Observational tests and non-linear structure formation

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Parametrisation

- Dark component

  We need to specify $(\delta P_E, \pi_E)$

- Parametrisation of Einstein equations

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho_m \Delta_m$$
$$\Phi = \eta(k,a)\Psi$$

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

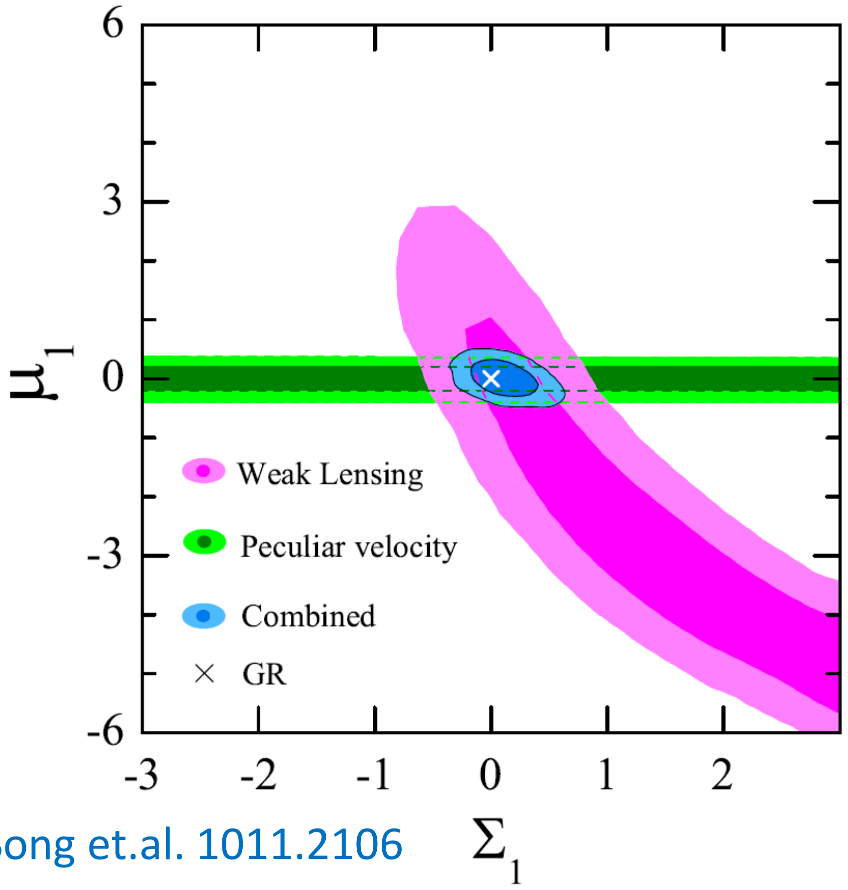
equivalently, we can also parametrise the lensing potential

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho_m \Delta_m$$
$$k^2\frac{(\Psi+\Phi)}{2} = -4\pi G a^2 \Sigma(k,a)\rho_m \Delta_m, \quad \Sigma = \frac{\mu(1+\eta)}{2}$$

$\mu = \eta = \Sigma = 1$ for smooth DE

Weak lensing and Redshift Distortions

- Combining WL ($\phi_W = (\Phi + \Psi)/2$) and RSD (θ_m) we can break the degeneracy



Song et.al. 1011.2106

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho_m \Delta_m$$

$$k^2(\Psi + \Phi) = -4\pi G a^2 \Sigma(k,a)\rho_m \Delta_m, \quad \Sigma = \frac{\mu(1+\eta)}{2}$$

$$\theta_m' + \mathcal{H}\left(1 + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\theta_m = \frac{k^2}{\mathcal{H}}\Psi$$

$$\Delta_m'' + \mathcal{H}\Delta_m' + 4\pi G a^2 \mu(k,a)\rho_m \Delta_m = 0$$

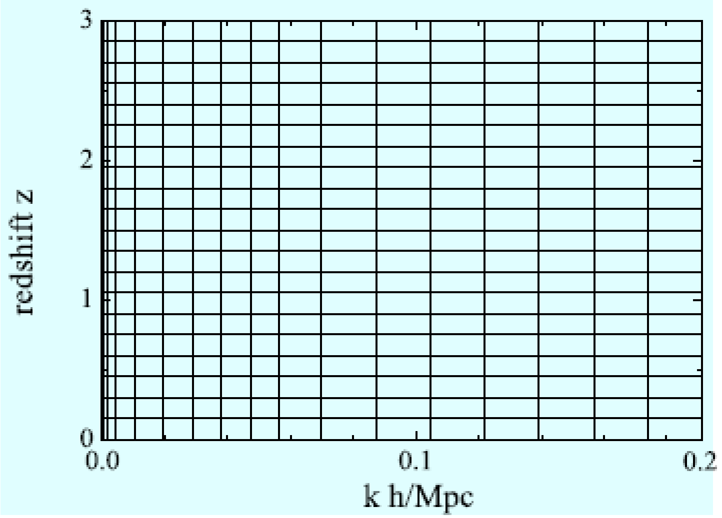
$$\mu(a) = 1 + \mu_1 a, \quad \Sigma(a) = 1 + \Sigma_1 a$$

Model independent constraints

- Make bins

  treat $\mu(k_i, z_i), \eta(k_i, z_i)$ in each bin as parameters

  Errors on these parameters are highly correlated



- Principal component analysis

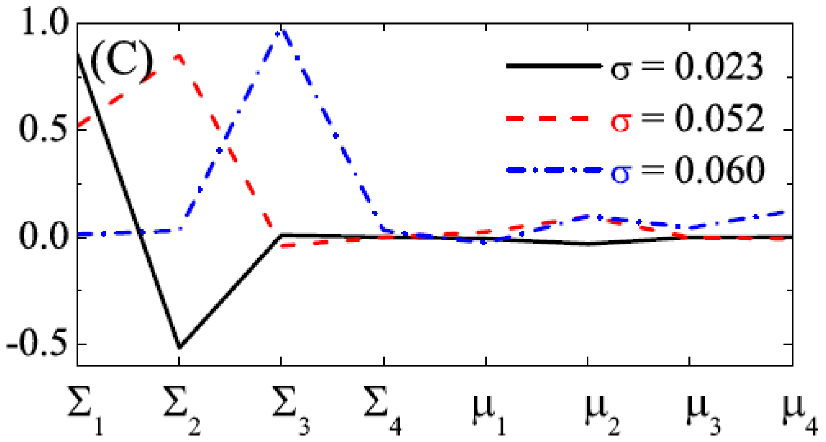
  Diagonalise the covariant matrix

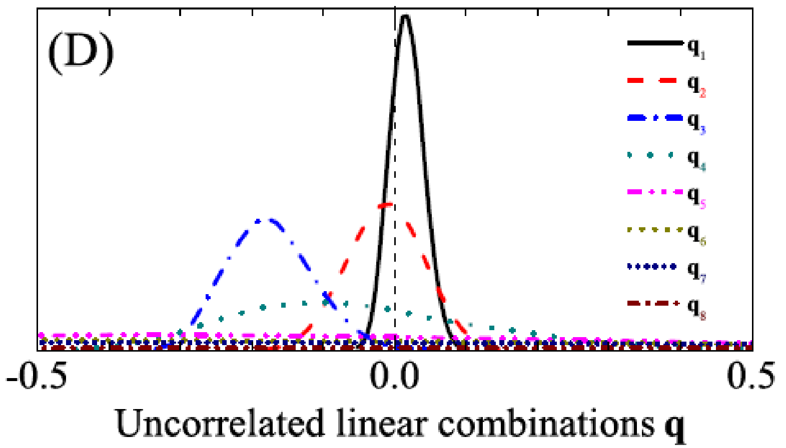
$$C_p = W\ \Lambda^{-1}\, W^T, \quad W = (\vec{e}_1, \vec{e}_2, \ldots,)$$

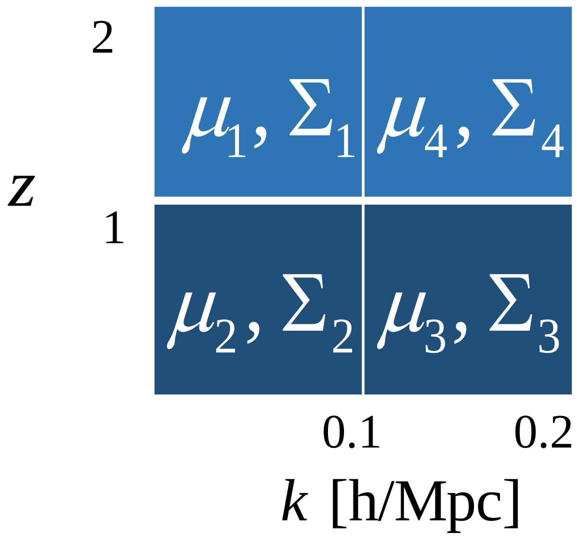
$$p = \{\mu_{1,} \ldots, \Sigma_1, \ldots.\}$$

  Uncorrelated parameter $q_i = -1 + \sum_j W_{ij} p_j \,/\, \sum_j W_{ij}$

Early attempts







q_1	$0.0 \pm 0.02 \pm 0.04$
q_2	$0.0 \pm 0.05 \pm 0.10$
q_3	$-0.17 \pm 0.06^{+0.13}_{-0.11}$
q_4	$-0.05 \pm 0.17^{+0.37}_{-0.28}$
q_5	$-0.10 \pm 0.52^{+1.1}_{-0.81}$
q_6	$-0.17 \pm 0.79^{+1.7}_{-1.2}$
q_7	$-0.02^{+1.1+2.1}_{-1.0-2.0}$
q_8	$-0.25 \pm 3.2^{+6.0}_{-5.2}$

$$q_i = -1 + \sum_j W_{ij} p_j / \sum_j W_{ij}$$

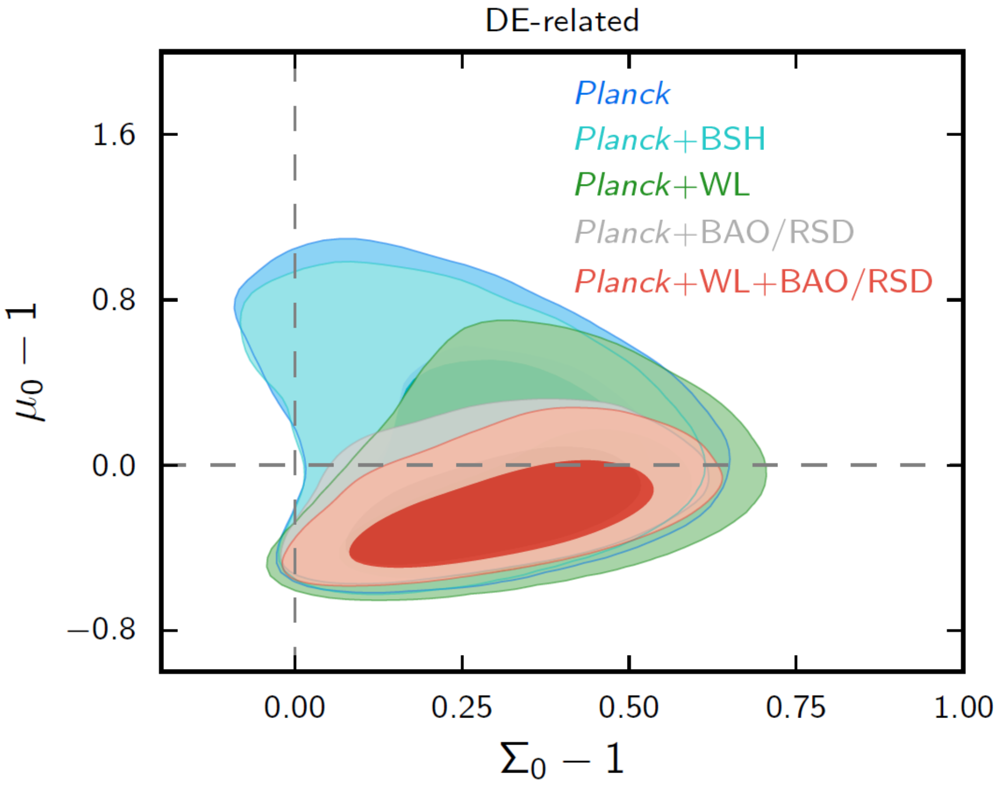
Zhao et.al. 1003.0001

Planck 2015 results

- Assuming LCDM background

$$\mu(a) = 1 + \bar{\mu}\,\Omega_{DE}(a), \quad \Sigma(a) = 1 + \bar{\Sigma}\,\Omega_{DE}(a)$$

