Structure formation and observational tests

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How to test DE/MG models

- Einstein equations

$$G_{\mu\nu} = \kappa^2 T^M_{\mu\nu} + \kappa^2 E_{\mu\nu}, \quad E_{\mu\nu} = T^{DE}_{\mu\nu} + G^{MG}_{\mu\nu} \qquad \nabla^\mu (T^M_{\mu\nu} + E_{\mu\nu}) = 0$$

- Background (homogeneity & Isotropy) $E^\mu_\nu = \mathrm{diag}\,(-\rho_E, P_E, P_E, P_E)$

  everything is determined by the equation of state $w_E = P_E / \rho_E$

- Small Inhomogeneity

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right] \qquad \mathcal{H} = \frac{a'}{a}$$

Linear scalar perturbations with respect to 3-space (assumed to be flat)

Cosmological perturbation theory

- Fourie transformation and Decomposition

Kodama & Sasaki
Mukhanov, Feldman, Brandenberger
Malik & Wands 0809.4944

$$\nabla^2 S = -k^2 S, \quad S \propto e^{i k_i x^i}$$

$$S_i = -i \hat{k}_i S, \quad \hat{k}_i = \frac{k_i}{k}$$

$$S_{ij} = \left(\frac{\delta_{ij}}{3} - \hat{k}_i \hat{k}_j \right) S, \quad A_{ij} = A_L \delta_{ij} + A_T S_{ij}, \quad S^i_i = 0$$

- Gauge fixing

We assumed the theory is invariant under diffeomorphism $x^\mu \to x^\mu + \xi^\mu$ and used the Longitudinal gauge. The gauge invariance can be always restored by introducing additional fields (Stuckelberg fields)

Matter content

- Energy momentum tensor

$$T^{\mu}_{I\nu} = \begin{pmatrix} -(\rho_I + \delta\rho_I), & (\rho_I + P_I) v_{I\,i} \\ -(\rho_I + P_I) v_I{}^i, & \left(P_I + \delta P_I\right)\delta^i_j - P_I\,\Pi_I{}^i{}_j \end{pmatrix}$$

$$v^i = v\,S^i,$$

$$\Pi^i_j = \Pi\,S^i_j : \text{ anisotropic stress}$$

- Conservation of energy momentum tensor

for now, we assume matter and dark component obeys the conservation independently

$$\frac{d\delta\rho_I}{d\eta} + 3\mathscr{H}(\delta\rho_I + \delta P_I) = -(\rho_I + P_I)(kv_I - 3\dot{\Phi})$$

$$\left(\frac{d}{d\eta} + 4\mathscr{H}\right)\left[\frac{(\rho_I + P_I)v_I}{k}\right] = \delta P_I - \frac{2}{3}P_I\,\Pi_I + (\rho_I + P_I)\Psi$$

$$w_m = P_m / \rho_m = 0, \quad \delta P_m = \Pi_m = 0,$$

$$w_E = P_E / \rho_E, \quad \delta P_E, \quad \Pi_E,$$

Equations for linear perturbations

- Einstein equations

$$k^2\Phi = -4\pi G a^2(\rho_m \Delta_m + \rho_E \Delta_E), \quad \rho_I \Delta_I = \delta\rho_I + 3(\rho_I + P_I)\frac{\mathcal{H}}{k} v_I$$

$$k^2(\Psi - \Phi) = -8\pi G a^2 P_E \, \Pi_E$$

- Conversation of energy momentum tensor for matter $\left(k/\mathcal{H}\right)^2 >> 1$

$$\Delta_m{}' = -\mathcal{H}\theta_m, \qquad \theta_m = \left(k/\mathcal{H}\right) v_m : \text{ velocity divergence}$$

$$\theta_m{}' + \mathcal{H}\left(1 + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\theta_m = \frac{k^2}{\mathcal{H}}\Psi \quad \Rightarrow \quad \Delta_m{}'' + \mathcal{H}\Delta_m{}' = -k^2\Psi$$

Evolution of matter is determined by the Newtonian potential

Dark component affects the evolution through the Newtonian potential

Equations for dark component

- Conservation of dark component (assumed to be held independently)

$$\frac{d\delta\rho_E}{d\eta} + 3\mathscr{H}(\delta\rho_E + \delta P_E) = -(\rho + P)(kv_E - 3\dot{\Phi})$$

$$\left(\frac{d}{d\eta} + 4\mathscr{H}\right)\left[\frac{\rho_E(1+\boxed{w_E})v_E}{k}\right] = \boxed{\delta P_E} - \frac{2}{3}P_E\boxed{\Pi_E} + (\rho_E + P_E)\Psi$$

- Sound speed

$$c_s^2 = \left.\frac{\delta P}{\delta\rho}\right|_{v=0}$$

$$\delta P = c_s^2\delta\rho + (c_s^2 - c_a^2)\rho'\frac{v}{k}, \quad c_a^2 = \frac{P'}{\rho'} = w - \frac{w'}{3\mathscr{H}(1+w)}: \quad \text{adiabatic sound speed}$$

Classification (1)

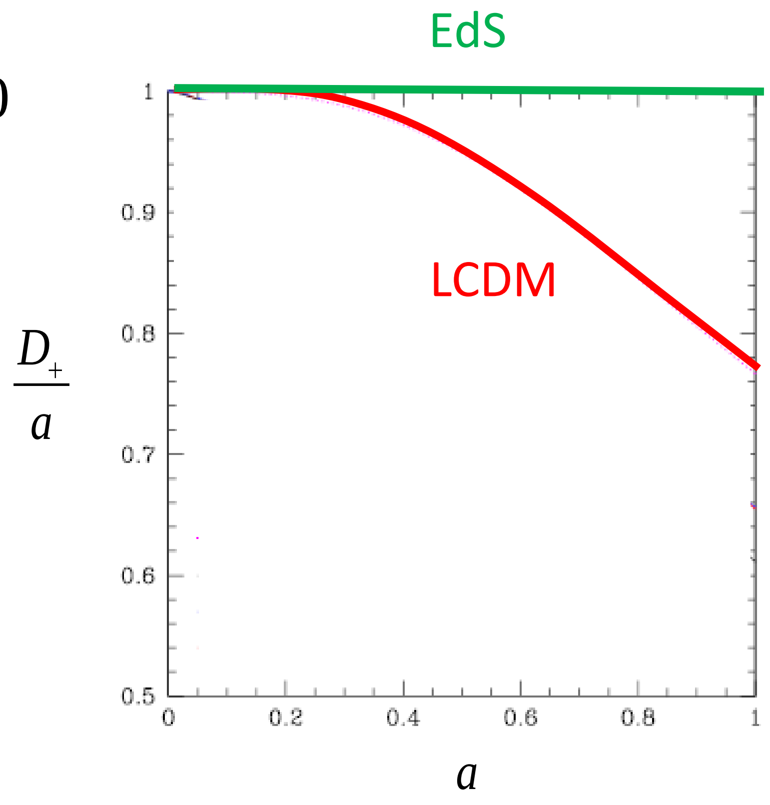
1) LCDM $E_{\mu\nu} = -\Lambda g_{\mu\nu}$

$$k^2\Phi = -4\pi G a^2 \rho_m \Delta_m$$
$$\Phi - \Psi = 0$$
$$\Rightarrow \quad \Delta_m'' + \mathscr{H}\Delta_m' - 4\pi G a^2 \rho_m \Delta_m = 0$$

We define the growth function D_+ as the growing mode solution for Δ_m

in MD era, $a \propto \eta^2, \mathscr{H}^2 = 8\pi G a^2 \rho_m / 3 \quad \Rightarrow \quad D_+ \propto a$

at late times, due to the cosmological constant, gravity becomes weaker $\mathscr{H}^2 = \dfrac{8\pi G a^2}{3}\left(\rho_m + \rho_\Lambda\right)$



Classification (2)

2) smooth DE $\delta\rho_E = \pi_E = 0$

using $N = \ln a$ $\quad \Delta_m'' + \mathcal{H}\Delta_m' - 4\pi G a^2 \rho_m \Delta_m = 0$

