Alfvén waves

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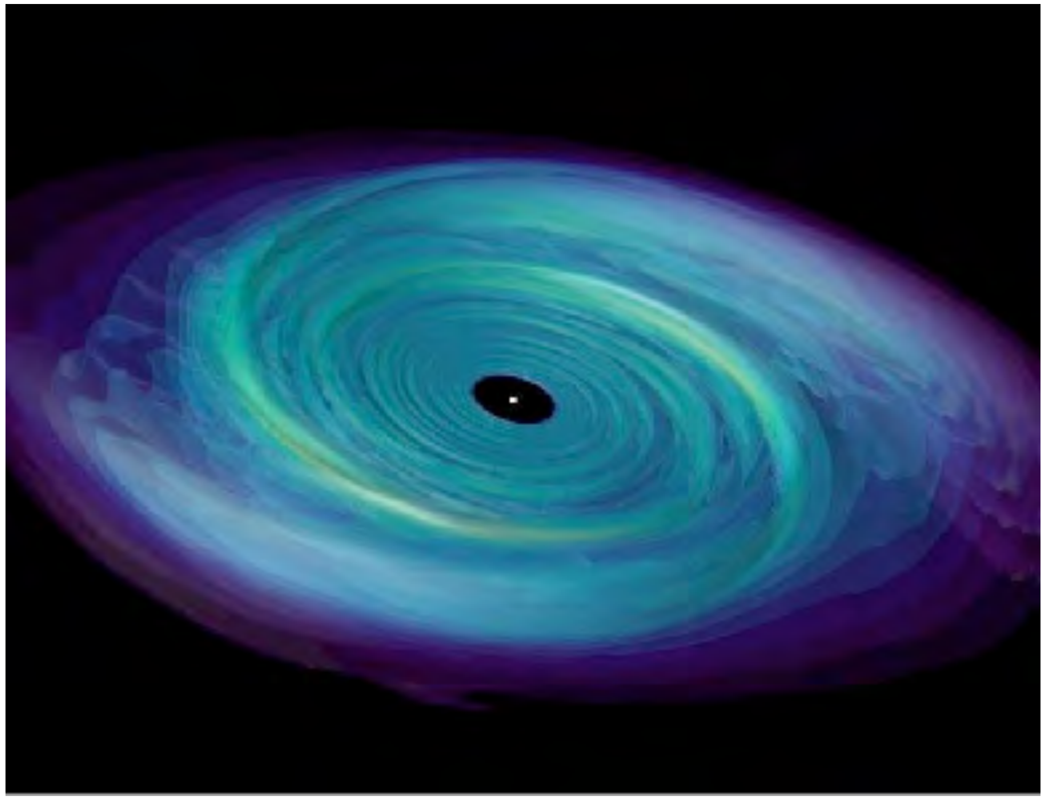
Importance of plasma waves

- Along with single particle motion, understanding of linear waves are foundation for physical intuition for behavior of plasmas
- Waves play direct role in important physical processes: RF heating in fusion plasmas, particle acceleration by waves in space plasmas, plasma turbulence in astrophysical objects

Importance of plasma waves

- Along with single particle motion, understanding of linear waves are foundation for physical intuition for behavior of plasmas
- Waves play direct role in important physical processes: RF heating in fusion plasmas, particle acceleration by waves in space plasmas, plasma turbulence in astrophysical objects
- Wave is collective response of plasma to perturbation, however, intuition for waves starts with considering single particle response to electric/magnetic fields that make up the wave
- Focus on magnetized plasmas: particle response is anisotropic, orientation of wave E-field wrt background magnetic field is essential in determining response

Wave equation, plasma dielectric model for linear waves

- Treat plasma as conducting medium; will lead to dielectric description (but start by treating plasma charge and currents as free)

$$\nabla \times \mathbf{B} = \mu_o \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_o \frac{\partial \mathbf{j}}{\partial t} = 0$$

- Plasma effects buried in current, need model to relate current to E

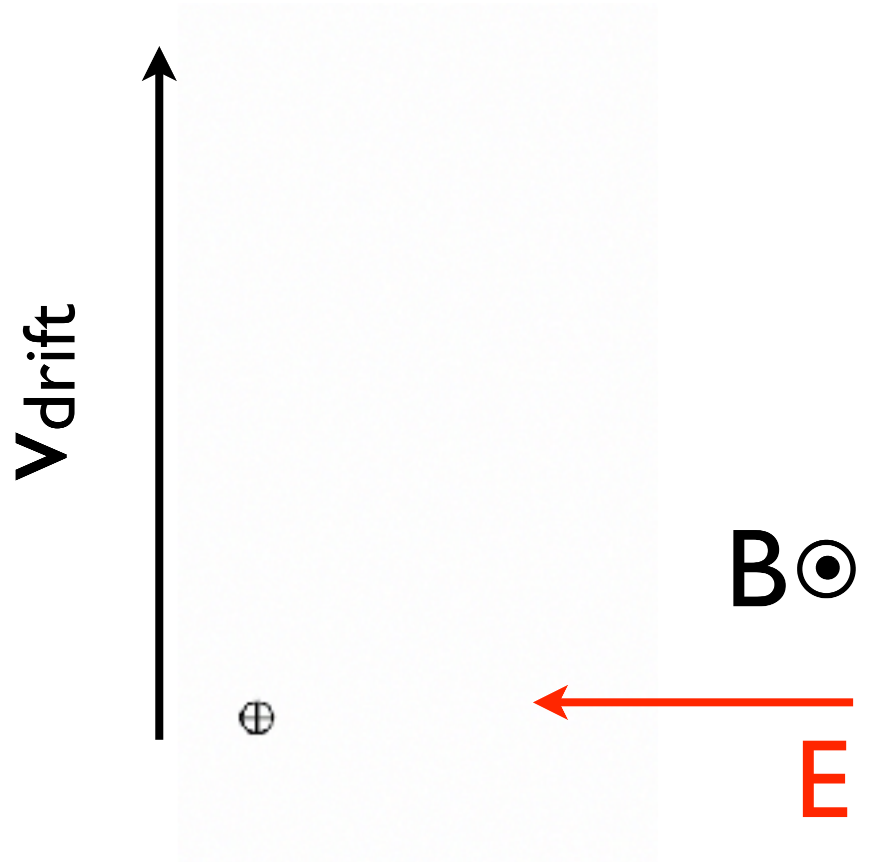
- Model plasma as cold fluid, will find a linear, tensor conductivity

$$\mathbf{j} = \sigma \cdot \mathbf{E}$$

Important intuition: Single particle response to wave fields

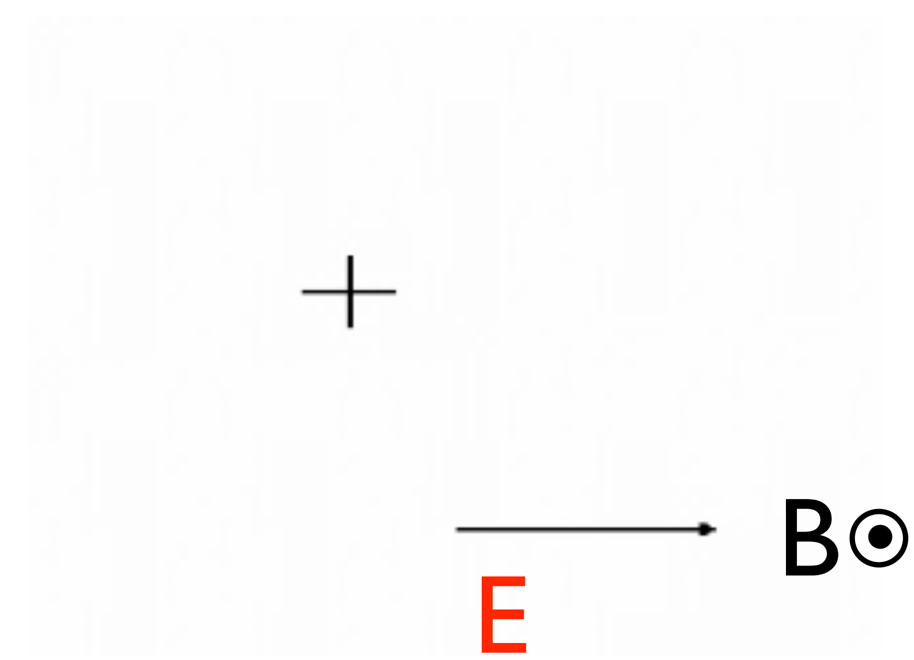
- Conductivity tensor tells us plasma response to applied electric field; useful to think about single particle orbits
- In particular for magnetized plasmas and wave electric fields that are perpendicular to B
- Two drifts matter (in uniform plasma): ExB drift and polarization drift
  - ExB drift is the dominant particle response for low frequency wave fields $\omega < \Omega_c$
  - Polarization drift is dominant at higher frequencies

ExB and Polarization Drifts



ExB drift, DC E Field

$$\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$



Polarization drift, ExB drift removed

$$\mathbf{v}_p = \frac{1}{\Omega} \frac{\partial}{\partial t} \frac{\mathbf{E}_\perp}{B}$$

- No currents from ExB at low freq (ions and electrons drift the same); above ion cyclotron freq, ions primarily polarize, no ExB, can get ExB current from electrons

Model for plasma conductivity

- Use cold, two-fluid model; formally cold means:

  $v_\phi \gg v_{\mathrm{th,e}}, v_{\mathrm{th,i}}$

$$
\begin{aligned}
n_s m_s \frac{d\mathbf{v}_s}{dt} &= n_s q_s \left(\mathbf{E} + \mathbf{v}_s \times \mathbf{B}\right) \\
\mathbf{j} &= \sum_s n_s q_s \mathbf{v}_s \equiv \sigma \cdot \mathbf{E}
\end{aligned}
$$

- Assume plane wave solution (uniform plasma), linearize the equations: $f(\mathbf{r}, t) = f \exp\left(i\mathbf{k}\cdot\mathbf{r} - i\omega t\right)$

$$
f = f_0 + f_1 + \ldots \quad ; \quad f_1 \ll f_o
$$

- Ignore terms higher than first order: arrive at equation that is the same for motion of a single particle (importance of understanding drifts!)

Plasma model, cont.

Choose $\mathbf{B} = B_0 \hat{z}$, $\mathbf{E} = \mathbf{E}_1 = E_x \hat{x} + E_z \hat{z}$

Ion momentum equation becomes:

$$\begin{aligned} -i\omega v_x - \Omega_i v_y &= \frac{eE_x}{m_i} \\ \Omega_i v_x - i\omega v_y &= 0 \end{aligned} \qquad \Omega_i = \frac{eB}{m_i}$$

Solve for v_x, v_y:

$$v_x = \frac{-i\omega}{\Omega_i^2 - \omega^2} \frac{e}{m_i} E_x \quad \text{(polarization)}$$

$$v_y = \frac{-\Omega_i}{\Omega_i^2 - \omega^2} \frac{e}{m_i} E_x \quad (\mathrm{E{\times}B})$$

For the parallel response: $v_z = \dfrac{ie}{\omega m_i} E_z$ (inertia-limited response)

Plasma model, cont.

Back to the wave equation, rewrite with plane wave assumption:

$$-\mathbf{k} \times \mathbf{k} \times \mathbf{E} - \frac{\omega^2}{c^2}\mathbf{E} - i\omega\mu_o\sigma \cdot \mathbf{E} = 0$$

Can rewrite in the following way:

$$\mathbb{M} \cdot \mathbf{E} = 0$$

$$\mathbb{M} = (\hat{k}\hat{k} - \mathbb{I})n^2 + \epsilon \qquad n^2 = \frac{c^2k^2}{\omega^2} \text{ index of refraction}$$

$$\epsilon = \mathbb{I} + \frac{i\sigma}{\epsilon_o\omega} \text{ dielectric tensor}$$

unit tensor

Cold plasma dispersion relation

Using the cold two-fluid model for σ, the dielectric tensor becomes:

$$\epsilon = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}$$

$$S = 1 - \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2} - \frac{\omega_{pe}^2}{\omega^2 - \Omega_e^2} \quad \text{(polarization)}$$

$$D = \frac{\Omega_i \omega_{pi}^2}{\omega(\omega^2 - \Omega_i^2)} - \frac{\Omega_e \omega_{pe}^2}{\omega(\omega^2 - \Omega_e^2)} \quad \text{(E}\times\text{B response)}$$

$$P = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} \quad \text{(inertial response)}$$

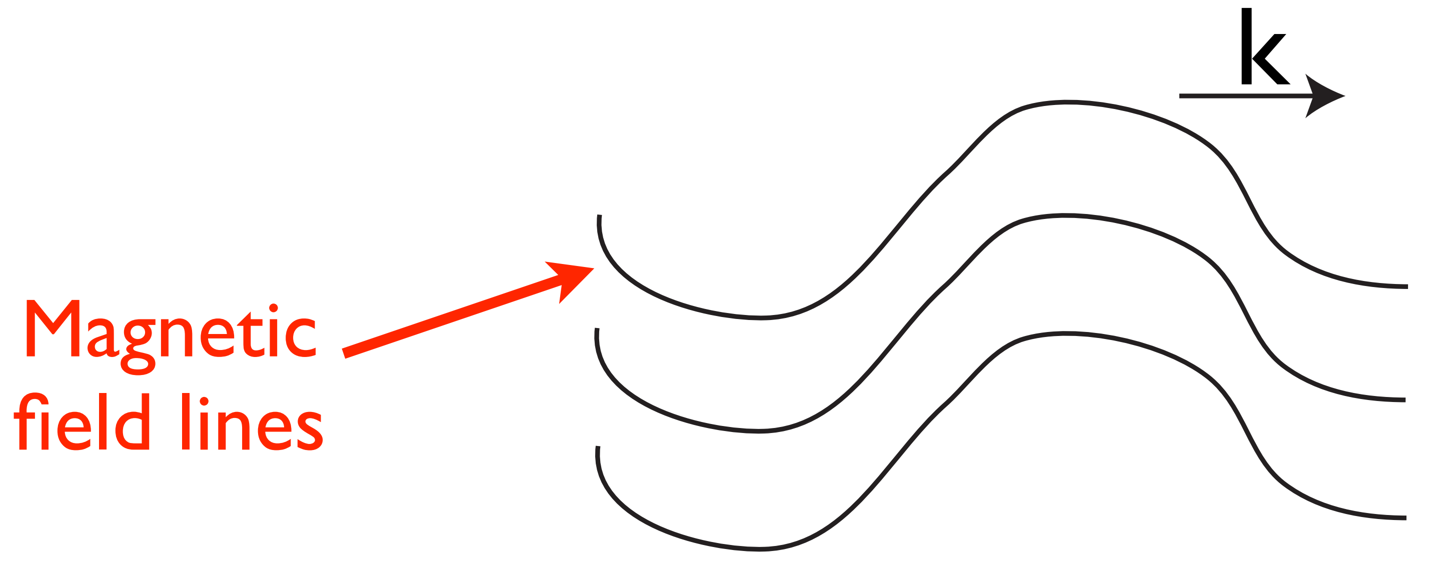
Defining θ to be the angle between $\mathbf{k}$ and $\mathbf{B_o}$, the wave equation becomes:

$$\begin{pmatrix} S - n^2 \cos^2\theta & -iD & n^2 \sin\theta\cos\theta \\ iD & S - n^2 & 0 \\ n^2 \sin\theta\cos\theta & 0 & P - n^2\sin^2\theta \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

$\det \mathbb{M} = 0$ provides dispersion relation for waves – allowable combinations of ω and $\mathbf{k}$

Low frequency waves: Alfvén waves

- For freq. much less than ion cyclotron frequency, primary waves are Alfvén waves



Shear Alfvén wave

- Primary motion: ExB motion of electrons and ions together (D→0)

- To pull this out of our cold plasma model:

$$\mathbf{k} = k_z \hat{z} \ (\theta = 0)$$

Shear wave in cold plasma model

$$\begin{pmatrix} S-n^2 & 0 & 0 \\ 0 & S-n^2 & 0 \\ 0 & 0 & P \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

$$n^2 = S = \cancel{1} - \frac{\omega_{pi}^2}{\cancel{\omega^2} - \Omega_i^2} - \cancel{\frac{\omega_{pe}^2}{\omega^2 - \Omega_e^2}} \approx \frac{\omega_{pi}^2}{\Omega_i^2} = \frac{c^2}{v_A^2}$$

$$\omega^2 = k_\parallel^2 v_A^2 \quad ; \quad v_A^2 = \frac{B^2}{\mu_o m_i n_i}$$

- Like wave on string: magnetic field plays role of tension, plasma mass → string mass

Alfvén waves from MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$ Continuity

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \vec{j} \times \vec{B}$$ Momentum

$$\vec{E} + \vec{v} \times \vec{B} = 0$$ Ohm's Law (electron momentum)

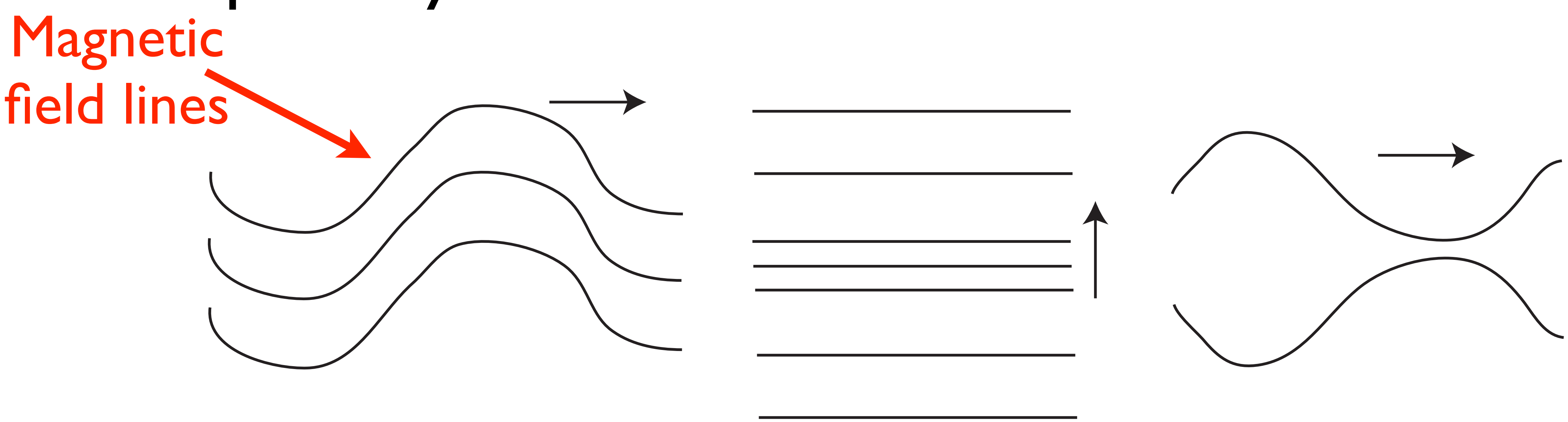
$$\frac{d}{dt} \left(\frac{p}{\rho^{\gamma}} \right) = 0$$ Pressure closure (adiabatic)

+ Maxwell's Equations

- Linearizing this system reveals four waves: fast and slow magnetosonic waves, the shear Alfvén wave, and the entropy wave

MHD Waves

- For freq. much less than ion cyclotron frequency, primary waves are Alfvén waves



Shear Alfvén wave

Compressional Alfvén wave (fast magnetosonic)

Slow magnetosonic

$$\omega^2 = k_{\parallel}^2 v_A^2$$

$$\omega^2 = \frac{k^2}{2}\left(c_s^2 + v_A^2 \pm \sqrt{c_s^4 + v_A^4 - 2c_s^2 v_A^2 \cos 2\theta}\right)$$

sound wave response (in fast/slow modes) not in our cold two-fluid model

Shear wave dispersion derivation

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+\vec{v}\cdot\nabla\vec{v}\right)=-\nabla p+\vec{j}\times\vec{B}=-\nabla\left(p+\underbrace{\frac{B^2}{2\mu_o}}_{\text{magnetic pressure}}\right)+\underbrace{\frac{\vec{B}\cdot\nabla\vec{B}}{\mu_o}}_{\text{magnetic tension}}$$

$$\frac{\partial\vec{B}}{\partial t}=-\nabla\times\vec{E}=\nabla\times\vec{v}\times\vec{B}$$

- We are looking for the shear wave, so we'll make appropriate assumptions:

$\vec{k}\cdot\delta\vec{v}=0$ incompressible motion

$\vec{B}=B_o\hat{z}+\delta B\hat{x}$ no field line compression, linearly polarized

$\delta B,\delta v\propto\exp(i\vec{k}\cdot\vec{r}-i\omega t)$ plane waves

$\delta p=0$ follows from the first assumption, adiabatic assumption

Shear wave dispersion derivation, cont

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \cancel{\vec{v}\cdot\nabla\vec{v}}\right) = -\nabla\left(\cancel{p} + \cancel{\frac{B^2}{2\mu_o}}\right) + \frac{\vec{B}\cdot\nabla\vec{B}}{\mu_o}$$

$$-i\omega\rho\delta\vec{v} = \frac{ik_{\|}B_o\delta B}{\mu_o}\hat{x}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times\vec{v}\times\vec{B}$$

$$-i\omega\delta B = ik_{\|}\delta v B_o$$

- Combine these two to get:

$$\boxed{\omega^2 = k_{\|}^2\frac{B^2}{\mu_o\rho} = k_{\|}^2 v_A^2}$$

Currents in MHD AW

$$\vec{j} = \frac{1}{\mu_o}\nabla \times \vec{B} = \frac{1}{\mu_o} i\vec{k} \times (\delta B\hat{x}) \qquad \delta\vec{E} = -\delta\vec{v} \times B_o\hat{z} = -\delta v B_o\hat{y}$$

$$\delta\vec{E} = \frac{k_{\|}B_o^2}{\omega\rho\mu_o}\delta B\hat{y}$$

$$\boxed{\vec{j} = -\frac{i\omega ne}{\Omega_i}\frac{\delta\vec{E}}{B_o} - \frac{ik_y\delta B}{\mu_o}\hat{z}}$$

Polarization current

- Current in $k_\perp$=0 AW is entirely due to ion polarization current: no field aligned current
- As $k_\perp$ is introduced, current closes along the field (inductively driven)

Finite $k_\perp$ introduces parallel current, electric field

$$\begin{pmatrix} S - n_\parallel^2 & 0 & n_\parallel n_\perp \\ 0 & S - n^2 & 0 \\ n_\parallel n_\perp & 0 & \epsilon_{zz} - n_\perp^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

- Shear Alfvén wave currents without $k_\perp$ are purely due to ion polarization and are cross-field
- With finite $k_\perp$, wave currents must close along the field: introduce parallel electric field and parallel particle response (easy to find departures from MHD…)
  - Ions carry current across field, electrons carry parallel current (ion parallel response important at higher β)
  - Electron parallel response introduces dispersion and damping to Alfvén wave

Kinetic and Inertial Alfvén waves: introduce dispersion and damping at finite $k_\perp$

- At finite $k_\perp$, wave obtains parallel electric field
- In low β plasma, key additional physics is parallel electron response

$$\frac{v_A^2}{v_{th,i}^2} = \frac{B^2 m_i}{\mu_o \rho T_i} = \frac{1}{\beta} \qquad\qquad \frac{v_A^2}{v_{th,e}^2} = \frac{1}{\beta}\frac{m_e}{m_i}$$

- Use kinetic electron response in parallel direction (ignore ion response)

$$\epsilon_{zz} \approx 1 + \frac{1}{k_\parallel^2 \lambda_D^2}\left(1 + \zeta_e Z(\zeta_e)\right) \quad ; \quad \zeta_e = \frac{\omega}{\sqrt{2} k_\parallel v_{\mathrm{th,e}}}$$

- For simplicity, assume cold ions, $k_\perp \rho_e << 1$; use cold perpendicular response

Kinetic and Inertial Alfvén waves

$$\begin{pmatrix} S - n_\parallel^2 & 0 & n_\parallel n_\perp \\ 0 & S - n^2 & 0 \\ n_\parallel n_\perp & 0 & \epsilon_{zz} - n_\perp^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

- Shear wave: $\mathbf{k}_\perp \parallel \mathbf{E}_\perp \quad , \quad E_y = 0$

$$\epsilon_{zz} \left(k_\parallel^2 - \frac{\omega^2}{c^2} S \right) = -k_\perp^2 S$$

$$S \approx \frac{c^2}{v_A^2}$$

$$\epsilon_{zz} \left(k_\parallel^2 - \frac{\omega^2}{v_A^2} \right) = -k_\perp^2 \frac{c^2}{v_A^2}$$

Kinetic and Inertial Alfvén waves

- Inertial Alfvén wave: cold electron response, $v_A >> v_{th,e}$

$$(\zeta_e \gg 1) \qquad \epsilon_{zz} \approx 1 - \frac{\omega^2}{\omega_{\rm pe}^2} + \frac{i\sqrt{\pi}\zeta_e}{k_\parallel^2 \lambda_D^2} \exp\left(-\zeta_e^2\right)$$

$$\omega_r^2 = \frac{k_\parallel^2 v_A^2}{(1 + k_\perp^2 \delta_e^2)} \quad , \quad \delta_e = \frac{c}{\omega_{\rm pe}}$$

- Kinetic Alfvén wave: hot electron response, $v_A << v_{th,e}$

$$(\zeta_e \ll 1) \qquad \epsilon_{zz} \approx 1 + \frac{1}{k_\parallel^2 \lambda_D^2} + \frac{i\sqrt{\pi}\zeta_e}{k_\parallel^2 \lambda_D^2} \exp\left(-\zeta_e^2\right)$$

$$\omega_r^2 = k_\parallel^2 v_A^2 \left(1 + k_\perp^2 \rho_s^2\right) \quad , \quad \rho_s = \frac{C_s}{\Omega_i}$$

Kinetic and Inertial Alfvén waves: Damping

- Landau damping rate for kinetic Alfvén wave:

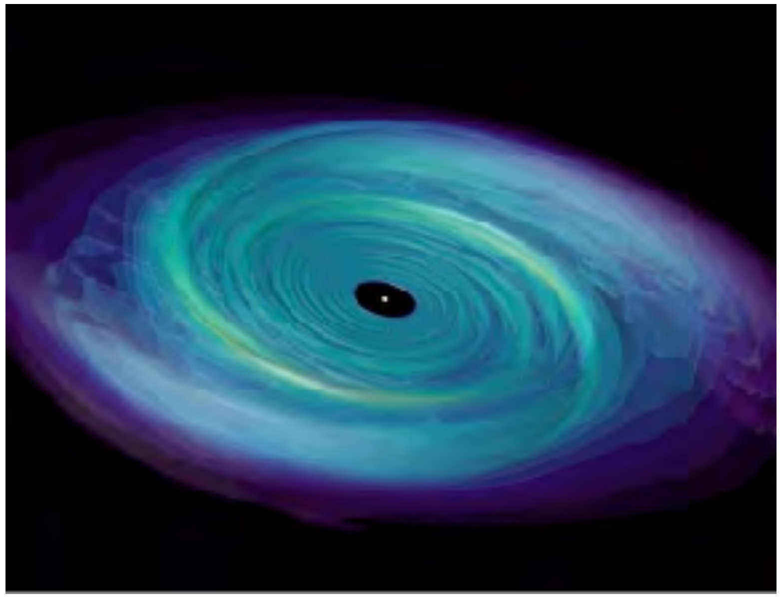
$$\omega_i = -k_\perp^2 \rho_s^2 \left(\frac{k_\parallel^2 v_A^2}{2\sqrt{2} k_\parallel v_{\mathrm{th,e}}} \right) \exp\left(-\frac{v_A^2}{2 v_{\mathrm{th,e}}^2} \left(1 + k_\perp^2 \rho_s^2\right) \right)$$

- Need finite $k_\perp \rho_s$ for Landau damping - generates finite $E_{||}$

$$\frac{E_\parallel}{E_\perp} = \frac{n_\parallel n_\perp}{n_\perp^2 - \epsilon_{zz}} \approx \beta \frac{\omega}{\Omega_i} \frac{k_\perp \rho_s}{1 + k_\perp^2 \rho_s^2}$$

- At higher β, collisionless damping on ions is important ($v_A \sim v_{th,i}$ for $\beta \sim 1$); Landau damping and transit-time magnetic pumping (called Barnes damping in astrophysical literature)

Importance of KAW and IAW



- Space plasmas: IAW/KAW observed in auroral zones, possible mechanism for auroral electron acceleration [Louarn, et al., GRL 21, 1847 (1994); Chaston, et al GRL 26, 647 (1999)]

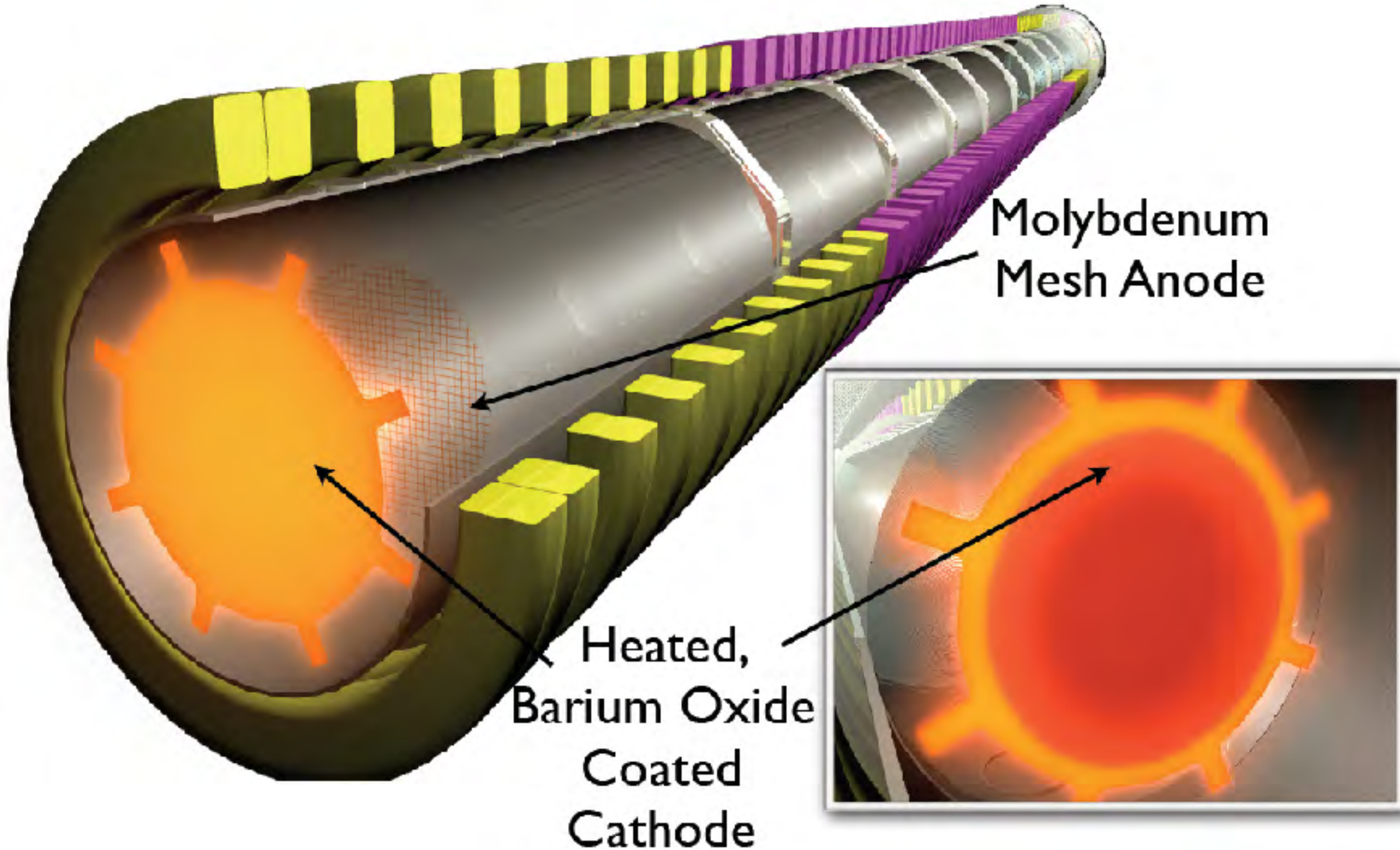
- Fusion devices: Alfvén wave heating [Mahajan Phys. Fluids 25, 652 (1982)], mode conversion to KAWs source of damping for Alfvén eigenmodes [Hasegawa and Chen, PRL 35, 370 (1975)]

- Space/Astro plasmas: KAWs terminate cascade in MHD/ Alfvénic turbulence [Bale, et al. PRL 94, 215002 (2005); Sahraoui, et al. PRL 102, 231102 (2009)].