$$\mu_0 = \mu(1), \quad \Sigma_0 = \Sigma(1)$$



Max. degeneracy	*Planck* TT+lowP	*Planck* TT+lowP +BSH	*Planck* TT+lowP +WL	*Planck* TT+lowP +BAO/RSD	*Planck* TT+lowP +WL+BAO/RSD
DE-related	$0.84^{+0.30}_{-0.40}$ (2.1σ)	$0.80^{+0.28}_{-0.39}$ (2.1σ)	$1.08^{+0.35}_{-0.42}$ (2.6σ)	$0.90^{+0.33}_{-0.37}$ (2.4σ)	1.03 ± 0.34 (3.0σ)
+ CMB lensing	$0.42^{+0.18}_{-0.34}$ (1.2σ)	$0.38^{+0.18}_{-0.28}$ (1.4σ)	$0.58^{+0.24}_{-0.37}$ (1.6σ)	$0.40^{+0.18}_{-0.28}$ (1.4σ)	$0.51^{+0.21}_{-0.30}$ (1.7σ)

tension with LCDM

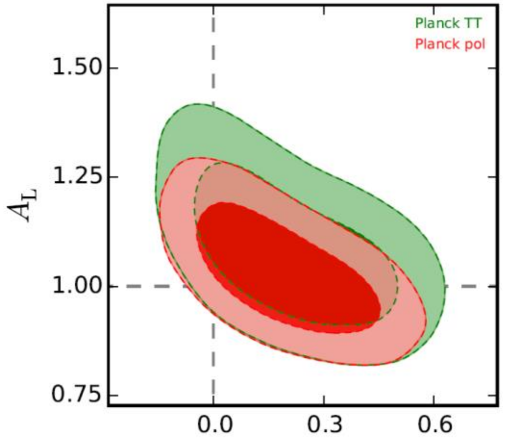
Planck 1502.01590

Tension with LCDM in Planck data

- Lensing amplitude

CMB lensing requires a larger amplitude than LCDM in the power spectrum $A_{lens} = 1.22 \pm 0.10$

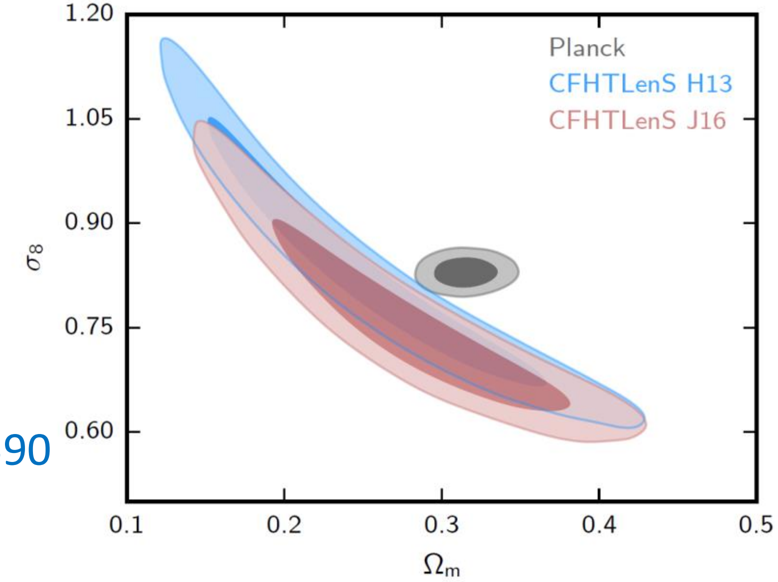
(cf. this tension does not exist for lensing measured from trispectrum)



Valentino et.al. 1509.07501

- Amplitude of fluctuations

The late time amplitude of fluctuations in LCDM predicted from primordial amplitude measured by CMB is larger than that measured by weak lensing (CFHTLS) Planck 1502.01590



Theory based parametrisation

- Effective theory approach

Consider a slowly varying scalar field.

We can define time using this scalar field (unitary gauge) $\phi = \text{const.}$

All information is contained in metric. We construct theory using quantities that respect 3D diffeomorphism invariance $x^i \to x^i + \xi^i$

K_{ij} : extrinsic curvature, R : 3D Ricci curvature, N : lapse

$$S^{(2)} = \int d^3x dt a^3 \frac{M^2}{2} \left[\delta K_{ij} \delta K^{ij} - \delta K^2 + (1+\alpha_T)\left(R \frac{\delta\sqrt{h}}{a^3} + \delta_2 R \right) + \alpha_K H^2 \delta N^2 + 4\alpha_B H \, \delta K \, \delta N + (1+\alpha_H) R \, \delta N \right]$$

$$\alpha_M \equiv \frac{1}{H} \frac{d}{dt} \ln M^2$$

Gleyzes et.al. 1411.3712

From theory to phenomenology

- Six free functions of time

  $M, \alpha_M, \alpha_K, \alpha_T, \alpha_B, \alpha_H$

- Phenomenological parameters can be related to these functions

$$\mu(a,k) \equiv -\frac{2H^2 k_H^2 \Psi}{\kappa^2 \rho_{\rm m} \Delta_{\rm m}} = \frac{1}{\kappa^2 M^2} \frac{\mu_{+2} k_H^2 + \mu_{+4} k_H^4 + \mu_{+6} k_H^6}{\mu_{-0} + \mu_{-2} k_H^2 + \mu_{-4} k_H^4 + \mu_{-6} k_H^6}$$

$$k_H = k / aH$$

$$\gamma(a,k) \equiv -\frac{\Phi}{\Psi} = \frac{\gamma_{+0} + \gamma_{+2} k_H^2 + \gamma_{+4} k_H^4}{\mu_{+2} + \mu_{+4} k_H^2 + \mu_{+6} k_H^4},$$

Lombriser & Taylor 1505.05915

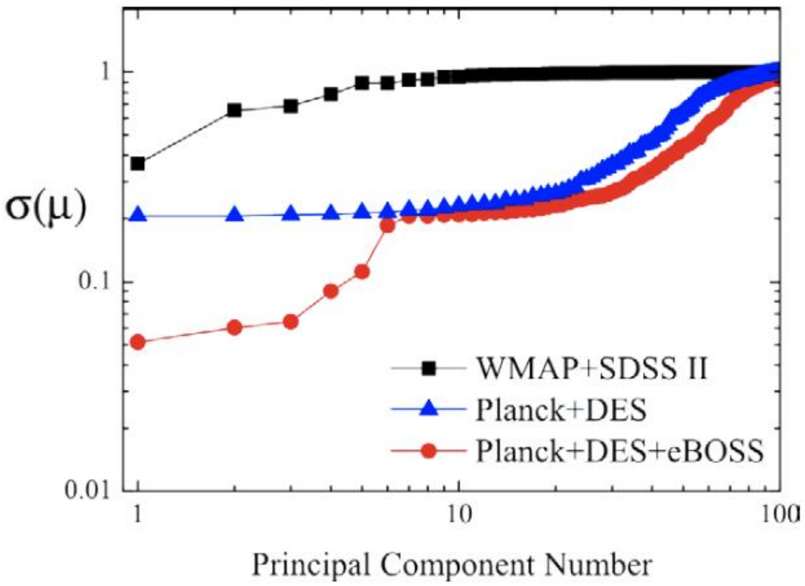
- Modified Boltzman codes are available

  (EFTCAMB: 1405.3590, hi_class arXiv:1605.06102)

