Turbulence and dissipation in magnetized space plasmas

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Outline

1. Part I: Turbulence in space plasmas (solar wind and planetary magnetosheaths): Introduction, space instrumentation, data analysis techniques
2. Part II: MHD Turbulence
3. Part III: Kinetic turbulence
Part I: Turbulence in space plasmas

1. Plasmas in the Universe

2. Space plasmas: Sun, Solar wind, Planetary magnetospheres

3. Why do we need to study plasma turbulence?

4. In-situ space instrumentation and related measurements

5. Multispacecraft data analysis techniques (e.g., the k-filtering)
Turbulence in the Univers

Turbulence is ubiquitous in the Univers – It covers all scales, from quantum to cosmological ones!

Observed heating and particle acceleration (i.e. jets) in astrophysical objects are caused by turbulence dissipation

Solar corona heating

Turbulence in galaxies & nebulas
Accretion disks of black holes

Matter spirals into the black hole, converting huge gravitational potential energy into heat:
- Magnetorotational Instability (MRI) drives turbulence [Balbus, 1992]
- Turbulence cascades nonlinearly to small scales
- Kinetic mechanisms damp turbulence and lead to plasma heating

Emitted radiations (e.g. X-rays) are function of the plasma heating
The Sun

Solar corona

Very strong heating in the transition region
The solar wind

The solar wind plasma is generally:

- Fully ionized (H\(^+\), e\(^-\))
- Non-relativistic (V\(_A\)\(<\)c), V\(~350-800\ km/s
- *Collisionless*
Sun-Earth coupling

SUN Magnetized planet

Solar Wind

Magnetic field & plasma particles

Magnetosphere

Magnetosheath
Planetary magnetospheres

- Mercury
  - MESSENGER
  - BEPICOLombo
- Earth
  - THEMIS
  - CLUSTER
  - MMS
- Jupiter
  - GALILEO
  - JUNO
  - JUICE
- Saturn
  - CASSINI

- d ~ 0.3 AU
  - $n_e \sim 60 \text{ cm}^3$
  - $B \sim 25 \text{ nT}$
  - $\rho_i \sim 5 \text{ Km}$

- d ~ 1 AU
  - $n_e \sim 10 \text{ cm}^3$
  - $B \sim 10 \text{ nT}$
  - $\rho_i \sim 100 \text{ Km}$

- d ~ 5.2 AU
  - $n_e \sim < 1 \text{ cm}^3$
  - $B \sim 2 \text{ nT}$
  - $\rho_i \sim 500 \text{ Km}$

- d ~ 9.5 AU
  - $n_e \sim < 1 \text{ cm}^3$
  - $B \sim 0.5 \text{ nT}$
  - $\rho_i \sim 1000 \text{ Km}$
Turbulence in fusion devices

Turbulence is the main obstacle to plasma confinement

A better understanding of turbulent transport → A better control → A longer confinement
Any common physics?

M100 galaxy $10^{23}$ m

Eagle nebula $10^{18}$ m

Performances limited by plasma turbulence

Strong pressure gradients $\rightarrow$ Instabilities

$\downarrow$

Turbulent system

Radial heat transport

$\rightarrow$ Prediction & control of turbulent transport
Near-Earth space plasmas

\[
\beta = \frac{\text{Pression thermique}}{\text{Pression magnétique}} \approx 0.4 \frac{NT}{B^2}
\]


Remote sensing (distant plasmas)

Bernard Lyot, the inventor of coronograph (photo Observatoire de Paris)

Bernard Lyot Telescope at Observatoire du Pic du Midi (photo P. Petit)
In-situ measurements (space plasmas)

Plasmas \(\rightarrow\) A coupled system of equations

\[
\begin{align*}
\nabla \times \mathbf{E} &= -\partial_t \mathbf{B} \\
\nabla \times \mathbf{B} &= \frac{1}{c^2} \partial_t \mathbf{E} + \mu_0 \mathbf{j} \\
\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\
\nabla \cdot \mathbf{B} &= 0
\end{align*}
\]

\[
\begin{align*}
\partial_t n_e + \nabla \cdot (n_e \mathbf{u}_e) &= 0 \\
n_e m_e \partial_t \mathbf{u}_e + n_e \mathbf{u}_e \nabla \cdot \mathbf{u}_e + \nabla p_e &= -n_e e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) \\
\partial_t n_i + \nabla \cdot (n_i \mathbf{u}_i) &= 0 \\
n_i m_i \partial_t \mathbf{u}_i + n_i \mathbf{u}_i \nabla \cdot \mathbf{u}_i + \nabla p_i &= n_i e (\mathbf{E} + \mathbf{u}_i \times \mathbf{B})
\end{align*}
\]

\[
\mathbf{j} = n_i e \mathbf{u}_i - n_e e \mathbf{u}_e \\
\rho = n_i e - n_e e
\]