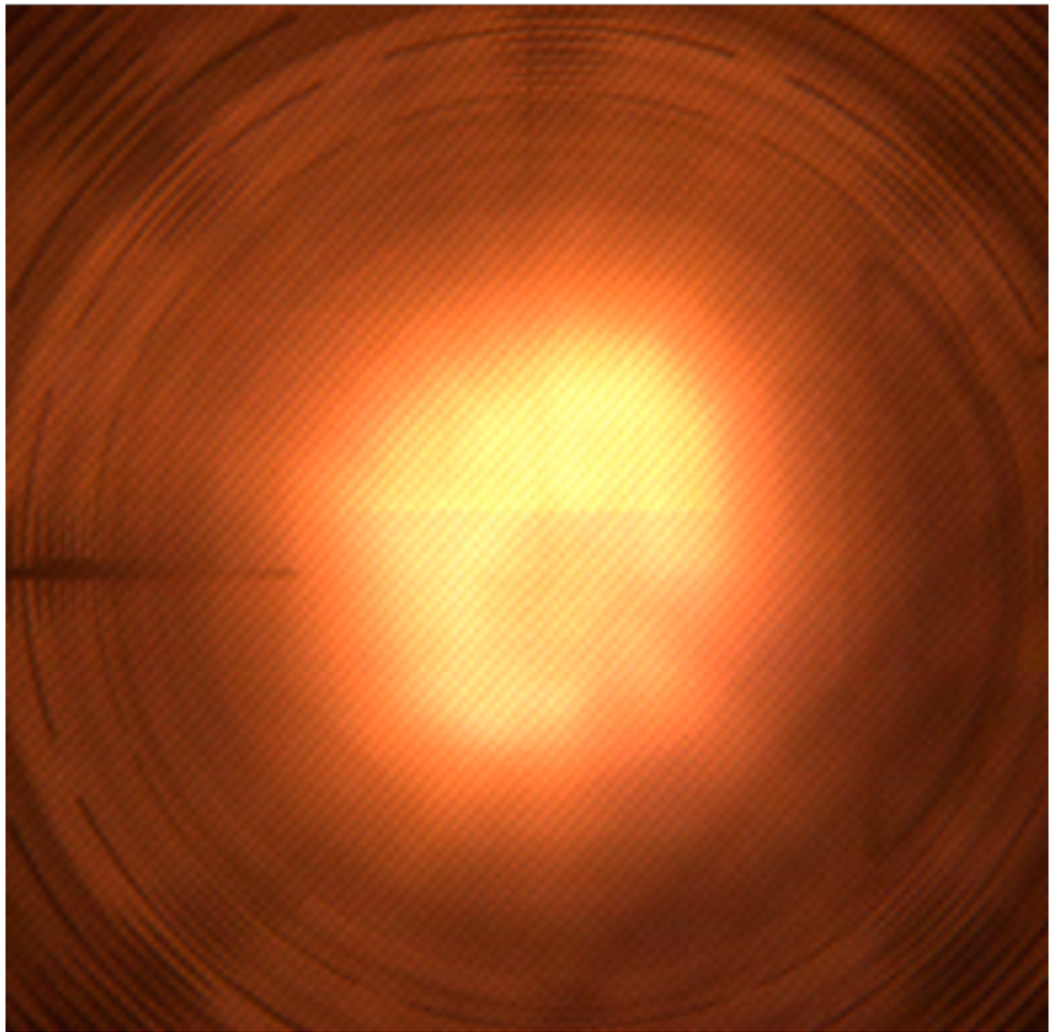
The LArge Plasma Device (LAPD) at UCLA

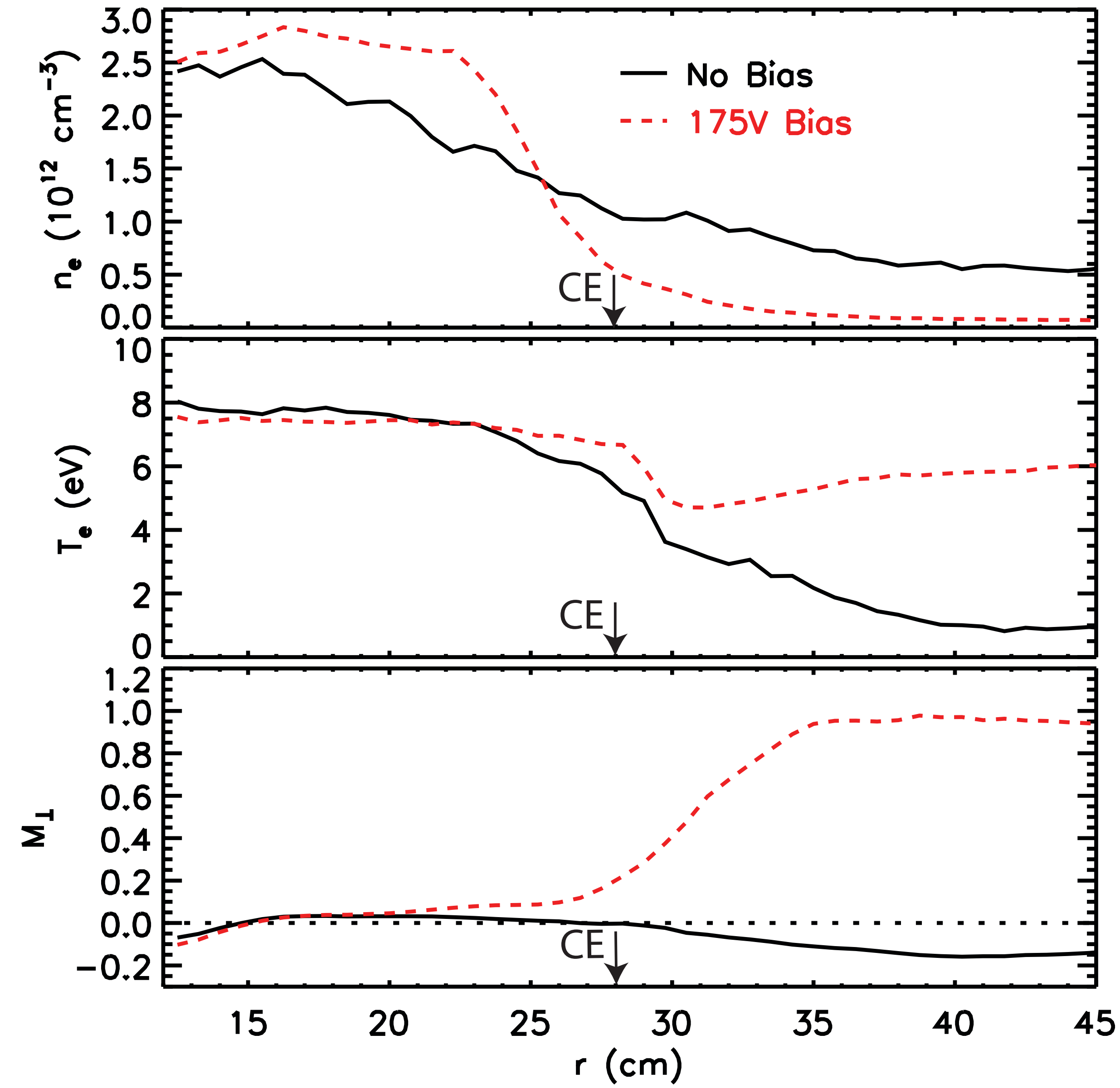
- Solenoidal magnetic field, cathode discharge plasma (BaO and LaB_6)
- BaO Cathode: n ~ 10^{12} cm^{-3}, T_e ~ 5-10 eV, $T_i \lesssim$ 1 eV
- LaB_6 Cathode: n ~ 5×10^{13} cm^{-3}, T_e ~ 10-15 eV, T_i ~ 6-10 eV
- B up to 2.5kG (with control of axial field profile)
- Large plasma size, 17m long, D~60cm (1kG: ~300 ρ_i, ~100 ρ_s)
- High repetition rate: 1 Hz

LAPD Plasma source



Example Plasma Profiles





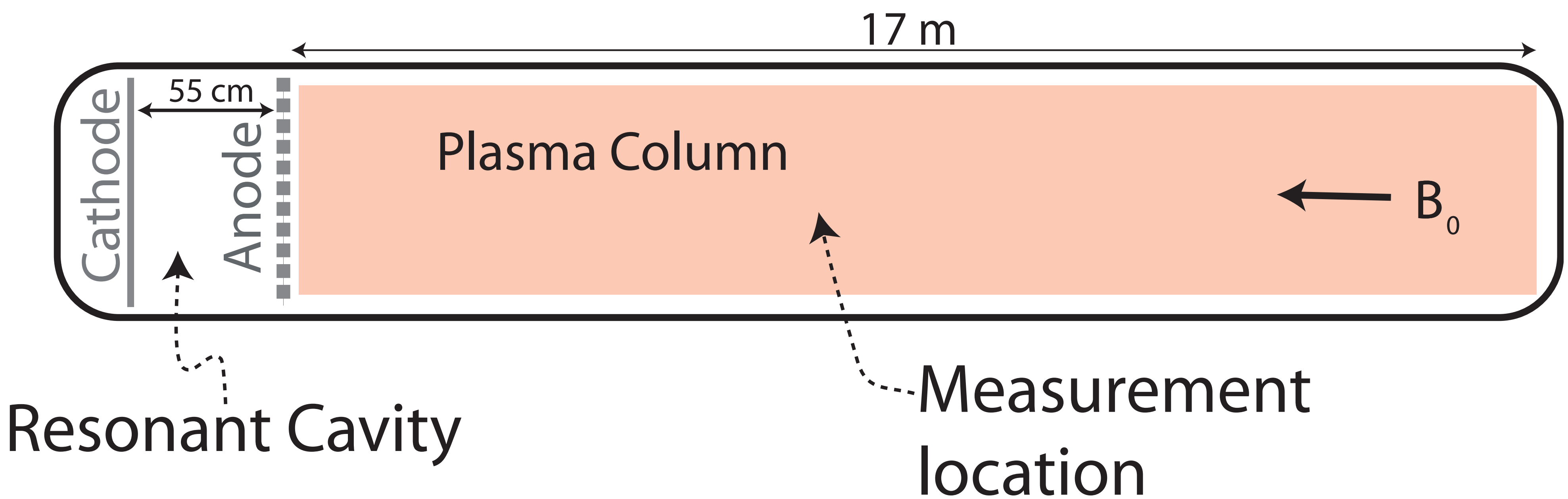
- Low field case (400G) (also shown: with particle transport barrier via driven flow*); generally get flat core region with D=30-50cm
- Broadband turbulence generally observed in the edge region

* Carter, et al, PoP 16, 012304 (2009)

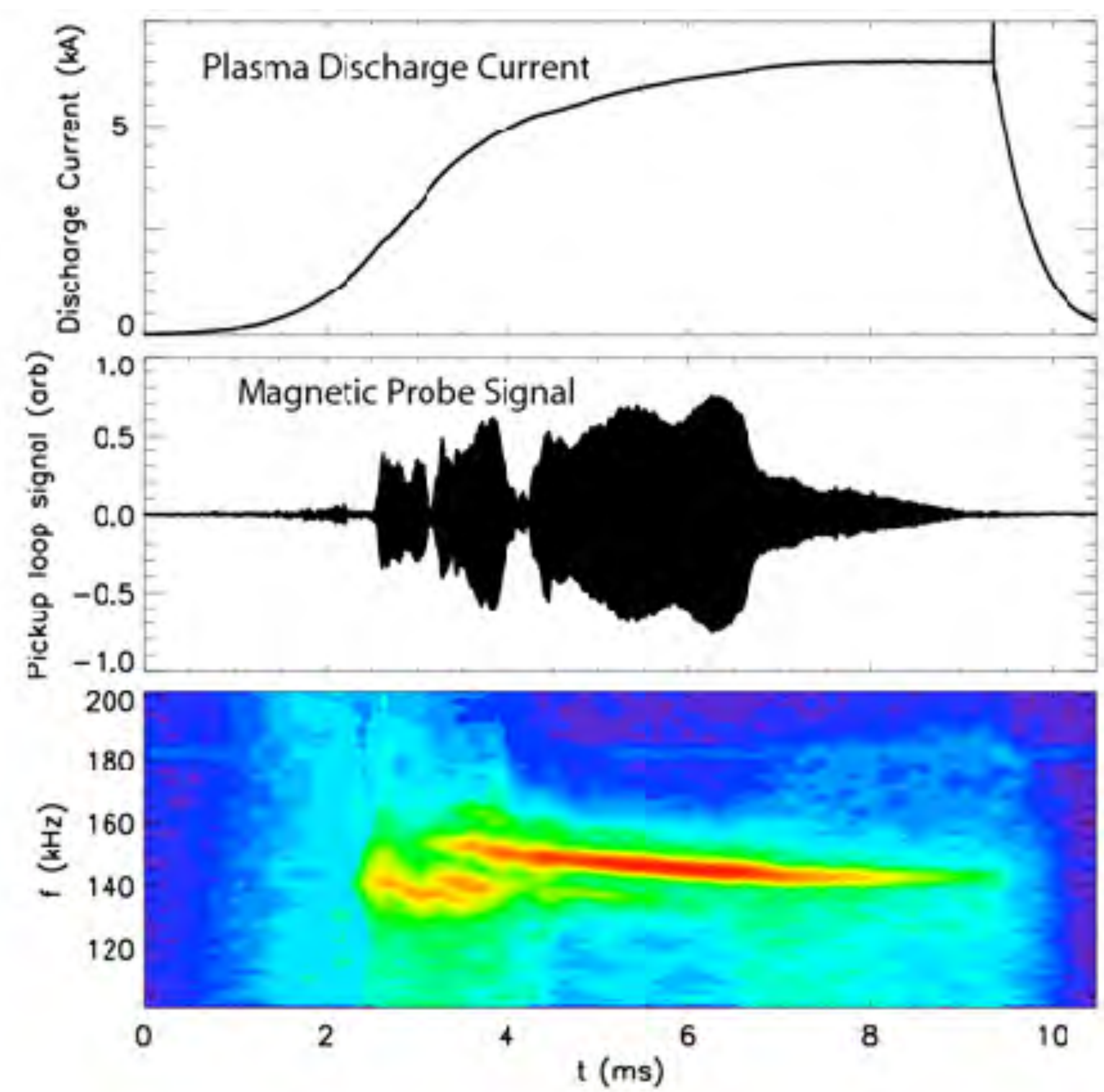
LAPD Parameters

$$\Omega_i \sim 400\text{kHz}$$
$$\nu_{ei} \sim 3\text{MHz}$$
$$\nu_{ii} \sim 300\text{kHz}$$
$$\omega_{\text{A}} \sim 200\text{kHz}$$
$$L_{\|} \sim 18\text{m}$$
$$L_{\perp} \sim 50\text{cm}$$
$$\lambda_{\text{mfp}} \sim 20\text{cm}$$
$$\rho_i \sim 2\text{mm}$$
$$\rho_s \sim 5\text{mm}$$
$$\delta_e \sim 5\text{mm}$$
$$v_{\text{th,e}} \sim 1 \times 10^8\text{cm/s}$$
$$v_{\text{A}} \sim 1 \times 10^8\text{cm/s}$$
$$\beta \sim m_e/m_i \sim 1 \times 10^{-4}$$

Example data: cylindrical Alfvén eigenmodes in LAPD



- Plasma source acts as resonant cavity for shear Alfvén waves
- Driven spontaneously by discharge current (thought to be inverse Landau damping on return current electrons)
- Alfvén wave "MASER"

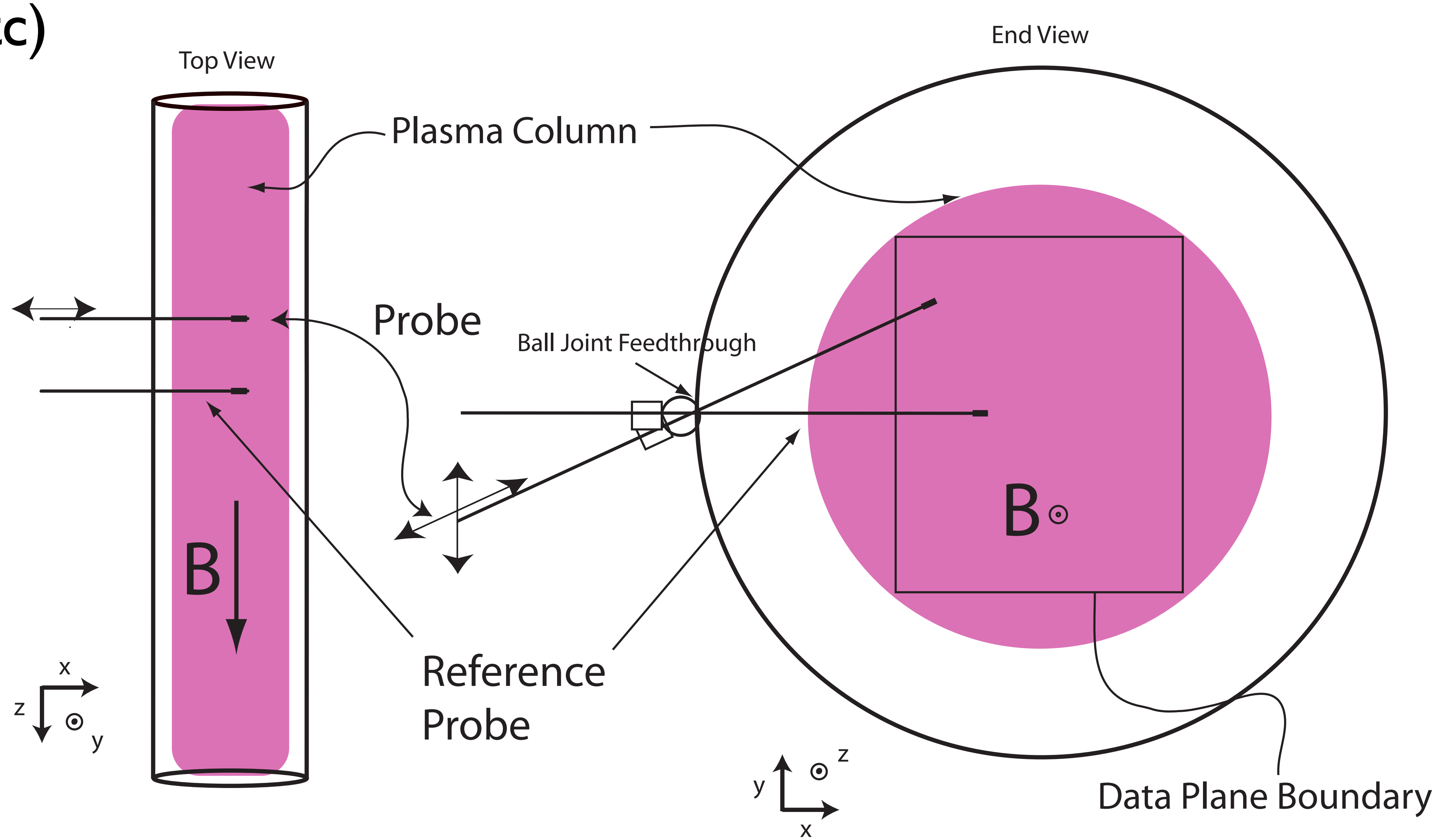


Maggs, Morales, PRL 91, 035004 (2003)

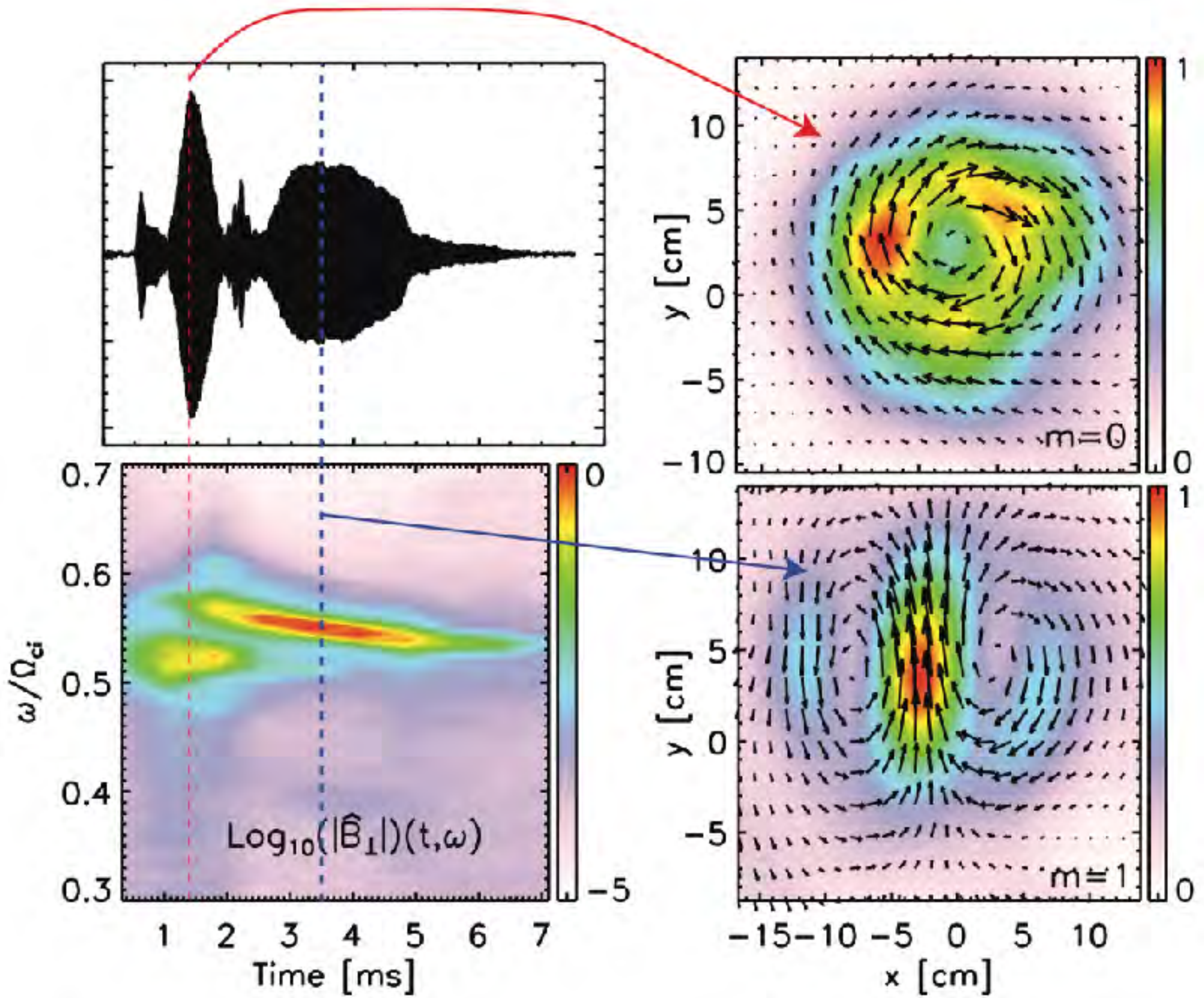
Maggs, Morales, Carter, PoP 12, 013103 (2005)

Measurement methodology in LAPD

- Use single probes to measure local density, temperature, potential, magnetic field, flow: move single probe shot-to-shot to construct average profiles
- Add a second (reference) probe to use correlation techniques to make detailed statistical measurements of turbulence (structure, etc)



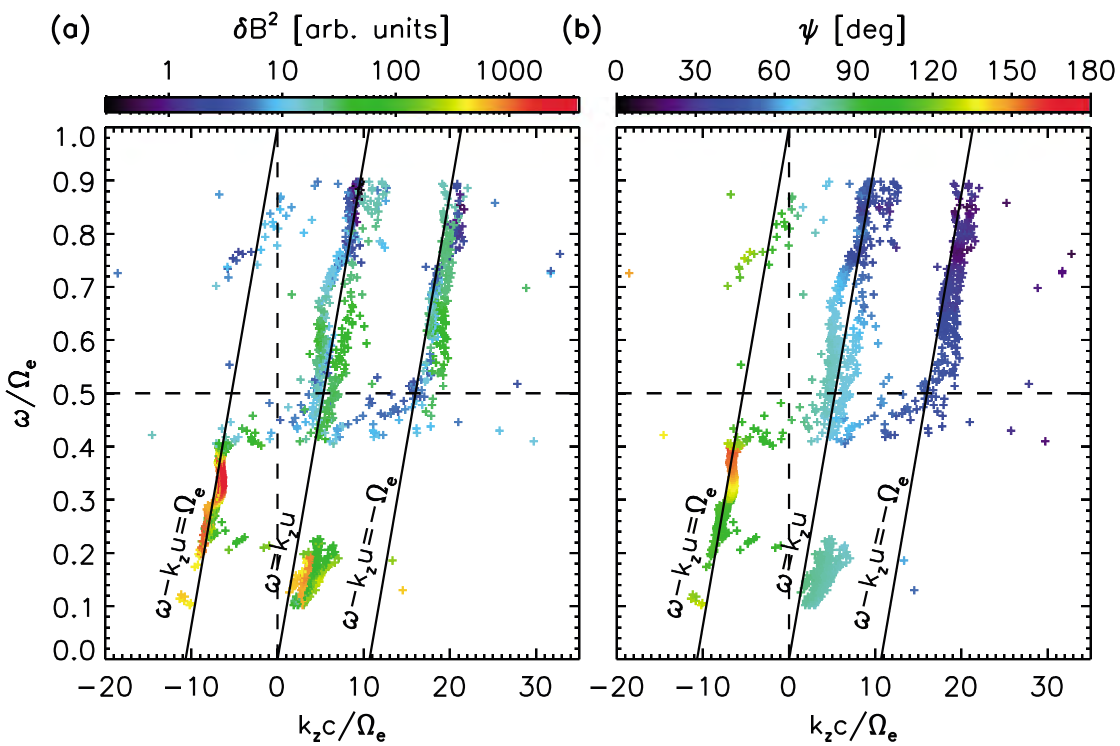
Measured structure of Alfvén eigenmodes in LAPD

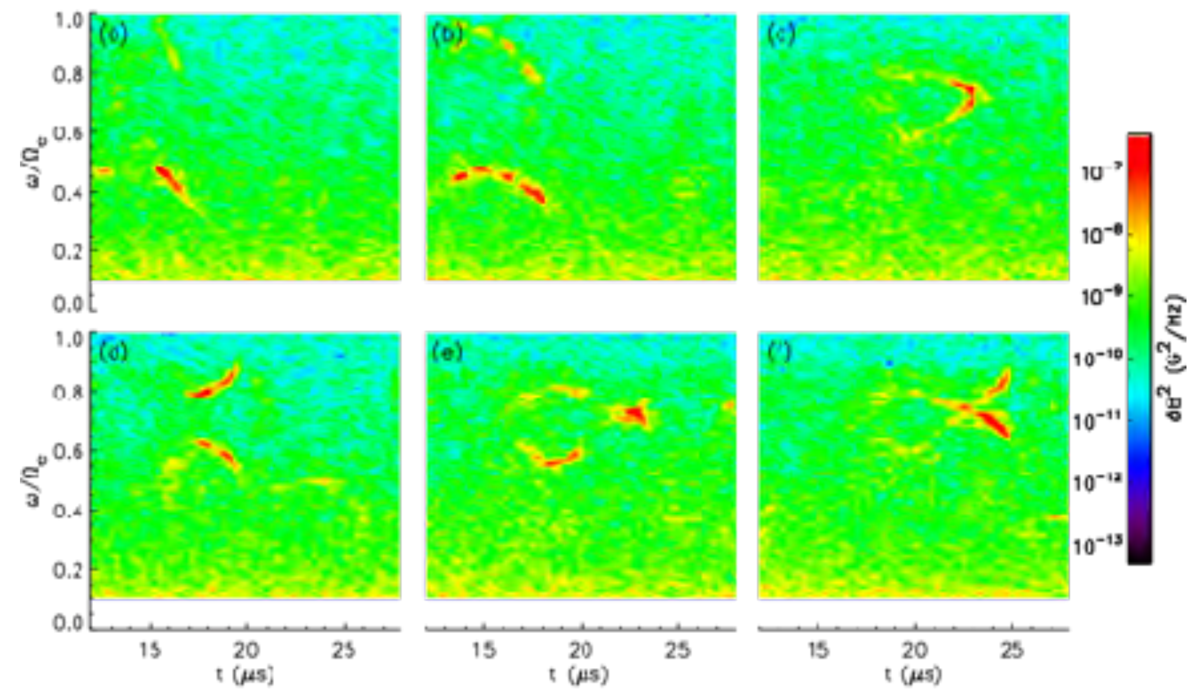


Example LAPD Users and Research Areas

- Basic Physics of Plasma Waves, e.g. linear properties of inertial and kinetic Alfvén waves (Gekelman, Morales, Maggs, Vincena…, Kletzing, Howes)
- Drift-wave turbulence and transport (Carter, Pace, Schaffner, Friedman, Popovich, Umansky, Maggs, Morales, Horton)
- Fast Waves/Physics of ICRF (D'Ippolito, Myra, Wright, Van Compernolle, Carter, Gekelman …)
- Wave-particle interactions (fast ions, fast electrons) (Colestock, Papadapoulous, Gekelman, Vincena, Zhou, Zhang, Heidbrink, Carter, Breizman, …)
- Reconnection (Gekelman, Van Compernolle, Daughton, …)
- Alfvén waves and shocks driven by laser blow-off (Niemann, Gekelman, Vincena, …)
- Nonlinear interactions between Alfvén waves (Carter, Dorfman, Howes, Kletzing, Skiff, Vincena, Boldyrev, ...)

