Higher spins in 3D: going from AdS to flat

Andrea Campoleoni

Université Libre de Bruxelles and
International Solvay Institutes

based on work with H.A. González, B. Oblak and M. Riegler

arXiv:1512.03353 & arXiv:1603.03812

Workshop on Topics in Three Dimensional Gravity, ICTP Trieste, 24/3/2016

(Higher-spin) BMS modules in 3D

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BMS symmetry

Bondi-**M**etzner-**S**achs group

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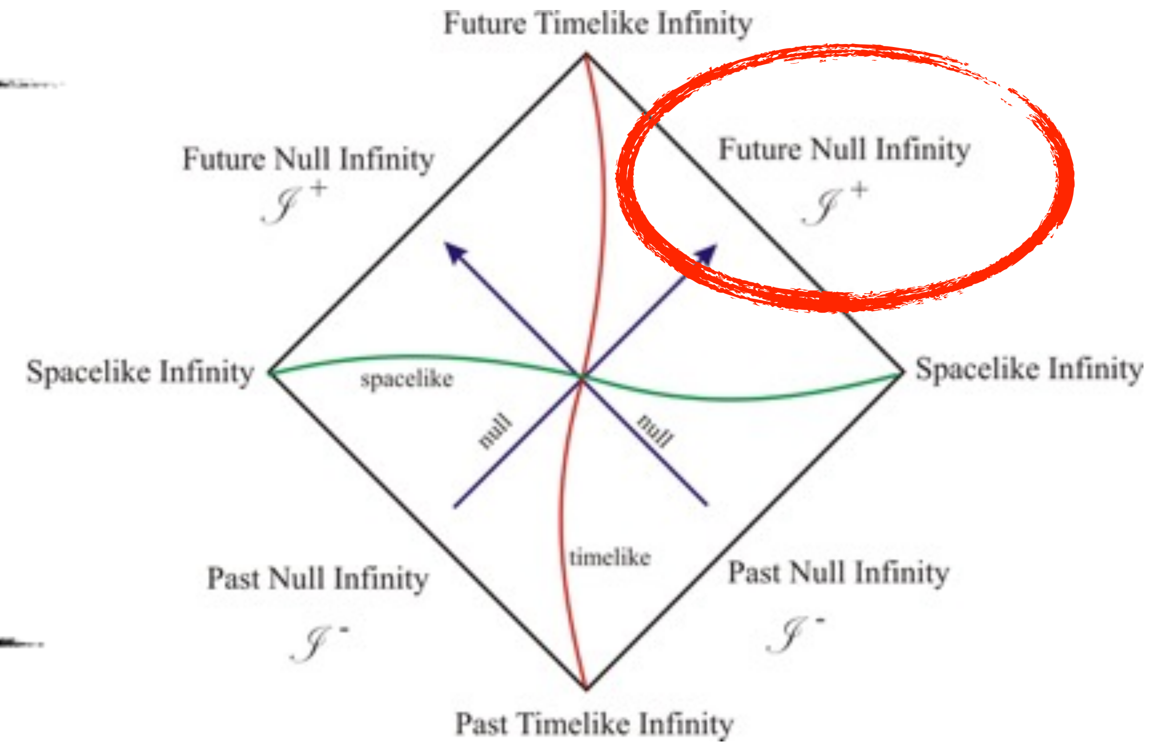
asymptotic symmetries at null ∞

of asymptotically flat gravity

Bondi, van der Burg, Metzner; Sachs (1962)



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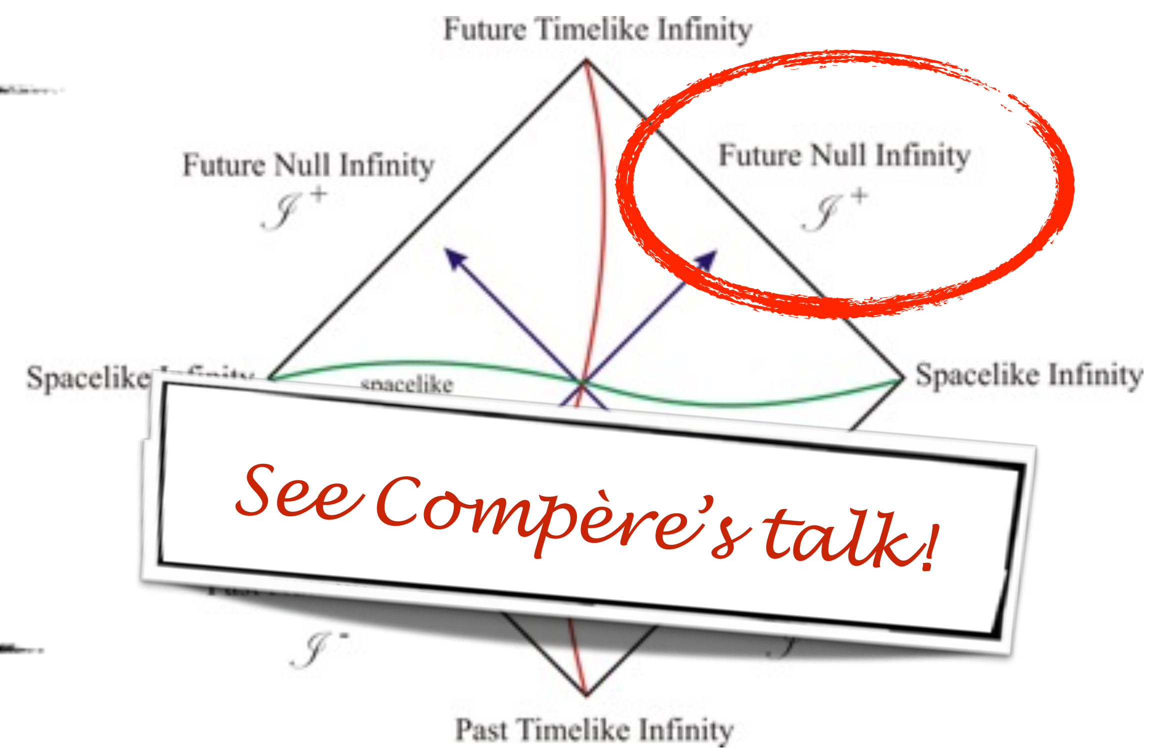

