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Holographically Viable Extensions of Topologically Massive and Minimal Massive Gravity?

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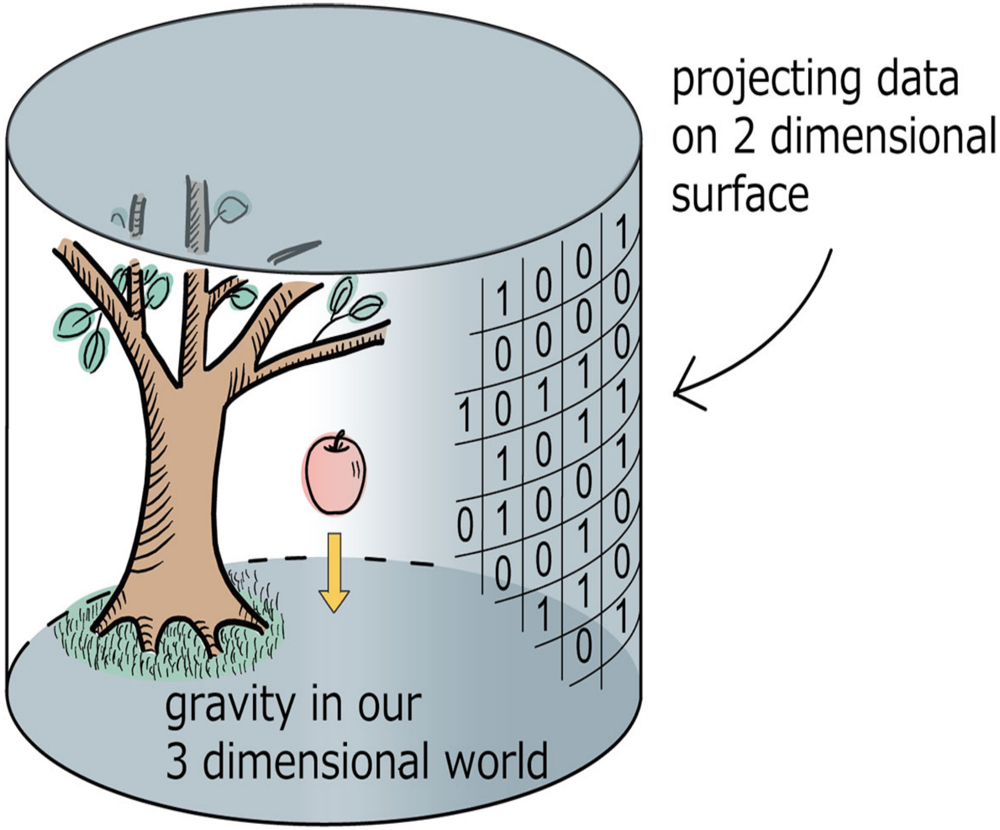
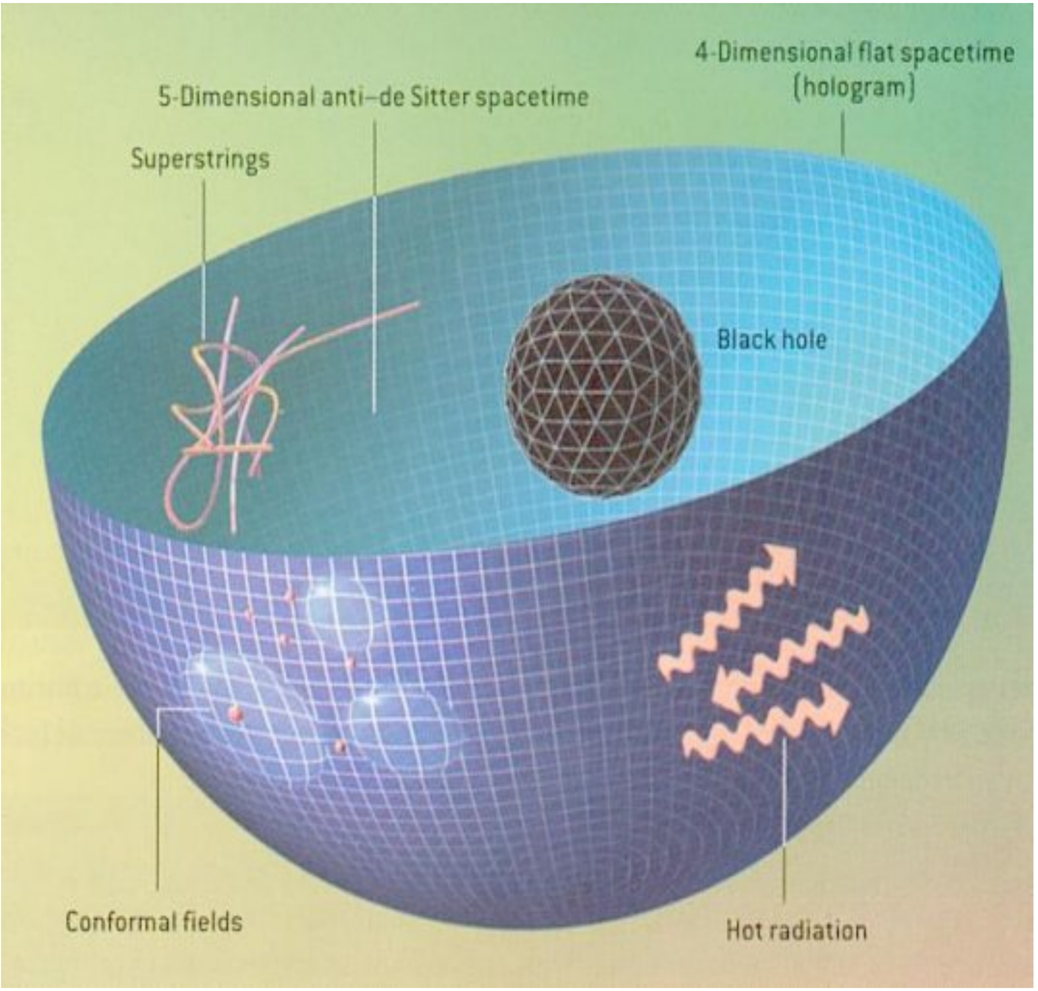
INTRODUCTION