Ideally, a space plasma physicist would like to measure:
- **B & E**: 3 components over a broad range of frequencies [DC, MHz]
- **N_{i,e}, V_{i,e}, T_{i,e}**: in 3D at all energies (eV, MeV) and with high resolutions
In-situ measurements

THOR spacecraft: 10 instruments (currently under phase A study at ESA)
**Instruments overview: fields (1)**

**Fluxgate magnetometer:** $B$ measurements in [DC, 1Hz]

- $B_{\text{ext}} = 0$
  - Two parallel bars are placed closely together
  - And the direction in which the coil is wrapped around the bar is reversed

- $B_{\text{ext}} \neq 0$
  - External magnetic field direction

![Graphs and diagrams showing the operation of a fluxgate magnetometer](image)
Search-coil magnetometer (SCM): [0.1Hz, ~1MHz]

Lenz’s law (induced voltage): \[ V_C = -N \frac{d\Phi}{dt} = -j2\pi f N S \mu_{\text{eff}} B \cos \theta \]

\( \mu_{\text{eff}} \) effective permeability of the core
A feedback reaction is needed to obtain a flat response function.

\[ G \equiv \frac{V}{B} \]

\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{L}{C}} \]

Dual band SCM:
LF [1Hz, 4kHz]
HF [1kHz, 1MHz]

Solar Orbiter/SCM (LPC2E)
BepiColombo/SCM (LPP-Univ. Kanasawa, JP)
Instruments overview: fields (3)

**Electric field: [DC, 1MHz]**

Spacecraft potential $V_{sc}$ → electron density $n_e$

Hadid et al., 2016b
Instruments overview: fields (4)

Onboard wave analyzers

THOR/FWP (Field and Wave Processing Unit – Courtesy THOR proposal)

Solar Orbiter/LF analyzer

TNR/HFR → High time resolution measurement of $N_e$ and $T_e$
Instruments overview: particles (1)
(Ion and electron) mass spectrometers

Energy selection

Angle selection

Departure of the Time Of Flight analysis

End of the flight: TOF gives the m/q ratio
Output measurements :

- The nature of the particles \((m/q)\)
- Their direction and energy/velocity
- Velocity distribution function (VDF)
- Moments of the VDF:
  - density, velocity, temperature

\[<v>\]
Instruments overview: particles (2)

Particle Processing Unit (PPU)

Solar Orbiter/PPU (Courtesy of TSD/RTI --from THOR proposal).

+Energetic particles + ASPOC + active sounder + …

THOR/PPU (Courtesy of TSD/RTI and THOR proposal).
Back to turbulence: phenomenology

NS equation:
\[ \partial_t V + F_i = -V \cdot \nabla V - \nabla P + \nu \nabla^2 V \]

- Hydro: Scale invariance down to the dissipation scale \( 1/k_d \)

- Collisionless Plasmas: - Breaking of the scale invariance at \( \rho_{i,e} d_{i,e} \)
  - Absence of the viscous dissipation scale \( 1/k_d \)

Inertial range

E (k)

\( k^{-5/3} \)

\( k_i \)

\( k_d \)

\( \text{Courtesy of A. Celani} \)

"Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity"

Lewis Fry Richardson (1920)
Solar wind turbulence

Typical power spectrum of magnetic energy at 1 AU

Does the energy cascade or dissipate below the ion scale \( \rho_i \)?

Leamon et al. 98; Goldstein et al. JGR, 94

Richardson & Paularena, GRL, 1995 (Voyager data)
How to analyse space turbulence?

Turbulence theories generally predict spatial spectra: K41 \((k^{-5/3})\); IK \((k^{-3/2})\), Anisotropic MHD turbulence \((k_{\perp}^{-5/3})\), Whistler turbulence \((k^{-7/3})\), ...

Example of measured spectra in the SW

But measurements provide only temporal spectra (generally with different power laws at different frequencies)

How to infer spatial spectra from temporal ones measured in the spacecraft frame? \(B^2 \sim \omega_{sc}^{-\alpha} \Rightarrow B^2 \sim k_{\parallel}^{-\beta} k_{\perp}^{-\gamma}\)?
The spatio-temporal ambiguity (1)

Spacecraft measurements show highly variable phenomena. With 1 point measurement one cannot distinguish space effects from temporal effects.
A minimum of 4 spacecraft is needed to sample the 3 directions of space (e.g., ESA/Cluster and NASA/MMS missions)
The Taylor frozen-in flow assumption

In the solar wind (SW) the Taylor’s hypothesis can be valid at MHD scales.