- It is a challenge to directly constrain these functions

Future forecasts

Next 3-5 years



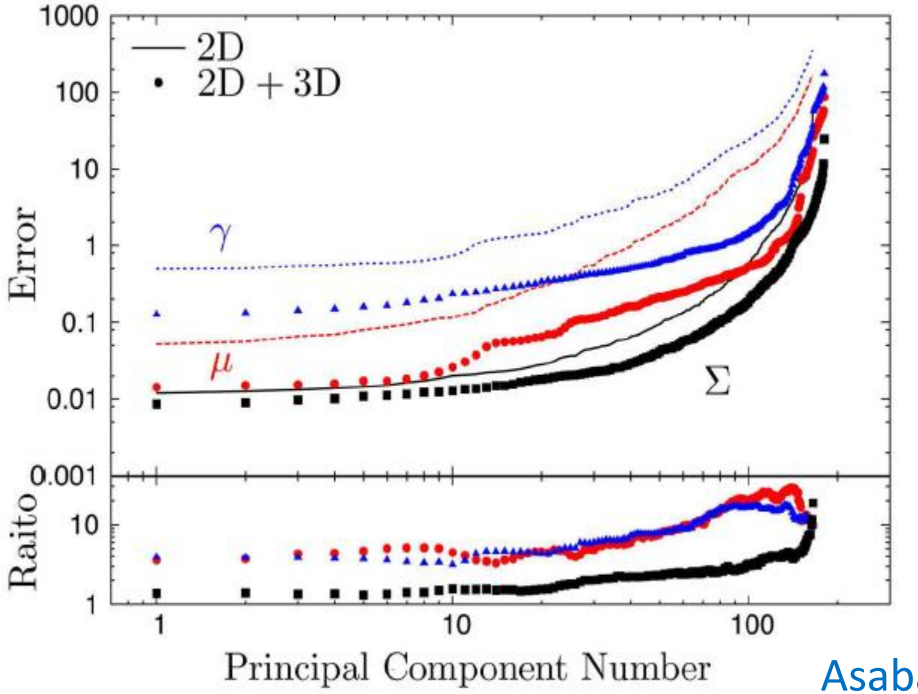
Zhao et.al. 1510.08216

DES (2012-2017) imaging

eBOSS (2014-2018) spectroscopic

Several parameters at the 5-10% level

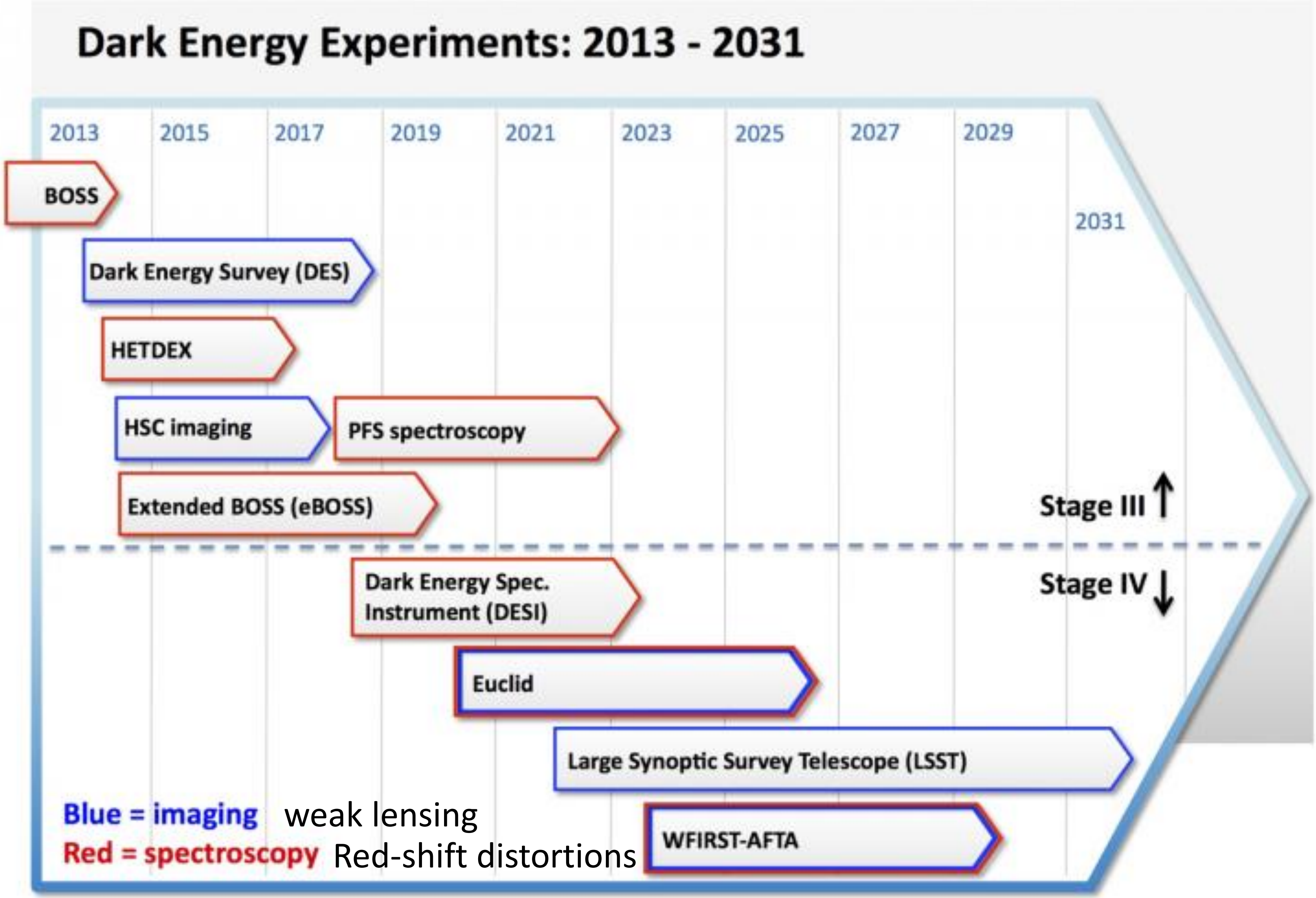
Next 5-10 years



Asaba et.al. 1306.2546

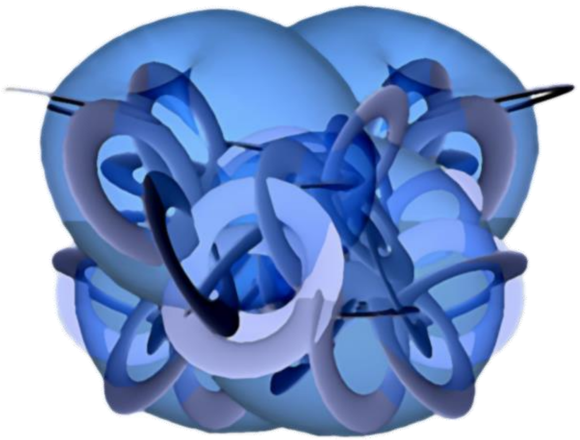
Euclid (2020-)

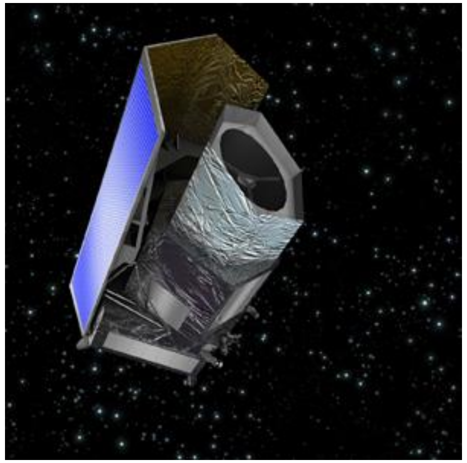
10 parameters at the 1% level



Abazajian et.al. Dark Energy and CMB

From theory to observations





initial conditions

fluid approach
Effective Field Theory

many free functions

$$\mu(k_i, z_i), \Sigma(k_i, z_i)$$

$$C_\ell^{IJ}(z)$$

galaxy count
Lensing
ISW
systematics

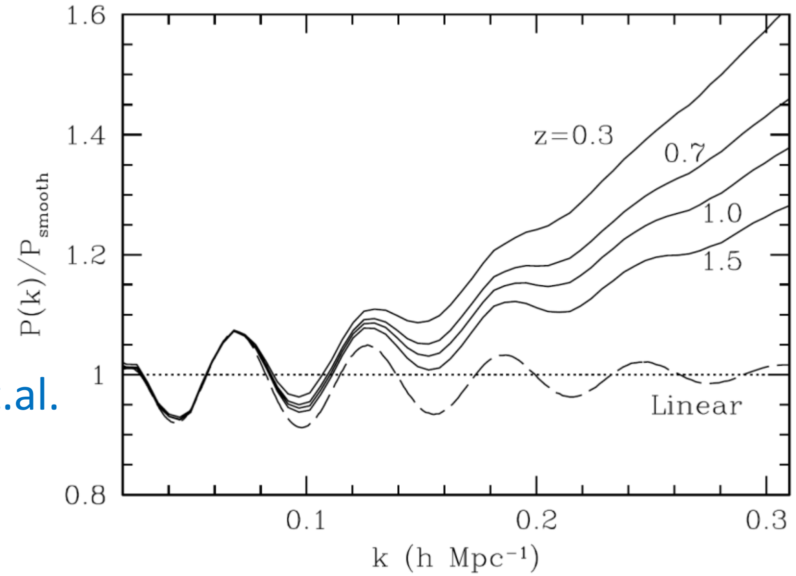
Non-linearity

- So far we only consider linear perturbations

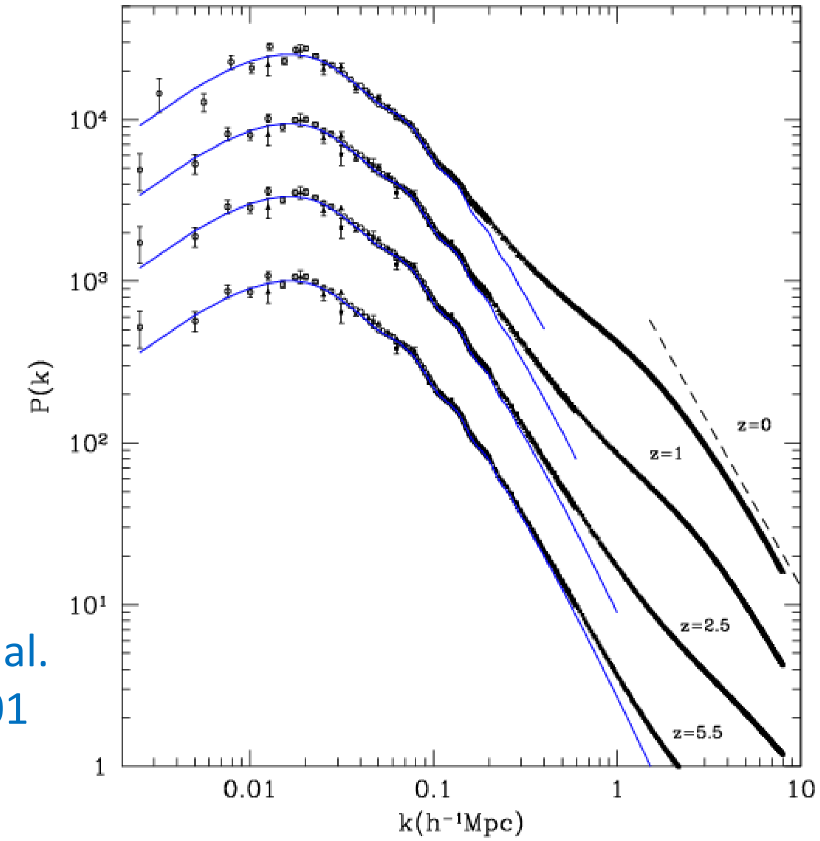
  density perturbations become non-linear at late times on small scales

  Power spectrum

$$\left\langle \Delta_m(\vec{k})\Delta_m(\vec{k}')\right\rangle = (2\pi)^3\,\delta^{(3)}(\vec{k}+\vec{k}')P(k)$$