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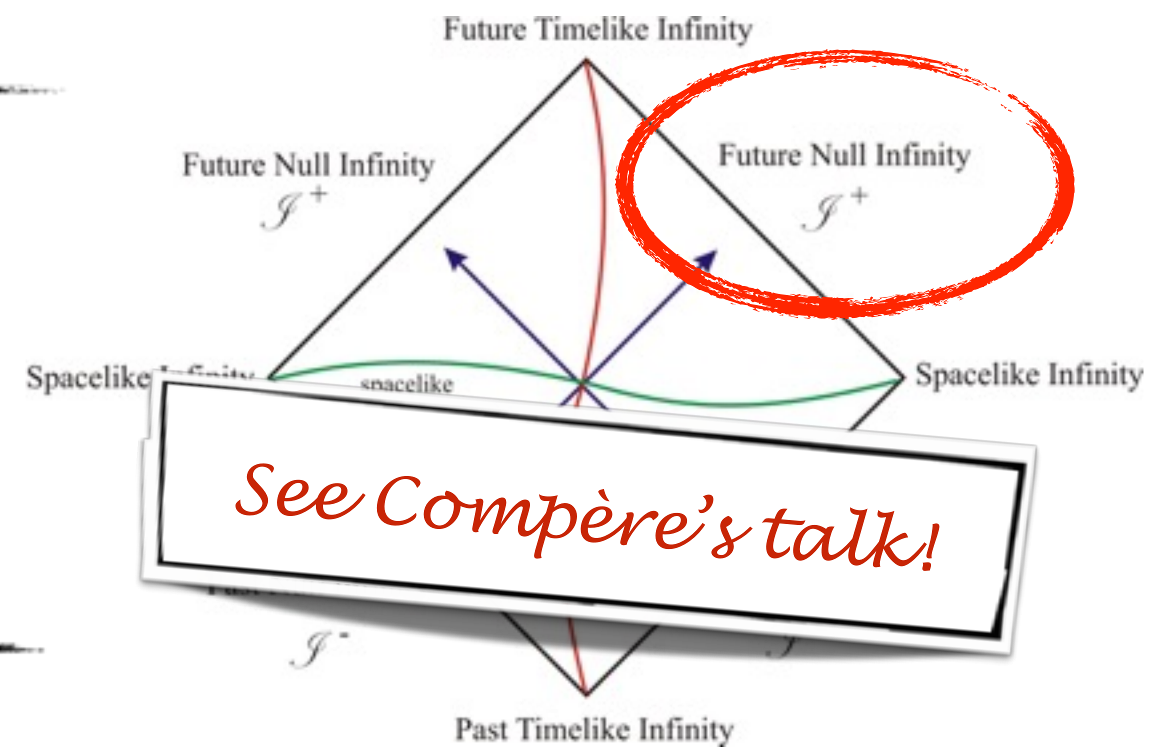

- Nice symmetry, but what about the *quantum* regime?
- (Unitary) representations of *local* BMS? Barnich, Troessaert (2009)

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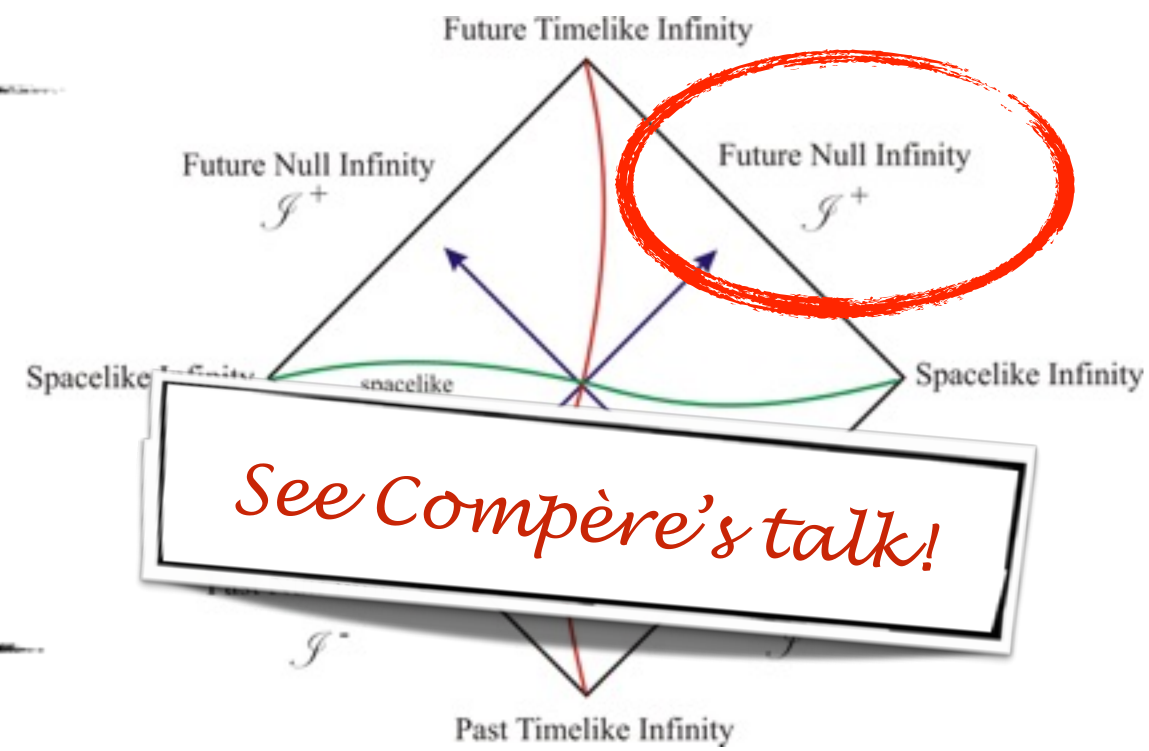

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- (Unitary) representations of *local* BMS_3 — Barnich, Troessaert (2009)
  - Induced representations — Barnich, Oblak (2014); Garbarz, Leston (2015)
  - Limit of CFT representations — Bagchi, Gopakumar, Mandal, Miwa (2010); Grumiller, Riegler, Rosseel (2014)

