Magnetic Self-Organization in the RFP

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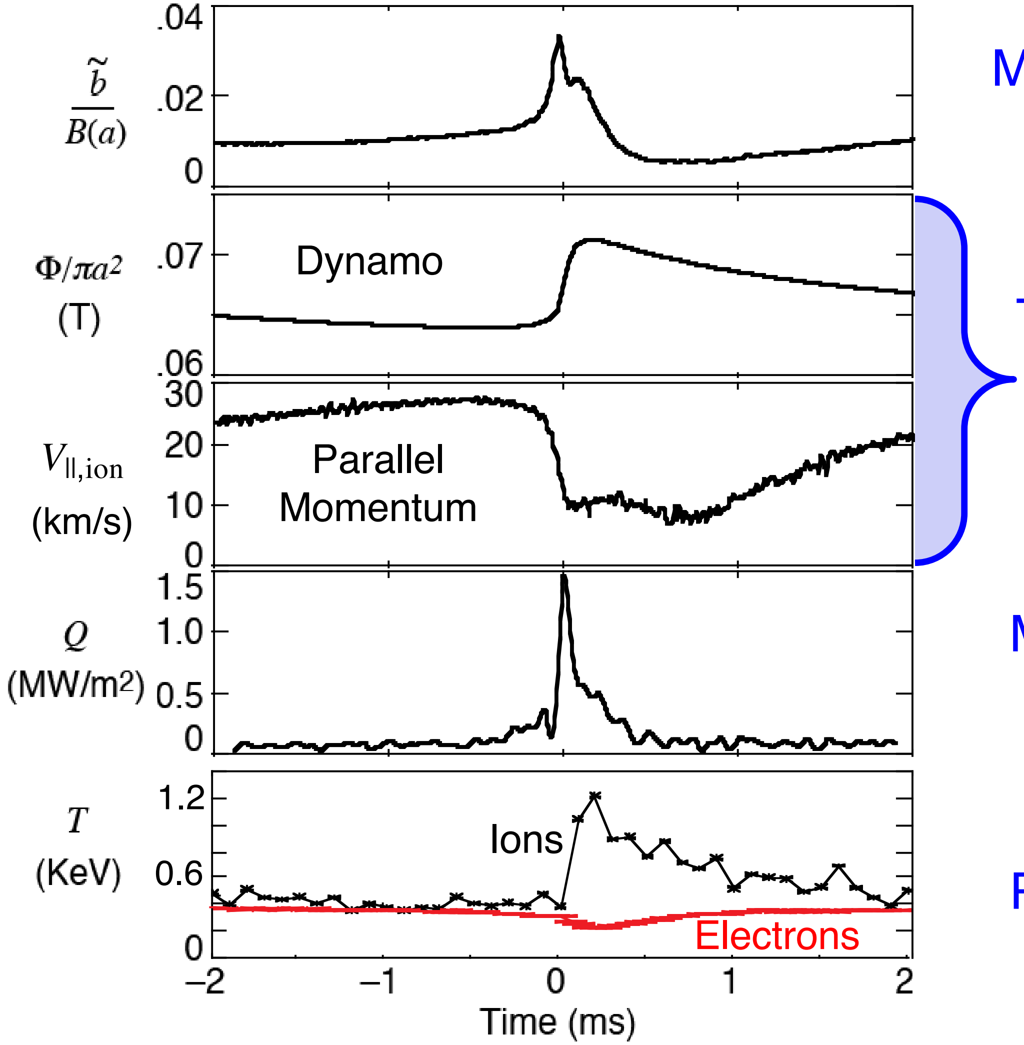
The RFP plasma exhibits a fascinating set of magnetic self-organization phenomena

Magnetic Relaxation Event Cycle



Magnetic Reconnection
(Tearing Instability)

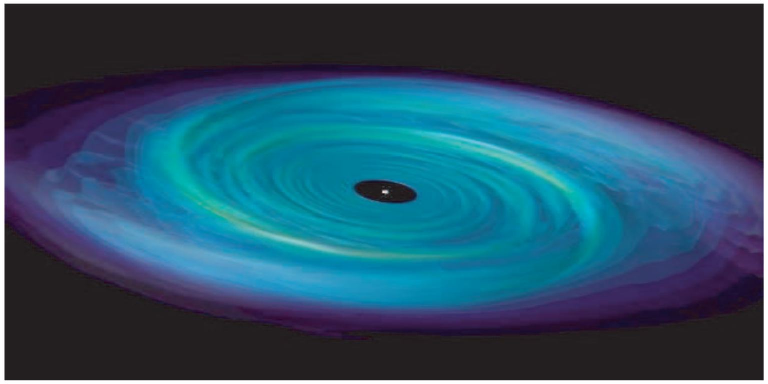
Two-Fluid Momentum
Relaxation
(current and ion flow)

Magnetic Turbulence
& Transport

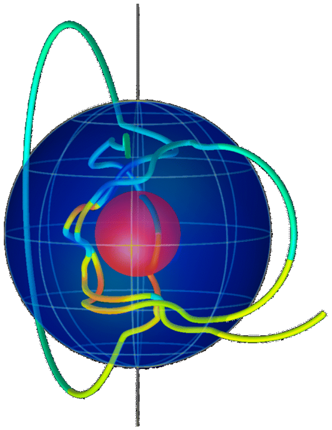
Non-Collisional
Particle Energization

Magnetic self-organization in natural plasmas

Momentum Transport

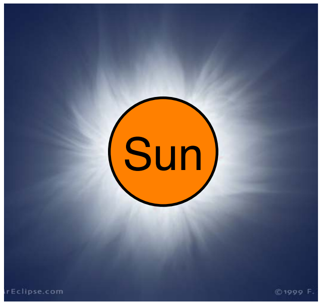


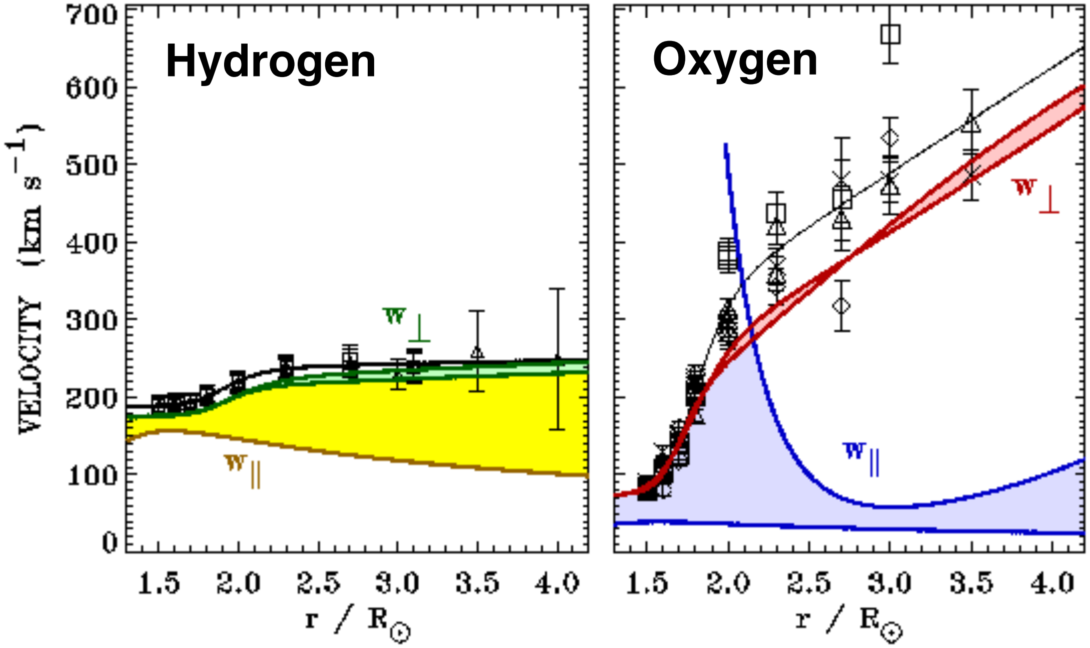
Solar/Geo Dynamo



Kuang & Bloxham, *Nature*, ’97

Ion Heating in the Solar Corona

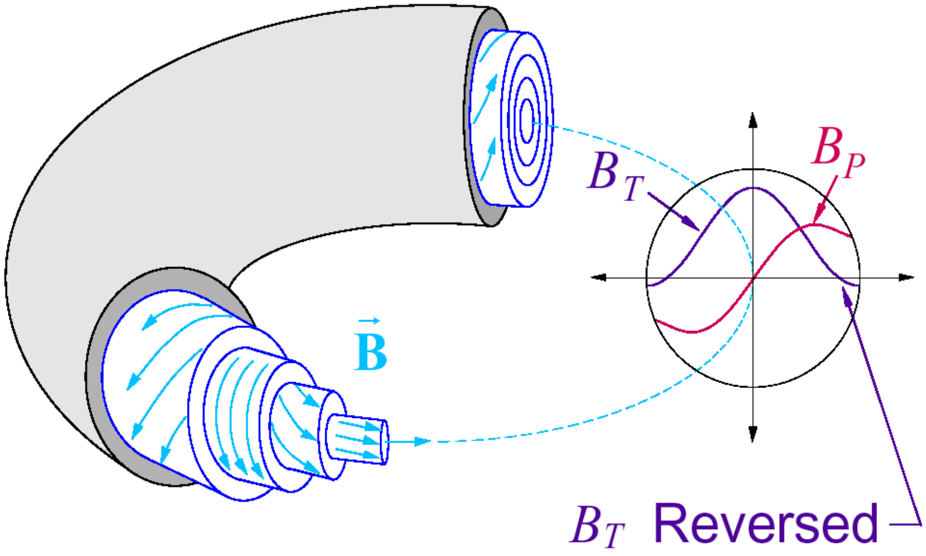


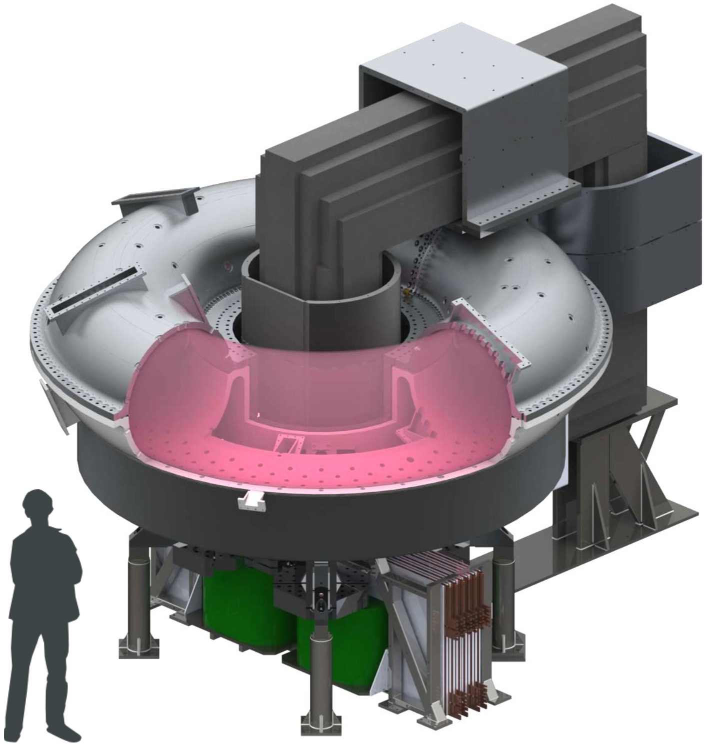


Cranmer et al., ApJ, **511**, 481 (1998)

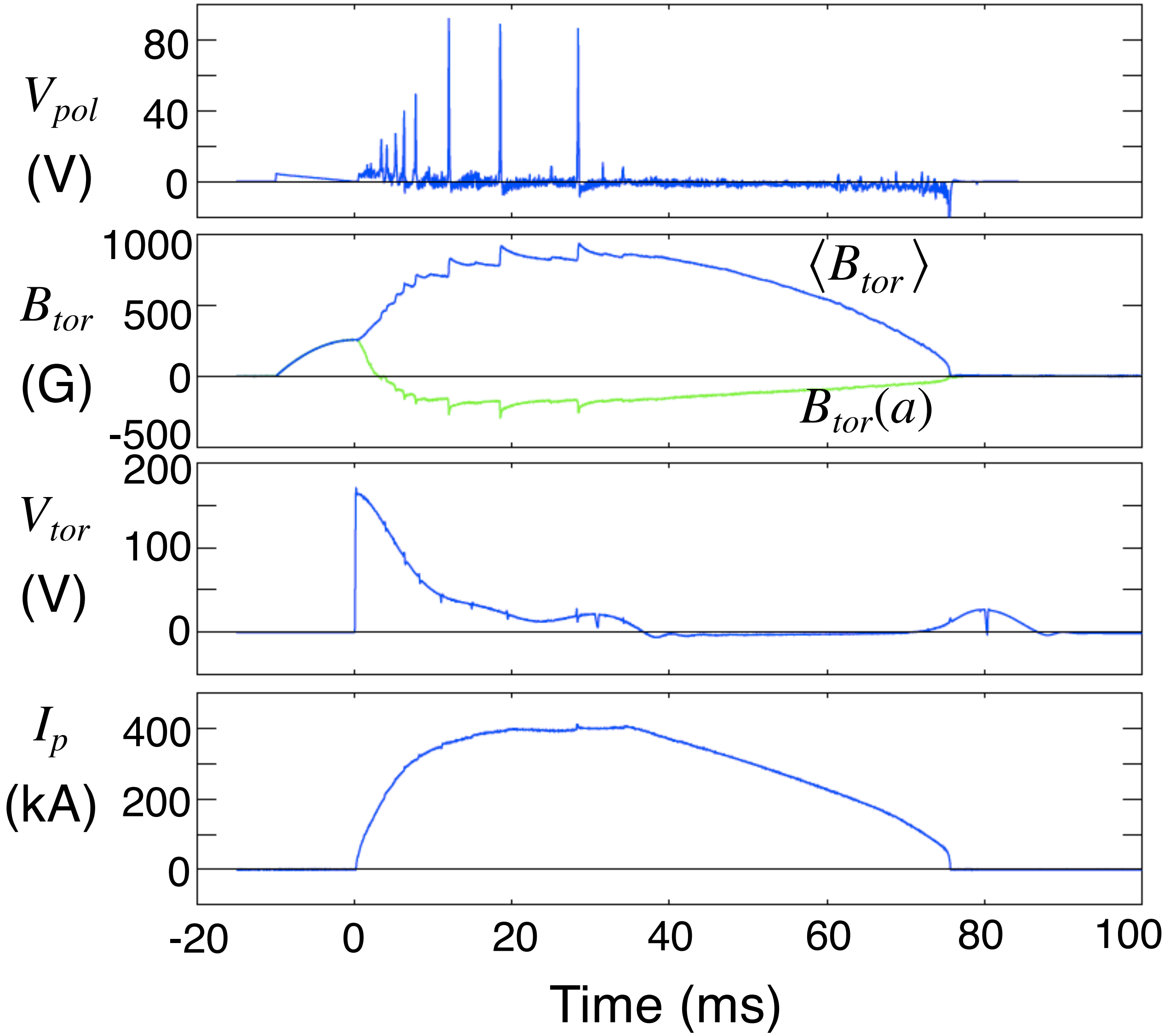
The MST RFP at UW-Madison

- Magnetic induction is used to drive a large current in the plasma
  - Plasma current, $I_p < 0.6$ MA ; $B < 0.5$ T
  - Externally applied inductive ohmic heating is 5-10 MW (input to electrons)
  - $T_\mathrm{i} \sim T_\mathrm{e} < 2$ keV, despite weak i-e collisional coupling ($n \sim 10^{19}$ m^{-3})
  - Minor radius, $a = 50$ cm ; ion gyroradius, $\rho_\mathrm{i} \approx 1$ cm ; $c/\omega_\mathrm{pi} \approx 10$ cm
    $\beta < 25\%$; Lundquist number $S = 5 \times 10^{5\text{-}6}$