Weinberg et.al. 1201.2434



Klypin et.al. 1411.4001

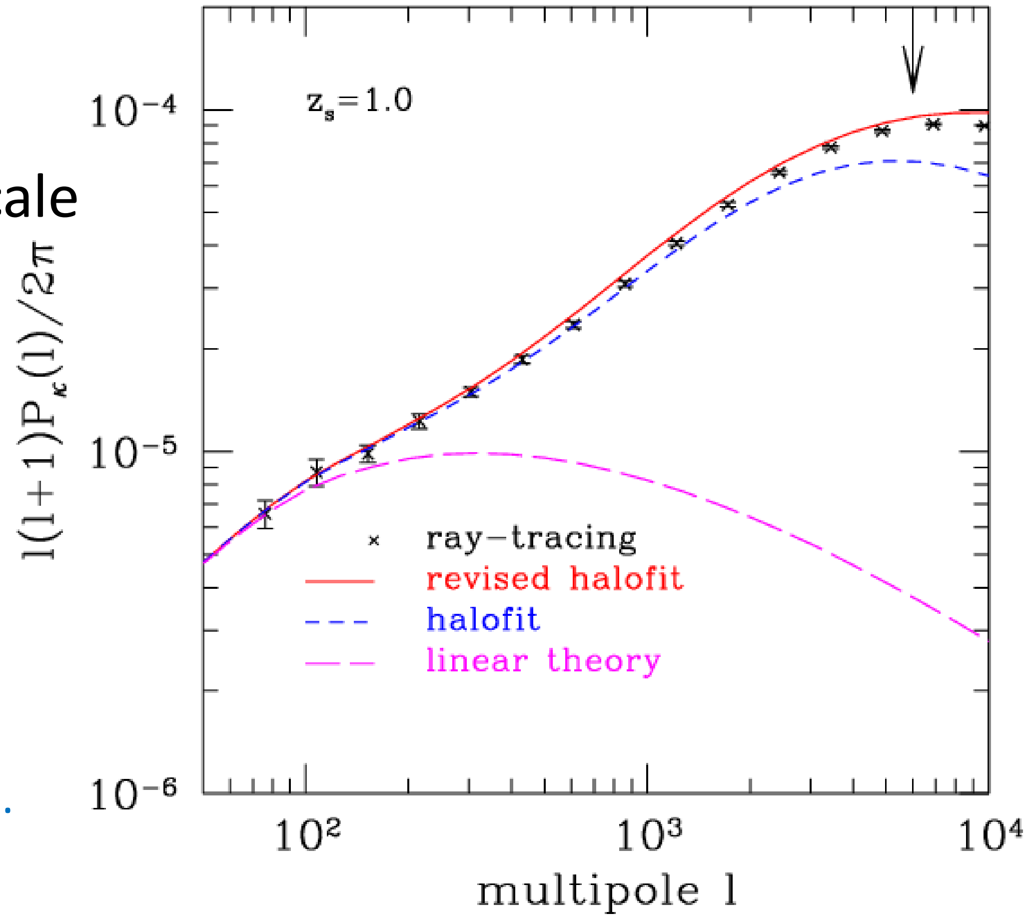
Importance of non-linearity

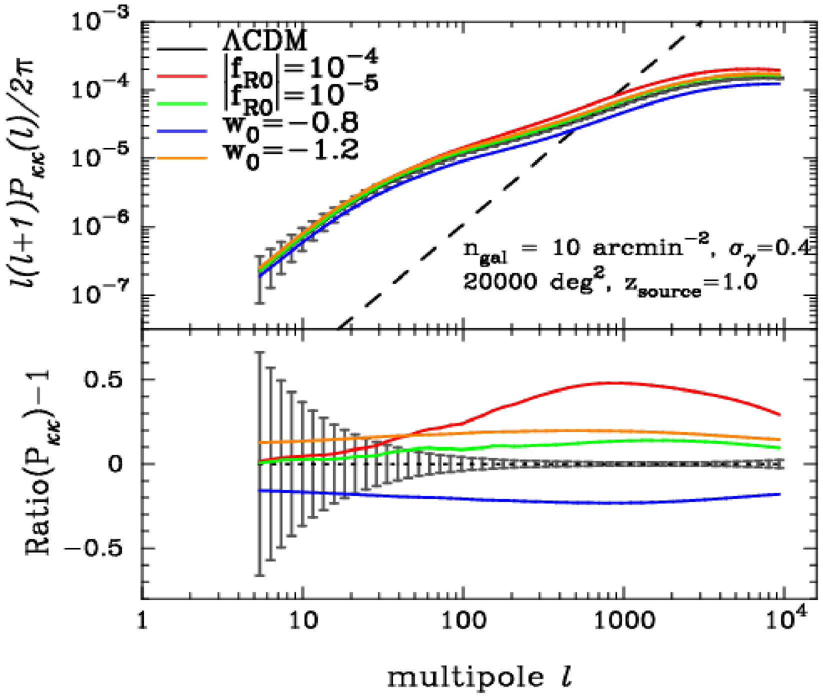
- Weak lensing

  Convergence power spectrum

  most information comes from non-linear scale

Takahashi et.al.
1208.2701





Shiraishi et.al.
1508.02104

Importance of non-linearity

Taruya et.al. 1006.0699

- Redshift distortions

Power spectrum in redshift space

$$P(k,\mu),\ \ \mu = \vec{n}\cdot\vec{k}/k$$

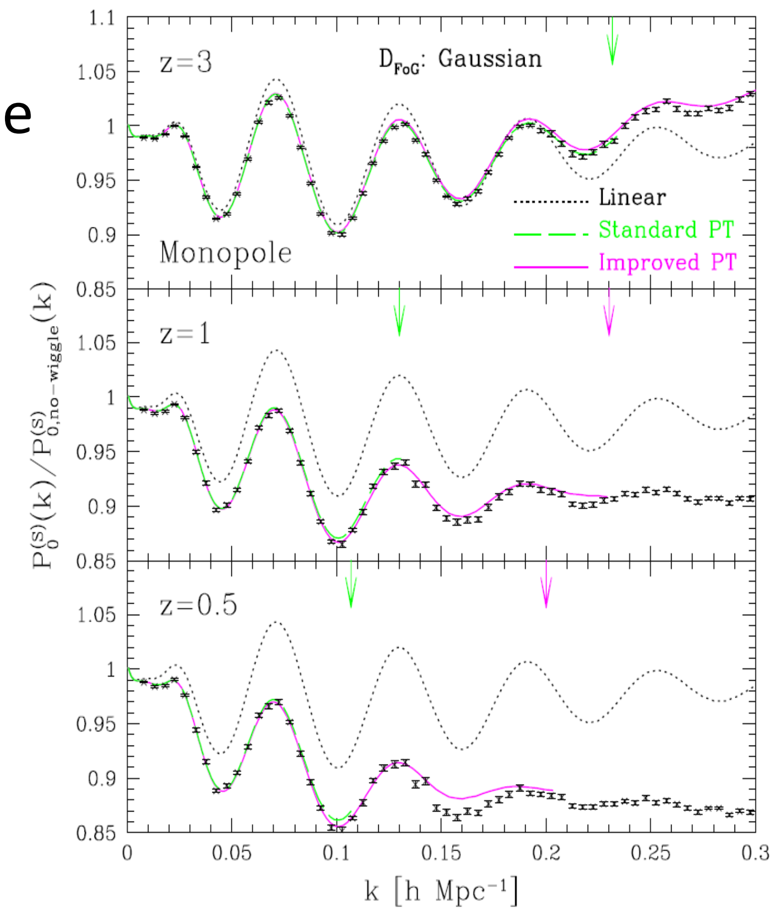
$$P(k,\mu) = \sum_{\ell} P_{\ell}(k) L_{\ell}(\mu)$$

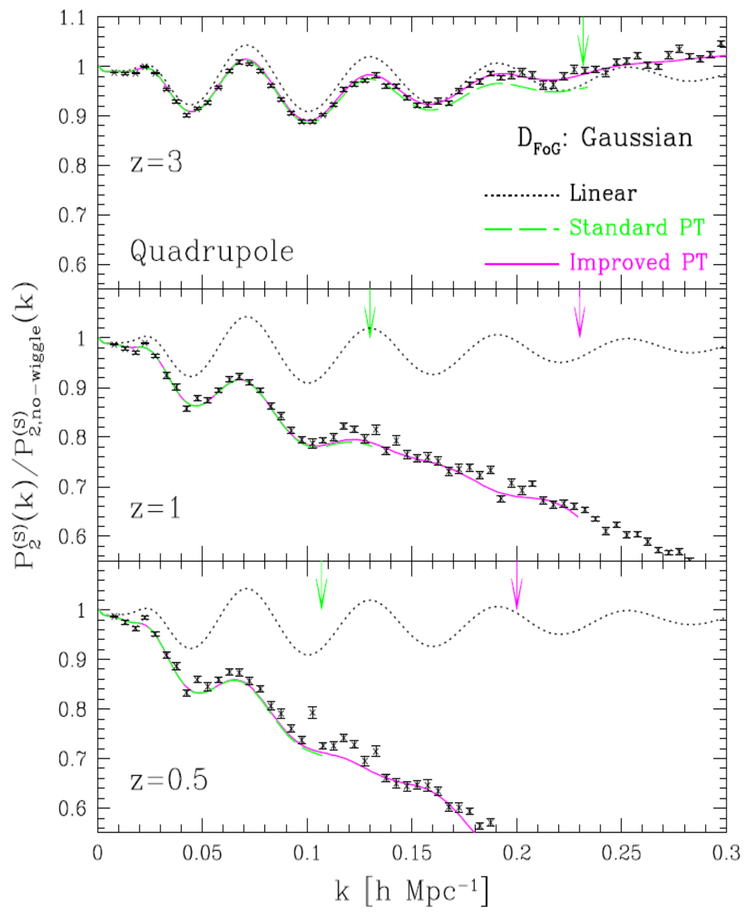
linear theory (Kaiser)

$$\delta^s(k) = \Delta_m(k)\left(1 + f\mu^2\right)$$

$$P_0(k) = \left(1 + \frac{2}{3}f + \frac{1}{5}f^2\right)P(k)$$

$$P_2(k) = \left(\frac{4}{3}f + \frac{4}{7}f^2\right)P(k)$$





Non-linear structure formation

- Structure becomes non-linear on small scales

We rely on the fact that GR can be approximated as Newtonian theory

- Fluid approximation

dark matter particles can be approximated as a pressure-less fluid (with no interactions)

$$\left(\frac{\partial \rho}{\partial t}\right)_r + \nabla_r \cdot (\rho \vec{u}) = 0,$$

$$\left(\frac{\partial \vec{u}}{\partial t}\right)_r + \left(\vec{u} \cdot \nabla_r\right)\vec{u} = -\nabla_r \Psi_N$$

Fluid equations

- Moving to the comoving coordinate $\vec{r} = a(t)\vec{x}$ and separate the background

$$\vec{u} = \dot{a}\,\vec{x} + \vec{v}(\vec{x},t), \quad \delta = \frac{\rho(\vec{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla\cdot(1+\delta)\vec{v} = 0,$$

$$\frac{\partial \vec{v}}{\partial t} + H\vec{v} + \frac{1}{a}\vec{v}\cdot\nabla\vec{v} = -\frac{1}{a}\nabla\Psi \qquad \Psi = \Psi_N + \frac{1}{2}\frac{\ddot{a}}{a}r^2$$

- Using velocity divergence and conformal time $\theta = \partial_i v^i / \mathscr{H}$

$$\delta' = -\mathscr{H}(1+\delta)\theta - \frac{v^i}{H}\partial_i\delta,$$

$$\theta' = -\mathscr{H}\left(1 + \frac{\mathscr{H}'}{\mathscr{H}^2}\right)\theta - \frac{1}{\mathscr{H}}\partial_i\left(v^j\partial_j v^i\right) - \frac{1}{\mathscr{H}}\nabla^2\Psi$$

Linear equations are the same as before

Spherical symmetry

- Total derivative with respect to $N = \log a \quad \frac{\partial \delta}{\partial N} = \frac{1}{\mathcal{H}} \frac{\partial \delta}{\partial \eta} \qquad \delta = \delta(\eta, x^i(\eta))$

$$\frac{d\delta}{dN} = -(1+\delta)\theta, \qquad \frac{d\delta}{dN} = \frac{\partial \delta}{\partial N} + \frac{v^i}{\mathcal{H}} \partial_i \delta : \text{ Lagrangian (convective) derivative}$$

$$\frac{d\theta}{dN} = -\left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\theta - \frac{1}{3}\theta^2 - \frac{1}{\mathcal{H}^2}\left[\left(\partial_i v^j\right)\left(\partial_j v^i\right) - \frac{1}{3}\theta^2\right] + \nabla^2 \Psi$$