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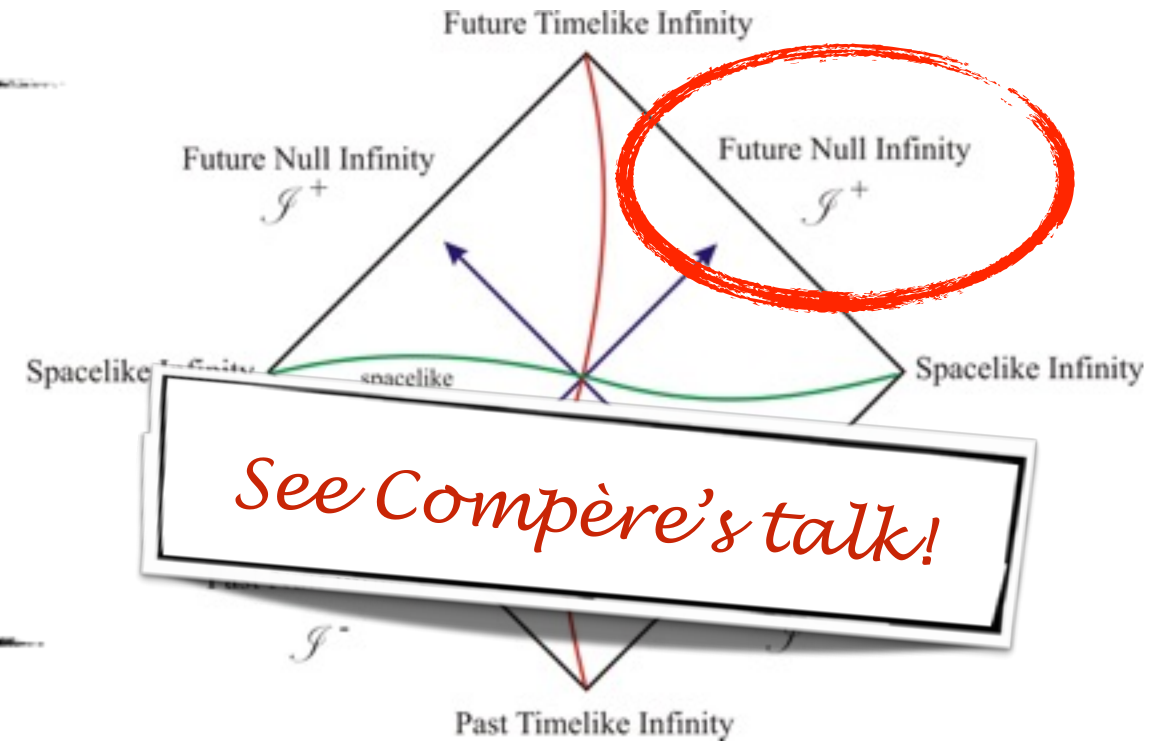

See Compère's talk!

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See also poster by T. Neogi

Why D=3? And why higher spins?

- Motivation I: *beauty*
  - In D = 3 the local BMS group is an Inonu-Wigner contraction of the AdS_3 local conformal symmetry at spatial infinity Brown, Henneaux (1986)

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- Motivation II: *…and the beast*
  - Several ways to obtain BMS as a limit of conformal symmetry: are they all equivalent?
  - Higher-spin fields → non-linear W algebras — Henneaux, Rey; A.C., Pfenninger, Fredenhagen, Theisen (2010)
  - Extension of the symmetry → more control over the flat limit!

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  - Extension of the symmetry → techniques that may be useful in D = 4?

Asymptotic symmetries in flat space

- Asymptotic symmetries at spatial infinity in AdS_3

Brown, Henneaux (1986)

$$
\begin{aligned}
[\mathcal{L}_m, \mathcal{L}_n] &= (m-n)\,\mathcal{L}_{m+n} + \frac{c}{12}\, m(m^2-1)\delta_{m+n,0} \\
[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] &= (m-n)\,\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}\, m(m^2-1)\delta_{m+n,0}
\end{aligned}
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- Define new generators and central charges

$$P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right), \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}$$

$$c_1 = c - \bar{c}$$
$$c_2 = \frac{c+\bar{c}}{\ell}$$

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[P_m, P_n] &= \ell^{-2}\,(\cdots)
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Asymptotic symmetries in flat space

- Asymptotic symmetries at null infinity in Minkowski$_3$

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[P_m, P_n] &= 0\,,
\end{aligned}$$

$$\ell \to \infty$$

- Define new generators and central charges

$$P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right), \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}$$

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$$

$$
c_1 = c - \bar{c}
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c_2 = \frac{c+\bar{c}}{\ell}
$$

- Same result directly from flat gravity — Barnich, Compere (2007)

- Everything extends to higher spins — Afshar, Bagchi, Fareghbal, Grumiller, Rosseel; Gonzalez, Matulich, Pino, Troncoso (2013)

Outline

The bms_3 algebra and its unitary irreps

Ultrarelativistic vs Galilean limits of CFT

Higher spins

Characters & partition functions

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The bms$_3$ algebra

- The centrally extended bms$_3$ algebra $(m \in \mathbb{Z})$

$$
\begin{aligned}
[J_m, J_n] &= (m-n)J_{m+n} + \frac{c_1}{12}\, m(m^2-1)\, \delta_{m+n,0} \\
[J_m, P_n] &= (m-n)P_{m+n} + \frac{c_2}{12}\, m(m^2-1)\, \delta_{m+n,0} \\
[P_m, P_n] &= 0
\end{aligned}
$$

- c_2 plays an important role in representation theory and doesn't vanish in gravity: $c_2 = \frac{3}{G}$

The bms_3 algebra

- The Poincaré subalgebra $(m = -1, 0, 1)$

$$
\begin{aligned}
[J_m, J_n] &= (m-n)J_{m+n} \quad \leftarrow \text{Lorentz} \\
[J_m, P_n] &= (m-n)P_{m+n} \\
[P_m, P_n] &= 0
\end{aligned}
$$

- P_m → translations; J_1 and J_{-1} → boosts; J_0 → rotations

How to build representations of bms_3?

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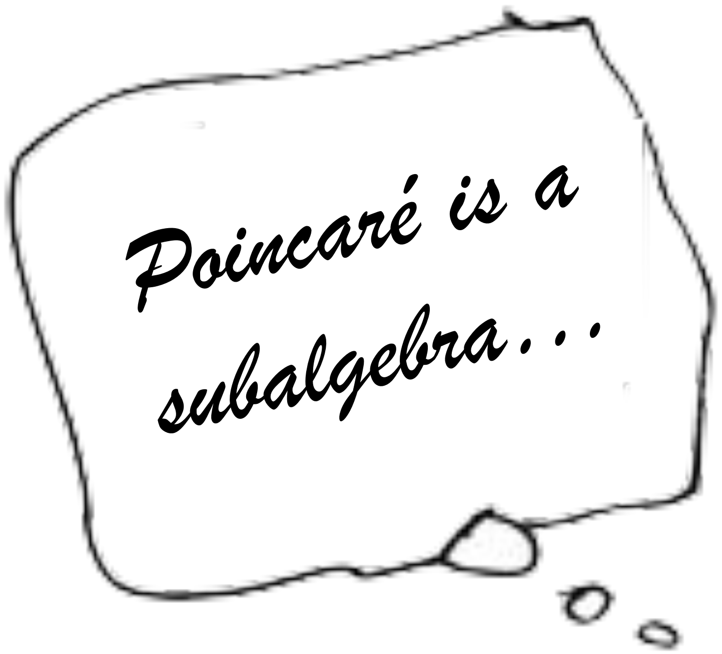

