

# Models of dark energy and modified gravity

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# Quintessence

- Scalar field

We now know a scale field exists (i.e. Higgs)

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + S_\phi$$

$$S_\phi = \int d^4x \left[ -\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right]$$

energy momentum tensor

$$T_{\mu\nu}^\phi = (\partial_\mu \phi)(\partial_\nu \phi) - g_{\mu\nu} \left[ \frac{1}{2} g^{\alpha\beta} (\partial_\alpha \phi)(\partial_\beta \phi) + V(\phi) \right]$$

$$T_{\mu\nu}^\phi = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_\phi)}{\delta g^{\mu\nu}}$$

scalar field equation

$$\frac{\delta S_\phi}{\delta \phi} = 0 \quad \nabla^\mu \nabla_\mu \phi - V'(\phi) = 0 \quad \longleftrightarrow \quad \nabla^\mu T_{\mu\nu}^\phi = 0$$

# Background

- Energy density and pressure

$$\rho_\phi = -T_0^0 = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad \phi = \phi(t)$$

$$P_\phi = \frac{1}{3} T_i^i = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

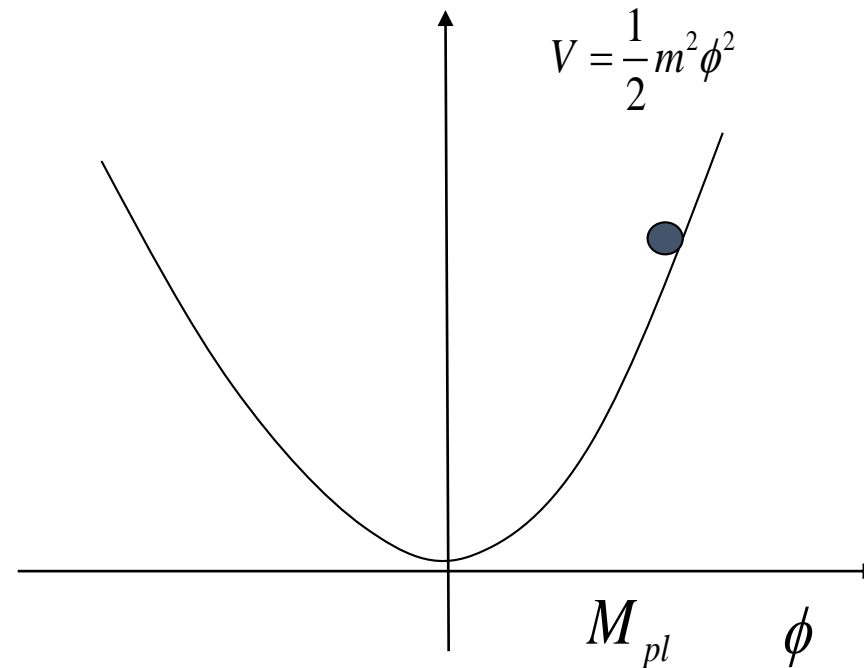
- Scalar field equation

$$\ddot{\phi} + \boxed{3H\dot{\phi}} + V'(\phi) = 0 \quad \longleftrightarrow \quad \dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$$

friction

Acceleration

$$\dot{\phi}^2 \ll V(\phi) \quad w_\phi = \frac{P_\phi}{\rho_\phi} \approx -1$$



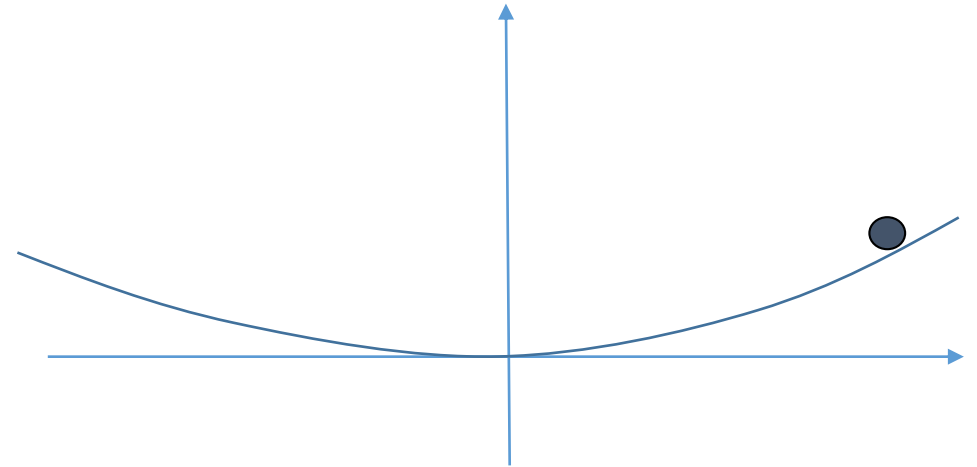
# Equation of state

- Equation of state

$$1 + w_\phi = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{V'^2}{9H^2(1 + \xi_\phi)^2 \rho_\phi} > 0, \quad \xi_\phi = \frac{\ddot{\phi}}{3H\dot{\phi}}$$

For  $w_\phi \approx -1$ ,  $\xi_\phi \ll 1$

$$1 + w_\phi \approx \frac{2}{3} \varepsilon_\phi \Omega_\phi(a) \quad \varepsilon_\phi = \frac{1}{2\kappa^2} \left( \frac{V'}{V} \right)^2, \quad \Omega_\phi = \frac{\rho_\phi}{\rho_\phi + \rho_m} \quad \kappa^2 = 8\pi G = \frac{1}{M_{pl}^2}$$



Analogous to slow-roll approximations for inflation but the scalar field does not completely dominates the energy density

