Chapter III
Elasticity: soft modes, bending and nonaffinity

I) A crash course in elasticity theory

1. **Stress**
2. **Mechanical equilibrium**
3. **Strain**
4. **Linear elasticity**

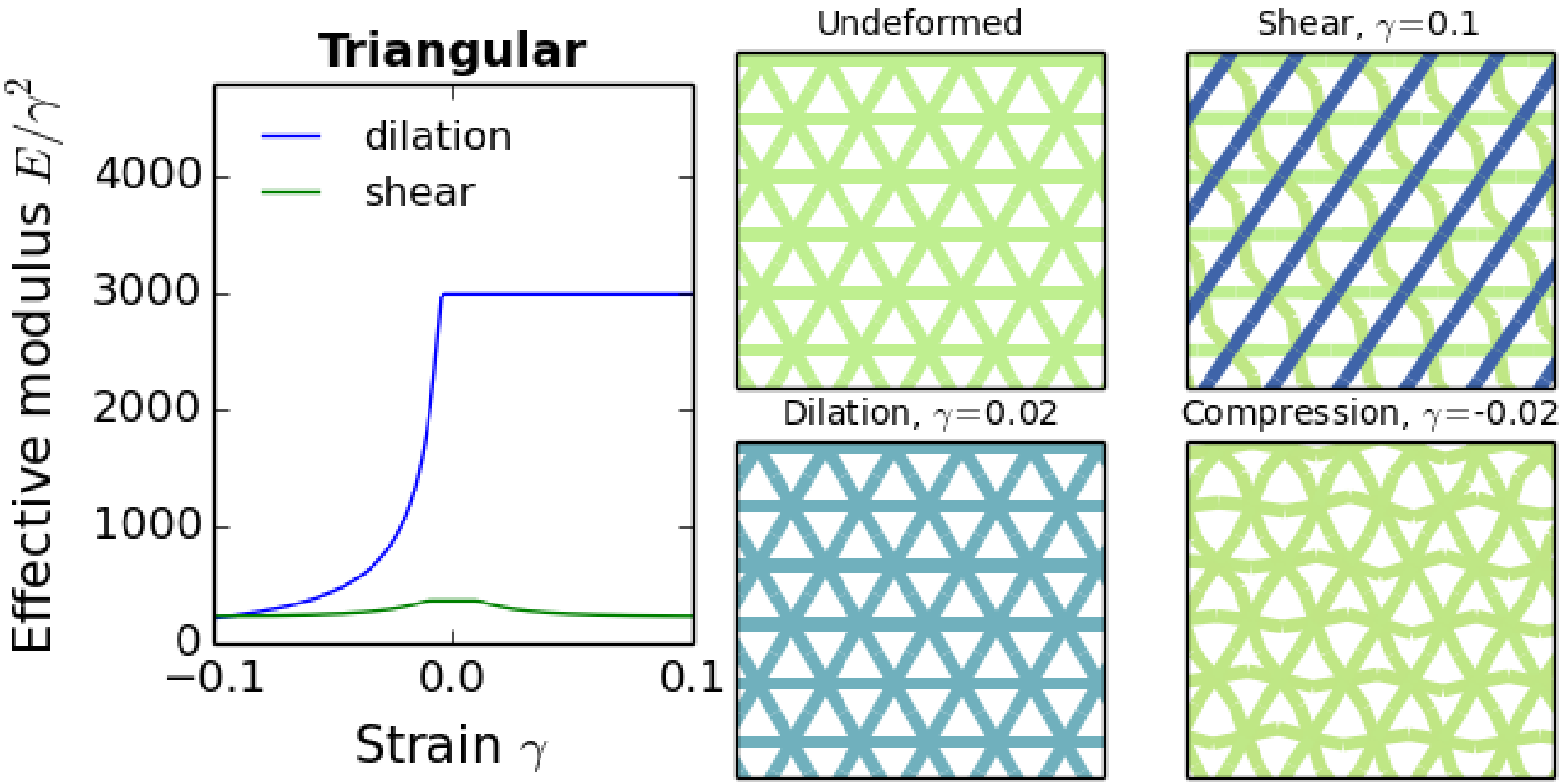
II) Affine deformation of a hyperstatic network

