Models of dark energy and modified gravity

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Quintessence

- Scalar field

  We now know a scale field exists (i.e. Higgs)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_\phi$$

$$S_\phi = \int d^4x\left[-\frac{1}{2}g^{\mu\nu}\left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) - V(\phi)\right]$$

energy momentum tensor

$$T^\phi_{\mu\nu} = \left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\left(\partial_\alpha\phi\right)\left(\partial_\beta\phi\right) + V(\phi)\right] \qquad T^\phi_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_\phi)}{\delta g^{\mu\nu}}$$

scalar field equation

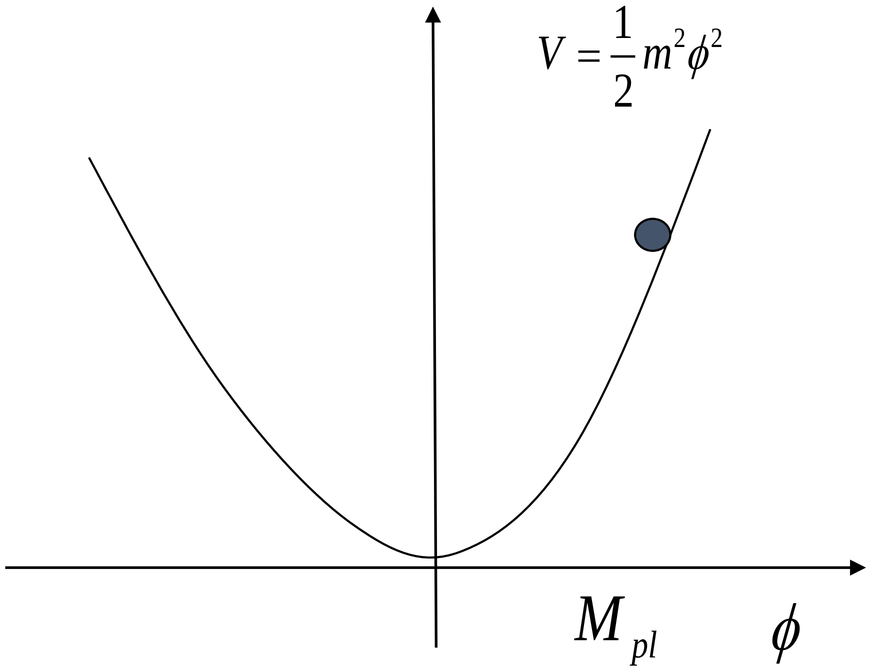
$$\frac{\delta S_\phi}{\delta\phi} = 0 \qquad \nabla^\mu\nabla_\mu\phi - V'(\phi) = 0 \Longleftrightarrow \nabla^\mu T^\phi_{\mu\nu} = 0$$

Background

- Energy density and pressure

$$\rho_\phi = -T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad \phi = \phi(t)$$

$$P_\phi = \frac{1}{3}T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$



- Scalar field equation

$$\ddot{\phi} + \boxed{3H\dot{\phi}} + V'(\phi) = 0 \iff \dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$$

friction

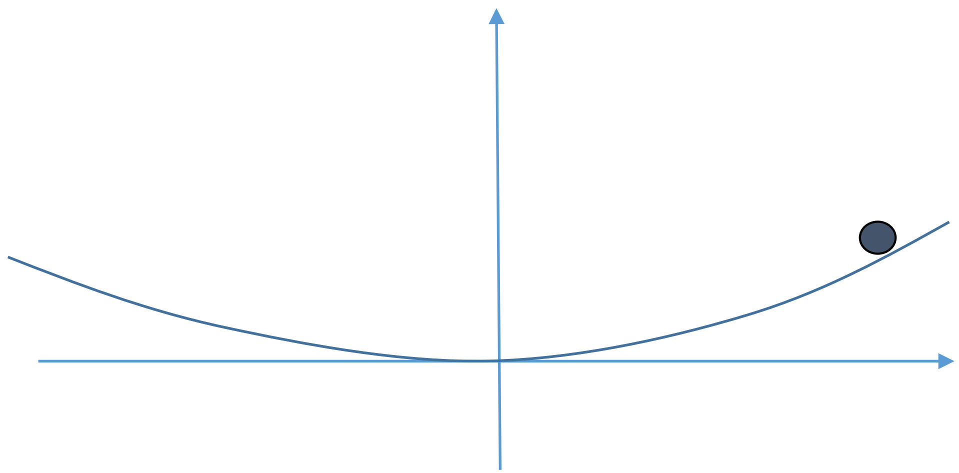
Acceleration

$$\dot{\phi}^2 << V(\phi) \qquad w_\phi = \frac{P_\phi}{\rho_\phi} \approx -1$$

Equation of state

- Equation of state

$$1 + w_\phi = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{V'^2}{9H^2(1+\xi_\phi)^2 \rho_\phi} > 0, \quad \xi_\phi = \frac{\ddot{\phi}}{3H\dot{\phi}}$$



For $w_\phi \approx -1, \quad \xi_\phi << 1$

$$1 + w_\phi \approx \frac{2}{3}\varepsilon_\phi \Omega_\phi(a) \quad \varepsilon_\phi = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2, \quad \Omega_\phi = \frac{\rho_\phi}{\rho_\phi + \rho_m} \quad \kappa^2 = 8\pi G = \frac{1}{M_{pl}^2}$$

Analogous to slow-roll approximations for inflation but the scalar field does not completely dominates the energy density