Reversed BT forms with sufficiently large plasma current, and persists as long as induction is maintained



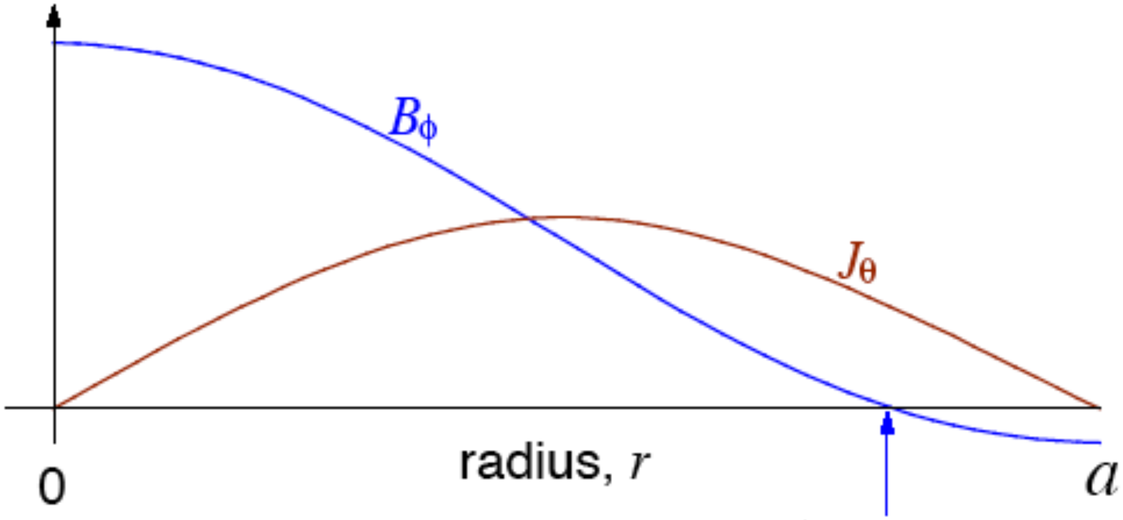
Poloidal loop voltage

Toroidal field

Toroidal loop voltage

Toroidal plasma current

However, a reversed-BT should not be an equilibrium



reversal implies: $(\nabla \times B)_\theta = \cancel{\frac{1}{R}\frac{\partial B_r}{\partial \phi}} - \frac{\partial B_\phi}{\partial r} \neq 0$

$\Rightarrow J_\theta \neq 0$, which for finite η

$$\Rightarrow E_\theta = -\frac{\partial}{\partial t}\int B_\phi \, dS \neq 0$$

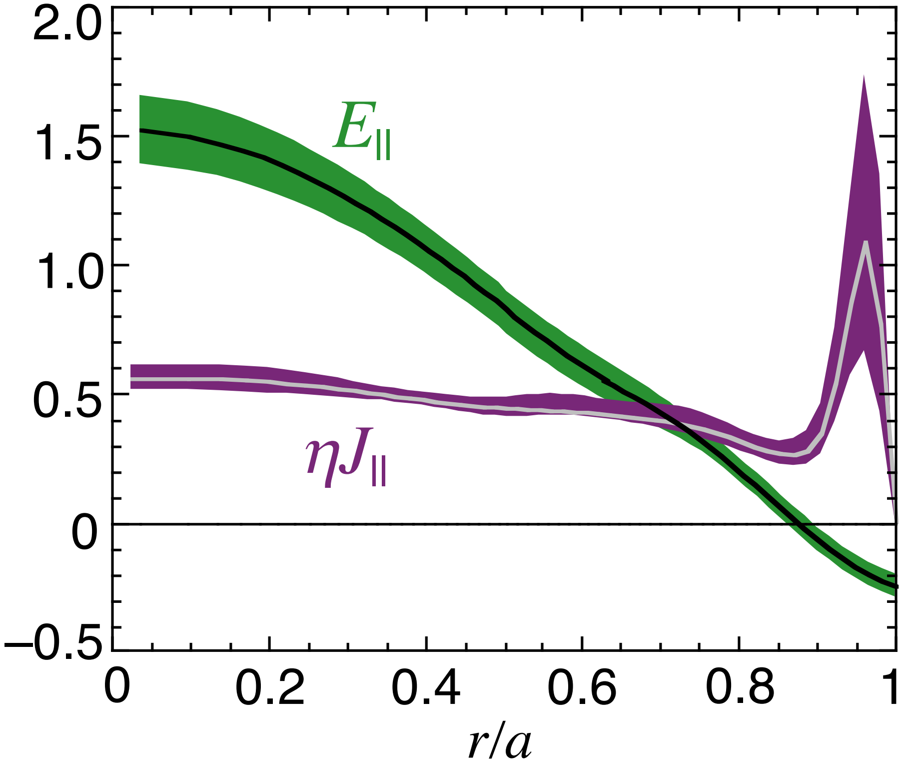
(non steady-state)

An imbalance in Ohm's law yields a similar conclusion

- Ohm's law: $\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \;\; \Rightarrow \;\; E_{||} = \eta J_{||}$ and $\mathbf{V}_{\perp} = \mathbf{E} \times \mathbf{B} / B^2$

- There is less current in the core than could be driven by $E_{||}$, and more current in the edge than should be driven by $E_{||}$

  ⇒ current profile is flatter than it "should" be

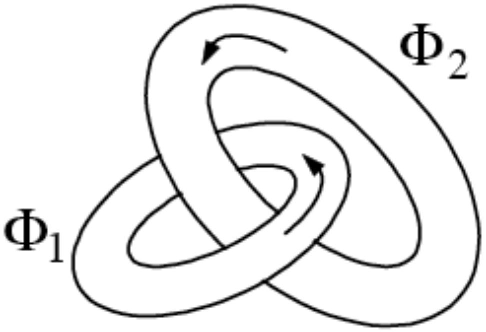


The RFP as a minimum energy configuration

- Minimize magnetic energy, with constrained global “magnetic helicity” $K = \int \mathbf{A} \cdot \mathbf{B} dV$ yields

$$\nabla \times \mathbf{B} = \lambda \mathbf{B} \quad \lambda = \text{ constant} \quad \text{(J.B. Taylor, 1974)}$$

$$K = 2\Phi_1\Phi_2$$



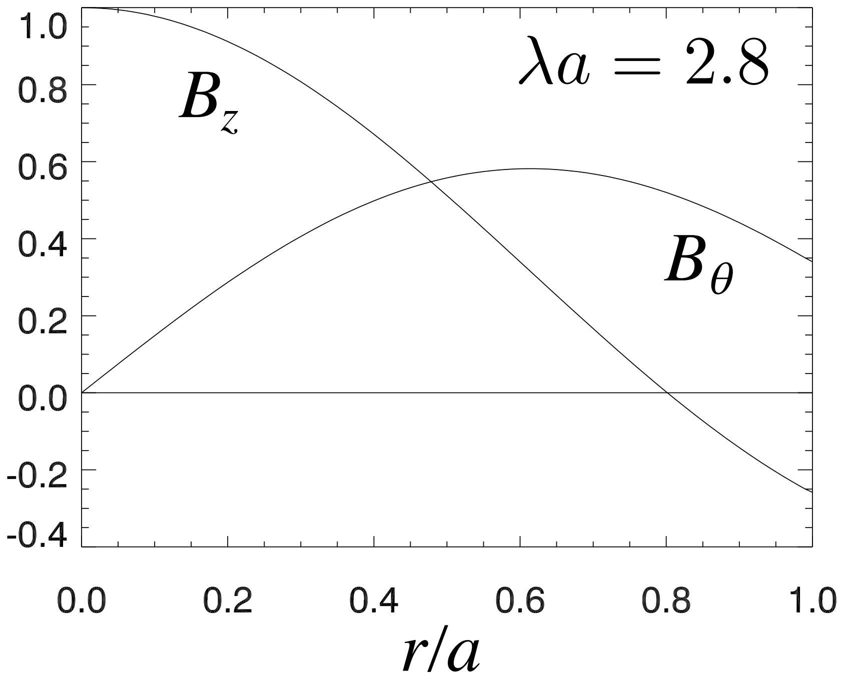
Solution in a cylinder: “Bessel Function Model”

$$B_z(r) = B_0 J_0(\lambda r)$$

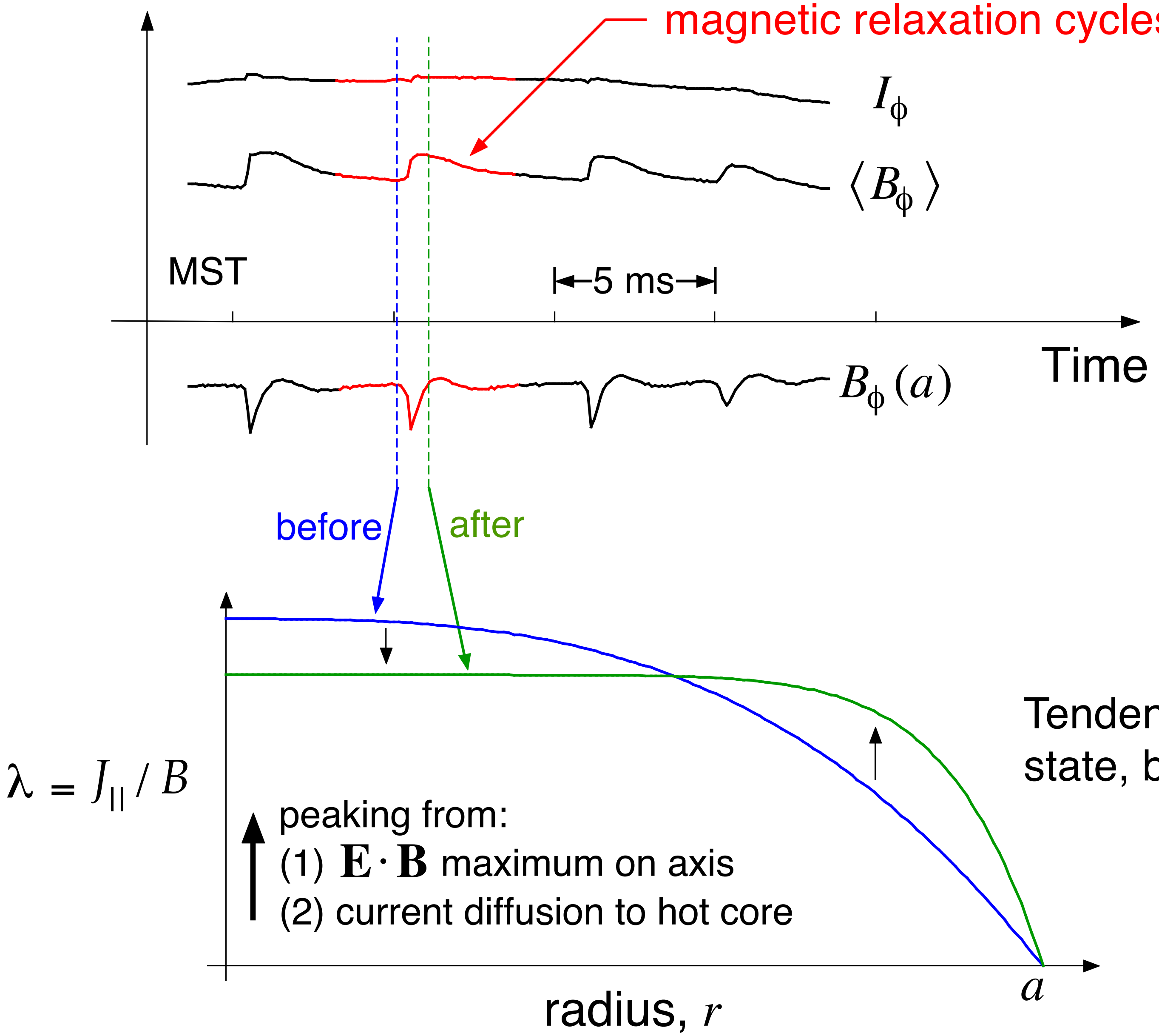
$$B_\theta(r) = B_0 J_1(\lambda r)$$

$J_0(\lambda a) < 0$ for $\lambda a > 2.4$

resembles an RFP equilibrium



Current profile exhibits a cycle of slow peaking followed by an abrupt flattening during impulsive relaxation events



peaking from:
(1) $\mathbf{E} \cdot \mathbf{B}$ maximum on axis
(2) current diffusion to hot core

Tendency toward a Taylor state, but not fully relaxed

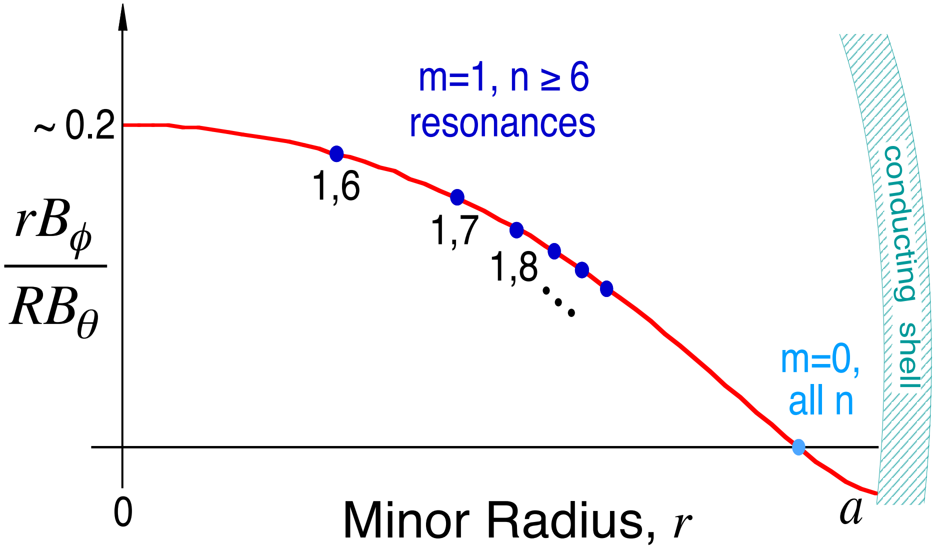
Relaxation cycles result from quasi-periodic impulsive magnetic reconnection events (a.k.a. sawteeth)