High SW speeds: \( V \sim 600 \text{km/s} \gg V_\phi \sim V_A \sim 50 \text{km/s} \Rightarrow \)

\[ \omega_{\text{spacecraft}} = \omega_{\text{plasma}} + k.V \approx k.V = kVV \]

\[ \Rightarrow \text{Inferring the } k\text{-spectrum is possible with one spacecraft} \]

But only along one single direction.
1. At MHD scales, even if the Taylor assumption is valid, inferring 3D $k$-spectra from an $\omega$-spectrum is impossible.

2. At sub-ion and electron scales, $V\phi$ can be larger than $V_{sw}$ ⇒ The Taylor’s hypothesis is invalid.

1 & 2 ⇒ Need to use multi-spacecraft measurements and appropriate methods to infer 3D $k$-spectra.
The k-filtering technique (1)

Goal: estimation of the spectral energy density $P(\omega,k)$ from the multipoint measurements of a turbulent field

Method: it uses a filter bank approach: the filter bank is constructed to absorb all signals, except those corresponding to plane waves with a specified frequency and wave vector, which pass unaffected.

By going through all frequencies and wave vectors, one gets an estimate of the wave-field energy distribution $P(\omega,k)$

[Pinçon & Lefeuvre, 1991; Sahraoui et al., 2003, 2004, 2006; 2010; Narita et al., 2010; Grison et al., 2005; Tjulin et al., 2005; Roberts et al., 2012]
The $k$-filtering technique (2)

$k$-filtering – notations

The wave field consists of $L$ real quantities:

$$A(r, t) = \begin{pmatrix} A_1(r, t) \\ A_2(r, t) \\ \vdots \\ A_L(r, t) \end{pmatrix}$$

The Fourier transformed measurements from $N$ spacecraft are put into one vector:

$$A(\omega) = \begin{pmatrix} A(r_1, \omega) \\ A(r_2, \omega) \\ \vdots \\ A(r_N, \omega) \end{pmatrix}$$

A spatial correlation matrix is defined:

$$M(\omega) = \langle A(\omega) A^{T*}(\omega) \rangle$$
The k-filtering technique (3)

The k-filtering equation

\[ M(\omega) = \langle A(\omega) A^\ast(\omega) \rangle \]

\[ A(\omega) = \begin{pmatrix} A(r_1, \omega) \\ A(r_2, \omega) \\ \vdots \\ A(r_N, \omega) \end{pmatrix} \]

\[ A(r, t) = \begin{pmatrix} A_1(r, t) \\ A_2(r, t) \\ \vdots \\ A_L(r, t) \end{pmatrix} \]

Wave-field energy density distribution

Spatial correlation matrix

Data from \( N \) different spacecraft

Wave-field of \( L \) real quantities

\[ P(\omega, k) = \text{Tr} \left[ C(\omega, k) C^\ast (\omega, k) H^\ast (k) M^{-1}(\omega) H(\omega) C(\omega, k) \right] \]

Constraining matrix

Matrix to keep track of the spacecraft positions

\[ k \cdot B = 0 \]

\[ \omega B = k \times E \]

\[ H(k) = \begin{pmatrix} I_L e^{ikr_1} \\ I_L e^{ikr_2} \\ \vdots \\ I_L e^{ikr_N} \end{pmatrix} \]

\[ L \times L \text{ unit matrix} \]
The k-filtering technique (4)

Simple 1D example

Two satellites (at \( x_1 \) and \( x_2 \)) measuring one field quantity \( \Phi(x,t) \).

The wave field is given by:

\[
\phi(x,\omega) = \phi_0(\omega) e^{ik_0x}
\]

The spatial correlation matrix is then:

\[
M(\omega) = |\phi_0(\omega)|^2 \begin{pmatrix}
1 & e^{ik_0(x_1-x_2)} \\
\frac{1}{e^{-ik_0(x_1-x_2)}} & 1
\end{pmatrix}
\]

This is not invertible, we must thus add some incoherent noise:

\[
M(\omega) = |\phi_0(\omega)|^2 \begin{pmatrix}
1 + \varepsilon & e^{ik(x_1-x_2)} \\
\frac{1}{e^{-ik(x_1-x_2)}} & 1 + \varepsilon
\end{pmatrix}
\]

The \( H \)-matrix in this case is:

\[
H(k) = \begin{pmatrix}
e^{ikx_1} \\
e^{ikx_2}
\end{pmatrix}
\]
The k-filtering technique (5)

Simple 1D example [Tjulin et al., 2005]

This gives (after some manageable algebra):

$$P(\omega, k) = \left| \phi_0(\omega) \right|^2 \frac{\varepsilon (2 + \varepsilon)}{2 \left( 1 + \varepsilon - \cos \left[ (k - k_0) (x_1 - x_2) \right] \right)}$$

Note that the result is periodic in $k$!
The k-filtering technique (6)