Quintessence potentials

- Freezing models

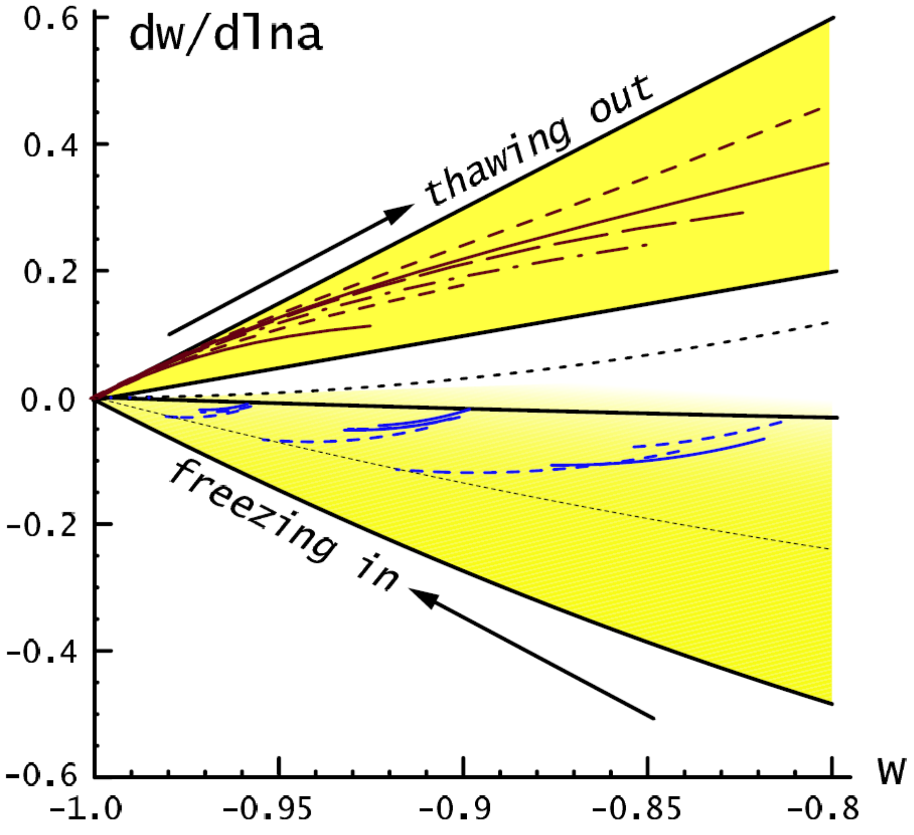
$$V(\phi) = M^{4+n}\phi^{-n}$$

The scalar field dynamics freezes at late time and approaches $w_\phi \approx -1$

- Thawing models

$$V(\phi) = M^4 \cos^2\left(\frac{\phi}{f}\right)$$

The scalar field is initially in the regions of $w_\phi \approx -1$ and later starts to roll down the potential



Linder & Caldwell astro-ph/0505494

Scaling solution

- Tracker solution

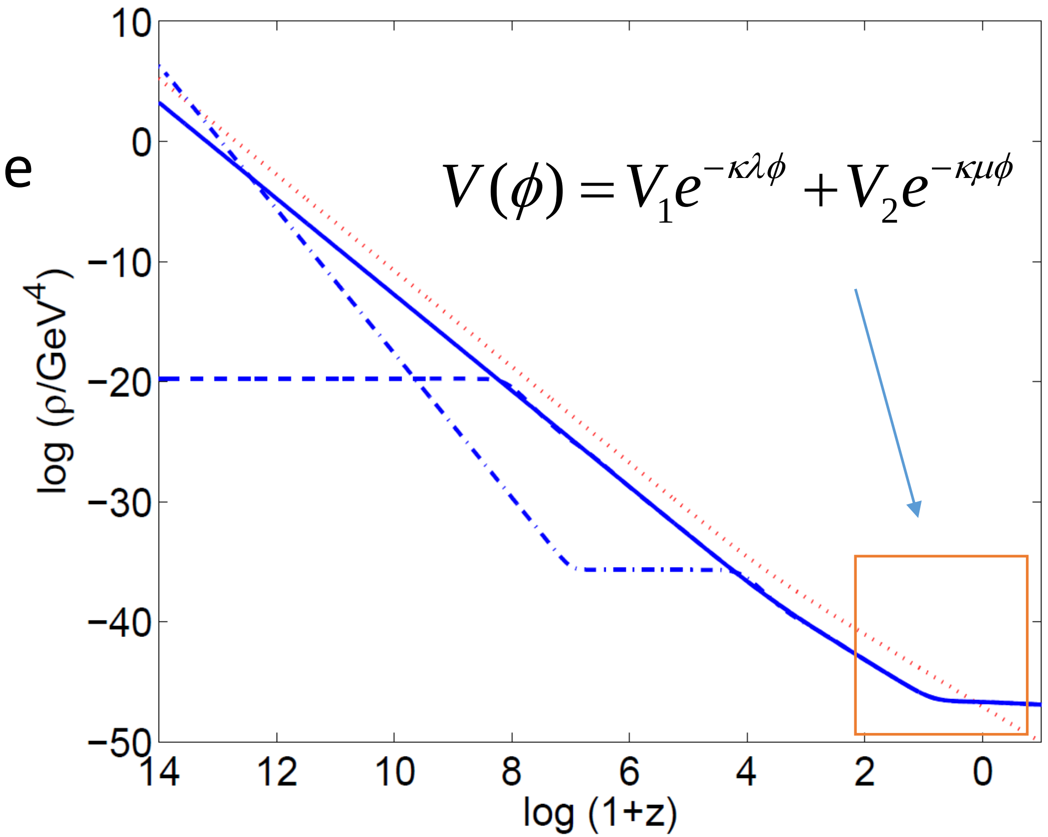
For some potentials, the energy density of the scalar field tracks that of matter/radiation

$$V(\phi) = V_0\, e^{-\kappa\lambda\phi}, \quad \kappa^2 = 8\pi G = \frac{1}{M_{pl}^2}$$

If $\lambda^2 > 3(1+w_M)$

there is an attractor where

$$w_\phi = w_M, \quad \Omega_\phi = \frac{3(1+w_M)}{\lambda^2}$$



Barreiro et.al. astro-ph/9910214

The late time solution is insensitive to the initial density of the scalar field (however we need $\lambda^2 < 2$ to have an accelerating solution)

Fundamental issues

- Consider a massive scalar field

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \qquad \varepsilon_\phi = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2 << 1 \quad \rightarrow \quad \phi > M_{pl} = 10^{18}\,\mathrm{GeV}$$

$$V(\phi_0) = \rho_{DE} = 10^{-48}\,\mathrm{GeV}^4 \quad \rightarrow \quad m = \left(\frac{\rho_{DE}}{\phi^2}\right)^{\frac{1}{2}} = 10^{-42}\,\mathrm{GeV} \approx H_0$$

Correction to the potential

$$V(\phi) = \frac{1}{4}\lambda\phi^4 < \rho_{DE} \quad \rightarrow \quad \lambda < 10^{-120}$$

these small numbers are very difficult to protect against quantum corrections

Symmetry

- Let's assume there is an exact symmetry with a massless scalar field.