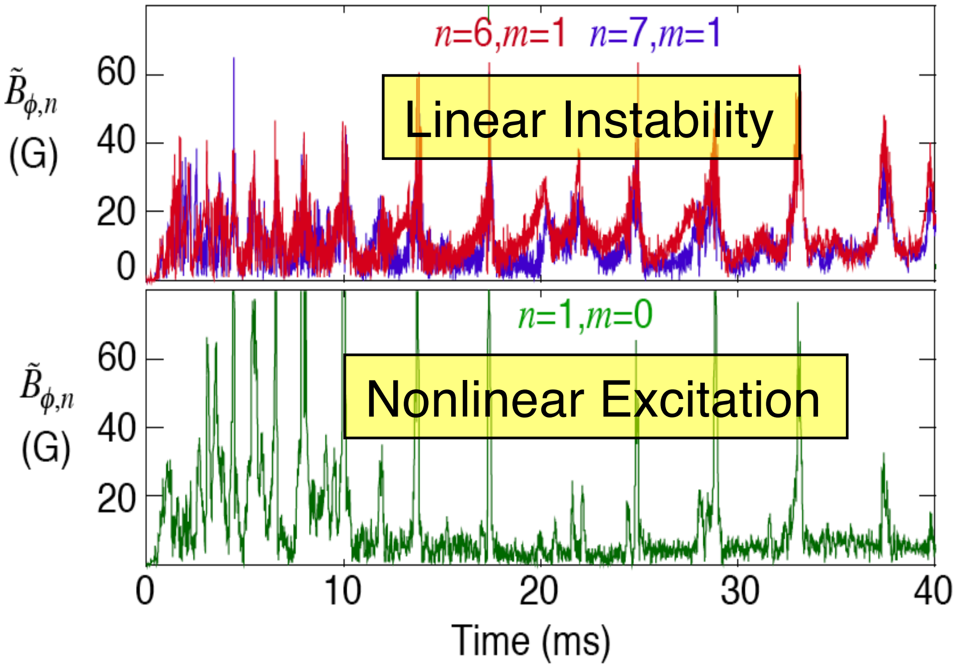
Toroidicity allows distinct $k_{||} = 0$ resonant modes at many radii in the plasma:

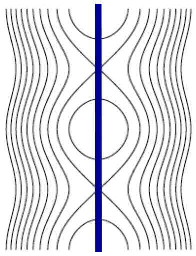
$$0 = \mathbf{k} \cdot \mathbf{B} = \frac{m}{r} B_\theta + \frac{n}{R} B_\phi$$

m = poloidal mode number

n = toroidal mode number







multiple
magnetic islands

A dynamo-like emf arrests the peaking tendency of the current profile, i.e., this is how tearing instability saturates in the RFP

- With non-axisymmetric quantities, (i.e., tearing instability):

$$\mathbf{B} = \langle \mathbf{B} \rangle + \tilde{\mathbf{B}}$$

toroidal surface average

spatial fluctuation

$$\tilde{B} \sim \tilde{b}(r) e^{i(m\theta - n\phi)}$$

$$\frac{\tilde{B}}{\langle B \rangle} << 1$$

- Then mean-field parallel Ohm's law becomes:

$$\langle E \rangle_{||} - \eta \langle J \rangle_{||} = \boxed{\langle \tilde{\mathbf{V}} \times \tilde{\mathbf{B}} \rangle_{||}}$$

dynamo-like emf
from tearing instability

Correlated product of fluctuations
□represents **nonlinear** saturation
at **equilibrium magnitude**

Nonlinear, resistive MHD provides a base model for the origin of the dynamo

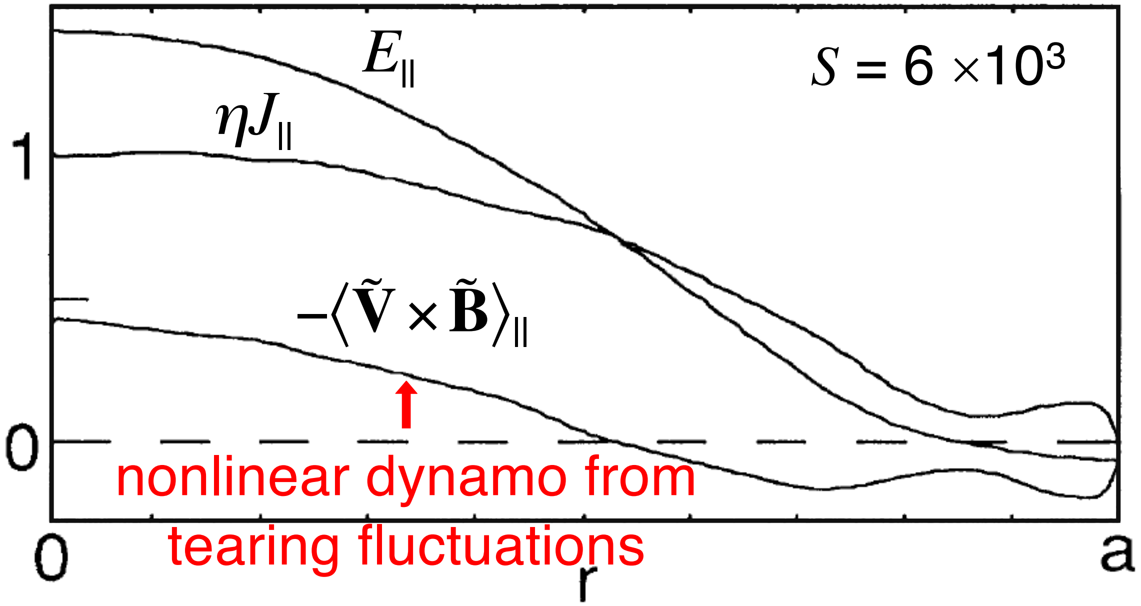
$$\mathbf{E} = \eta\mathbf{J} - S\,\mathbf{V}\times\mathbf{B}$$

$$S = \frac{\tau_R}{\tau_A} =$$ Lundquist number

$$\rho\frac{\partial\mathbf{V}}{\partial t} = -S\rho\mathbf{V}\cdot\nabla\mathbf{V} + S\mathbf{J}\times\mathbf{B} + P_m\nabla^2\mathbf{V}$$

$P_m = \nu/\eta =$ Magnetic Prandtl number

Ohm's Law



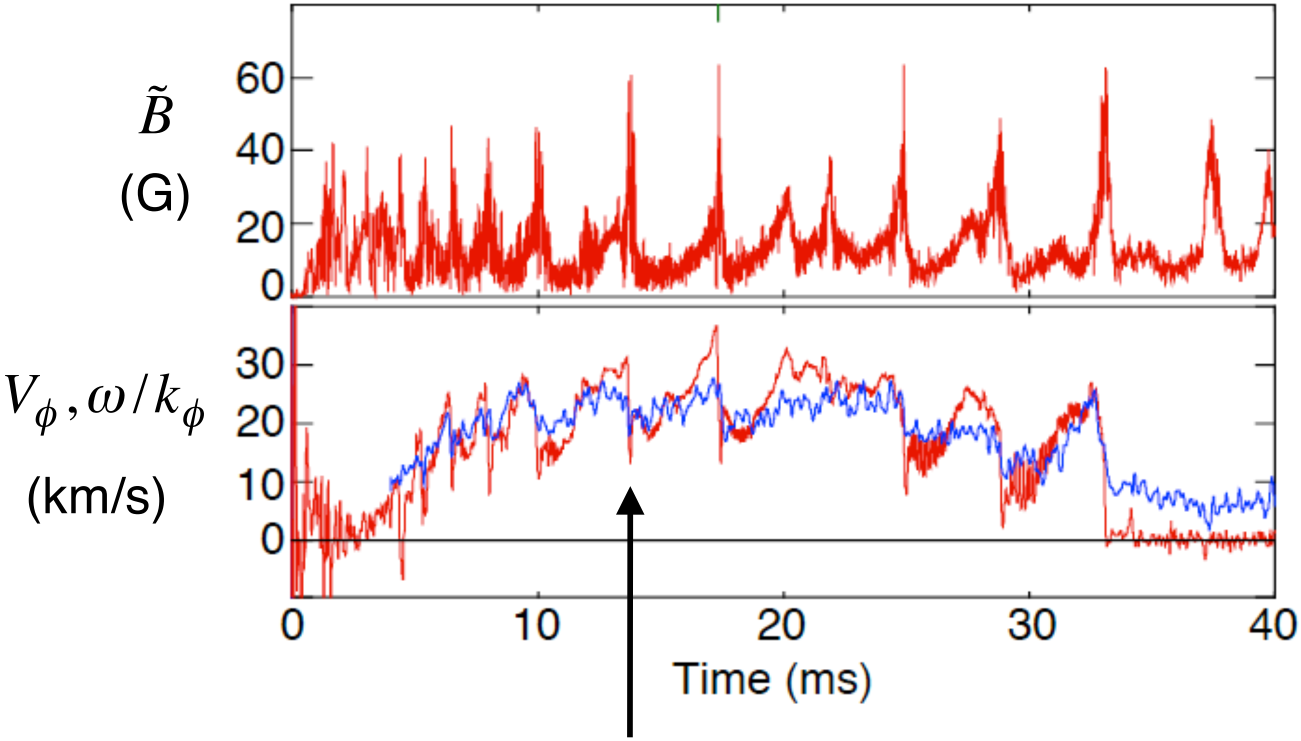
nonlinear dynamo from tearing fluctuations

Dynamo emf maintains the current profile close to marginal stability.

$\tilde{\mathbf{V}}, \tilde{\mathbf{B}} =$ fluctuations associated with tearing modes

Plasma (ion) flow also affected during relaxation events

- Implies coupled electron and ion momentum relaxation

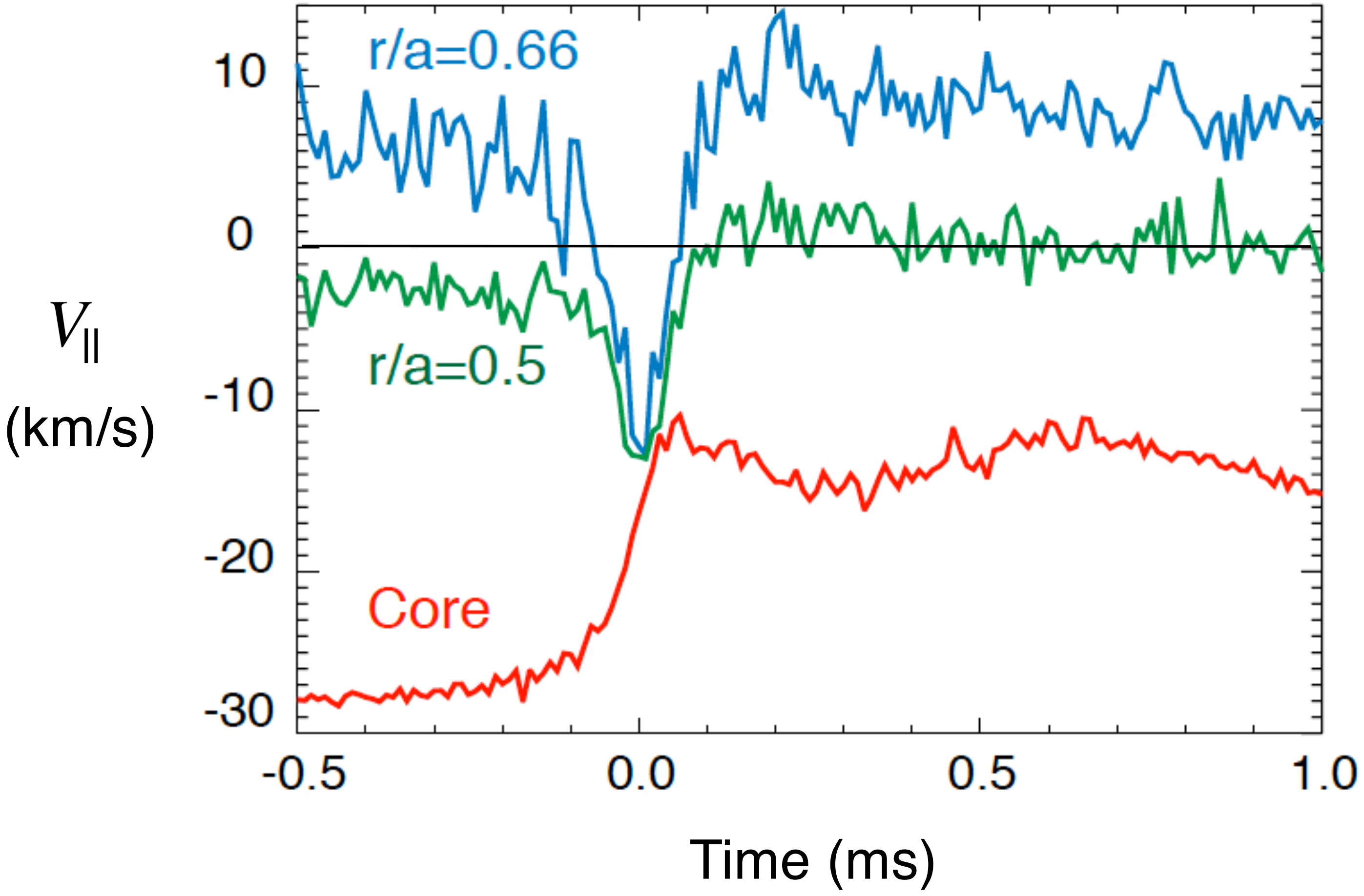


Tearing fluctuation

Plasma flow and mode rotation

Typical relaxation event

Profile of the parallel flow also flattens during relaxation events



Computational model for tearing-relaxation recently extended to include two-fluid effects

- Nonlinear multi-mode evolution solved using NIMROD

Ohm's law: $$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \frac{1}{ne}\mathbf{J} \times \mathbf{B} - \frac{1}{ne}\nabla p_e + \eta\mathbf{J} + \frac{m_e}{ne^2}\frac{\partial \mathbf{J}}{\partial t}$$

Momentum: $$nm_i\frac{d\mathbf{V}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_{gyro} - \nabla \cdot \nu nm_i\mathbf{W}$$

Relaxation process couples electron and ion momentum balance

Generalized Ohm's law permits several possible mechanisms for dynamo action

- The MHD and Hall mechanisms are measured to be significant, summing together in a way that has not been completely diagnosed

$$\mathbf{E} - \eta \mathbf{J} =$$

$$-\mathbf{V} \times \mathbf{B} + \frac{1}{en} \mathbf{J} \times \mathbf{B} - \frac{1}{en} \nabla p_e$$

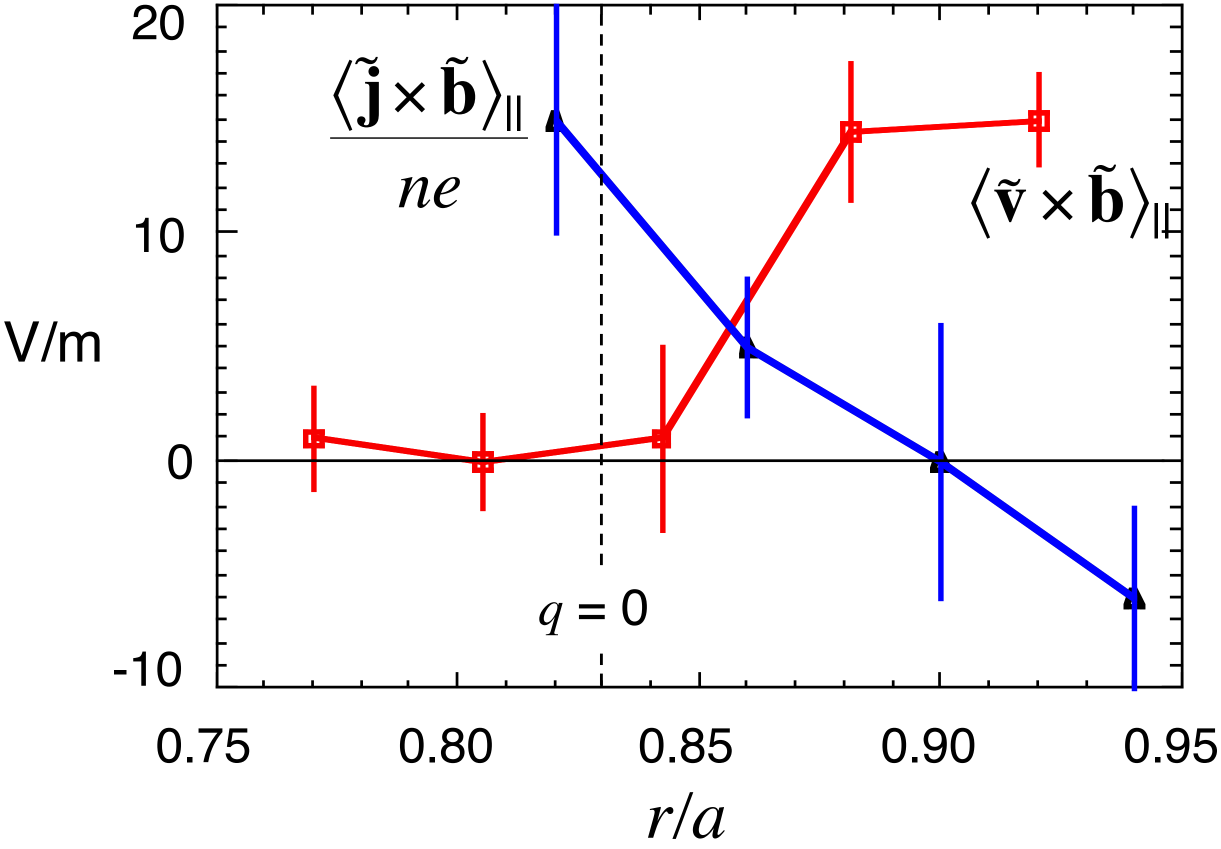
↑ "MHD"