Whistler modes excited by energetic electrons



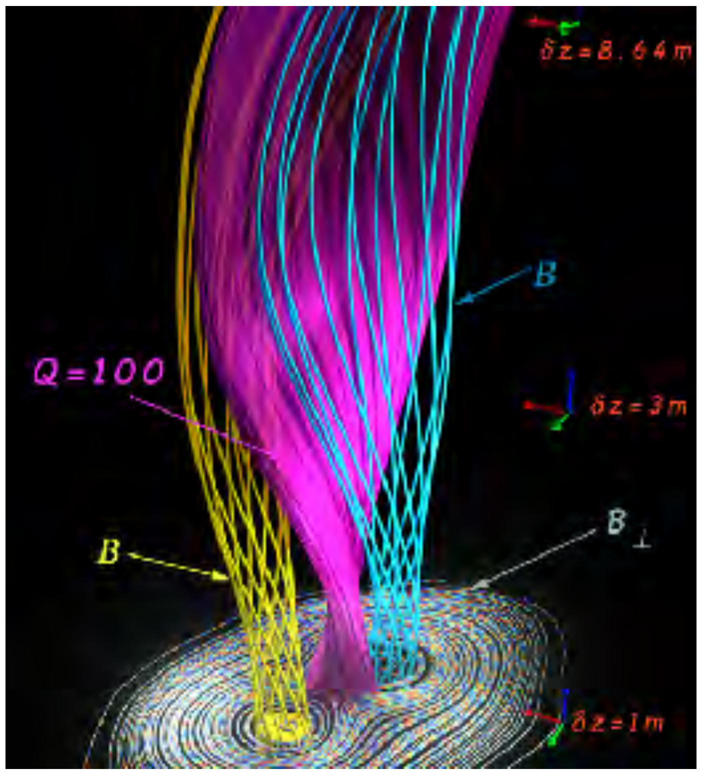


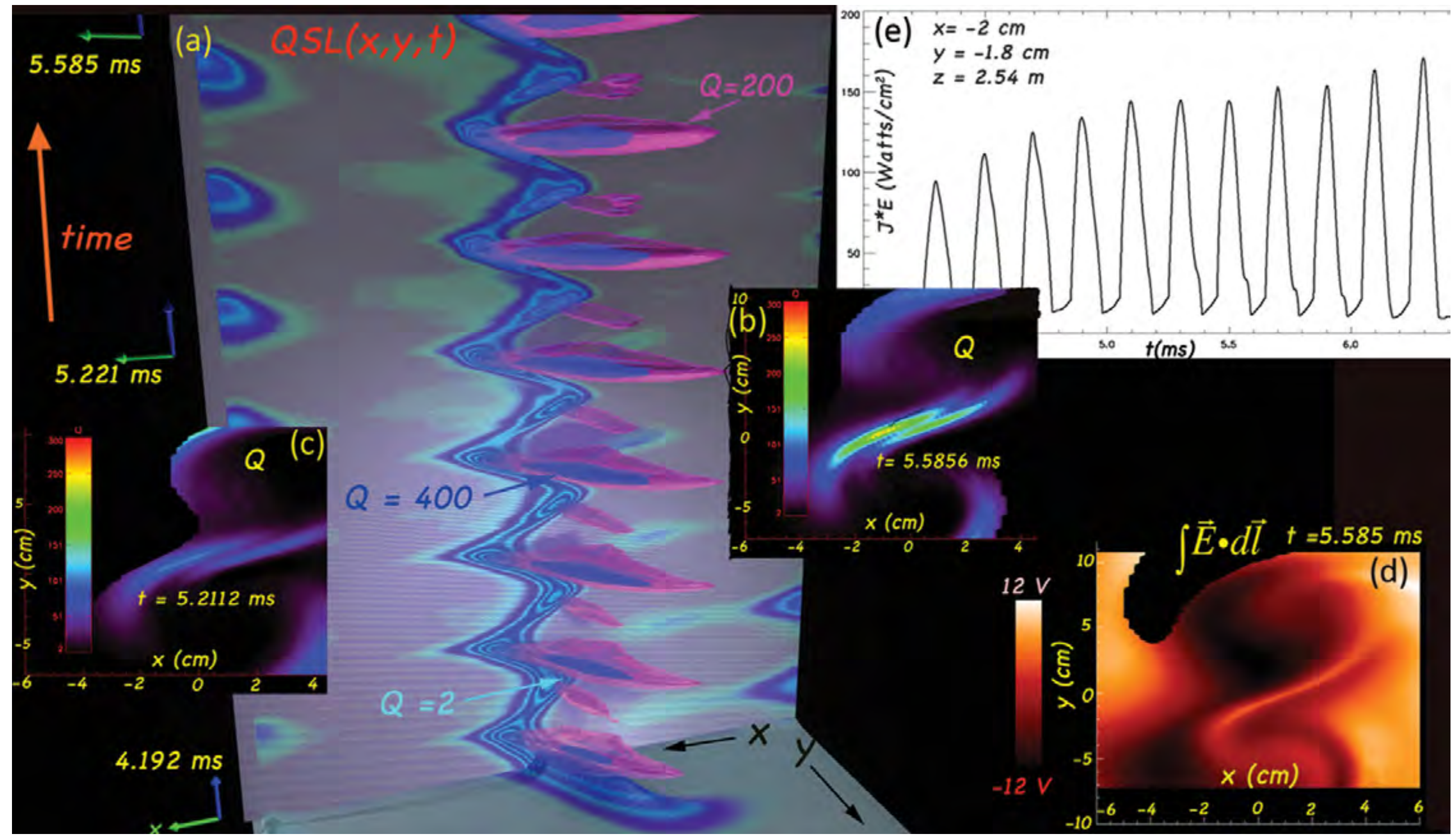
- Excitation of whistler waves by energetic electron beam (project led by J. Bortnik, R. Thorne)

- See "chirping" emission, similar to whistler chorus in magnetosphere (tied to transport/loss of radiation belt electrons)

X. An, et al., Geophys. Res. Lett., 43 (2016)

Three-dimensional reconnection in flux ropes



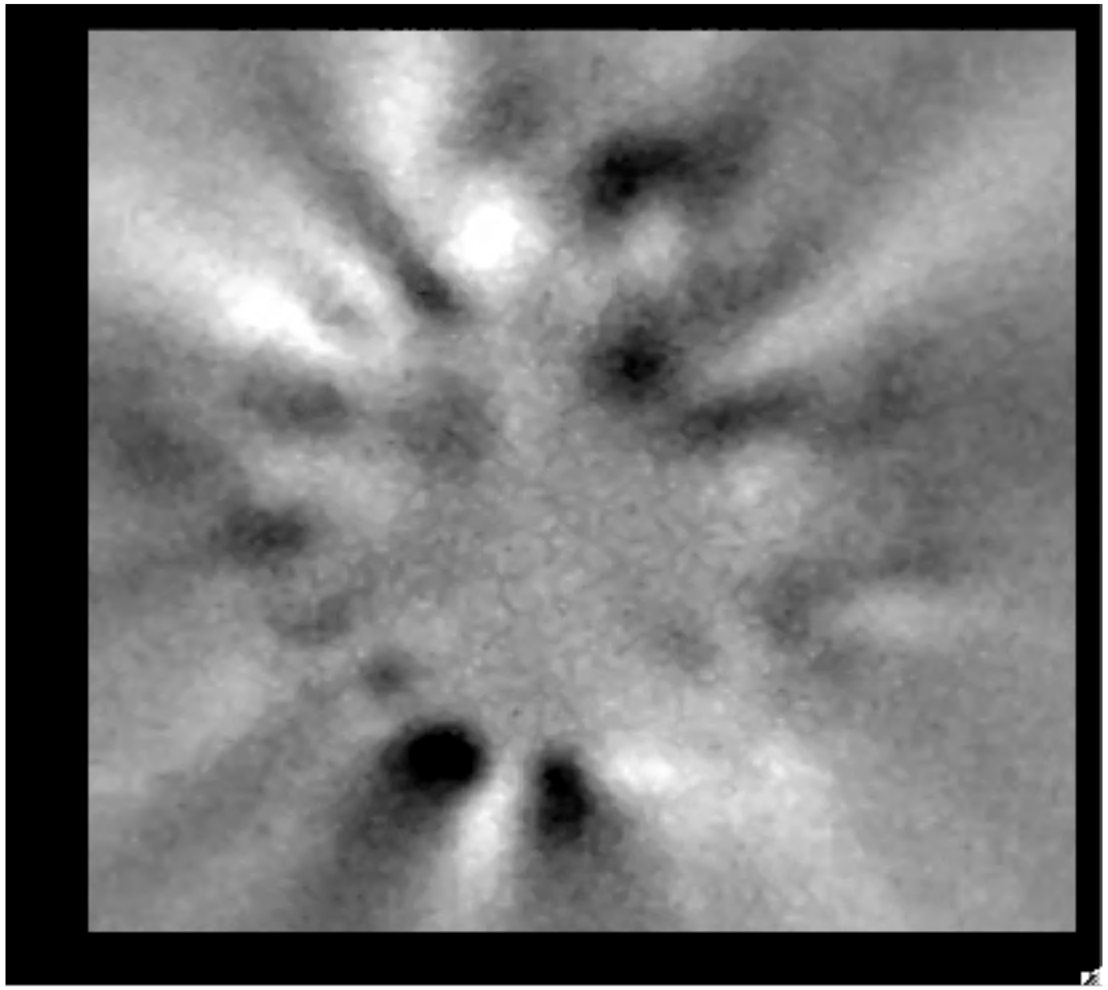


- Kink-unstable current carrying structures (flux ropes) interact and reconnect in LAPD, see periodic/pulsating reconnection

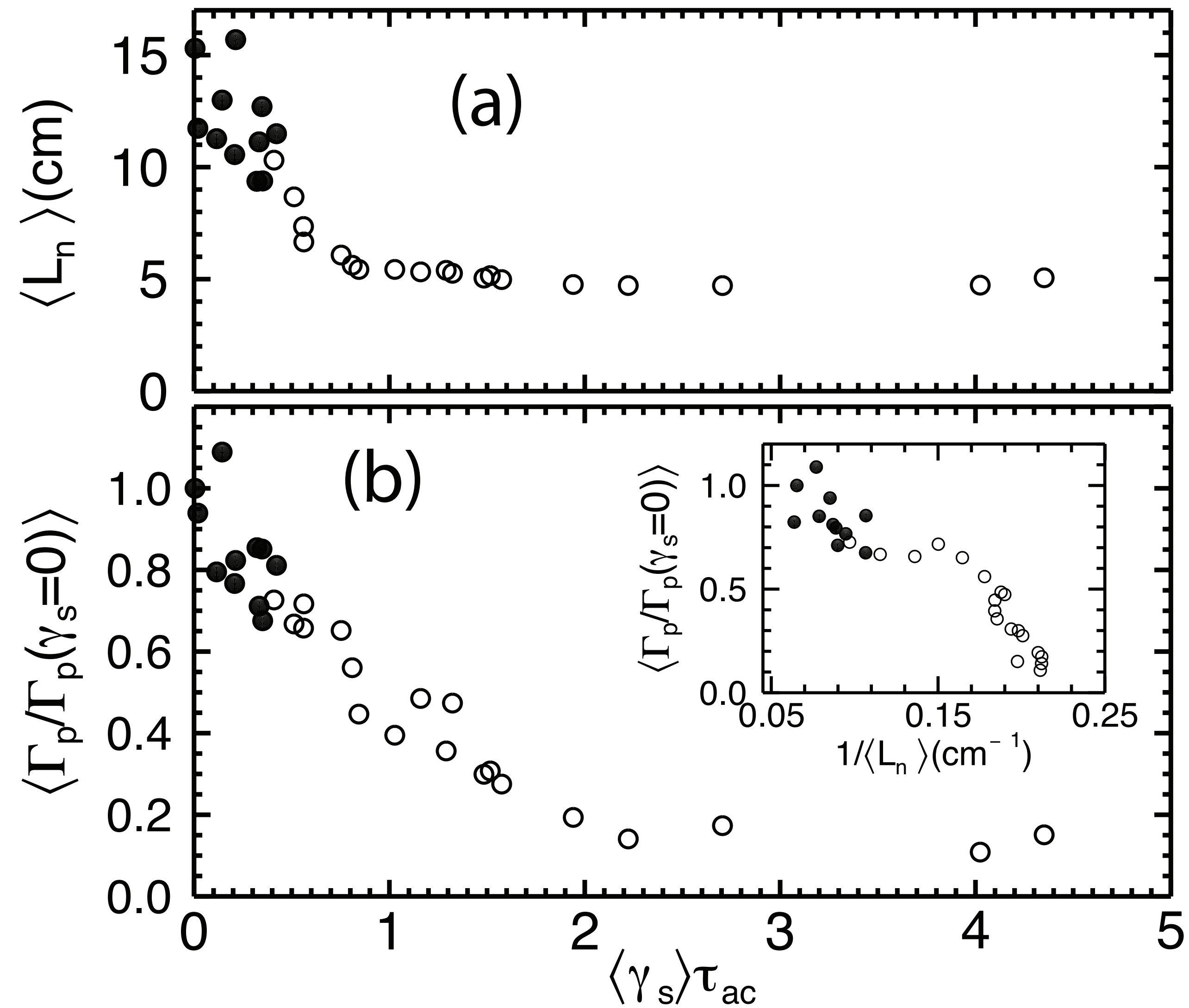
- First time "squashing factor"/presence of quasi-separatrix-layer (QSL) quantitatively linked to the reconnection rate

Gekelman, et al., Phys. Rev. Lett. 116, 235101 (2016)

Shear suppression of turbulent transport in LAPD



Visible light (40k frames/sec)

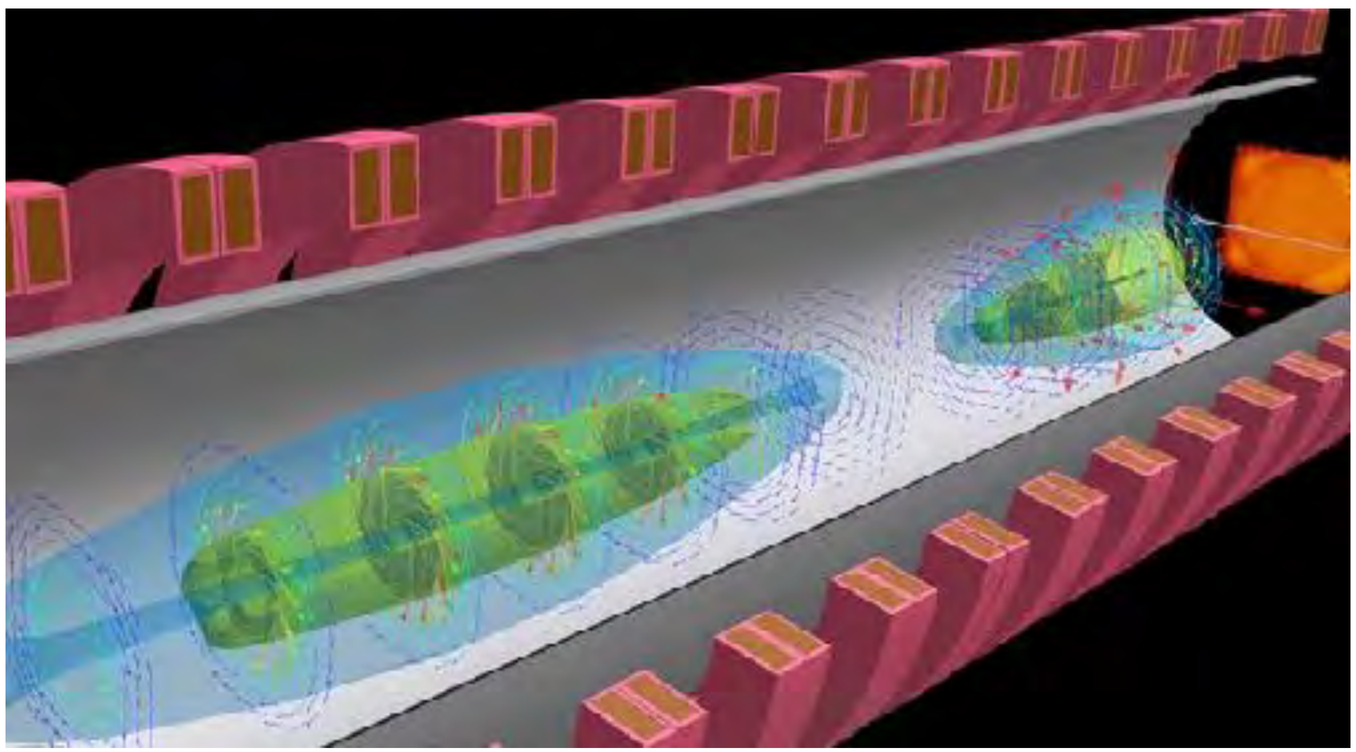


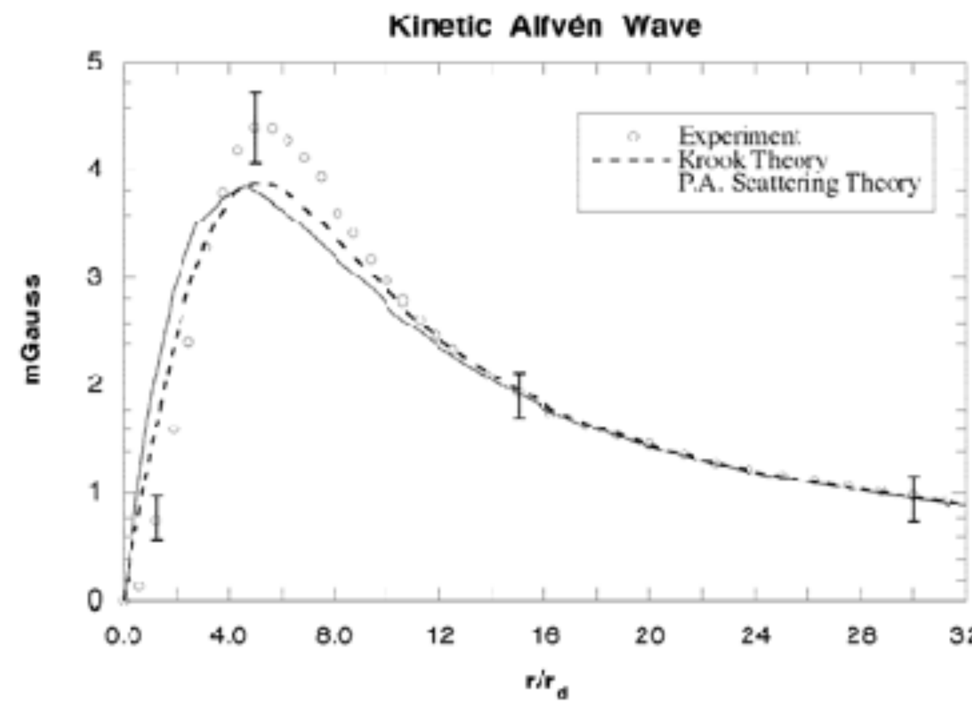
- Limiter biasing used to control edge flow: can reverse flow direction, zero-out spontaneous rotation

- Documented response of turbulence and transport to continuous variation in shear [Schaffner et al., PRL 109, 135002 (2012)]; compared to decorrelation models [Schaffner, et al.,PoP 2013]

IAW/KAW wave studies in LAPD

- LAPD created to enable AW research need length to fit parallel wavelength (~few meters)
- Below: 3D AW pattern from a small antenna (comparable to skin depth, sound gyroradius)



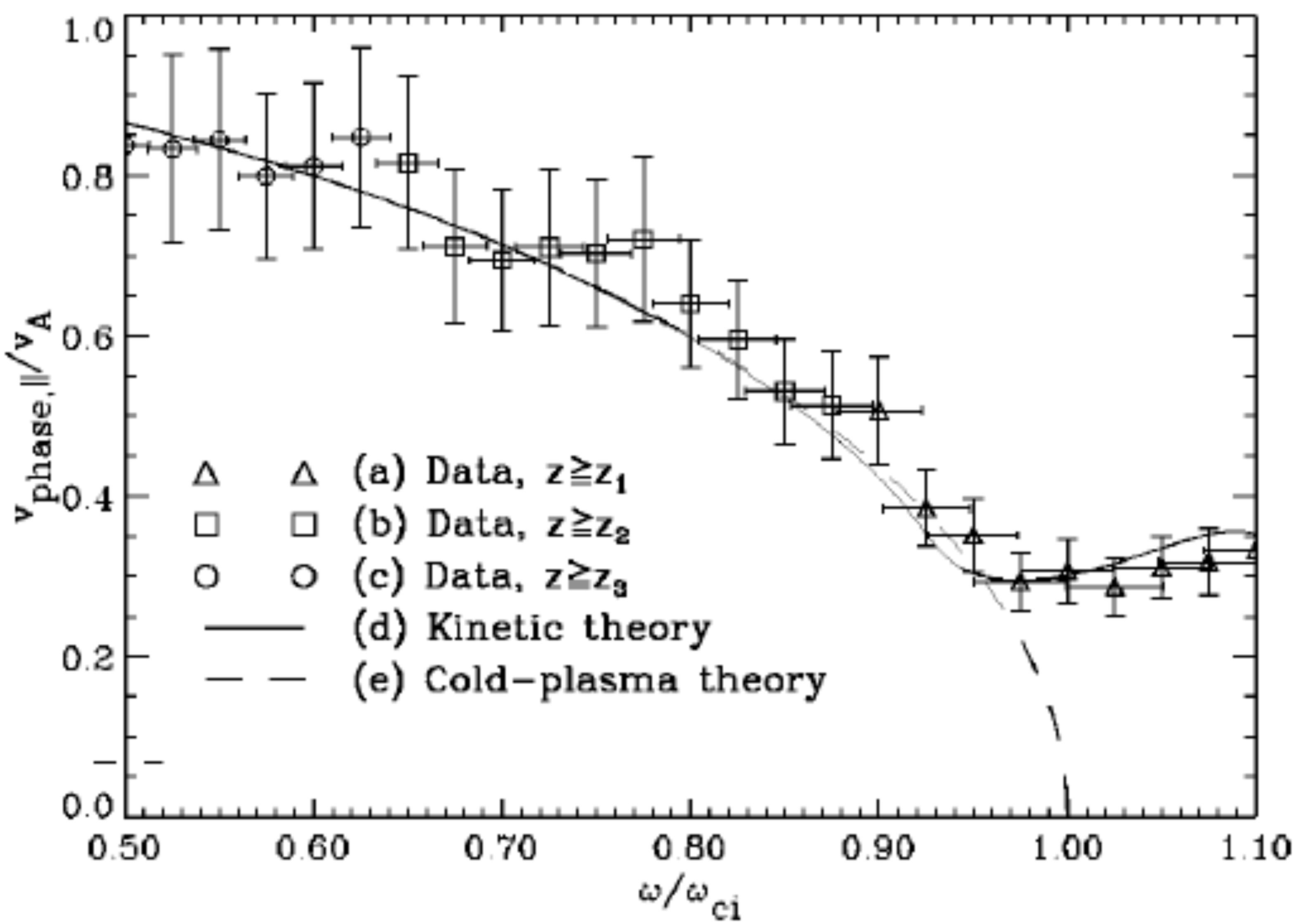


- A number of issues studied over the years: radiation from small source, resonance cones, field line resonances, wave reflection, conversion from KAW to IAW on density gradient… [UCLA LAPD group: Gekelman, Maggs, Morales, Vincena, et al]

Review: Gekelman, et al., PoP 18, 055501, (2011)

Details, publication list at http://plasma.physics.ucla.edu

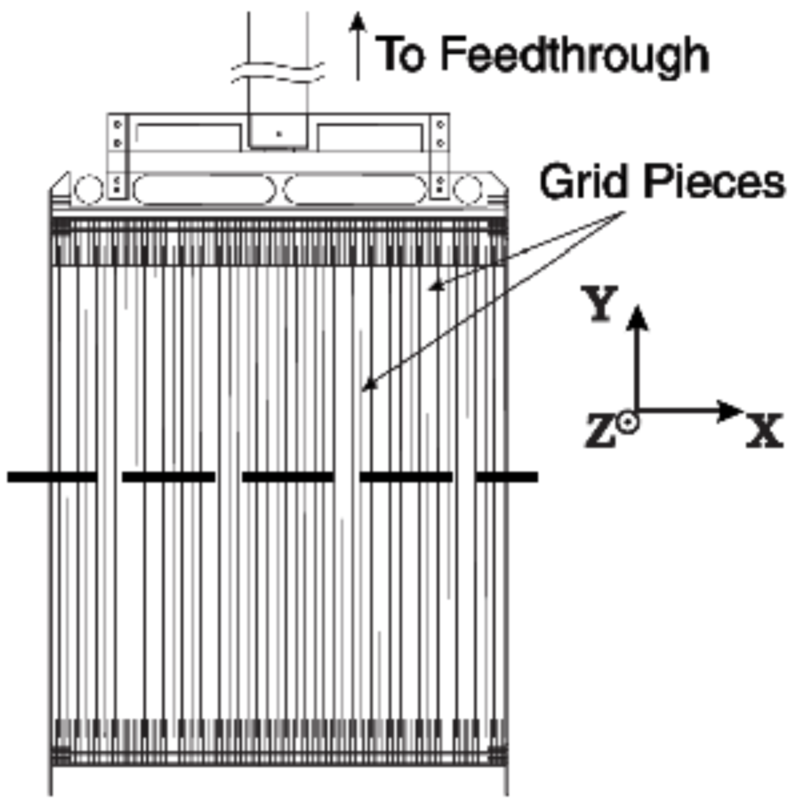
Finite frequency dispersion relation for KAWs

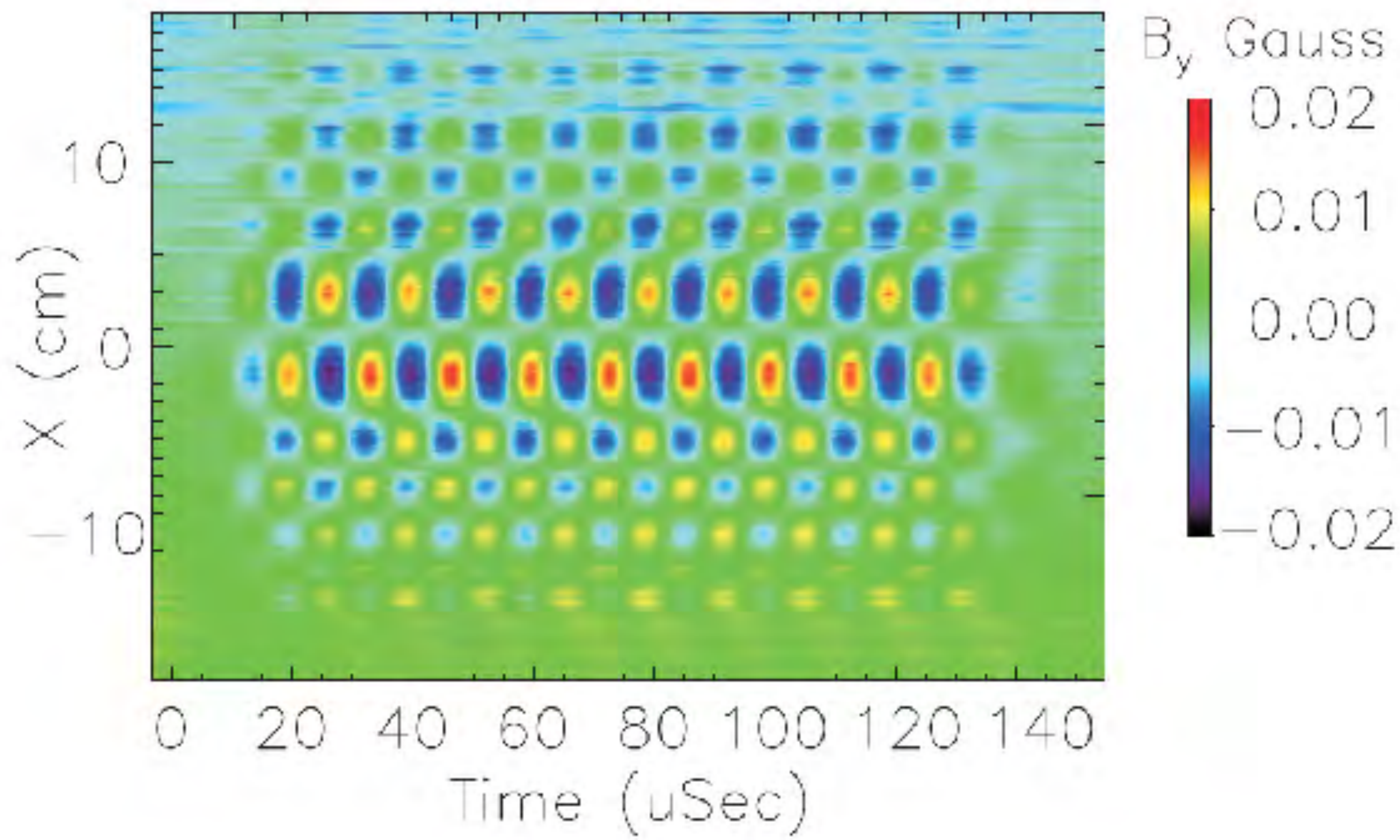


Vincena, et al. PoP 8, 3884 (2001)

- Need kinetic theory to explain observations around Ω_i
- Nice study of absorption of KAW in “magnetic beach”

Study of IAW/KAW dispersion & damping



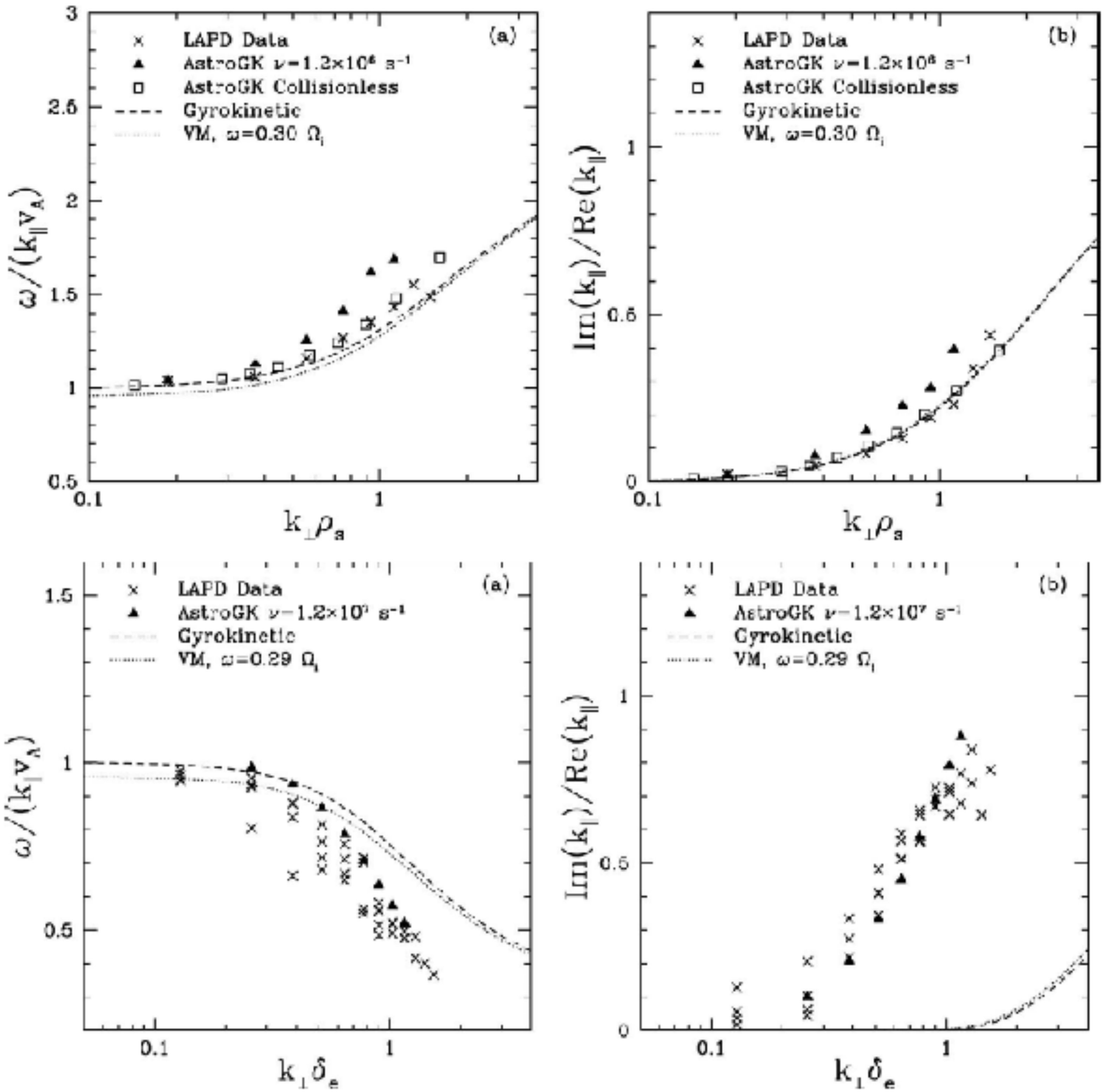


- Special antenna built to create plane-wave-like AWs with control over k to do detailed dispersion/damping measurements [U. Iowa group, Kletzing, Skiff + students]

Kletzing, et al, PRL 104, 095001 (2010)

Measured dispersion and damping, GK modeling

- Measurements compared to AstroGK simulations, including collisions (crucial to get inertial AW dispersion/damping right)