Holographic Principle is about encoding information from (D+1)-dimensional space onto D-dimensional space.
In 1997, Juan Maldacena [1] developed the holographic idea further. In essence, he showed that quantum math describing physics in three spatial dimensions without gravity can be equivalent to math describing a four-dimensional space with gravity.

[1] J. Maldacena, The Large N Limit of Superconformal Field Theories and Supergravity, International Journal of Theoretical Physics 38,4 (1999).

anti-de Sitter (AdS)/conformal field theory(CFT) correspondence



One of the most promising approaches to a quantum theory of gravity is via the anti-de Sitter (AdS)/conformal field theory (CFT) correspondence.

(AdS)/ (CFT) correspondence, is a relationship between two kinds of physical theories. On one side are anti-de Sitter spaces (AdS), which are used in theories of quantum gravity on the other side of the correspondence are conformal field theories (CFT).

In 3D how does it works?

Einstein's gravity: It has a healthy boundary structure but suffers from bulk triviality.

Topologically Massive Gravity (TMG): It does not have a unitary dual CFT in asymptotically AdS spacetimes. In the sense of AdS / CFT correspondence it is not viable as a quantum gravity.

New Massive Gravity (NMG): It also has the bulk/ boundary unitarity clash and hence does not posses the expected holographic description.

Minimal Massive Gravity (MMG): It is unitary both in the bulk and on the boundry.

TMG [2] field equations derived from the action:

$$S = \int d^3x \sqrt{-g}(R - 2\Lambda) + \frac{1}{4\mu}\epsilon^{\lambda\mu\nu}\Gamma^{\rho}_{\lambda\sigma}\left(\partial_\mu \Gamma^{\sigma}_{\rho\nu} + \frac{2}{3}\Gamma^{\sigma}_{\mu\alpha}\Gamma^{\alpha}_{\nu\rho}\right)$$

as,

$$\frac{1}{\mu}C_{\mu\nu} + \sigma G_{\mu\nu} + \Lambda_0 g_{\mu\nu} = 0$$

This model modifies the field equations of general relativity by adding a new term with three derivatives.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

$$C_{\mu\nu} = \eta_{\nu}{}^{\alpha\beta}\nabla_{\alpha}S^{\nu}_{\beta}$$

$$S_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R$$

[2] S. Deser, R. Jackiw and S. Templeton, Topologically Massive Gauge Theories, Ann. Phys. (N.Y.) 140 372 (1982); 185 406(E) (1988).