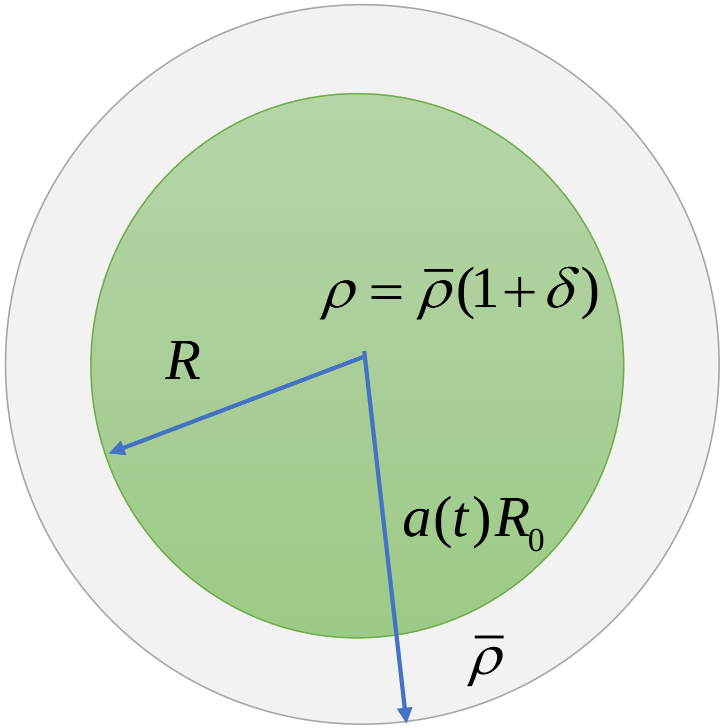
- Spherical symmetry $v^i = \frac{v}{\sqrt{3}}(1,1,1) \qquad \left(\partial_i v^j\right)\left(\partial_j v^i\right) - \frac{1}{3}\theta^2 = 0$

$$\frac{d^2\delta}{dN^2} + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\frac{d\delta}{dN} + (1+\delta)\nabla^2\Psi = \frac{4}{3}\frac{1}{1+\delta}\left(\frac{d\delta}{dN}\right)^2$$

Spherical collapse

- Dynamics of spherical over-density

$$\frac{\ddot{R}}{R}\vec{x} = -\vec{\nabla}\Psi_N = -\vec{\nabla}\left(\Psi - \frac{1}{2}\frac{\ddot{a}}{a}r^2\right), \quad \delta = \left(\frac{a R_0}{R}\right)^3 - 1$$



- Matter dominated universe in LCDM $a(t) = a_0(t/t_0)^{2/3}$

$$\ddot{R} = -\frac{GM(R)}{R^2}, \quad M(R) = \frac{4\pi\rho R^3}{3} \qquad \dot{R}^2 = \frac{2GM}{R} - C$$

$$t = C^{3/2}GM(\tau - \sin\tau),$$
$$R = GM(1-\cos\tau)/C$$

$$\delta = \frac{9(\tau - \sin\tau)^2}{2(1-\cos\tau)^3} - 1, \quad \delta(\tau = 0) = 0$$

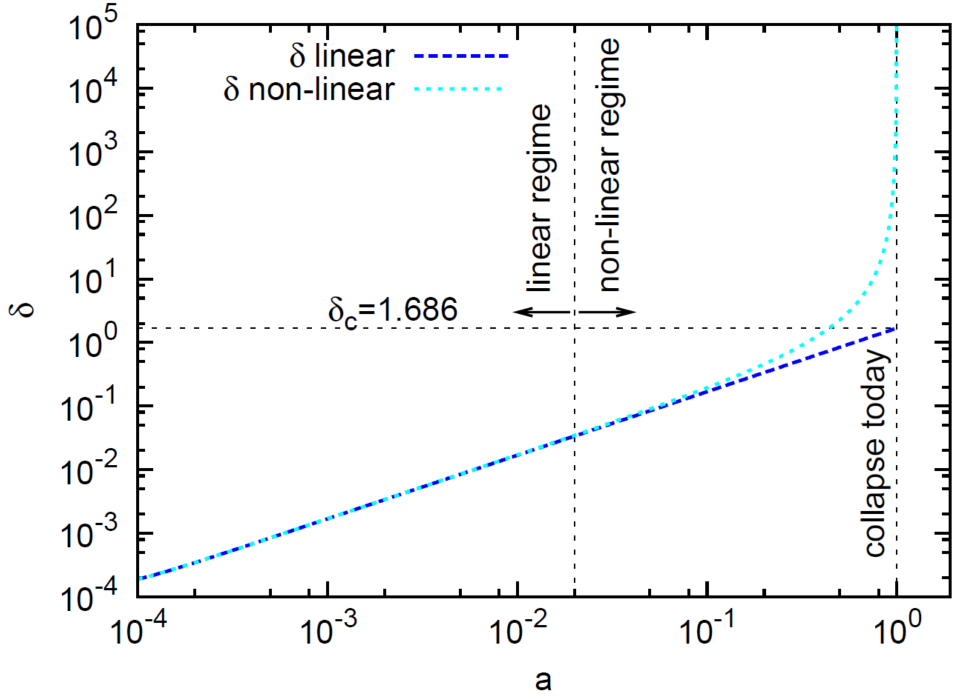
Spherical collapse in EdS

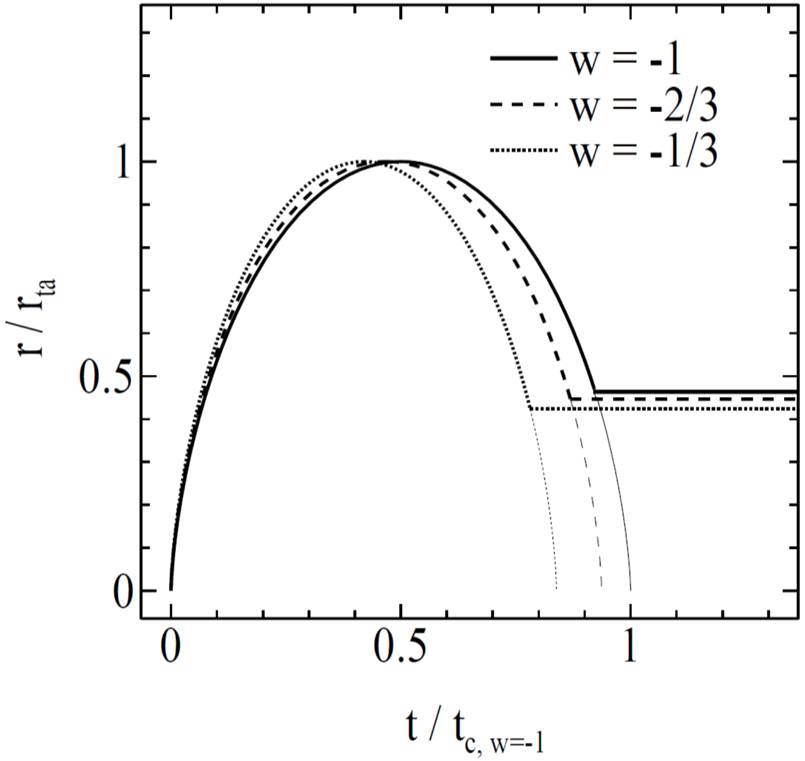
Weinberg et.al. astro-ph/0210134

- Linear density

$$\delta_L = \frac{3}{5}\left[\frac{3}{4}(\tau - \sin\tau)\right]^{2/3} \propto t^{2/3}$$

Pace et.al. 1005.0233





$$\delta_c = \delta_L(t = t_c) = \frac{3}{5}\left(\frac{3\pi}{2}\right)^{2/3} = 1.686$$

Virialisation

In reality, it does not collapse to singularity but rather virialises once the kinetic energy and the potential energy satisfy

$$K = \frac{R}{2}\frac{\partial U}{\partial R} \rightarrow U + 2K = 0 \qquad U \propto R^{-1}$$