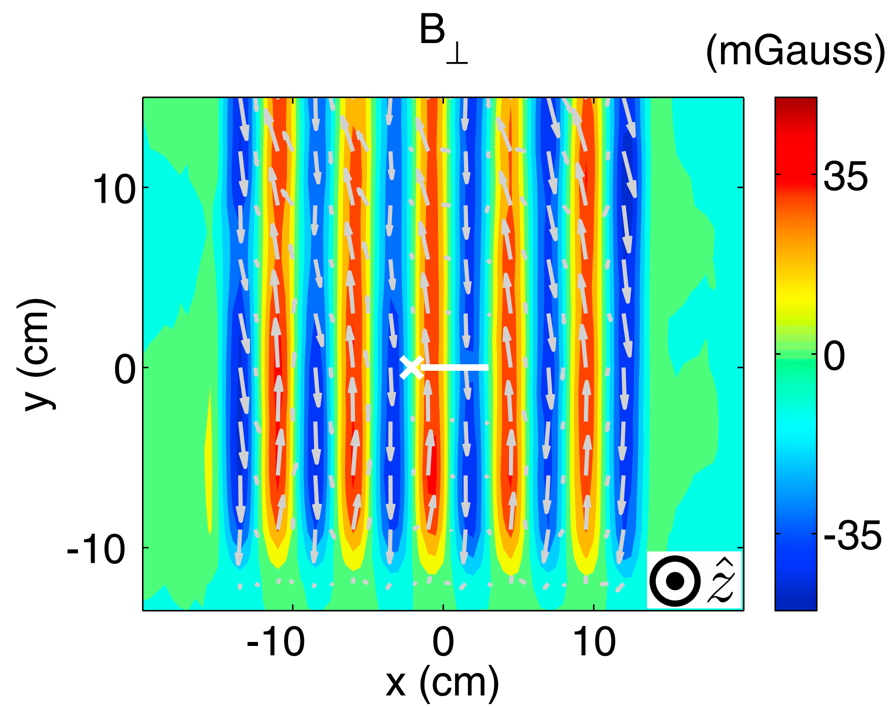


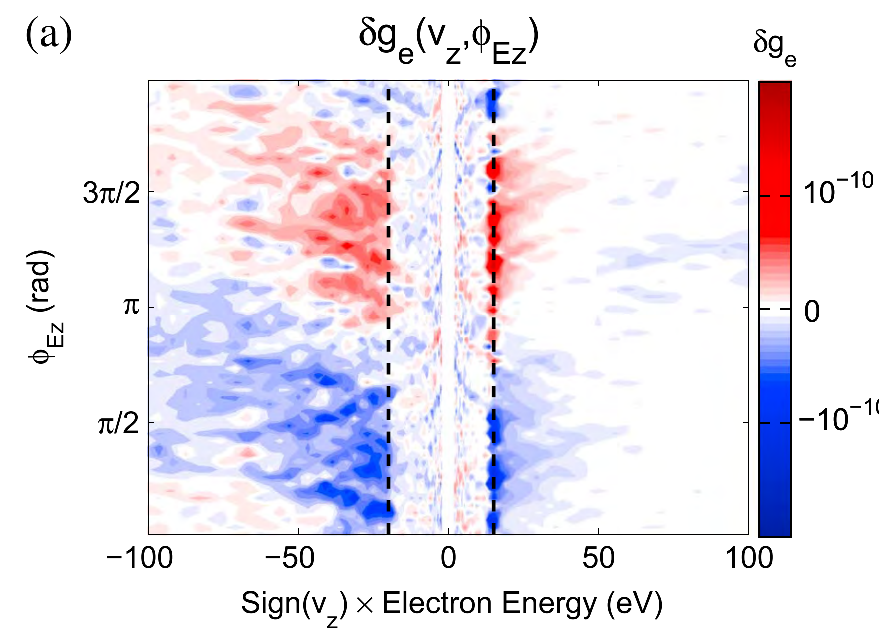
kinetic AW

inertial AW

Nielson, Howes, et al, PoP **17**, 022105 (2010)

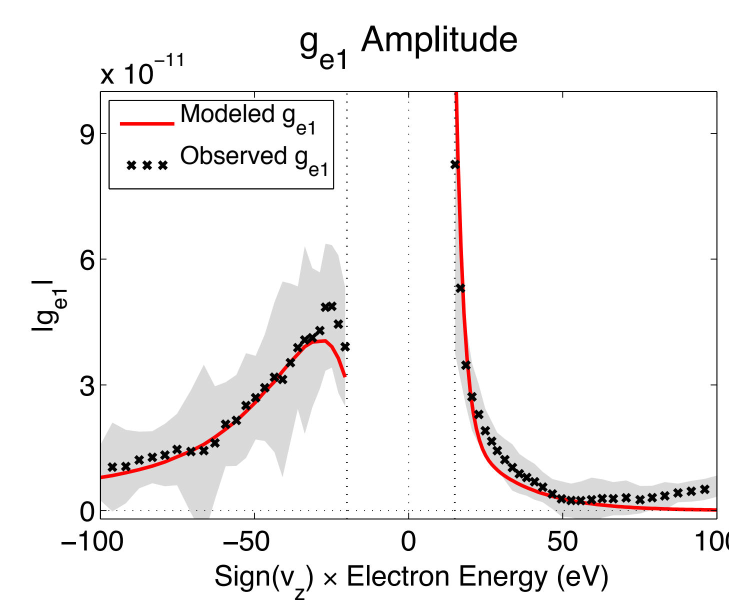
Electron response to inertial Alfvén wave





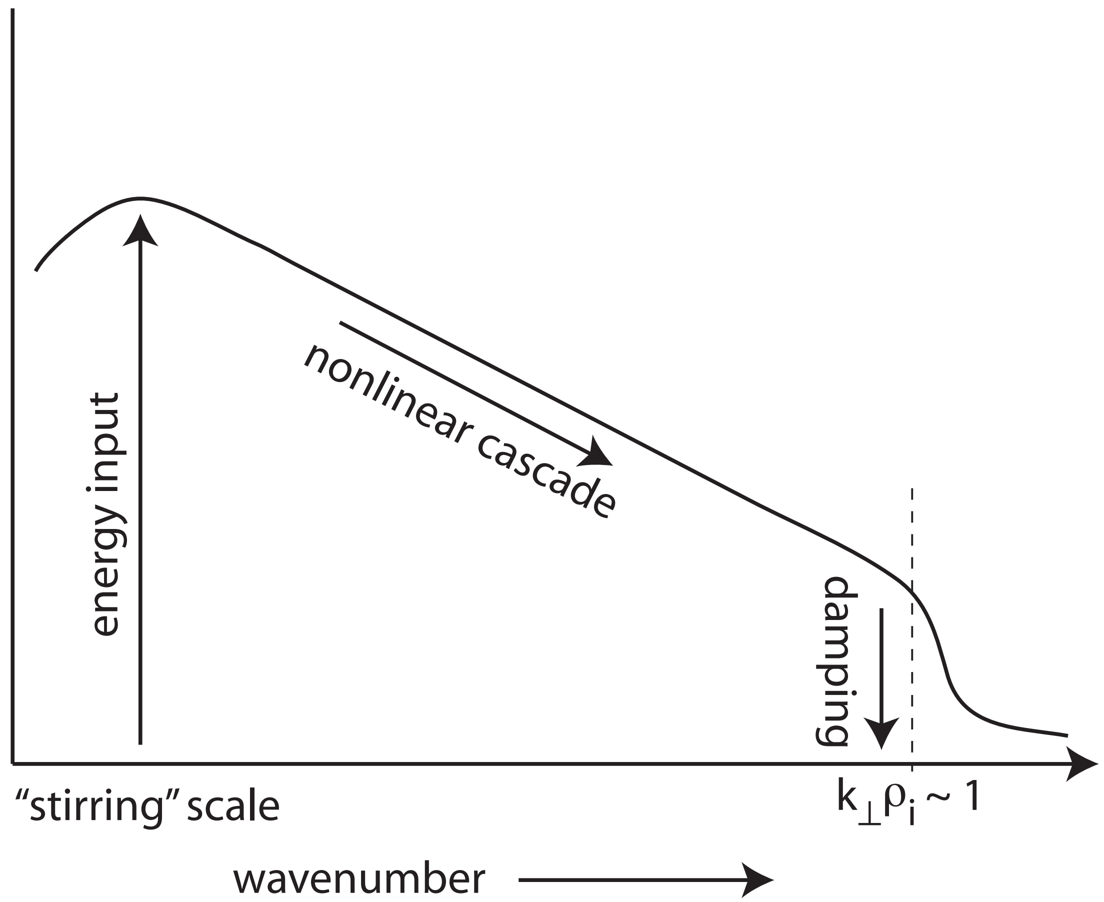
- U. Iowa group: interest in understanding electron acceleration by Alfvén waves; relevance to generation of Aurora

- Used novel electron distribution diagnostic (whistler wave absorption) to study oscillation in electron distribution function in presence of inertial AW



Schroeder, et al., Geophys. Res. Lett. 43, 4701 (2016)

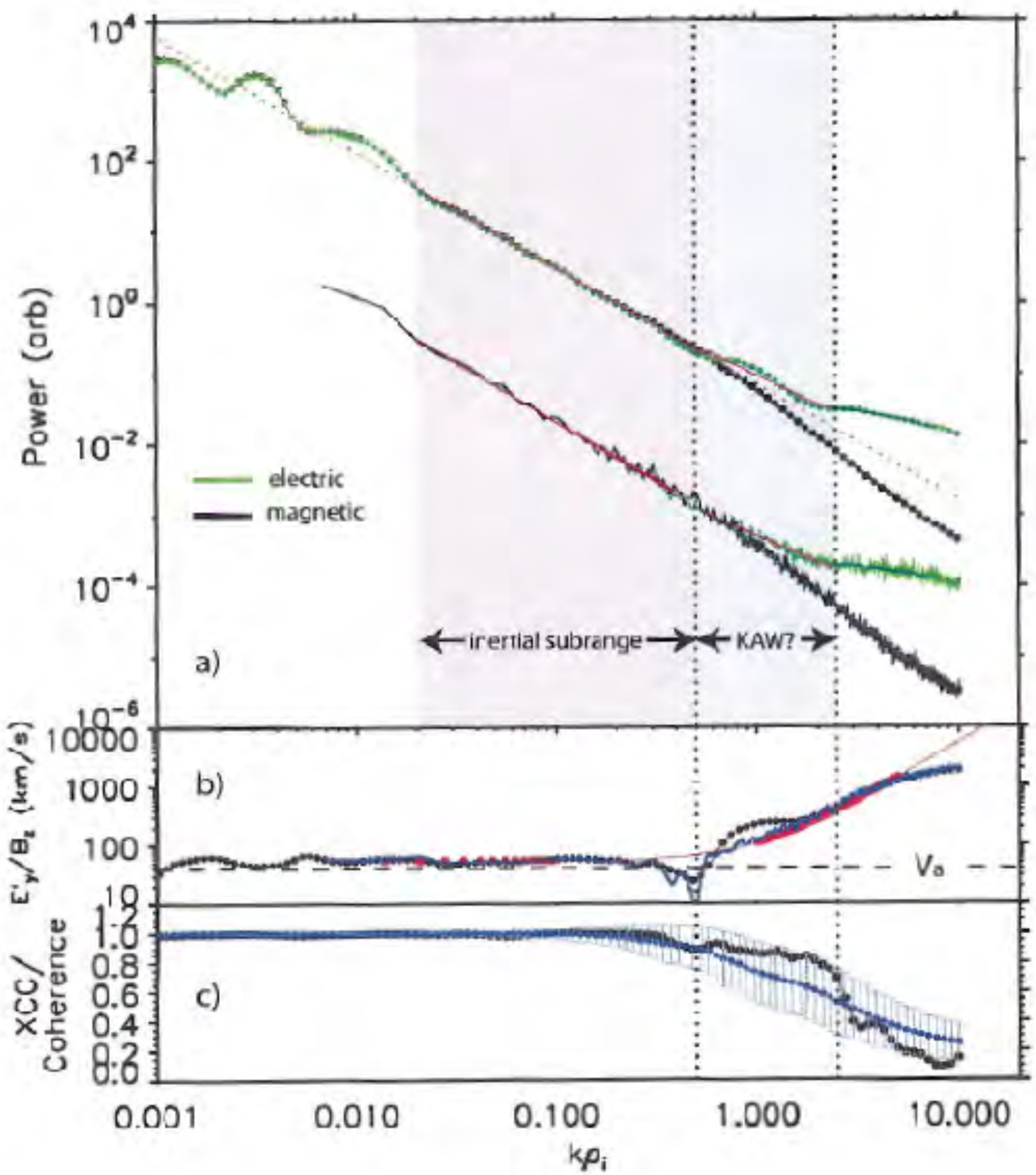
On to nonlinear processes: motivation from MHD turbulence



- Low frequency turbulence in magnetized plasma (e.g. solar wind, accretion disk)
- Energy is input at "stirring" scale (e.g. MRI in accretion disk, tearing mode or Alfvén Eigenmode in tokamak or RFP) and cascades nonlinearly to dissipation scale

- From a weak turbulence point of view, cascade is due to interactions between linear modes: shear Alfvén waves
- Motivates laboratory study of wave-wave interactions among Alfvén waves

Turbulent Cascade in the Solar wind



Bale, 2005

Theory of the Alfvénic cascade

- Kraichnan: nonlinear perturbations arise through interaction between counter-propagating shear Alfvén waves (ideal, incompressible MHD)

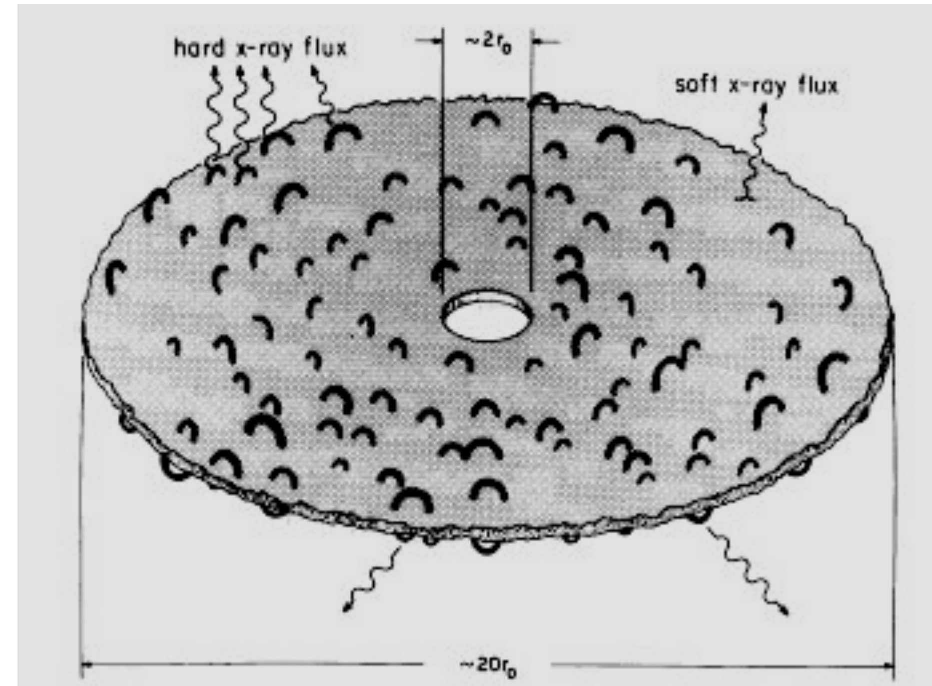
$$\frac{\partial \mathbf{w}_+}{\partial t} + v_A \frac{\partial \mathbf{w}_+}{\partial z} = -\mathbf{w}_- \cdot \nabla \mathbf{w}_+ - \nabla P$$
$$\frac{\partial \mathbf{w}_-}{\partial t} - v_A \frac{\partial \mathbf{w}_-}{\partial z} = -\mathbf{w}_+ \cdot \nabla \mathbf{w}_- - \nabla P$$
$$P = \int \frac{d^3x'}{4\pi} \frac{\nabla \mathbf{w}_+ : \nabla \mathbf{w}_-}{x - x'}$$

(Above from MHD equations: $\mathbf{w}_+ = v_A \hat{\mathbf{z}} + \mathbf{v} - \mathbf{b}$ and $\mathbf{w}_- = -v_A \hat{\mathbf{z}} + \mathbf{v} + \mathbf{b}$)

- Cascade is highly anisotropic, primarily in the perpendicular direction (follows from three wave matching rules) [Shebalin, Matthaeus, Goldreich, Sridhar, Bhattachargee, et al]
- Physically: right-going wave follows the perturbed field lines of left-going wave, shear each other apart to produce smaller-scale structure
- Non-ideal effects (compressibility, FLR, etc) allow three wave interactions involving other modes, copropagating interactions

“Classical” accretion: drag provided by collisions among the plasma particles in the disk

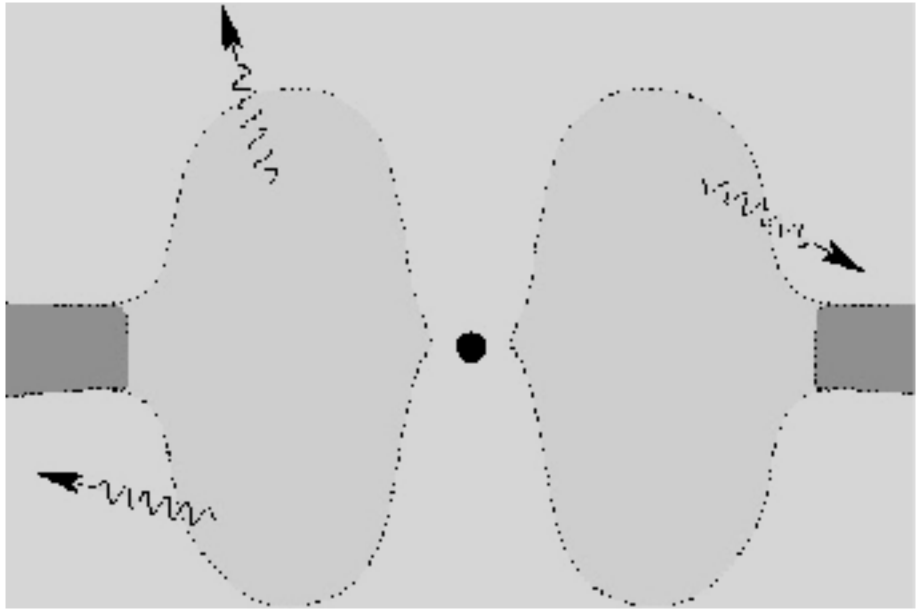
- Only happens in “cool” disks (remember plasmas become “collisionless” as they get hot)
- In classical disk, energy gets transferred to light particles via collisions: electrons are heated



- Electrons radiate this energy away very effectively (x-rays due to synchrotron radiation); keeps disk cool, results in “thin”disk (relevant to protostar, planetary disks, some BH)

Problem with “hot” disks: collisions too infrequent to explain observed accretion rates

- Radiatively inefficient disks are often observed: not enough radiation to cool disk as matter accretes, energy gets stored in thermal energy, get puffed-up, thick disk



Problem with "hot" disks: collisions too infrequent to explain observed accretion rates

- Radiatively inefficient disks are often observed: not enough radiation to cool disk as matter accretes, energy gets stored in thermal energy, get puffed-up, thick disk
- Because plasma is very hot, collisions are too infrequent to explain observed rates of accretion!
- Turbulence to the rescue? Problem: disks are hydrodynamically stable (no "linear" instability in Keplerian flow of neutral gas)

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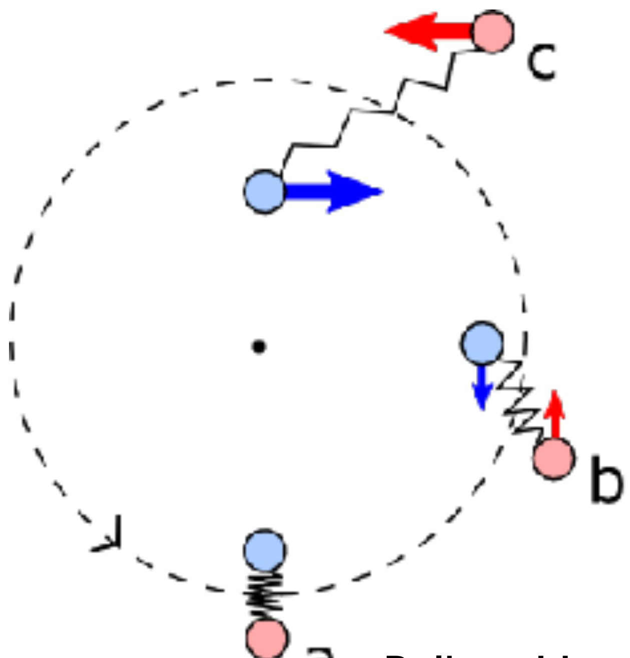
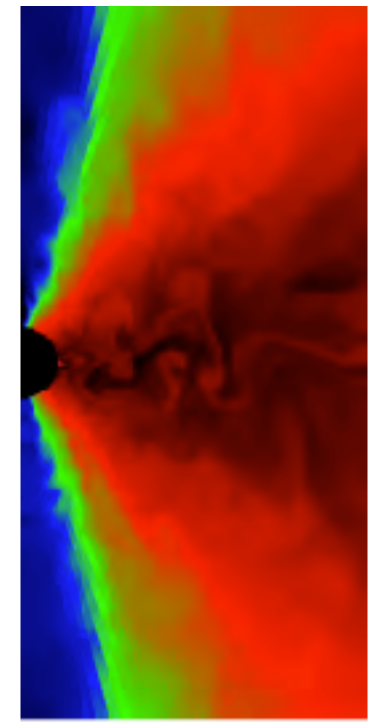
➡ However, if you acknowledge this “gas” is a plasma, and that magnetic fields can be present, there is an instability: Magnetorotational Instability (MRI) [Velikhov, Chandrasekhar, Balbus, Hawley]

Magnetorotational instability (MRI): transports momentum, but where does energy go?



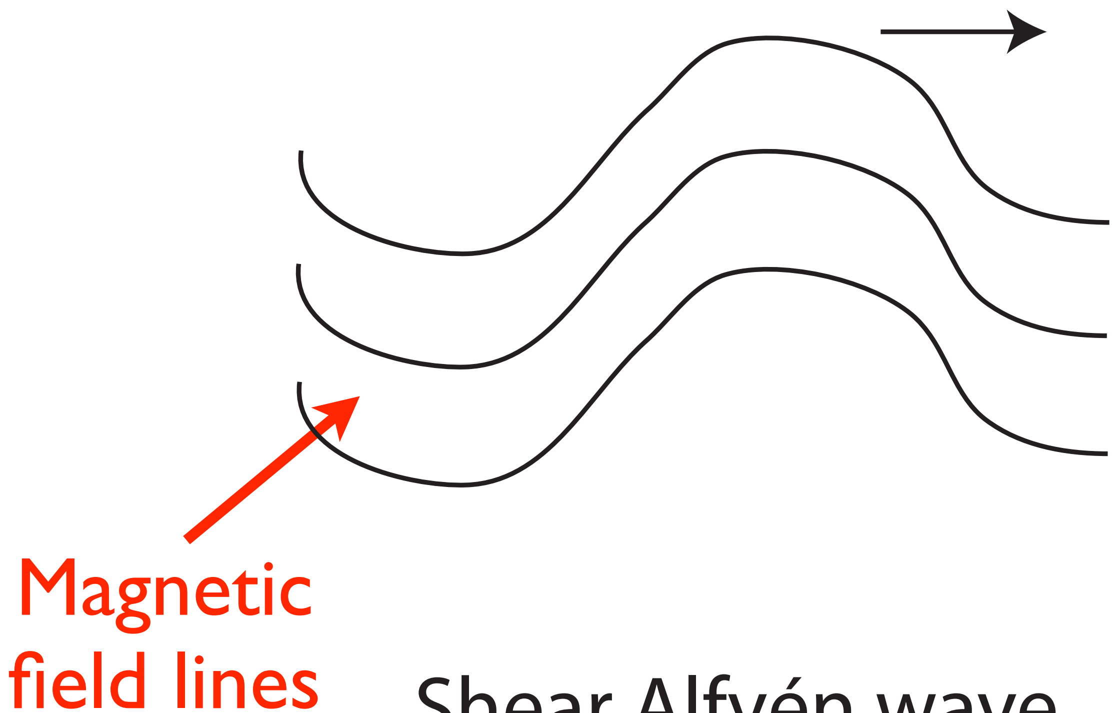
Balbus, Hawley, Rev. Mod. Phys. 70, 1–53 (1998)



MRI simulation (Stone)

- Presence of weak magnetic field allows instability: angular momentum transported outward, matter inward
- Instability provides "anomalous" viscosity, accretion can occur
- Energy released in accretion gets taken up by turbulent magnetic fields which grow as part of the instability: where does this energy go and why isn't it radiated away?

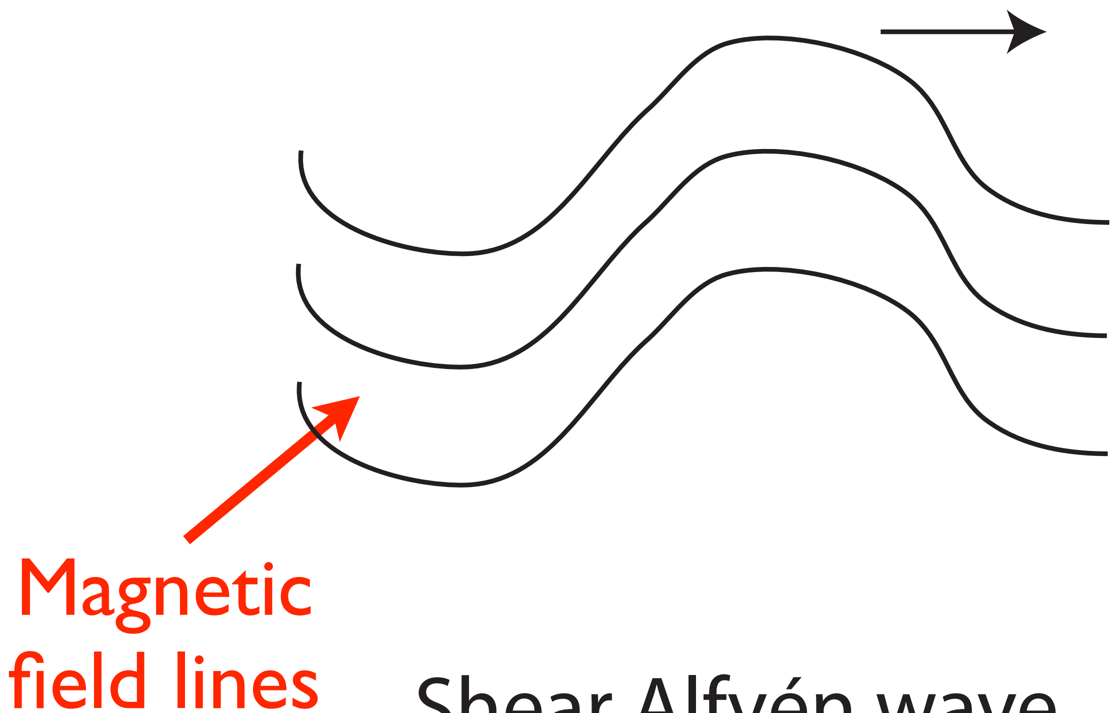
Energy in MRI can drive turbulent cascade of Alfvén waves



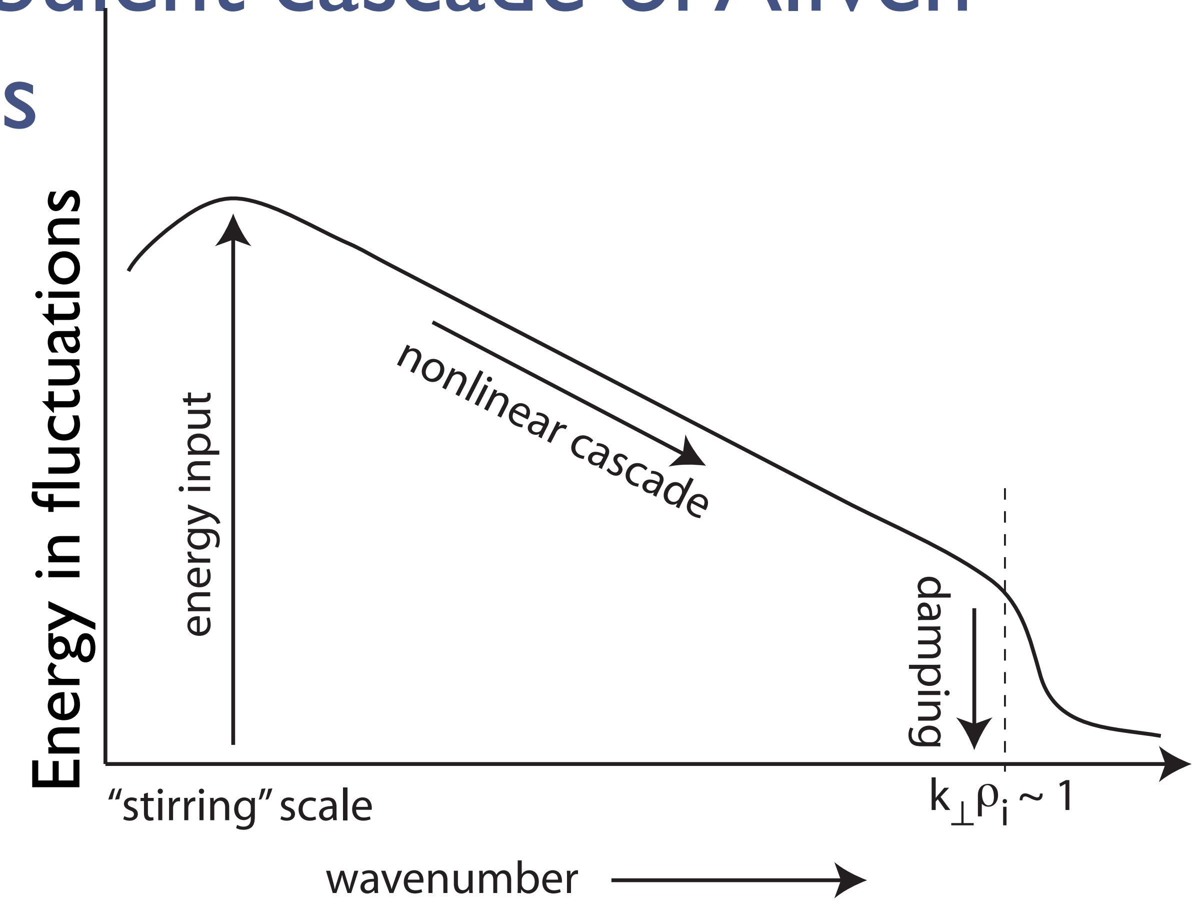
Shear Alfvén wave

- Shear Alfvén wave: analogous to wave on string, tension provided by field line, mass by plasma

Energy in MRI can drive turbulent cascade of Alfvén waves



Shear Alfvén wave

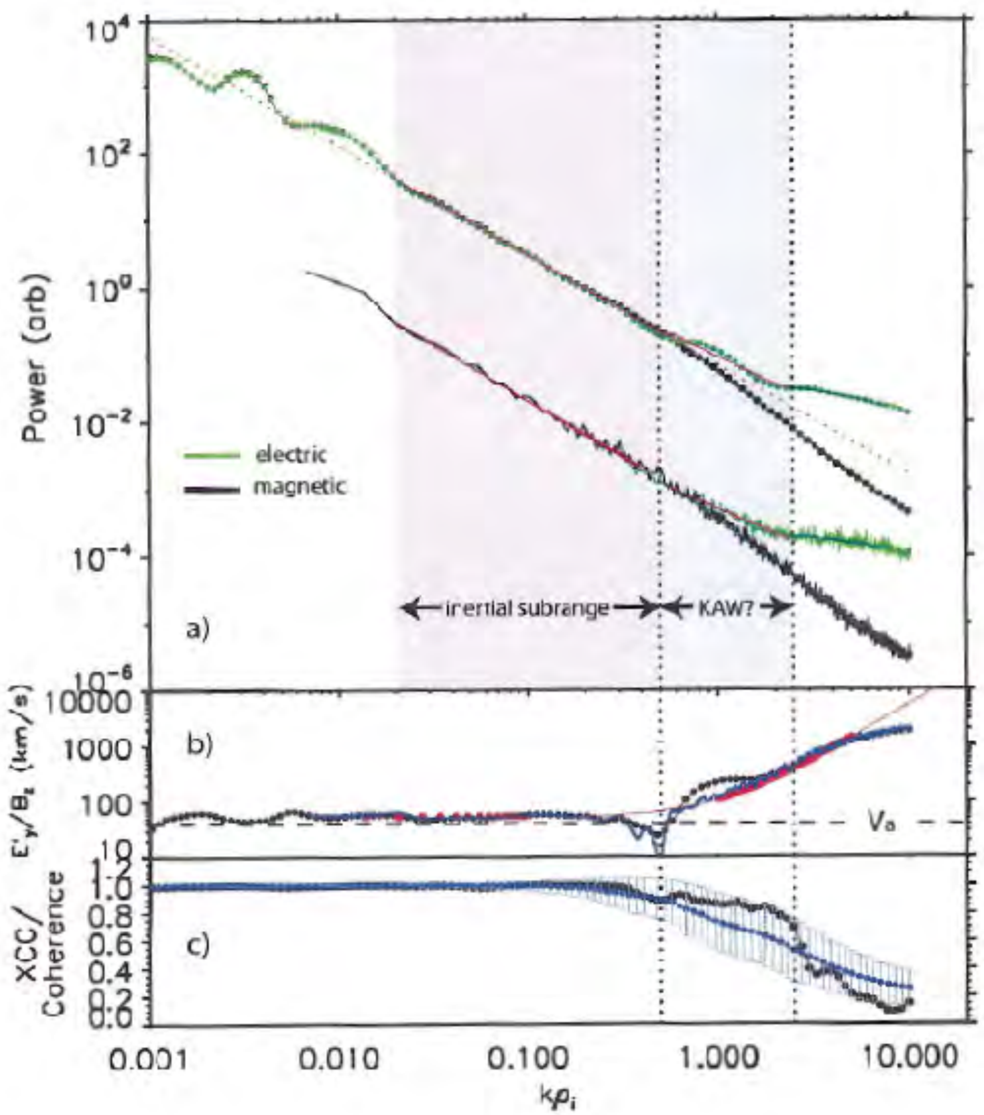


- Shear Alfvén wave: analogous to wave on string, tension provided by field line, mass by plasma
- MRI acts as large scale "stirring"; instability perturbations are like large-scale Alfvén waves
- Nonlinear interaction among waves generates daughter waves at smaller spatial scales; cascade down to dissipation scales where energy dissipated into plasma thermal energy