Introducing a symmetry breaking term, quantum corrections generate a potential (cf. QCD axion)

$$V(\phi) = \mu^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right), \quad m_\phi \approx \frac{\mu^2}{f}$$

$$f \approx M_{pl,} \quad \mu \approx (H_0 M_{pl})^{1/2} \approx \rho_{DE}^{1/4} \approx (10^{-3}\,\mathrm{eV}), \quad m_\phi \approx H_0$$

small μ is technically natural as (we assumed that) there is an exact symmetry in the limit $\mu \to 0$ thus quantum corrections are $O(\mu^4)$ (Pseudo Nambu-Goldstone bosons)

Modified Gravity

- Lovelock theorem

4D theory of metric and the equations of motion is given by

$$S_{\mu\nu}(g) = \kappa^2 T_{\mu\nu}$$

we require $S_{\mu\nu}(g)$ is

1. a tensor made from the metric and its first and second derivatives
2. symmetric
3. divergence free

$$\Rightarrow \quad S_{\mu\nu}(g) = a\, G_{\mu\nu} + b\, g_{\mu\nu}$$

Beyond Einstein

- Lovelock theorem indicates that if you modify Einstein theory then you must do one or more of the following

  - Consider other fields (scalars, vectors, etc)

  - Accept higher derivatives

  - Work in a space with more than four dimensions

  - Non-local, …

Higher derivative theories

- Ostrogradski ghost

Theories with higher derivatives $L(g, \dot{g}, \ddot{g})$

$$\frac{d^2}{d^2 t}\left(\frac{\partial L}{\partial \ddot{g}}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{g}}\right) + \left(\frac{\partial L}{\partial g}\right) = 0$$

We treat $\dot{g}$ as a new variable and $\ddot{g}$ as a new velocity (we need four initial conditions)

If the velocity $\ddot{g}$ can be expressed in terms of momenta $\partial L / \partial \ddot{g}$ (non-degeneracy condition), it can be shown that the Hamiltonian is not bounded from below Woodard astro-ph/0601672

Ghost

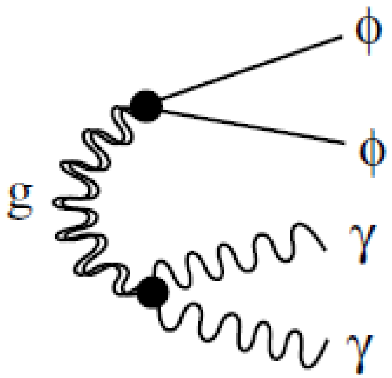
- If the kinetic term has a wrong sign

    The Hamiltonian is not positive definite

$$S_\phi = \int d^4x \left[\frac{1}{2} g^{\mu\nu} \left(\partial_\mu \phi\right)\left(\partial_\nu \phi\right) - V(\phi) \right]$$

$$\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$$ "Phantom DE"

quantum mechanically, particles can be created from
vacuum without costing any energy ⇨ instability of vacuum



Cline et.al. hep-ph/0311312

there is no time scale for
instability in Lorentz invariant theory
the decay is instantaneous

Unless the mass of ghost is above the cut-off scale of the theory,
ghost poses a serious consistency problem

f(R) gravity

- Generalise the Einstein-Hilbert action

$$S=\int d^4x\sqrt{-g}F(R)+S_m\left[g\right],\quad F(R)=R+f(R)$$

- The equations of motion is fourth order

$$F'(R)R_{\mu\nu}-\frac{1}{2}F(R)g_{\mu\nu}-\left(\nabla_\mu\nabla_\nu+g_{\mu\nu}\Box\right)F'(R)=\kappa^2T_{\mu\nu}$$

but this theory violates the assumption of Ostrogradski theorem (i.e. non-degeneracy)

In fact the theory can be rewritten as scalar-tensor gravity

$$S=\int d^4x\sqrt{-g}\left(F(\phi)+(R-\phi)F'(\phi)\right)\qquad\frac{\delta S}{\delta\phi}=(R-\phi)F''(\phi)=0\ \Rightarrow\ \phi=R\quad(F''(R)\neq0)$$

$$S=\int d^4x\left(\psi R-V(\psi)\right)\qquad\psi=F'(\phi),\quad V=RF'-F$$

Brans-Dicke gravity

- Brans-Dicke (BD) gravity (Jordon frame)

$$S = \int d^4x\sqrt{-g}\left(\psi R - \frac{\omega_{BD}}{\psi}\left(\nabla\psi\right)^2 + V(\psi)\right) + S_M[g_{\mu\nu}]$$

f(R) gravity $\quad \omega_{BD} = 0$

- Einstein frame

Conformal transformation $\quad \tilde{g}_{\mu\nu} = \frac{2\psi}{M_{pl}^2} g_{\mu\nu}, \quad \frac{\phi}{M_{pl}} = \frac{1}{2\alpha}\ln\left(\frac{2\psi}{M_{pl}^2}\right), \quad 2w_{BD} + 3 = 1/2\alpha^2$

$$S = \int d^4x\sqrt{-\tilde{g}}\left(\frac{M_{pl}^2}{2}\tilde{R} - \frac{1}{2}\left(\tilde{\nabla}\phi\right)^2 - \tilde{V}(\phi)\right) + S_M[A^2(\phi)\tilde{g}_{\mu\nu}] \qquad A(\phi) = \exp\left(\frac{\alpha\,\phi}{M_{pl}}\right)$$