↑ "Hall"

↑ "Diamagnetic" ($\nabla_\perp \tilde{p}_e$)

There's also a "kinetic" dynamo, i.e., stochastic transport of current

Probe measurements in the edge region show that both the MHD and Hall dynamo emf terms are important

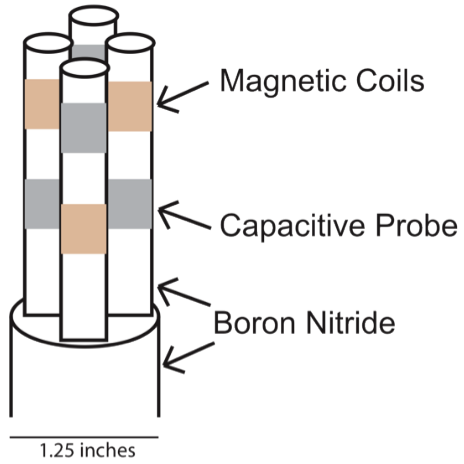


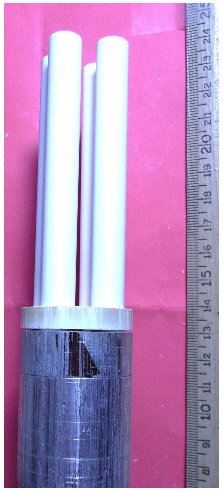
Measurements of the "total" dynamo emf show a balance in Ohm's law

$$\langle E\rangle_{||} - \eta\langle J\rangle_{||} = -\langle \tilde{\mathbf{V}}_e \times \tilde{\mathbf{B}}\rangle_{||} \approx \langle \tilde{\mathbf{E}}\cdot\tilde{\mathbf{B}}\rangle_{||} / B$$

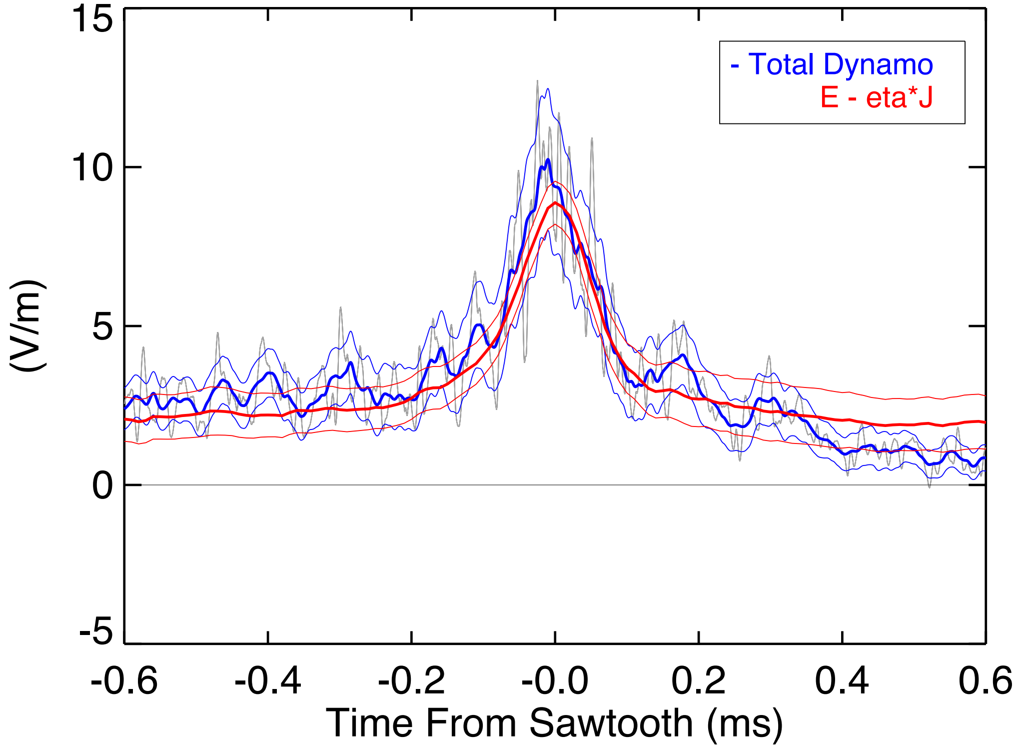
↑

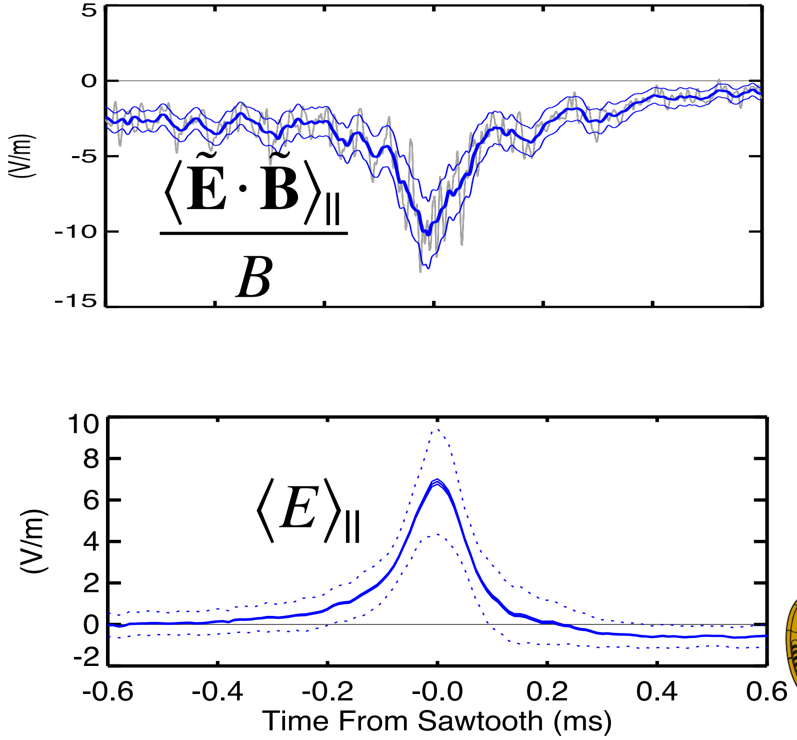
"total" dynamo emf





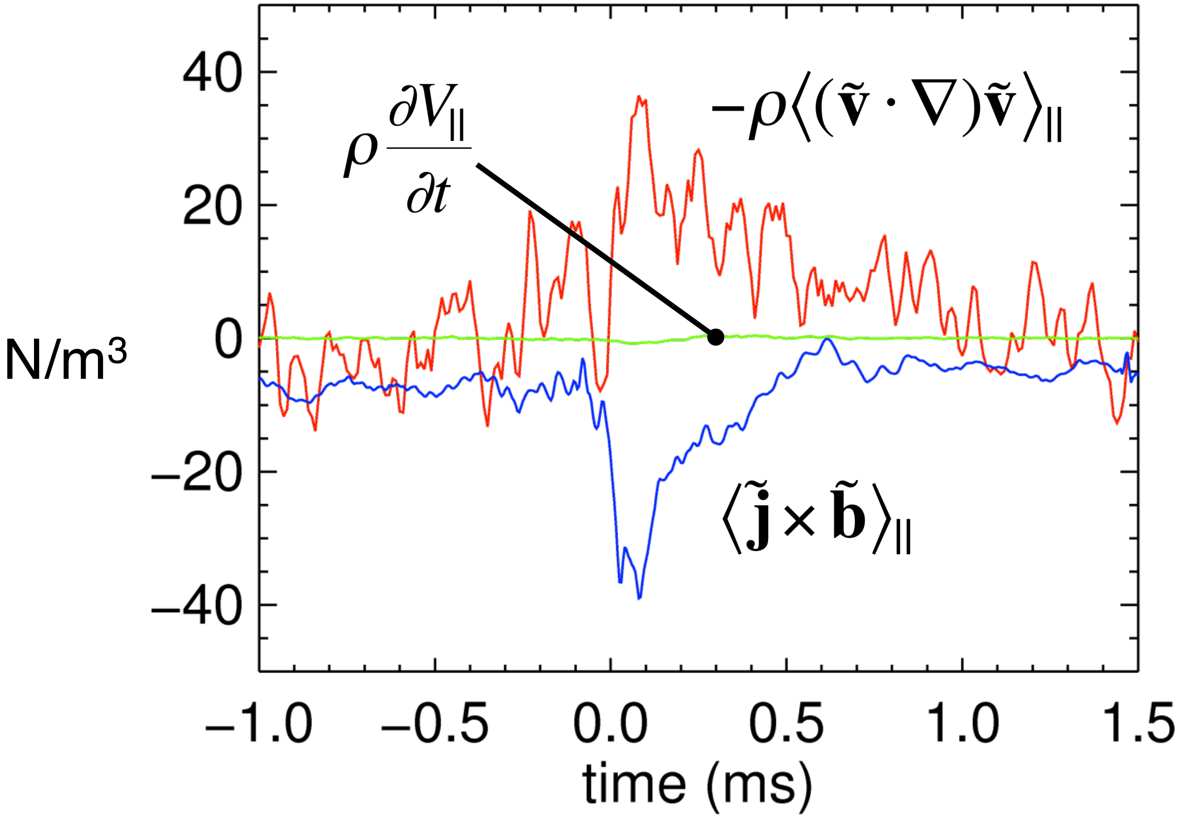
Measured Balance of Parallel Ohm's Law





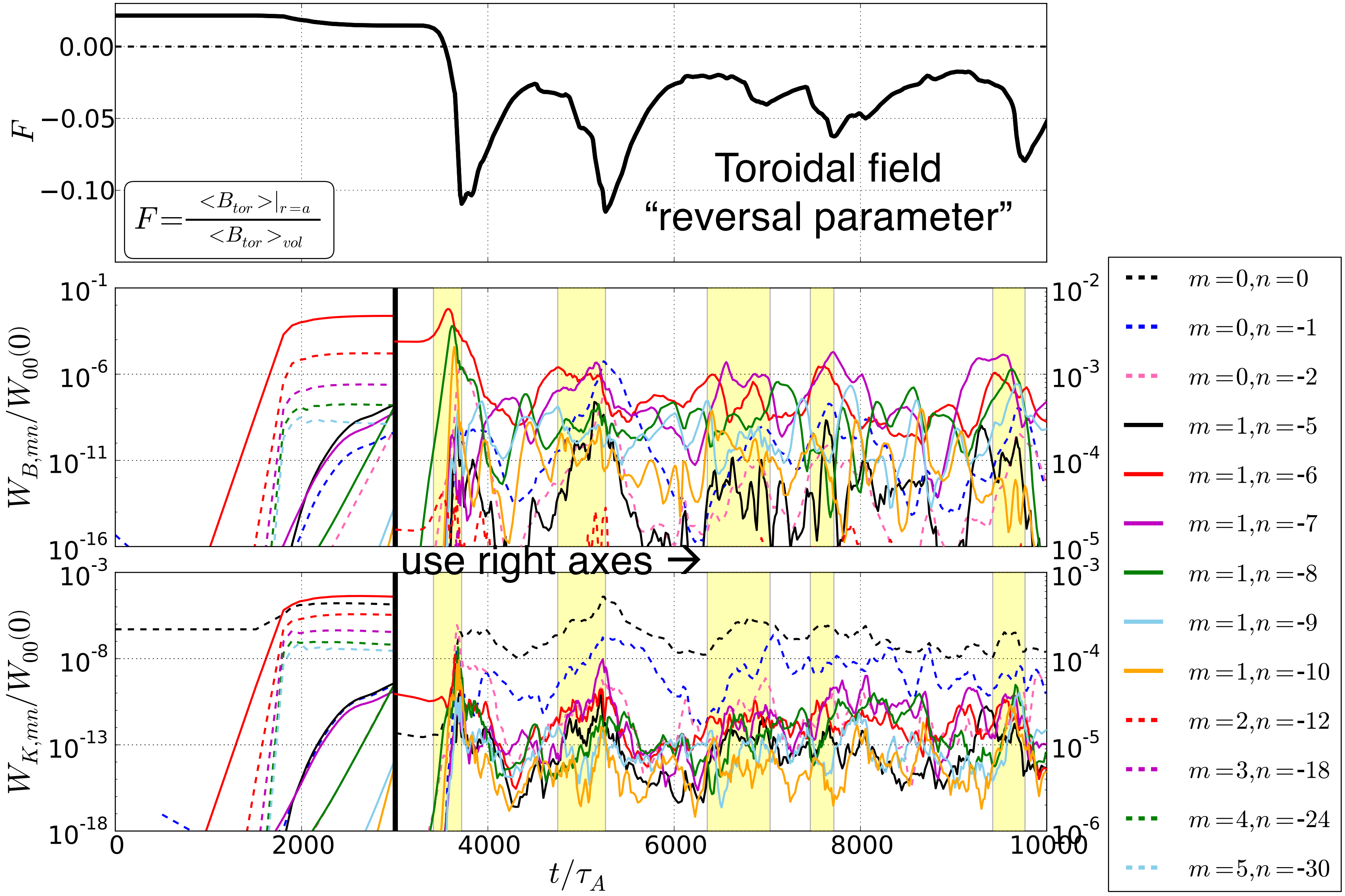
The Reynolds stress bursts in opposition to Hall emf, which is the Maxwell stress in parallel momentum balance

$$\rho \frac{\partial V_{||}}{\partial t} = \langle \tilde{\mathbf{j}} \times \tilde{\mathbf{b}} \rangle_{||} - \rho \langle (\tilde{\mathbf{v}} \cdot \nabla) \tilde{\mathbf{v}} \rangle_{||} - \nabla_{||} p + \nu \rho \nabla^2 V_{||}$$



Probe measurements
r/a=0.85
(edge region)

Relaxation events similar to those in MST are seen in NIMROD extended MHD simulations