Quataert *ApJ* **500** 978 (1998)

- Direct ion heating possible at dissipation scale: could explain observations

Turbulent Alfvénic cascade observed in the solar wind

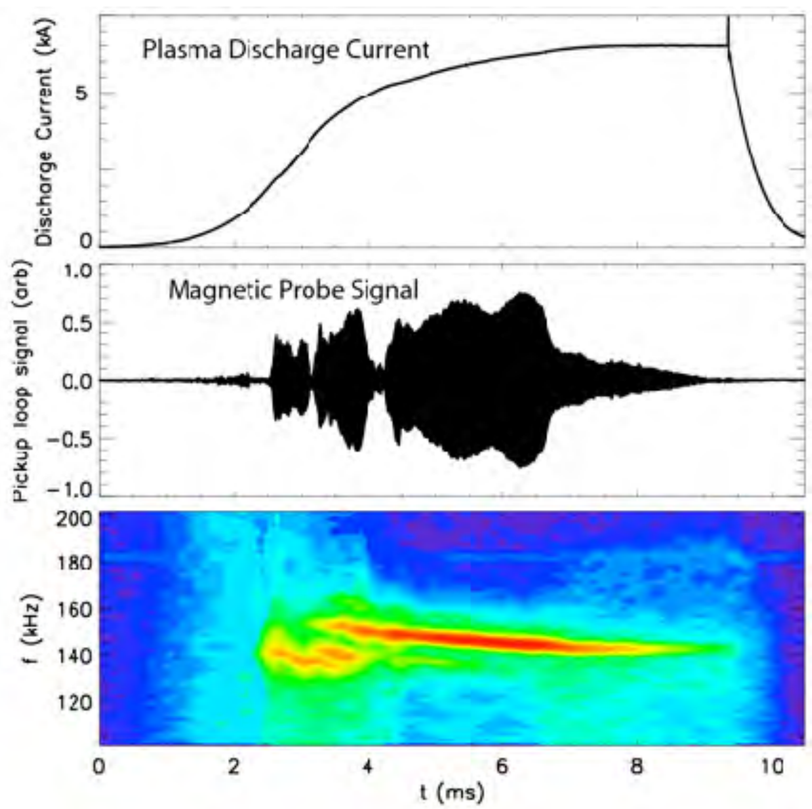


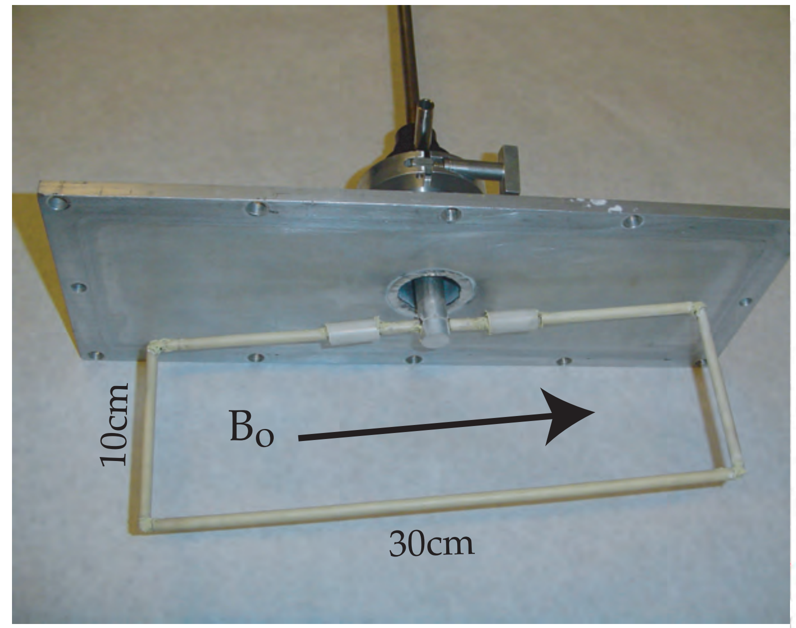
Bale, et al. PRL 94, 215002 (2005)

- "Stirring" comes from strong flows, AWs that originate at the sun
- Satellite measurements of electric and magnetic field fluctuations reveals turbulent spectrum

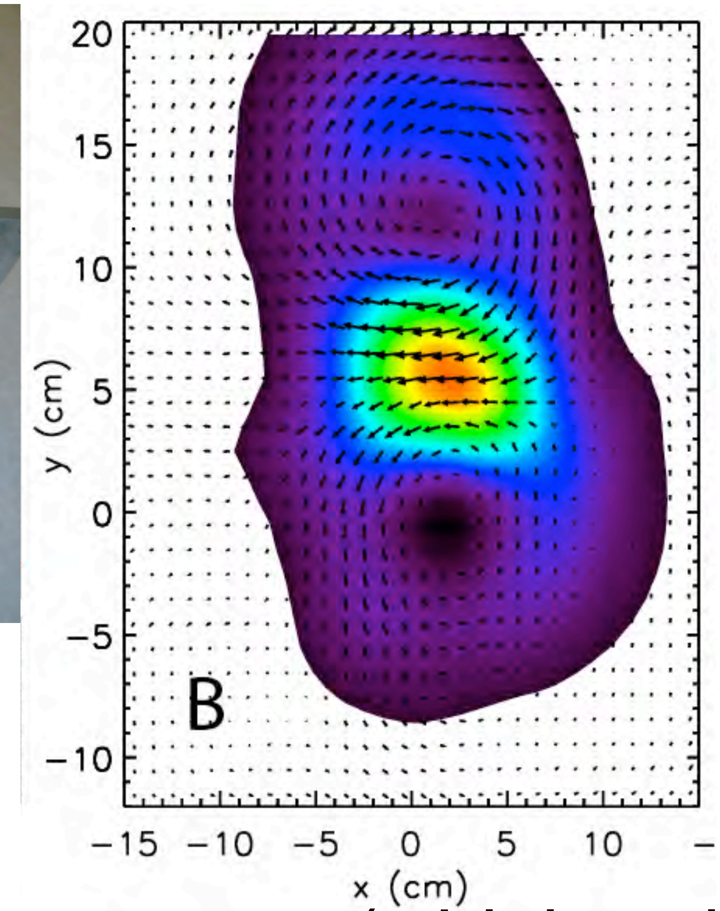
- Questions raised: what sets shape of spectrum (power law observed, close to Komolgorov); how is energy dissipated
- Motivates laboratory study of wave-wave interactions among Alfvén waves

Large amplitude wave sources: MASER and Antenna



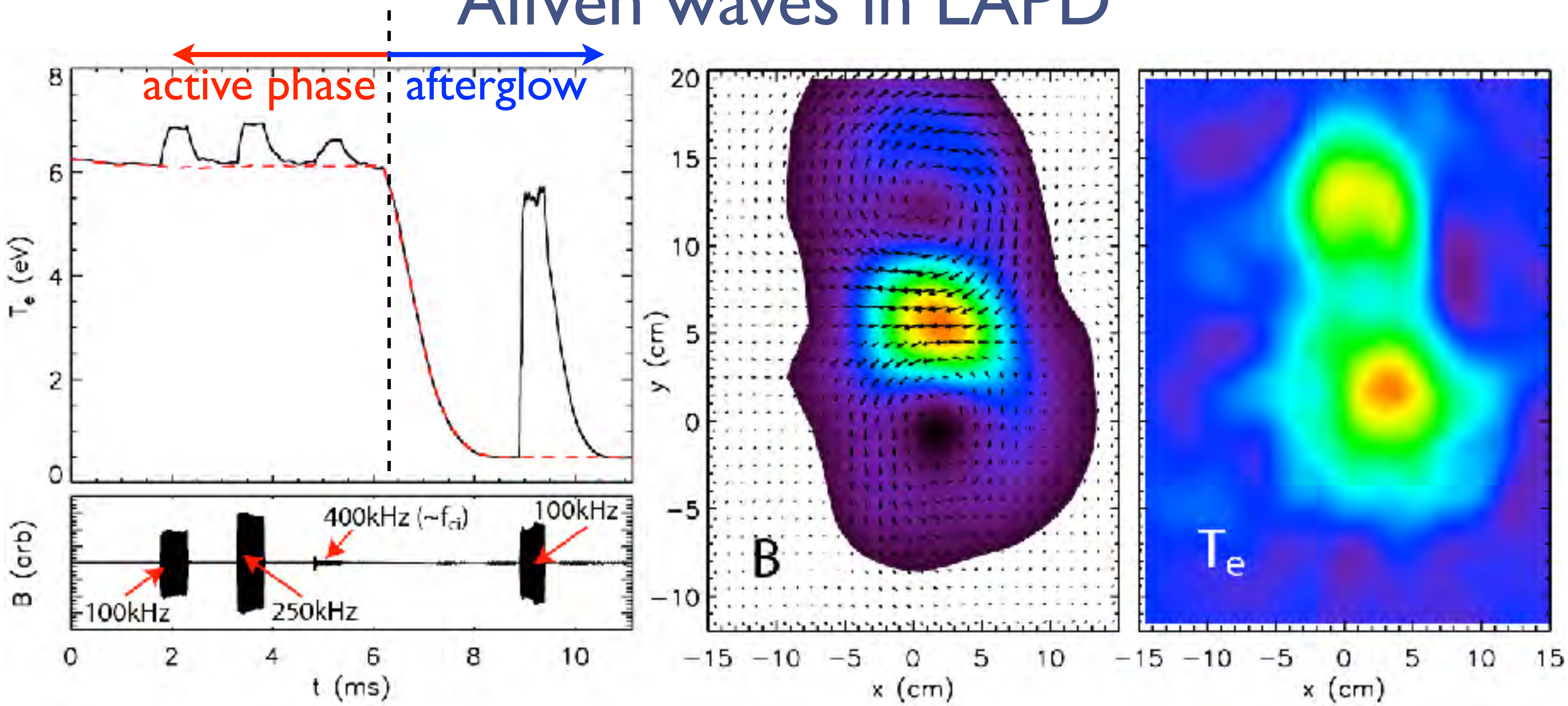


I ~ 1kA, V ~ 1kV



- Resonant cavity (MASER, narrowband), loop antenna (wideband)
- Both can generate AWs with $\delta B/B \sim 1\%$ (~10G or 1mT); large amplitude from several points of view:
  - Wave beta is of order unity $$\beta_w = \frac{2\mu_o p}{\langle \delta B^2 \rangle} \approx 1$$
  - Wave Poynting flux ~ 200 kW/m^2, same as discharge heating power density
  - From GS theory: stronger nonlinearity for anisotropic waves; here $k_{||}/k_{\perp} \sim \delta B/B$

Strong electron heating by large amplitude Alfvén waves in LAPD



- Localized heating observed, on wave current channel (collisional and Landau damping: Note damping length is comparable to machine length!)
- Results in structuring of plasma (additionally see parallel outflows, density, potential modification, cross-field flows)

Three-wave interactions with two "pump" Alfvén waves

- Three-wave matching conditions must be satisfied (arise from quadratic nonlinearities (e.g. ∇B^2))

$$\omega_1 + \omega_2 = \omega_3$$
$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3$$

- For three IDEAL AWs (MHD cascade interaction), must have counter-propagating waves with the third "wave" having $k_\parallel = 0$ (leads to perpendicular cascade)

- This constraint is removed if we allow for different third mode (e.g. sound wave) and/or include dispersion (KAW, IAW): e.g. co-propagating interaction allowed

  - In LAPD experiments, waves have $k_\perp \rho_s \sim 1$, $\omega/\Omega_i \sim 1$: dispersive kinetic or inertial Alfvén waves

  - Co-propagating interaction allowed (waves can pass through one another)

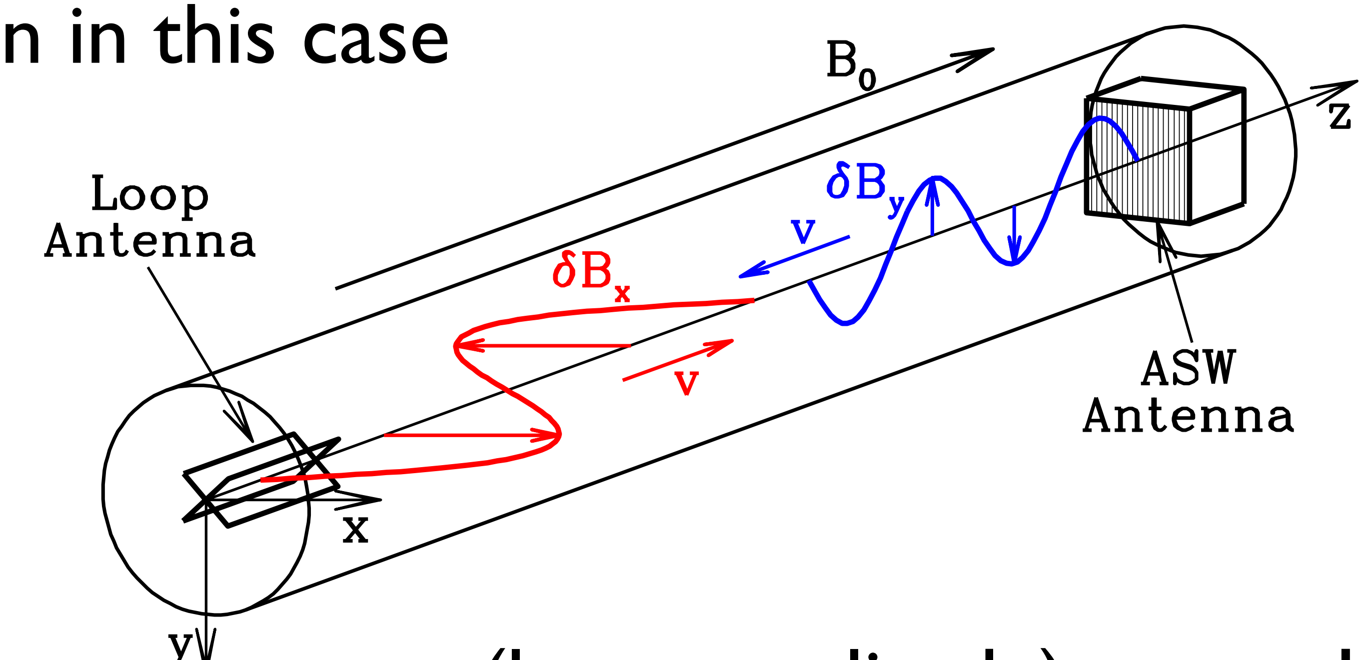
  - Decay instabilities possible (parametric, modulational)

  - LAPD experiments with dispersive KAW/IAW

MHD-cascade relevant collisions: AW+AW → AW

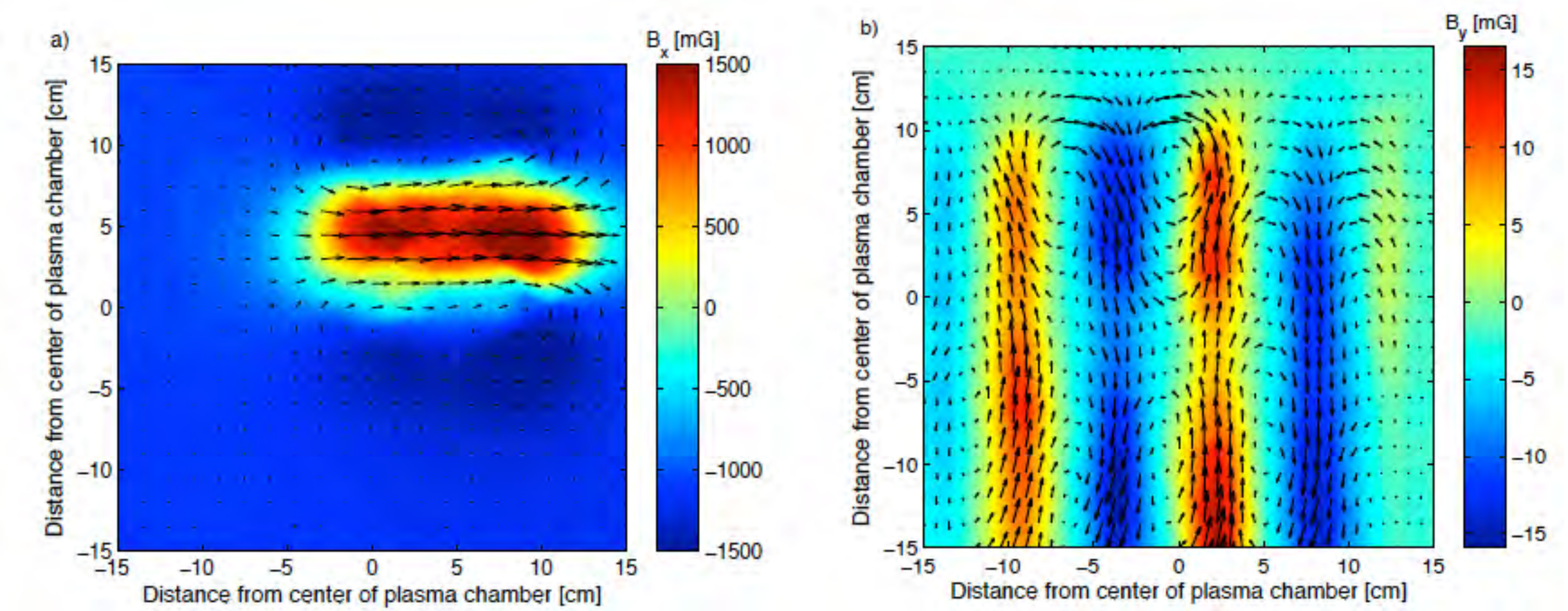
- Initial attempts in LAPD (Carter, Boldyrev, et al.): no strong evidence for daughter wave production/cascade (instead see beat waves, heating, harmonic generation, etc). Used local interaction, trying to look for perp. cascade.

- New idea (Howes): have one of the two interacting (pump) waves be $k_\parallel \approx 0$, theoretical prediction for stronger NL interaction in this case



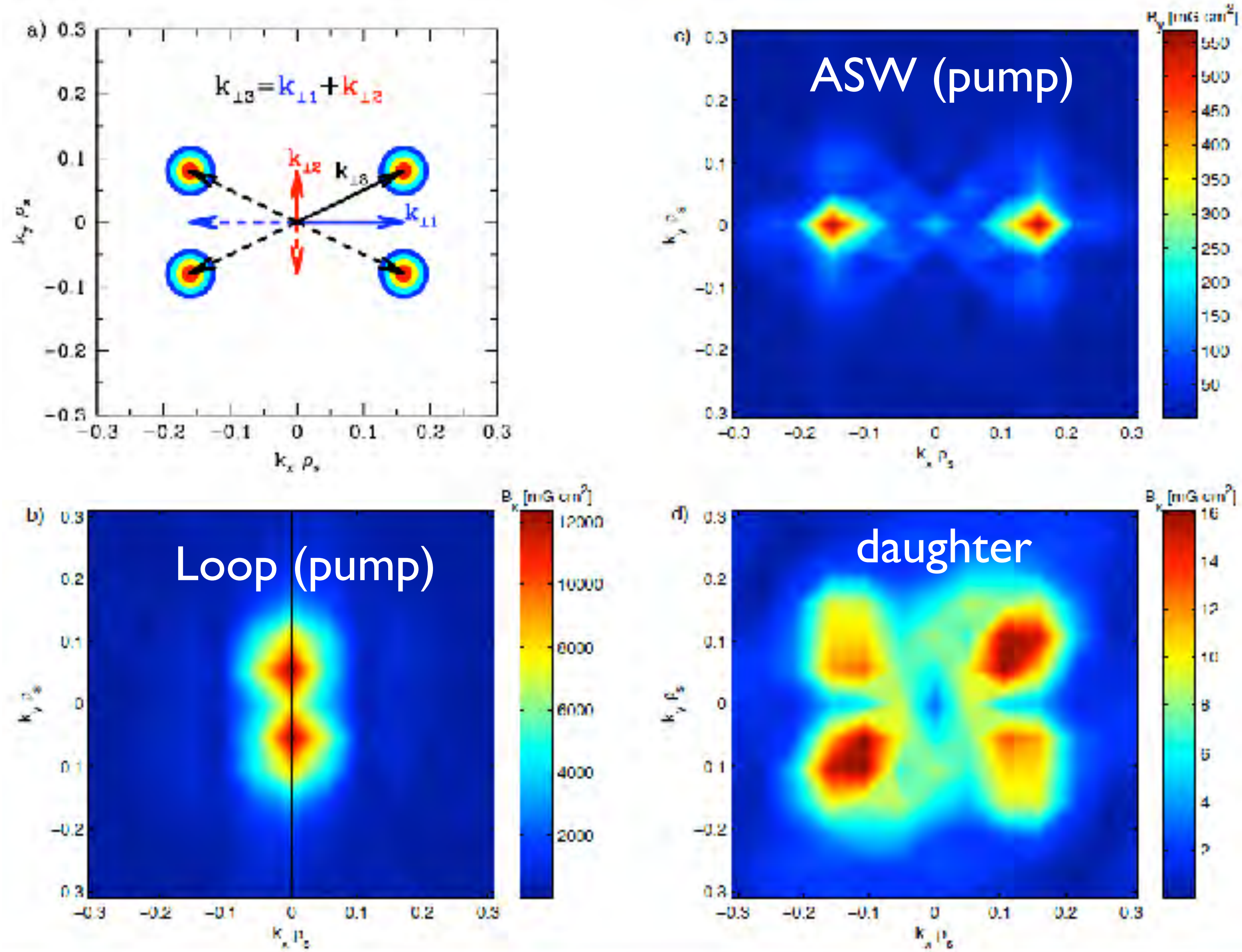
- UCLA Loop antenna (large amplitude) versus U. Iowa ASW antenna (small amplitude but precise $k_\perp$ control)

Interaction maximized, sensitivity to daughter wave enhanced through linearly polarized pumps



- Loop antenna: B_x only, low frequency wave (60 kHz), ~1.5G amplitude
- ASW antenna: B_y only, 270kHz ($f/f_{ci} \approx 0.5$, picked to avoid harmonics of loop antenna), ~15mG amplitude
- Cross-polarization maximizes interaction; look for generation of B_x fluctuations at 270kHz

First laboratory observation of daughter AW production: consistent with weak turbulence theory

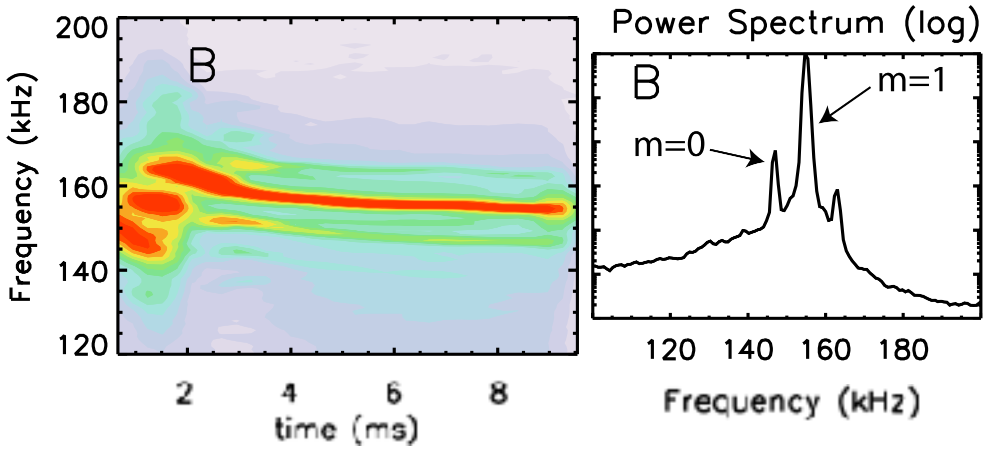


Howes et al., PRL 109, 255001 (2012)

- Perpendicular wavenumber spectrum consistent with three-wave matching ($k_1 + k_2 = k_3$)

First observation of three wave interaction in LAPD: production of quasimodes by co-propagating AWs

- Spontaneous multimode emission by the cavity is often observed, e.g. m=0 and m=1



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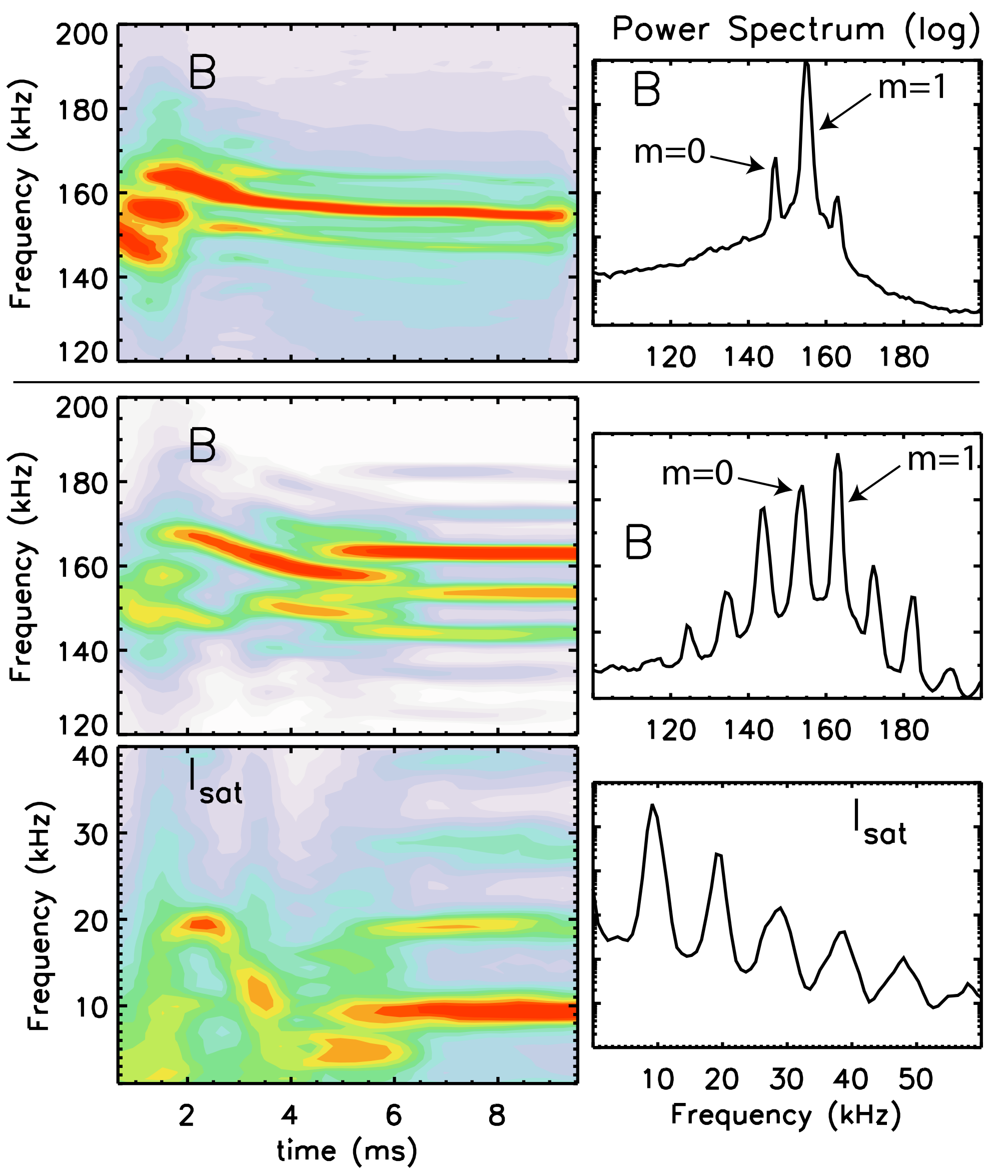
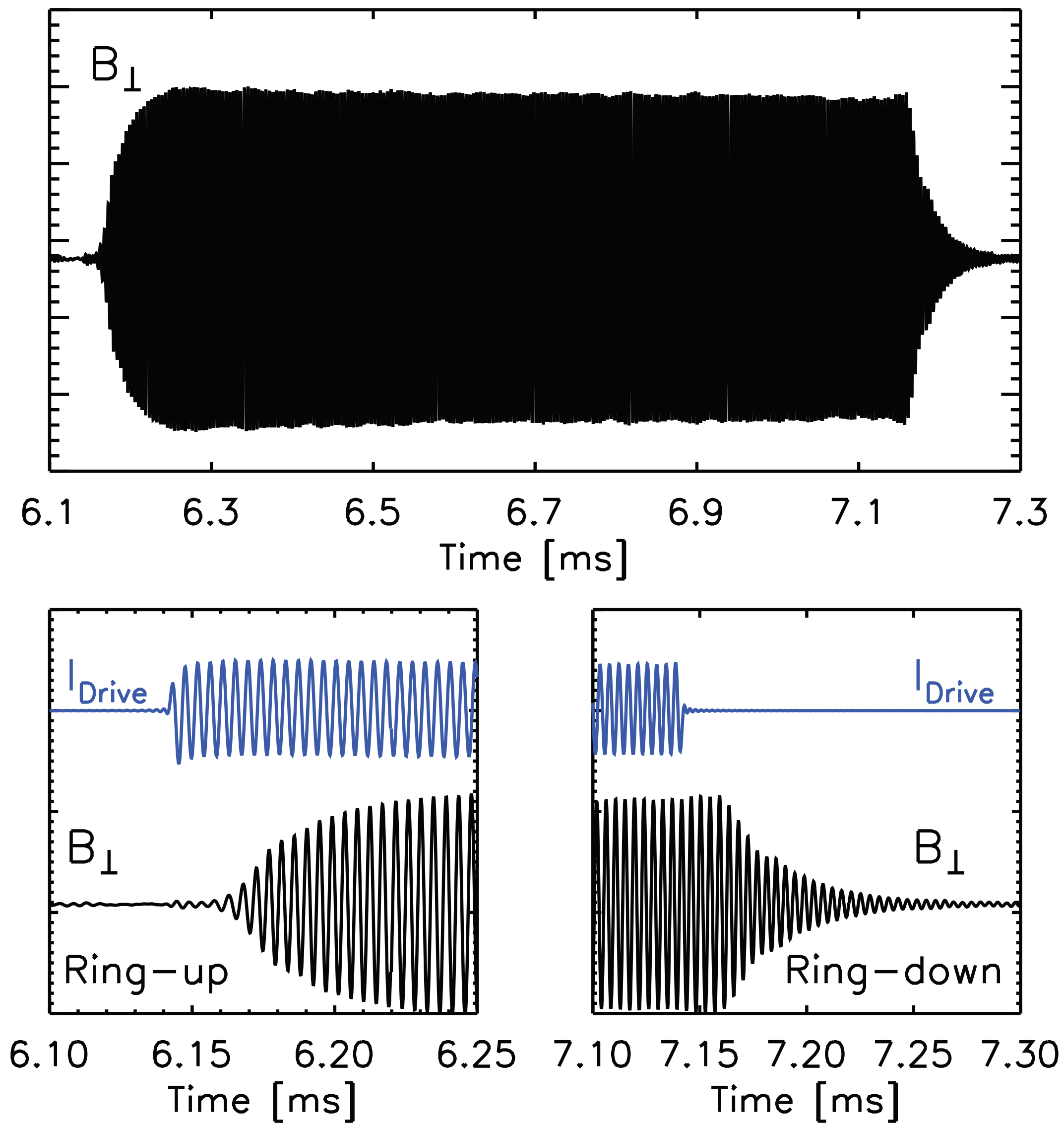