MMG [3] field equations have an additional symmetric J -tensor in it.

$$E_{\mu\nu} \equiv \bar{\sigma} G_{\mu\nu} + \bar{\Lambda}_0 \, g_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \frac{\gamma}{\mu^2} J_{\mu\nu} = 0$$

$$J^{\mu\nu} \equiv -\frac{1}{2\det g} \varepsilon^{\mu\rho\sigma} \varepsilon^{\nu\tau\eta} S_{\rho\tau} S_{\sigma\eta} = G^{\rho}_{\mu} G_{\rho\nu} - \frac{1}{2} g_{\mu\nu} G_{\rho\sigma} G^{\rho\sigma} + \frac{1}{4} G_{\mu\nu} R + \frac{1}{16} g_{\mu\nu} R^2$$

$$\nabla_\mu E^{\mu\nu} \equiv \nabla_\mu (\bar{\sigma} G^{\mu\nu} + \bar{\Lambda}_0 \, g^{\mu\nu} + \frac{1}{\mu} C^{\mu\nu} + \frac{\gamma}{\mu^2} J^{\mu\nu}) = \nabla_\mu J^{\mu\nu}$$

$$\nabla_\mu J^{\mu\nu} = \eta^{\nu\rho\sigma} S_\rho{}^\tau C_{\sigma\tau},$$

The MMG field equations do not obey the Bianchi Identity and therefore cannot be obtained from an action with the metric being the only variable. But the covariant divergence vanishes for metrics that are solutions to the full MMG equations. Therefore, one has an ”on-shell Bianchi Identity.

[3] E. Bergshoeff, O. Hohm, W. Merbis, A. J. Routh and P. K. Townsend, Minimal Massive 3D Gravity, Class. Quantum Grav. 31, 145008 (2014).

Since MMG has remarkable properties which the other three dimensional theories lack, it bring to mind:

- Can we find any different rank two tensor, which satisfy on shell conservation at quadratic order?
- Is MMG unique or is it part of a large class of theories?
- Are there any deformations of TMG or MMG at the cubic, quartic and higher orders?

1) QUADRATIC EXTENSIONS OF TMG

Let $\varepsilon_{\mu\nu} = 0$,be the field equations.

$$\varepsilon_{\mu\nu} = \sigma G_{\mu\nu} + \Lambda g_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \gamma Y_{\mu\nu},$$

and the trace equation:

$$-\sigma \frac{R}{2} + 3\Lambda + \gamma Y = 0$$

The main question is to find all possible Y-tensors, which satisfy the on-shell conservation.

$$\nabla_\mu \varepsilon^{\mu\nu} = \gamma \nabla_\mu Y^{\mu\nu} = 0$$

The most general quadratic tensor:

$$Y^{\mu\nu} = aS_2^{\mu\nu} + bg^{\mu\nu}S_2 + cS^{\mu\nu}S + dg^{\mu\nu}S^2$$

$$\mathcal{S}_2^{\mu\nu} \equiv S^\mu_\rho S^{\rho\nu}$$

$$\mathcal{S}_2 \equiv S_{\mu\nu}S^{\mu\nu}$$

The trace and divergence are respectivly,

$$Y = (a+3b)\mathcal{S}_2 + (c+3d)S^2,$$

$$\nabla_\mu Y^{\mu\nu} = \Big((a+c)S_\rho{}^\nu + (2d+c)S\delta^\nu_\rho\Big)\nabla^\rho S + S^{\mu\rho}\Big(a\nabla_\mu S_\rho{}^\nu + 2b\nabla^\nu S_{\mu\rho}\Big).$$

In order to get a on shell conserved tensor,we must write the last term in terms of Cotton tensor.

By using the definition of the 3 index Cotton tensor in any dimension,

$$C_{\alpha\mu\nu} = \nabla_\alpha R_{\mu\nu} - \nabla_\mu R_{\alpha\nu} - \frac{1}{2(n-1)}\Big(g_{\mu\nu}\nabla_\alpha R - g_{\alpha\nu}\nabla_\mu R\Big).$$

in 3 dimensions one has,

$$C_{\alpha\mu\nu} = \nabla_\alpha S_{\mu\nu} - \nabla_\mu S_{\alpha\nu} = \eta_{\lambda\mu\alpha} C^{\lambda}_{\nu}$$

$$a = -2b$$

$$\nabla_\mu Y^{\mu\nu} = \Big((a+c)S_\rho{}^\nu + (2d+c)S\delta^\nu_\rho\Big)\nabla^\rho S + a\eta_\lambda{}^{\nu\mu} S_\mu{}^\rho C_\rho{}^\lambda,$$

$$Y = b\mathcal{S}_2 + (c+3d)S^2.$$

i) b≠0

Second term with Cotton tensor vanishes on shell, but first term does not, but if we set,

$$a + c = 0, \qquad 2d + c = 0,$$

the first term will vanishes too.