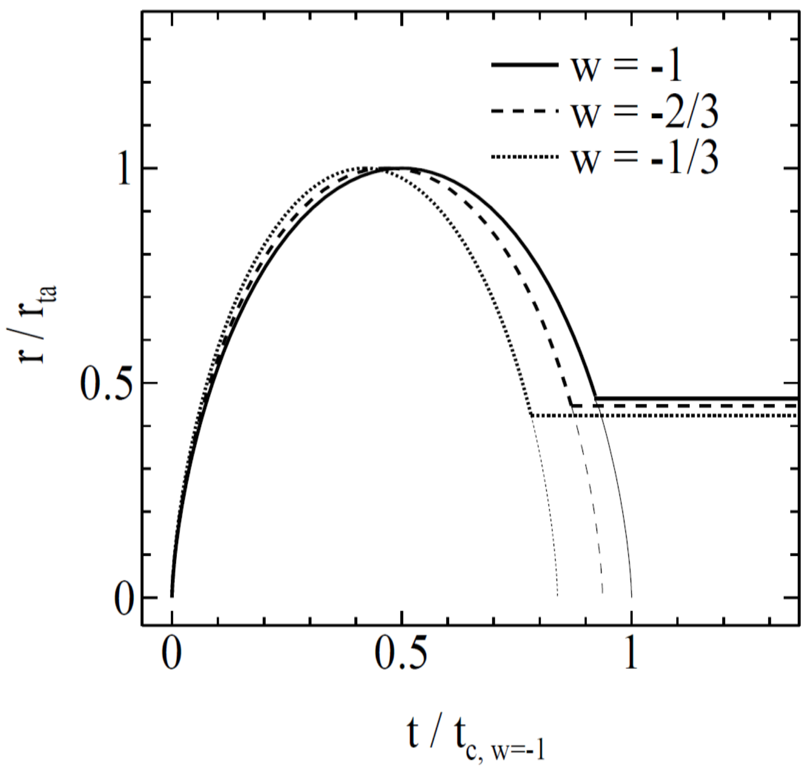
Conservation of energy

$$E = U + K = U(\tau_{TA}) = U(\tau_V) + K(\tau_V) = \frac{1}{2}U(\tau_V)$$

$$R_V = \frac{1}{2}R_T, \quad \delta(\tau_V) = 178$$

vilialisation creates dark matter halos

Weinberg et.al. astro-ph/0210134

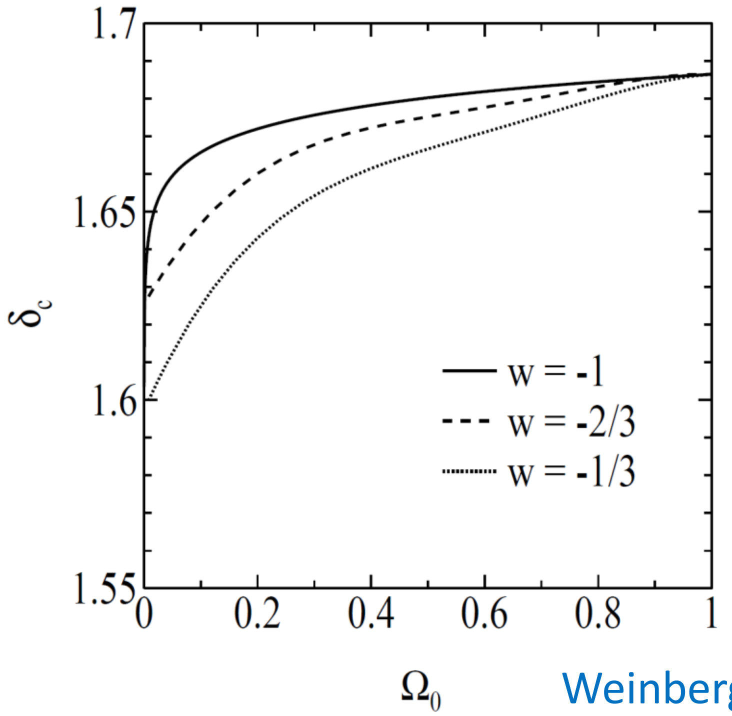


Smooth DE

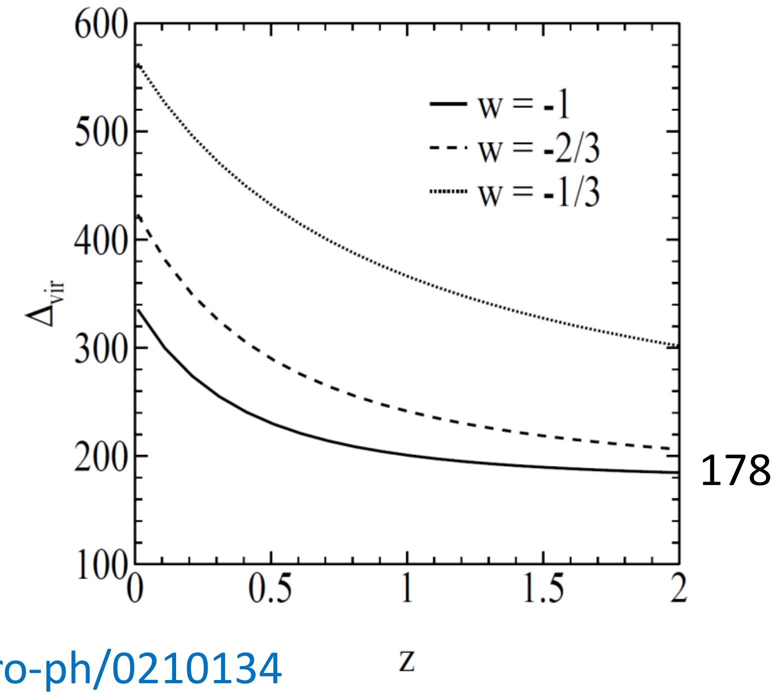
- Smooth DE

$$\frac{\ddot{R}}{R}\vec{x} = -\vec{\nabla}\Psi_N = -\vec{\nabla}\left(\Psi - \frac{1}{2}\frac{\ddot{a}}{a}r^2\right)$$

$$\frac{\ddot{R}}{R} = -\frac{4}{3}\pi G\left(\rho_m + (1+3w_{DE})\rho_{DE}\right)R$$



Weinberg et.al. astro-ph/0210134



Clustering DE/Modified gravity

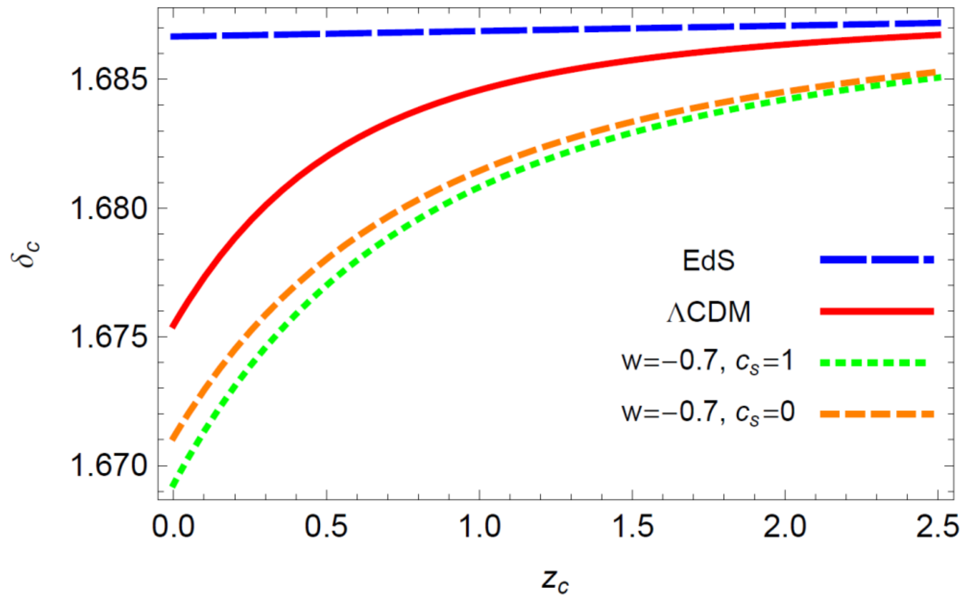
- Clustering DE

  if the sound speed is small, DE clusters and co-move with dark matter

- Modified gravity

  dynamics of spherical top-hat over density can be affected by what happens outside

  (Birkoff's theorem can be violated)



Creminelli et.al. 0911.2701

Perturbation theory

- Non-linear fluid equation

$$\delta' = -\mathscr{H}(1+\delta)\theta - \frac{v^i}{H}\partial_i\delta,$$

$$\theta' = -\mathscr{H}\left(1+\frac{\mathscr{H}'}{\mathscr{H}^2}\right)\theta - \frac{1}{\mathscr{H}}\partial_i\left(v^j\partial_j v^i\right) - \frac{1}{\mathscr{H}}\nabla^2\Psi$$

- Solve these equations perturbatively $\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)}, \theta = \theta^{(1)} + \theta^{(2)} + \theta^{(3)}$

  clustering DE/MG we need to find $\Psi = A_1\delta + A_2\delta^2 + A_3\delta^3 + \ldots$

  1-loop corrections

$$P(k) = P_{lin}(k) + P_{22}(k) + P_{13}(k)$$

$$\left\langle \delta^{(2)}(\vec{k})\delta^{(2)}(\vec{k}') \right\rangle = \left(2\pi\right)^3 \delta^{(3)}(\vec{k}+\vec{k}')P_{22}(k)$$

$$\left\langle \delta^{(1)}(\vec{k})\delta^{(3)}(\vec{k}') + \delta^{(3)}(\vec{k})\delta^{(1)}(\vec{k}') \right\rangle = \left(2\pi\right)^3 \delta^{(3)}(\vec{k}+\vec{k}')P_{13}(k)$$

Breakdown of fluid approximation

- Fluid approximation breaks down when velocity dispersion becomes important

We need to solve collisionless Boltzman equation

$$\frac{df(\eta,\vec{x},\vec{p})}{d\eta}=\frac{\partial f}{\partial\eta}+v^i\nabla_i f+\frac{dp^i}{d\eta}\frac{\partial f}{\partial p^i}=0$$

$$\int d^3p\ f=\rho,$$

$$\int d^3p\,\frac{p^i}{m}f=\rho\,v^i,$$

$$\int d^3p\,\frac{p_i p_j}{m^2}f=\rho\,v_i v_j+\sigma_{ij},$$

$$\left(\frac{\partial\rho}{\partial t}\right)_r+\nabla_r\cdot(\rho\vec{u})=0,$$

$$\left(\frac{\partial\vec{u}}{\partial t}\right)_r+\left(\vec{u}\cdot\nabla_r\right)\vec{u}=-\nabla_r\Psi_N-\frac{1}{\rho}\nabla^j\sigma_{ij}$$

Velocity dispersions

N-body simulations

- Solve collisionless Boltzman equation using many particles
  (assuming they obey geodesics)

$$\vec{x}'' + \mathcal{H} x' = -\nabla\Psi \qquad \rho(\vec{x}) = a^{-3}\sum_i m_i \delta(\vec{x} - \vec{x}_i)$$

- LCDM/smooth DE $\nabla^2\Psi = 4\pi G a^2 \rho, \quad \Psi(x) = -a^{-1}\sum_i \frac{G m_i}{|x - x_i|}\delta(\vec{x} - \vec{x}_i)$

- Clustering DE/ MG
  computation of the Newton potential is challenging

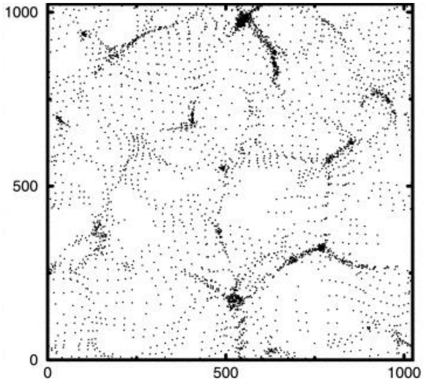
$$\nabla^2\Psi = 4\pi G a^2 \rho - \frac{1}{2}\nabla^2\varphi \qquad 3\nabla^2\varphi + r_c^2\left\{\left(\nabla^2\varphi\right)^2 - \partial_i\partial_j\varphi\, \partial^i\partial^j\varphi\right\} = 8\pi G a^2 \rho$$

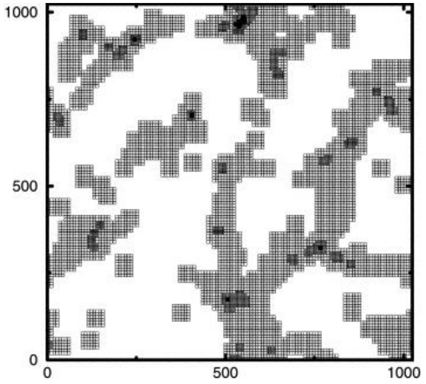
N-body Simulations for MG

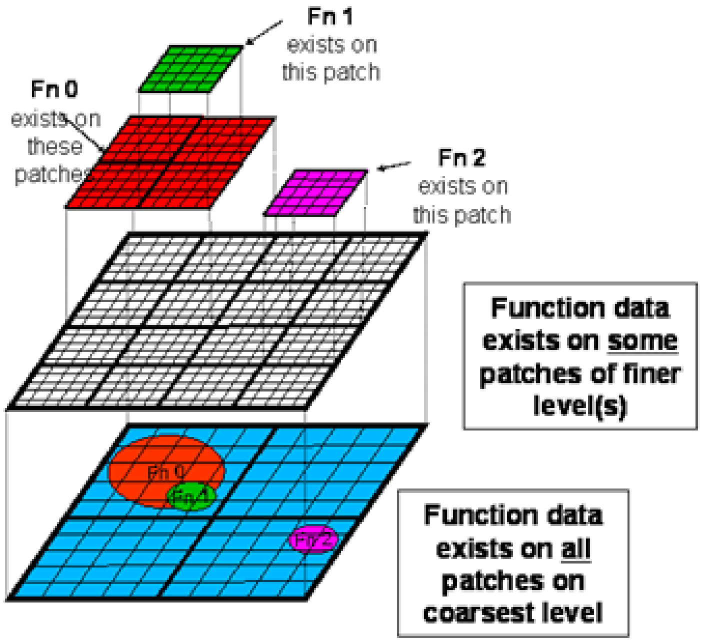
- Multi-level adaptive mesh refinement
- solve Poisson equation using a linear Gauss-Seidel relaxation
- add a scalar field solver using a non-linear Gauss Seidel relaxation

ECOSMOG Li, Zhao, Teyssier, KK JCAP1201 (2012) 051
MG-GADGET   Puchwein, Baldi, Springel   MNRAS (2013) 436 348
ISIS Llinares, Mota, Winther A&A (2014) 562 A78
DGPM, Schmidt PRD80, 043001

Modified Gravity Simulations comparison project
Winther, Shcmidt, Barreira et.al. arXiv: 1506.06384







Example f(R)

$$f(R) = R - 2\Lambda - |f_{R0}|\frac{R_0^2}{R} \quad |f_{R0}| = 10^{-4}, 10^{-5}, 10^{-6}$$

- Full f(R) simulations

  solve the non-linear scalar equation

$$\nabla^2 \delta f_R = -\frac{a^2}{3}[\delta R(f_R) + 8\pi G \delta\rho_{\rm M}]$$