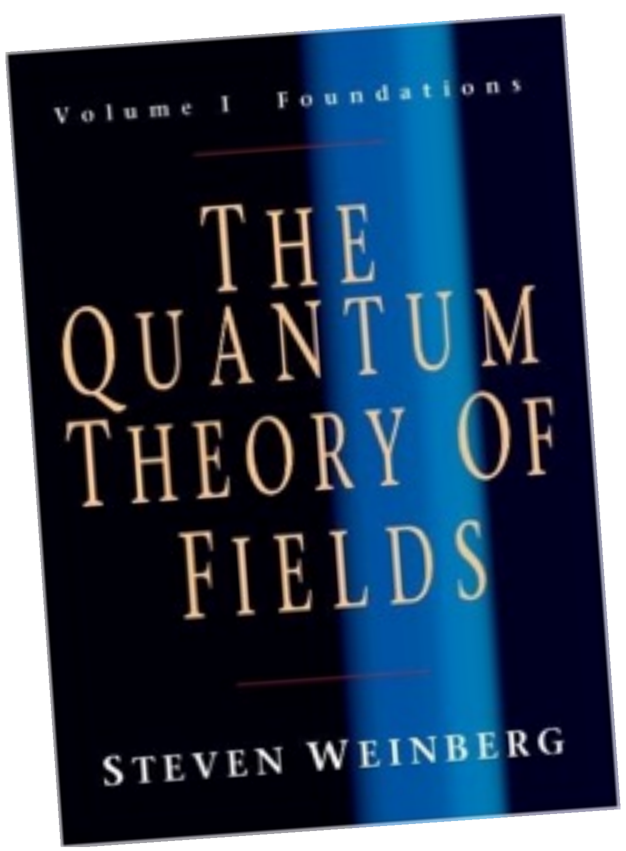

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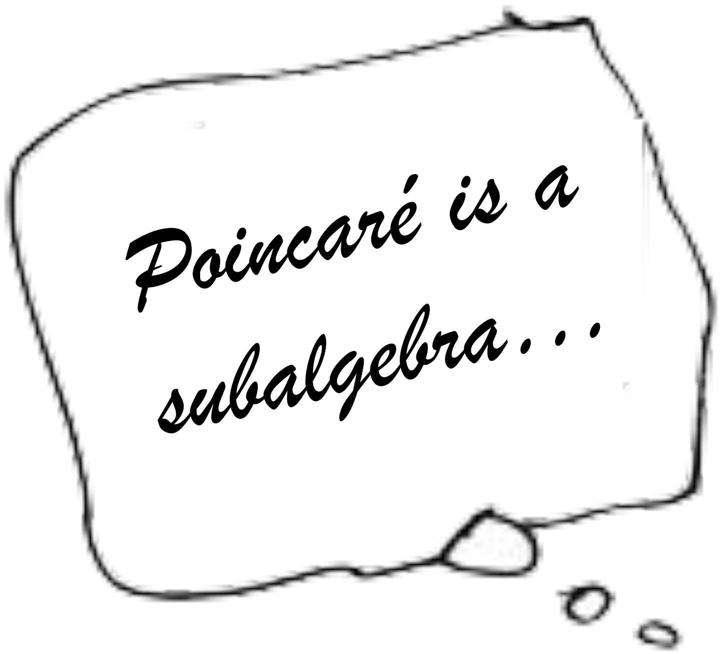