Also a=-1 choice gives us the J-tensor.

$$Y^{\mu\nu} = J^{\mu\nu} = R^{\mu\rho}R_{\rho}^{\nu} - \frac{1}{2}g^{\mu\nu}R_{\rho\sigma}R^{\rho\sigma} - \frac{3}{4}RR^{\mu\nu} + \frac{5}{16}g^{\mu\nu}R^{2}$$

ii) b=0

$$Y_{\mu\nu} = cS_{\mu\nu}S + dg_{\mu\nu}S^2,$$

$$Y = (c + 3d)S^2$$

gives a shift of TMG parameters.

2) UNIQUENESS OF MMG

Suppose X-tensor is divergence free and symmetric

$$\nabla_\mu X^{\mu\nu} = 0$$

Using this tensor we can build symmetric Y-tensor (is not in the most general quadratic form) as,

$$Y^{\mu\nu} \equiv \frac{1}{2}\eta^{\mu\rho\sigma}\eta^{\nu\tau\eta}\widetilde{X}_{\rho\tau}\widetilde{X}_{\sigma\eta} ,$$

where, $\widetilde{X}_{\sigma\eta} = X_{\sigma\eta} + a\, g_{\sigma\eta}X$ .By using the identity,

$$\eta^{\mu\sigma\rho}\eta_{\nu\alpha\beta} = -\delta^\mu{}_\nu\left(\delta^\sigma{}_\alpha\delta^\rho{}_\beta - \delta^\sigma{}_\beta\delta^\rho{}_\alpha\right) + \delta^\mu{}_\alpha\left(\delta^\sigma{}_\nu\delta^\rho{}_\beta - \delta^\sigma{}_\beta\delta^\rho{}_\nu\right) - \delta^\mu{}_\beta\left(\delta^\sigma{}_\nu\delta^\rho{}_\alpha - \delta^\sigma{}_\alpha\delta^\rho{}_\nu\right),$$

$$\varepsilon_{\mu\nu} = X_{\mu\nu} + Y_{\mu\nu} = 0$$

we can write Y-tensor in the form:

$$Y_{\mu\nu} \equiv X^\rho_\mu X_{\rho\nu} + c_1 g_{\mu\nu}X_{\rho\sigma}X^{\rho\sigma} + c_2 X_{\mu\nu}X + c_3 g_{\mu\nu}X^2.$$

$$\nabla_\mu Y^{\mu\nu} = \eta^{\mu\rho\sigma}\eta^{\nu\tau\eta}\widetilde{X}_{\rho\tau}\nabla_\mu \widetilde{X}_{\sigma\eta} \equiv \eta^{\nu\eta\tau}\widetilde{X}_{\rho\tau}Z_\eta{}^\rho.$$

we defined a new tensor as,

$$Z^{\mu\nu} = \eta^{\mu\alpha\beta}\nabla_\alpha \widetilde{X}_\beta{}^\nu$$

$$\varepsilon_{\mu\nu} = X_{\mu\nu} + Y_{\mu\nu} + Z_{\mu\nu} = 0$$

Z-tensor is traceless but it is not symmetric.

$$\eta_{\mu\nu\sigma}Z^{\mu\nu} = -\nabla_\nu \widetilde{X}_\sigma{}^\nu + \nabla_\sigma \widetilde{X}.$$

$$\nabla_\sigma \widetilde{X}^{\sigma\eta} = \frac{a}{(1+3a)}\nabla^\eta \widetilde{X}.$$

With the choice a=-1/2 it becomes symmetric.With this choice

$$\nabla_\mu Z^{\mu\nu} = \eta^{\nu\alpha\beta}R_{\alpha\lambda}\widetilde{X}_\beta{}^\lambda = \eta^{\nu\alpha\beta}G_{\alpha\lambda}\widetilde{X}_\beta{}^\lambda = \eta^{\nu\alpha\beta}S_{\alpha\lambda}\widetilde{X}_\beta{}^\lambda;$$