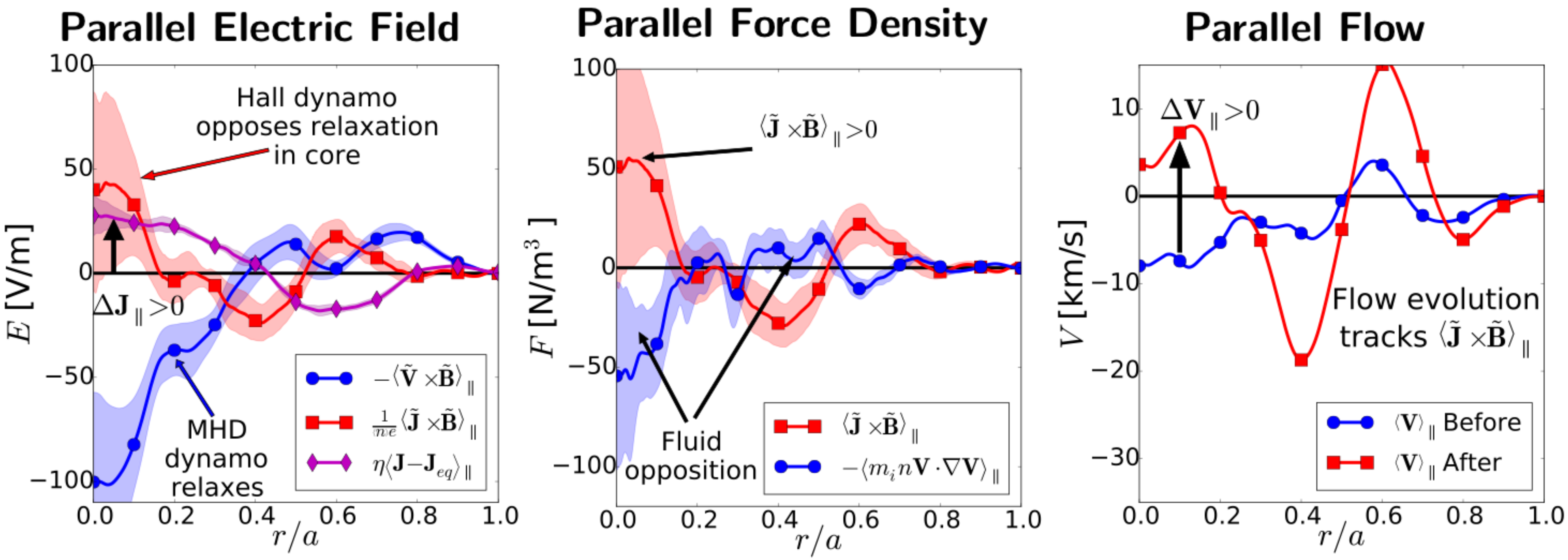


$F = \frac{<B_{tor}>|_{r=a}}{<B_{tor}>_{vol}}$

Toroidal field “reversal parameter”

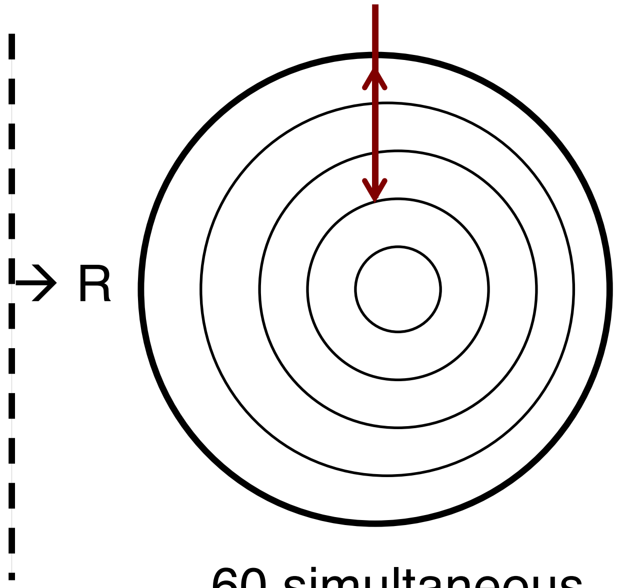
use right axes →

NIMROD simulations reveal the same tendencies as observed in MST plasmas



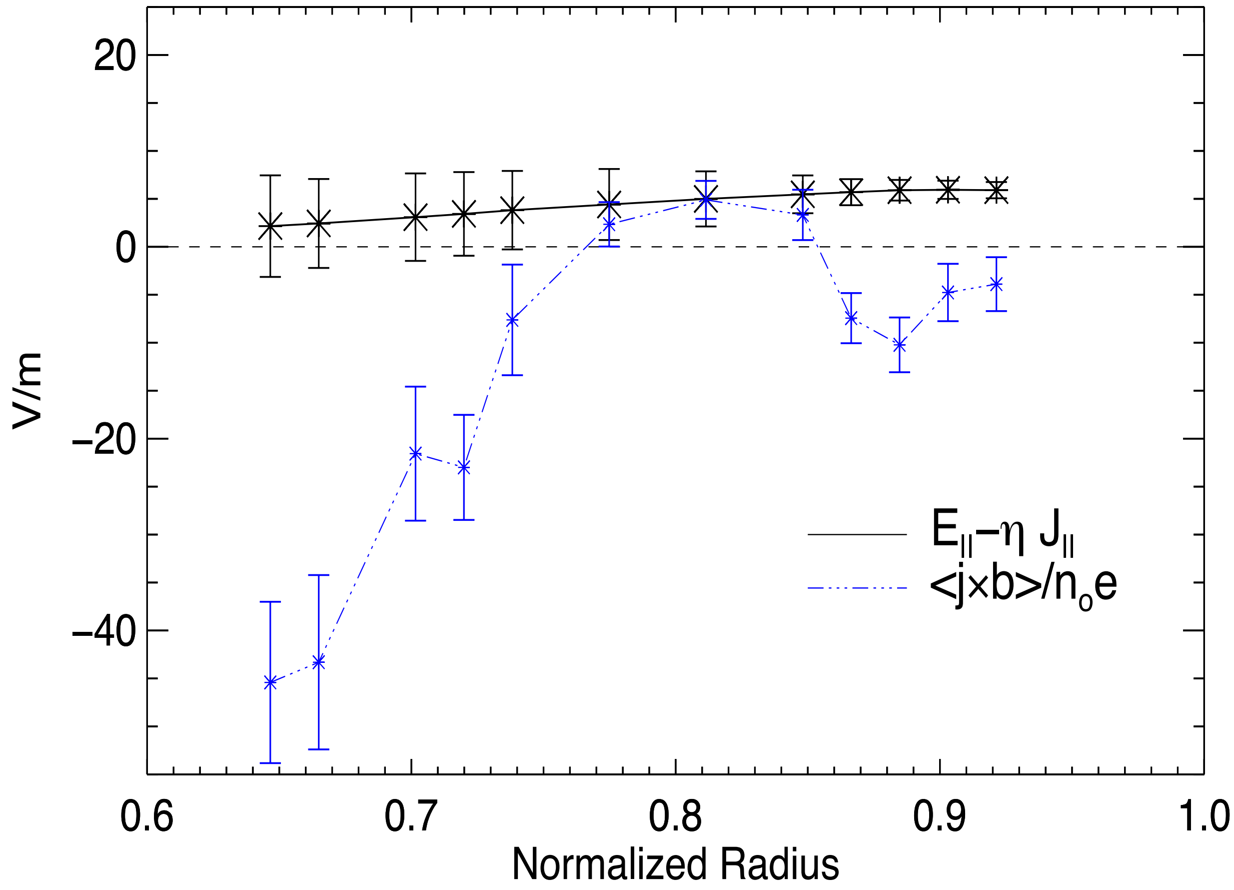
NIMROD simulations motivated probe measurements of the Hall dynamo over a larger portion of the plasma

- A deep-insertion capacitive probe for the total dynamo is in development



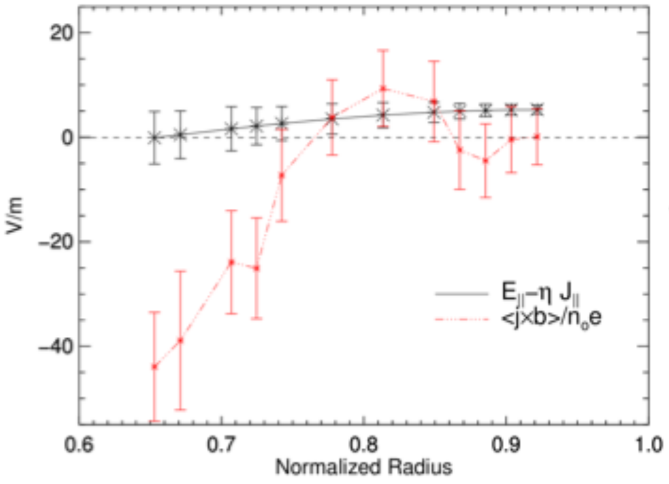
“Deep insertion” magnetic probe

60 simultaneous measurements



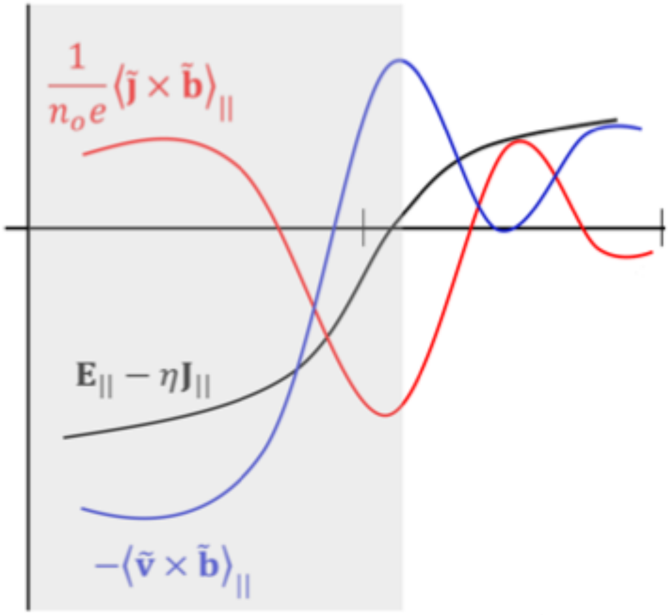
The measurements in MST are qualitatively similar to NIMROD predictions

Exp. Probe Data

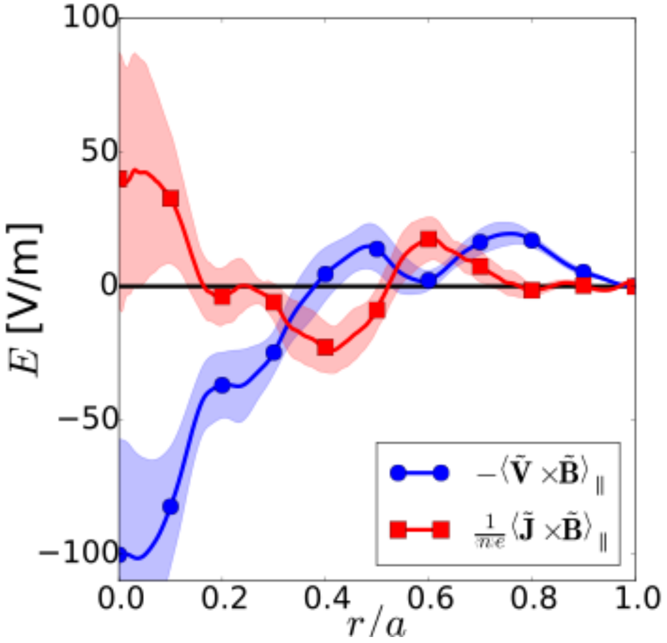


Courtesy of J. Triana

Sketch of Inferred Dynamo



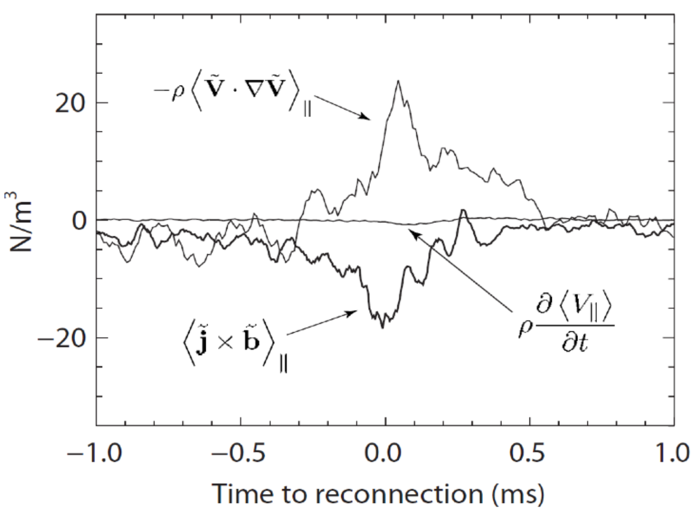
Computational Results



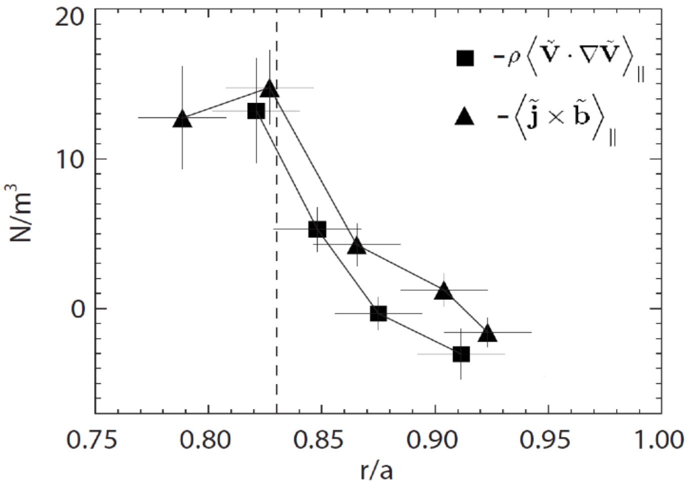
$$\langle\mathbf{E}\rangle_{||}-\eta\langle\mathbf{J}\rangle_{||}\approx\underbrace{-\left\langle\tilde{\mathbf{V}}\times\tilde{\mathbf{B}}\right\rangle_{||}}_{\text{MHD dynamo}}+\underbrace{\frac{1}{\langle n\rangle e}\left\langle\tilde{\mathbf{J}}\times\tilde{\mathbf{B}}\right\rangle_{||}}_{\text{Two-fluid Hall dynamo}}$$

Relaxation of parallel flow is also in good qualitative agreement

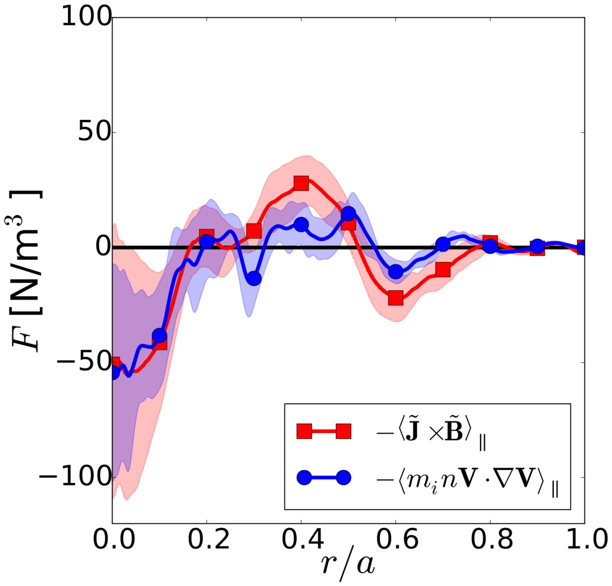
Exp. Data at $r/a = 0.83$



Exp. Data At Crash



Computational Results



$$m_i\,\langle n\rangle\,\frac{\partial\langle\mathbf{V}\rangle_\parallel}{\partial t}\approx\underbrace{-m_i\,\langle n\mathbf{V}\cdot\boldsymbol{\nabla}\mathbf{V}\rangle_\parallel}_{\text{Reynolds force}}+\underbrace{\left\langle\tilde{\mathbf{J}}\times\tilde{\mathbf{B}}\right\rangle_\parallel}_{\text{Lorentz force}}$$