III) Bending-dominated elasticity of a hypostatic network

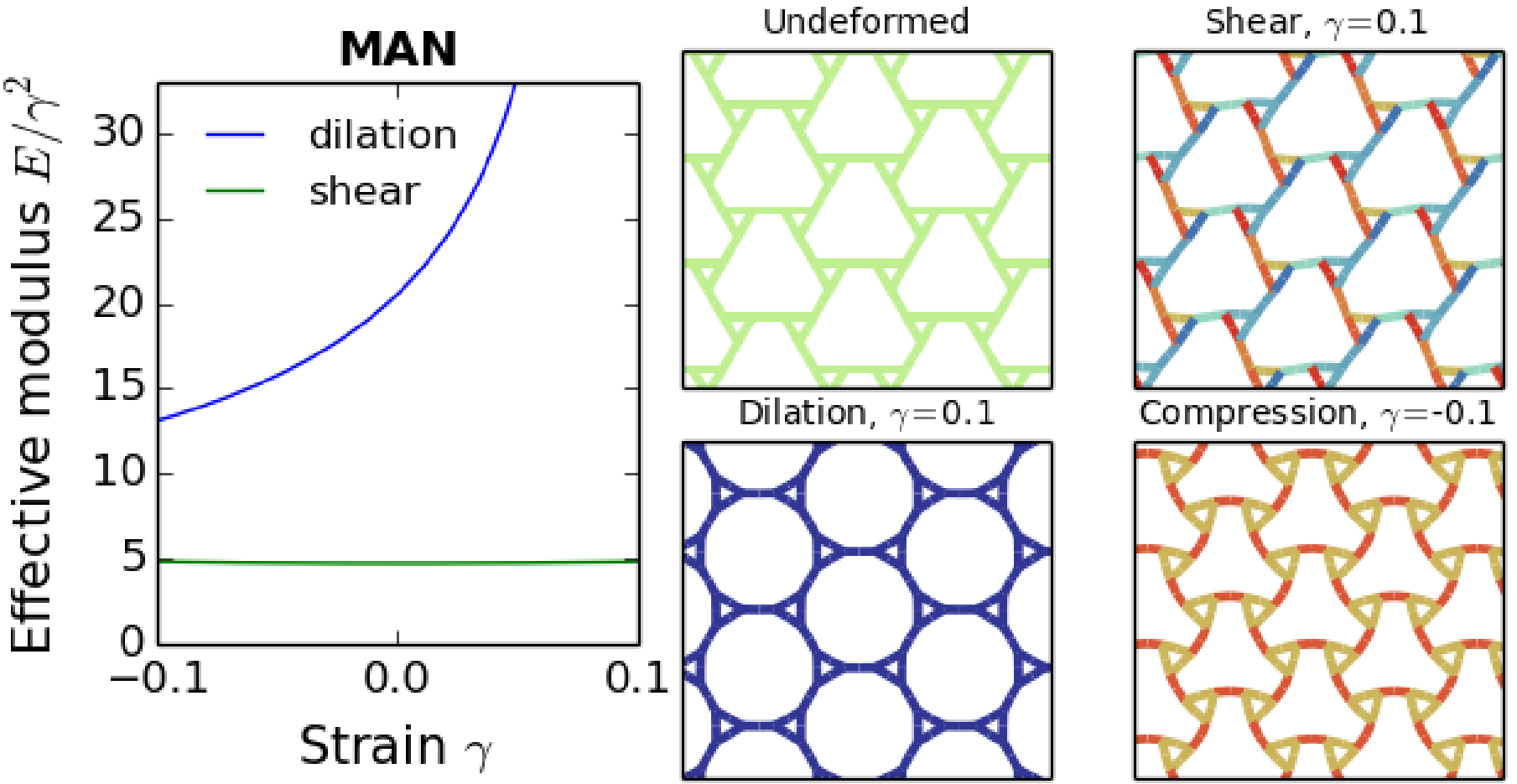
IV) Conclusions

References:
Landau & Lifschitz, *Theory of elasticity*, Butterworth Heinemann (1986)
R. Aris, *Vectors, tensors, and the basic equations of fluid mechanics*, Dover (1989)
Broedersz *et al. Nat. Phys.* **7**, 983 (2011)

