Models of dark energy and modified gravity

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Quintessence

• Scalar field

We now know a scale field exists (i.e. Higgs)

\[
S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} R + S_\phi
\]

\[
S_\phi = \int d^4 x \left[ -\frac{1}{2} g^{\mu\nu} \left( \partial_\mu \phi \right) \left( \partial_\nu \phi \right) - V(\phi) \right]
\]

energy momentum tensor

\[
T^\phi_{\mu\nu} = \left( \partial_\mu \phi \right) \left( \partial_\nu \phi \right) - g_{\mu\nu} \left[ \frac{1}{2} g^{\alpha\beta} \left( \partial_\alpha \phi \right) \left( \partial_\beta \phi \right) + V(\phi) \right]
\]

scalar field equation

\[
\frac{\delta S_\phi}{\delta \phi} = 0 \quad \nabla^\mu \nabla_\mu \phi - V'(\phi) = 0 \quad \nabla^\mu T^\phi_{\mu\nu} = 0
\]
Background

• Energy density and pressure

\[ \rho_\phi = -T_{00} = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad \phi = \phi(t) \]

\[ P_\phi = \frac{1}{3} T_i^i = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]

• Scalar field equation

\[ \ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0 \quad \leftrightarrow \quad \dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0 \]

\[ \text{Acceleration} \]

\[ \dot{\phi}^2 \ll V(\phi) \quad w_\phi = \frac{P_\phi}{\rho_\phi} \approx -1 \]
Equation of state

- Equation of state

\[ 1 + w_\phi = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{V'^2}{9H^2(1 + \xi_\phi)^2 \rho_\phi} > 0, \quad \xi_\phi = \frac{\dot{\phi}}{3H\phi} \]

For \( w_\phi \approx -1, \quad \xi_\phi \ll 1 \)

\[ 1 + w_\phi \approx \frac{2}{3} \epsilon_\phi \Omega_\phi(a), \quad \epsilon_\phi = \frac{1}{2\kappa^2} \left( \frac{V'}{V} \right)^2, \quad \Omega_\phi = \frac{\rho_\phi}{\rho_\phi + \rho_m} \quad \kappa^2 = 8\pi G = \frac{1}{M_{pl}^2} \]

Analogous to slow-roll approximations for inflation but the scalar field does not completely dominates the energy density
Quintessence potentials

• Freezing models

\[ V(\phi) = M^{4+n} \phi^{-n} \]

The scalar field dynamics freezes at late time and approaches \( w_\phi \approx -1 \)

• Thawing models

\[ V(\phi) = M^4 \cos^2 \left( \frac{\phi}{f} \right) \]

The scalar field is initially in the regions of \( w_\phi \approx -1 \) and later starts to roll down the potential

Linder & Caldwell astro-ph/0505494
Scaling solution

• Tracker solution

For some potentials, the energy density of the scalar field tracks that of matter/radiation

\[ V(\phi) = V_0 e^{-\kappa \phi}, \quad \kappa^2 = 8\pi G = \frac{1}{M_{\text{pl}}^2} \]

If \( \lambda^2 > 3(1 + w_M) \)

there is an attractor where

\[ w_\phi = w_M, \quad \Omega_\phi = \frac{3(1 + w_M)}{\lambda^2} \]

The late time solution is insensitive to the initial density of the scalar field (however we need \( \lambda^2 < 2 \) to have an accelerating solution)
Fundamental issues

• Consider a massive scalar field

\[ V(\phi) = \frac{1}{2} m^2 \phi^2 \quad \varepsilon_\phi = \frac{1}{2\kappa^2} \left( \frac{V'}{V} \right)^2 \ll 1 \quad \rightarrow \quad \phi > M_{pl} = 10^{18} \text{GeV} \]

\[ V(\phi_0) = \rho_{DE} = 10^{-48} \text{GeV}^4 \quad \rightarrow \quad m = \left( \frac{\rho_{DE}}{\phi^2} \right)^{\frac{1}{2}} = 10^{-42} \text{GeV} \approx H_0 \]

Correction to the potential

\[ V(\phi) = \frac{1}{4} \lambda \phi^4 < \rho_{DE} \quad \rightarrow \quad \lambda < 10^{-120} \]

these small numbers are very difficult to protect against quantum corrections
Symmetry