Spatial Aliasing effect: Two satellites cannot distinguish between $k_1$ and $k_2$ if: $\Delta k \cdot r_{12} = 2\pi n \ (n=1,2, \ldots)$

For Cluster: $\Delta k = n_1 \Delta k_1 + n_2 \Delta k_2 + n_3 \Delta k_3$

with: $\Delta k_1 = (r_{31} \times r_{21})2\pi/V$,
$\Delta k_2 = (r_{41} \times r_{21})2\pi/V$,
$\Delta k_3 = (r_{41} \times r_{31})2\pi/V$

$V = r_{41} \cdot (r_{31} \times r_{21})$ [Neubaur & Glassmeir, 1990]
END of Part I
PART II: MHD scale turbulence

1. Theoretical models: HD, incompressible and compressible MHD

2. Turbulence at MHD scales in the solar wind
   - Energy cascade rate: incompressible vs compressible models
   - Spatial anisotropy and scaling properties

3. Comparisons with magnetosheath turbulence
Incompressible HD turbulence

\[ \partial_t u + (u \cdot \nabla)u = -\nabla p + \nu \Delta u \]

\[ E(k) = u_k^2 \sim k^{-5/3} \]

\[ S_3(l) = < \delta u_l^3 > = < [u(x + l) - u(x)]^3 > = -\frac{4}{5} \epsilon l \]

\( \varepsilon \) is the energy cascade (dissipation) rate

Kolmogorov's phenomenological theory

Andrei N. Kolmogorov (1941a)

The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers†

By A. N. Kolmogorov

The first hypothesis of similarity. For the locally isotropic turbulence the distributions \( F_\nu \) are uniquely determined by the quantities \( \nu \) and \( \varepsilon \).

The second hypothesis of similarity. If the models of the vectors \( \dot{y}^\nu \) and of their differences \( \dot{y}^\nu - \dot{y}^\mu \) (where \( k \neq \mu \)) are large in comparison with \( \lambda \), then the distribution laws \( F_\nu \) are uniquely determined by the quantity \( \varepsilon \) and do not depend on \( x \).

\[ B_{ad}(r) \sim C_0^2 r^4 \]

\[ S_L(r) = \langle [u(x + r) - u(x)] \cdot \hat{r}\rangle^2 \sim C e^{\frac{3}{2}} r^{\frac{5}{2}} \]

\[ E(k) = C'e^{\frac{3}{2}} k^{-\frac{5}{3}} \]
Incompressible MHD turbulence: equations and phenomenology (1)

Incompressible MHD equations

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla \left( p + \frac{B^2}{8\pi} \right) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi}
\]

\[
\nabla \cdot \mathbf{B} = 0
\]

\[
\nabla \cdot \mathbf{v} = 0
\]

Elsässer variables:

\[
\mathbf{z}^\pm = \mathbf{v} \pm \frac{\mathbf{B}}{\sqrt{4\pi \rho_0}} = \mathbf{v} \pm \mathbf{v}_A
\]
Incompressible MHD turbulence: third order law and energy cascade rate (2)

\[ \partial_t z^\pm \mp v_A \cdot \nabla z^\pm + \mp \cdot \nabla z^\pm = -\nabla p \]

Linear term: \( k_\parallel v_A z^\pm \)

Nonlinear term: \( k_\perp v_\perp z^\pm \)

Ratio of nonlinear to linear terms \[
\chi = \frac{k_\perp v_\perp}{k_\parallel v_A}
\]

\( \chi \sim 1 \Rightarrow \) Critically Balanced turbulence

Third order law
[Politano & Pouquet, PRE, 1998]

\[
\langle (\delta z^\pm)^2 \delta z_\mp \rangle = \frac{4}{3} \varepsilon^\pm l
\]

\( z^\pm = v^\pm \frac{B}{\sqrt{4\pi \rho_0}} \)

\( E^\pm = \frac{1}{2} z^\pm \mp \)

is the cascade rate of the pseudo-energies
Compressible isothermal MHD turbulence: Equations

Compressible MHD equations

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla \left( p + \frac{B^2}{8\pi} \right) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad \nabla \cdot \mathbf{B} = 0
\]

\[P = C_s^2 \rho\]

Isothermal closure

\[C_s \text{ sound speed (constant)}\]
Compressible MHD turbulence: linear solutions

\[ \frac{\omega}{\omega_{ci}} \]

- **Fast mode**
- **Intermediate mode**
- **Slow mode**
- **Mode d’Alfvén**
Compressible isothermal MHD turbulence: 3rd order law and energy cascade rate (3)