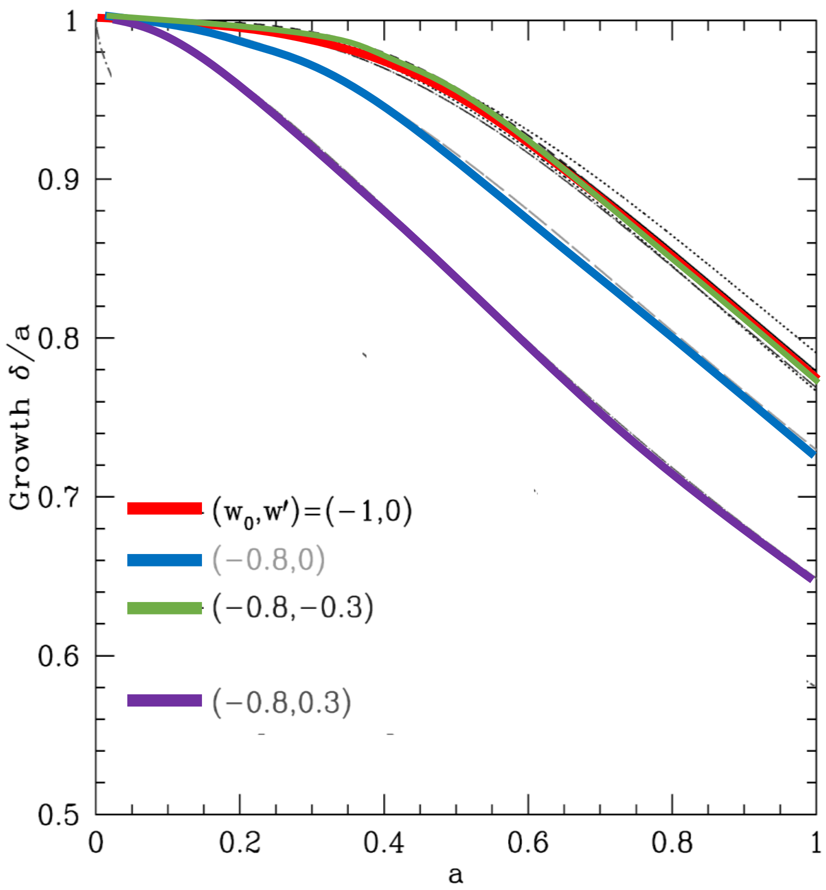
$$\ddot{D}_+ + \frac{1}{2}(1 - 3\Omega_{DE} w_{DE})\dot{D}_+ - \frac{3}{2}\Omega_m D_+ = 0$$

$$\dot{\Omega}_m = 3 w_{DE}(1 - \Omega_m)\Omega_m, \quad \Omega_{DE} = 1 - \Omega_m$$

For a fixed present-day Ω_{DE} ,if $w_{DE} > -1$,DE density is larger in the past suppressing the growth compared with LCDM

$$w_{DE}(z) = w_0 + w_a \frac{z}{1+z}$$

$$\frac{D_+}{a}$$



Linder astro-ph/0305286

Growth rate

- Growth rate

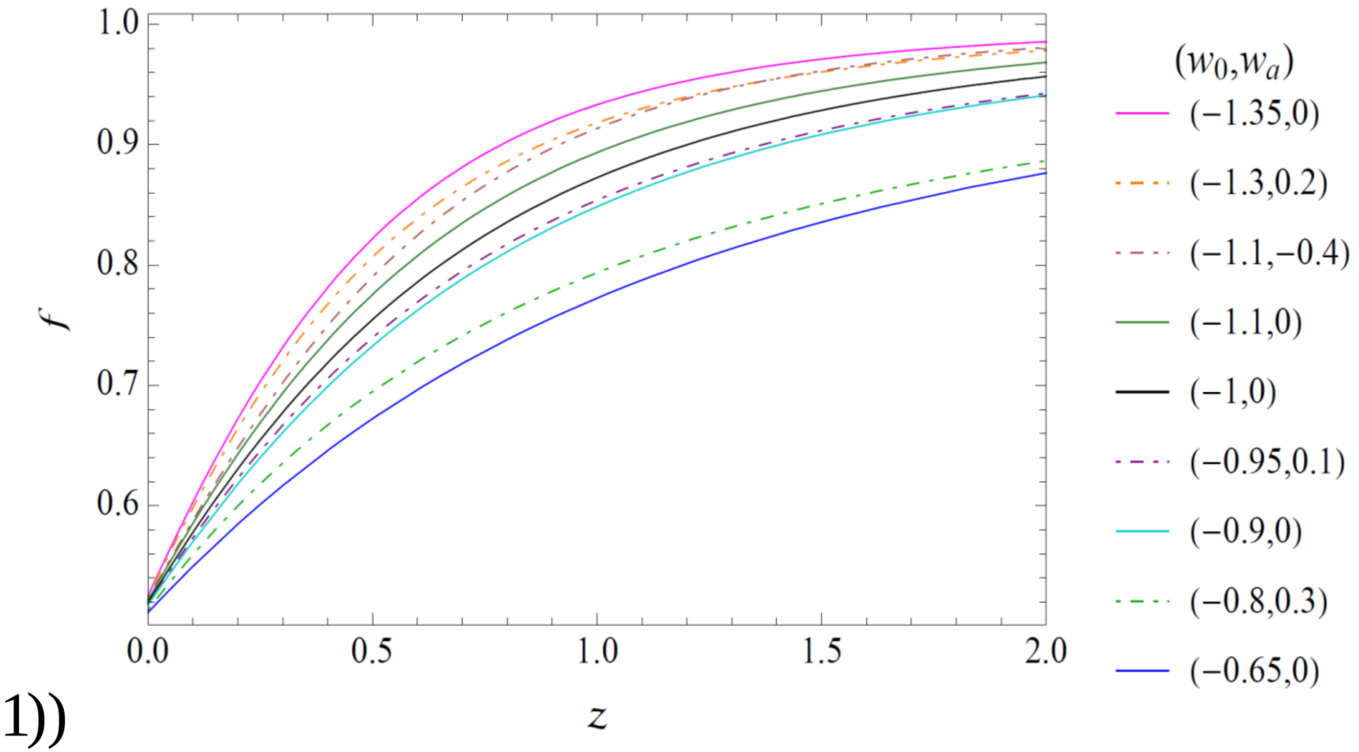
$$f = \frac{d \ln \Delta_m}{d \ln a} = \frac{a}{D_+}\frac{dD_+}{da}$$

$$\dot{f} + f^2 + \left[\frac{1}{2} - \frac{3}{2} w_{DE}(1-\Omega_m)\right] f = \frac{3}{2}\Omega_m$$

$$f = \Omega_m^{\gamma}, \quad \gamma = 0.545 + 0.05(1 + w_{DE}(z=1))$$

γ is insensitive to the equation of state w_{DE}

(but the growth rate depends on w_{DE} through Ω_m)



Dossett & Ishak 1311.0726

Classification (3)

- Clustering DE $\delta\rho_E \neq 0 \quad (\pi_E = 0)$

Let's consider a toy model for dark component with non-zero sound speed

$$\delta P_E = c_{sE}^2 \delta\rho_E$$

assuming that the dark component dominates the universe

$$\Delta_E'' + \mathcal{H}\Delta_E' + \left(c_{sE}^2 k^2 - 4\pi G a^2 \rho_E\right)\Delta_E = 0$$

For $k > k_J = \sqrt{\dfrac{4\pi G a^2 \rho_E}{c_{sE}}}$ pressure wins over gravity and Δ_E does not grow

clustering DE requires small sound speed $\left(k_J / a\right)^{-1}$: Jean's length

Quintessence

- Sound speed

$$c_{s\phi}^2 \equiv \left.\frac{\delta P_\phi}{\delta \rho_\phi}\right|_{v_\phi=0} = 1, \quad v_\phi \propto \delta\phi$$

propagation speed

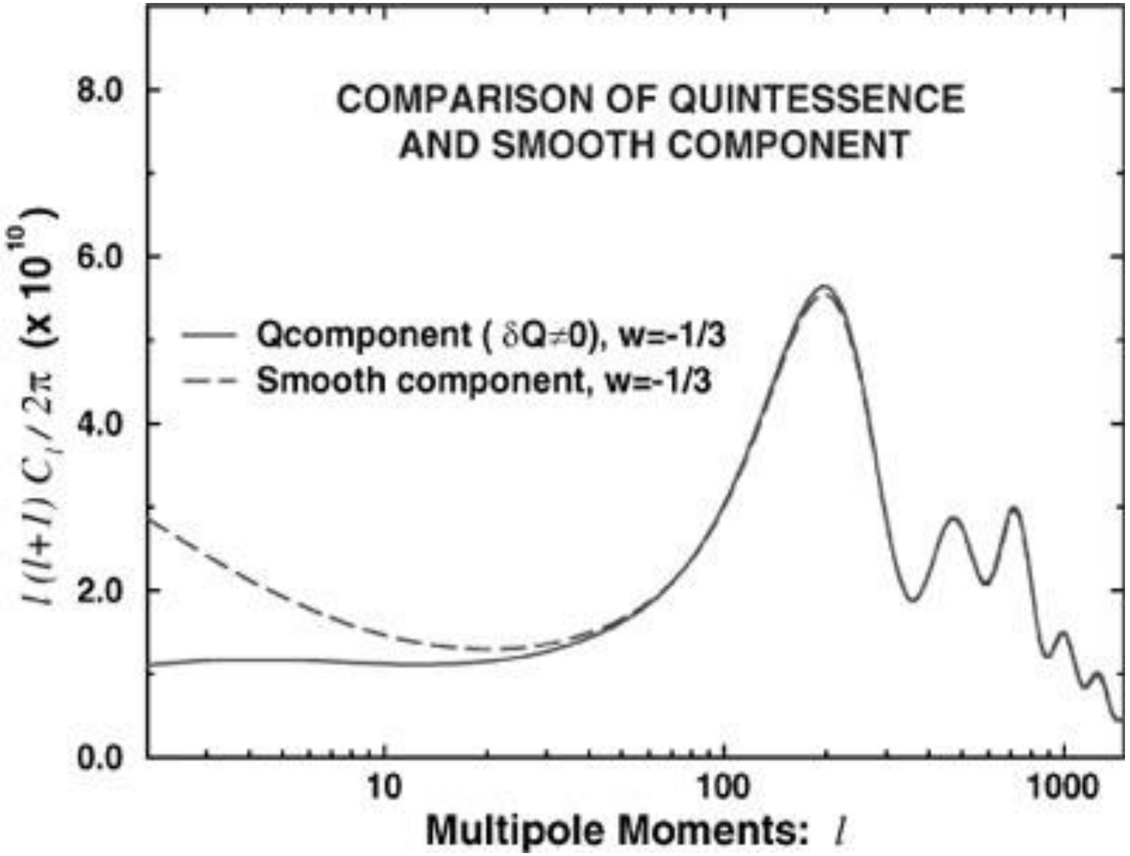
scalar field equation of motion $S_\phi = \int d^4x \left[-\frac{1}{2} g^{\mu\nu} \left(\partial_\mu \phi\right)\left(\partial_\nu \phi\right) - V(\phi) \right]$