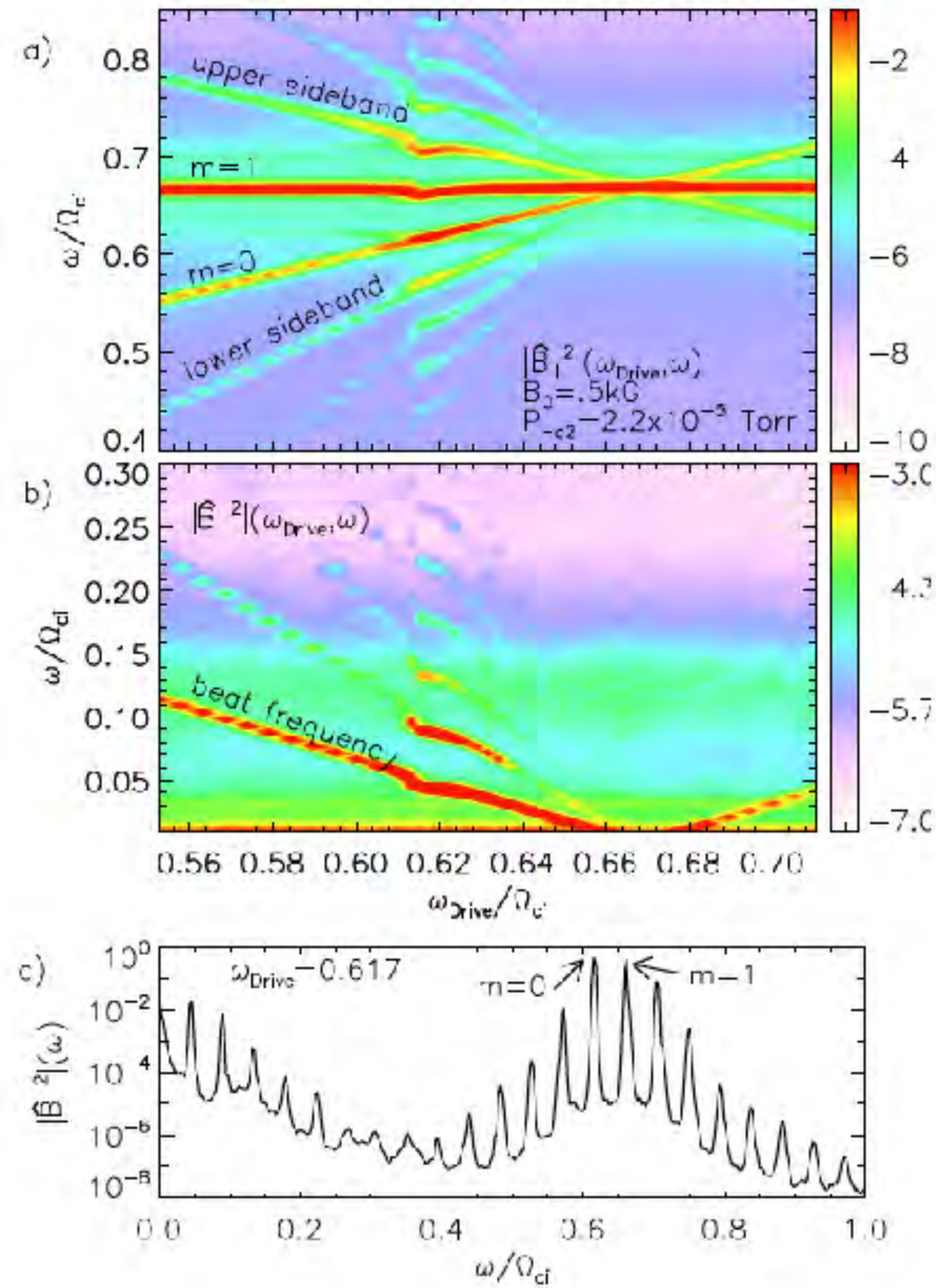
- Can control multimode emission (e.g. current, shortening the plasma column)
- With two strong primary waves, observe beat driven quasimode which scatters pump waves, generating sidebands
- Strong interaction: "pump" $\delta B/B \sim 1\%$, QM $\delta n/n \sim 10\%$

T.A. Carter, B. Brugman, et al., PRL 96, 155001 (2006)

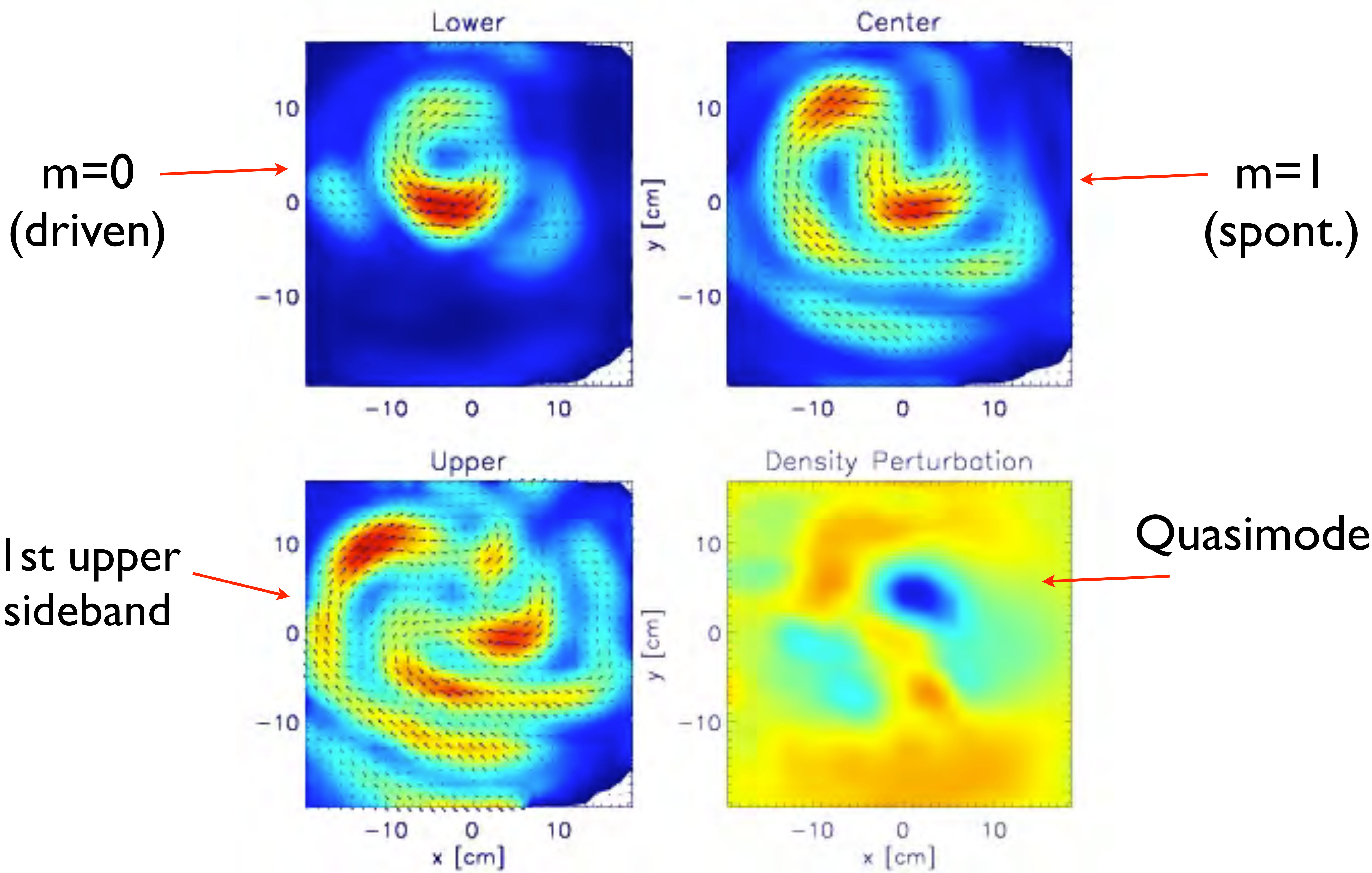
Driven cavity, antenna launched waves used to study properties of interaction



Driven cavity: can produce QMs with range of beat frequencies (limited by width of cavity resonance for driven m=0)



Structure of interacting modes



Beat driven wave is off-resonance Alfvén wave; theory consistent with observed amplitude, resonant behavior

- Nonlinear Braginskii fluid theory, $k_\perp >> k_{||}$, $\omega/\Omega_{ci} \sim 1$

$$\frac{\delta n}{n_o} = \frac{\delta k_\perp v_A}{\Omega_{ci}} \frac{k_{\|,1} v_A}{\Omega_{ci}} \frac{k_{\|,2} v_A}{\Omega_{ci}} \frac{\left(\frac{(\delta k_\perp + 2k_{\perp,1})\, v_A}{\Omega_{ci}} \left(1 + 2\frac{\Omega_{ci}}{\delta\omega}\right) - \frac{\delta k_\perp v_A}{\Omega_{ci}} \right)}{\left(1 - \left(\frac{\delta\omega}{\delta k_{\|} v_A}\right)^2\right)} \left[\frac{B_1^* B_2}{B_o^2}\right]$$

- Exhibits resonant behavior (for Alfvénic beat wave) - reasonable agreement with experiments

- Ignoring resonant demoninator, $\delta n/n \sim$ 1-2% for LAPD parameters

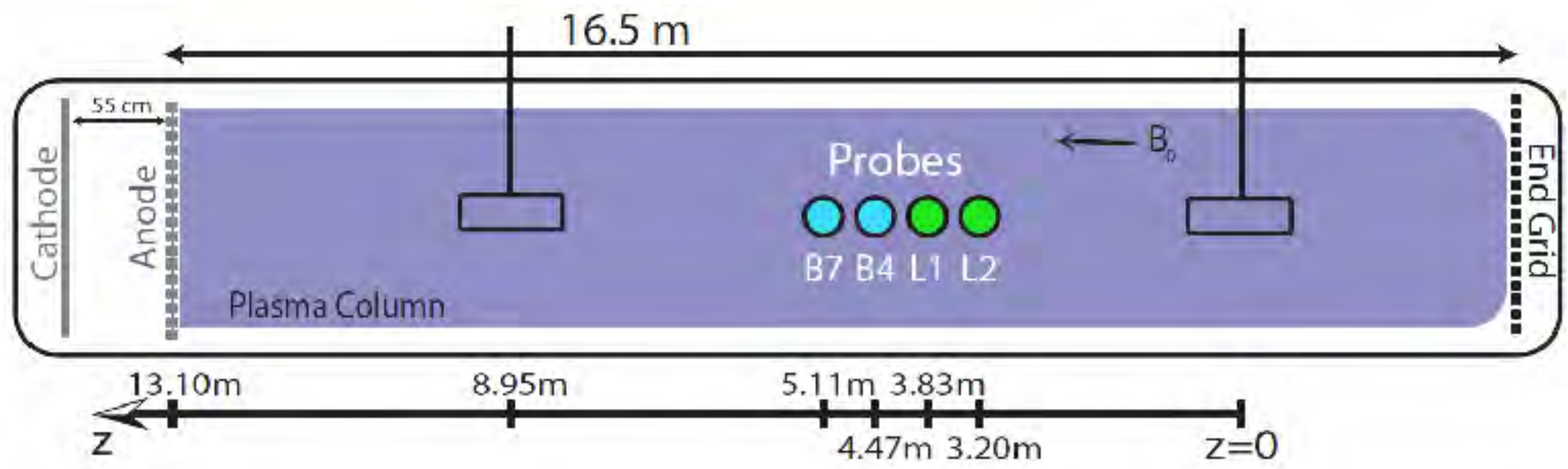
- Dominant nonlinear forcing is perpendicular (NL polarization drift): easier to move ions across the field to generate density response due to $k_\perp >> k_{||}$

Nonlinear excitation of sound waves by Alfvén waves

- Parametric decay instability: decay of large amplitude AW to sound wave and backward-propagating AW
  - Might be important in solar wind (how do you generate counter-propagating AW spectrum starting with AWs propagating from the sun?) and fusion plasmas (ICRF)
- In LAPD, decay growth rate slower than AW transit time (hard to see without larger amplitude, but we are looking)
- Instead, study three-wave interaction at heart of the instability: two counter-propagating AWs which beat together to drive a sound wave

Nonlinear excitation of sound waves by AWs

- Study three-wave process at heart of parametric decay by interacting two frequency-detuned, counter-propagating AWs



[Dorfman & Carter, PRL 110, 195001 (2013)]

Nonlinear excitation of sound waves by AWs

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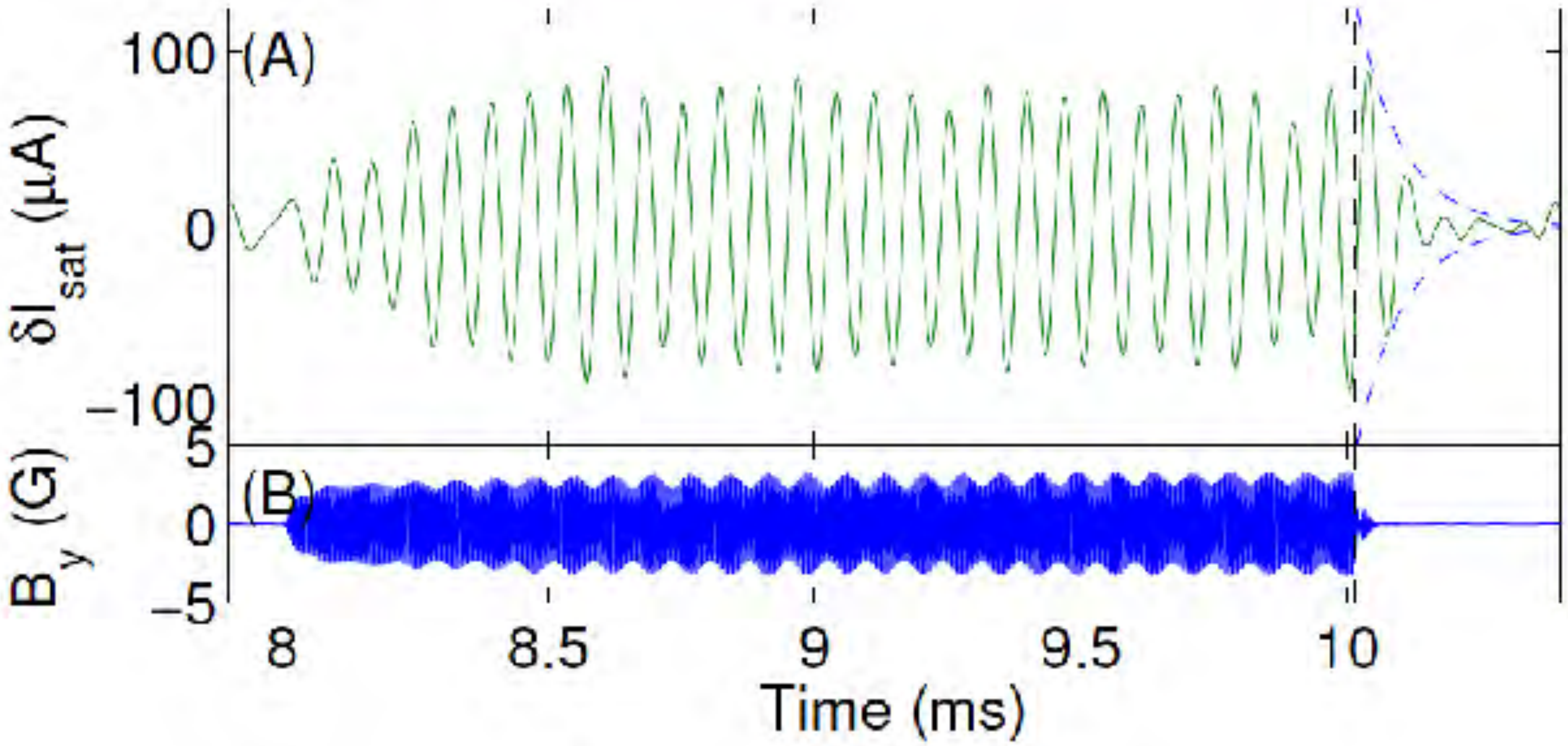
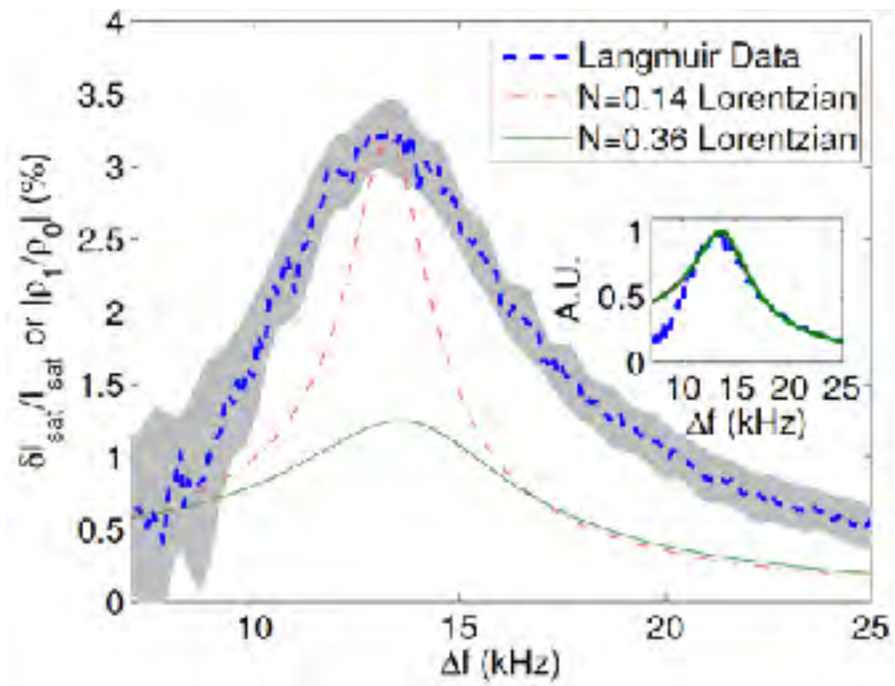
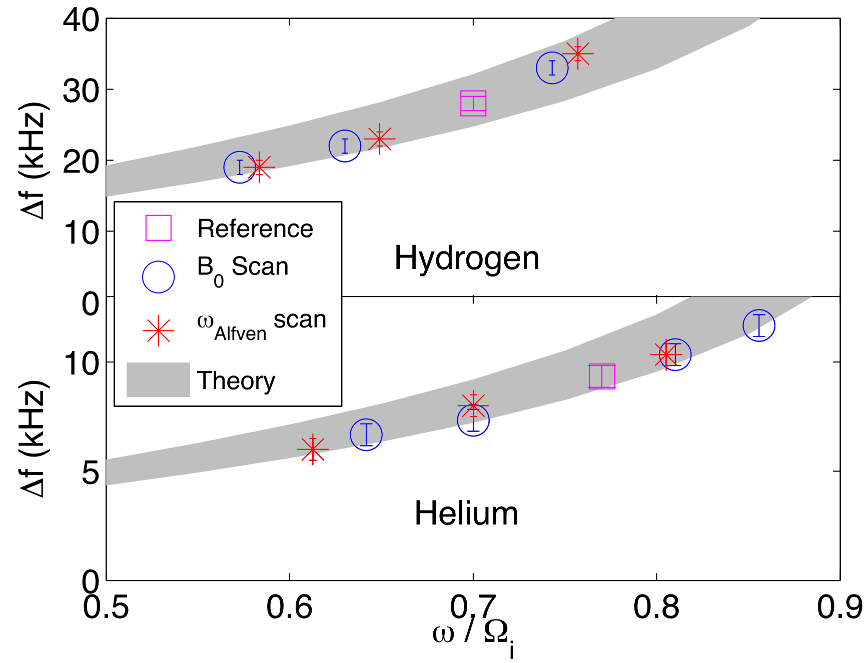


- Nonlinear response at beat frequency observed; response persists after nonlinear drive is turned off: evidence for excitation of damped linear wave

[Dorfman & Carter, PRL 110, 195001 (2013)]

Resonant response observed; consistent with simple model of nonlinear sound wave drive, though damping not fully explained





- Beat-wave response peaks at beat frequency consistent with simple fluid model (three-wave matching AW + AW → IAW)

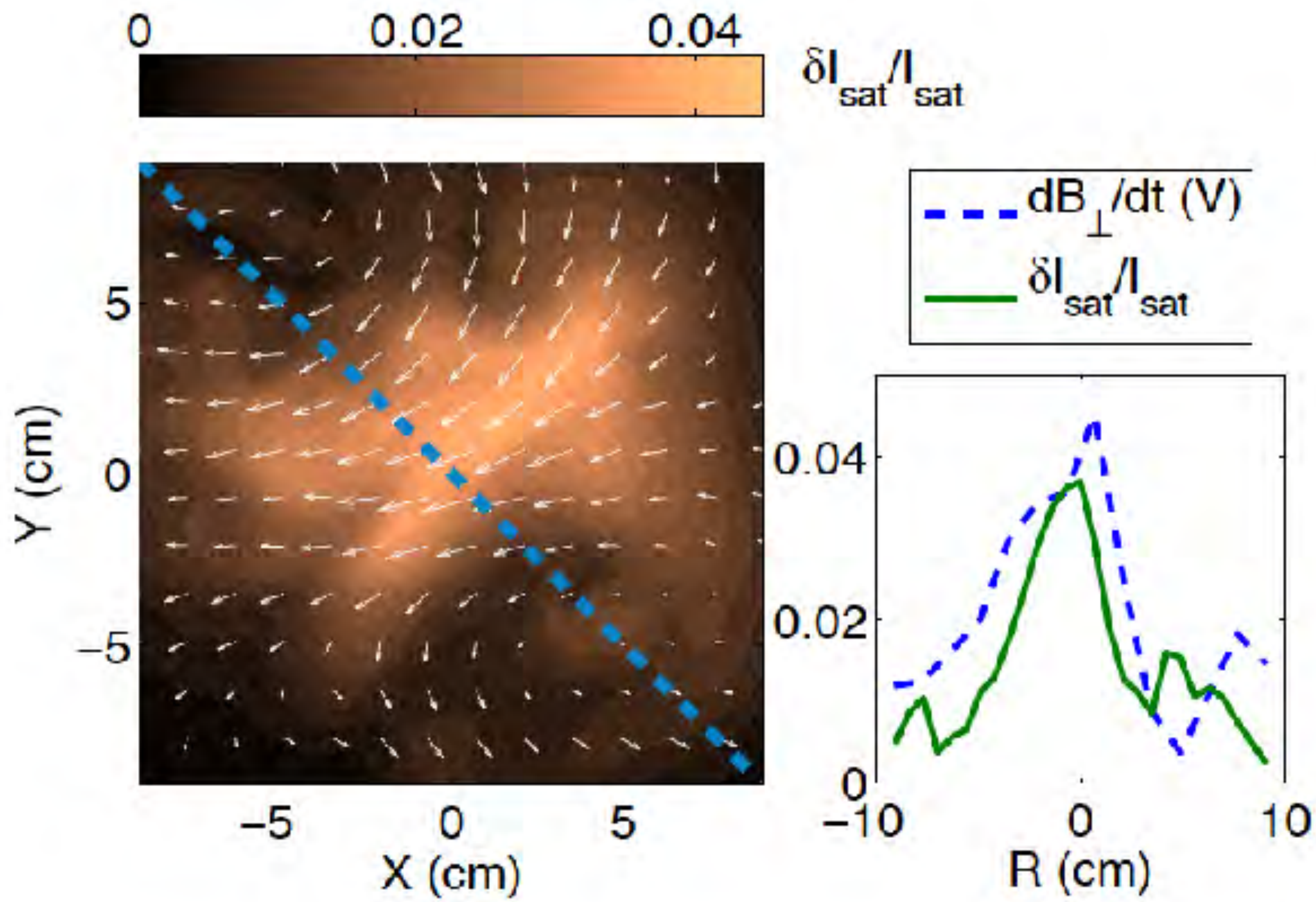
$$\Delta\omega = \frac{2\omega\sqrt{\beta}}{\sqrt{1+(k_\perp\rho_s)^2-\left(\frac{\omega}{\Omega_i}\right)^2}}$$

$$\frac{\partial^2\rho}{\partial t^2} + \nu\frac{\partial\rho}{\partial t} - C_s{}^2\frac{\partial^2\rho}{\partial z^2} = \frac{\partial^2}{\partial z^2}\left[\frac{b_{\perp 1}\cdot b_{\perp 2}}{4\pi}\right]$$

$$\left|\frac{\rho_1}{\rho_D}\right| = \frac{1}{\sqrt{\left(1-\Omega_D{}^2\right)^2+N^2\Omega_D{}^2}}$$

- Amplitude of peak predicted by theory (damping via ion-neutral collisions), but width not matched

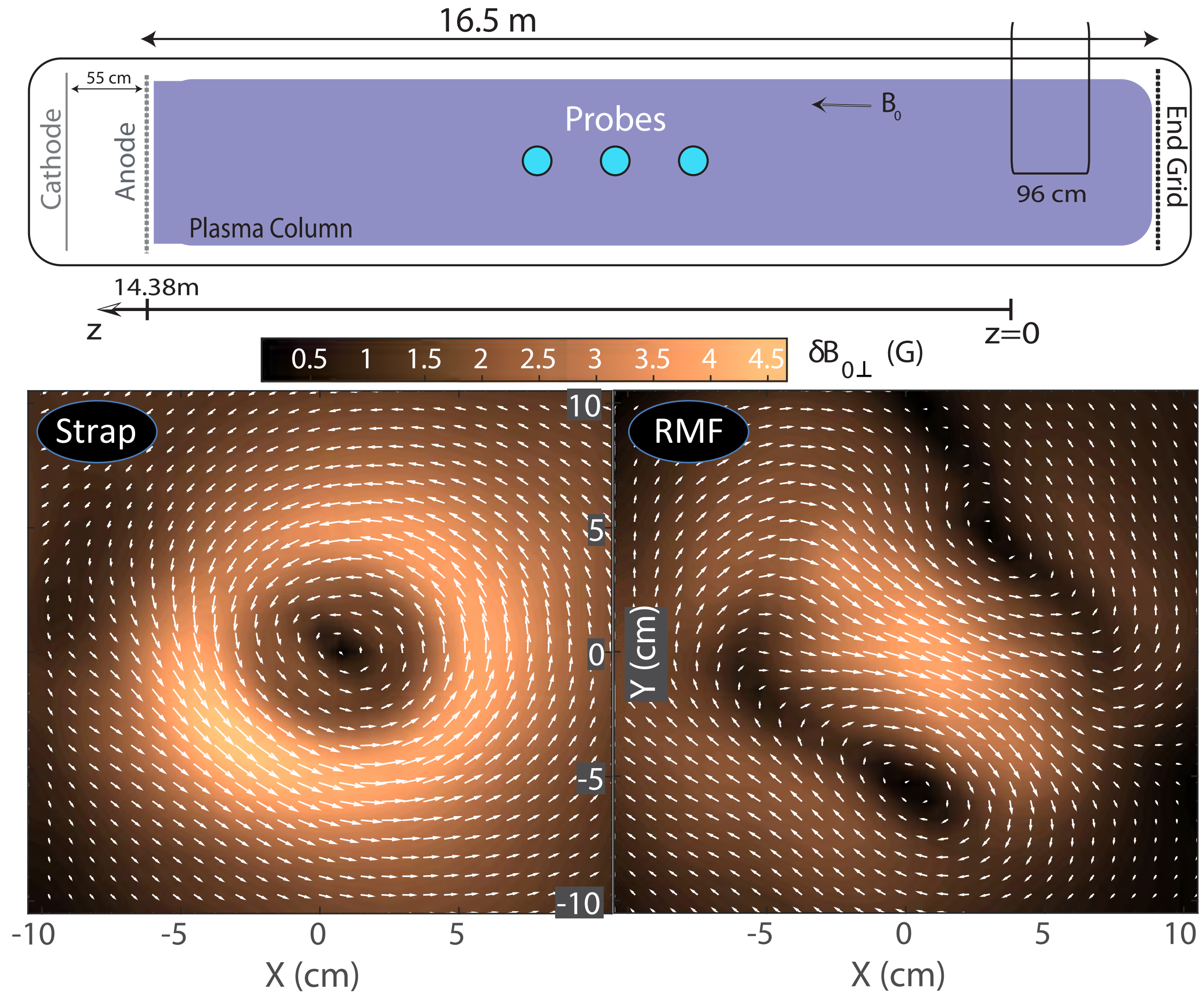
Spatial pattern of driven wave consistent with parallel ponderomotive drive



- Driven mode peaks near spatial maximum of magnetic field fluctuation of beating Alfvén waves

Observation of a parametric instability of KAWs

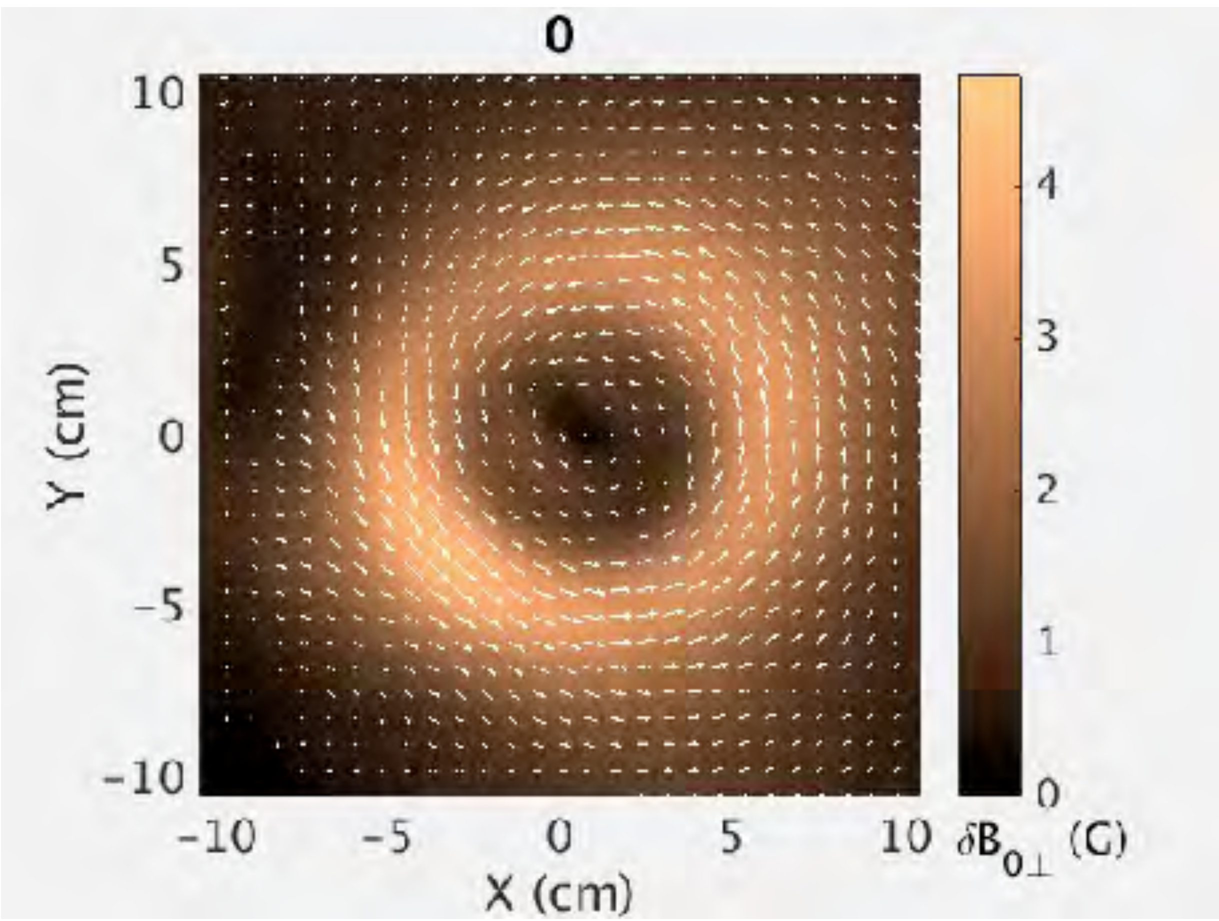
Pump wave spatial patterns (two different kinds of antennas)

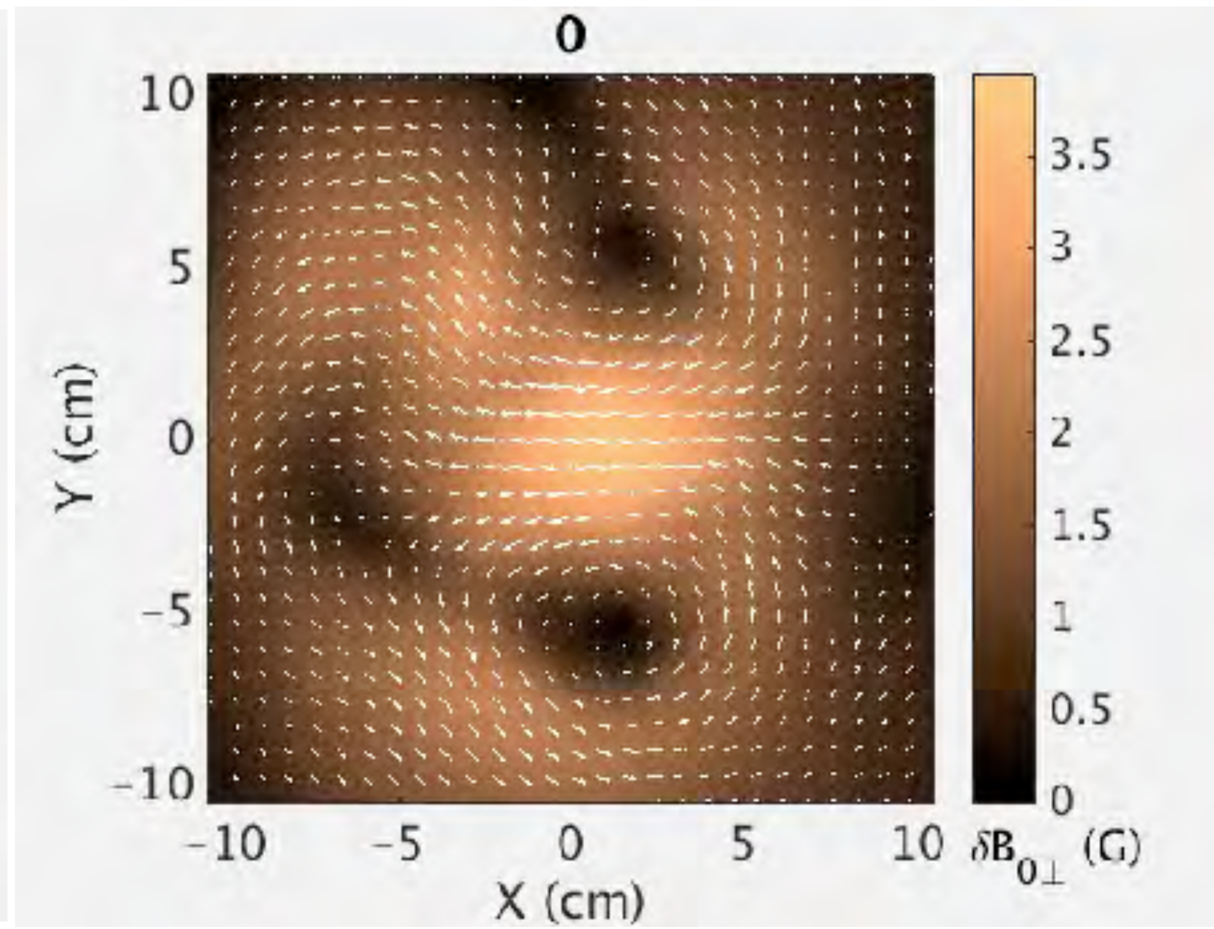


- Single, large amplitude KAW launched. Above an amplitude threshold and frequency, observe production of daughter modes.