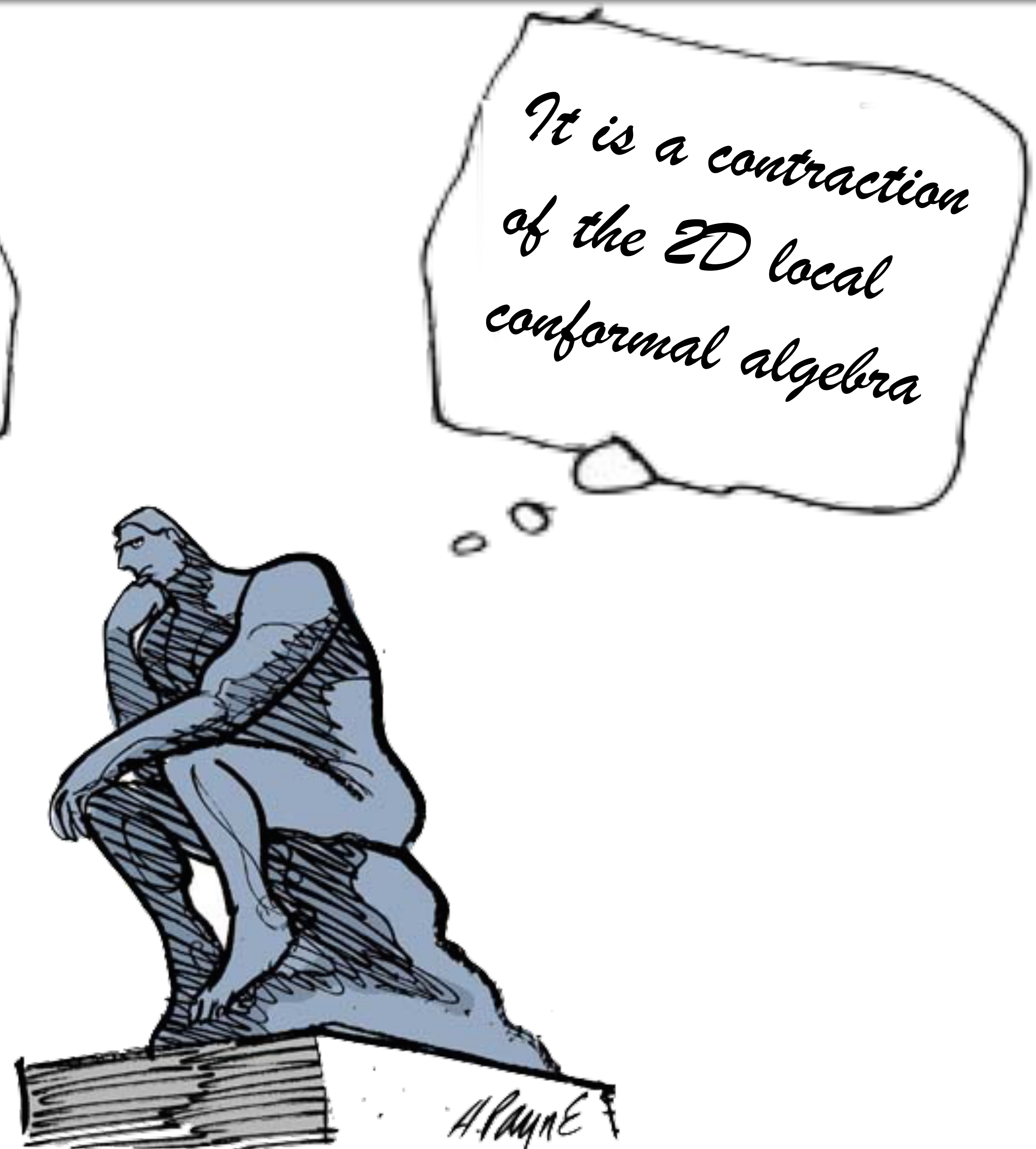
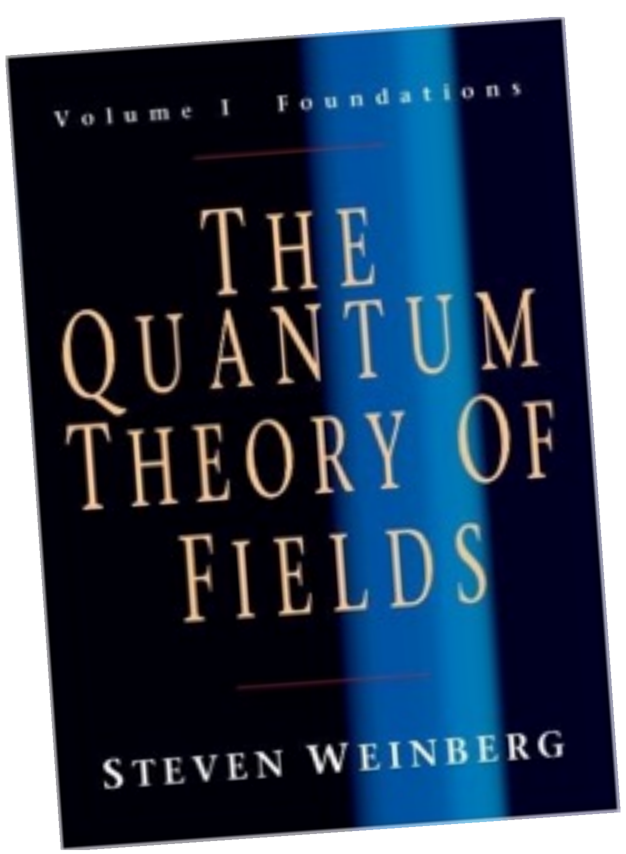
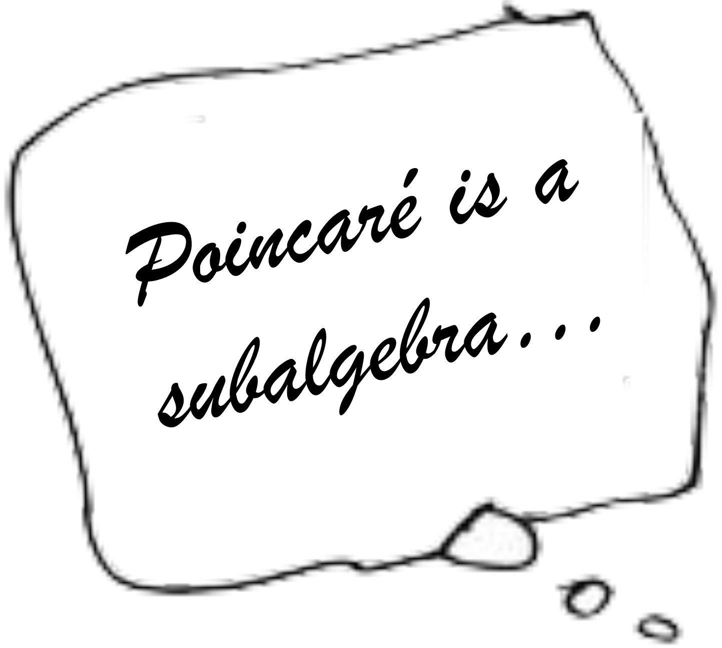
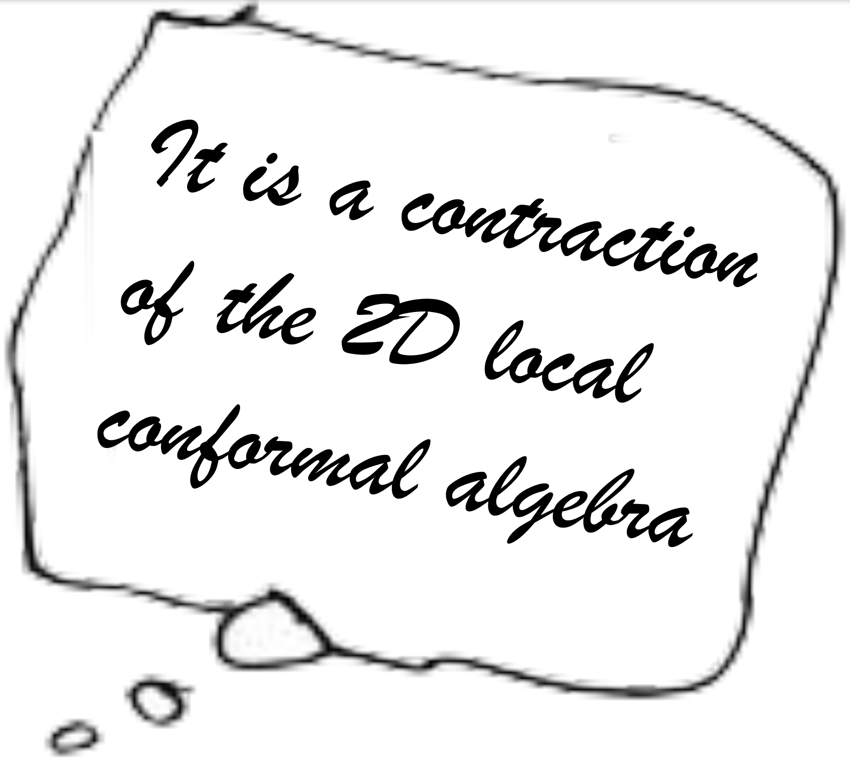