# Quintessence potentials

- Freezing models

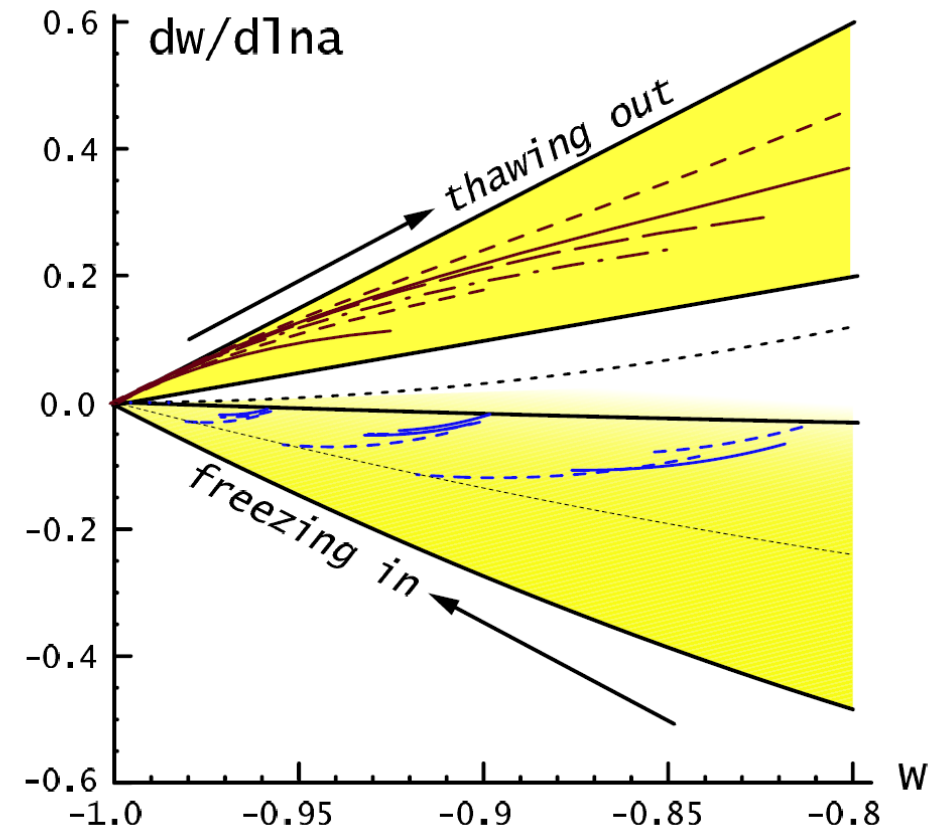
$$V(\phi) = M^{4+n} \phi^{-n}$$

The scalar field dynamics freezes at late time and approaches  $w_\phi \approx -1$

- Thawing models

$$V(\phi) = M^4 \cos^2\left(\frac{\phi}{f}\right)$$

The scalar field is initially in the regions of  $w_\phi \approx -1$  and later starts to roll down the potential



Linder & Caldwell astro-ph/0505494

# Scaling solution

- Tracker solution

For some potentials, the energy density of the scalar field tracks that of matter/radiation

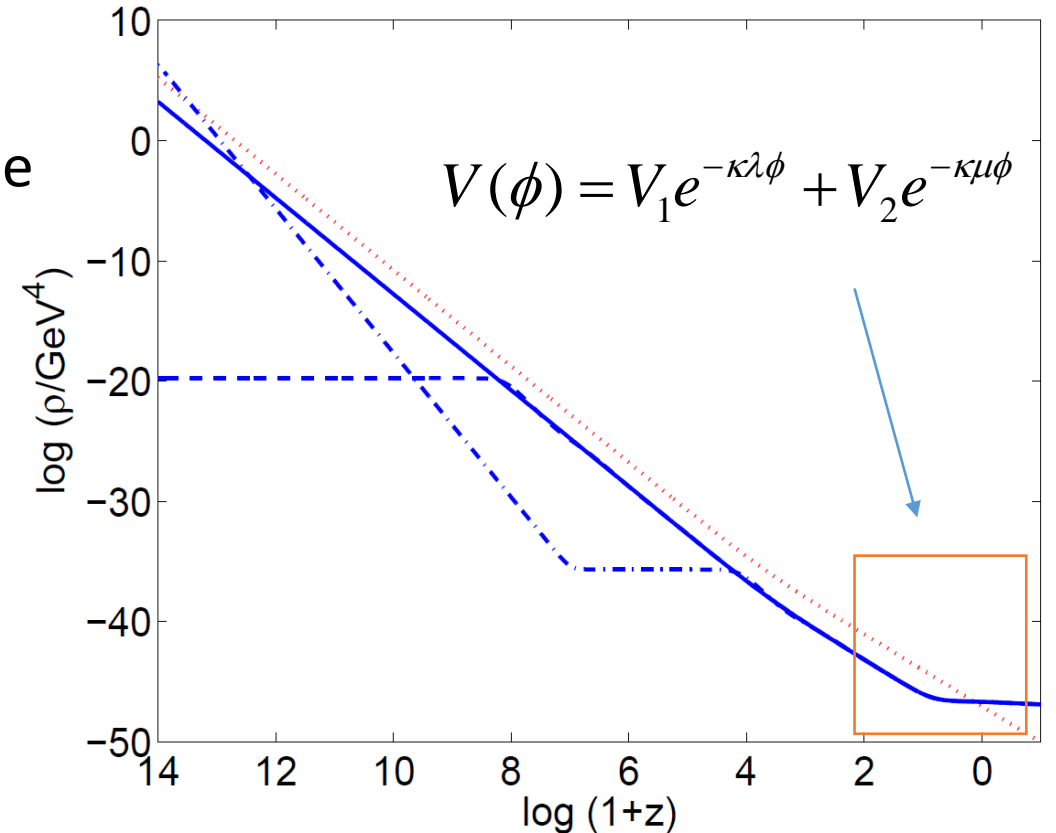
$$V(\phi) = V_0 e^{-\kappa\lambda\phi}, \quad \kappa^2 = 8\pi G = \frac{1}{M_{pl}^2}$$

If  $\lambda^2 > 3(1 + w_M)$

there is an attractor where

$$w_\phi = w_M, \quad \Omega_\phi = \frac{3(1 + w_M)}{\lambda^2}$$

The late time solution is insensitive to the initial density of the scalar field (however we need  $\lambda^2 < 2$  to have an accelerating solution)



