Quantum ergodicity and Benjamini-Schramm convergence of hyperbolic surfaces

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Quantum Ergodicity Theorem of Shnirelman, Zelditch and Colin de Verdière is an equidistribution result of eigenfunctions of the Laplacian in high frequencies on a compact Riemannian manifold with an ergodic geodesic flow. We complement this work by introducing a Quantum Ergodicity theorem on compact hyperbolic surfaces, where instead of going to high frequencies we fix an interval of frequencies and vary the geometric parameters of the surface such as volume, injectivity radius and genus. In particular, we are interested in such results under Benjamini-Schramm convergence of hyperbolic surfaces. This applies for example in the case of congruence coverings of compact arithmetic surfaces.

Our work is inspired by analogous results for holomorphic cusp forms and eigenfunctions for large regular graphs. The proof uses the mixing properties of the geodesic flow together with a wave propagation approach recently considered by Brooks, Le Masson and Lindenstrauss (2015) on discrete graphs. It does not use any microlocal analysis, making it quite different from the usual proof of quantum ergodicity in the high frequency limit. We replace the wave propagator with renormalised averaging operators over discs, which simplifies the analysis and allows us to make use of a general quantitative ergodic theorem of Nevo.

Joint work with Etienne Le Masson (Bristol).