$$\delta\ddot{\phi} + 3H\delta\phi + c_{p\phi}^2 \frac{k^2}{a^2}\delta\phi + \ldots = 0$$

for standard kinetic term, $c_{p\phi}^2 = c_{s\phi}^2 = 1$ thus the scalar field does no cluster below the horizon scale thus can be approximated as smooth DE $\delta\rho_E = \pi_E = 0$

Quintessence

- Note that this does not mean we can ignore the perturbations of scalar field entirely



Caldwell: An introduction to quintessence

K-essence/massive scalar field

- K-essence

$$S = \int d^4x \sqrt{-g} K(X), \quad X = -\frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right)$$

$$c_{s\phi}^2 \equiv \frac{K_{,X}}{K_{,X} + 2X\, K_{,XX}} << 1 \quad (\text{if } K_{,X} << X\, K_{,XX})$$

- Massive scalar field

scalar field perturbations oscillate with frequency $\omega = \sqrt{m^2 + \left(k / a\right)^2}$

averaging order many oscillations $\left\langle \delta\dot{\phi}^2 \right\rangle = \omega(k)^2$

$$c_{s\phi}^2 \approx \frac{k^2}{4a^2m^2} \quad (k^2 < a^2m^2)$$

Effects on growth

- Einstein equations

$$k^2\Phi = -4\pi a^2 G\left(1+\frac{\rho_E\Delta_E}{\rho_m\Delta_m}\right)\rho_m\Delta_m$$

$$\Phi - \Psi = 0$$

$$D_+'' + \mathscr{H} D_+' + 4\pi G a^2\left(1+\frac{\rho_E\Delta_E}{\rho_m\Delta_m}\right)\rho_m D_+ = 0$$

clustering DE acts like modifications of gravity for dark matter

- Anisotropic stress

scalar field does not have anisotropic stress $\Pi_E = 0$

normal matter has small anisotropic stress compared with density

$$\Pi \approx O(\Psi / G\rho) \approx O(\Delta\, a^2 / k^2) << \Delta$$

Classification (4)

- Brasn-Dicke gravity $\omega_{BD} = 0$

$$S = \int d^4x \sqrt{-g}\, \psi R + S_M[g_{\mu\nu}], \quad \psi = \frac{1}{16\pi G}(1+\varphi)$$

$$k^2\Phi = -4\pi G a^2 \rho_m \Delta_m - \frac{1}{2}k^2\varphi$$

$$\Psi - \Phi = \varphi$$

$\Leftrightarrow$

$$k^2\Phi = -4\pi G a^2 (\rho_m \Delta_m + \rho_E \Delta_E)$$

$$k^2(\Psi - \Phi) = -8\pi G a^2 P_E \Pi_E$$

$\Rightarrow$

$$4\pi G a^2 P_E \Pi_E = -4\pi G a^2 \rho_E \Delta_E = -\frac{1}{2}k^2\varphi$$

- f(R) gravity example

$$S = \int d^4x \sqrt{-g}\left[\frac{F(R)}{16\pi G} + L_m\right]$$

$$3\nabla^2\varphi = 3a^2\bar{\mu}^2\varphi - 8\pi G a^2 \rho_m \delta_m, \quad \bar{\mu}^2 = \frac{F_{,R}}{3F_{,RR}}$$

Growth in f(R) gravity

- Poisson equation

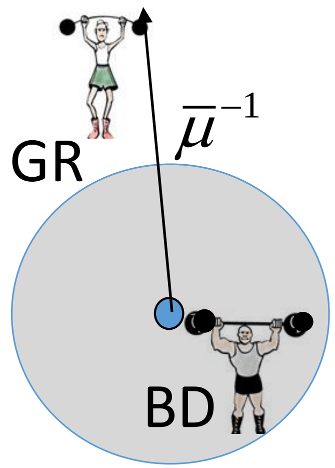
$$k^2\Psi = -4\pi G\left[\frac{4+\left(a^2\bar{\mu}^2/k^2\right)}{3+\left(a^2\bar{\mu}^2/k^2\right)}\right]a^2\rho_m\delta_m$$

large scales

$$k^2\Psi = -4\pi G a^2\rho_m\delta_m$$

Small scales

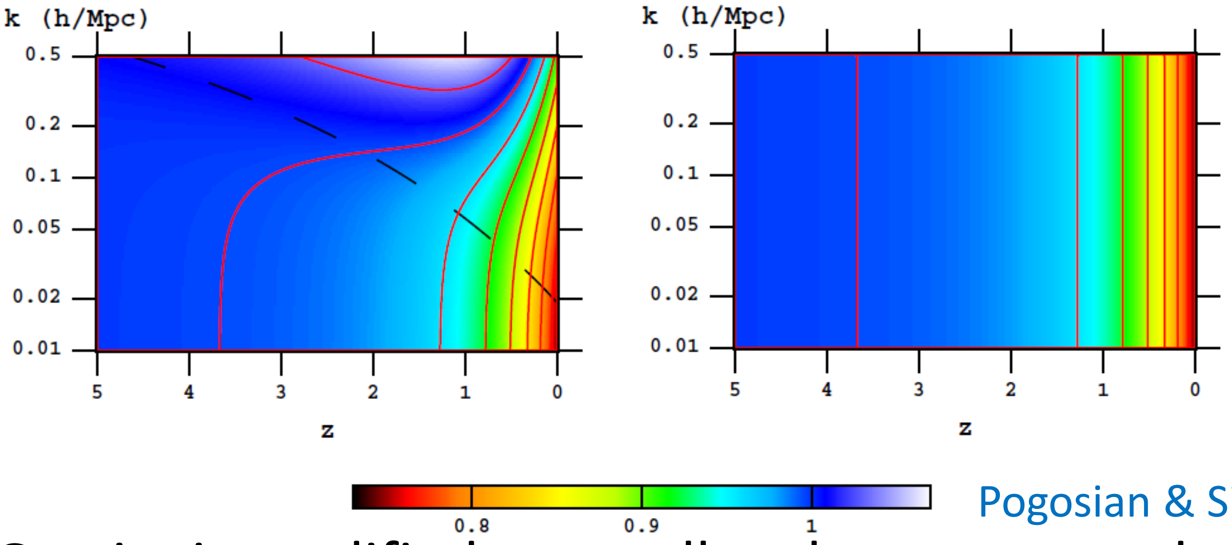
$$k^2\Psi = -\frac{16}{3}\pi G a^2\rho_m\delta_m$$

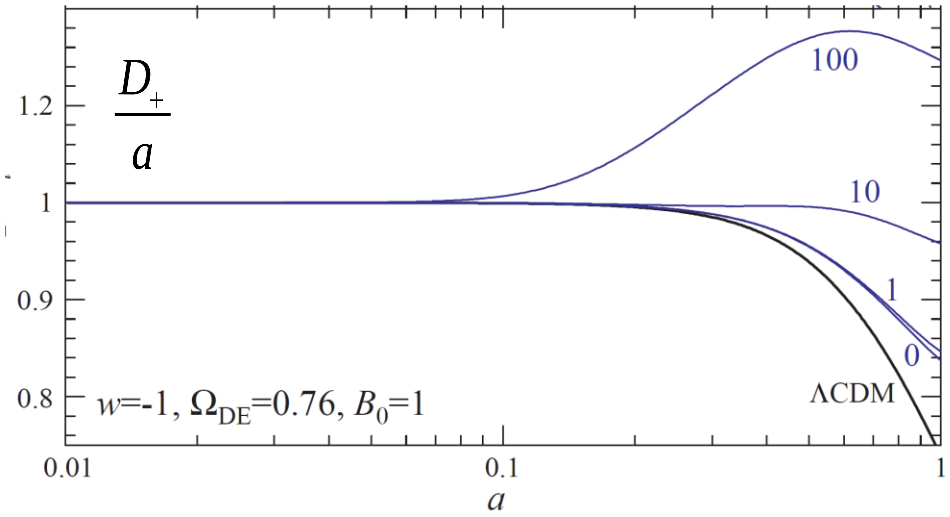


Song et.al. astro-ph/0610532

Small scales

Large scales





Pogosian & Silvestri 0709.0296

- Gravity is modified on small scales so we need screening mechanism

Classification (5)