Magnetic self-organization creates the possibility to sustain a steady-state fusion plasma using induction

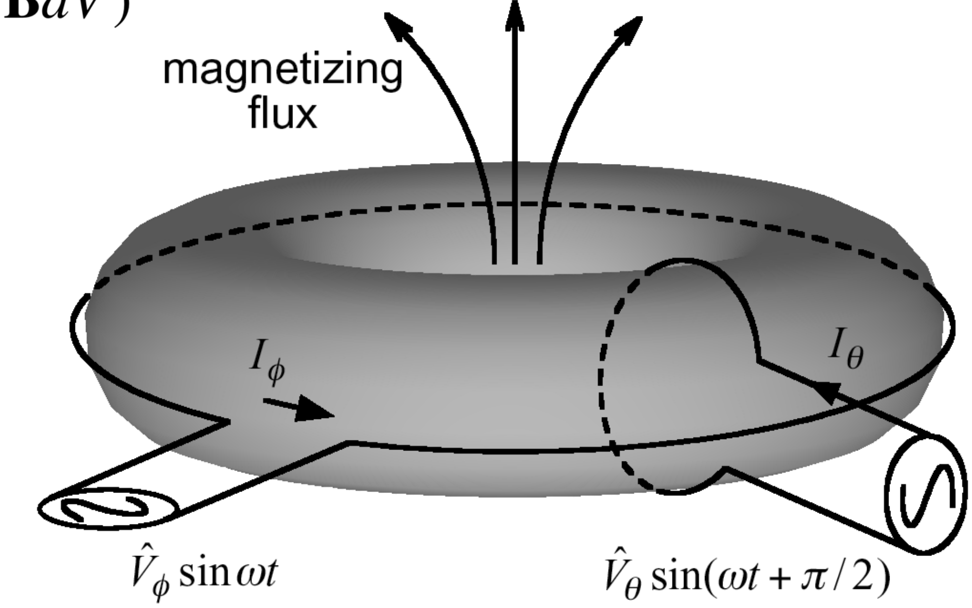
- Magnetic helicity balance motivated by success of Taylor relaxation
- Conventional induction maintains helicity balance with constant V_ϕ & Φ
- “Oscillating field current drive” (OFCD) generates DC helicity injection using purely AC loop voltages

$$\frac{\partial K}{\partial t} = 2\underset{\uparrow}{V_\phi}\Phi - 2\int \mathbf{E}\cdot\mathbf{B}dV \qquad \left(K = \int \mathbf{A}\cdot\mathbf{B}dV\right)$$

apply oscillating V_ϕ & Φ :

$$V_\phi = \hat{V}_\phi \sin\omega t \quad \& \quad \Phi = \frac{\hat{V}_\theta}{\omega}\sin\omega t + \Phi_{dc}$$

$$\langle 2V_\phi\Phi\rangle = \frac{\hat{V}_\phi\hat{V}_\theta}{2\omega}\sin\delta \qquad \delta = \text{Phase}[V_\theta, V_\phi]$$



Energy balance with "relaxed" current profile for modeling OFCD

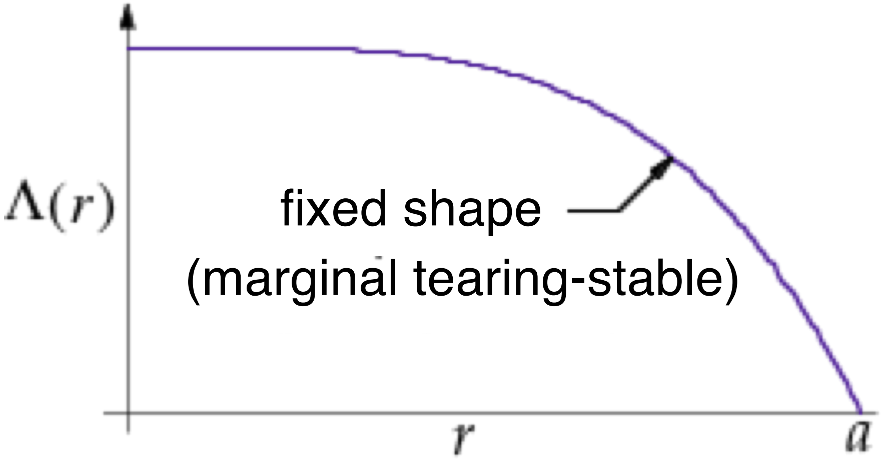
$$\frac{\partial}{\partial t}\int \frac{1}{2\mu_0}B^2 dV = \underbrace{I_\varphi V_\varphi + I_\theta V_\theta}_{\Uparrow} - \int \eta J^2 dV$$

prescribe AC loop voltages
in global power balance

- Evolve 1D equilibrium:

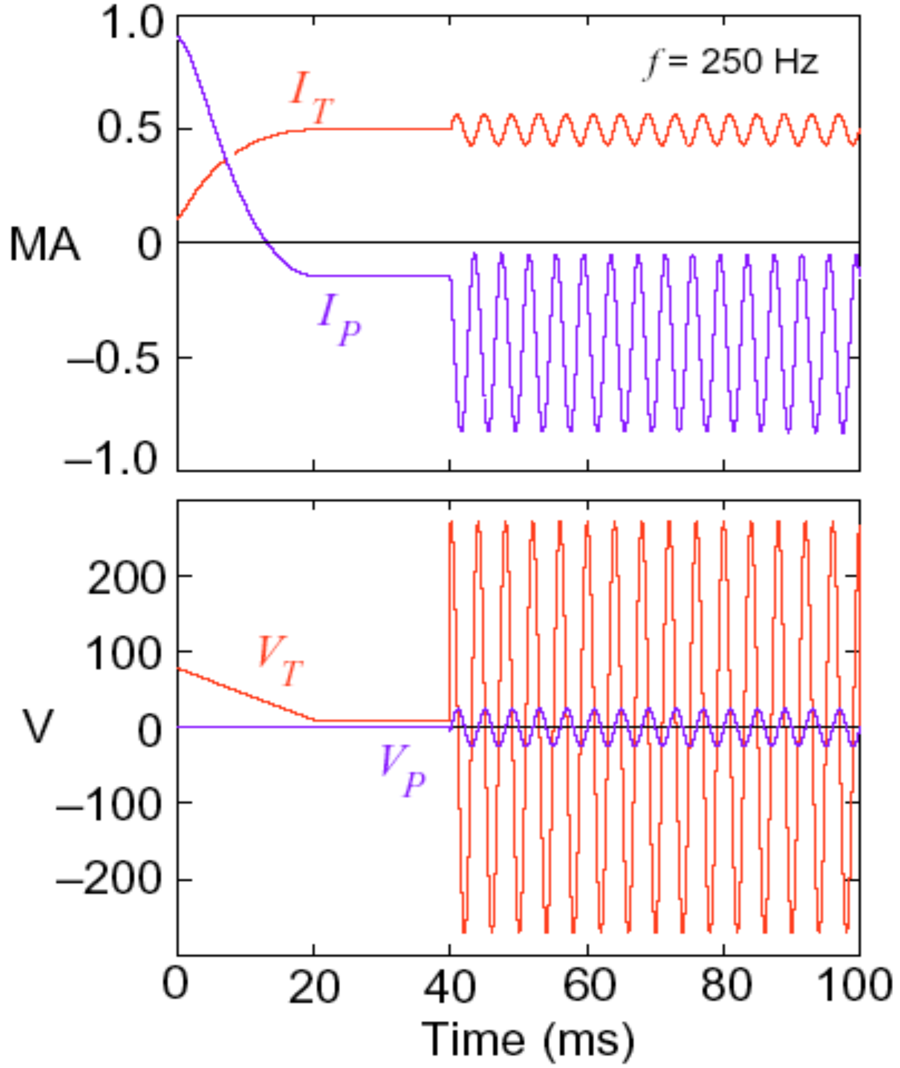
$$\nabla \times \mathbf{B} = \lambda(r,t)\mathbf{B} + \mathbf{B} \times \nabla p / B^2$$

$$\lambda(r,t) = \lambda_o(t)\Lambda(r)$$

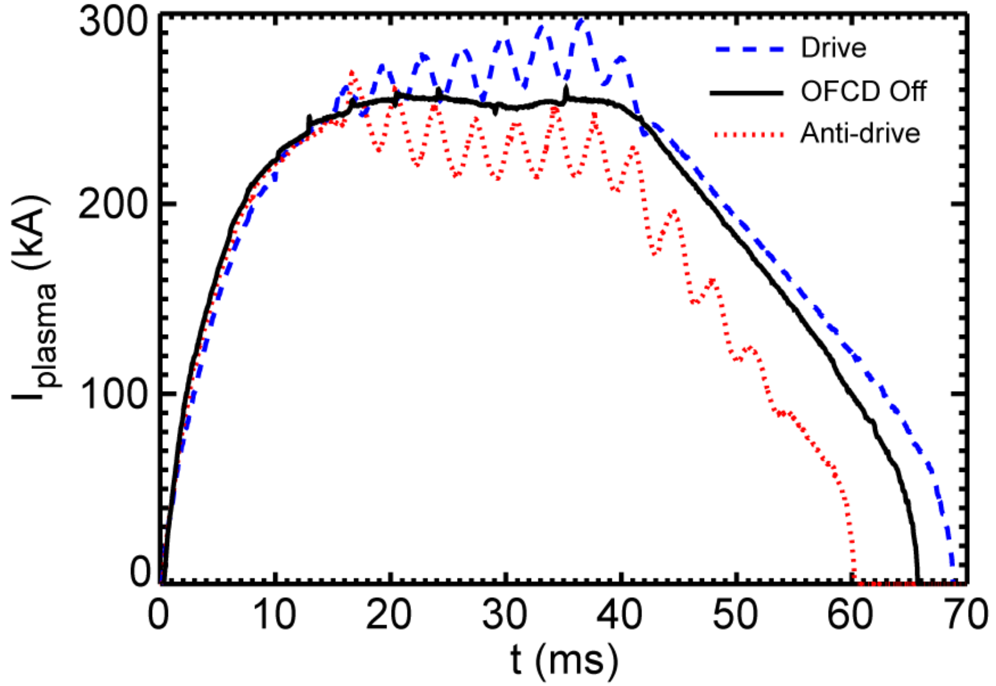


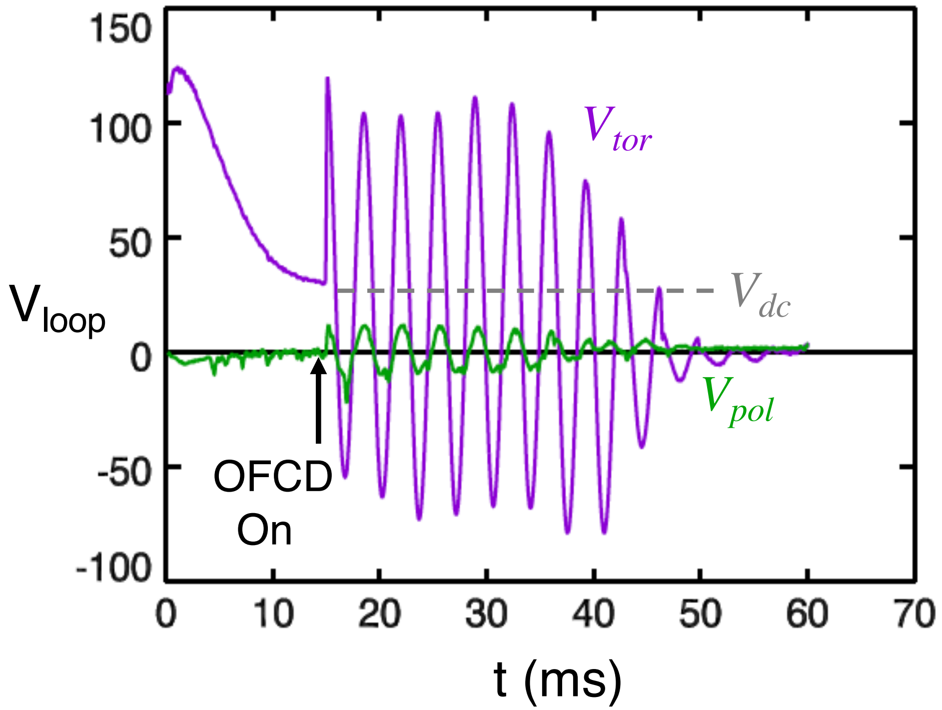
fixed shape
(marginal tearing-stable)

Simulated OFCD



OFCD on MST produces 10% increase in plasma current, as much as expected

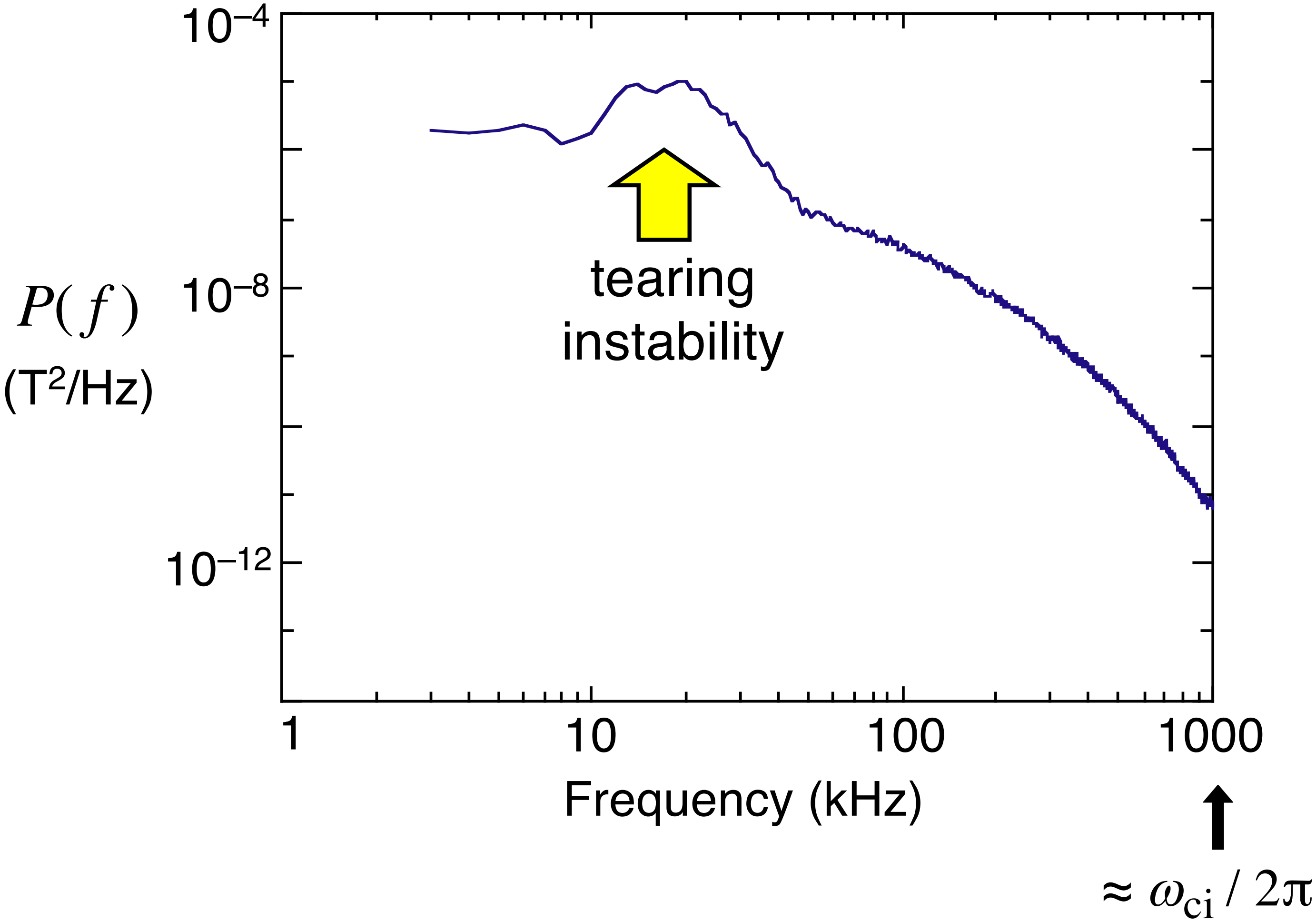




OFCD current drive efficiency measured the same as for steady induction (≈ 0.1 A/W)