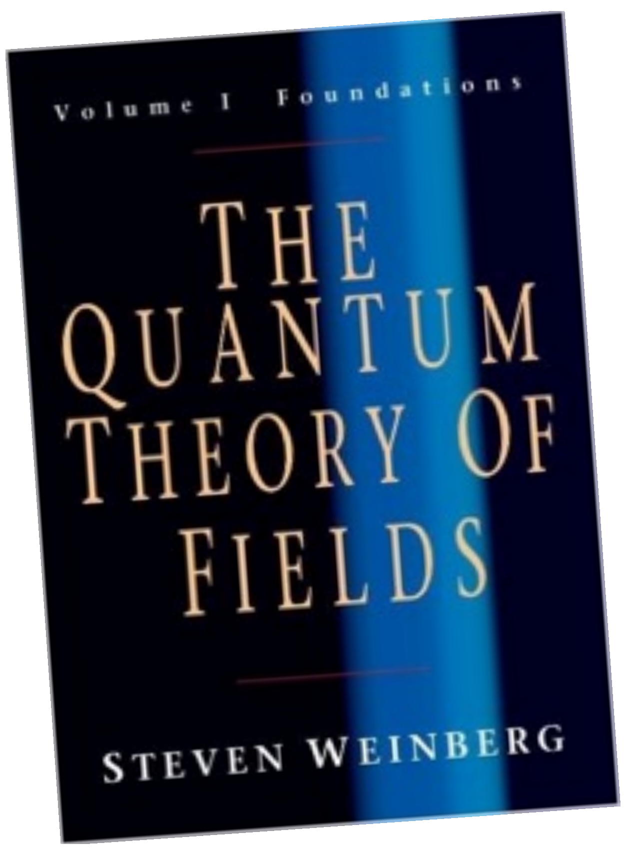


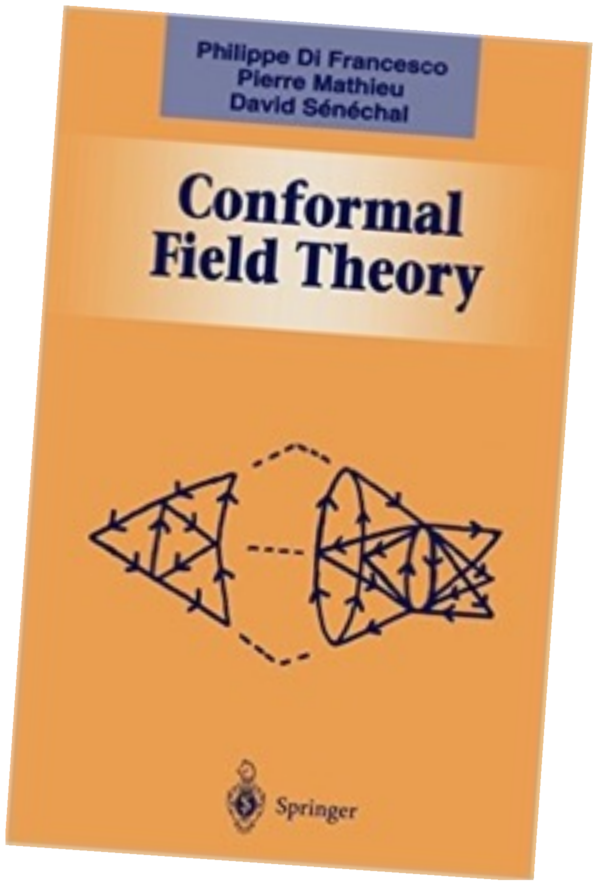


How to build representations of bms₃?









Poincaré unitary irreps in a nutshell

- Irreps of Poincaré *group* classified by orbits of momenta
  - all p^μ that satisfy $p^2 = -M^2$ for some mass M
- P_0 gives the energy and P_1,P_{-1} commute with it
  - build a basis of eigenstates of momentum: $|p^\mu, s\rangle$
- All plane waves can be obtained from a given one via

$$U(\Lambda)|p^\mu, s\rangle = e^{is\theta}|\Lambda^\mu{}_\nu p^\nu, s\rangle$$

$U(\omega) = \exp\left[\, i\left(\omega J_1 + \omega^* J_{-1}\right)\right]$ is a unitary operator

Rest-frame state & Poincaré modules

- Massive representations
  - Representative for the momentum orbit $k^\mu = (M,0,0)$
  - The corresponding plane wave $|M,s\rangle$ satisfies

$$P_0|M,s\rangle = M|M,s\rangle\,, \quad P_{-1}|M,s\rangle = P_1|M,s\rangle = 0\,, \quad J_0|M,s\rangle = s|M,s\rangle$$

  - $|M,s\rangle$ is annihilated by all P_n aside P_0!

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Save the info!

Rest-frame state & Poincaré modules

- *Rest-frame state*:

$$P_0|M,s\rangle = M|M,s\rangle\,, \quad P_{-1}|M,s\rangle = P_1|M,s\rangle = 0\,, \quad J_0|M,s\rangle = s|M,s\rangle$$

- Irreps of the Poincaré <u>*algebra*</u> built upon $|M,s\rangle$
  - Basis of the representation space: $|k,l\,\rangle = (J_{-1})^k (J_1)^l |M,s\rangle$
  - P_n and J_n act linearly on these states
  - <u>Irreducible?</u> Yes, Casimirs commute with all J_n
  - <u>Unitary?</u> Change basis! $|p^\mu,s\rangle = U(\Lambda)|M,s\rangle \longrightarrow \langle\, p^\mu, s\,|\,q^\mu, s\,\rangle = \delta_\mu(p,q)$

$\mathfrak{bms}_3$ modules

- Representation theory of BMS_3 *group* — Barnich, Oblak (2014)
  - Irreps again classified by orbits of supermomentum $p(\varphi) = \sum_{n\in\mathbb{Z}} p_n e^{in\varphi}$
  - It exists a basis $|p(\varphi), s\rangle$ of eigenstates of supermomentum
  - Orbits with a constant $p(\varphi) = M - c_2/24$ → *rest-frame state*!

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- Given the rest-frame state — A.C., Gonzalez, Oblak, Riegler (2016)

$$P_0|M,s\rangle = M|M,s\rangle\,, \quad P_m|M,s\rangle = 0 \text{ for } m \neq 0\,, \quad J_0|M,s\rangle = s|M,s\rangle$$

one can build a representation of the bms$_3$ *algebra* on

$$J_{n_1} J_{n_2} \cdots J_{n_N} |M,s\rangle \quad \text{with} \quad n_1 \geq n_2 \geq \ldots \geq n_N$$

$\mathfrak{bms}_3$ modules

- Group theory techniques do not apply neither to higher spins nor in D = 4

see however
A.C. , Gonzalez, Oblak, Riegler (2015)

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bms$_3$ modules

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see however
A.C., Gonzalez, Oblak, Riegler (2015)

- Unitarity and irreducibility not clear in this basis
→ turn to a basis of eigenstates of momentum!