So far, we assumed that matter and dark component obey the conservation equation independently, but this is not necessarily the case.

$$\nabla^{\mu}(T^{m}_{\mu\nu} + E_{\mu\nu}) = 0 \qquad \nabla^{\mu}T^{m}_{\mu\nu} = Q_{\nu}$$

$$\nabla^{\mu}E_{\mu\nu} = -Q_{\nu}$$

Example: $Q_{\mu} = -\alpha\,\rho_m \nabla_{\mu}\phi$

$$S = \int d^4x\sqrt{-\tilde{g}}\left(\tilde{R} - \frac{1}{2}\left(\tilde{\nabla}\phi\right)^2 - \tilde{V}(\phi)\right) + S_M[A^2(\phi)\tilde{g}_{\mu\nu}]$$

$$\Delta_m{}' = -\mathscr{H}\theta_m$$

$$\theta_m{}' + \mathscr{H}\left(1 + \frac{\mathscr{H}'}{\mathscr{H}^2} + \alpha\frac{\phi'}{\mathscr{H}}\right)\theta_m = \frac{k^2}{\mathscr{H}}(\Psi + \alpha\,\delta\phi)$$

$$A(\phi) = \exp\left(\frac{\alpha\phi}{M_{pl}}\right)$$

Although we do not modify gravity, this looks like modified gravity

(we can evade local constraints by breaking strong equivalence principle)

Zoology of DE/MG models

LCDM $\Lambda\ (\Omega_E)$

Clustering DE $(\Omega_E, w_E, \delta\rho_E)$
K-essence

Modified gravity $(\Omega_E, w_E, \delta\rho_E, \pi_E)$
(screening mechanisms)

Smooth DE (Ω_E, w_E)
Quintessence

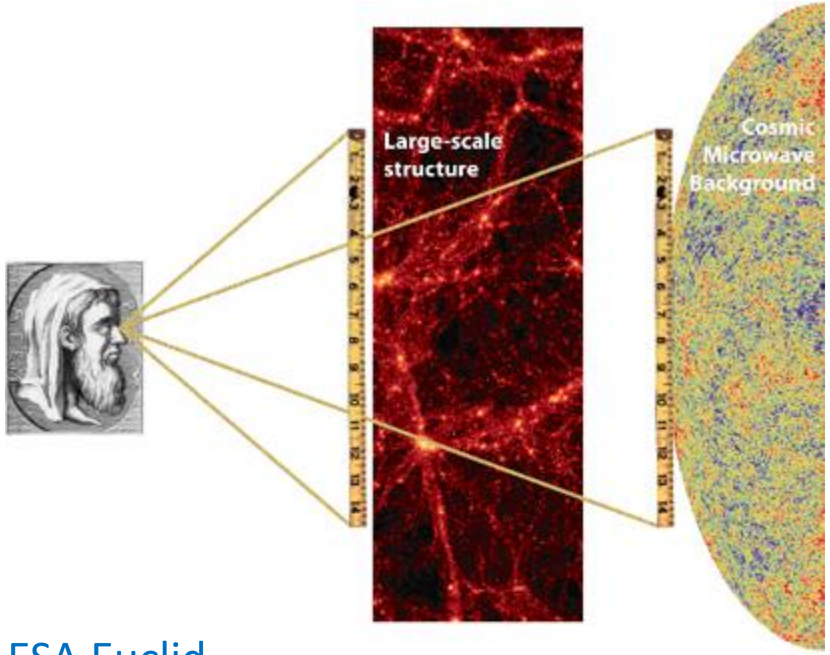
Interacting DE $(\Omega_E, w_E, \delta\rho_E, Q_E^\mu)$
(violation of equivalence principle)

Observations

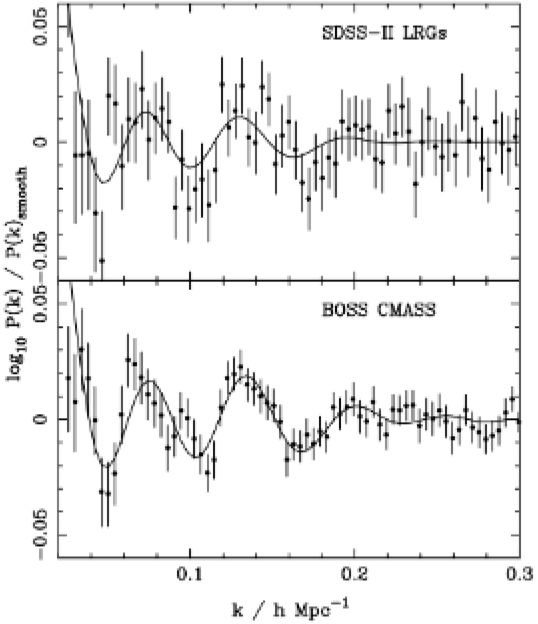
- Background $H(z)$

Supernovae: luminosity distance

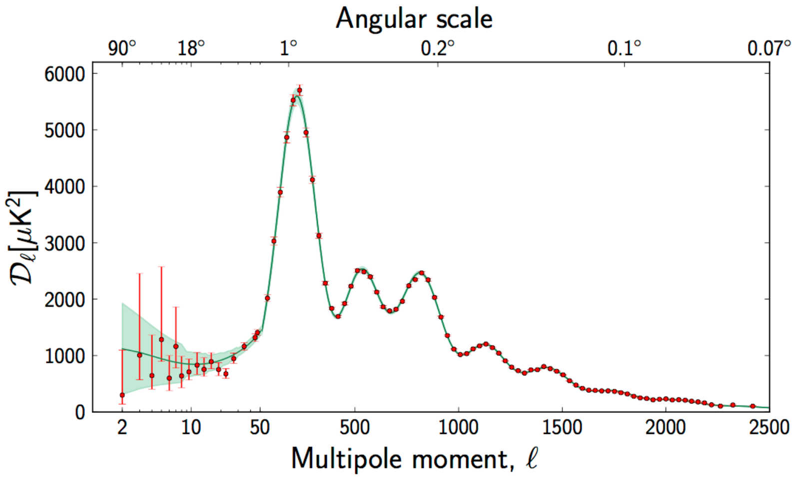
CMB/Baryon Acoustic Oscillation (BAO): angular diameter distance



ESA Euclid



BOSS



ESA Planck

Observations

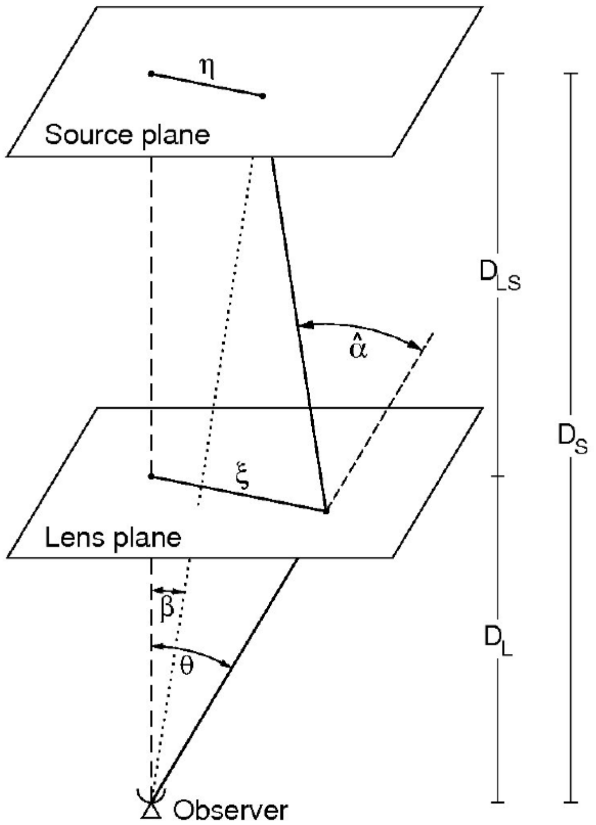
- Weak lensing

$$ds^2 = a^2\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

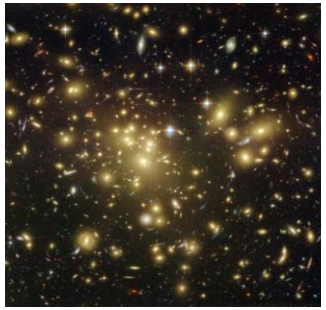
Convergence (photons follow geodesic)