Barreiro et.al. astro-ph/9910214

# Fundamental issues

- Consider a massive scalar field

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \quad \varepsilon_\phi = \frac{1}{2\kappa^2} \left( \frac{V'}{V} \right)^2 \ll 1 \quad \rightarrow \quad \phi > M_{pl} = 10^{18} \text{ GeV}$$

$$V(\phi_0) = \rho_{DE} = 10^{-48} \text{ GeV}^4 \quad \rightarrow \quad m = \left( \frac{\rho_{DE}}{\phi^2} \right)^{\frac{1}{2}} = 10^{-42} \text{ GeV} \approx H_0$$

Correction to the potential

$$V(\phi) = \frac{1}{4} \lambda \phi^4 < \rho_{DE} \quad \rightarrow \quad \lambda < 10^{-120}$$

these small numbers are very difficult to protect against quantum corrections

# Symmetry

- Let's assume there is an exact symmetry with a massless scalar field. Introducing a symmetry breaking term, quantum corrections generate a potential (cf. QCD axion)

$$V(\phi) = \mu^4 \left( 1 + \cos\left(\frac{\phi}{f}\right) \right), \quad m_\phi \approx \frac{\mu^2}{f}$$

$$f \approx M_{pl}, \quad \mu \approx (H_0 M_{pl})^{1/2} \approx \rho_{DE}^{1/4} \approx (10^{-3} \text{ eV}), \quad m_\phi \approx H_0$$

small  $\mu$  is technically natural as (we assumed that) there is an exact symmetry in the limit  $\mu \rightarrow 0$  thus quantum corrections are  $O(\mu^4)$  (Pseudo Nambu-Goldstone bosons)



# Modified Gravity

- Lovelock theorem

4D theory of metric and the equations of motion is given by

$$S_{\mu\nu}(g) = \kappa^2 T_{\mu\nu}$$

we require  $S_{\mu\nu}(g)$  is

1. a tensor made from the metric and its first and second derivatives
2. symmetric
3. divergence free

➔  $S_{\mu\nu}(g) = a G_{\mu\nu} + b g_{\mu\nu}$

# Beyond Einstein

- Lovelock theorem indicates that if you modify Einstein theory then you must do one or more of the following
  - Consider other fields (scalars, vectors, etc)
  - Accept higher derivatives
  - Work in a space with more than four dimensions
  - Non-local, ...

# Higher derivative theories

- Ostrogradski ghost

Theories with higher derivatives  $L(g, \dot{g}, \ddot{g})$

$$\frac{d^2}{dt^2} \left( \frac{\partial L}{\partial \ddot{g}} \right) - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{g}} \right) + \left( \frac{\partial L}{\partial g} \right) = 0$$

We treat  $\dot{g}$  as a new variable and  $\ddot{g}$  as a new velocity (we need four initial conditions)

If the velocity  $\dot{g}$  can be expressed in terms of momenta  $\partial L / \partial \dot{g}$

(non-degeneracy condition), it can be shown that the Hamiltonian is not

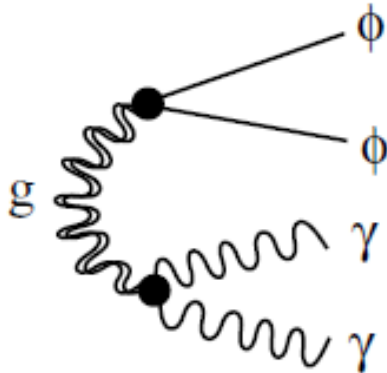
bounded from below [Woodard astro-ph/0601672](https://arxiv.org/abs/astro-ph/0601672)

# Ghost

- If the kinetic term has a wrong sign  
The Hamiltonian is not positive definite

$$S_\phi = \int d^4x \left[ \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right]$$
$$\rho_\phi = -\frac{1}{2} \dot{\phi}^2 + V(\phi) \quad \text{“Phantom DE”}$$

quantum mechanically, particles can be created from vacuum without costing any energy → instability of vacuum



there is no time scale for  
instability in Lorentz invariant theory  
the decay is instantaneous

Unless the mass of ghost is above the cut-off scale of the theory,  
ghost poses a serious consistency problem

# f(R) gravity

- Generalise the Einstein-Hilbert action

$$S = \int d^4x \sqrt{-g} F(R) + S_m[g], \quad F(R) = R + f(R)$$

- The equations of motion is fourth order

$$F'(R)R_{\mu\nu} - \frac{1}{2}F(R)g_{\mu\nu} - (\nabla_\mu \nabla_\nu + g_{\mu\nu} \square) F'(R) = \kappa^2 T_{\mu\nu}$$

but this theory violates the assumption of Ostrogradski theorem (i.e. non-degeneracy)

In fact the theory can be rewritten as scalar-tensor gravity

$$S = \int d^4x \sqrt{-g} (F(\phi) + (R - \phi)F'(\phi)) \quad \frac{\delta S}{\delta \phi} = (R - \phi)F''(\phi) = 0 \quad \Rightarrow \quad \phi = R \quad (F''(R) \neq 0)$$

$$S = \int d^4x (\psi R - V(\psi)) \quad \psi = F'(\phi), \quad V = RF' - F$$

# Brans-Dicke gravity

- Brans-Dicke (BD) gravity (Jordan frame)

$$S = \int d^4x \sqrt{-g} \left( \psi R - \frac{\omega_{BD}}{\psi} (\nabla \psi)^2 + V(\psi) \right) + S_M[g_{\mu\nu}]$$

f(R) gravity  $\omega_{BD} = 0$

- Einstein frame

Conformal transformation  $\tilde{g}_{\mu\nu} = \frac{2\psi}{M_{pl}^2} g_{\mu\nu}, \quad \frac{\phi}{M_{pl}} = \frac{1}{2\alpha} \ln \left( \frac{2\psi}{M_{pl}^2} \right), \quad 2\omega_{BD} + 3 = 1 / 2\alpha^2$

$$S = \int d^4x \sqrt{-\tilde{g}} \left( \frac{M_{pl}^2}{2} \tilde{R} - \frac{1}{2} (\tilde{\nabla} \phi)^2 - \tilde{V}(\phi) \right) + S_M[A^2(\phi) \tilde{g}_{\mu\nu}] \quad A(\phi) = \exp \left( \frac{\alpha \phi}{M_{pl}} \right)$$