In the inertial zone we obtain (Banerjee & Galtier, PRE, 2013)

\[-2\varepsilon = \frac{1}{2} \nabla_r \cdot \left( \frac{1}{2} \delta (p \zeta^-) \cdot \delta \zeta^- + \delta \rho \delta e \right) \delta \zeta^+ + \frac{1}{2} \delta (p \zeta^+) \cdot \delta \zeta^+ + \delta \rho \delta e \delta (e + \frac{v_A^2}{2} \delta (p \zeta^- + p \zeta^+)) \]

\[-\frac{1}{4} \left( \frac{1}{\beta^2} \nabla' \cdot (p \zeta^+ e') + \frac{1}{\beta} \nabla' \cdot (\rho' \zeta^+ e) + \frac{1}{\beta} \nabla' \cdot (\rho' \zeta^- e') + \frac{1}{\beta} \nabla' \cdot (\rho' \zeta^- e) \right) \]

\[+ \langle (\nabla \cdot \mathbf{v}) [R_E - E - \frac{8}{3} (\mathbf{v}_A \cdot \mathbf{v}_A) - \frac{p'}{2} + \frac{\beta H'}{2} \rangle \rangle + \langle (\nabla' \cdot \mathbf{v}') [R_E - E - \frac{8}{3} (\mathbf{v}_A \cdot \mathbf{v}_A') - \frac{p'}{2} + \frac{\beta H'}{2} \rangle \rangle \]

\[+ \langle (\nabla' \cdot \mathbf{v}_A) [R_H - R_H' + H' - \frac{8}{3} \rho (\mathbf{v} \cdot \mathbf{v}_A)] \rangle + \langle (\nabla' \cdot \mathbf{v}_A') [R_H - R_H' + H' - \frac{8}{3} \rho (\mathbf{v} \cdot \mathbf{v}_A')] \rangle \]

where

\[E = \rho (\mathbf{v} \cdot \mathbf{v} + \mathbf{v}_A \cdot \mathbf{v}_A) / 2 + \rho e, \quad E' = \rho' (\mathbf{v}' \cdot \mathbf{v}' + \mathbf{v}_A' \cdot \mathbf{v}_A') / 2 + \rho' e'; \]

\[R_E = \rho (\mathbf{v} \cdot \mathbf{v} + \mathbf{v}_A \cdot \mathbf{v}_A) / 2 + \rho e', \quad R_E' = (\rho' \mathbf{v}' \cdot \mathbf{v} + \mathbf{v}_A \cdot \mathbf{v}_A) / 2 + \rho' e'; \]

\[R_H = \rho (\mathbf{v} \cdot \mathbf{v}_A + \mathbf{v}_A \cdot \mathbf{v}_A) / 2, \quad R_H' = \rho' (\mathbf{v} \cdot \mathbf{v}_A + \mathbf{v} \cdot \mathbf{v}_A') / 2 \]

\[H = \rho \mathbf{v} \cdot \mathbf{v}_A, \quad H' = \rho' \mathbf{v}' \cdot \mathbf{v}_A'; \quad \beta = 2 C_S^2 / \mathbf{v}_A^2; \quad \beta' = 2 C_S'^2 / \mathbf{v}_A'^2 \]
Additional assumptions:

- Neglect the source terms
- Isotropy
- Uniform $\beta$

\[ \Phi = \frac{1}{\beta} \nabla \ell \cdot \langle \rho \mathbf{v} e' - \rho' \mathbf{v}' e \rangle = -\frac{2}{\beta} \nabla \ell \cdot \langle \delta e \delta (\rho \mathbf{v}) \rangle \]

\[ -\frac{4}{3} \varepsilon_c \ell = \mathcal{F}_{C+\Phi}(\ell) \]

\[ \mathcal{F}_{C+\Phi}(\ell) = \mathcal{F}_1(\ell) + \mathcal{F}_2(\ell) + \mathcal{F}_3(\ell) \]

\[ \mathcal{F}_1(\ell) = \left\langle \frac{1}{2} \left[ \delta (\rho \mathbf{z}^-) \cdot \delta \mathbf{z}^- \right] \delta z^+ \ell + \frac{1}{2} \left[ \delta (\rho \mathbf{z}^+) \cdot \delta \mathbf{z}^+ \right] \delta z^- \ell \right\rangle, \]

\[ \mathcal{F}_2(\ell) = \left\langle 2 \delta \rho \delta e \delta v_\ell \right\rangle, \]

\[ \mathcal{F}_3(\ell) = \left\langle 2 \delta \left[ \left( 1 + \frac{1}{\beta} \right) e + \frac{v_A^2}{2} \right] \delta (\rho_1 v_\ell) \right\rangle . \quad (8) \]
The solar wind

The solar wind plasma is generally:

- Fully ionized (H\(^+\), e\(^-\))
- Non-relativistic (\(V_A < < c\)), \(V \sim 350-800\) km/s
- Collisionless