• Let’s assume there is an exact symmetry with a massless scalar field. Introducing a symmetry breaking term, quantum corrections generate a potential (cf. QCD axion)

\[ V(\phi) = \mu^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right), \quad m_\phi \approx \frac{\mu^2}{f} \]

\[ f \approx M_{pl}, \quad \mu \approx (H_0 M_{pl})^{1/2} \approx \rho_{DE}^{1/4} \approx (10^{-3} \text{ eV}), \quad m_\phi \approx H_0 \]

small \( \mu \) is technically natural as (we assumed that) there is an exact symmetry in the limit \( \mu \to 0 \) thus quantum corrections are \( O(\mu^4) \) (Pseudo Nambu-Goldstone bosons)
Modified Gravity

• Lovelock theorem

4D theory of metric and the equations of motion is given by

\[ S_{\mu\nu}(g) = \kappa^2 T_{\mu\nu} \]

we require \( S_{\mu\nu}(g) \) is

1. a tensor made from the metric and its first and second derivatives
2. symmetric
3. divergence free

\[ S_{\mu\nu}(g) = a \, G_{\mu\nu} + b \, g_{\mu\nu} \]
Beyond Einstein

• Lovelock theorem indicates that if you modify Einstein theory then you must do one or more of the following

  • Consider other fields (scalars, vectors, etc)

  • Accept higher derivatives

  • Work in a space with more than four dimensions

  • Non-local, ...
Higher derivative theories

• Ostrogradski ghost

Theories with higher derivatives \( L(g, \dot{g}, \ddot{g}) \)

\[
\frac{d^2}{d^2t} \left( \frac{\partial L}{\partial \ddot{g}} \right) - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{g}} \right) + \left( \frac{\partial L}{\partial g} \right) = 0
\]

We treat \( \dot{g} \) as a new variable and \( \ddot{g} \) as a new velocity (we need four initial conditions)

If the velocity \( \ddot{g} \) can be expressed in terms of momenta \( \partial L / \partial \ddot{g} \)

(non-degeneracy condition), it can be shown that the Hamiltonian is not bounded from below  

Woodard astro-ph/0601672
Ghost

• If the kinetic term has a wrong sign
  The Hamiltonian is not positive definite
  quantum mechanically, particles can be created from vacuum without costing any energy → instability of vacuum
  
  $S_\phi = \int d^4x \left[ \frac{1}{2} g^{\mu\nu} \left( \partial_\mu \phi \right) \left( \partial_\nu \phi \right) - V(\phi) \right]$
  $
  \rho_\phi = -\frac{1}{2} \dot{\phi}^2 + V(\phi) \quad \text{“Phantom DE”}
  $

  there is no time scale for instability in Lorentz invariant theory
  the decay is instantaneous

  Unless the mass of ghost is above the cut-off scale of the theory, ghost poses a serious consistency problem

Cline et.al. hep-ph/0311312
f(R) gravity

• Generalise the Einstein-Hilbert action

\[ S = \int d^4 x \sqrt{-g} F(R) + S_m [g], \quad F(R) = R + f(R) \]

• The equations of motion is fourth order

\[ F'(R) R_{\mu\nu} - \frac{1}{2} F(R) g_{\mu\nu} - \left( \nabla_\mu \nabla_\nu + g_{\mu\nu,\square} \right) F'(R) = \kappa^2 T_{\mu\nu} \]

but this theory violates the assumption of Ostrogradski theorem (i.e. non-degeneracy)
In fact the theory can be rewritten as scalar-tensor gravity

\[ S = \int d^4 x \sqrt{-g} \left( F(\phi) + (R - \phi) F'(\phi) \right), \quad \frac{\delta S}{\delta \phi} = (R - \phi) F''(\phi) = 0 \quad \Rightarrow \quad \phi = R \quad (F''(R) \neq 0) \]

\[ S = \int d^4 x (\psi R - V(\psi)) \quad \psi = F'(\phi), \quad V = RF' - F \]
Brans-Dicke gravity