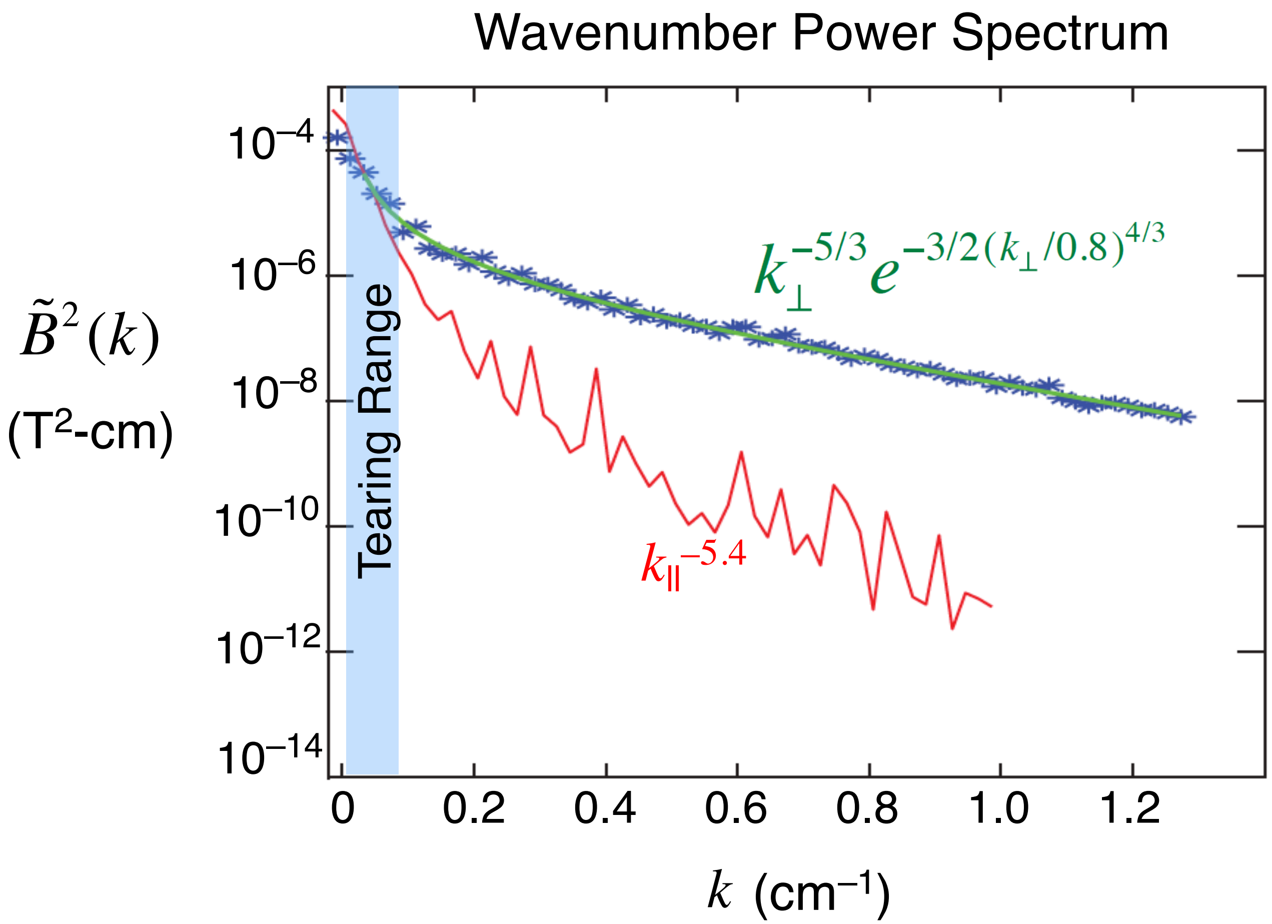
Tearing instability at the global scale drives a cascade to gyro-scale turbulence

Power Spectrum of Magnetic Fluctuations



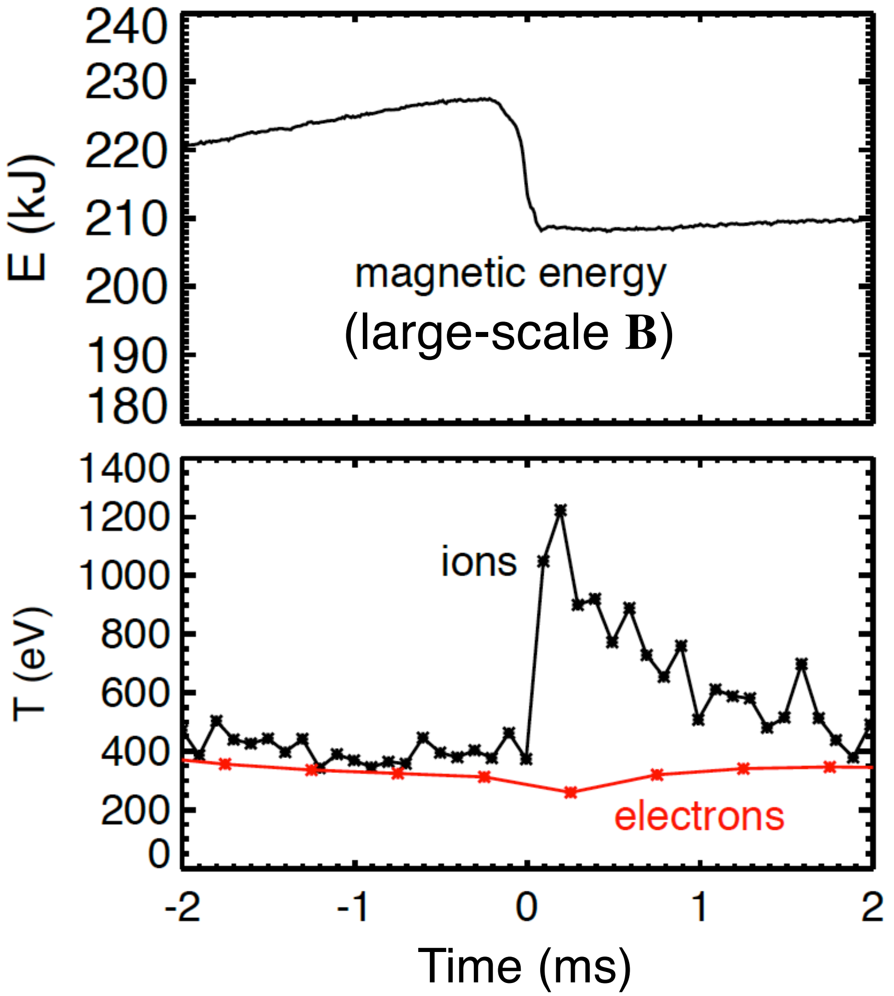
The cascade is anisotropic and hints at a non-classical dissipation mechanism

- The $k_\perp$ spectrum is well-fit by a dissipative cascade model (P. Terry, PoP 2009)
- Onset of exponential decay occurs at a smaller $k_\perp$ than expected for classical dissipation



Powerful ion energization occurs during the impulsive magnetic reconnection events

- Instantaneous heating rate can be as large as 10 MeV/s (50 MW!)

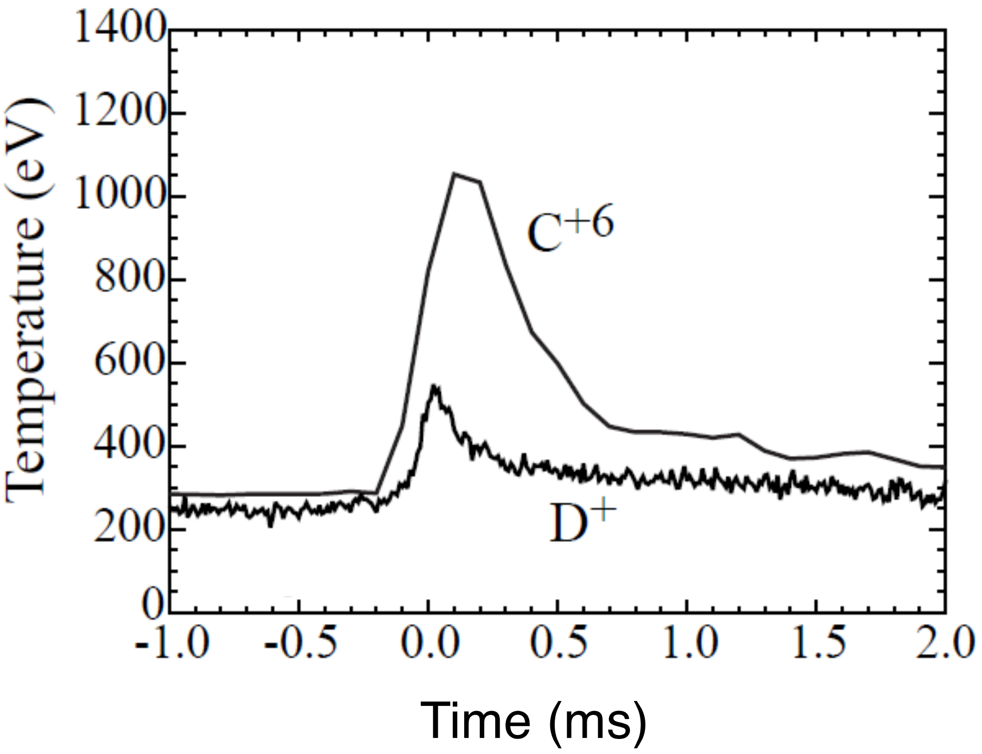


Relative to Reconnection Event

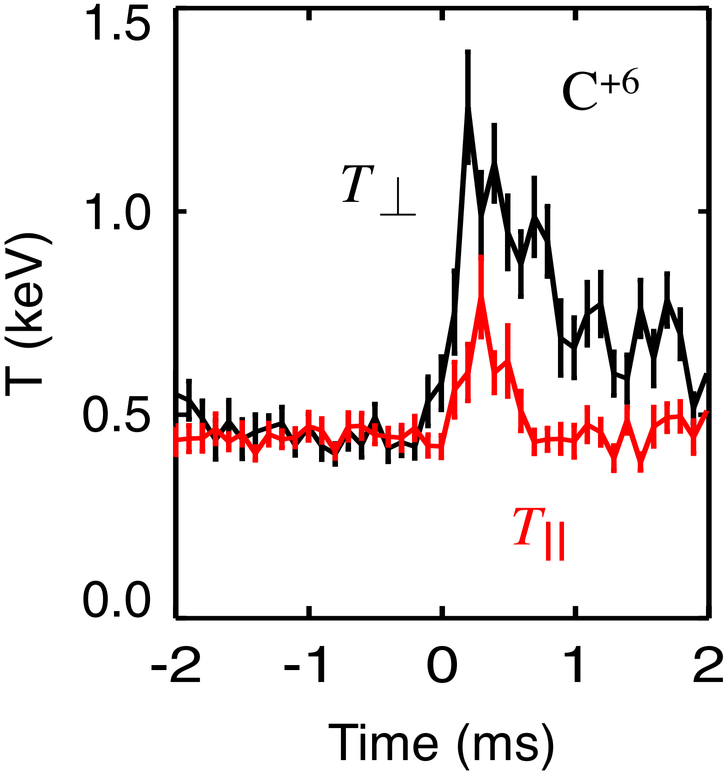
Heating is anisotropic and species dependent

- MST is equipped with several ion temperature diagnostics:
  - Rutherford scattering for majority ion temperature
  - Charge-exchange recombination spectroscopy (CHERS) for minority ions
  - Neutral particle energy analyzers (energetic neutral loss from plasma)

Minority Ions Hotter than Majority Ions

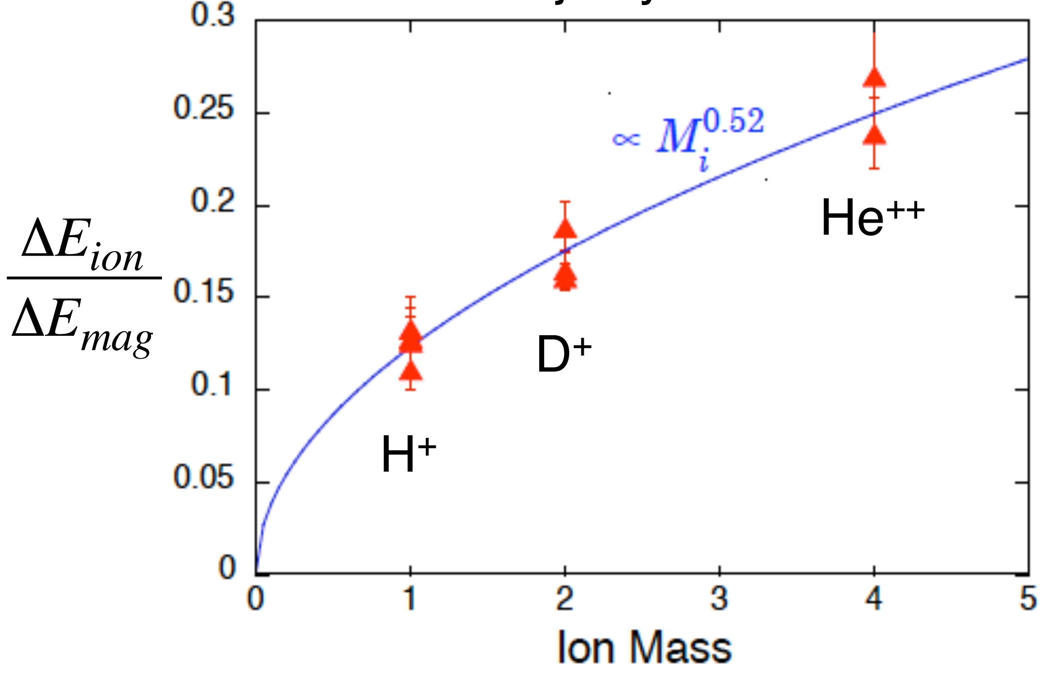


Heating is Anisotropic



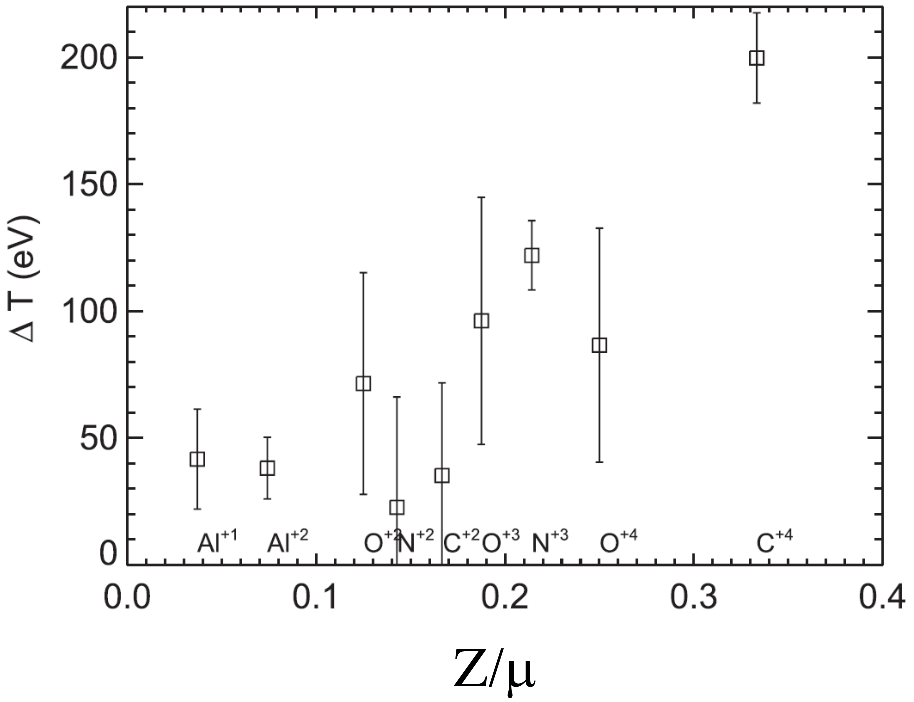
Heating depends on mass and charge

Majority Ions



(varied fueling gas)