- Given the rest-frame state

A.C., Gonzalez, Oblak, Riegler (2016)

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Ultrarelativistic vs Galilean limits of CFT

Higher spins

Characters & partition functions

Ultrarelativistic limit

- New generators: $$P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right), \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}$$
- In the limit $\ell \to \infty$ the conformal algebra becomes bms_3

Ultrarelativistic limit

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- In the limit $\ell \to \infty$ the conformal algebra becomes bms₃
- What happens to *highest-weight representations*?
  - HW state: $\mathcal{L}_n|h,\bar{h}\rangle = 0\,, \qquad \bar{\mathcal{L}}_n|h,\bar{h}\rangle = 0 \quad \text{when } n>0$
  - Verma module: $\mathcal{L}_{-n_1}\cdots\mathcal{L}_{-n_k}\bar{\mathcal{L}}_{-\bar{n}_1}\cdots\bar{\mathcal{L}}_{-\bar{n}_l}|h,\bar{h}\rangle$

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  - Verma module: $\mathcal{L}_{-n_1}\cdots\mathcal{L}_{-n_k}\bar{\mathcal{L}}_{-\bar{n}_1}\cdots\bar{\mathcal{L}}_{-\bar{n}_l}|h,\bar{h}\rangle$
  - New quantum numbers of the HW state: $M \equiv \frac{h+\bar{h}}{\ell}\,, \qquad s \equiv h - \bar{h}$
  - Rewrite $\mathcal{L}_{-n_1}\cdots\mathcal{L}_{-n_k}\bar{\mathcal{L}}_{-\bar{n}_1}\cdots\bar{\mathcal{L}}_{-\bar{n}_l}|h,\bar{h}\rangle$ in the new basis as $J_{n_1}J_{n_2}\cdots J_{n_N}|M,s\rangle$ with $n_1 \geq n_2 \geq \ldots \geq n_N$
  - J_n don't annihilate the vacuum → invertible change of basis!

Ultrarelativistic limit

- Matrix elements of P_n and J_n

A.C. , Gonzalez, Oblak, Riegler (2016)

$$P_n\,|k_1,\ldots,k_N\rangle \;=\; \sum_{k'_i} \mathsf{P}^{(n)}_{k'_i;\,k_j}(M,s,\ell)\,|k'_1,\ldots,k'_N\rangle$$

$$J_n\,|k_1,\ldots,k_N\rangle \;=\; \sum_{k'_i} \mathsf{J}^{(n)}_{k'_i;\,k_j}(M,s)\,|k'_1,\ldots,k'_N\rangle$$

  - ℓ comes from the “old" CFT HW conditions: $\left(P_{\pm n} \pm \frac{1}{\ell} J_{\pm n}\right)|h,\bar{h}\rangle = 0$
  - only negative powers of ℓ appear: limit exists!

- If $h = \frac{M\ell + s}{2} + \lambda + \mathcal{O}(\ell^{-1}), \quad \bar{h} = \frac{M\ell - s}{2} + \lambda + \mathcal{O}(\ell^{-1})$

the highest-weight state $|h,\bar{h}\rangle$ satisfies in the limit

$$P_0|M,s\rangle = M|M,s\rangle\,, \quad P_m|M,s\rangle = 0 \text{ for } m \neq 0\,, \quad J_0|M,s\rangle = s|M,s\rangle$$

Galilean limit

- Alternative contraction conformal → bms3

Bagchi, Gopakumar, Mandal, Miwa (2010)

$$M_n \equiv \epsilon\left(\bar{\mathcal{L}}_n - \mathcal{L}_n\right), \qquad L_n \equiv \bar{\mathcal{L}}_n + \mathcal{L}_n$$

$$c_L = \bar{c} + c$$
$$c_M = \epsilon\left(\bar{c} - c\right)$$

$$\begin{aligned}
[L_m, L_n] &= (m-n)\,L_{m+n} + \frac{c_L}{12}\,m(m^2-1)\,\delta_{m+n,0} \\
[L_m, M_n] &= (m-n)\,M_{m+n} + \frac{c_M}{12}\,m(m^2-1)\,\delta_{m+n,0} \\
[M_m, M_n] &= \epsilon^2\,(\cdots)
\end{aligned}$$

- What happens to highest-weight reps?

$$\Delta = \bar{h} + h\,, \qquad \xi = \epsilon\left(\bar{h} - h\right)$$

$$L_n|\Delta,\xi\rangle = 0\,, \qquad M_n|\Delta,\xi\rangle = 0\,, \quad n > 0$$

$$|\{\mathfrak{l}_i\},\{\mathfrak{m}_j\}\rangle = L_{-\mathfrak{l}_1}\dots L_{-\mathfrak{l}_i} M_{-\mathfrak{m}_1}\dots M_{-\mathfrak{m}_j}|\Delta,\xi\rangle \qquad \mathfrak{m}_1 \geq \dots \geq \mathfrak{m}_j > 0$$

Galilean limit

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$$L_n|\Delta,\xi\rangle = 0\,, \qquad M_n|\Delta,\xi\rangle = 0\,, \quad n>0$$

$$|\{\mathfrak{l}_i\},\{\mathfrak{m}_j\}\rangle = L_{-\mathfrak{l}_1}\dots L_{-\mathfrak{l}_i} M_{-\mathfrak{m}_1}\dots M_{-\mathfrak{m}_j}|\Delta,\xi\rangle$$

- HW reps are mapped into other HW reps
  - Cool! We can define a scalar product using $(M_m)^\dagger = M_{-m}$ $(L_m)^\dagger = L_{-m}$
  - These reps are typically non-unitary and reducible
  - Ok for condensed matter applications but bad for gravity!

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Gravity in D = 2+1

- Einstein-Hilbert action

$$I = \frac{1}{16\pi G}\int \epsilon_{abc}\left(e^a \wedge R^{bc} + \frac{1}{3l^2}\, e^a \wedge e^b \wedge e^c\right)$$

- A couple of useful tricks…
  - $\omega_\mu{}^a = \frac{1}{2}\,\epsilon^a{}_{bc}\,\omega_\mu{}^{b,c}$
  - so(2,1) ≃ sl(2,R) : $[J_a, J_b] = \epsilon_{abc} J^c$

Gravity in D = 2+1

- Einstein-Hilbert action

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3l^2}\, e\wedge e\wedge e\right) \quad \text{with} \quad \begin{cases} e = e^a J_a \\ \omega = \omega^a J_a \end{cases}$$

- A couple of useful tricks…
  - $\omega_\mu{}^a = \frac{1}{2}\epsilon^a{}_{bc}\,\omega_\mu{}^{b,c}$
  - so(2,1) ≃ sl(2,R) : $[J_a, J_b] = \epsilon_{abc} J^c$

“Higher spins” in D = 2+1

- What happens with other gauge algebras? E.g. sl(3,R)?