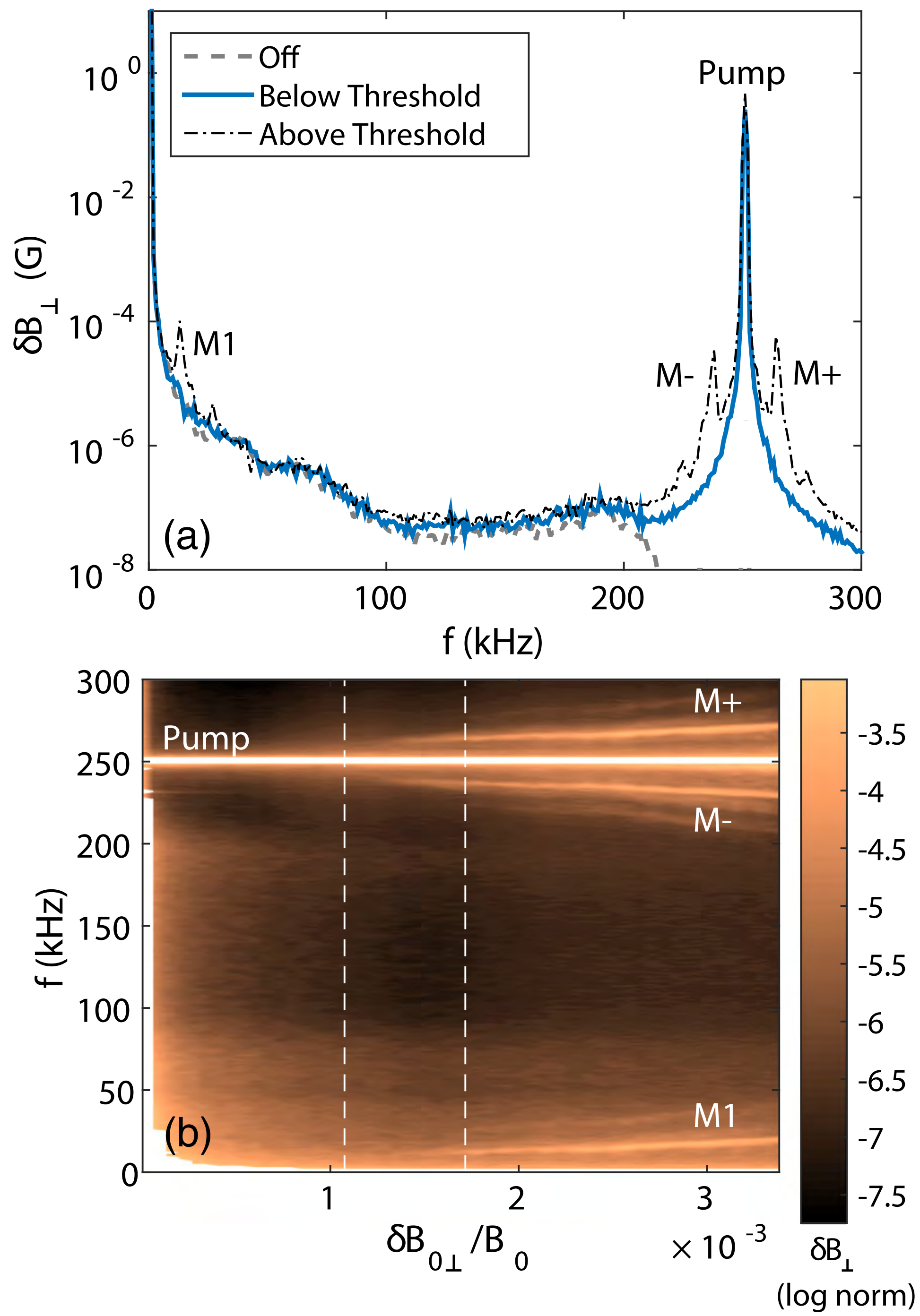
[Dorfman & Carter, PRL, 116, 195002 (2016)]

Pump waves: linearly and circularly polarized



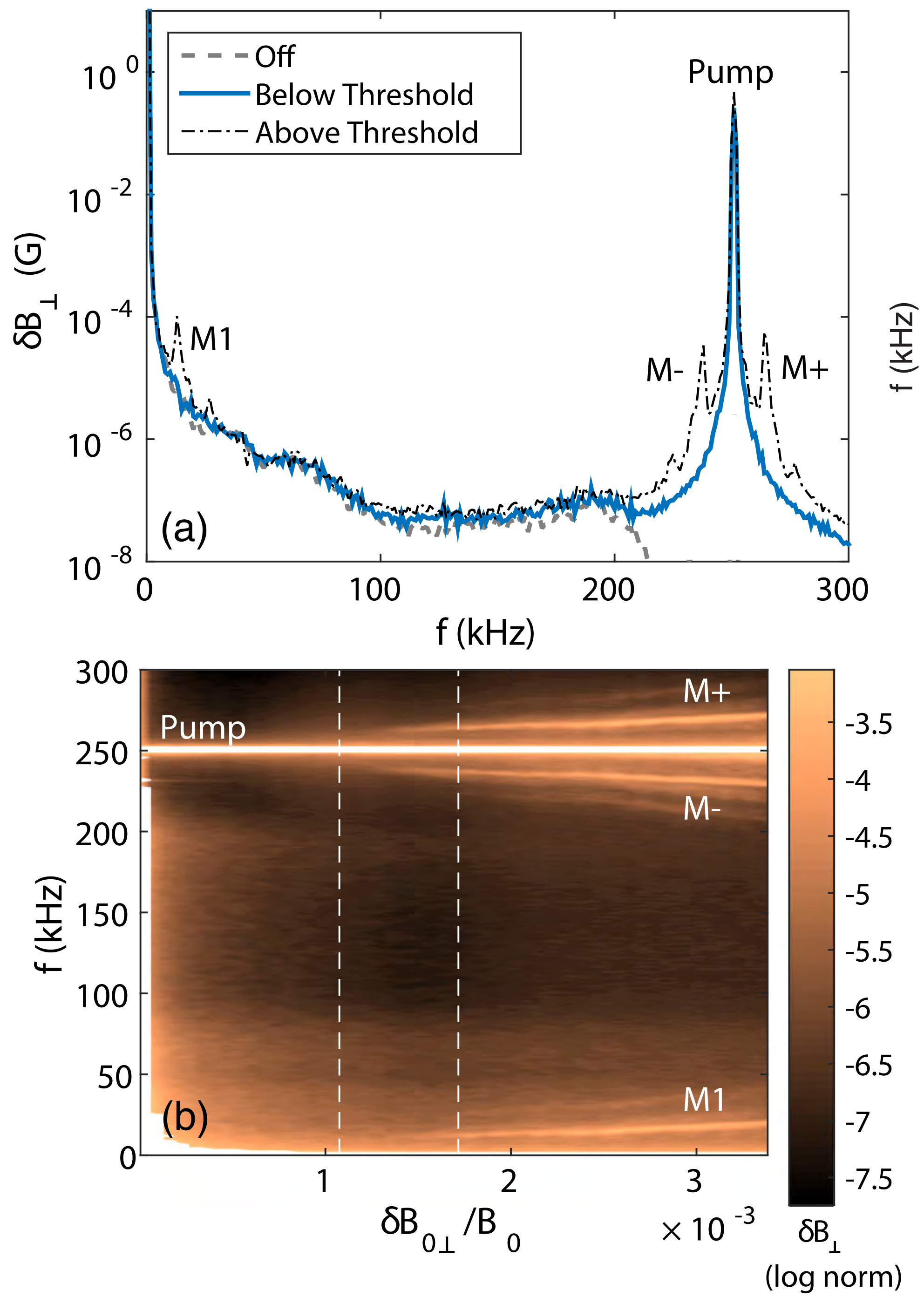


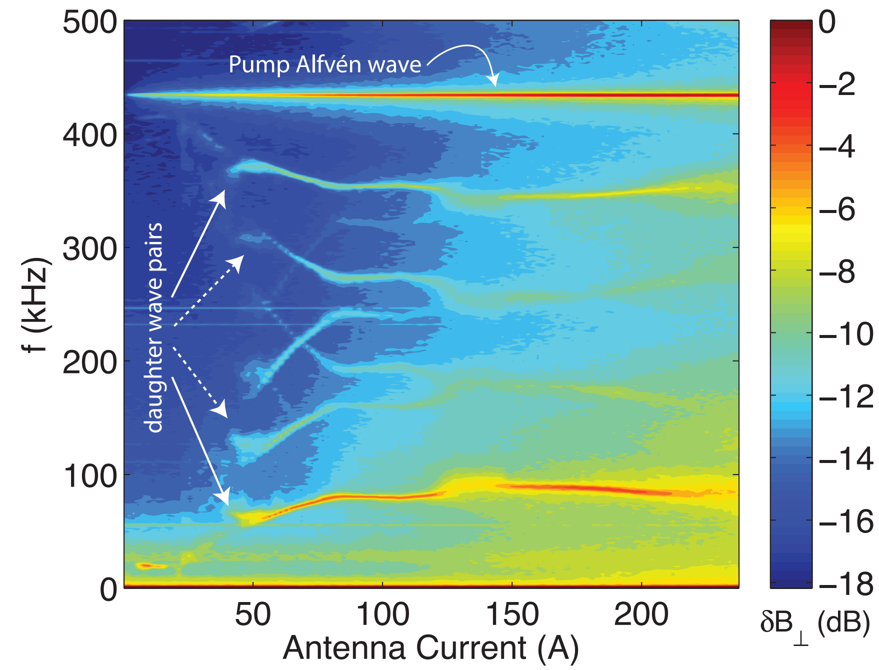
Production of sidebands and low frequency mode



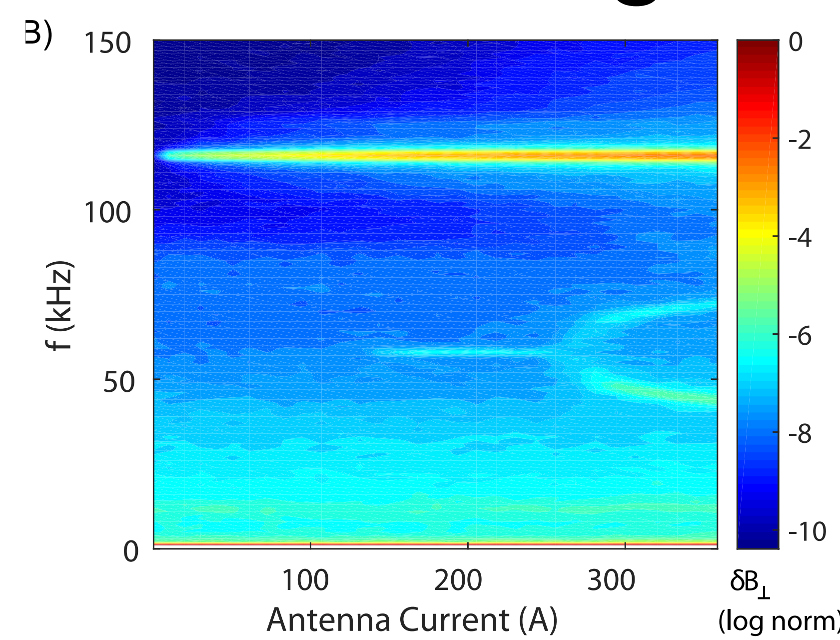
- Production of daughter waves observed: threshold both in wave amplitude and in frequency (only observed for $f \gtrsim 0.5\ f_{ci}$)
- All three daughter waves co-propagating with pump (need dispersive AWs)
  - Modes satisfy three-wave matching rules

Production of sidebands and low frequency mode

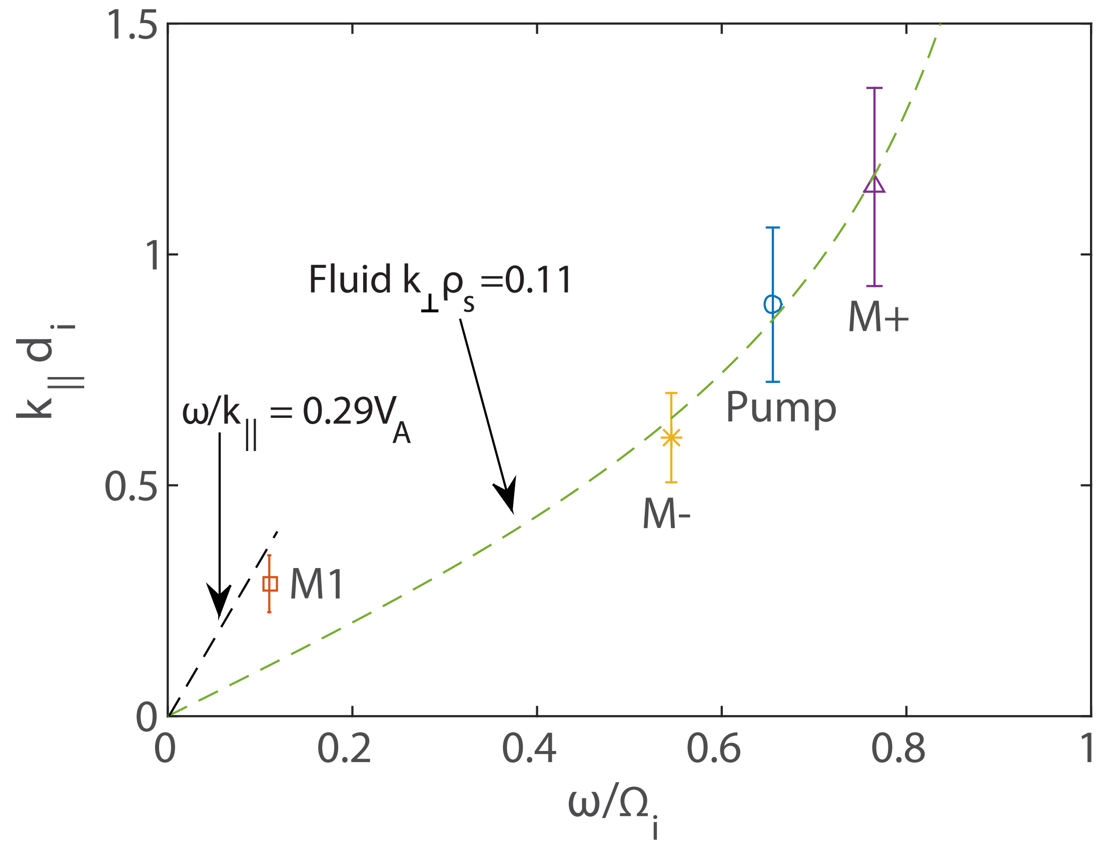


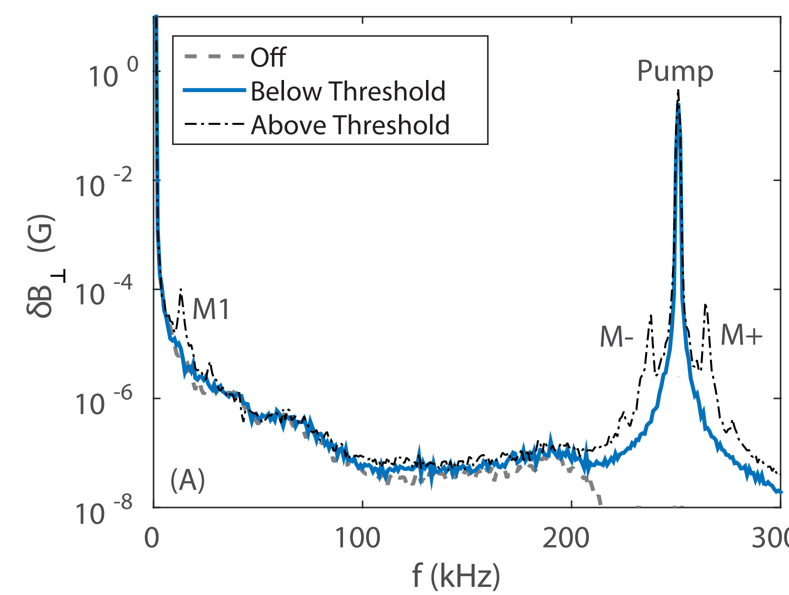


Variety of behaviors observed as plasma parameters are changed



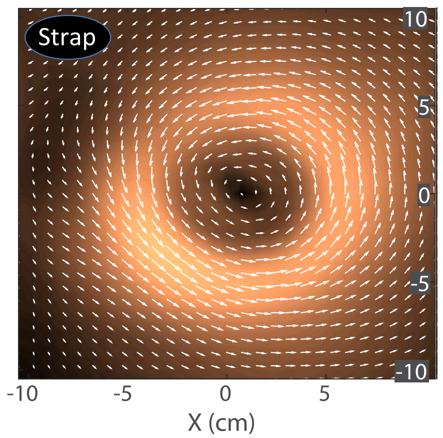
Sidebands are KAWs, low frequency mode is quasimode



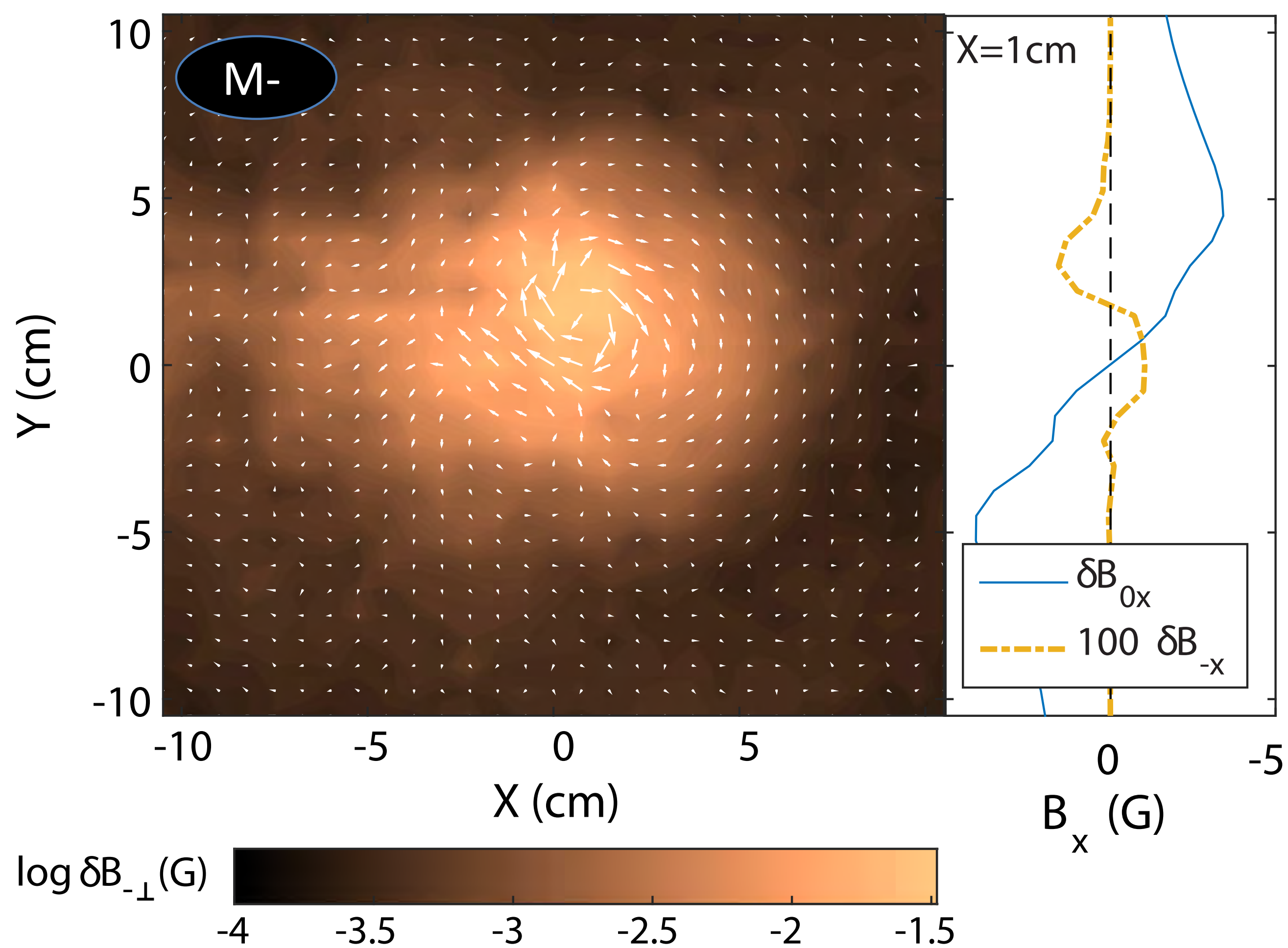


- Sideband waves are consistent with KAW dispersion relation
- Low frequency mode is a non-resonant mode/quasimode: phase speed inconsistent with sound wave or KAW
- Participant modes consistent with **modulational decay instability**

Daughter quasimode located on pump current channel, inconsistent with parallel ponderomotive drive



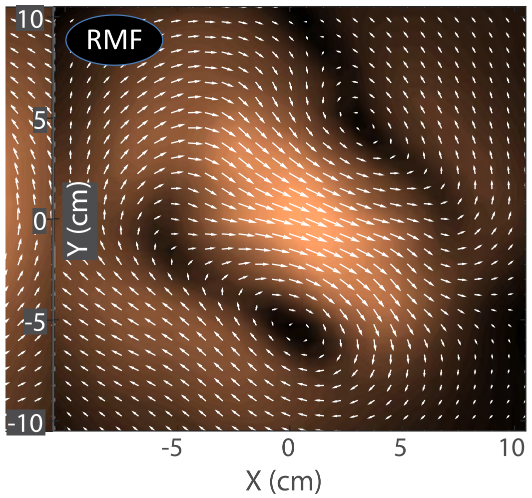
Pump



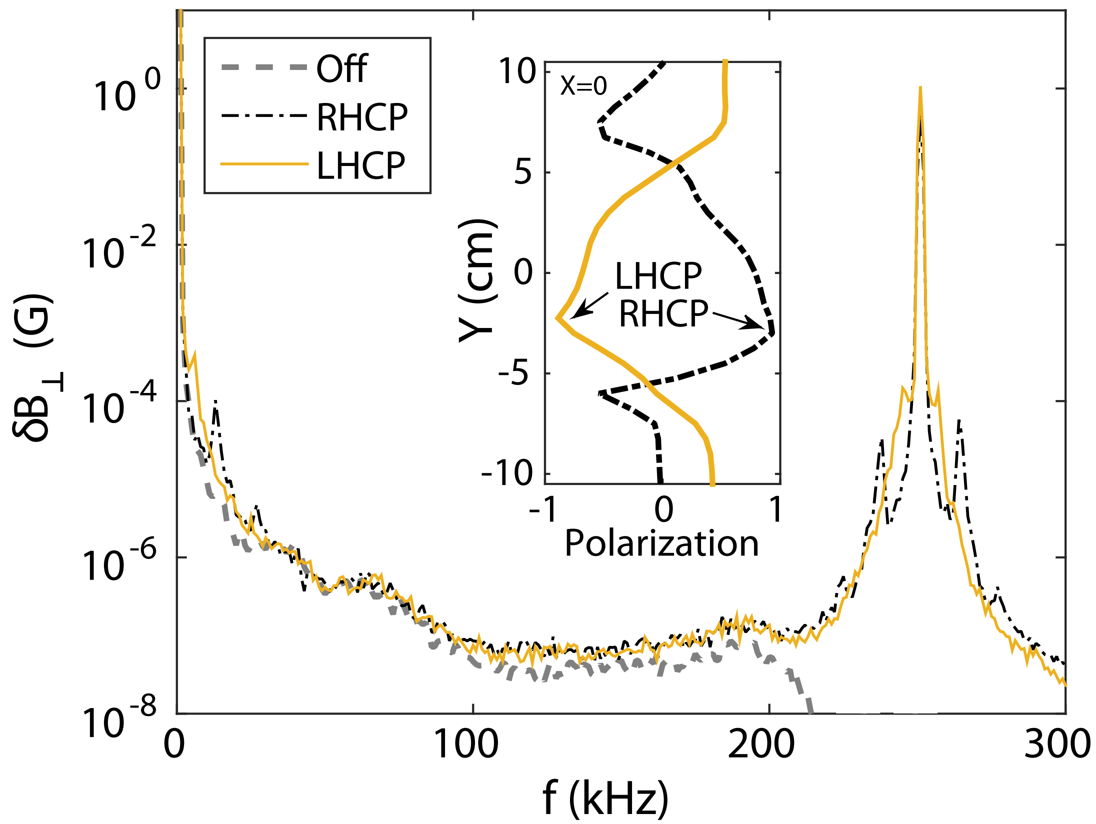
daughter

- Perpendicular nonlinearity? Importance of $k_\perp$ of pump, daughters

Parametric instability changes with pump polarization

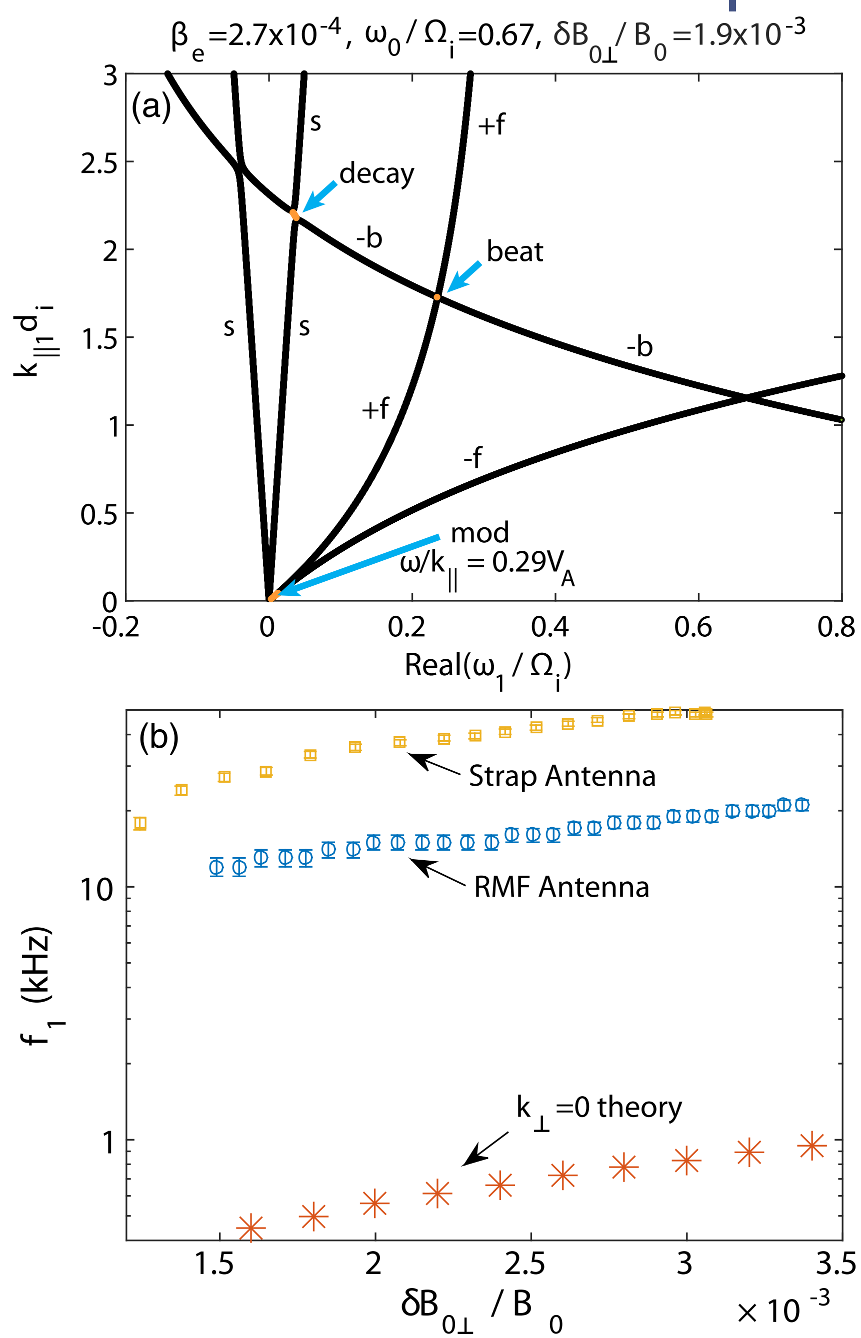


Pump



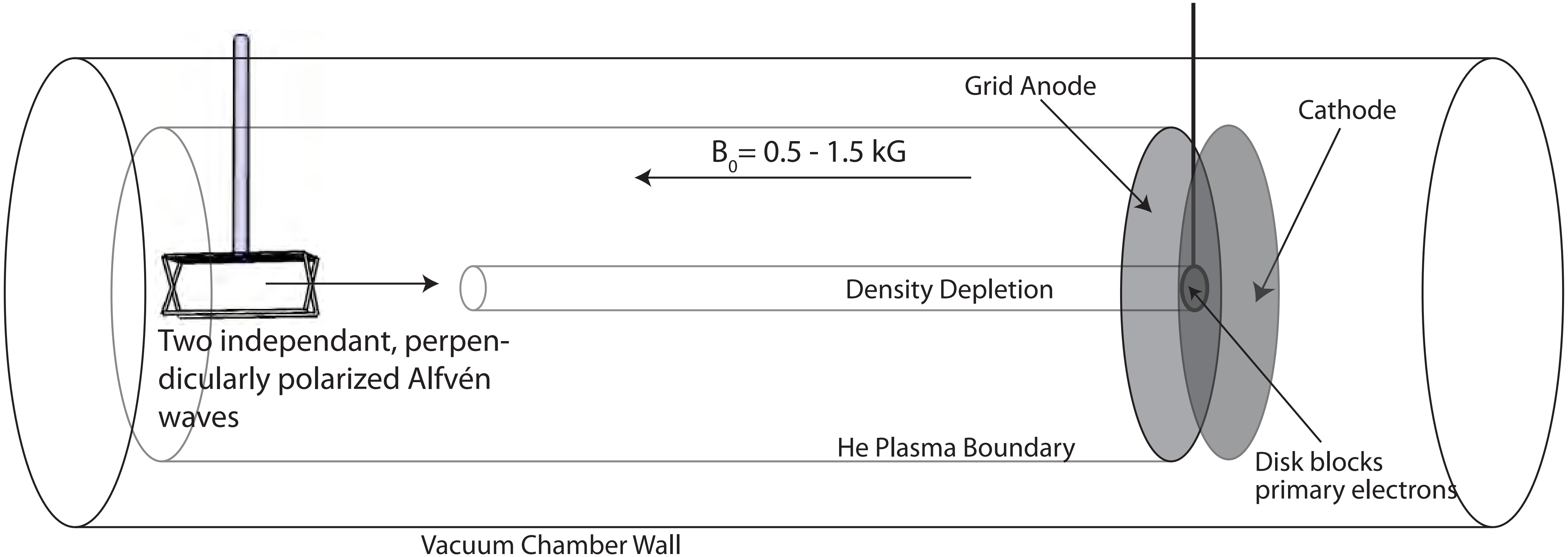
- Change in daughter frequency/amplitude with change from dominant LHCP to RHCP

