Random matrix theory and infinite-dimensional stochastic differential equations

Hirofumi Osada (Kyushu University)

We talk about stochastic dynamics whose (unlabeled) equilibrium states are point processes appearing in random matrix theory. These dynamics are called interacting Brownian motions (IBMs).

We give various examples of IBMs related to random matrices. That is, sine, Airy, Bessel IBMs in one space dimension, and Ginibre IBM in two dimensions. We construct stochastic dynamics as a pathwise unique strong solution of an infinite-dimensional stochastic differential equation (ISDE). We also prove a convergence of N particle systems to solutions of ISDEs.

Our method is analytic, and based on stochastic analysis. We present a sequence of general theorems to solve ISDEs. We establish a new formulation of solutions of ISDEs in terms of tail $ ¥sigma $-fields of labeled path spaces consisting of trajectories of infinitely many particles. These formulations are equivalent to the original notions of solutions of ISDEs, and more feasible to treat in infinite dimensions.

When inverse temperature $\beta=2$ and space dimension d=1, there exists another construction of stochastic dynamics based on space-time correlation functions, called the algebraic construction. Our method yields the same stochastic dynamics obtained by the algebraic construction.