If ,

$$\widetilde{X}_\beta{}^\lambda = a_0\delta_\beta{}^\lambda + a_1 S_\beta{}^\lambda + a_2\mathcal{S}_{2\beta}{}^\lambda + a_3\mathcal{S}_{3\beta}{}^\lambda + a_4\mathcal{S}_{4\beta}{}^\lambda + \sum_{i=5}^{\infty} a_i\mathcal{S}_{i\beta}{}^\lambda$$

divergence of Z-tensor vanishes, then X tensor is

$$X_{\sigma\eta} = -2g_{\sigma\eta}a_0 + a_1(S_{\sigma\eta} - g_{\sigma\eta}S) + a_2\big(\mathcal{S}_{2\sigma\eta} - g_{\sigma\eta}\mathcal{S}_2\big),$$

we assumed that the covariant derivative of the X tensor vanishes.It is possible only if a_2=0.

$$Z^{\mu\nu} = a_1 C^{\mu\nu}$$

$$\nabla_\mu Y^{\mu\nu} = \eta^{\nu\eta\tau}\widetilde{X}_{\rho\tau}Z_\eta{}^\rho = a_1\eta^{\nu\eta\tau}(a_0 g_{\rho\tau} + a_1 S_{\rho\tau})C_\eta{}^\rho$$

which vanishes on shell for the field equations.

$$C_{\mu\nu} = c_1 g_{\mu\nu} + c_2 S_{\mu\nu} + c_3 Y_{\mu\nu}$$

$$Y^{\mu\nu} = J^{\mu\nu}$$

This calculations shows uniqueness of the MMG at the quadratic order.

3) EXTENSIONS OF TMG AND MMG

Suppose we have the following deformation of TMG and MMG

$$\sigma G_{\mu\nu} + \Lambda g_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \gamma_1 J_{\mu\nu} + \gamma_2 \mathcal{K}_{\mu\nu} = 0,$$

$$-\sigma \frac{R}{2} + 3\Lambda + \gamma_1 J + \gamma_2 K = 0$$, trace equation.

A) CUBIC EXTENSIONS

The most general two-tensor which can build form powers of Ricci tensor is

$$K^{\mu\nu} = a_1 R^{\mu\rho} R_{\rho\alpha} R^{\alpha\nu} + a_2 g^{\mu\nu} R^{\rho\alpha} R_{\alpha\beta} R^{\beta}{}_{\rho} + a_3 R R^{\mu\rho} R^{\nu}{}_{\rho} + a_4 R^{\mu\nu} R_{\alpha\beta}{}^2 + a_5 g^{\mu\nu} R R_{\alpha\beta}{}^2 + a_6 R^{\mu\nu} R^2 + a_7 g^{\mu\nu} R^3$$

$$\mathcal{K} = (a_1 + 3a_2)\mathcal{R}_3 + (a_3 + a_4 + 3a_5) R \mathcal{R}_2 + (a_6 + 3a_7) R^3,$$

its covariant divergence is;

$$\nabla_\mu \mathcal{K}^{\mu\nu} = \nabla_\mu R \Bigg((a_1 + a_3)\mathcal{R}_2^{\mu\nu} + \Big(\frac{3a_3}{4} + 2a_6 + \frac{a_4}{2}\Big) R R^{\mu\nu} + \Big(\frac{a_4}{2} + a_5 + \frac{3a_2}{4}\Big) g_{\mu\nu}\mathcal{R}_2$$

$$+\Big(\frac{a_5}{2} + \frac{a_6}{2} + 3a_7\Big) g_{\mu\nu} R^2 \Bigg) + \mathcal{R}_2^{\alpha\rho}\Big(3a_2 \nabla^\nu S_{\rho\alpha} + a_1 \nabla_\alpha S_\rho{}^\nu\Big) + R^{\mu\nu} R_{\alpha\beta}\Big(a_1 \nabla_\alpha S_{\beta\mu} + 2a_4 \nabla_\mu S_{\beta\alpha}\Big)$$

$$+ R R^{\mu\rho}\Big(a_3 \nabla_\mu S_\rho^\nu + 2a_5 \nabla^\nu S_{\mu\rho}\Big).$$

Again, in order to get an on shell conserved tensor, we must write Cotton tensor in the expression.

$$a_1 + 2a_4 = 0, \qquad a_1 + 3a_2 = 0, \qquad a_3 + 2a_5 = 0,$$

$$a_1 + a_3 = k, \quad a_3 + \frac{8}{3}a_6 + \frac{2}{3}a_4 = -k, \quad a_4 + 2a_5 + \frac{3}{2}a_2 = -k \quad a_6 + a_5 + 6a_7 = \frac{5}{8}k.$$

