1. Cubulate $\langle a, b, c \mid a^2 b^2 c^2 bac \rangle$.

2. Find an inifinite NPC cube complex with two embedded hyperplanes.

3. Find an infinite NPC cube complex with one hyperplane.

4. Find the link of vertex in a product of two

   (a) trees
   (b) CAT(0) cube complexes.

5. ($\star$) Show that a CAT(0) cube complex is a product of two trees if and only if the link of each vertex is a complete bipartite graph.

6. Show that the Salvetti complex is nonpositively curved.

7. Are the hyperplanes in the Salvetti complex the Salvetti complexes? If so, what is the corresponding graph?

8. ($\star\star$) Show that a finitely generated torsion-group cannot act on a finite dimensional CAT(0) cube complex without a global fixed point.

9. Let α, β, γ be ultrafilters and let

   $$med(\alpha, \beta, \gamma) = (\alpha \cup \beta) \cap (\alpha \cup \gamma) \cap (\beta \cup \gamma) = (\alpha \cap \beta) \cup (\alpha \cap \gamma) \cup (\beta \cap \gamma).$$

   Show that $med(\alpha, \beta, \gamma)$ is an ultrafilter.

10. Let $[\alpha, \beta] = \{\gamma \mid med(\alpha, \beta, \gamma) = \gamma\}$. Show that when α, β are vertices of X then $[\alpha, \beta]$ consists of the vertices of X which lie on 1-skeleton geodesics between α and β.

11. ($\star$) Show that when X is of dimension n then $[\alpha, \beta]$ embeds in $\mathbb{E}^n$.

12. Describe the cube complex corresponding to the space with walls (three families of parallel lines on the plane):
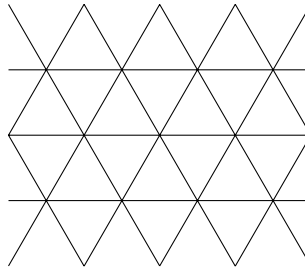
13. Let α be a (self-intersecting) geodesic closed curve on a closed surface of genus $g \geq 2$. Show that there exists a bound on the number of lifts of α to $\mathbb{H}^2$ that pairwise intersect.

14. Show that H is separable if and only if for every $g \in G - H$ there exists a finite group F and a homomorphism $\phi : G \to F$ such that $\phi(g) \notin \phi(H)$.

15. Show that $\mathcal{B} = \{\text{cosets of finite index subgroups}\}$ is a basis for a topology (called *profinite topology*).

16. Show that H is separable if and only if H is closed in the profinite topology.

17. If G is residually finite and H is a virtual retract then H is separable.

18. Let $f : X \to Y$ be a a local isometry of cube complexes. Show that if Y is NPC then so is X. Show that $f_* : \pi_1 X \to \pi_1 Y$ is injective.

19. Hallucinate about the canonical completion and retraction.