the scalar field is coupled to matter

# Dvali-Gadadze-Porrati (DGP) braneworld model

- 5D braneworld model

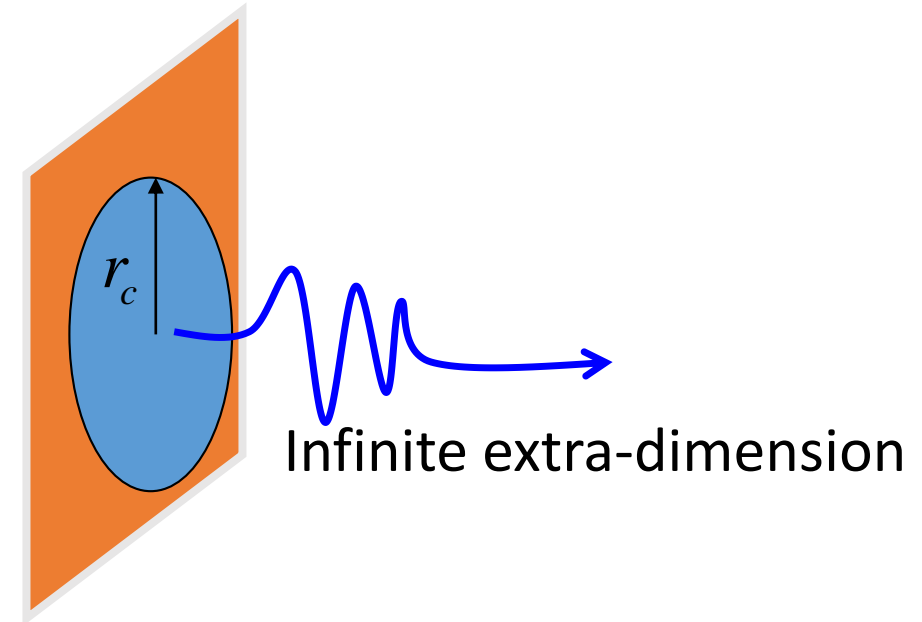
Standard model particles are confined to a 4D brane in a 5D bulk spacetime

$$S = \frac{1}{32\pi G r_c} \int d^5x \sqrt{-g_{(5)}} R_{(5)} + \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} L_m$$

cross over scale  $r_c$

$$\pm \frac{H}{r_c} = H^2 - \frac{8\pi G}{3} \rho$$

in the + branch, the expansion of the universe accelerates without the cosmological constant (self-accelerating solution)



# Fundamental issues

- Theoretical consistency

modified gravity models often suffer from instabilities

(cf. the self-accelerating solution in DGP model suffers from ghost instabilities)

- Small numbers

cf. viable  $f(R)$  models

$$F(R) = R - 2\Lambda_{eff} - \frac{R_0^{n+1}}{R^n}, \quad \Lambda_{eff} \approx R_0 \approx H_0^2$$

DGP models  $r_c \approx H_0^{-1}$



# Ultra-light scalar field

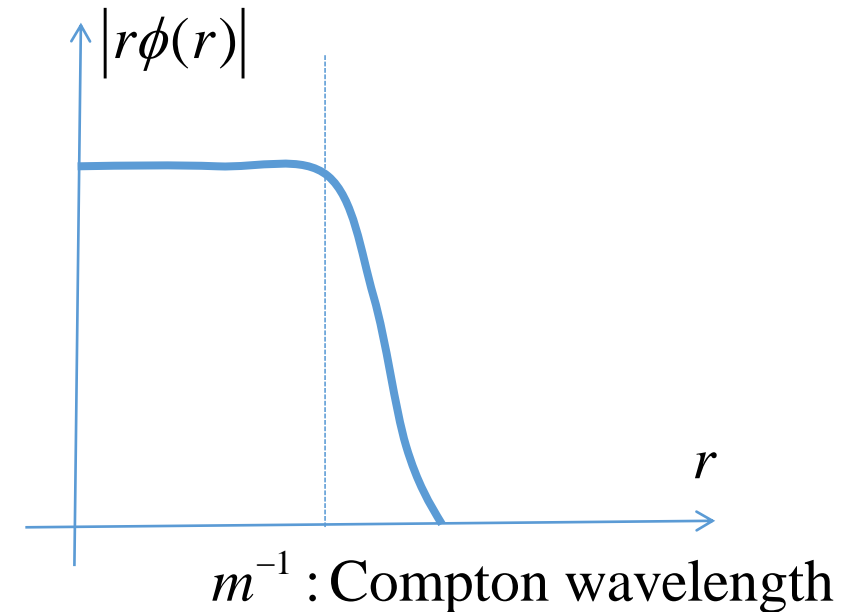
- In most of DE/MG models, there exists a scalar field with tiny mass

$$S = \int d^4x \left[ -\frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 \right] - \frac{\alpha}{M_{pl}} \int d^4x \phi \rho$$

static source

$$(\nabla^2 - m^2)\phi = \frac{\alpha}{M_{pl}} \rho, \quad \frac{\phi}{M_{pl}} = -\left( \frac{\alpha}{4\pi M_{pl}^2} \right) \frac{M}{r} \exp(-mr)$$

→ 
$$\begin{cases} \phi = -\left( \frac{\alpha}{4\pi M_{pl}^2} \right) \frac{M}{r}, & r < m^{-1} \\ \phi \rightarrow 0, & r > m^{-1} \end{cases}$$



if  $m^{-1} \approx H_0^{-1}$  the scalar field mediates a long range force

# Fifth force

- Due to the coupling between the scalar and matter, the geodesic equation is modified as

$$\ddot{a} = -\nabla\Psi - \frac{\alpha}{M_{pl}} \nabla\phi = F_G + \boxed{F_5} \quad \text{Fifth force}$$

$$F_5 = \frac{2\alpha^2}{8\pi M_{pl}^2} \frac{M}{r^2} \quad \text{for } r < m^{-1}$$

$$F_G = \frac{1}{8\pi M_{pl}^2} \frac{M}{r^2}$$

Fifth force is strongly constrained in the solar system

$$\alpha^2 < 10^{-5}, \quad w_{BD} > 40,000 \quad (3 + 2w_{BD}) = 1 / 2\alpha^2$$