These reduce the divergence of the K-tensor to

$$\nabla_\mu \mathcal{K}^{\mu\nu} = kJ^{\mu\nu}\nabla_\mu R + a_1 \eta_\lambda{}^{\nu\alpha} R_{\alpha\beta} R^\beta{}_\rho C^{\lambda\rho} + a_1 \eta_{\lambda\mu\alpha} R^{\mu\nu} R_{\alpha\beta} C^{\lambda\beta} + a_3 \eta_{\lambda\mu}{}^\nu R R^{\mu\rho} C^\lambda{}_\rho,$$

the third term vanishes because of symmetries.By using the three dimensional identity

$$\eta^{\lambda\nu\alpha}\xi^\rho = g^{\lambda\rho}\eta^{\beta\nu\alpha}\xi_\beta + g^{\nu\rho}\eta^{\lambda\beta\alpha}\xi_\beta + g^{\alpha\rho}\eta^{\lambda\nu\beta}\xi_\beta,$$

we can combine the remaining two terms and we get,

$$\nabla_\mu \mathcal{K}^{\mu\nu} = k\left(J^{\mu\nu}\nabla_\mu R + \eta_{\lambda\mu}{}^\nu R R^{\mu\rho} C^\lambda{}_\rho\right).$$

$$\mathcal{K} = -\frac{k}{2} R\left(R^2_{\alpha\beta} - \frac{3}{8}R^2\right) = -kRJ.$$

i) k=0

K tensor is conserved and traceless without using the field equations.

$$\mathcal{K}^{\mu\nu} = \mathcal{R}_3^{\mu\nu} - \frac{1}{3}g^{\mu\nu}\mathcal{R}_3 - R\mathcal{R}_2^{\mu\nu} - \frac{1}{2}R^{\mu\nu}\mathcal{R}_2 + \frac{1}{2}g^{\mu\nu}R\mathcal{R}_2 + \frac{1}{2}R^{\mu\nu}R^2 - \frac{1}{6}g^{\mu\nu}R^3.$$

From Cayley-Hamilton theorem

$$A^3 - (\mathrm{Tr}A)A^2 + \frac{1}{2}\left[(\mathrm{Tr}A)^2 - \mathrm{Tr}(A^2)\right]A - \det(A)I_3 = 0.$$

$$\det A = \frac{1}{6}\left[(\mathrm{Tr}A)^3 - 3\mathrm{Tr}(A^2)(\mathrm{Tr}A) + 2\mathrm{Tr}(A^3)\right].$$

$$A = (R^\mu_\nu)$$

this tensor is identically zero.

ii) k≠0

The second term vanishes both for TMG and MMG mass shell, but the first term does not. From the trace equation,

$$-\sigma \frac{R}{2} + 3\Lambda + \gamma_1 J + \gamma_2 K = 0$$

$$J = \frac{1}{2}(R_{\rho\sigma}R^{\rho\sigma} - \frac{1}{16}R^2)$$

R is not constant.

There does not exist an on shell conserved tensor for TMG and MMG, which build from the third powers of Ricci tensor.

B)QUARTIC EXTENSIONS

$$\sigma G_{\mu\nu} + \Lambda g_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \gamma_1 J_{\mu\nu} + \gamma_2 L_{\mu\nu} = 0$$

$$-\sigma \frac{R}{2} + 3\Lambda + \gamma_1 J + \gamma_2 L = 0$$, trace equation.

The most general rank two tensor, which can build from powers of Ricci tensor, is,

$$\mathcal{L}^{\mu\nu} = a_1 \mathcal{R}_4^{\mu\nu} + a_2 \mathcal{R}_2^{\mu\nu} \mathcal{R}_2 + a_3 R \mathcal{R}_3^{\mu\nu} + a_4 R^2 \mathcal{R}_2^{\mu\nu} + a_5 R^{\mu\nu} R^3 + a_6 R^{\mu\nu} \mathcal{R}_3$$
$$+ a_7 R^{\mu\nu} R \mathcal{R}_2 + a_8 g^{\mu\nu} \mathcal{R}_4 + a_9 g^{\mu\nu} R \mathcal{R}_3 + a_{10} g^{\mu\nu} R^2 \mathcal{R}_2 + a_{11} g^{\mu\nu} R^4 + a_{12} g^{\mu\nu} \mathcal{R}_2^2.$$

and its covariant derivative:

$$\nabla_\mu \mathcal{L}^{\mu\nu} = \Bigg((\frac{5}{4}a_1 + a_3)\mathcal{R}_3^{\mu\nu} + (\frac{3}{4}a_2 + \frac{3}{4}a_6 + a_7)R^{\mu\nu}\mathcal{R}_2^{\mu\nu} + (a_3 + 2a_4 + \frac{1}{2}a_2)R\mathcal{R}_2^{\mu\nu}$$
$$+(\frac{3}{4}a_4 + \frac{1}{2}a_7 + 3a_5)R^2R^{\mu\nu} + (\frac{1}{2}a_5 + 4a_{11} + \frac{1}{2}a_{10})g^{\mu\nu}R^3 +$$
$$(\frac{1}{2}a_6 + a_9 + a_8)g^{\mu\nu}\mathcal{R}_3 + (\frac{1}{2}a_7 + a_{12} + 2a_{10} + \frac{3}{4}a_9)g^{\mu\nu}R\mathcal{R}_2 \Bigg)\nabla_\mu R$$
$$+R^{\mu\alpha}\mathcal{R}_2(a_2\nabla_\mu S_\alpha{}^\nu + 4a_{12}\nabla^\nu S_{\mu\alpha}) + R\mathcal{R}_2{}^\mu{}_\beta(a_3\nabla_\mu S^{\beta\nu} + 3a_9\nabla^\nu S_\mu{}^\beta)$$
$$+RR^{\mu\alpha}R^{\beta\nu}(a_3\nabla_\mu S_{\alpha\beta} + 2a_7\nabla_\beta S_{\alpha\mu}) + R^2R^{\mu\alpha}(a_4\nabla_\mu S_\alpha{}^\nu + 2a_{10}\nabla^\nu S_{\mu\alpha})$$
$$+\mathcal{R}_2{}^\mu{}_\beta R_\rho{}^\nu(a_1\nabla_\mu S^{\beta\rho} + 3a_6\nabla^\rho S_\mu{}^\beta) + \mathcal{R}_3^{\mu\rho}(a_1\nabla_\mu S_\rho{}^\nu + 4a_8\nabla^\nu S_{\mu\rho})$$
$$+R^{\mu\alpha}\mathcal{R}_2^{\nu\beta}(a_1\nabla_\mu S_{\alpha\beta} + 2a_2\nabla_\beta S_{\alpha\mu}).$$

The terms in the last four line can be written in terms of Cotton tensor by relating the parameters as;

$$a_2 = -\frac{a_1}{2}, \quad a_6 = -\frac{a_1}{3}, \quad a_7 = -\frac{a_3}{2}, \; a_8 = -\frac{a_1}{4}, \; a_9 = -\frac{a_3}{3}, \; a_{10} = -\frac{a_4}{2}, \; a_{12} = \frac{a_1}{8}.$$

and for simplicity, we can choose the other parameters as;

$$\frac{5a_1}{4}+a_3 = -k \quad a_3-\frac{1}{4}a_1+2a_4 = \frac{3k}{4}, \quad -\frac{1}{4}a_3+3a_5+\frac{3}{4}a_4 = -\frac{5k}{16}, \quad -\frac{1}{4}a_4+\frac{1}{2}a_5+4a_{11} = \frac{17}{192}k$$

$$\nabla_\mu \mathcal{L}^{\mu\nu} = k\left(R^{\mu\alpha}J_\alpha{}^{\nu} - \frac{1}{3}g^{\mu\nu}(R^{\alpha\beta}J_{\alpha\beta} - \frac{1}{8}RJ)\right)\nabla_\mu R + \left(\frac{1}{2}a_1\mathcal{R}_2 + (\frac{k}{8}+\frac{a_1}{2})R^2\right)\nabla_\mu J^{\mu\nu}$$
$$-a_1 R R^{\mu\nu}\nabla_\alpha J^\alpha_\mu + a_1\eta_k{}^{\nu}{}_{\mu}\mathcal{R}_3^{\mu\rho}C^k{}_{\rho},$$

$$\mathcal{L} = \frac{k}{8}R^2 J.$$

i) k=0

We get an on shell conserved tensor as;

$$\mathcal{L}^{\mu\nu} = \mathcal{R}_4^{\mu\nu} - \frac{1}{2}\mathcal{R}_2^{\mu\nu}\mathcal{R}_2 - \frac{5}{4}R\mathcal{R}_3^{\mu\nu} + \frac{3}{4}R^2\mathcal{R}_2^{\mu\nu} - \frac{7}{24}R^{\mu\nu}R^3 - \frac{1}{3}R^{\mu\nu}\mathcal{R}_3 + \frac{5}{8}R^{\mu\nu}R\mathcal{R}_2 - \frac{1}{4}g^{\mu\nu}\mathcal{R}_4 + \frac{5}{12}g^{\mu\nu}R\mathcal{R}_3 - \frac{3}{8}g^{\mu\nu}R^2\mathcal{R}_2 - \frac{1}{12}g^{\mu\nu}R^4 + \frac{1}{8}g^{\mu\nu}\mathcal{R}_2^2.$$

which is identically zero.