the scalar field is coupled to matter

Dvali-Gadadze-Porrati (DGP) braneworld model

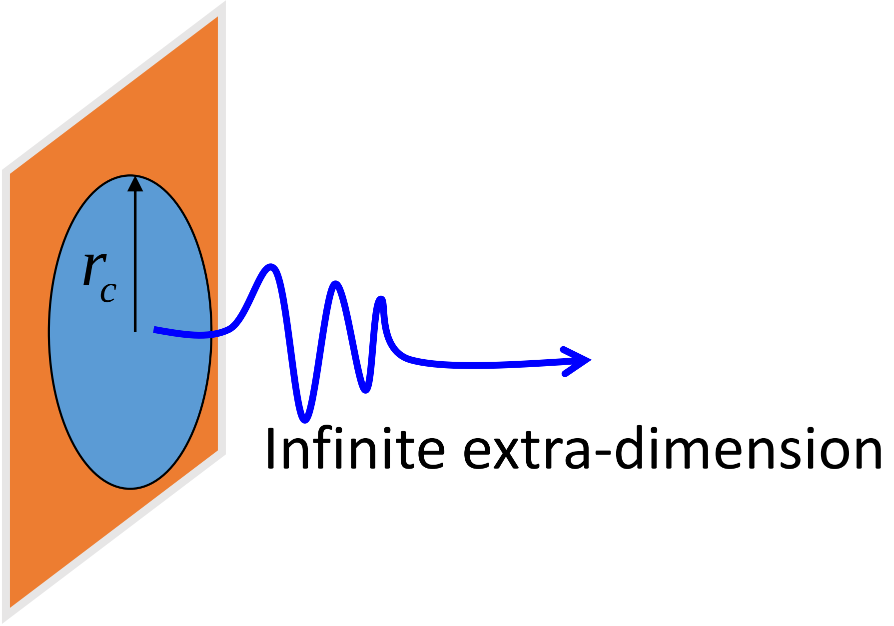
- 5D braneworld model

  Standard model particles are confined to a 4D brane in a 5D bulk spacetime

$$S = \frac{1}{32\pi G r_c}\int d^5x\sqrt{-g_{(5)}}R_{(5)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}L_m$$

cross over scale r_c

$$\pm\frac{H}{r_c} = H^2 - \frac{8\pi G}{3}\rho$$



in the + branch, the expansion of the universe accelerates without the cosmological constant (self-accelerating solution)

Fundamental issues

- Theoretical consistency

  modified gravity models often suffer from instabilities

  (cf. the self-accelerating solution in DGP model suffers from ghost instabilities)

- Small numbers

  cf. viable f(R) models

$$F(R) = R - 2\Lambda_{eff} - \frac{R_0^{n+1}}{R^n}, \quad \Lambda_{eff} \approx R_0 \approx H_0^2$$

DGP models $r_c \approx H_0^{-1}$

Ultra-light scalar field

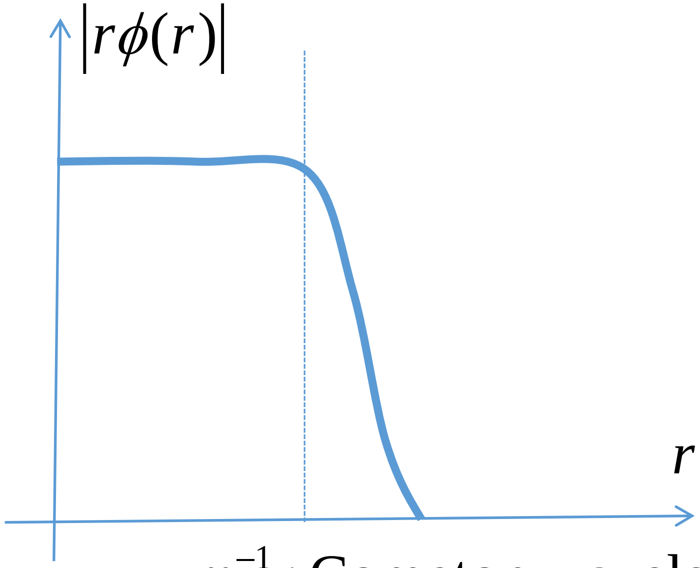
- In most of DE/MG models, there exists a scalar field with tiny mass

$$S=\int d^4x\left[-\frac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right)-\frac{1}{2}m^2\phi^2\right]-\frac{\alpha}{M_{pl}}\int d^4x\,\phi\,\rho$$

static source

$$(\nabla^2-m^2)\phi=\frac{\alpha}{M_{pl}}\rho,\qquad \frac{\phi}{M_{pl}}=-\left(\frac{\alpha}{4\pi M_{pl}^2}\right)\frac{M}{r}\exp(-m\,r)$$

$$\Rightarrow\quad \begin{cases}\phi=-\left(\dfrac{\alpha}{4\pi M_{pl}^2}\right)\dfrac{M}{r}, & r<m^{-1}\\ \phi\to 0, & r>m^{-1}\end{cases}$$



m^{-1} : Compton wavelength

if $\quad m^{-1}\approx H_0^{-1}\quad$ the scalar field mediates a long range force

Fifth force

- Due to the coupling between the scalar and matter, the geodesic equation is modified as

$$\ddot{\vec{a}} = -\nabla\Psi - \frac{\alpha}{M_{pl}}\nabla\phi = F_G + \boxed{F_5}$$

Fifth force

$$F_5 = \frac{2\alpha^2}{8\pi M_{pl}^2}\frac{M}{r^2} \quad \text{for} \quad r < m^{-1}$$

$$F_G = \frac{1}{8\pi M_{pl}^2}\frac{M}{r^2}$$

Fifth force is strongly constrained in the solar system

$$\alpha^2 < 10^{-5}, \quad w_{BD} > 40,000 \quad (3 + 2w_{BD}) = 1/2\alpha^2$$

Way out

- Quintessence

  the coupling to matter is assumed to be zero $\alpha^2 = 0$

- Violation of strong equivalence principle

  the solar system constraints come from coupling to baryons $\alpha_{baryon}^2 = 10^{-5}$

  the coupling to cold dark matter can be large $\alpha_{cdm}^2 = O(1)$

  interacting dark energy

$$S = \int d^4x \sqrt{-\tilde{g}} \left(\frac{M_{pl}^2}{2} \tilde{R} - \frac{1}{2} \left(\tilde{\nabla} \phi \right)^2 - \tilde{V}(\phi) \right) + S_{cdm}[A^2(\phi) \tilde{g}_{\mu\nu}] + S_{baryon}[\tilde{g}_{\mu\nu}]$$