# Way out

- Quintessence

the coupling to matter is assumed to be zero  $\alpha^2 = 0$

- Violation of strong equivalence principle

the solar system constraints come from coupling to baryons  $\alpha_{baryon}^2 = 10^{-5}$

the coupling to cold dark matter can be large  $\alpha_{cdm}^2 = O(1)$

interacting dark energy

$$S = \int d^4x \sqrt{-\tilde{g}} \left( \frac{M_{pl}^2}{2} \tilde{R} - \frac{1}{2} (\tilde{\nabla} \phi)^2 - \tilde{V}(\phi) \right) + S_{cdm} [A^2(\phi) \tilde{g}_{\mu\nu}] + S_{baryon} [\tilde{g}_{\mu\nu}]$$

# Screening mechanism

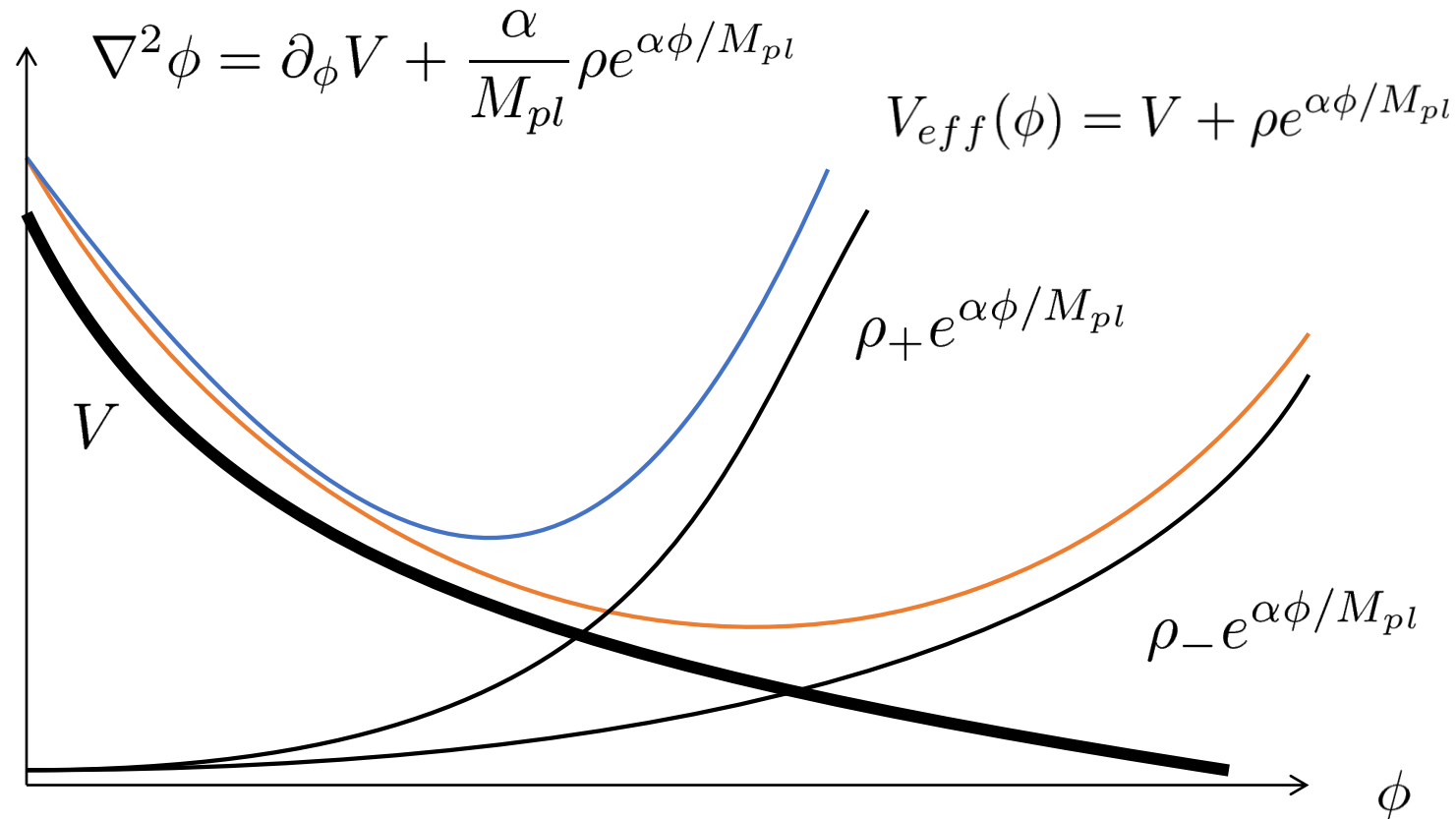
- The coupling constant or mass need to be scale dependent

$$\begin{array}{ll} m|_{\text{cosmo}} = O(H_0) & m|_{\text{local}} \gg H_0 \\ \alpha^2|_{\text{cosmo}} = O(1) & \alpha^2|_{\text{local}} \rightarrow 0 \end{array}$$

- Two representative mechanisms
  1. Chameleon mechanism
  2. Vainshtein mechanism

# Chameleon mechanism

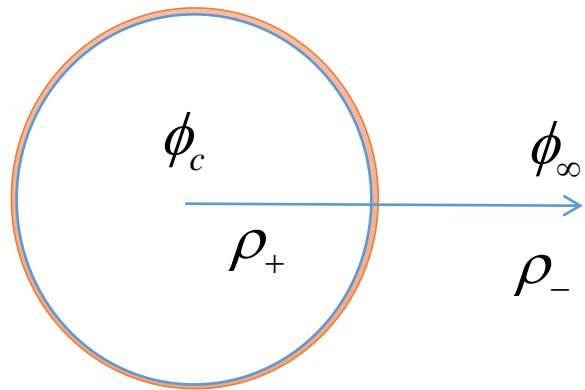
$$S = \int d^4x \sqrt{-\tilde{g}} \left( \frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right) + S_m[A^2(\phi)g_{\mu\nu}] \quad A(\phi) = \exp\left(\alpha \frac{\phi}{M_{pl}}\right)$$