- Non-Chameleon simulations

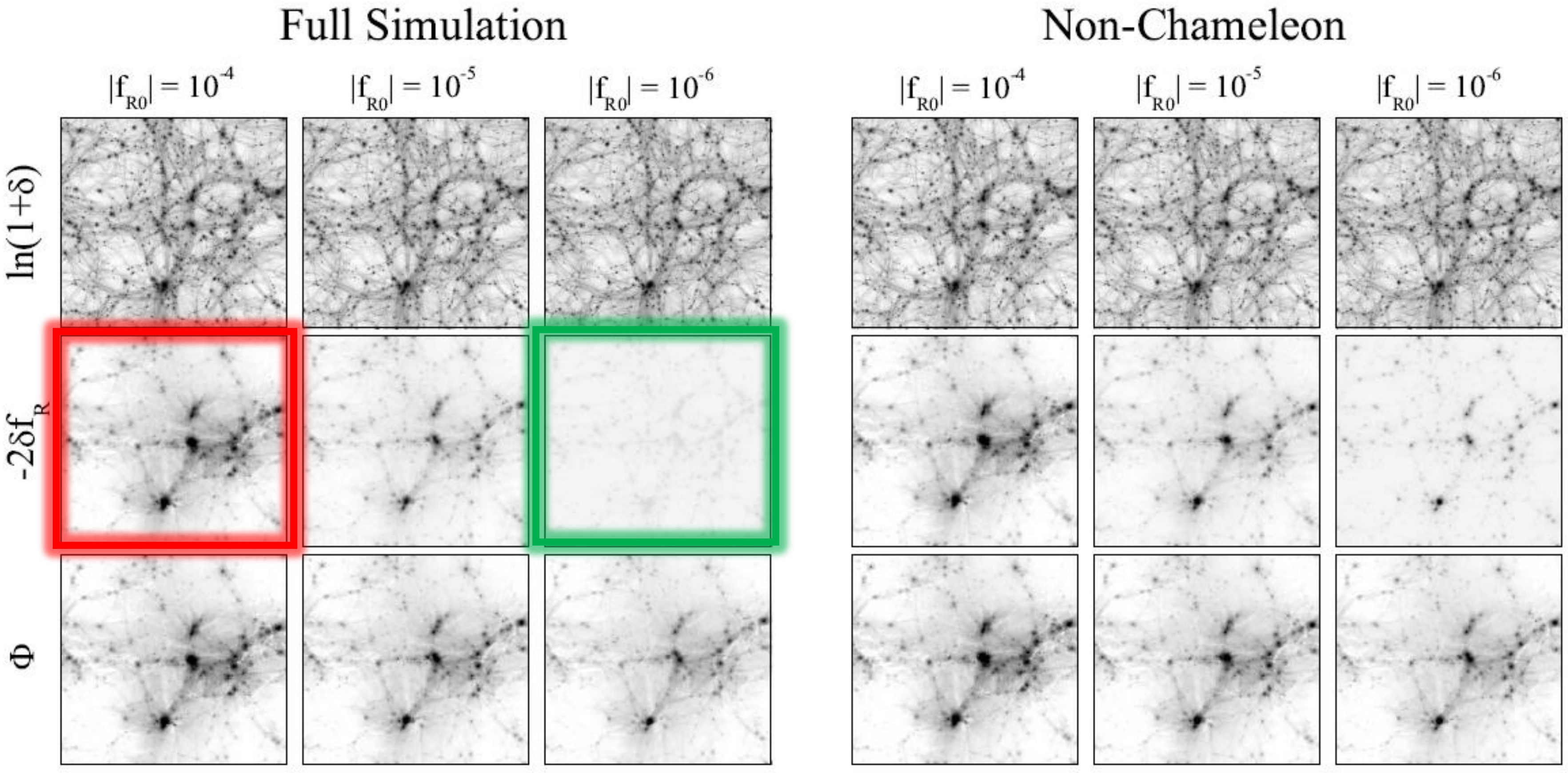
  artificially suppress the Chameleon by linearising the scalar equation to remove the Chameleon effect

$$\nabla^2 \delta f_R = a^2 \bar{\mu}^2 \delta f_R - \frac{8\pi G}{3} a^2 \delta\rho_{\rm M}$$