Theory: qualitatively consistent with $k_\perp = 0$ modulation decay theory (with important quantitative differences)



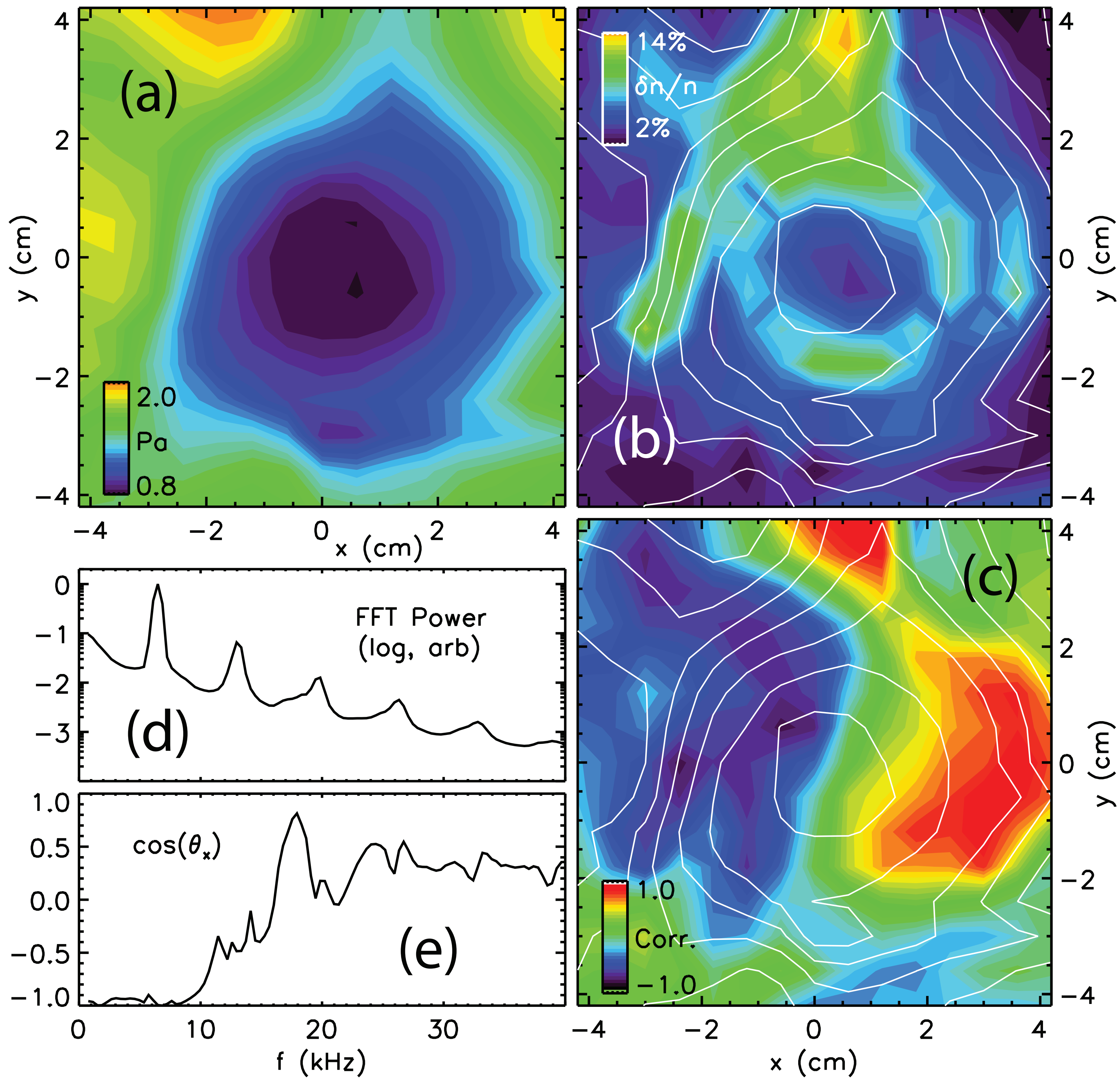
- Theory for $k_\perp$=0 parametric instabilities (Wong & Goldstein; Hollweg) solved for LAPD parameters
- Modulational decay instability predicted to be unstable with consistent phase velocity for M1 (low frequency daughter)
- Mode frequency and growth rate too low for experiment, but scales consistently with amplitude (importance of finite $k_\perp$?)
- **Parametric decay (sound wave production) predicted to have higher growth rate but we have not observed it!**

Exciting/controlling drift waves via beating AWs



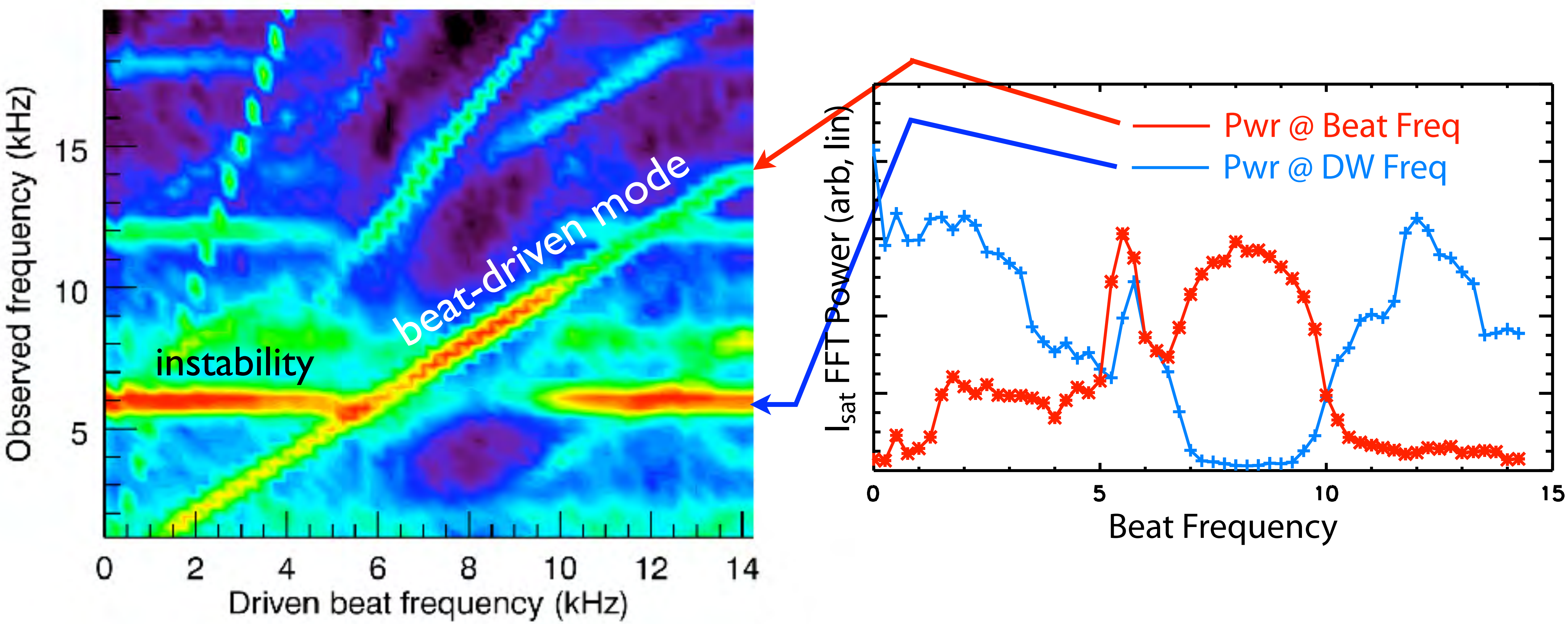
- Density depletion formed by inserting blocking disk into anode-cathode region, blocking primary electrons therefore limiting plasma production in its shadow
- Instability grows on periphery of striation/depletion (drift-Alfvén waves studied in depth [Burke, Peñano, Maggs, Morales, Pace, Shi…])
- Launch KAWs into depletion, look for interaction

Unstable fluctuations observed on depletion



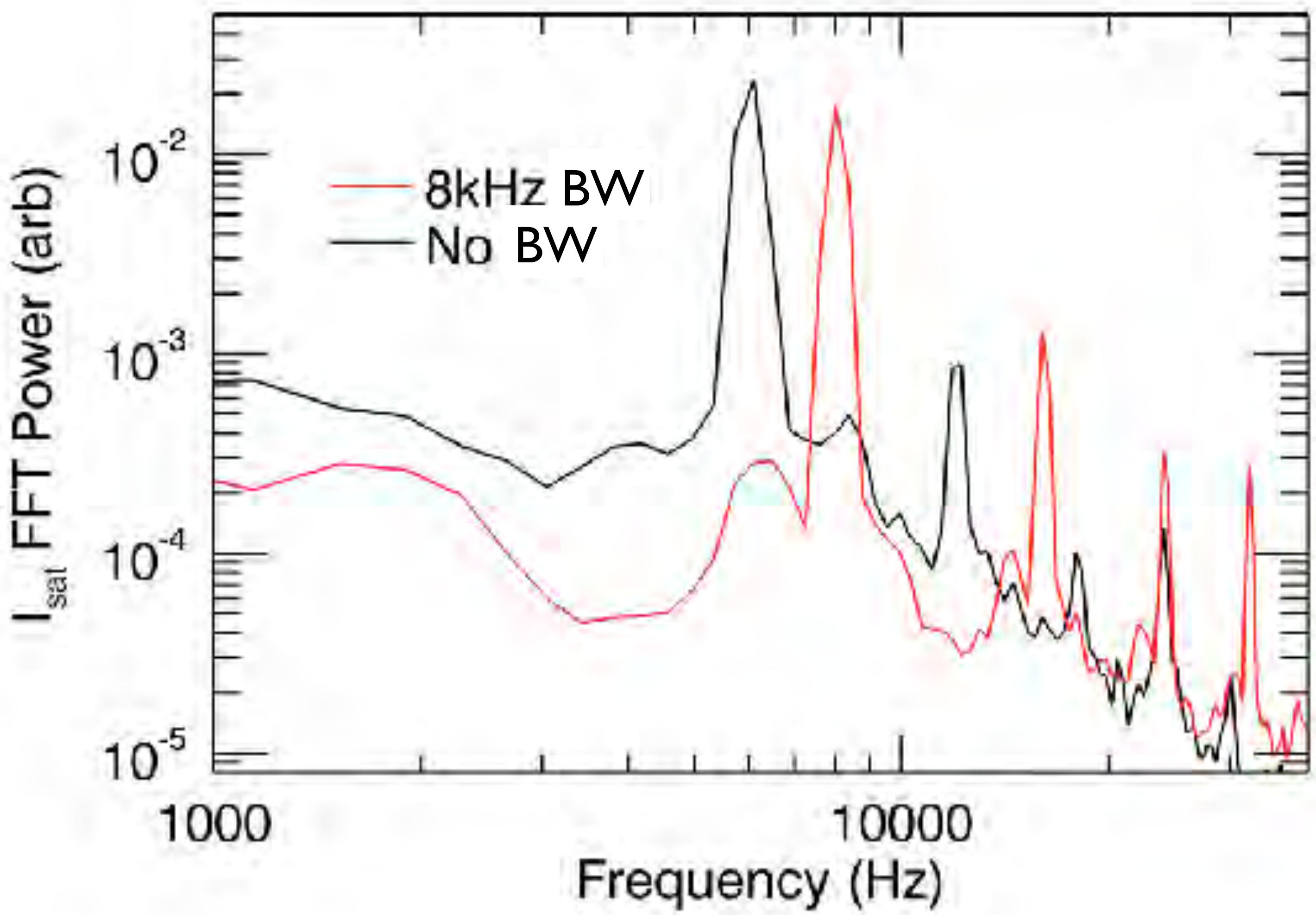
- m=1 coherent fluctuation observed localized to pressure gradient
- Sheared cross-field flow also present in filament edge: Drift-wave instability modified by shear (coupling to KH)

Resonant drive and mode-selection/suppression of instability



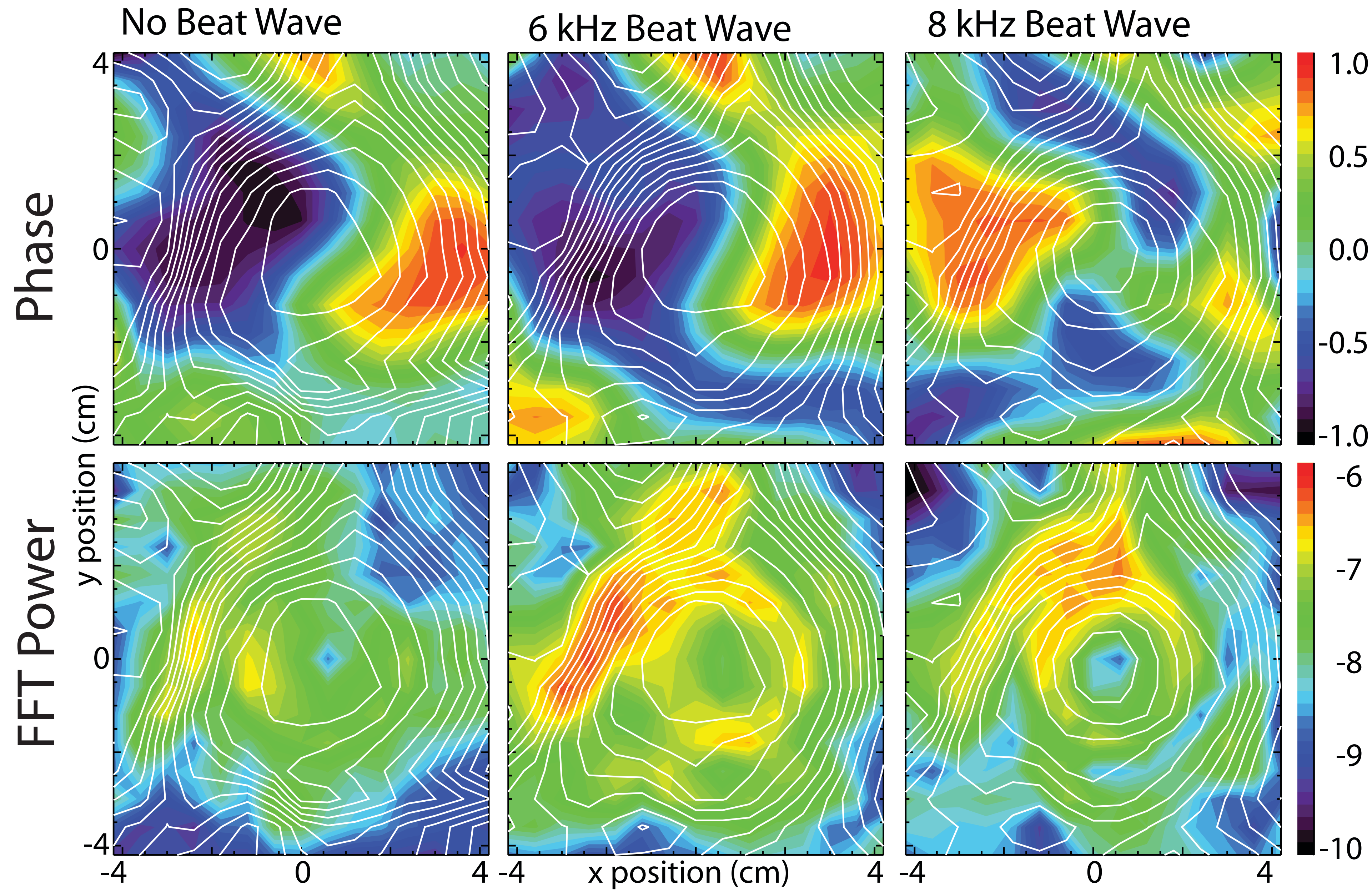
- Beat response significantly stronger than uniform plasma case
- Resonance at (downshifted) instability frequency observed, suppression of the unstable mode observed above (and slightly below)
- Instability returns at higher beat frequency

BW controls unstable mode and reduces broadband noise



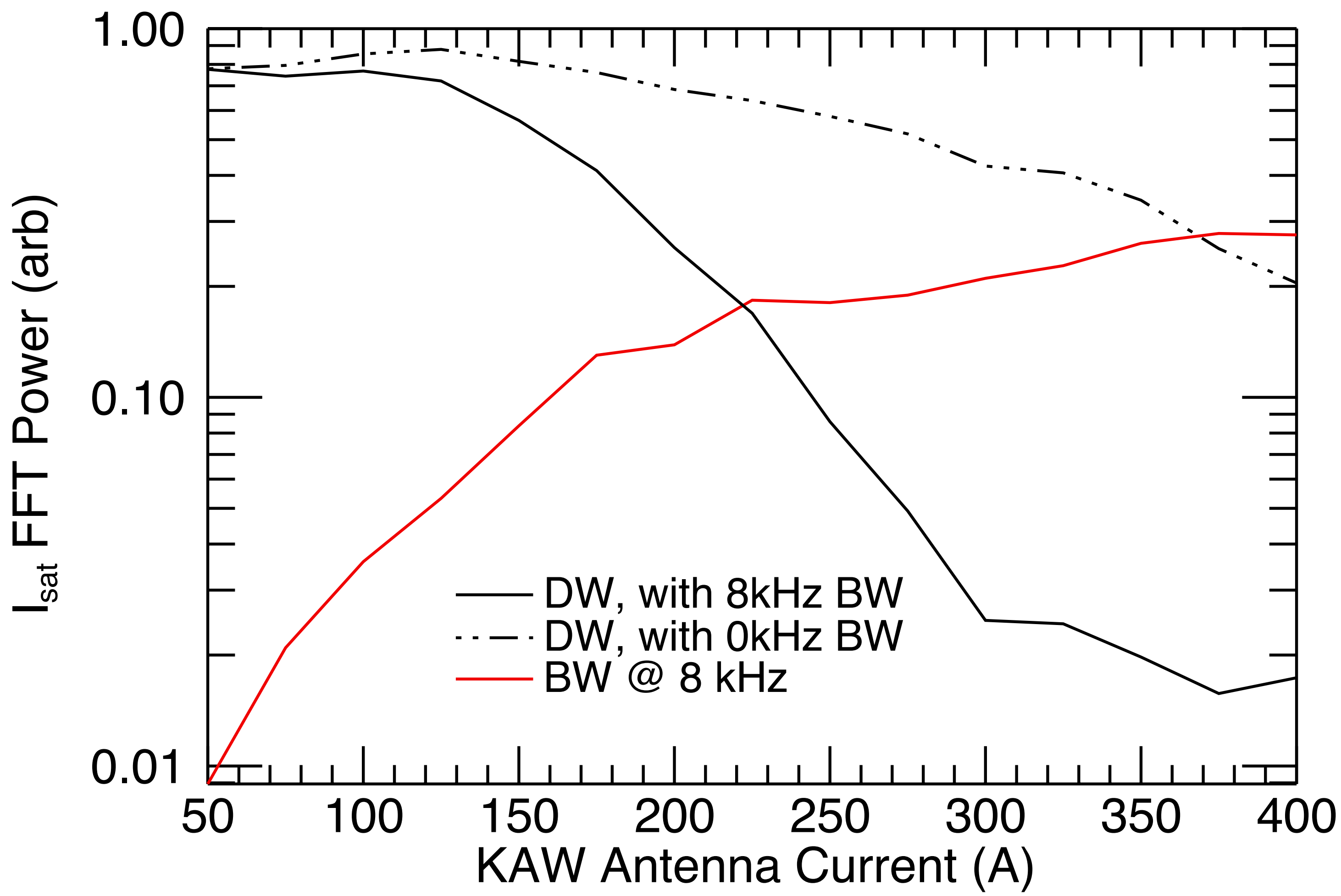
- Threshold for control: beat-driven mode has comparable (but less) amplitude than original unstable mode
- With beat wave, quieter at wide range of frequencies (previously generated nonlinearly by unstable mode)

Structure of beat-driven modes suggest coupling to linear modes



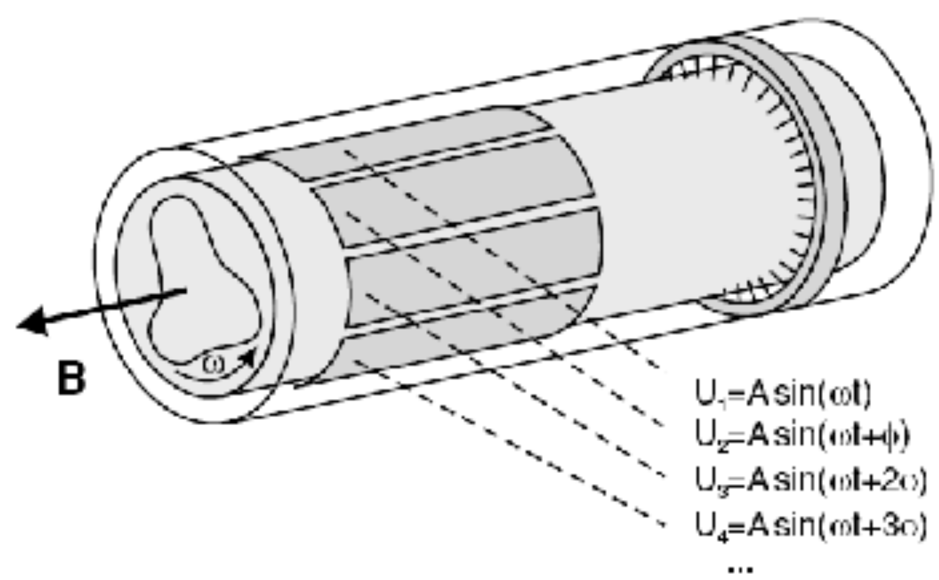
- Beat wave has m=1 (6 kHz peak), m=2 (8 kHz peak)
- Rotation in electron diamagnetic direction (same as instability)

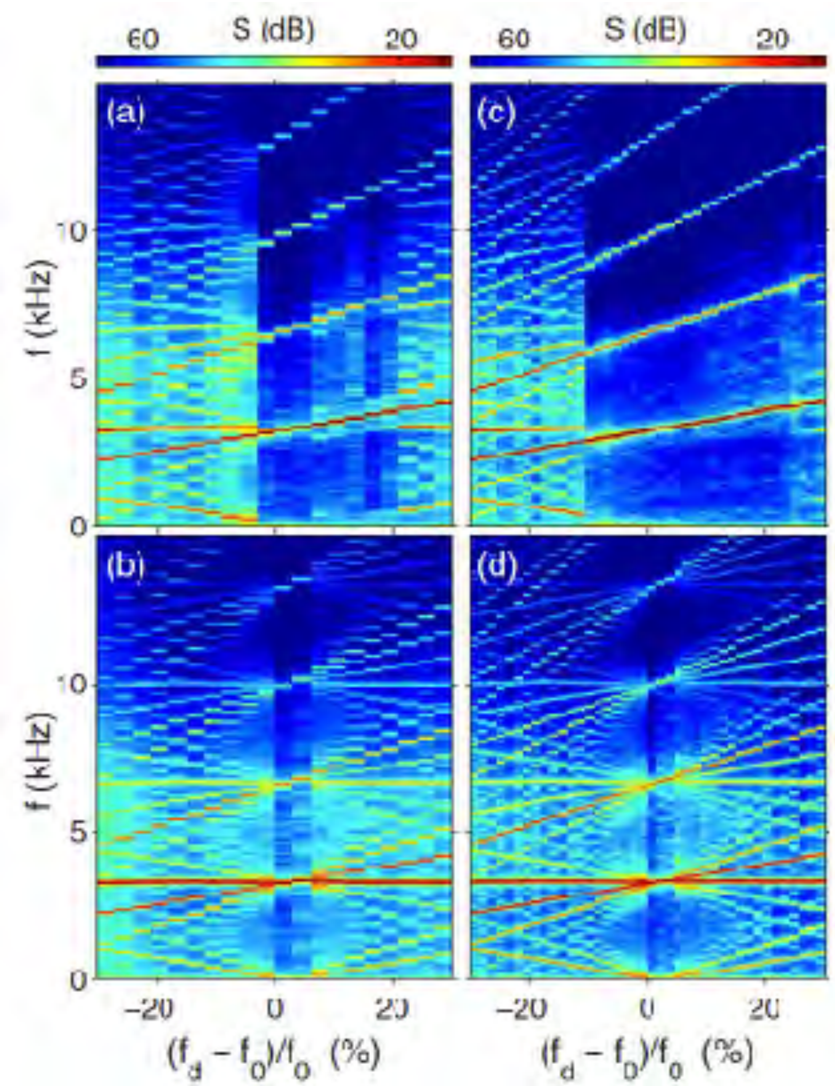
Threshold for control, saturation of BW observed

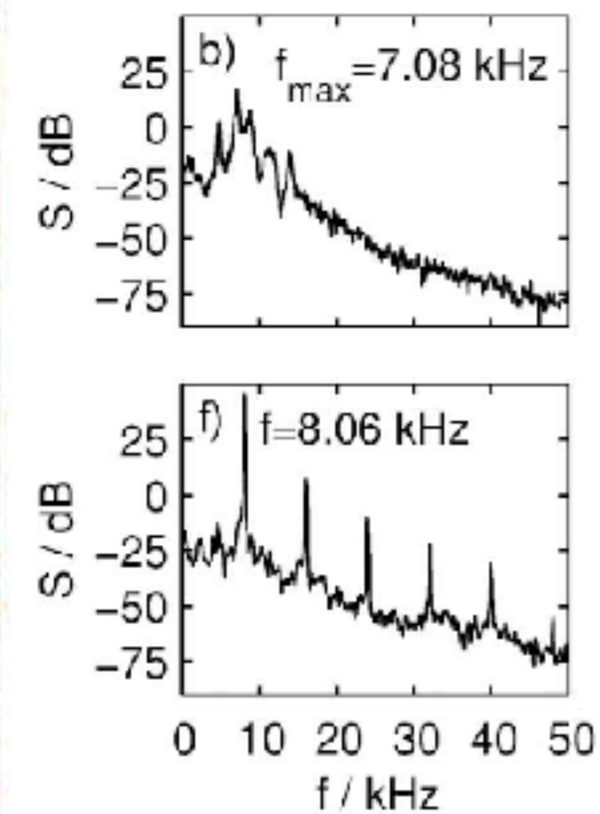


- Modification of DW seen starting at PBW/PDW ~ 10%; maximum suppression for comparable BW power
- Two effects: electron heating from KAWs modifies profiles, causing some reduction in amplitude without BW
- BW response seems to saturate as DW power bottoms out

Similar behavior seen using external antenna to excite drift-waves



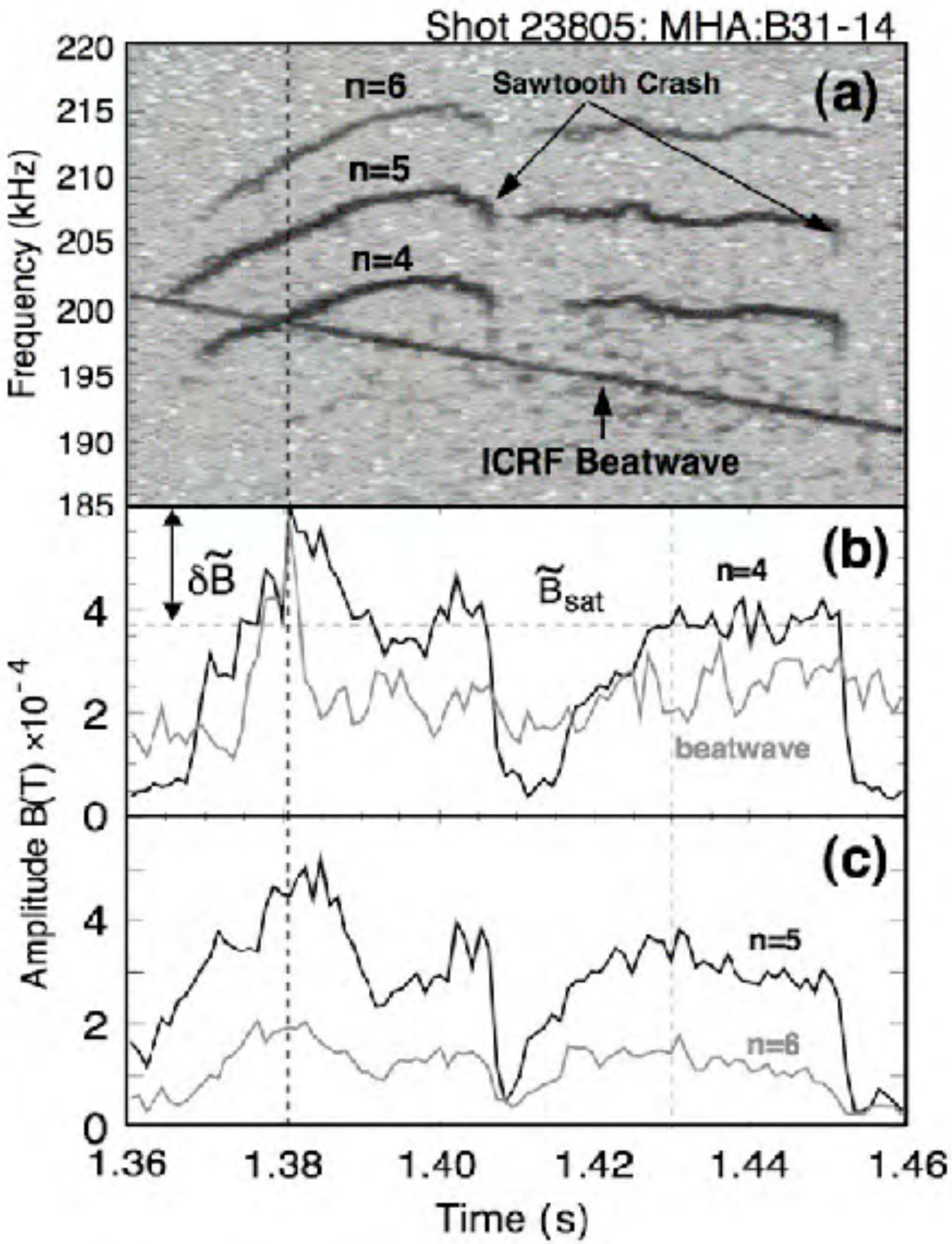




Schroeder, et al PRL 2001
Brandt, et al, PoP 2010

- Used external antenna structure on MIRABELLE, VINETA to try to directly excite drift-waves
- Saw collapse of spectrum onto coherent drift-wave at the driven frequency (+ harmonics), transport modified

ICRF beat waves used to drive AEs



Sassenberg, et al., NF 50, 052003 (2010)

- ICRF BWs used to excited TAEs in JET [Fasoli, et al.] and ASDEX [Sassenberg, et al.]
- Could use ICRF to interact with control lower frequency modes (drift-type, ELMs, etc)