# Thin shell condition

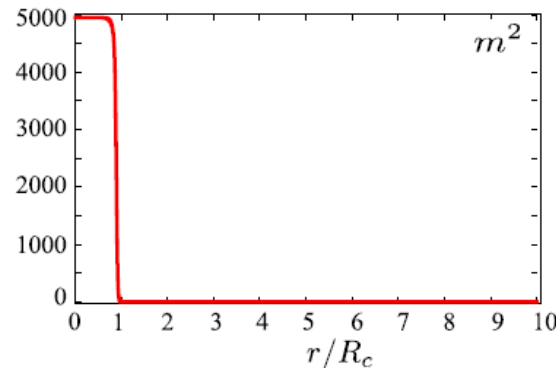
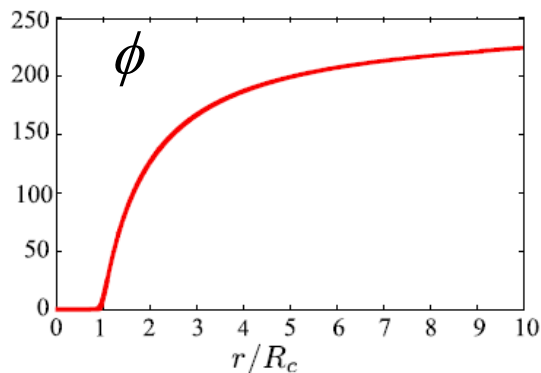
- If the thin shell condition is satisfied, only the shell of the size  $\Delta R_c$  contributes to the fifth force [Khoury & Weltman astro-ph/0309411](#)

$$V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}}$$



$$\phi(r) = -\left(\frac{\alpha}{4\pi M_{pl}}\right)\left(\frac{3\Delta R_c}{R_c}\right)\frac{M \exp(-m_\infty(r - R_c))}{r} + \phi_\infty$$

$$\frac{\Delta R_c}{R_c} = \frac{(\phi_\infty - \phi_c) / M_{pl}}{6\alpha\Psi_c} \ll 1 \quad \Psi_c = \frac{GM}{R_c}$$



*Screening is determined by the gravitational potential of the object*

# Solar system constraints

- Solar system constraints

$$\rho_{\odot} \sim 10 \text{ g cm}^{-3}$$

$$\rho_{gal} \sim 10^{-24} \text{ g cm}^{-3}$$

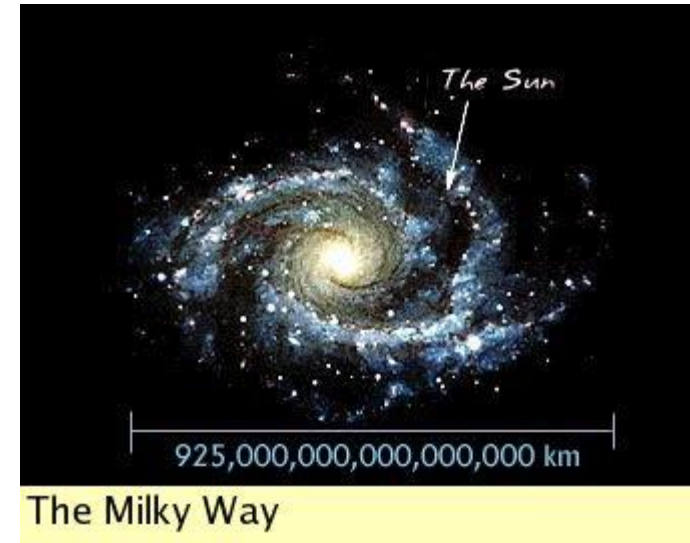
$$\frac{\Delta R_c}{R_c} = \frac{\phi_{gal} - \phi_{\odot}}{6\alpha M_{pl} \Psi_{\odot}} \sim \frac{\phi_{gal}}{6\alpha M_{pl} \Psi_{\odot}}$$

The sun has a potential  $\Psi_{\odot} \sim 10^{-6}$

The thin shell suppression eases the constraints  $\alpha = O(1)$

$$\frac{\Delta R_c}{R_c} \leq 10^{-5} \quad \rightarrow \quad \frac{\phi_{gal}}{M_{pl}} < 5 \times 10^{-11}$$

This is a model (potential) independent constraint



# From galaxy to cosmology

- Example

$$V = V_0 - M^4 \left( \phi / M_{pl} \right)^{1/2} \quad \frac{\phi_{gal}}{M_{pl}} = \left( \frac{M^4}{\alpha \rho_{gal}} \right)^2 \quad \rho_{gal} \sim 10^{-24} \text{ g cm}^{-3}$$

$$\rho_{crit} \sim 10^{-29} \text{ g cm}^{-3}$$

## Solar system constraints

$$\frac{\phi_{gal}}{M_{pl}} < 10^{-11} \quad \frac{\phi_{cosmo}}{M_{pl}} = \left( \frac{M^4}{\alpha \rho_{crit}} \right)^2 \approx 10^{10} \frac{\phi_{gal}}{M_{pl}} \leq 10^{-1} \quad M \approx 10^{-3} \text{ eV}$$

Galaxy

$$\frac{\Delta R_c}{R_c} = \frac{\phi_{cosmo} - \phi_{gal}}{6\alpha M_{pl} \Psi_{gal}} \sim \frac{\phi_{cosmo}}{6\alpha M_{pl} \Psi_{gal}}$$

The Milky way galaxy  $\Psi_{Milky} \sim 10^{-6}$

in order to screen the Milky way, we need  $\frac{\phi_{cosmo}}{M_{pl}} < 10^{-6}$