- Brans-Dicke (BD) gravity (Jordon frame)

\[ S = \int d^4x \sqrt{-g} \left( \psi R - \frac{\omega_{BD}}{\psi} (\nabla \psi)^2 + V(\psi) \right) + S_M [g_{\mu\nu}] \]

f(R) gravity \( \omega_{BD} = 0 \)

- Einstein frame

Conformal transformation \( \tilde{g}_{\mu\nu} = \frac{2\psi}{M_{pl}^2} g_{\mu\nu} \), \( \frac{\phi}{M_{pl}} = \frac{1}{2\alpha} \ln \left( \frac{2\psi}{M_{pl}^2} \right) \), \( 2w_{BD} + 3 = 1/2\alpha^2 \)

\[ S = \int d^4x \sqrt{-\tilde{g}} \left( \frac{M_{pl}^2}{2} \tilde{R} - \frac{1}{2} (\tilde{\nabla} \phi)^2 - \tilde{V}(\phi) \right) + S_M [A^2(\phi) \tilde{g}_{\mu\nu}] \quad A(\phi) = \exp \left( \frac{\alpha \phi}{M_{pl}} \right) \]

the scalar field is coupled to matter
Dvali-Gadadze-Porrati (DGP) braneworld model

• 5D braneworld model

Standard model particles are confined to a 4D brane in a 5D bulk spacetime

\[ S = \frac{1}{32\pi G r_c} \int d^5x \sqrt{-g}\, R(5) + \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, R + \int d^4x \sqrt{-g}\, L_m \]

cross over scale \( r_c \)

\[ \pm \frac{H}{r_c} = H^2 - \frac{8\pi G}{3} \rho \]

in the + branch, the expansion of the universe accelerates without the cosmological constant (self-accelerating solution)
Fundamental issues

• Theoretical consistency
  
  modified gravity models often suffer from instabilities
  
  (cf. the self-accelerating solution in DGP model suffers from ghost instabilities)

• Small numbers
  
  cf. viable f(R) models

\[ F(R) = R - 2\Lambda_{\text{eff}} - \frac{R_0^{n+1}}{R^n}, \quad \Lambda_{\text{eff}} \approx R_0 \approx H_0^2 \]

DGP models \( r_c \approx H_0^{-1} \)
Ultra-light scalar field

- In most of DE/MG models, there exists a scalar field with tiny mass

\[
S = \int d^4x \left[ -\frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 \right] - \frac{\alpha}{M_{pl}} \int d^4x \phi \rho
\]

static source

\[
(\nabla^2 - m^2)\phi = \frac{\alpha}{M_{pl}} \rho,
\]

\[
\frac{\phi}{M_{pl}} = -\left( \frac{\alpha}{4\pi M_{pl}^2} \right) \frac{M}{r} \exp(-m r)
\]

\[
\begin{cases}
\phi = -\left( \frac{\alpha}{4\pi M_{pl}^2} \right) \frac{M}{r}, & r < m^{-1} \\
\phi \to 0, & r > m^{-1}
\end{cases}
\]

if \( m^{-1} \approx H_0^{-1} \) the scalar field mediates a long range force

\( m^{-1} : \text{Compton wavelength} \)
Fifth force

- Due to the coupling between the scalar and matter, the geodesic equation is modified as

\[ \ddot{a} = -\nabla\Psi - \frac{\alpha}{M_{\text{pl}}} \nabla \phi = F_G + F_5 \]

Fifth force

\[ F_5 = \frac{2\alpha^2}{8\pi M_{\text{pl}}^2} \frac{M}{r^2} \quad \text{for} \quad r < m^{-1} \]

\[ F_G = \frac{1}{8\pi M_{\text{pl}}^2} \frac{M}{r^2} \]

Fifth force is strongly constrained in the solar system

\[ \alpha^2 < 10^{-5}, \quad w_{BD} > 40,000 \quad (3 + 2w_{BD}) = 1/2\alpha^2 \]
Way out