Stretching-dominated network

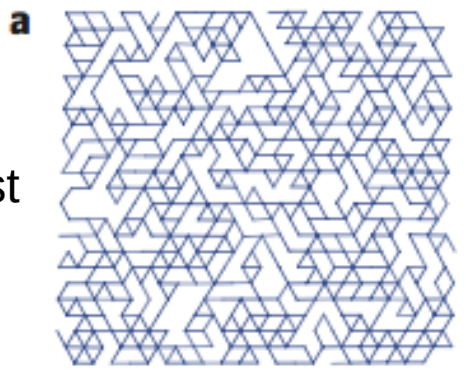


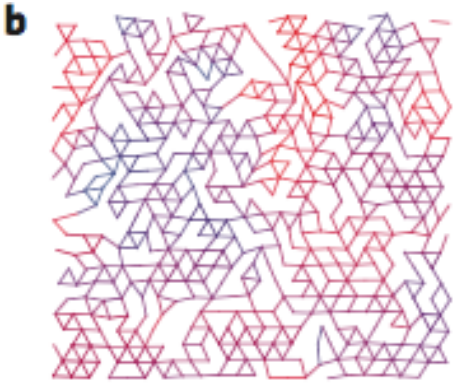
Bending-dominated network



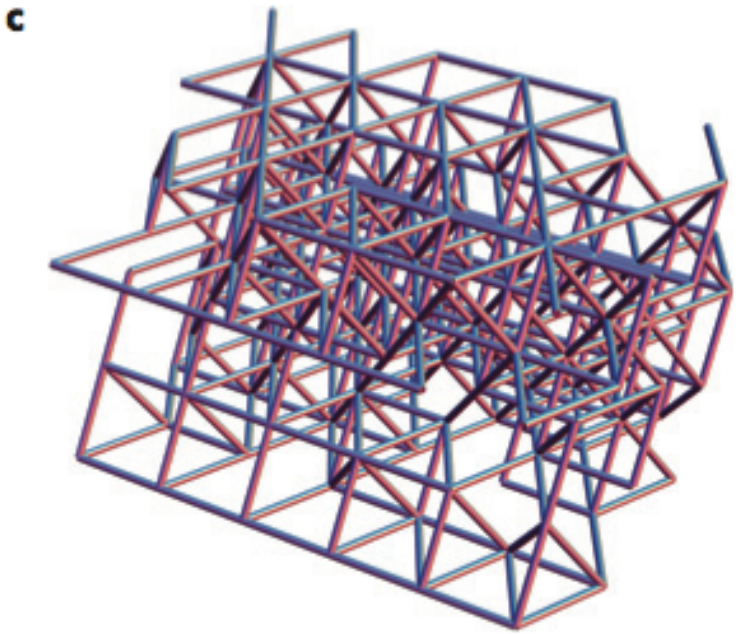
Criticality and isostaticity in fibre networks