# Vainshtein mechanism

- Vainshtein mechanism

originally discussed in massive gravity

rediscovered in DGP brane world model

linear theory  $\omega_{BD} = 0$

$$3\nabla^2\varphi = -8\pi G\rho$$

$$\nabla^2\Psi = 4\pi G\rho - \frac{1}{2}\nabla^2\varphi$$

even if gravity is weak, the scalar can be non-linear  $r_c \sim m^{-1} \sim H_0^{-1}$

$$3\nabla^2\varphi + r_c^2 \left\{ (\nabla^2\varphi)^2 - \partial_i\partial_j\varphi \partial^i\partial^j\varphi \right\} = 8\pi G a^2 \rho$$

# Vainshtein radius

- Spherically symmetric solution for the scalar

$$\frac{d\phi}{dr} = \frac{r_g}{r^2} \Delta(r), \quad \Delta(r) = \frac{2}{3} \left( \frac{r}{r_V} \right)^3 \left( \sqrt{1 + \left( \frac{r_V}{r} \right)^3} - 1 \right) \quad r_V = \left( \frac{8r_c^2 r_g}{9} \right)^{\frac{1}{3}}, \quad r_g = 2GM$$

$r_g$

$r_V$

$r_c$



Screening is determined by the curvature of the object

$$\Phi = \frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}},$$

$$\Psi = -\frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}}$$

$$\Phi = \frac{r_g}{2r} \left( \frac{2}{3} \right),$$

$$\Psi = -\frac{r_g}{2r} \left( \frac{4}{3} \right)$$

$$\left( \frac{r_V}{r} \right)^{-3/2} \propto \left( \frac{GM}{r^3} r_c^2 \right)^{-3/2}$$

2.95km

0.1 kpc

3000Mpc for the Sun

# Solar system constraints

- The fractional change in the gravitational potential  $\varepsilon = \frac{\delta\Psi}{\Psi}$   
The anomalous perihelion precession

$$\delta\phi = \pi r \frac{\partial}{\partial r} \left[ r^2 \frac{\partial}{\partial r} \left( \frac{\varepsilon}{r} \right) \right]$$

The vainshtein radius is shorter for a smaller object

Lunar laser ranging: the Earth-moon distance  $r_{E-M} = 4.1 \times 10^5 \text{ km}$

$$\delta\phi = \frac{3\pi}{4} \left( \frac{r_{E-M}^3}{2GM_{\oplus} r_c^2} \right)^{1/2} < 2.4 \times 10^{-11} \quad \Rightarrow \quad r_c > H_0^{-1}$$

# Horndeski theory

- DGP non-linear interactions

$$L = \square\pi (\partial_\mu\pi)(\partial^\mu\pi) \quad (\square\pi)^2 - (\partial_\mu\partial_\nu\pi)(\partial^\mu\partial^\nu\pi) = 0$$

this action has “galileon” symmetry  $\partial_\mu\pi \rightarrow \partial_\mu\pi + c_\mu$

in 4D, there are only two more galileon terms that give 2<sup>nd</sup> order equations of motion

- Covariant theory (Horndeski theory)

$$\mathcal{L}_3 = -G_3(\phi, X)\square\phi \quad X = -\frac{1}{2}(\partial\phi)^2$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X} \left[ (\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2 \right],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6} \left[ (\square\phi)^3 - 3(\square\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3 \right]$$

# Screening mechanisms

- They look contrived but some theories naturally have these mechanisms

## **f(R) gravity**

$$F(R) = R - 2\Lambda_{\text{eff}} - \frac{R_0^{n+1}}{R^n}$$

we expect to recover GR in the limit  $R / R_0 \gg 1$

In fact, in Einstein frame this is nothing but chameleon with  $V = V_0 - M^4 \left( \phi / M_{pl} \right)^{1/2}$

## **DGP gravity**

$$S = \frac{1}{32\pi G r_c} \int d^5 x \sqrt{-g_{(5)}} R_{(5)} + \frac{1}{16\pi G} \int d^4 x \sqrt{-g} R + \int d^4 x \sqrt{-g} L_m$$

we expect to recover GR in the limit  $r / r_c \gg 1$  and this is realised by Vainshtein

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**Dynamics of dark energy**  
**hep-th/0603057**

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# Modified Gravity and Cosmology