• Quintessence
  the coupling to matter is assumed to be zero $\alpha^2 = 0$

• Violation of strong equivalence principle
  the solar system constraints come from coupling to baryons $\alpha_{\text{baryon}}^2 = 10^{-5}$
  the coupling to cold dark matter can be large $\alpha_{\text{cdm}}^2 = O(1)$
  interacting dark energy

$$S = \int d^4x \sqrt{-\tilde{g}} \left( \frac{M_{\text{pl}}^2}{2} \tilde{R} - \frac{1}{2} \left( \tilde{\nabla} \phi \right)^2 - \tilde{V}(\phi) \right) + S_{\text{cdm}}[A^2(\phi) \tilde{g}_{\mu\nu}] + S_{\text{baryon}}[\tilde{g}_{\mu\nu}]$$
Screening mechanism

• The coupling constant or mass need to be scale dependent

\[ m_{\text{cosmo}} = O(H_0) \quad m_{\text{local}} \gg H_0 \]

\[ \alpha^2_{\text{cosmo}} = O(1) \quad \alpha^2_{\text{local}} \to 0 \]

• Two representative mechanisms

1. Chameleon mechanism
2. Vainshtein mechanism
Chameleon mechanism

\[ S = \int d^4x \sqrt{-\tilde{g}} \left( \frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right) + S_m[A^2(\phi)g_{\mu\nu}] \quad A(\phi) = \exp \left( \alpha \frac{\phi}{M_{pl}} \right) \]

\[ \nabla^2 \phi = \partial_\phi V + \frac{\alpha}{M_{pl}} \rho e^{\alpha\phi/M_{pl}} \]

\[ V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}} \]

Khoury & Weltman astro-ph/0309300
Thin shell condition

• If the thin shell condition is satisfied, only the shell of the size $\Delta R_c$ contributes to the fifth force

$$V_{eff}(\phi) = V + \rho e^{\alpha \phi/M_{pl}}$$

$$\phi(r) = -\left( \frac{\alpha}{4\pi M_{pl}} \right) \left( \frac{3\Delta R_c}{R_c} M \exp\left(-m_\infty(r-R_c)\right) \frac{1}{r} \right) + \phi_\infty$$

$$\frac{\Delta R_c}{R_c} = \frac{(\phi_\infty - \phi_c) / M_{pl}}{6\alpha \Psi_c} \ll 1 \quad \Psi_c = \frac{GM}{R_c}$$

Screening is determined by the gravitational potential of the object

Silvestri 1103.4013

Khoury & Weltman astro-ph/0309411
Solar system constraints

- Solar system constraints

\[
\rho_\odot \sim 10 \text{ g cm}^{-3}
\]

\[
\rho_{\text{gal}} \sim 10^{-24} \text{ g cm}^{-3}
\]

\[
\frac{\Delta R_c}{R_c} = \frac{\phi_{\text{gal}} - \phi_\odot}{6\alpha M_{\text{pl}} \Psi_\odot} \sim \frac{\phi_{\text{gal}}}{6\alpha M_{\text{pl}} \Psi_\odot}
\]

The sun has a potential \( \Psi_\odot \sim 10^{-6} \)

The thin shell suppression eases the constraints \( \alpha = O(1) \)

\[
\frac{\Delta R_c}{R_c} \leq 10^{-5} \quad \Rightarrow \quad \frac{\phi_{\text{gal}}}{M_{\text{pl}}} < 5 \times 10^{-11}
\]

This is a model (potential) independent constraint
From galaxy to cosmology

• Example

\[ V = V_0 - M^4 \left( \frac{\phi}{M_{pl}} \right)^{1/2} \]

\[ \frac{\phi_{gal}}{M_{pl}} = \left( \frac{M^4}{\alpha \rho_{gal}} \right)^2 \]

\[ \rho_{gal} \sim 10^{-24} \text{ g cm}^{-3} \]

\[ \rho_{crit} \sim 10^{-29} \text{ g cm}^{-3} \]

Solar system constraints

\[ \frac{\phi_{gal}}{M_{pl}} < 10^{-11} \]

\[ \rho_{cosmo} = \left( \frac{M^4}{\alpha \rho_{crit}} \right)^2 \approx 10^{10} \frac{\phi_{gal}}{M_{pl}} \leq 10^{-1} \]

\[ M \approx 10^{-3} \text{ eV} \]

Galaxy

\[ \frac{\Delta R_c}{R_c} = \frac{\phi_{cosmo} - \phi_{gal}}{6\alpha M_{pl} \Psi_{gal}} \sim \frac{\phi_{cosmo}}{6\alpha M_{pl} \Psi_{gal}} \]