Screening mechanism

- The coupling constant or mass need to be scale dependent

$$m\big|_{\cos mo} = O(H_0) \qquad m\big|_{local} \gg H_0$$

$$\alpha^2\big|_{\cos mo} = O(1) \qquad \alpha^2\big|_{local} \to 0$$

- Two representative mechanisms

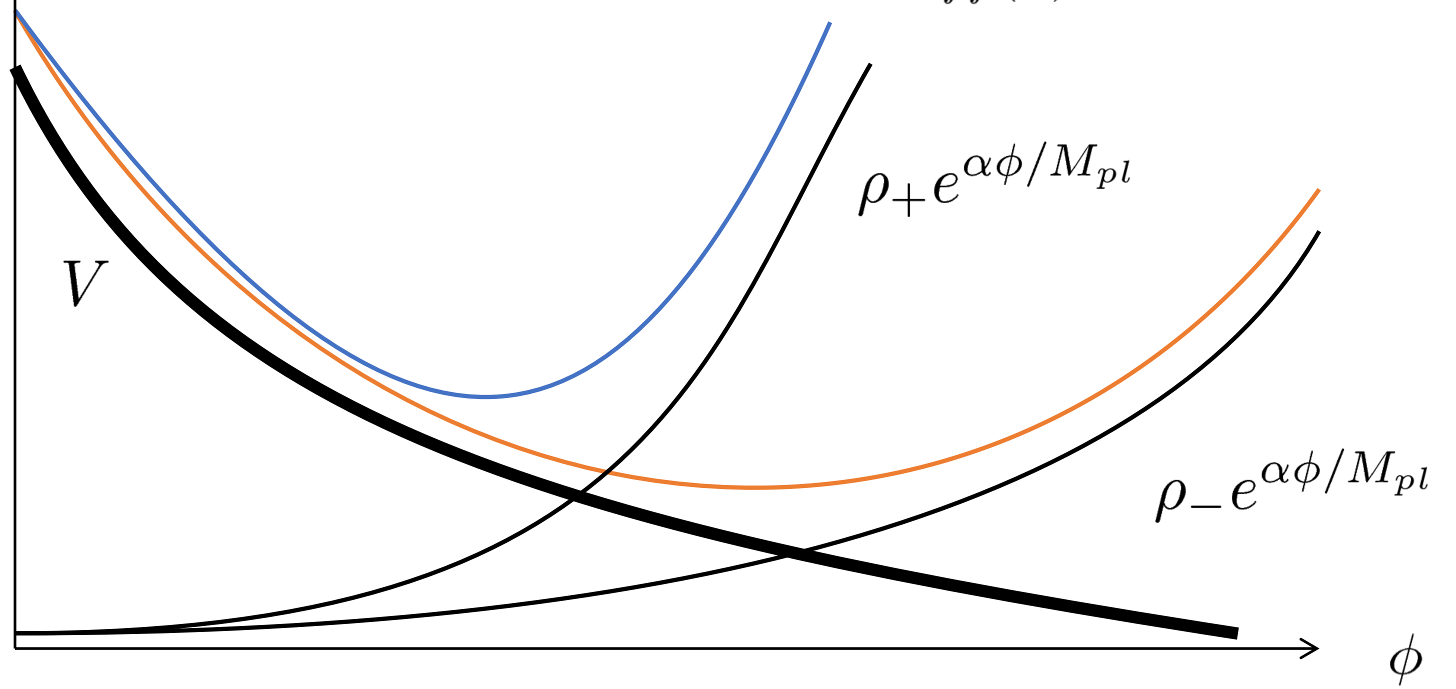
1. Chameleon mechanism
2. Vainshtein mechanism

Chameleon mechanism

$$S=\int d^4x\sqrt{-\tilde{g}}\left(\frac{M_{pl}^2}{2}R-\frac{1}{2}(\nabla\phi)^2-V(\phi)\right)+S_m[A^2(\phi)g_{\mu\nu}] \qquad A(\phi)=\exp\left(\alpha\frac{\phi}{M_{pl}}\right)$$

$$\nabla^2\phi=\partial_\phi V+\frac{\alpha}{M_{pl}}\rho e^{\alpha\phi/M_{pl}}$$

$$V_{eff}(\phi)=V+\rho e^{\alpha\phi/M_{pl}}$$



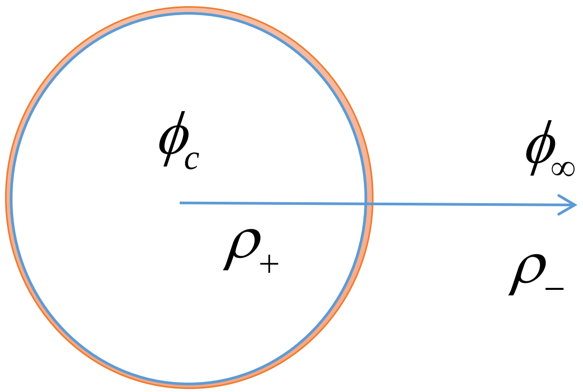
Khoury & Weltman astro-ph/0309300

Thin shell condition

- If the thin shell condition is satisfied, only the shell of the size ΔR_c contributes to the fifth force