$$\kappa(\vec{n}) = \int d\chi \boxed{\frac{(\chi_s - \chi)\chi}{\chi_s}} \nabla_\perp{}^2 \phi_W(\eta_0 - \chi, \chi\vec{n}), \quad \phi_W = \frac{1}{2}(\Psi + \Phi)$$

geometry

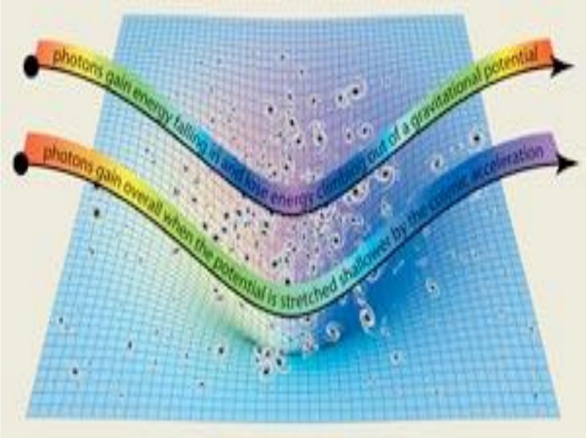


Galaxy shape is determined by shear which can be computed from convergence



Bartelmann & Schneider astro-ph/9912508

Observations



http://cmbcorrelations.pbworks.com

- CMB

  Integrated Sachs-Wolfe (ISW) effect

  The time variation of lensing potential causes

  a shift of photon temperature

$$\Theta_\ell(k) = \int_0^{\eta} d\eta \, \frac{\partial \phi_W(k,\eta)}{\partial \eta} \, j_\ell[k(\eta_0 - \eta)] \qquad \phi_W = \frac{1}{2}(\Psi + \Phi)$$

  lensing

  CMB is also lensed

$$\Theta_{lensed}(\vec{n}) = \Theta(\vec{n} + \vec{d})$$

$$\vec{d} = \vec{\nabla}\psi, \quad \psi(\vec{n}) = -2\int d\chi \, \frac{(\chi_{LSS} - \chi)\chi}{\chi_{LSS}} \phi_W(\eta_0 - \chi, \chi\vec{n}), \quad \phi_W = \frac{1}{2}(\Psi + \Phi)$$

Observations

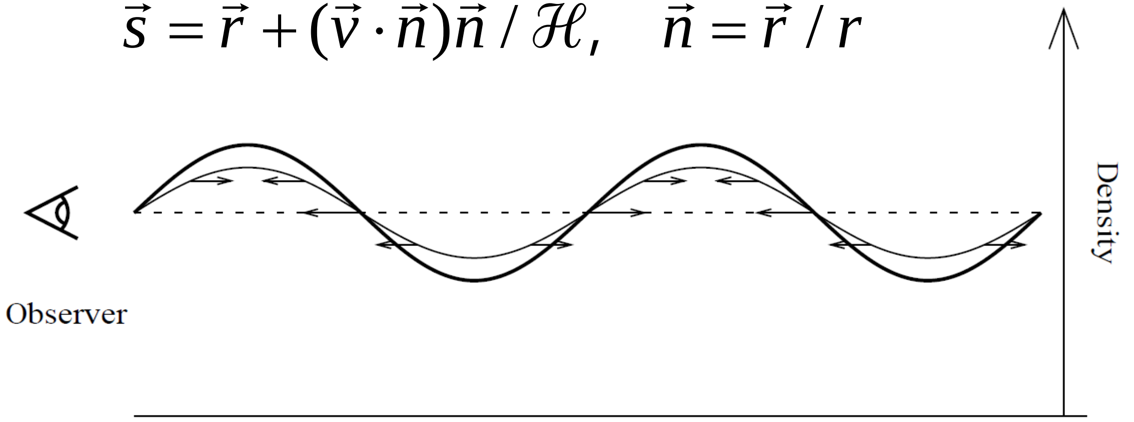
$$\vec{s} = \vec{r} + (\vec{v}\cdot\vec{n})\vec{n}/\mathscr{H}, \quad \vec{n} = \vec{r}/r$$

- Redshift distortions

  galaxies have peculiar velocities

  clustering of galaxies in redshift space

  is enhanced along the line of sight



Hamilton astro-ph/9708102

$$\delta^s(k,\mu) = \Delta_m(k) - \mu^2\theta(k), \quad \mu^2 = \frac{\left(\vec{k}\cdot\vec{n}\right)^2}{k^2}$$

If the continuity equation holds, the velocity dispersion is related to the growth rate

$$\delta^s(k,\mu) = \Delta_m(k)\left(1-\mu^2\frac{\theta(k)}{\Delta_m(k)}\right) = \Delta_m(k)\left(1+\mu^2 f\right)$$

$$\Delta_m{}' = -\mathscr{H}\theta_m$$

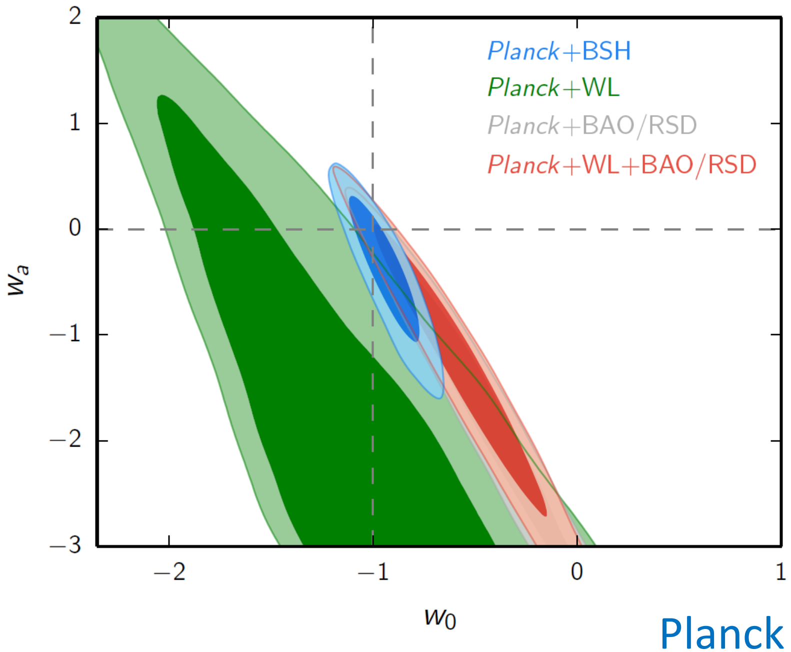
$$f = \frac{d\ln\Delta_m}{d\ln a} = \frac{a}{D_+}\frac{dD_+}{da}$$