The Milky way galaxy \( \Psi_{Milk} \sim 10^{-6} \)

in order to screen the Milky way, we need \( \frac{\phi_{cosmo}}{M_{pl}} < 10^{-6} \)
Vainshtein mechanism

- Vainshtein mechanism
  - originally discussed in massive gravity
  - rediscovered in DGP brane world model
  - linear theory \( \omega_{BD} = 0 \)

\[
3 \nabla^2 \varphi = -8\pi G \rho \\
\nabla^2 \Psi = 4\pi G \rho - \frac{1}{2} \nabla^2 \varphi
\]

- even if gravity is weak, the scalar can be non-linear

\[
3 \nabla^2 \varphi + r_c^2 \left\{ \left( \nabla^2 \varphi \right)^2 - \partial_i \partial_j \varphi \partial^i \partial^j \varphi \right\} = 8\pi G a^2 \rho
\]

\[
r_c \sim m^{-1} \sim H_0^{-1}
\]
### Vainshtein radius

- Spherically symmetric solution for the scalar

\[
\frac{d\varphi}{dr} = \frac{r_g}{r^2} \Delta(r), \quad \Delta(r) = \frac{2}{3} \left( \frac{r}{r_V} \right)^3 \left( \sqrt{1 + \left( \frac{r_V}{r} \right)^3} - 1 \right)
\]

\[
r_V = \left( \frac{8r_c^2r_g}{9} \right)^{\frac{1}{3}}, \quad r_g = 2GM
\]

<table>
<thead>
<tr>
<th>(R_g)</th>
<th>(R_V)</th>
<th>(R_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4D Einstein</td>
<td>4D BD</td>
<td>Screening is determined by the curvature of the object</td>
</tr>
</tbody>
</table>

\[
\Phi = \frac{r_g}{2r} + \frac{r_g r}{2r_c^{\frac{3}{2}}}, \quad \Phi = \frac{r_g}{2r} \left( \frac{2}{3} \right), \quad \Phi = \frac{r_g}{2r} \left( \frac{4}{3} \right)
\]

\[
\Psi = -\frac{r_g}{2r} + \frac{r_g r}{2r_c^{\frac{3}{2}}}, \quad \Psi = -\frac{r_g}{2r} \left( \frac{4}{3} \right)
\]

2.95 km \hspace{1cm} 0.1 kpc \hspace{1cm} 3000 Mpc \hspace{1cm} for the Sun
Solar system constraints

- The fractional change in the gravitational potential
  \[ \varepsilon = \frac{\delta \Psi}{\Psi} \]

The anomalous perihelion precession

\[ \delta \phi = \pi r \frac{\partial}{\partial r} \left[ r^2 \frac{\partial}{\partial r} \left( \frac{\varepsilon}{r} \right) \right] \]

The vainshtein radius is shorter for a smaller object

Lunar laser ranging: the Earth-moon distance \( r_{E-M} = 4.1 \times 10^5 \) km

\[ \delta \phi = \frac{3\pi}{4} \left( \frac{r_{E-M}^3}{2GM_\oplus r_c^2} \right)^{1/2} < 2.4 \times 10^{-11} \quad \Rightarrow \quad r_c > H_0^{-1} \]
Horndeski theory

• DGP non-linear interactions

\[ L = \Box \pi \left( \partial_{\mu} \pi \right) \left( \partial^{\mu} \pi \right) - \left( \partial_{\mu} \partial_{\nu} \pi \right) \left( \partial^{\mu} \partial^{\nu} \pi \right) = 0 \]

this action has “galileon” symmetry \( \partial_{\mu} \pi \rightarrow \partial_{\mu} \pi + c_{\mu} \)

in 4D, there are only two more galileon terms that give 2\textsuperscript{nd} order equations of motion