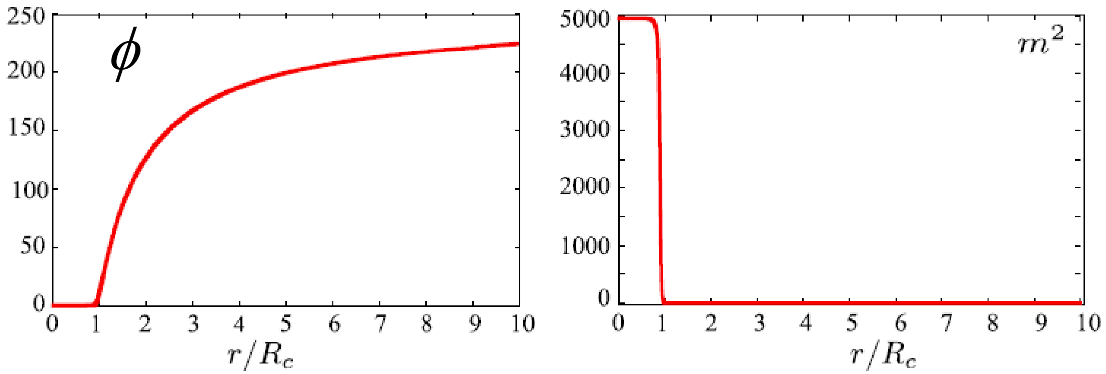
Khoury & Weltman astro-ph/0309411

$$V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}}$$



$$\phi(r) = -\left(\frac{\alpha}{4\pi M_{pl}}\right)\left(\frac{3\Delta R_c}{R_c}\right)\frac{M\exp\left(-m_\infty(r-R_c)\right)}{r} + \phi_\infty$$

$$\frac{\Delta R_c}{R_c} = \frac{(\phi_\infty - \phi_c)/M_{pl}}{6\alpha\Psi_c} << 1 \qquad \Psi_c = \frac{GM}{R_c}$$



Silvestri 1103.4013

Screening is determined by the gravitational potential of the object

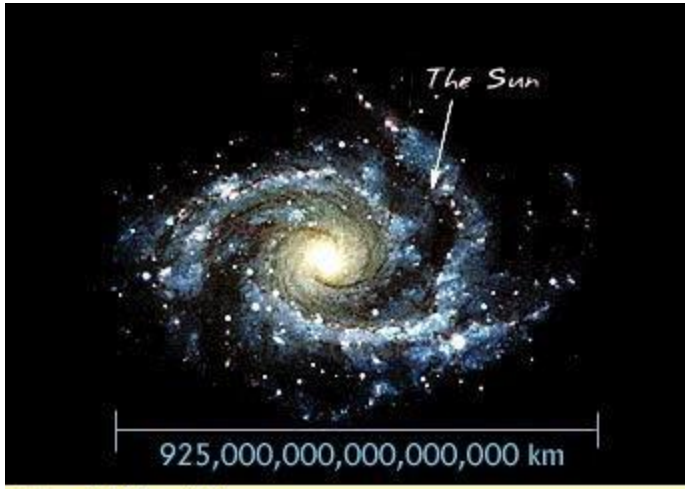
Solar system constraints

- Solar system constraints

$$\rho_\odot \sim 10 \text{ g cm}^{-3}$$

$$\rho_{gal} \sim 10^{-24} \text{g cm}^{-3}$$

$$\frac{\Delta R_c}{R_c} = \frac{\phi_{gal} - \phi_\odot}{6\alpha M_{pl}\Psi_\odot} \sim \frac{\phi_{gal}}{6\alpha M_{pl}\Psi_\odot}$$



The Milky Way

The sun has a potential $\Psi_\odot \sim 10^{-6}$

The thin shell suppression eases the constraints $\alpha = O(1)$

$$\frac{\Delta R_c}{R_c} \leq 10^{-5} \quad \Rightarrow \quad \frac{\phi_{gal}}{M_{pl}} < 5\times 10^{-11}$$

This is a model (potential) independent constraint

From galaxy to cosmology

- Example

$$V = V_0 - M^4 \left(\phi / M_{pl}\right)^{1/2} \qquad \frac{\phi_{gal}}{M_{pl}} = \left(\frac{M^4}{\alpha \rho_{gal}}\right)^2$$

$$\rho_{gal} \sim 10^{-24}\,\mathrm{g\ cm}^{-3}$$

$$\rho_{crit} \sim 10^{-29}\,\mathrm{g\ cm}^{-3}$$

Solar system constraints

$$\frac{\phi_{gal}}{M_{pl}} < 10^{-11} \qquad \frac{\phi_{\cos mo}}{M_{pl}} = \left(\frac{M^4}{\alpha \rho_{crit}}\right)^2 \simeq 10^{10} \frac{\phi_{gal}}{M_{pl}} \leq 10^{-1} \qquad M \simeq 10^{-3}\mathrm{eV}$$

Galaxy $\frac{\Delta R_c}{R_c} = \frac{\phi_{\cos mo} - \phi_{gal}}{6\alpha M_{pl} \Psi_{gal}} \sim \frac{\phi_{\cos mo}}{6\alpha M_{pl} \Psi_{gal}}$

The Milky way galaxy $\Psi_{Milk} \sim 10^{-6}$

in order to screen the Milky way, we need $\frac{\phi_{\cos mo}}{M_{pl}} < 10^{-6}$

Vainshtein mechanism

- Vainshtein mechanism

originally discussed in massive gravity

rediscovered in DGP brane world model

linear theory $\omega_{BD} = 0$

$$3\nabla^2\varphi = -8\pi G\rho$$

$$\nabla^2\Psi = 4\pi G\rho - \frac{1}{2}\nabla^2\varphi$$

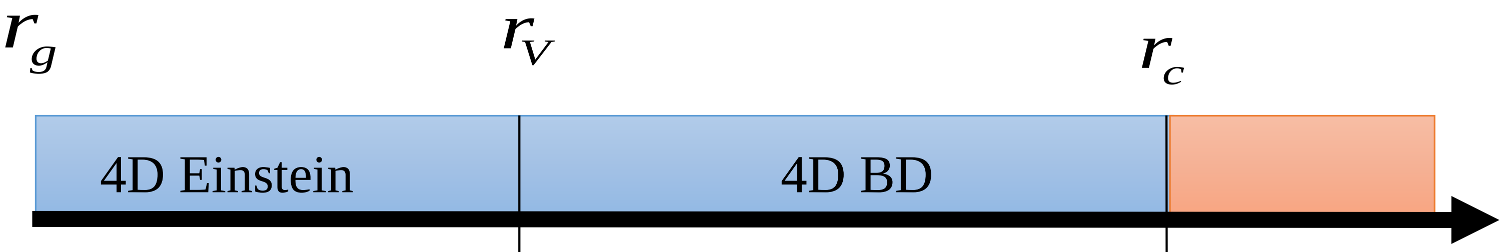
even if gravity is weak, the scalar can be non-linear $r_c \sim m^{-1} \sim H_0^{-1}$

$$3\nabla^2\varphi + r_c^2\left\{\left(\nabla^2\varphi\right)^2 - \partial_i\partial_j\varphi\,\partial^i\partial^j\varphi\right\} = 8\pi G a^2\rho$$

