Genus-one Gromov–Witten Invariants of Threefolds

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Abstract: Enumerative questions have a very long history in Mathematics and have been revolutionised in the nineties with the construction of the moduli space of stable maps and the machinery allowing us to integrate on these very sin- gular spaces. However, moduli spaces of stable maps have many "unwanted" components which are reflected in the intersection numbers.

I will discuss the Li–Zinger formula which expresses genus-one Gromov–Witten invariants of hypersurfaces in projective spaces as the sum of *reduced* invariants and certain genus-zero Gromov–Witten invariants of the hypersur- face. The reduced invariants are closely related to the "enumerative" numbers (or the Gopakumar–Vafa invariants) of the threefold, while the genus zero Gromov–Witten invariants are contributions from certain "unwanted" compo- nents of the moduli space of stable maps. This formula was proved using symplectic methods by Li and Zinger, and algebraically by Chang and Li for the quintic threefold. I will explain a new algebraic proof of this result, based on a gluing method for virtual classes.