[Richardson & Paularena, 1995]
Typical magnetic power spectrum at 1AU

Kiyani et al., 2015
Estimation of the energy cascade rate: compressible vs incompressible MHD model (1)

\[ z^\pm = v^\pm \frac{B}{\sqrt{4\pi \rho_0}} \]

\[ \left\langle (\delta z^\pm)^2 \delta z_l^\pm \right\rangle = \frac{4}{3} \epsilon^\pm l \]
Estimation of the energy cascade rate: compressible vs incompressible MHD model (2)

Fast wind
2011-01-09 14:26-15:01

Slow wind
2009-11-20 03:33-04:08

Estimation of the energy cascade rate: cross helicity and turbulent Mach number

\[ \sigma_c = \frac{H_c}{(1/2)\langle \delta v^2 + \delta b^2 \rangle} \]

\[ H_c = \langle \delta v \cdot \delta b \rangle \]
Spatial anisotropy

In MHD turbulence, the presence of a mean magnetic field makes the turbulence anisotropic at small scales [Montgomery et al., 1983]

\[ l / L_0 \sim 1 \quad l / L_0 \sim 1/10 \quad l / L_0 \sim 1/100 \]

3D Electron -MHD simulations [Meyrand & Galtier, 2013]
Anisotropy and the critical balance conjecture

The critical balance conjecture [Goldreich & Sridhar, 1995]:
Linear (Alfvén) time ~ nonlinear (turnover) time
\[ \omega \sim k_{\parallel} V_A \sim k_{\perp} u_{\perp} \]
\[ \Rightarrow k_{\parallel} \sim k_{\perp}^{2/3} \]


Single satellite analysis → use of the Taylor assumption:

\[ \omega_{sc} \sim k \cdot V_{sw} \sim k_v V_{sw} \]

\[ V_{//} B \rightarrow k_v = k_{//} \quad \text{Assumes axisymmetry around } B \]

\[ V_{\perp} B \rightarrow k_v = k_{\perp} \]

\[ \Theta_{BV} \rightarrow 0 \Rightarrow B^2 \sim k_{//}^{-2} \Rightarrow \text{Partial evidence of the critical balance } [\text{Horbury et al., PRL, 2008}] \]
See also Chen et al., PRL, 2010
The ESA/Cluster mission

The first multispacecraft mission: 4 identical satellites

Objectives:

- **3D exploration** of the Earth magnetosphere boundaries (magnetopause, bow shock, magnetotail) & SW

- **Measurements of 3D quantities:** \( \mathbf{J} = \nabla \times \mathbf{B}, \ldots \)

- **Fundamental physics:** turbulence, reconnection, particle acceleration, \ldots

Different orbits and separations (10^2 to 10^4 km) depending on the scientific goal
The 4 satellites before launch
The \( k \)-filtering technique

Interferometric method: it provides, by using a NL filter bank approach, an optimum estimation of the 4D spectral energy density \( P(\omega,k) \) from simultaneous multipoints measurements [Pinçon & Lefeuvre; Sahraoui et al., 03, 04, 06, 10; Narita et al., 03, 06, 09]

We use \( P(\omega,k) \) to calculate

1. 3D \( \omega\)-\( k \) spectra \( \Rightarrow \) plasma mode identification e.g. Alfvén, whistler
2. 3D \( k \)-spectra (anisotropies, scaling, …)
Measurable spatial scales

Given a spacecraft separation $d$ only one decade of scales $2d < \lambda < 30d$ can be correctly determined

- $\lambda_{\text{min}} \approx 2d$, otherwise spatial aliasing occurs.
- $\lambda_{\text{max}} \approx 30d$, because larger scales are subject to higher uncertainties

$\omega_{\text{sat}} \sim kV \Rightarrow f_{\text{max}} \sim k_{\text{max}} V/\lambda_{\text{min}} \ (V \sim 500\text{km/s})$

- $d \sim 10^4 \text{ km} \Rightarrow$ MHD scales
- $d \sim 10^2 \text{ km} \Rightarrow$ Sub-ion scales
- $d \sim 1 \text{ km} \Rightarrow$ Electron scales \textit{(but not accessible with Cluster: $d > 100$)}
1- MHD scale solar wind turbulence

Position of the Quartet on March 19, 2006
Data overview

FGM data (CAA, ESA)

Ion plasma data from CIS (AMDA, CESR)
To compute reduced spectra we integrate over

1. all frequencies $f_{sc}$:  \[ \tilde{P}(\mathbf{k}) = \sum_{k_y,k_z} P(f_{sc}, \mathbf{k}) \]

2. all $k_{i,j}$:  \[ \tilde{P}(k_x) = \sum_{k_y,k_z} \tilde{P}(k_x, k_y, k_z) \]
Anisotropy of MHD turbulence along $B_0$ and $V_{sw}$

Turbulence is not axisymmetric (around $B$) [see also Sahraoui, PRL, 2006]

[Narita et al., PRL, 2010]

The anisotropy ($\perp B$) is along $V_{sw} \rightarrow SW$

expansion effect?[Saur & Bieber, JGR, 1999]
Mirror mode turbulence

Compressible, anisotropic and non-axisymmetric turbulence (along $B_0$, the magnetopause normal $n$, and the flow $v$) [Sahraoui+, PRL, 2006]
PART III:
Kinetic (sub-ion scale) turbulence
Kinetic turbulence

1. Kinetic scales in the SW: Some hotly debated question vs Cluster observations
   - The nature of the cascade or dissipation below $\rho_i$: KAW? whistler? Others?
   - The nature of the dissipation: wave-particle interactions? Current sheets/Reconnection?