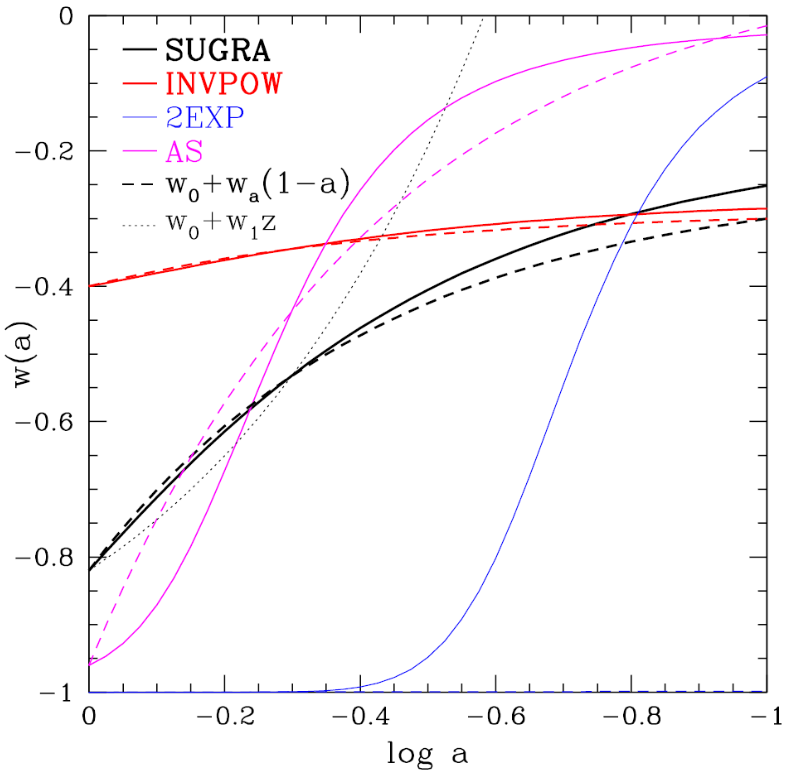
Background expansion history

- Background expansion is determined by the equation of state

Parametrisation

$$w_{DE} = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z}$$





Planck collaboration 1502.01590

Linder astro-ph/0311403

Model independent approach

- Principal component analysis

Approximate $w_{DE}(z)$ with many stepwise constant values

$$1+w_{DE}(z)=\sum_{i=1}^{N} w_i\theta_i(z)$$

Errors on w_i are highly correlated. We diagonalise the covariant matrix

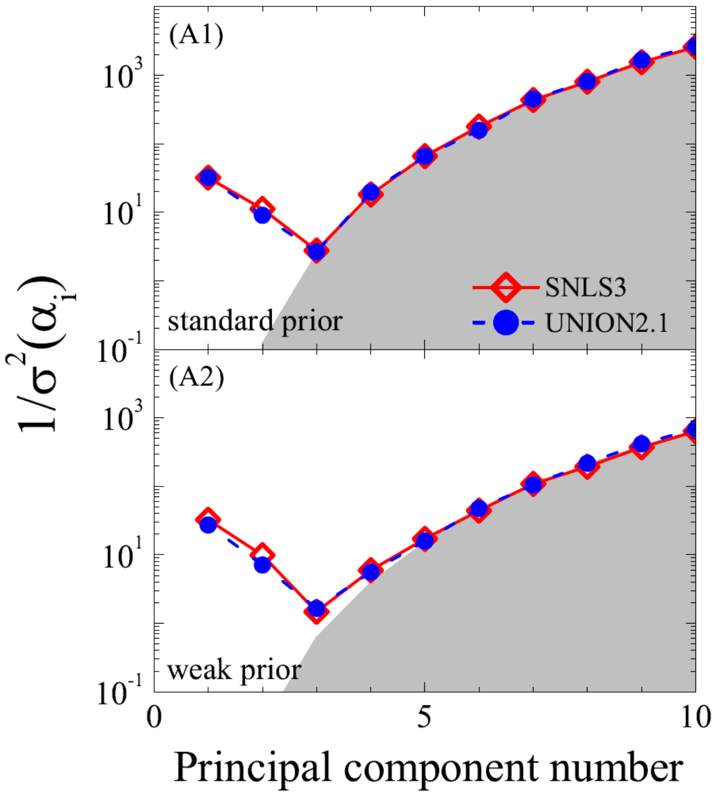
$$C_{ij}=\left\langle (w_i-\bar{w}_i)(w_j-\bar{w}_j)\right\rangle \quad C=W\,\Lambda^{-1}\,W^T, \quad W=(\vec{e}_1,\vec{e}_2,...,), \quad \Lambda_{ij}=\lambda_i\delta_{ij}$$

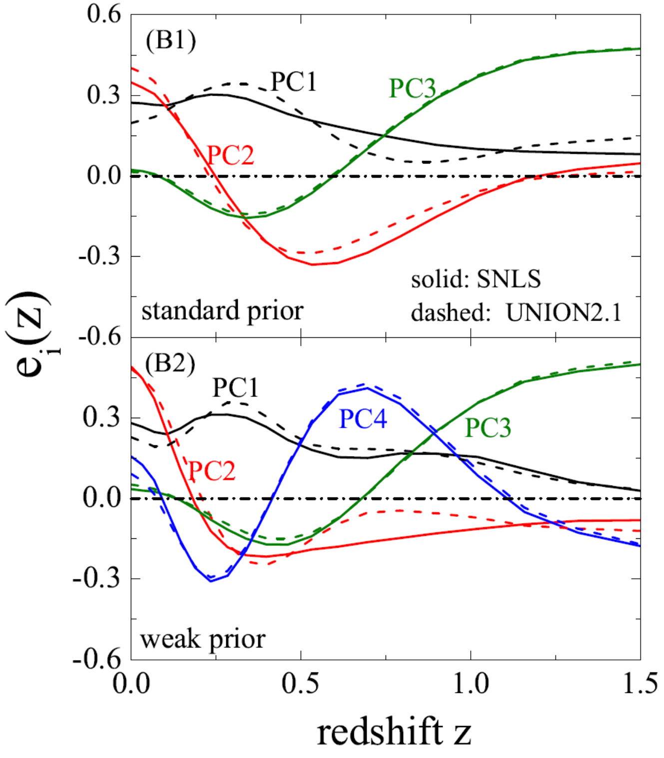
$$1+w_{DE}(z)=\sum_{i=1}^{N}\alpha_i e_i(z)$$

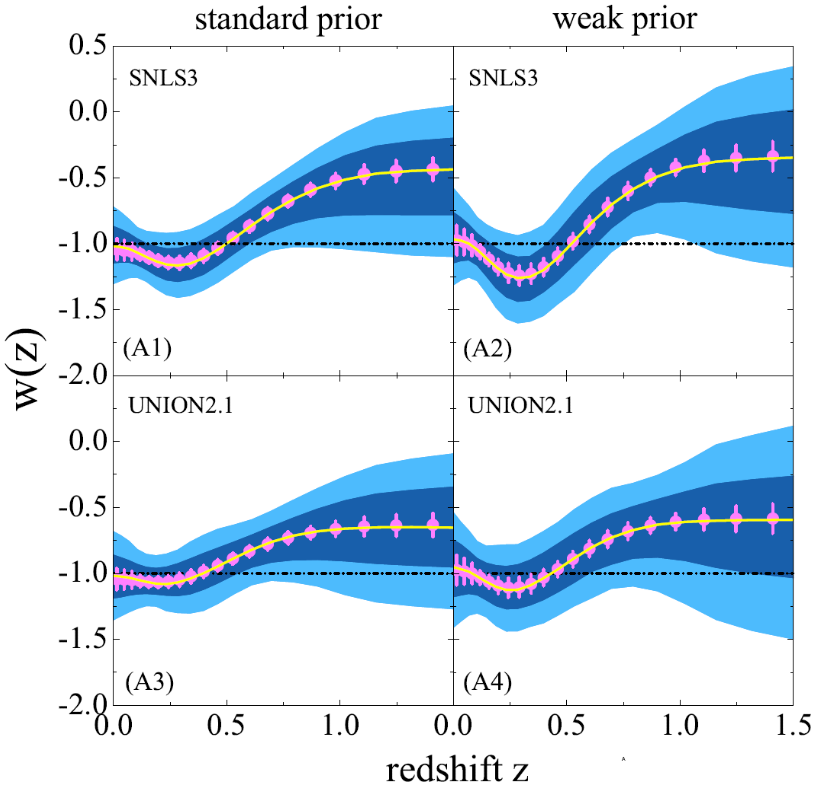
Errors on new parameters $\alpha_i=\sum_j W_{ij}(w_j-\bar{w}_j)$ are uncorrelated and given by $\sqrt{\lambda_i}$

Principal component

- Requires a prior to truncate poorly determined eigen modes







Zhao et.al. 1207.3804

Model based parametrisation

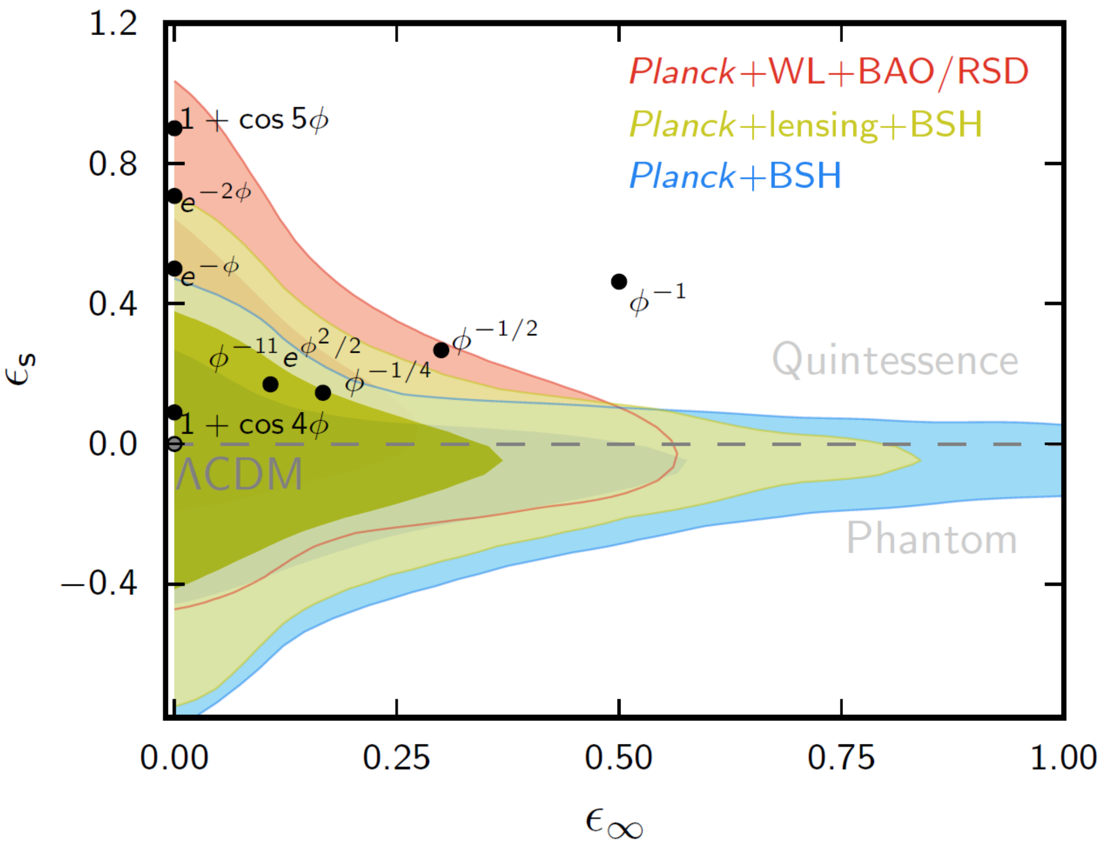
- Parametrisation

$$1+w_\phi \approx \frac{2}{3}\varepsilon_\phi \Omega_\phi(a)$$

$$\varepsilon_\phi = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2, \quad \Omega_\phi = \frac{\rho_\phi}{\rho_\phi + \rho_m}$$