network at rest



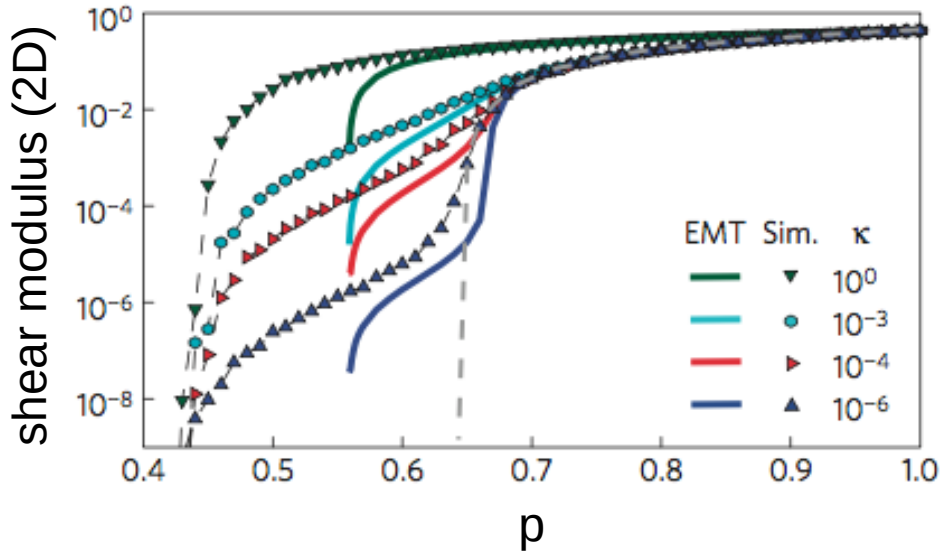


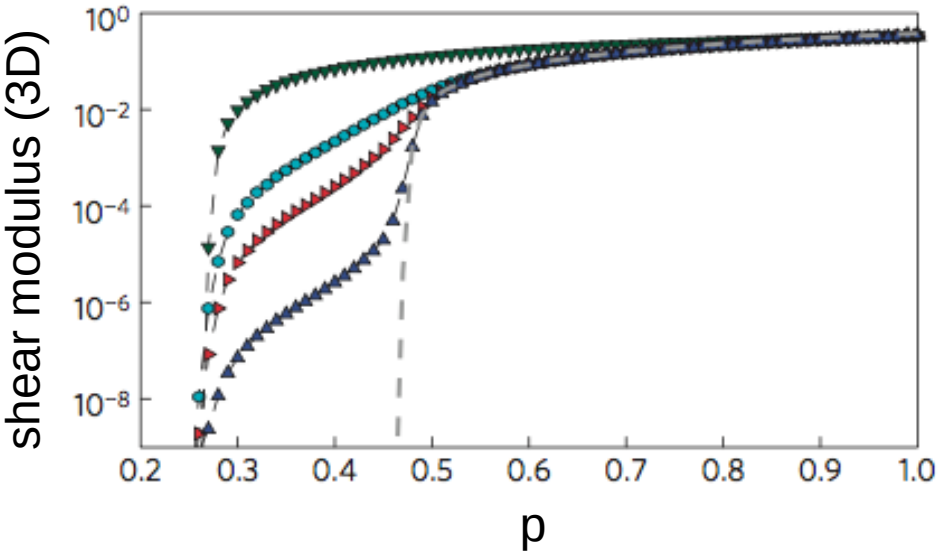
nonaffine deformation
under simple shear
(linear response)