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sl(3,R) algebra:

$$[J_a, J_b] = \epsilon_{abc} J^c$$

$$[J_a, T_{bc}] = \epsilon^m{}_{a(b} T_{c)m}$$

$$[T_{ab}, T_{cd}] = -\left(\eta_{a(c}\epsilon_{d)bm} + \eta_{b(c}\epsilon_{d)am}\right) J^m$$

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no problems in defining the flat limit

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- Yet another trick: *Einstein-Hilbert* ↔ *Chern-Simons*
  - AdS: so(2,2) ≃ sl(2,R) ⊕ sl(2,R) Chern-Simons action   Achúcarro, Townsend (1986)

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  - <u>Flat space</u>: iso(2,1) = sl(2,R) ⊕ sl(2,R)$_{Ab}$ Chern-Simons action   Witten (1988)

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  - AdS: so(2,2) ≃ sl(2,R) ⊕ sl(2,R) Chern-Simons action — Achúcarro, Townsend (1986)
  - Flat space: iso(2,1) = sl(2,R) ⊕ $sl(2,R)_{Ab}$ Chern-Simons action — Witten (1988)
  - Higher spins: sl(N,R) ⊕ / ⊕ sl(N,R) Chern-Simons theories — Blencowe (1989)

Spin-3 extension of the conformal algebra

$$[\mathcal{L}_m,\, \mathcal{L}_n] = (m-n)\,\mathcal{L}_{m+n} + \frac{c}{12}\,(m^3-m)\,\delta_{m+n,\,0}\,,$$

$$[\mathcal{L}_m,\, \mathcal{W}_n] = (2m-n)\,\mathcal{W}_{m+n}\,,$$

$$[\mathcal{W}_m,\, \mathcal{W}_n] = (m-n)(2m^2+2n^2-mn-8)\,\mathcal{L}_{m+n} + \frac{96}{c+\frac{22}{5}}\,(m-n)\,:\!\mathcal{L}\mathcal{L}\!:_{m+n}$$
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- Normal ordering now needed:

$$:\mathcal{L}\mathcal{L}:_m = \sum_{p\geq -1} \mathcal{L}_{m-p}\mathcal{L}_p + \sum_{p<-1} \mathcal{L}_p\mathcal{L}_{m-p} - \frac{3}{10}(m+3)(m+2)\mathcal{L}_m$$

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- Ultrarelativistic contraction:

$$P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right), \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}$$
$$W_m \equiv \mathcal{W}_m - \bar{\mathcal{W}}_{-m}, \qquad Q_m \equiv \frac{1}{\ell}\left(\mathcal{W}_m + \bar{\mathcal{W}}_{-m}\right)$$

Spin-3 extension of bms_3

- Limit $\ell \to \infty$: bms_3 algebra plus…

$$
\begin{aligned}
[W_m, W_n] &= (m-n)(2m^2+2n^2-mn-8)J_{m+n} + \frac{96}{c_2}(m-n)\Lambda_{m+n} \\
&\quad - \frac{96\, c_1}{c_2^2}(m-n)\Theta_{m+n} + \frac{c_1}{12}(m^2-4)(m^3-m)\,\delta_{m+n,0}\,, \\
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&\quad + \frac{c_2}{12}(m^2-4)(m^3-m)\,\delta_{m+n,0}\,, \\
[Q_m, Q_n] &= 0\,,
\end{aligned}
$$

- Non-linearities survive in the limit!

$$
\Theta_m \equiv \sum_{p=-\infty}^{\infty} P_{m-p}P_p\,, \qquad \Lambda_m \equiv \sum_{p=-\infty}^{\infty} (P_{m-p}J_p + J_{m-p}P_p)
$$

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Spin-3 extension of bms$_3$

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$$[W_m,\, Q_n] = (m-n)(2m^2+2n^2-mn-8)P_{m+n} + \frac{96}{c_2}(m-n)\Theta_{m+n} + \frac{c_2}{12}(m^2-4)(m^3-m)\,\delta_{m+n,\,0}\,,$$

$$[Q_m,\, Q_n] = 0\,,$$

Galilean limit
=
different ordering!

- Non-linearities survive in the limit!

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Grumiller, Riegler, Rosseel (2014)

Higher-spin modules

- Representations as for bms3 and Poincaré
  - Introduce a rest-frame state $P_m|M, q_0\rangle = 0\,, \qquad Q_m|M, q_0\rangle = 0 \quad \text{for } m \neq 0$
  - Build the vector space which carries the representation as

    $$W_{k_1} \cdots W_{k_m} J_{l_1} \cdots J_{l_n}|M, q_0\rangle \qquad k_1 \geq \cdots \geq k_m \qquad l_1 \geq \cdots \geq l_n$$

  - No problems with non-linearities (construction based on the universal enveloping algebra)

- Construction compatible with normal ordering:
  - $\langle 0|\Theta_n|0\rangle = \langle 0|\Lambda_n|0\rangle = 0$
  - Not true if one uses “Galilean" highest-weight reps!

Outline

The bms_3 algebra and its unitary irreps

Ultrarelativistic vs Galilean limits of CFT

Higher spins

Characters & partition functions

Vacuum characters

A.C., Gonzalez, Oblak, Riegler (2015)

- One-loop partition function for a field of spin s

$$Z_{M,s}[\beta,\vec{\theta}] = \exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\chi_{M,s}[n\vec{\theta}, in\beta]\right]$$

characters of the Poincaré group

See González's talk!

- Vacuum character for a "flat" W_N algebra

$$\chi_{\text{vac}}[\theta,\beta] = e^{\frac{\beta}{8G}}\prod_{s=2}^{N}\left(\prod_{n=s}^{\infty}\frac{1}{|1-e^{in(\theta+i\epsilon)}|^2}\right)$$

- The vacuum character matches the product of partition functions of spin 2,3,…,N

Conclusions & outlook

- Higher-spin extensions of the $\mathfrak{bms}_3$ algebra admit unitary representations (no *"no-go"* as claimed earlier)
- Realised as induced modules
  - existence relies on very mild assumptions
  - unitarity $\Leftrightarrow$ plane wave basis
- Check: characters vs one-loop partition functions
- Towards a sensible $\mathfrak{bms}_3$ quantum theory?
- Hints for representation theory in four dimensions?