$$\varepsilon_s = \varepsilon_\phi(\rho_m = \rho_{DE})$$

$$\varepsilon_\infty = \varepsilon_\phi \Omega_\phi(a \to 0)$$

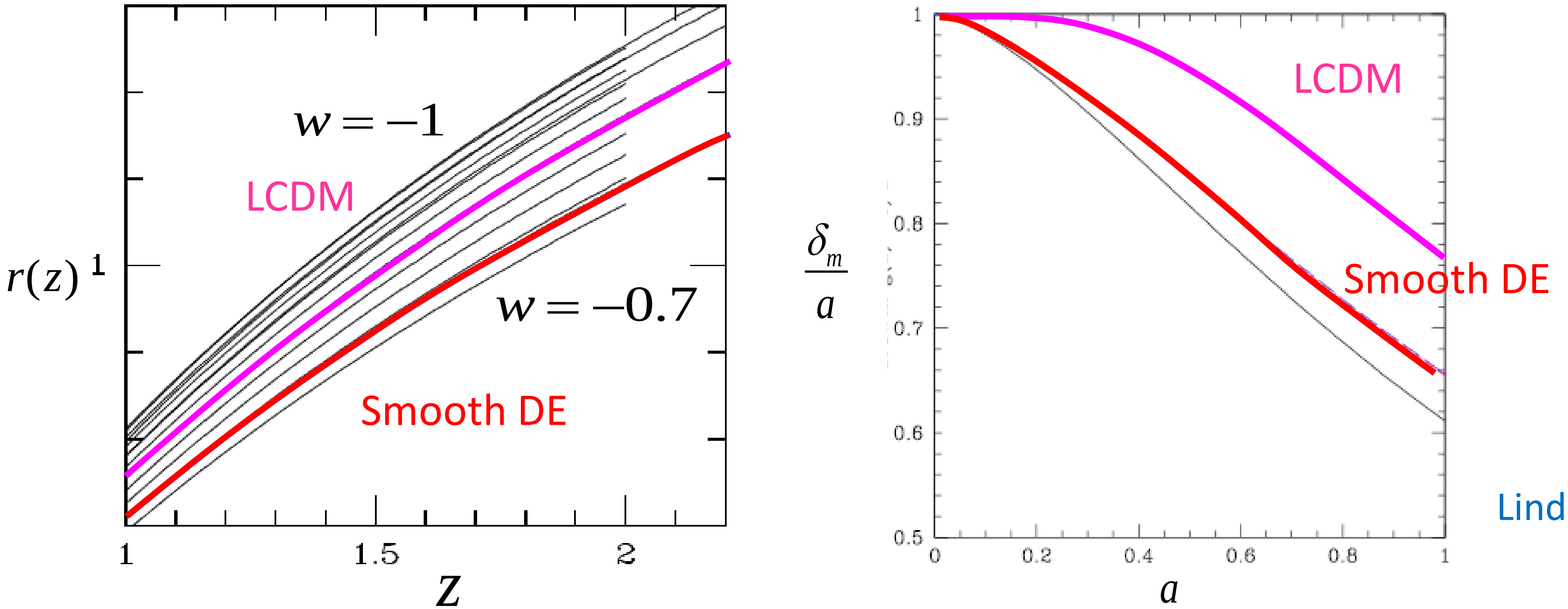


Planck 1502.01590

Expansion history v structure growth

- LCDM/Smooth DE

There is a one-to-one correspondence between background expansion history and growth of structure



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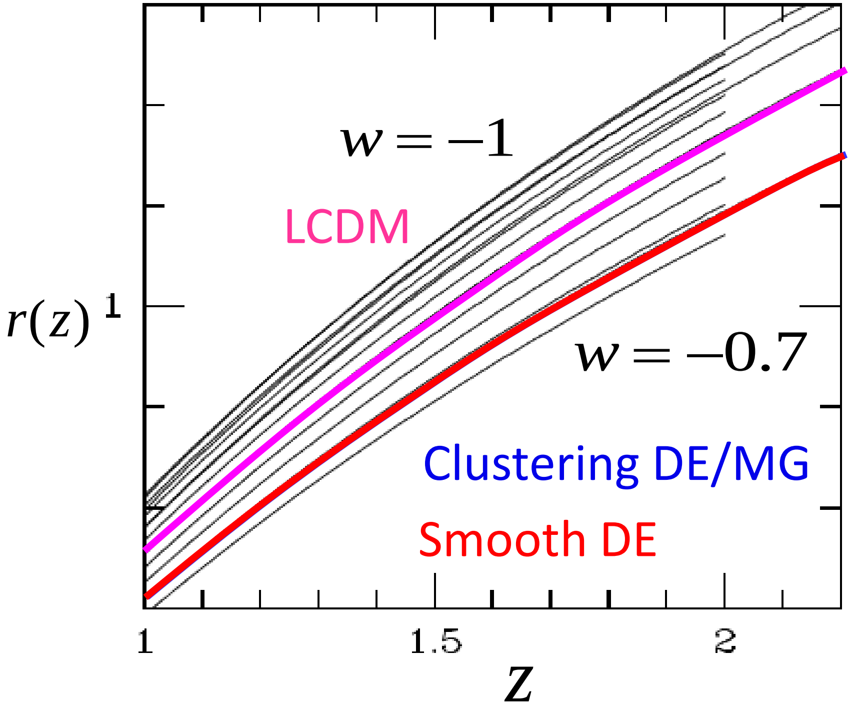
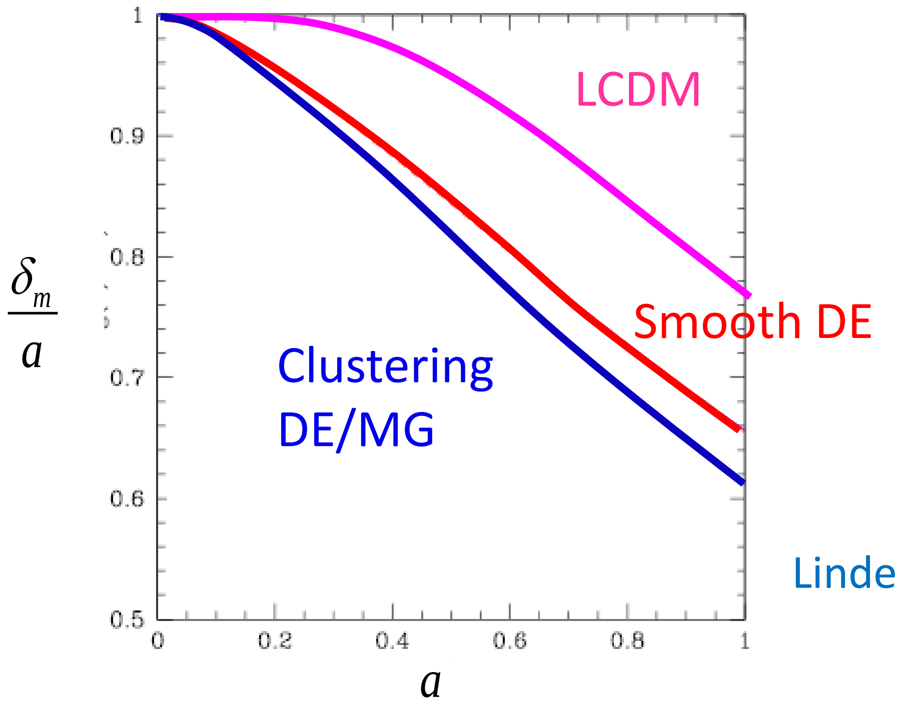
Expansion history v structure growth

- Clustering DE/MG

structure growth is controlled also by $\delta\rho_E$

Even if it has the same expansion history as smooth DE, structure growth is different