Snapshots at z=0

Zhao et.al. 1011.1257

- If the fifth force is not suppressed, we have $-2\delta f_R = \Phi.$

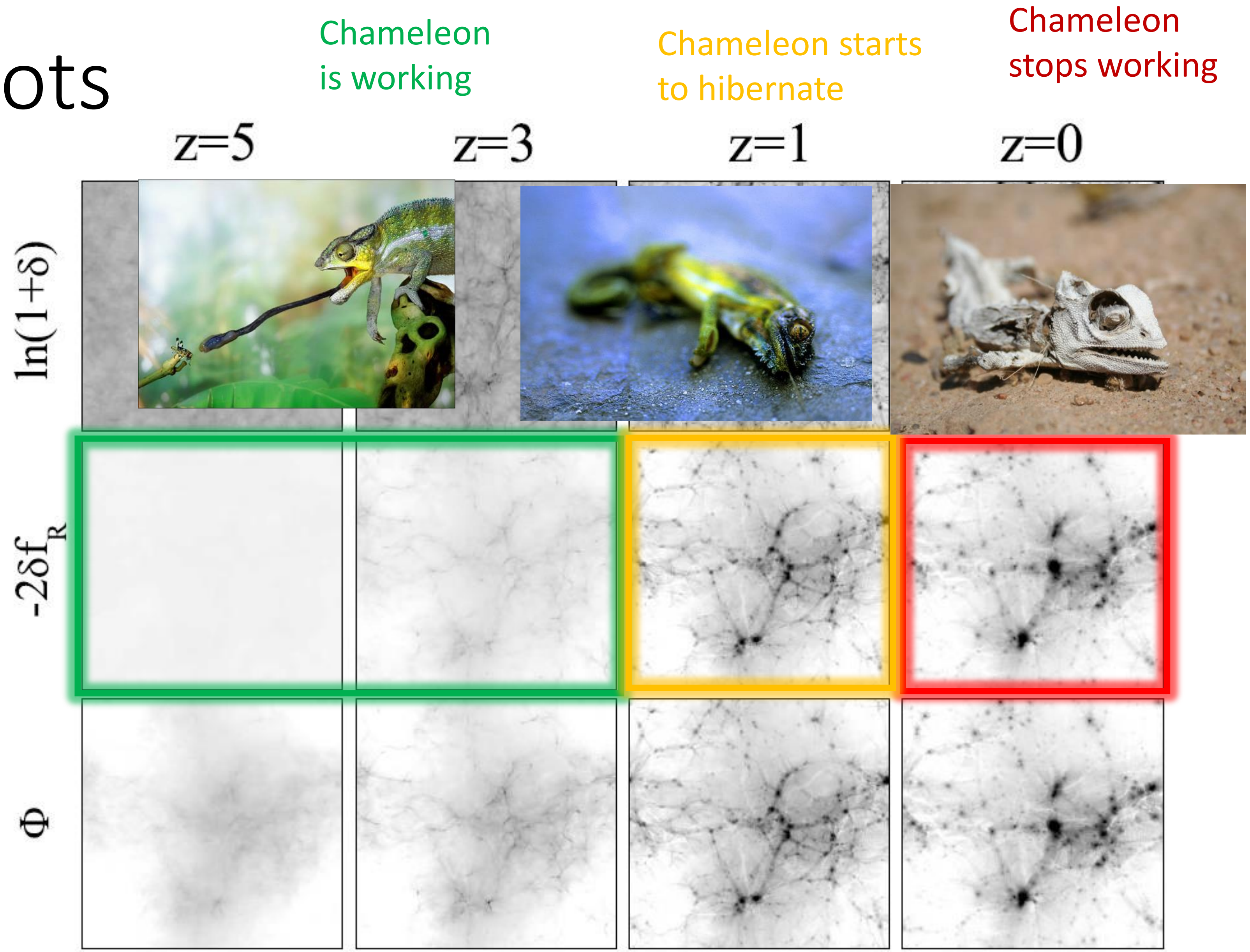


Fifth force is not suppressed

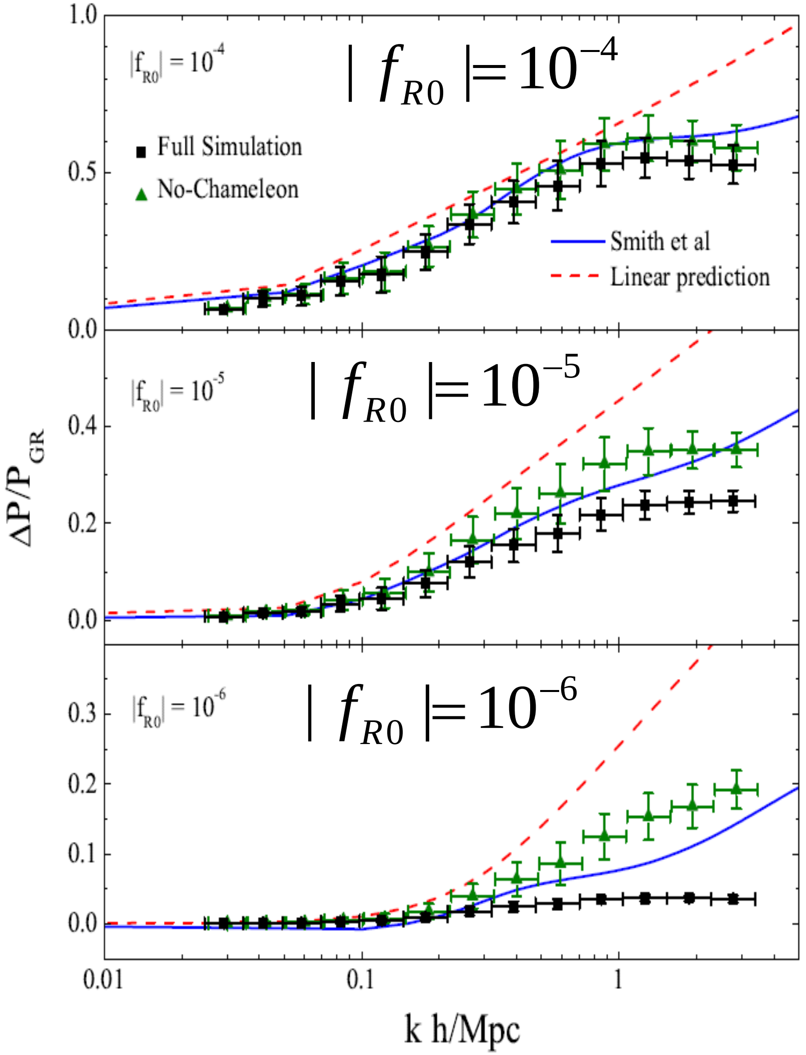
Chameleon is working

Snapshots

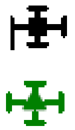
$|f_{R0}| = 10^{-4}$

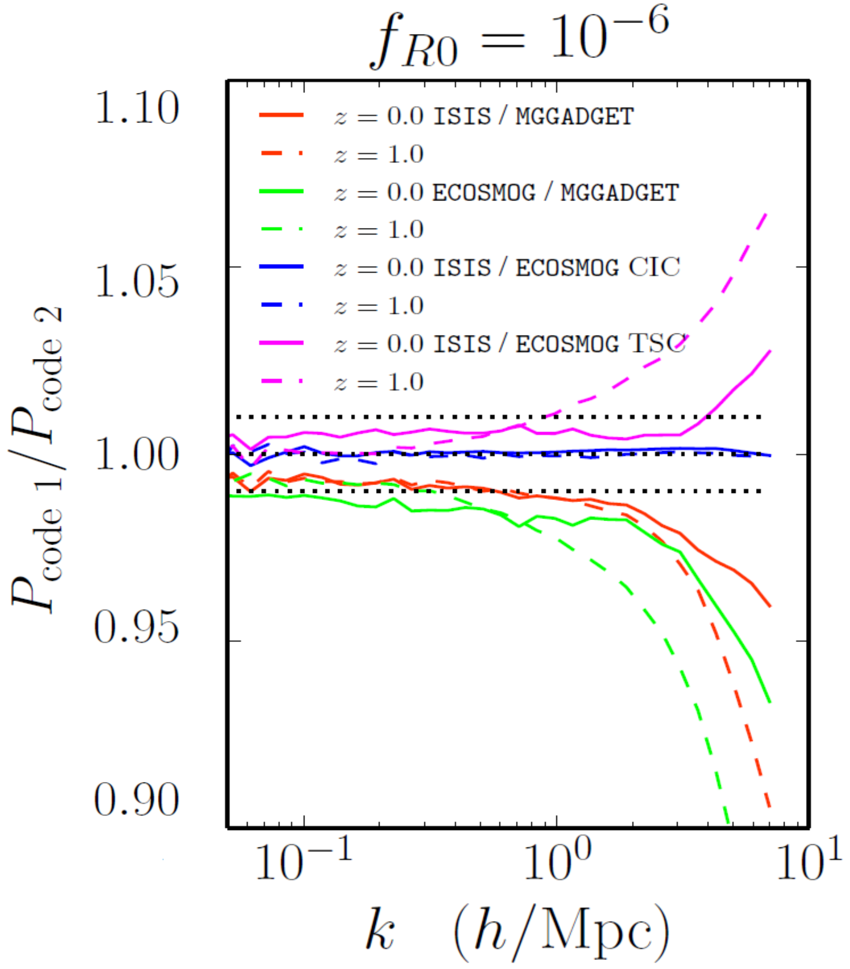


Power spectrum (z=0)



Zhao, Li, KK 1011.1257





Winther et.al. 1506.06384

Redshift distortions

- Finger of God effects

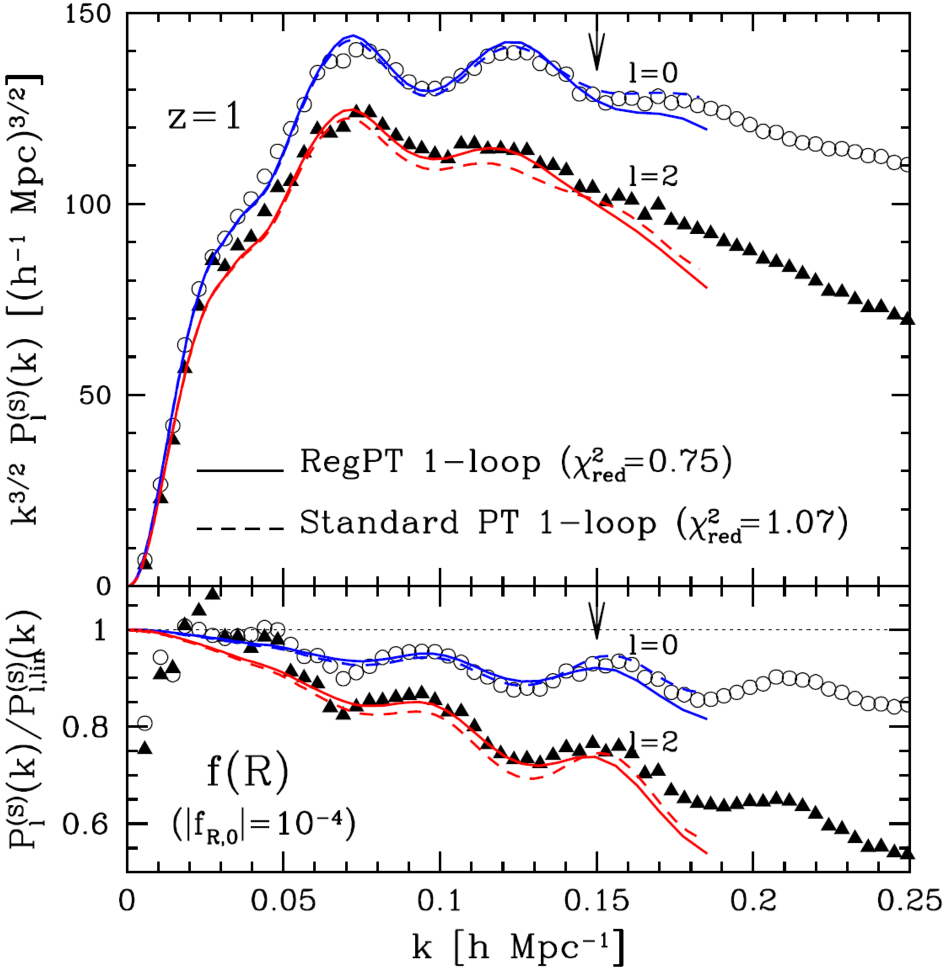
On small scales, redshift space power spectrum is suppressed by random velocities

This effect is stronger in f(R) due to the enhanced gravity and this needs to be Modelled properly

constraint from SDSS DR12

$$| f_{R0} | < 8 \times 10^{-4}$$

Song et.al. 1507.01592



Taruya et.al. 1408.4232

Mass function

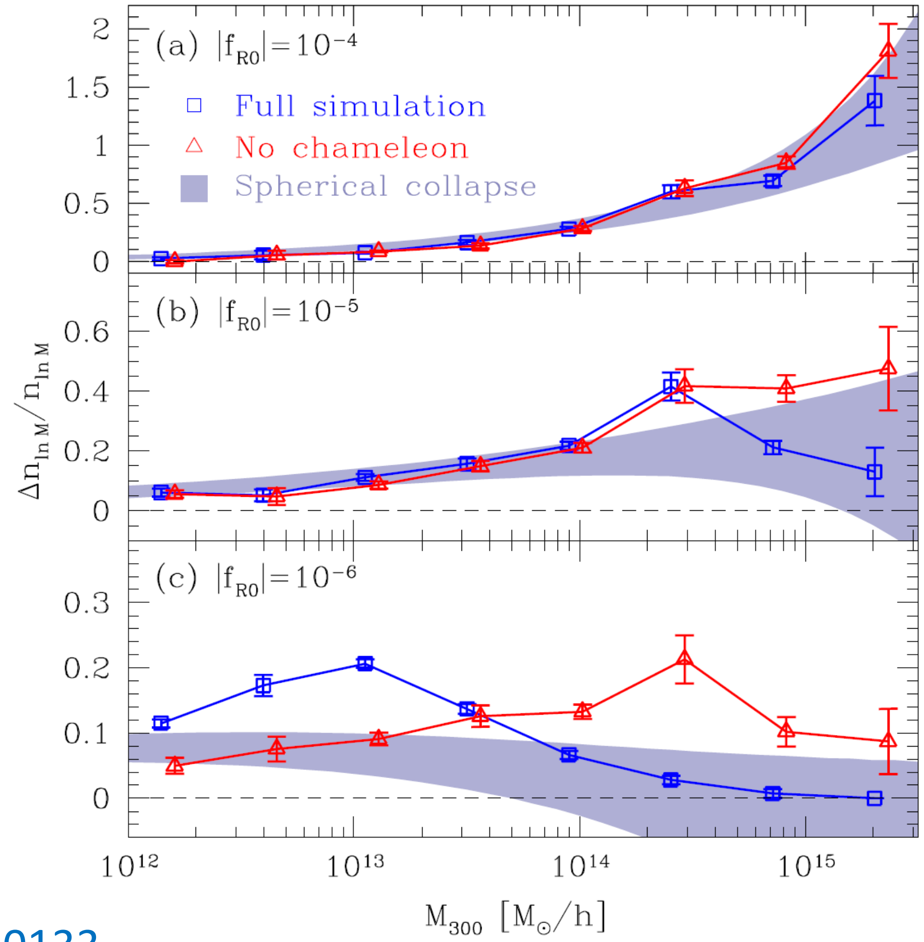
Schmidt et.al. 0812.0545

- Mass function

number density of dark matter halos of mass M

Enhanced gravity creates more massive halos

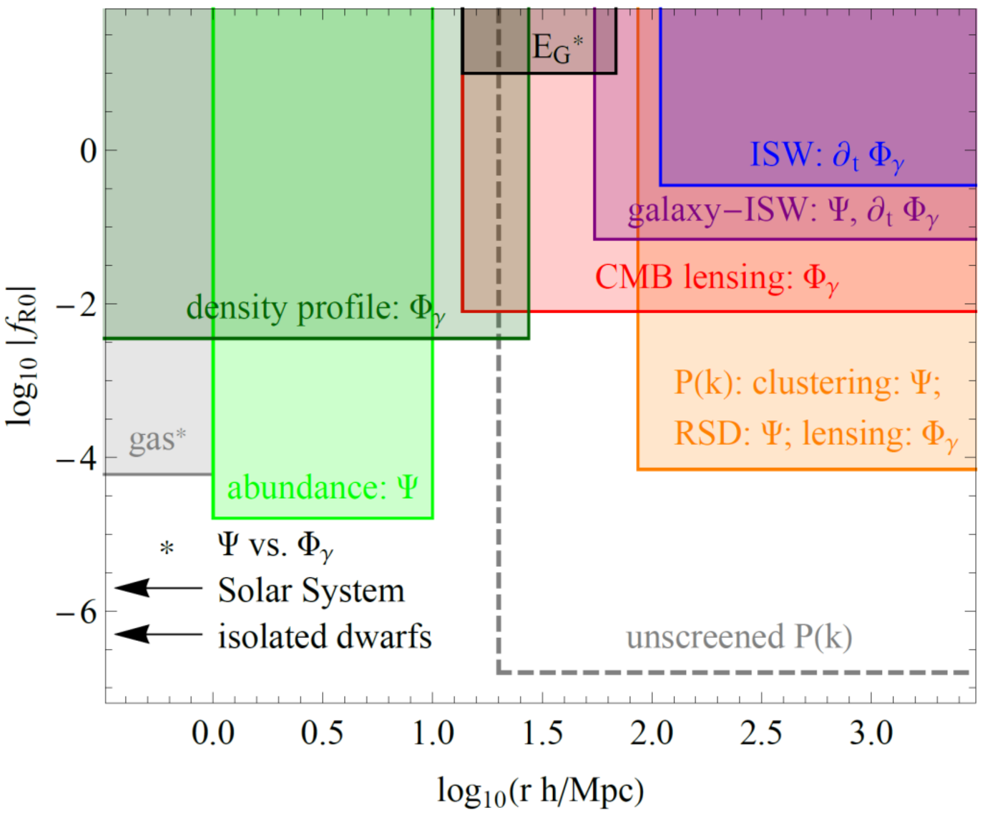
If the chameleon mechanism works, it suppresses the enhanced gravity for massive halos.



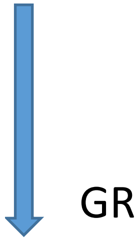
constraint from abundance of clusters

$|f_{R0}| < 1.62\times 10^{-5}$ Cataneo et.al. 1412.0133

Constraints on f(R) gravity



Joyce et.al. 1601.06133



Non-linear regime is powerful for constraining chameleon gravity