2. Conclusions & perspectives (turbulence & the future space missions)
I- Theoretical predictions on small scale turbulence

1. Fluid models (Hall-MHD)
   - Whistler turbulence (E-MHD): (Biskamp et al., 99, Galtier, 08)
   - Weak Turbulence of Hall-MHD (Galtier, 06; Sahraoui et al., 07)

2. Gyrokinetic theory: $k_{//} \ll k_{\perp}$ and $\omega \ll \omega_{ci}$ (Schekochihin et al. 06; Howes et al., 11)
Other numerical predictions on electron scale turbulence

2D PIC simulations gave evidence of a power law dissipation range at $k\rho_e > 1$

Figure 4. Spectrum of magnetic fluctuation $|\delta B|^2/|B_0|^2$ in the parallel direction $k_{||}\rho_e$. The noise level curve is in red. The power-law best fits are superimposed. (A color version of this figure is available in the online journal.)

Figure 5. Spectrum of magnetic fluctuation $|\delta B|^2/|B_0|^2$ in the perpendicular direction $k_{\perp}\rho_e$. The noise level curve is in red. The power-law best fits are superimposed.

3D PIC simulations of whistler turbulence: $k^{-4.3}$ at $kd_e > 1$

Chang & Gary, GRL 2011
First evidence of a cascade from MHD to electron scale in the SW

1. Two breakpoints corresponding to $\rho_i$ and $\rho_e$ are observed.

2. A clear evidence of a new inertial range $\sim f^{-2.5}$ below $\rho_i$.

3. First evidence of a dissipation range $\sim f^{-4}$ near the electron scale $\rho_e$.

Sahraoui et al., PRL, 2009
Whistler or KAW turbulence?

1. Large (MHD) scales ($L > \rho_i$): strong correlation of $E_y$ and $B_z$ in agreement with $E = -V \times B$

2. Small scales ($L < \rho_i$): steepening of $B^2$ and enhancement of $E^2$ (however, strong noise in $E_y$ for $f > 5$ Hz)

⇒ Good agreement with GK theory of Kinetic Alfvén Wave turbulence

Howes et al.
PRL, 11

See also Bale et al., PRL, 2005
Theoretical interpretation: KAW turbulence

Linear Maxwell-Vlasov solutions: \( \Theta_{kB} \approx 90^\circ, \beta_i \approx 2.5, T_i/T_e \approx 4 \)

The Kinetic Alfvén Wave solution extends down to \( kp_e \approx 1 \) with \( \omega_r < \omega_{ci} \)


\[ \omega_r = k_//V_A k_\perp \rho_i / \sqrt{\beta_i + 2/(1+T_i/T_e)} \]
**E/B : KAW theory vs observations**

\[ \omega_i = k // V_A k_\perp \rho_i / \sqrt{\beta_i + 2/(1 + T_i / T_e)} \]

- Lorentz transform: \( E_{\text{sat}} = E_{\text{plas}} + V \times B \)
- Taylor hypothesis to transform the spectra from \( f \) (Hz) to \( k \rho_i \)

1. Large scale (\( k \rho_i < 1 \)): \( \delta E / \delta B \sim V_A \)
2. Small scale (\( k \rho_i > 1 \)): \( \delta E / \delta B \sim k^{1.1} \) ⇒ in agreement with GK theory of KAW turbulence \( \delta E^2 \sim k_\perp^{-1/3} \) & \( \delta B^2 \sim k_\perp^{-7/3} \) ⇒ \( \delta E / \delta B \sim k \)
3. The departure from linear scaling (\( k \rho_i \gg 10 \)) is due to noise in \( E_y \) data

\[ \Theta_{kb} \sim 90 \]

- Sahraoui et al., PRL, 2009
Magnetic compressibility

Additional evidence of KAW at $k \rho_i > 1$


[Cluster/STAFF-SC data]

[KAW, $\Theta_{kB} = 89.9$]

3D $k$-spectra at sub-proton scales of SW turbulence

**Conditions required:**

1. Quiet SW: NO electron foreshock effects
2. Shorter Cluster separations (~100km) to analyze sub-proton scales
3. Regular tetrahedron to infer actual 3D $k$-spectra [Sahraoui et al., JGR, 2010]
4. High SNR of the STAFF data to analyse HF (>10Hz) SW turbulence.

