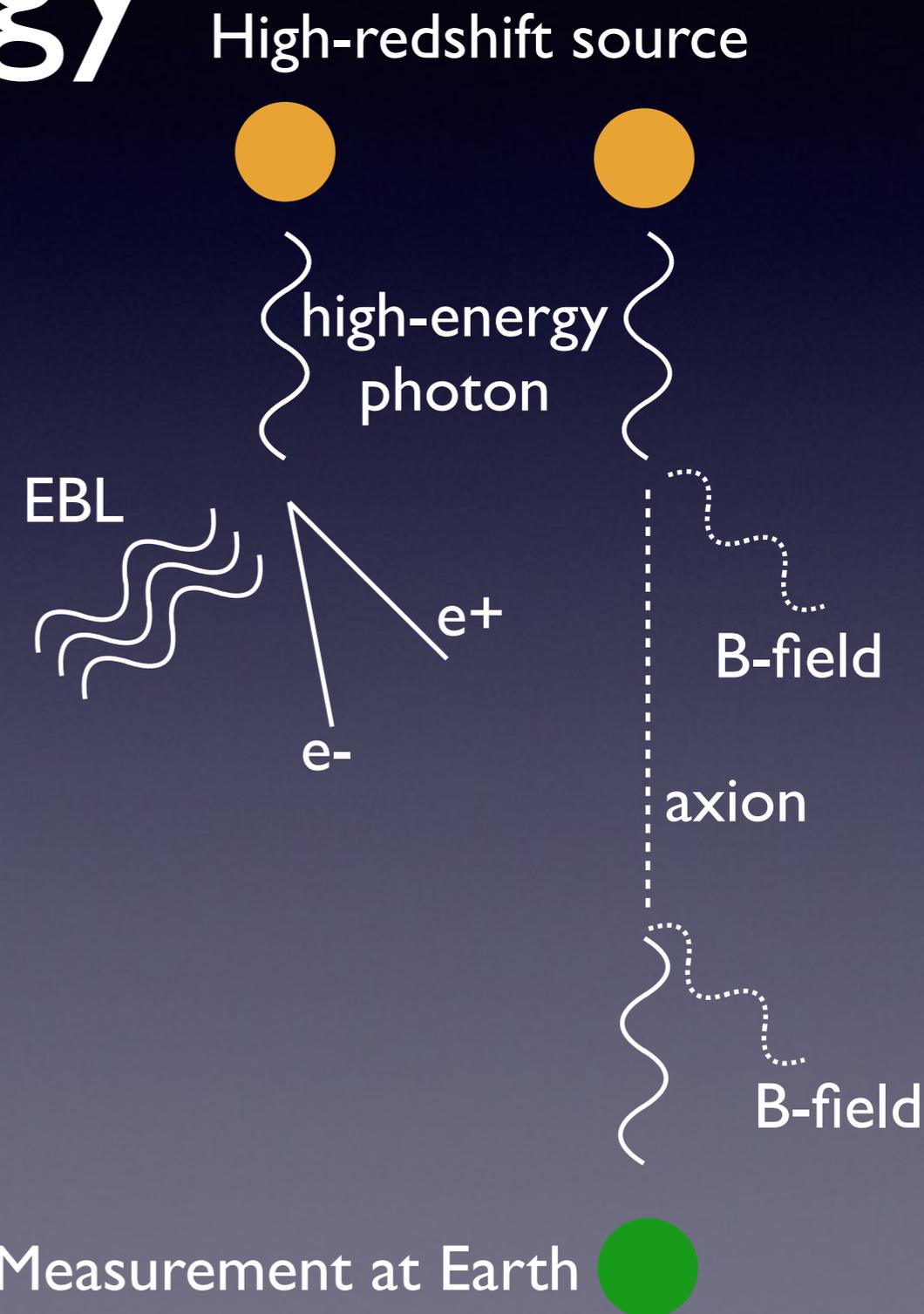
- ADMX current limits just miss the edge of the region interesting for CDM (for QCD axion)
- Timescale for ADMX-Gen2 to cover full CDM region below  $\sim 10$  GHz, IF  $O(1)$  misalignment angle, is  $O(5 \text{ years})$ .



“too much dark matter”  
= need small misalignment angle

# Axion conversion and cosmology

- There are small but non-zero magnetic fields in the space between clusters
- Photons propagating to us from high redshifts could potentially spend some of their time as axions
- In general this is a small effect - but can be important for very high-energy photons.
- Pair production on the extragalactic background light (EBL) generally stops these photons from traveling far.
- Becomes an issue when there is enough energy for pair production in COM frame:  $E_\gamma E_{\text{EBL}} \gtrsim m_e^2$
- Depends sensitively on the spectrum of the EBL - but observation of sufficiently high-energy photons from sufficiently high redshift could point to the existence of axions or similar particles.



# Summary

- We can probe the properties of dark matter particles with terrestrial experiments:
  - Underground WIMP searches - carving deep into supersymmetric parameter space for spin-independent scattering
  - Collider searches - attempt to produce DM directly, see it via missing energy/momentum
  - Axion searches - attempt to force photons to convert to axions, or capture cosmological axions and produce visible photons/E&M fields
- Another topic I haven't talked about is dark photon searches:
  - relatively low-energy accelerators can probe new regions of parameter space for light particles weakly coupled to the Standard Model
  - would not be the DM itself, but might be coupled to it.

# Conclusions

- Dark matter is 80% of the universe's matter, and we don't know what it is.
- We have a wide array of gravitational probes that:
  - tell us the DM is cold and  $\sim$ collisionless
  - may offer hints of discrepancies between purely cold collisionless DM and observation - but more data and better analysis tools needed
- We have no shortage of ideas for particle dark matter candidates - WIMPs and axions are two of the most popular. Several completely independent possibilities for achieving cold dark matter with the right relic density.
- Dark matter annihilation, decay or other interactions could leave visible imprints on the cosmological history, or in present-day astrophysical observations
- There is a broad ongoing experimental effort to probe particle physics models of dark matter, at colliders, direct detection experiments and in dedicated axion searches.
- I hope I've given you a flavor of what's going on, and pointed you to some useful tools for understanding it.

Thanks for listening, and  
for all the questions!