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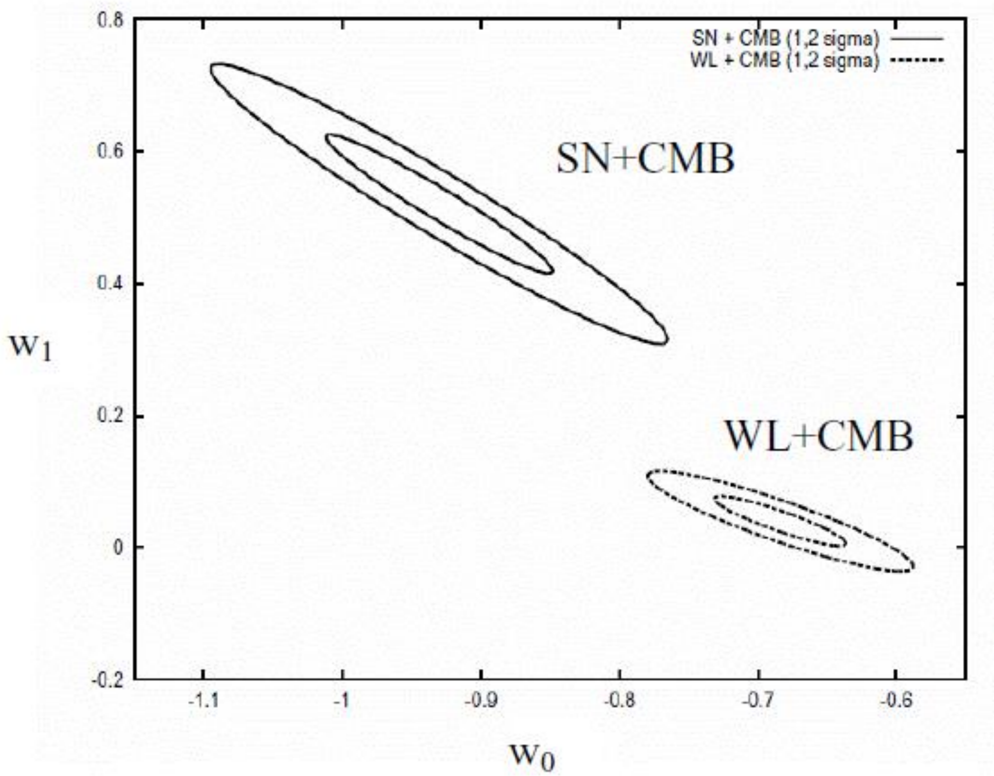
Consistency test

Assume that the Universe is described by a clustering DE/MG model but we still try to fit the date using smooth DE

$$w(z) = w_0 + w_1 z,$$



SNe+CMB

SNe+weak lensing

Inconsistent!

Ishak et.al. astro-ph/0507184

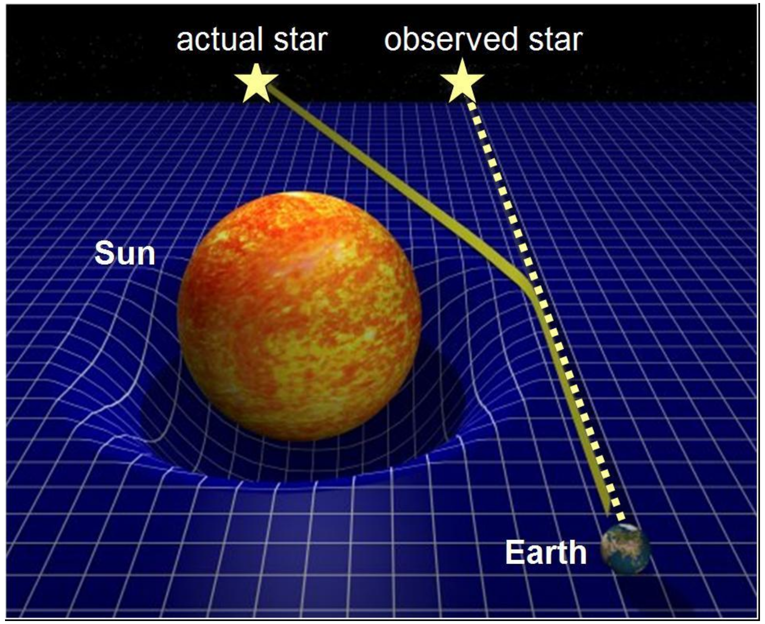
Clustering DE v Modified gravity

- Modified gravity models have anisotropic stress

cf. BD gravity with $\omega_{BD} = 0$ $\quad 4\pi G a^2 P_E \Pi_E = -4\pi G a^2 \rho_E \Delta_E = -\frac{1}{2} k^2 \varphi$

this creates a difference between lensing potential and Newton potential

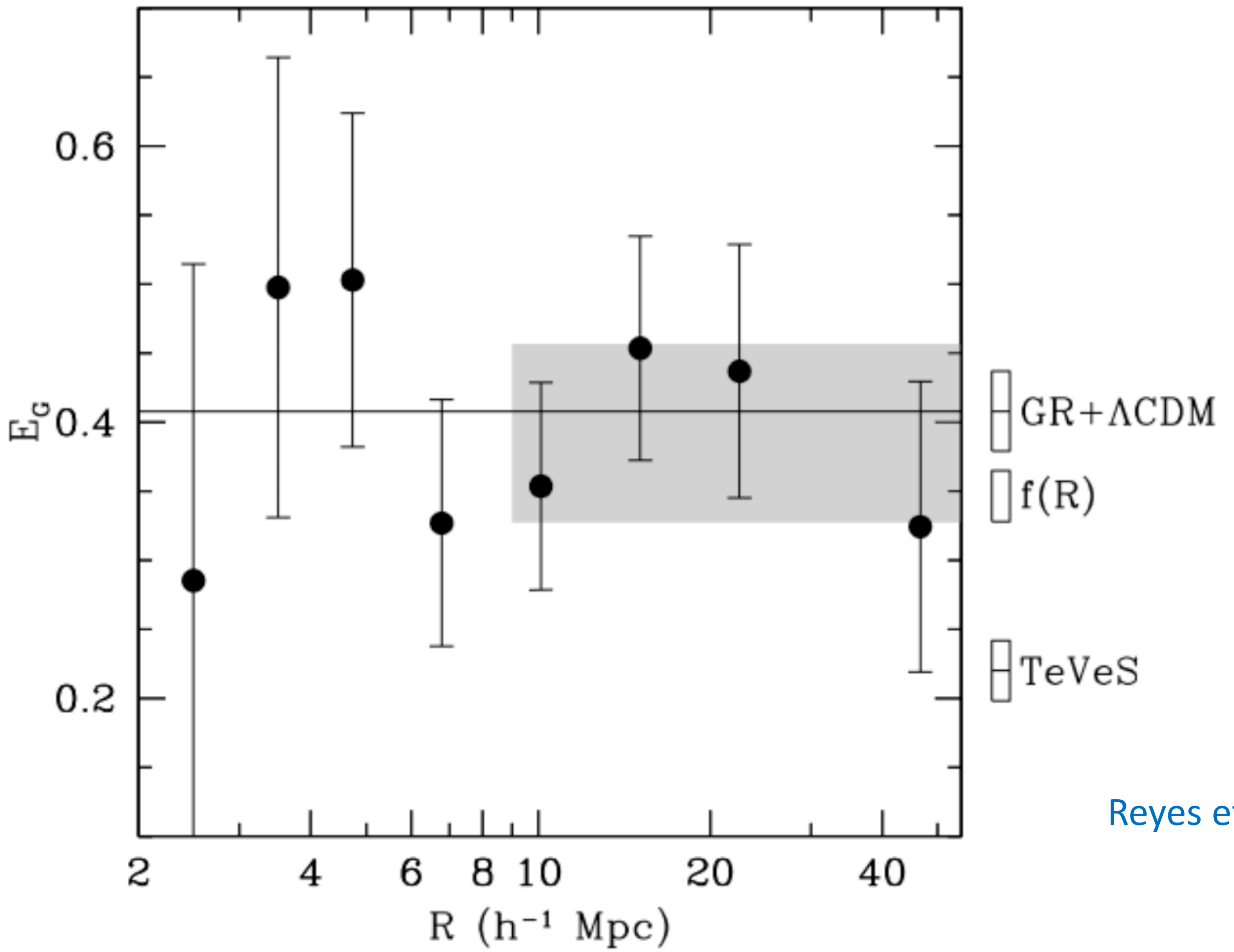
$$\phi_W = \frac{1}{2}(\Phi + \Psi) \neq \Psi \qquad k^2(\Psi - \Phi) = -8\pi G a^2 P_E \, \Pi_E$$



lensing mass is not the same as dynamical mass

ex.) Eg parameter

$$\langle E_G \rangle = \frac{\nabla^2(\Psi + \Phi)}{-3H_0^2 a^{-1} \theta_m} \qquad \theta_m' + \mathcal{H}\left(1 + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\theta_m = \frac{k^2}{\mathcal{H}}\Psi$$



Reyes et.al. 1003.2185