20040110, 06h05-06h55
3D $k$-spectra at sub-proton scales

We use the $k$-filtering technique to estimate the 4D spectral energy density $P(\omega,k)$.

We use $P(\omega,k)$ to calculate

1. 3D $\omega$-$k$ spectra
2. 3D $k$-spectra (anisotropies, scaling, …)
Comparison with the Vlasov theory

\[ \beta_i \sim 2 \quad T_i/T_e = 3 \quad 85^\circ < \Theta_{kB} < 89^\circ \]

Turbulence cascades following the Kinetic Alfvén mode (KAW) as proposed in Sahraoui et al., PRL, 2009

→ Rules out the cyclotron heating

→ Heating by p-Landau and e-Landau resonances

[Sahraoui et al., PRL, 2010]

Limitation due to the Cluster separation (d~200km)
3D $k$-spectra at sub-ion scales

1. First direct evidence of the breakpoint near the proton gyroscale in $k$-space (no additional assumption, e.g. Taylor hypothesis, is used)

2. Strong steepening of the spectra below $\rho_i \rightarrow$ A Transition Range to dispersive/electron cascade
Journey of the energy cascade through scales

1. Turbulence
2. e-Acceleration & Heating
3. Reconnection

Dissipation range
Another interpretation in Meyrand & Galtier, 2010
Dissipation through reconnection/current sheets

Large scale laminar current sheet: reconnection can occur and the can be heated or accelerated (e.g. jets)

[Zhong+, Nature Physics, 2010]
Turbulent current sheets

[Lazarian & Vishniac, 1999]

2D Hall-MHD simulation of turbulence: evidence of a large number of reconnecting regions

[e.g., Retinò+, Nature Physics, 2007]
Dissipation by wave-particle interaction or via reconnection?

Good correlation between enhanced $T_p$ and threshold of linear kinetic instabilities

Good correlation between enhanced high shear $B$ angles and the threshold of linear instabilities!!

Osman et al., PRLs, 2012a,b
Higher order statistics and intermittency (1)

8.2 Self-similar and intermittent random functions

Fig. 8.1. A portion of the graph of the Brownian motion curve, enlarged twice, illustrating its self-similarity.

Self-similar signal

Fig. 8.2. The Devil’s staircase: an intermittent function.

intermittent signal

[Frisch, 1995]
Higher order statistics and intermittency (2)
2. Monfractality vs multifractality in the dispersive range:

\[ n_e \sim 4 \, \text{cm}^{-3} \quad \beta_i \sim 2 \]

\[ V_A \sim 50 \, \text{km s}^{-1} \]

\[ T_i \sim 103 \, \text{eV} \quad |B| \sim 4 \, \text{nT} \]

[Kiyani et al., PRL, 2009]
Evidence of **monofractality (self-similarity)** at sub-proton scales, while MHD-scales are **multifractal (intermittent)**

[See also Alexandrova et al., ApJ, 2008]
Conclusions

The Cluster data helps understanding crucial problems of astrophysical turbulence:

- Its nature and anisotropies in $k$-space at MHD and sub-ion scales
- Its cascade and dissipation down to the electron gyroscale $\rho_e \Rightarrow$ electron heating and/or acceleration by turbulence
- Strong evidences of KAW turbulence ($\omega \ll \omega_{ci}$, $k_// \ll k_\perp$)\Rightarrow Heating by e-p-Landau dampings (no cyclotron heating)
- Importance of kinetic physics in SW turbulence
- Turbulence & dissipation are at the heart of the future space missions: ESA/Solar Orbiter (2018), NASA/Solar Probe Plus (2018), THOR (2026 ?)
Turbulence and the future space missions

**Magnetospheric Multiscale**
A Solar-Terrestrial Probe

- 4 NASA satellites, launch 2015
- Higher resolution instrumentations
- Small separations (~10km)
- Equatorial orbit
⇒ Need of multi-scale measurements with appropriate spacecraft separations

Narita et al. PRL, 2010

Sahraoui et al. PRL, 2010
Solar Orbiter

Exploring the Sun-Heliosphere Connection

Launch 2017

Distance : 0.28 AU

In-situ measurements & remote sensing
Launch 2019
Distance: ~0.03 AU
In-situ measurements & remote sensing

Solar Probe
Plus

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland 20771
THOR
Turbulent Heating Observer

Turbulent energy dissipation and particle energization

- SNSB (2012)
- ESA (S1 Call, 2012)
- CNES (TOR/TWINS, call for ideas, 2013)
- ESA (M4 call, 2015): **under Phase A study. Final decision 2017**