• Covariant theory (Horndeski theory)

\[ L_3 = -G_3(\phi, X) \Box \phi \quad X = -\frac{1}{2} \left( \partial \phi \right)^2 \]

\[ L_4 = G_4(\phi, X) R + G_{4X} \left[ \left( \Box \phi \right)^2 - \left( \nabla_{\mu} \nabla_{\nu} \phi \right)^2 \right] . \]

\[ L_5 = G_5(\phi, X) G_{\mu\nu} \nabla^{\mu} \nabla^{\nu} \phi - \frac{G_{5X}}{6} \left[ \left( \Box \phi \right)^3 - 3 \left( \Box \phi \right) \left( \nabla_{\mu} \nabla_{\nu} \phi \right)^2 + 2 \left( \nabla_{\mu} \nabla_{\nu} \phi \right)^3 \right] \]
Screening mechanisms

• They look contrived but some theories naturally have these mechanisms

**f(R) gravity**

\[
F(R) = R - 2\Lambda_{\text{eff}} - \frac{R_0^{n+1}}{R^n}
\]

we expect to recover GR in the limit \( R / R_0 \gg 1 \)

In fact, in Einstein frame this is nothing but chameleon with \( V = V_0 - M^4 \left( \phi / M_{\text{pl}} \right)^{1/2} \)

**DGP gravity**

\[
S = \frac{1}{32\pi G r_c} \int d^5 x \sqrt{-g} R_{(5)} + \frac{1}{16\pi G} \int d^4 x \sqrt{-g} R + \int d^4 x \sqrt{-g} L_m
\]

we expect to recover GR in the limit \( r / r_c \gg 1 \) and this is realised by Vainshtein
V. Scalar-field models of dark energy
   A. Quintessence 20
   B. R-essence 20
   C. Tachyon field 23
   D. Phantom (ghost) field 24
   E. Dilatonic dark energy 25
   F. Chaplygin gas 26

VI. Cosmological dynamics of scalar fields in the presence of a barotropic perfect fluid 26
   A. Autonomous system of scalar-field dark energy models 27
      1. Fixed or critical points 27
      2. Stability around the fixed points 27
   B. Quintessence 28
      1. Constant $\lambda$ 28
      2. Dynamically changing $\lambda$ 30
   C. Phantom fields 30
   D. Tachyon fields 30
      1. Constant $\lambda$ 31
      2. Dynamically changing $\lambda$ 31
   E. Dilatonic ghost condensate 33

VII. Scaling solutions in a general Cosmological background 34
   A. General Lagrangian for the existence of scaling solution 34
   B. General properties of scaling solutions 35
   C. Effective potential corresponding to scaling solutions 36
      1. Ordinary scalar fields 36
      2. Tachyon 36
      3. Dilatonic ghost condensate 36
   D. Autonomous system in Einstein gravity 37

VIII. The details of quintessence 37
   A. No-goorevits constraint 37
   B. Exit from a scaling regime 38
   C. Assisted quintessence 38
   D. Particle physics models of Quintessence 39
      1. Supergravity inspired models 39
      2. Pseudo-Nambu-Goldstone models 42
   E. Quintessential inflation 43

IX. Coupled dark energy 44
   A. Critical points for coupled Quintessence 45
   B. Stability of critical points 45
      1. Ordinary field ($\epsilon = +1$) 46
      2. Phantom field ($\epsilon = -1$) 47
   C. General properties of fixed points 48
   D. Can we have two scaling regimes ? 48
   E. Varying mass neutrino scenario 50
   F. Dark energy through boson-bulk energy exchange 50