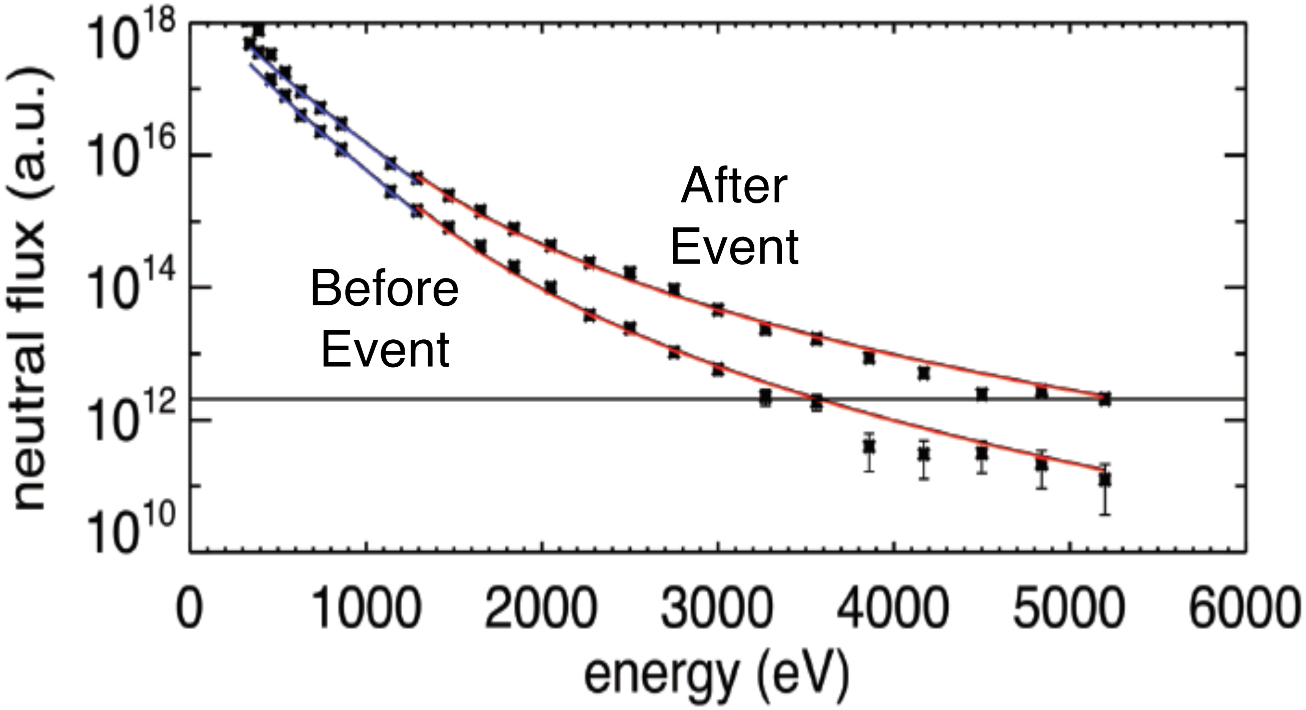
Minority Ions



An energetic ion tail is generated and reinforced at each reconnection event

- Distribution is well-fit by a Maxwellian plus a power-law tail
- Reminiscent of power laws observed for astrophysical energetic particles

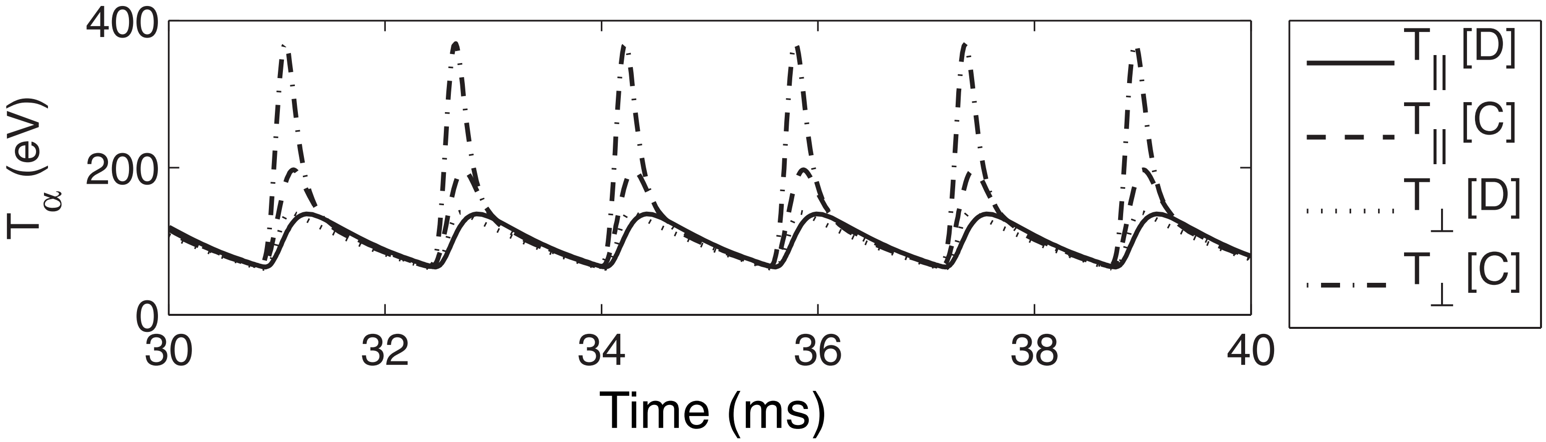
$$f_{D+}(E) = A\, e^{-E/kT} + B\, E^{-\gamma}$$



Proposed Ion Heating Mechanisms

Existing models for ion heating in the RFP are based on distinct mechanisms

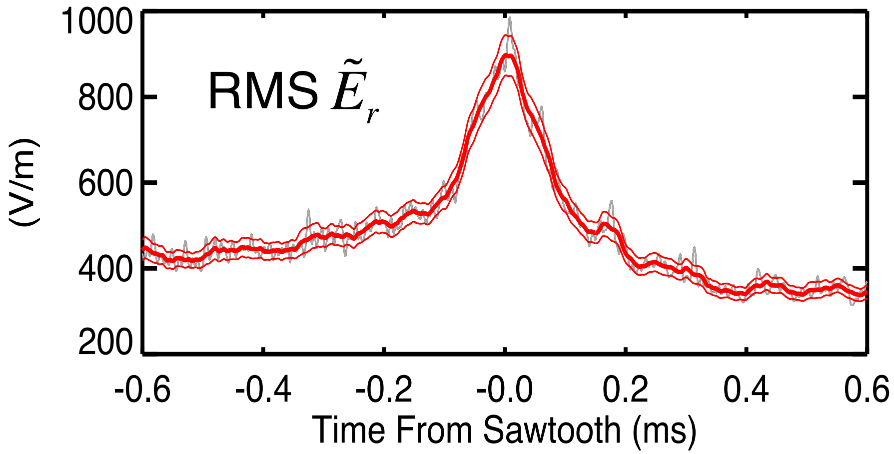
- **Cyclotron-resonant heating**:
  - Feeds off the turbulent cascade to gyro-scale
  - Preferential perpendicular heating, but with collisional relaxation
  - Preferential minority ion heating, since $\tilde{B}^2(\omega_{ci})$ is larger where ω_{ci} is smaller
  - Mass scaling is predicted with dominant minority heating and collisional relaxation

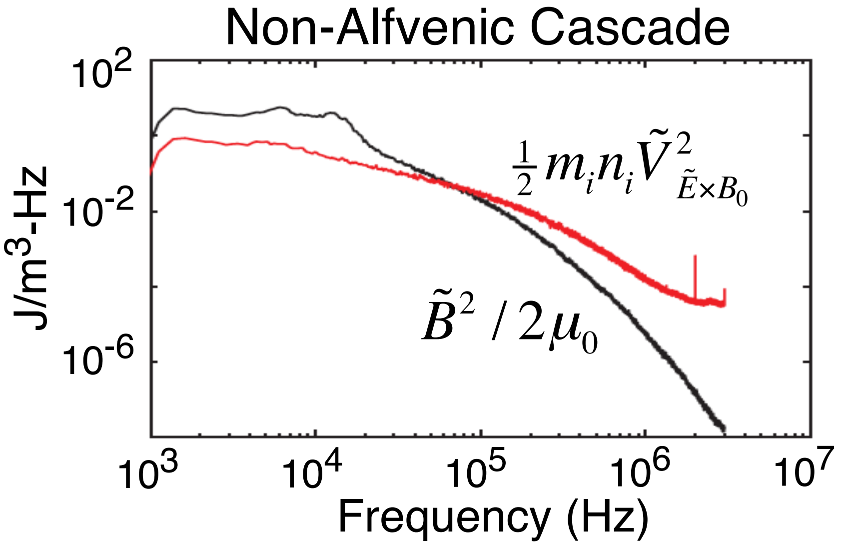


Tangri et al., PoP **15** (2008)
(similar to Cranmer et al)

Existing models for ion heating in the RFP are based on distinct mechanisms

- **Stochastic heating**:
  - Feeds off large electrostatic electric field fluctuations and the distinct stochastic magnetic diffusion process
  - Monte Carlo modeling yields MST-like heating rates (Fiksel et al, PRL 2009)
  - Predicts mass scaling close to that observed



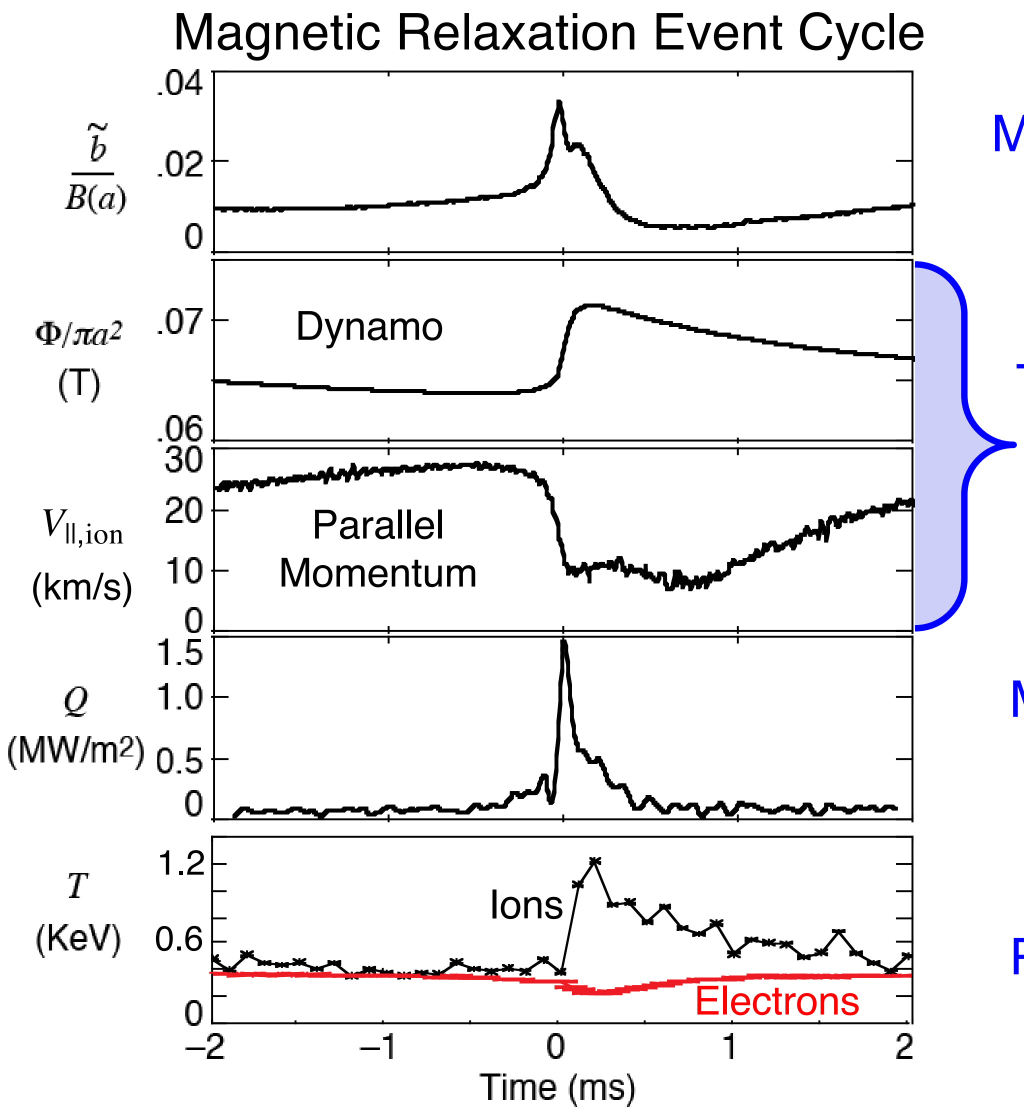

Non-Alfvenic Cascade

- Emerging story: measurements not shown here suggest the electrostatic fluctuations for $f \gtrsim$100 kHz are drift waves excited in the turbulent cascade
  - Importance of non-uniformity and gradients at the system scale and coupling of different types of modes/waves

Existing models for ion heating in the RFP are based on distinct mechanisms

- **Viscous heating**:
  - No clear experimental evidence for the required large sheared flow
  - Perpendicular flow is dominant for tearing modes for which the classical viscosity is small
  - A “reliable” dissipation mechanism, but difficult to achieve the large heating rates seen in MST plasmas
  - See, e.g., Svidzinski et al, PoP **15** (2009)

The RFP plasma exhibits a fascinating set of magnetic self-organization phenomena



Magnetic Reconnection
(Tearing Instability)

Two-Fluid Momentum
Relaxation
(current and ion flow)

Magnetic Turbulence
& Transport

Non-Collisional
Particle Energization