$$\mathcal{R}_4^{\mu\nu} = \frac{1}{3}R^{\mu\nu}\mathcal{R}_3 + R\mathcal{R}_3^{\mu\nu} + \frac{1}{2}\mathcal{R}_2^{\mu\nu}\mathcal{R}_2 - \frac{1}{2}R^{\mu\nu}R\mathcal{R}_2 - \frac{1}{2}\mathcal{R}_2^{\mu\nu}R^2 + \frac{1}{6}R^{\mu\nu}R^3,$$

(we get this identity by using K- tensor)

$$\delta^{\nu_1\nu_2\nu_3\nu_4}_{[\mu_1\mu_2\mu_3\mu_4]}\widetilde{R}^{\mu_2}_{\nu_1}\widetilde{R}^{\mu_3}_{\nu_2}\widetilde{R}^{\mu_4}_{\nu_3}\widetilde{R}^{\mu_1}_{\nu_4} = \frac{1}{4}\widetilde{\mathcal{R}}_4 - \frac{1}{8}\widetilde{\mathcal{R}}_2^2 = 0,$$

(Schouten identity)

ii) k≠0

As a result of the trace equation

$$-\sigma \frac{R}{2} + 3\Lambda + \gamma_1 J + \gamma_2 L = 0$$

$$\mathcal{L} = \frac{k}{8} R^2 J.$$

$$J = \frac{1}{2}(R_{\rho\sigma} R^{\rho\sigma} - \frac{1}{16} R^2)$$

R is not constant, since the J tensor has the square of the Ricci tensor in it. The first term in the divergence of the L-tensor does not vanish.

There does not exist an on shell conserved tensor for TMG and MMG, which build from fourth powers of Ricci tensor.

C) HIGHER ORDER EXTENSIONS

By using the Schouten identities, we can write higher order powers of Ricci tensor in terms of the lower ones.

$$\delta^{\nu_1\nu_2\nu_3\nu_4}_{[\mu_1\mu_2\mu_3\mu_4]} \widetilde{R}^{\mu_2}_{\nu_1} \widetilde{R}^{\mu_3}_{\nu_2} \widetilde{R}^{\mu_4}_{\nu_3} \widetilde{R}^{\mu}_{\nu_4} \widetilde{R}^{\mu_1}_{\nu} = \frac{1}{4}(\widetilde{\mathcal{R}}_5)^{\mu}_{\nu} - \frac{1}{8}\widetilde{\mathcal{R}}_2(\widetilde{\mathcal{R}}_3)^{\mu}_{\nu} - \frac{1}{12}\widetilde{\mathcal{R}}_3(\widetilde{\mathcal{R}}_2)^{\mu}_{\nu} = 0,$$

$$\delta^{\nu_1\nu_2\nu_3\nu_4}_{[\mu_1\mu_2\mu_3\mu_4]} \widetilde{R}^{\mu_2}_{\nu_1} \widetilde{R}^{\mu_3}_{\nu_2} \widetilde{R}^{\mu_4}_{\nu_3} \widetilde{R}^{\mu}_{\nu_4} \widetilde{R}^{\mu_1}_{\nu} = \frac{1}{4}(\widetilde{\mathcal{R}}_5)^{\mu}_{\nu} - \frac{1}{8}\widetilde{\mathcal{R}}_2(\widetilde{\mathcal{R}}_3)^{\mu}_{\nu} - \frac{1}{12}\widetilde{\mathcal{R}}_3(\widetilde{\mathcal{R}}_2)^{\mu}_{\nu} = 0,$$

There does not exist an on shell conserved rank two tensor except the quadratic one.

CONCLUSION

In this work,

we prove that at the quadratic order in the curvature MMG is the only deformation of TMG and it is a uniqe theory.

We have also showed that there does not exist a deformation of TMG or MMG on to the qubic and quartic orders.

It is diffucult to construct on shell conserved rank two tensors in three dimensions.

THANK YOU FOR YOUR LISTENING!