Vainshtein radius

- Spherically symmetric solution for the scalar

$$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r), \quad \Delta(r) = \frac{2}{3}\left(\frac{r}{r_V}\right)^3\left(\sqrt{1+\left(\frac{r_V}{r}\right)^3}-1\right) \qquad r_V = \left(\frac{8r_c^2 r_g}{9}\right)^{\frac{1}{3}}, \quad r_g = 2GM$$



Screening is determined by the curvature of the object

$$\Phi = \frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}}, \qquad \Psi = -\frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}}$$

$$\Phi = \frac{r_g}{2r}\left(\frac{2}{3}\right), \qquad \Psi = -\frac{r_g}{2r}\left(\frac{4}{3}\right)$$

$$\left(\frac{r_V}{r}\right)^{-3/2} \propto \left(\frac{GM}{r^3}r_c^2\right)^{-3/2}$$

2.95km 0.1 kpc 3000Mpc for the Sun

Solar system constraints

- The fractional change in the gravitational potential $\varepsilon = \frac{\delta\Psi}{\Psi}$

  The anomalous perihelion precession

  $$\delta\phi = \pi r \frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(\frac{\varepsilon}{r} \right) \right]$$

  The vainshtein radius is shorter for a smaller object

  Lunar laser ranging: the Erath-moon distance $r_{E-M} = 4.1 \times 10^5 \mathrm{km}$

  $$\delta\phi = \frac{3\pi}{4}\left(\frac{r_{E-M}{}^3}{2GM_\oplus r_c^2} \right)^{1/2} < 2.4 \times 10^{-11} \quad \Rightarrow \quad r_c > H_0^{-1}$$

Horndeski theory

- DGP non-linear interactions

$$L = \Box\pi\left(\partial_\mu\pi\right)\left(\partial^\mu\pi\right) \qquad \left(\Box\pi\right)^2 - \left(\partial_\mu\partial_\nu\pi\right)\left(\partial^\mu\partial^\nu\pi\right) = 0$$

this action has "galileon" symmetry $\partial_\mu\pi \to \partial_\mu\pi + c_\mu$

in 4D, there are only two more galileon terms that give 2nd order equations of motion

- Covariant theory (Horndeski theory)

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi \qquad X = -\frac{1}{2}\left(\partial\phi\right)^2$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]$$

Screening mechanisms

- They look contrived but some theories naturally have these mechanisms

f(R) gravity

$$F(R) = R - 2\Lambda_{eff} - \frac{R_0^{n+1}}{R^n}$$

we expect to recover GR in the limit $R / R_0 \gg 1$

In fact, in Einstein frame this is nothing but chameleon with $V = V_0 - M^4 \left(\phi / M_{pl}\right)^{1/2}$

DGP gravity

$$S = \frac{1}{32\pi G r_c} \int d^5x \sqrt{-g_{(5)}} R_{(5)} + \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} L_m$$

we expect to recover GR in the limit $r / r_c \gg 1$ and this is realised by Vainshtein

Dynamics of dark energy
hep-th/0603057

3

Modified Gravity and Cosmology
arXiv:1106.2476

Summary

- DE/MG models

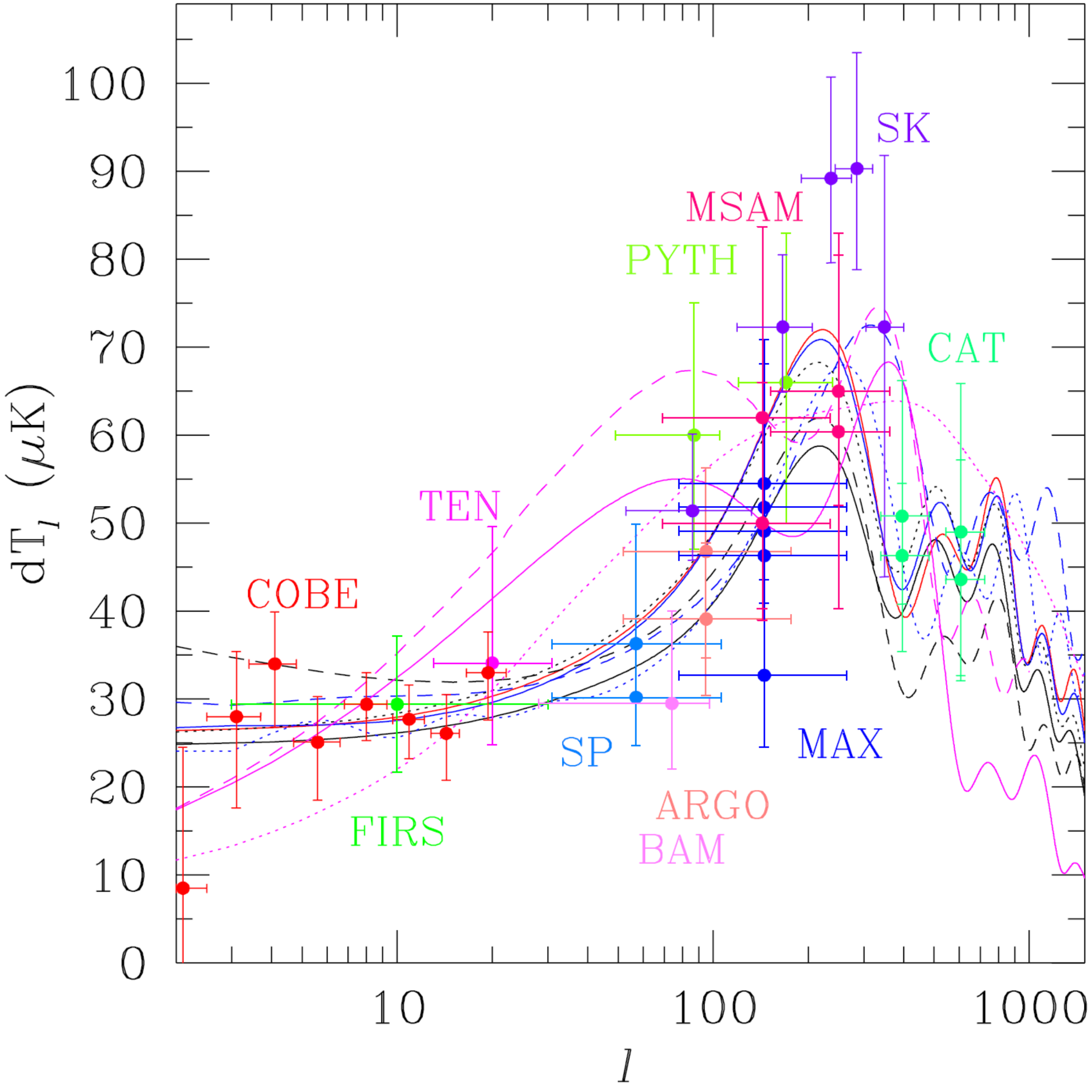
There are many models but we still do not have a compelling alternative to LCDM