X. Dark energy and varying alpha 51
   A. Varying alpha from quintessence 51
   B. Varying alpha from tachyon fields 52
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Alternative Theories of Gravity with Extra Fields</td>
<td>49</td>
</tr>
<tr>
<td>3.1</td>
<td>Scalar-Tensor Theories</td>
<td>49</td>
</tr>
<tr>
<td>3.1.1</td>
<td>Action, field equations, and conformal transformations</td>
<td>49</td>
</tr>
<tr>
<td>3.1.2</td>
<td>Brans-Dicke theory</td>
<td>52</td>
</tr>
<tr>
<td>3.1.3</td>
<td>General scalar-tensor theories</td>
<td>52</td>
</tr>
<tr>
<td>3.1.4</td>
<td>The chancellor mechanism</td>
<td>66</td>
</tr>
<tr>
<td>3.2</td>
<td>Einstein-Scalar Theories</td>
<td>68</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Modified Newtonian dynamics</td>
<td>69</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Action and field equations</td>
<td>69</td>
</tr>
<tr>
<td>3.2.3</td>
<td>FLRW solutions</td>
<td>70</td>
</tr>
<tr>
<td>3.2.4</td>
<td>Cosmological perturbations</td>
<td>70</td>
</tr>
<tr>
<td>3.2.5</td>
<td>Observations and constraints</td>
<td>73</td>
</tr>
<tr>
<td>3.3</td>
<td>Bimetric Theories</td>
<td>75</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Rosen’s theory, and non-dynamical metrics</td>
<td>76</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Drummond’s theory</td>
<td>77</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Massive gravity</td>
<td>77</td>
</tr>
<tr>
<td>3.3.4</td>
<td>Bigravity</td>
<td>79</td>
</tr>
<tr>
<td>3.3.5</td>
<td>Bimetric MOND</td>
<td>81</td>
</tr>
<tr>
<td>3.4</td>
<td>Tensor-Vector-Scalar Theories</td>
<td>81</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Actions and field equations</td>
<td>82</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Newtonian and MOND limits</td>
<td>84</td>
</tr>
<tr>
<td>3.4.3</td>
<td>Homogeneous and isotropic cosmology</td>
<td>86</td>
</tr>
<tr>
<td>3.4.4</td>
<td>Cosmological perturbation theory</td>
<td>91</td>
</tr>
<tr>
<td>3.4.5</td>
<td>Cosmological observations and constraints</td>
<td>93</td>
</tr>
<tr>
<td>3.5</td>
<td>Other Theories</td>
<td>96</td>
</tr>
<tr>
<td>3.5.1</td>
<td>The Einstein-Cartan-Sciama-Kibble Theory</td>
<td>96</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Scalar-Tensor-Vector Theory</td>
<td>99</td>
</tr>
<tr>
<td>4</td>
<td>Higher Derivative and Non-Local Theories of Gravity</td>
<td>101</td>
</tr>
<tr>
<td>4.1</td>
<td>(\mathcal{K}) Theories</td>
<td>104</td>
</tr>
<tr>
<td>4.1.1</td>
<td>Action, field equations and transformations</td>
<td>104</td>
</tr>
<tr>
<td>4.1.2</td>
<td>Weak-field limit</td>
<td>106</td>
</tr>
<tr>
<td>4.1.3</td>
<td>Exact solutions, and general behaviour</td>
<td>111</td>
</tr>
<tr>
<td>4.1.4</td>
<td>Cosmology</td>
<td>114</td>
</tr>
<tr>
<td>4.1.5</td>
<td>Stability issues</td>
<td>120</td>
</tr>
<tr>
<td>4.2</td>
<td>General combinations of Ricci and Riemann curvature</td>
<td>123</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Action and field equations</td>
<td>123</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Weak-field limit</td>
<td>125</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Exact solutions, and general behaviour</td>
<td>126</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Physical cosmology and dark energy</td>
<td>128</td>
</tr>
<tr>
<td>4.2.5</td>
<td>Other topics</td>
<td>131</td>
</tr>
<tr>
<td>4.3</td>
<td>Horava-Wisniewsky Gravity</td>
<td>137</td>
</tr>
<tr>
<td>4.3.1</td>
<td>The projectable theory</td>
<td>141</td>
</tr>
<tr>
<td>4.3.2</td>
<td>The non-projectable theory</td>