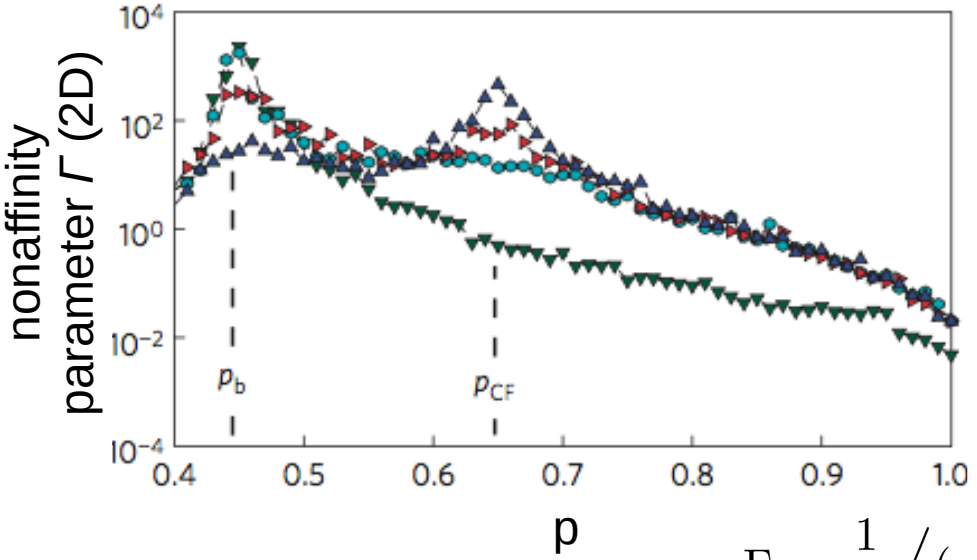


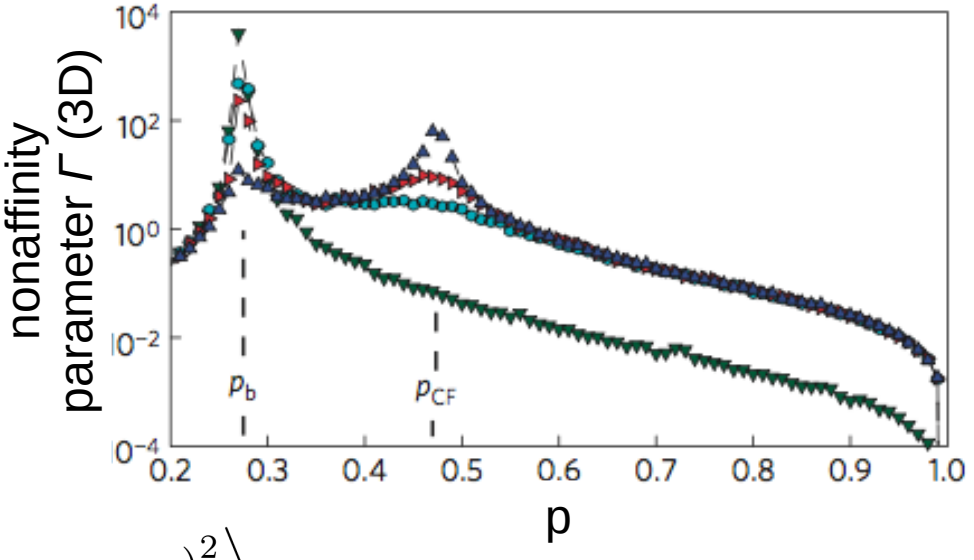
three-dimensional version

Mechanics and non-affine strain fluctuations



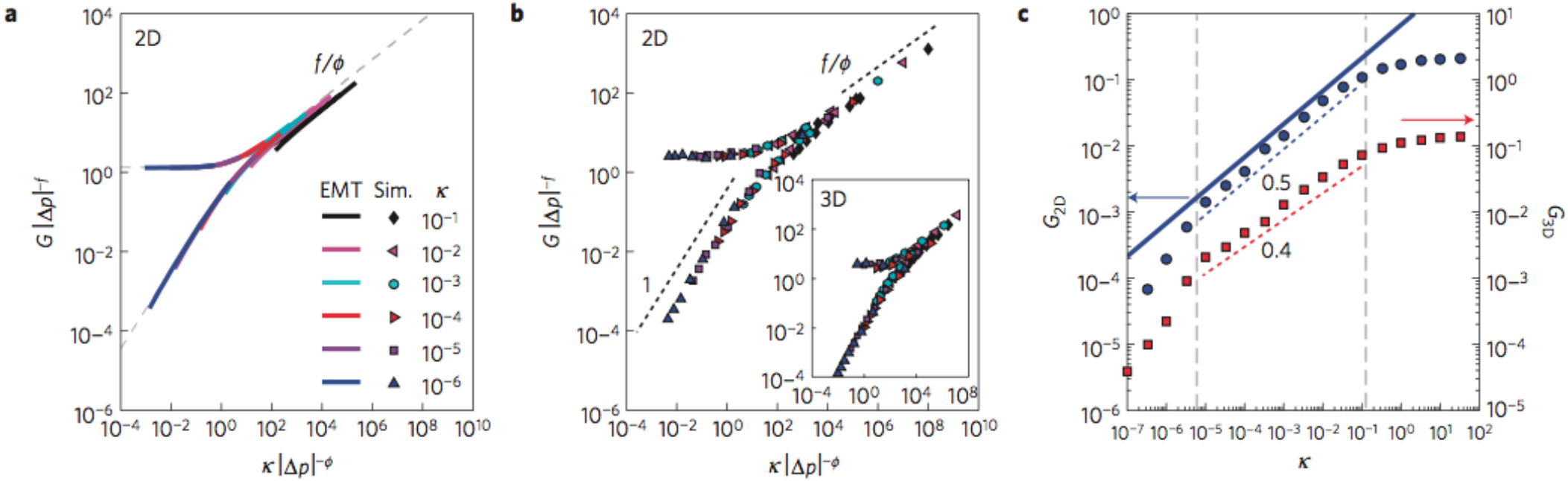




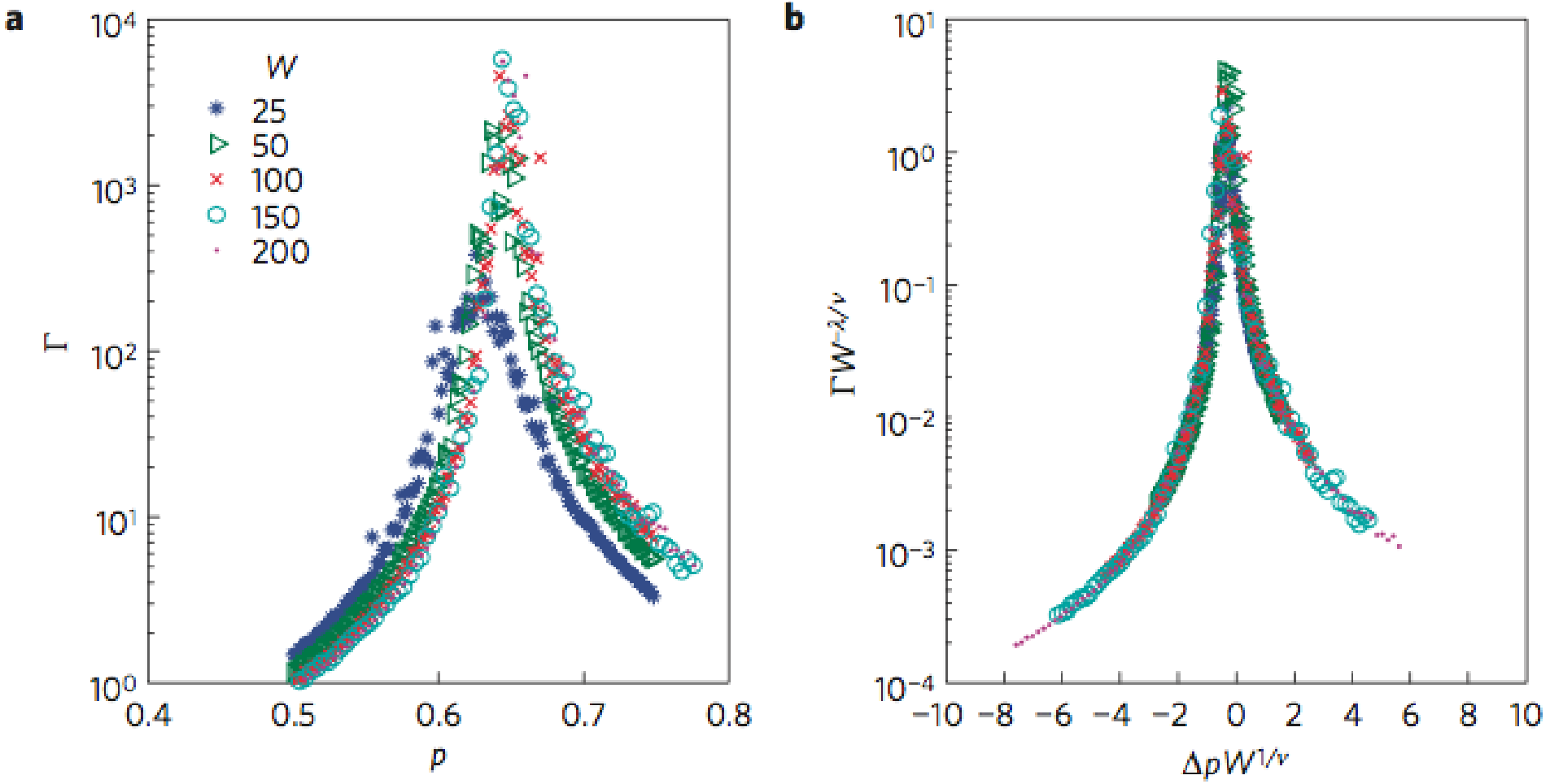


$$\Gamma \propto \frac{1}{\gamma^2}\left\langle (\mathbf{u}-\mathbf{u}_{\text{affine}})^2 \right\rangle$$

Scaling analysis of the mechanics and anomalous elasticity



Finite-size scaling



Phase diagram