## arXiv:1106.2476

# Summary

- DE/MG models

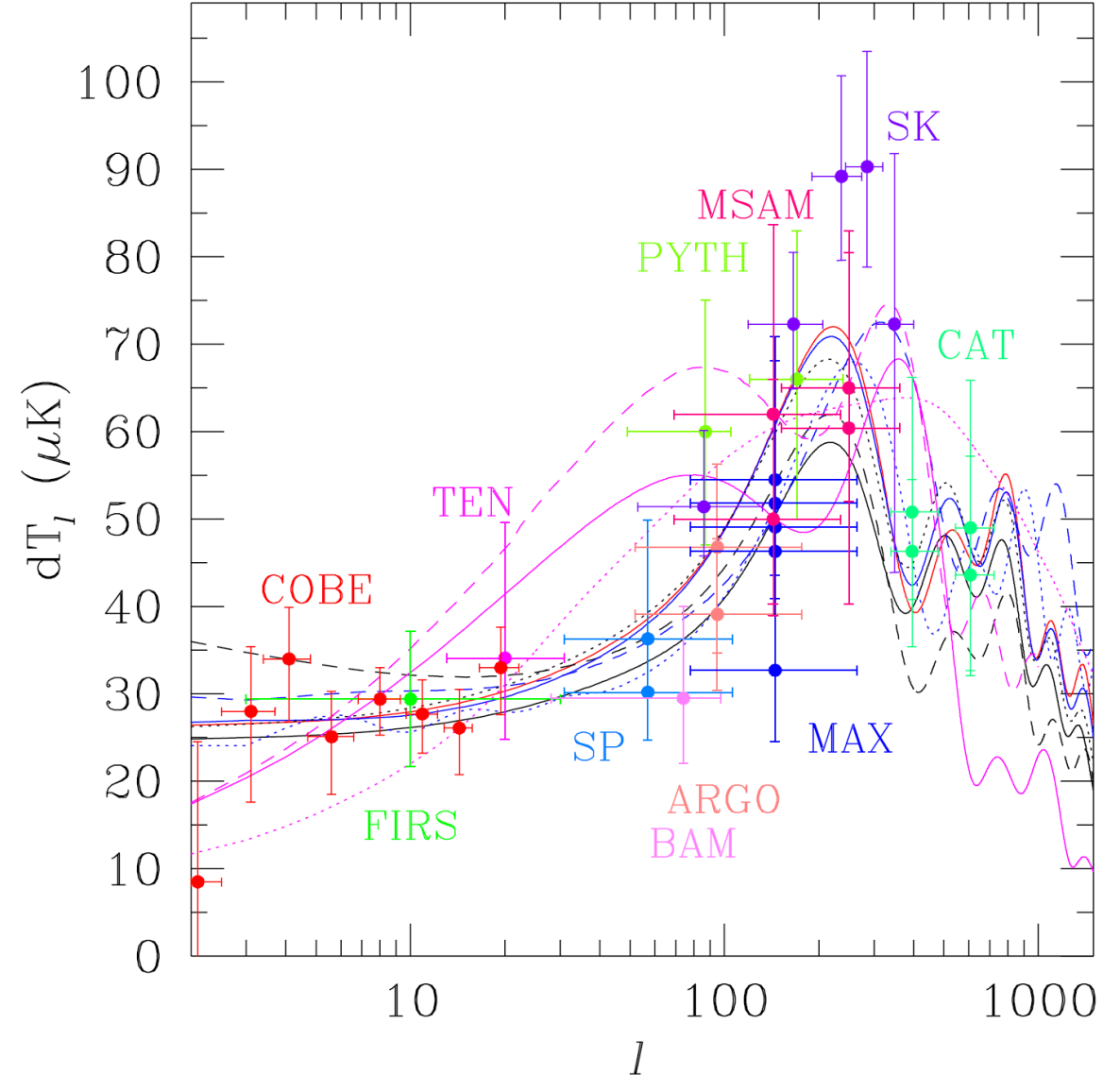
There are many models but we still do not have a compelling alternative to LCDM

Observation may give us a clue:

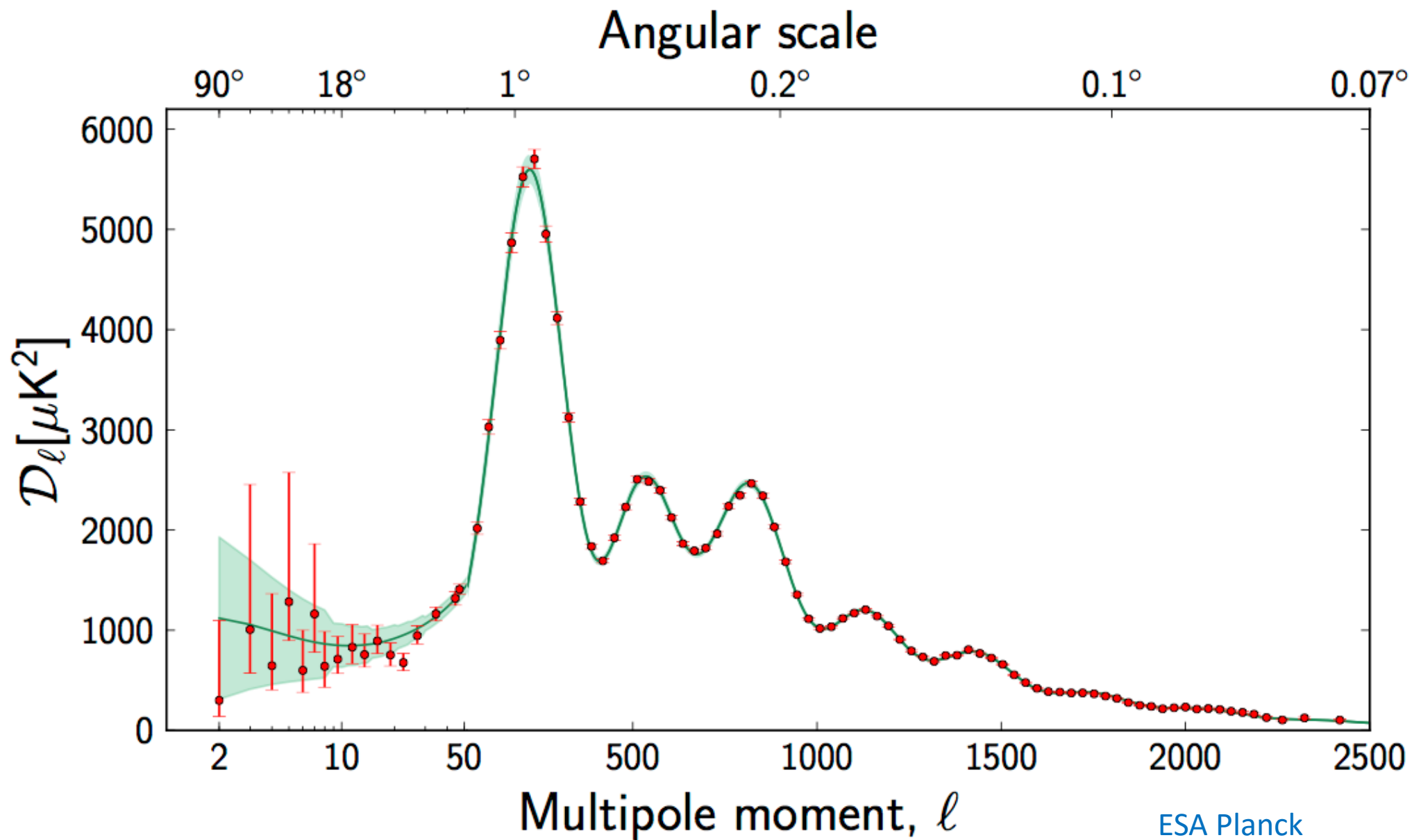
Is it the cosmological constant or a light degree of freedom?

Does it cluster or couple to matter?

cf. CMB (before WMAP)







- Lecture 3 Structure formation and observational tests
- Lecture 4 Observational tests and non-linear structure formation