Observation may give us a clue:

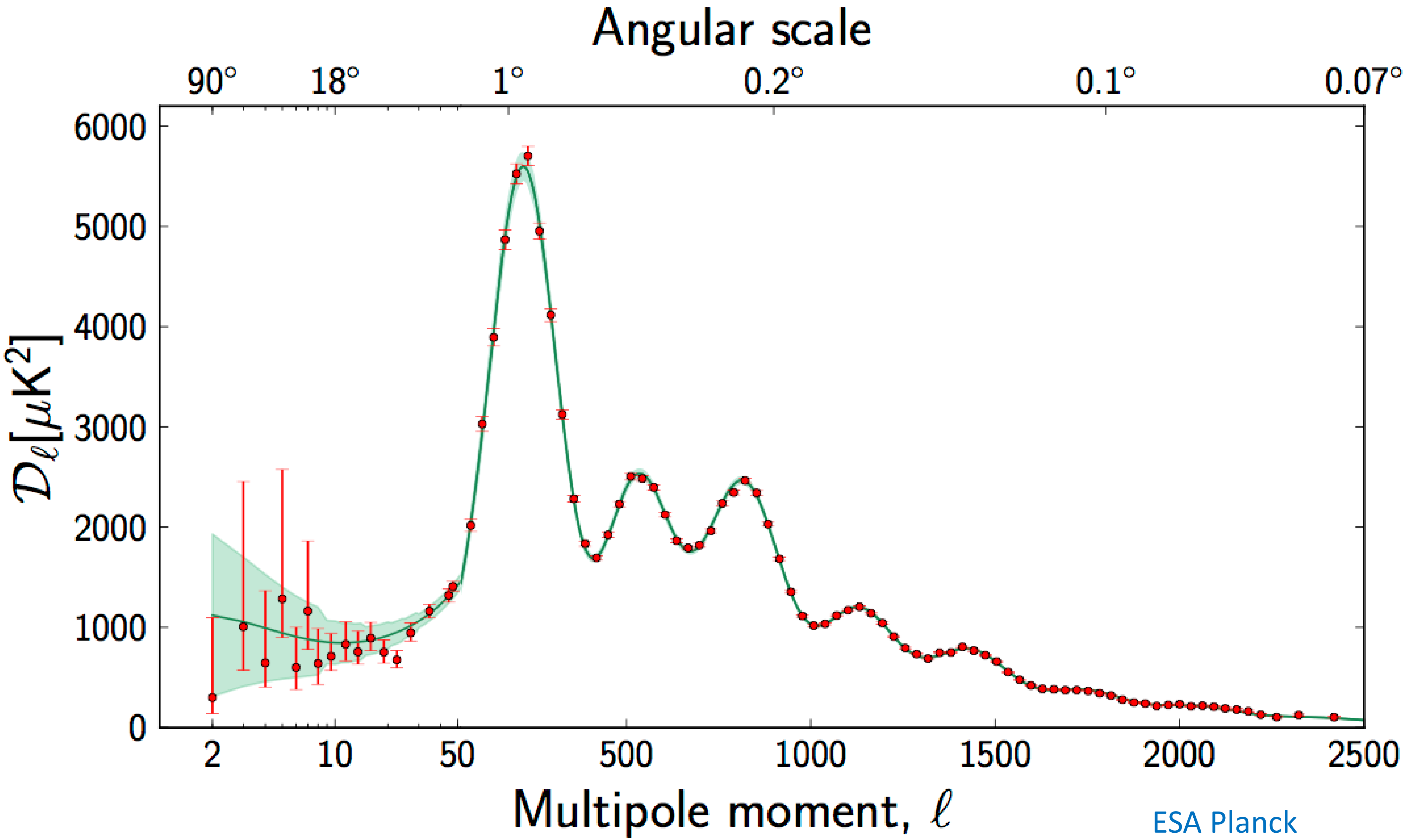
Is it the cosmological constant or a light degree of freedom?

Does it cluster or couple to matter?

cf. CMB (before WMAP)



Gawiser & Silk astro-ph/9806197



Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
$\mathcal{D}_\ell [\mu K^2]$
6000
5000
4000
3000
2000
1000
0
2
10
50
500
1000
1500
2000
2500
Multipole moment, ℓ

ESA Planck

- Lecture 3 Structure formation and observational tests
- Lecture 4 Observational tests and non-linear structure formation