<td>144</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Aspects of Horava-Wisniewsky cosmology</td>
<td>146</td>
</tr>
<tr>
<td>4.3.4</td>
<td>The (\mathcal{K})CDM model</td>
<td>148</td>
</tr>
<tr>
<td>4.3.5</td>
<td>HMT-da Silva theory</td>
<td>148</td>
</tr>
<tr>
<td>4.4</td>
<td>Galileons</td>
<td>150</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Galileon modification of gravity</td>
<td>151</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Courant galileon</td>
<td>157</td>
</tr>
<tr>
<td>4.4.3</td>
<td>DHF galileon</td>
<td>158</td>
</tr>
<tr>
<td>4.4.4</td>
<td>Galileon cosmology</td>
<td>160</td>
</tr>
<tr>
<td>4.4.5</td>
<td>Multi-galileons</td>
<td>161</td>
</tr>
<tr>
<td>4.5</td>
<td>Other Theories</td>
<td>164</td>
</tr>
<tr>
<td>4.5.1</td>
<td>Ghost condensates</td>
<td>164</td>
</tr>
<tr>
<td>4.5.2</td>
<td>Non-metric gravity</td>
<td>166</td>
</tr>
<tr>
<td>4.5.3</td>
<td>Dark energy from curvature corrections</td>
<td>169</td>
</tr>
<tr>
<td>5</td>
<td>Higher Dimensional Theories of Gravity</td>
<td>172</td>
</tr>
<tr>
<td>5.1</td>
<td>Kaluza-Klein Theories of Gravity</td>
<td>172</td>
</tr>
<tr>
<td>5.1.1</td>
<td>Kaluza-Klein compactifications</td>
<td>173</td>
</tr>
<tr>
<td>5.1.2</td>
<td>Kaluza-Klein cosmology</td>
<td>174</td>
</tr>
<tr>
<td>5.2</td>
<td>The Brane World Paradigm</td>
<td>179</td>
</tr>
<tr>
<td>5.2.1</td>
<td>The ADD model</td>
<td>189</td>
</tr>
<tr>
<td>5.3</td>
<td>Randall-Sundrum Gravity</td>
<td>181</td>
</tr>
<tr>
<td>5.3.1</td>
<td>The RS1 model</td>
<td>182</td>
</tr>
<tr>
<td>5.3.2</td>
<td>The RS2 model</td>
<td>184</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Other RS-like models</td>
<td>185</td>
</tr>
<tr>
<td>5.3.4</td>
<td>Action and equations of motion</td>
<td>188</td>
</tr>
<tr>
<td>5.3.5</td>
<td>Linear perturbations in RS1 and RS2</td>
<td>189</td>
</tr>
<tr>
<td>5.4</td>
<td>Brane Cosmology</td>
<td>195</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Brane based formalism - covariant formulation</td>
<td>197</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Bulk based formalism - moving branes in a static bulk</td>
<td>199</td>
</tr>
<tr>
<td>5.4.3</td>
<td>Cosmological perturbations</td>
<td>209</td>
</tr>
<tr>
<td>5.4.5</td>
<td>Linear perturbations on the normal branch</td>
<td>209</td>
</tr>
<tr>
<td>5.4.6</td>
<td>Linear perturbations (and ghosts) on the self-accelerating branch</td>
<td>211</td>
</tr>
<tr>
<td>5.4.7</td>
<td>From strong coupling to the Veneziano mechanism</td>
<td>214</td>
</tr>
<tr>
<td>5.4.8</td>
<td>DGP cosmology</td>
<td>221</td>
</tr>
<tr>
<td>5.6</td>
<td>Higher Co-Dimension Braneworlds</td>
<td>228</td>
</tr>
<tr>
<td>5.6.1</td>
<td>Cascading gravity</td>
<td>230</td>
</tr>
<tr>
<td>5.6.2</td>
<td>Degradation</td>
<td>235</td>
</tr>
<tr>
<td>5.7</td>
<td>Einstein Gauss-Bonnet Gravity</td>
<td>236</td>
</tr>
<tr>
<td>5.7.1</td>
<td>Action, equations of motion, and vacua</td>
<td>237</td>
</tr>
<tr>
<td>5.7.2</td>
<td>Kaluza-Klein reduction of EGB gravity</td>
<td>240</td>
</tr>
<tr>
<td>5.7.3</td>
<td>Co-dimension one branes in EGB gravity</td>
<td>240</td>
</tr>
<tr>
<td>5.7.4</td>
<td>Co-dimension two branes in EGB gravity</td>
<td>244</td>
</tr>
</tbody>
</table>
Summary

• DE/MG models
  There are many models but we still do not have a compelling alternative to LCDM

Observation may give us a clue:
  Is it the cosmological constant or a light degree of freedom?
  Does it cluster or couple to matter?

  cf. CMB (before WMAP)
• Lecture 3  Structure formation and observational tests
• Lecture 4  Observational tests and non-linear structure formation